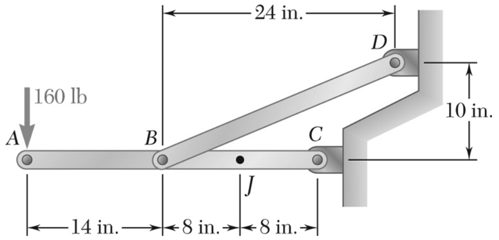
CHAPTER 7



PROBLEM 7.1

Determine the internal forces (axial force, shearing force, and bending moment) at Point J of the structure indicated.

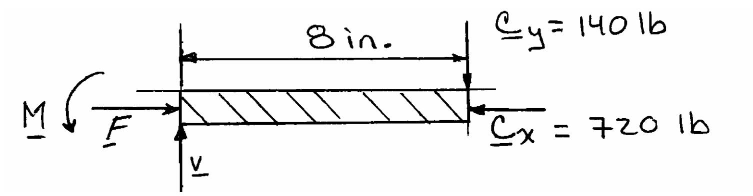
Frame and loading of Problem 6.75.

SOLUTION

From Problem 6.75: $\mathbf{C}_x = 720 \text{ lb} \longleftarrow$

$\mathbf{C}_y = 140 \text{ lb} \downarrow$

FBD of JC:



$$\xrightarrow{+} \Sigma F_x = 0: \quad F - 720 \text{ lb} = 0$$

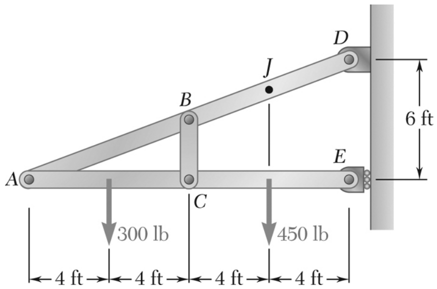
$$F = +720 \text{ lb} \qquad \mathbf{F} = 720 \text{ lb} \longrightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad V - 140 \text{ lb} = 0$$

$$V = 140 \text{ lb} \qquad \mathbf{V} = 140.0 \text{ lb} \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (140 \text{ lb})(8 \text{ in.}) = 0$$

$$M = +1120 \text{ lb} \cdot \text{in.} \qquad \mathbf{M} = 1120 \text{ lb} \cdot \text{in.} \circlearrowleft \blacktriangleleft$$



PROBLEM 7.2

Determine the internal forces (axial force, shearing force, and bending moment) at Point J of the structure indicated.

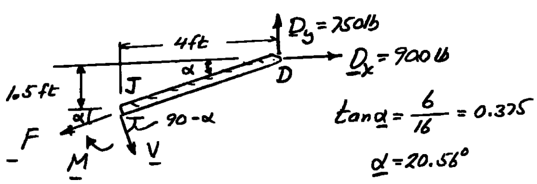
Frame and loading of Problem 6.78.

SOLUTION

From Problem 6.78: $\mathbf{D}_x = 900\text{ lb} \longrightarrow$

$\mathbf{D}_y = 750\text{ lb}\uparrow$

FBD of JD:



$$+\circlearrowleft \Sigma M_J = 0: \quad -M + (750\text{ lb})(4\text{ ft}) - (900\text{ lb})(1.5\text{ ft}) = 0$$

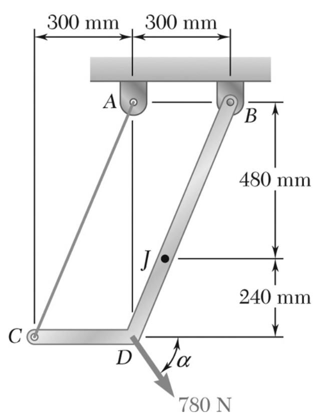
$$M = +1650\text{ lb}\cdot\text{ft} \qquad \mathbf{M} = 1650\text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

$$+\nwarrow \Sigma F = 0: \quad -V + (750\text{ lb})\cos 20.56° - (900\text{ lb})\sin 20.56° = 0$$

$$V = +386.2\text{ lb} \qquad \mathbf{V} = 386\text{ lb} \searrow 69.4° \blacktriangleleft$$

$$+\swarrow \Sigma F = 0: \quad F - (750\text{ lb})\sin 20.56° + (900\text{ lb})\cos 20.56° = 0$$

$$F = +1106.1\text{ lb} \qquad \mathbf{F} = 1106\text{ lb} \swarrow 20.6° \blacktriangleleft$$

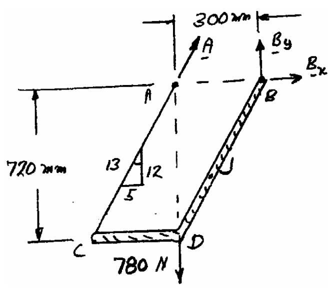


PROBLEM 7.3

Determine the internal forces at Point J when $\alpha = 90°$.

SOLUTION

Reactions $(\alpha = 90°)$



$\Sigma M_A = 0$: $\mathbf{B}_y = 0$

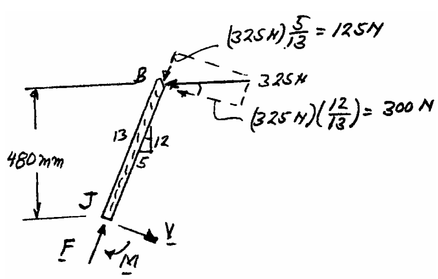
$+\!\uparrow \Sigma F_y = 0$: $A\left(\dfrac{12}{13}\right) - 780\text{ N} = 0$

$A = 845\text{ N}$ $\qquad \mathbf{A} = 845\text{ N} \nearrow$

$\xrightarrow{+} \Sigma F_x = 0$: $(845\text{ N})\dfrac{5}{13} + B_x = 0$

$B_x = -325\text{ N}$ $\qquad \mathbf{B}_x = 325\text{ N} \longleftarrow$

FBD BJ:

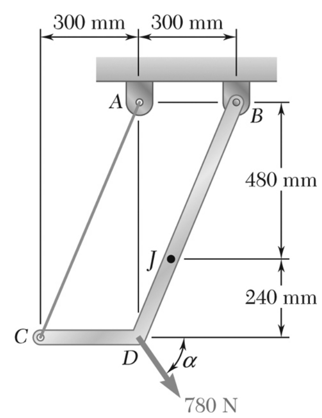


$+\swarrow \Sigma F = 0$: $125\text{ N} - F = 0$ $\qquad \mathbf{F} = 125.0\text{ N} \measuredangle 67.4°$ ◀

$+\searrow \Sigma F = 0$: $V - 300\text{ N} = 0$ $\qquad \mathbf{V} = 300\text{ N} \searrow 22.6°$ ◀

$+\circlearrowright \Sigma M = 0$: $(325\text{ N})(0.480\text{ m}) - M = 0$

$M = +156\text{ N}\cdot\text{m}$ $\qquad \mathbf{M} = 156.0\text{ N}\cdot\text{m} \circlearrowright$ ◀

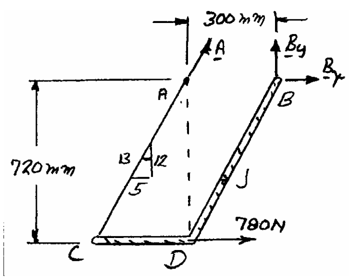


PROBLEM 7.4

Determine the internal forces at Point J when $\alpha = 0$.

SOLUTION

Reactions ($\alpha = 0$)



$$+\circlearrowleft \Sigma M_A = 0: \quad (780 \text{ N})(0.720 \text{ m}) + B_y(0.3 \text{ m}) = 0$$

$$B_y = -1872 \text{ N} \qquad \mathbf{B}_y = 1872 \text{ N} \downarrow$$

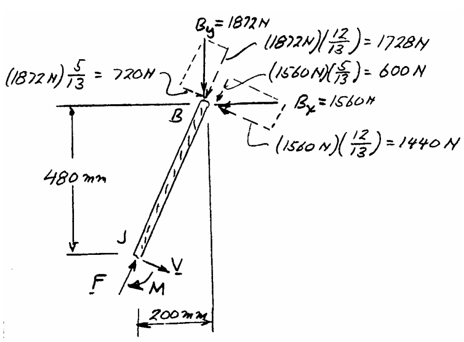
$$+\uparrow \Sigma F_y = 0: \quad A\left(\frac{12}{13}\right) - 1872 \text{ N} = 0$$

$$\mathbf{A} = 2028 \text{ N} \nearrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad (2028 \text{ N})\left(\frac{5}{13}\right) + 780 \text{ N} + B_x = 0$$

$$B_x = -1560 \text{ N} \qquad \mathbf{B}_x = 1560 \text{ N} \longleftarrow$$

FBD BJ:



$$\swarrow \Sigma F = 0: \quad 1728 \text{ N} + 600 \text{ N} - F = 0$$

$$F = +2328 \text{ N} \qquad \mathbf{F} = 2330 \text{ N} \measuredangle 67.4° \blacktriangleleft$$

$$\overset{+}{\searrow} \Sigma F = 0: \quad 720 \text{ N} - 1440 \text{ N} + V = 0$$

$$V = +720 \text{ N} \qquad \mathbf{V} = 720 \text{ N} \searrow 22.6° \blacktriangleleft$$

$$BJ = \sqrt{480^2 + 200^2} = 520 \text{ mm}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (1440 \text{ N})(0.520 \text{ m}) - (720 \text{ N})(0.520 \text{ m}) - M = 0$$

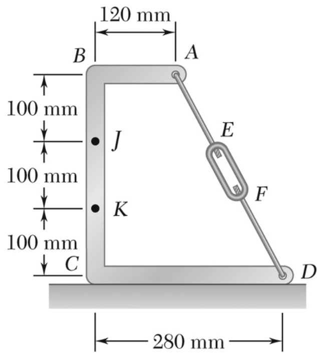
$$M = +374.4 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 374 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

Alternate

Computation of M using $\mathbf{B}_x + \mathbf{B}_y$:

$$+\circlearrowleft \Sigma M_J = 0: \quad (1560 \text{ N})(0.48 \text{ m}) - (1872 \text{ N})(0.2 \text{ m}) - M = 0$$

$$M = +374.4 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 374 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

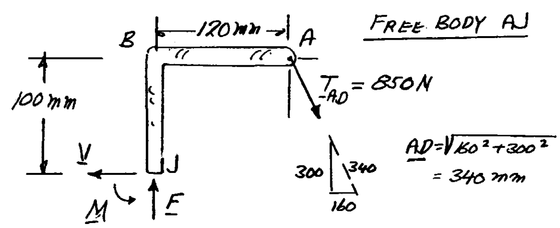


PROBLEM 7.5

Knowing that the turnbuckle has been tightened until the tension in wire *AD* is 850 N, determine the internal forces at point indicated:

Point *J*.

SOLUTION



$$\xrightarrow{+} \Sigma F_x = 0: \quad -V + \left(\frac{160}{340}\right)(850 \text{ N}) = 0$$

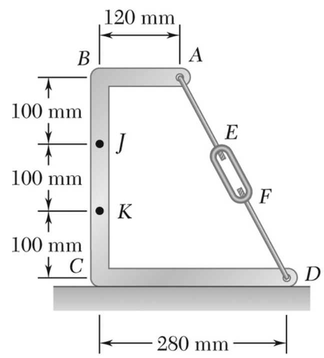
$$V = +400 \text{ N} \qquad \mathbf{V} = 400 \text{ N} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F - \left(\frac{300}{340}\right)(850 \text{ N}) = 0$$

$$F = +750 \text{ N} \qquad \mathbf{F} = 750 \text{ N} \uparrow \blacktriangleleft$$

$$+\circlearrowright \Sigma M_J = 0: \quad M - \left(\frac{300}{340}\right)(850 \text{ N})(120 \text{ mm}) - \left(\frac{160}{340}\right)(850 \text{ N})(100 \text{ mm}) = 0$$

$$M = +130 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 130 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



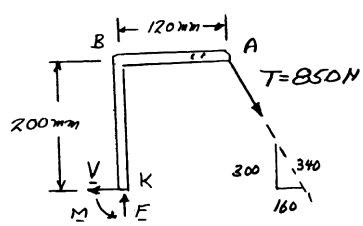
PROBLEM 7.6

Knowing that the turnbuckle has been tightened until the tension in wire *AD* is 850 N, determine the internal forces at point indicated:

Point *K*.

SOLUTION

<u>Free body *AK*</u>:



$$AD = \sqrt{160^2 + 300^2}$$
$$= 340 \text{ mm}$$

<u>On portion *KBA*</u>:

$$\xrightarrow{+} \Sigma F_x = 0: \quad -V + \left(\frac{160}{340}\right)(850 \text{ N}) = 0$$

$$V = +400 \text{ N} \qquad \mathbf{V} = 400 \text{ N} \longleftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F - \left(\frac{300}{340}\right)(850 \text{ N}) = 0$$

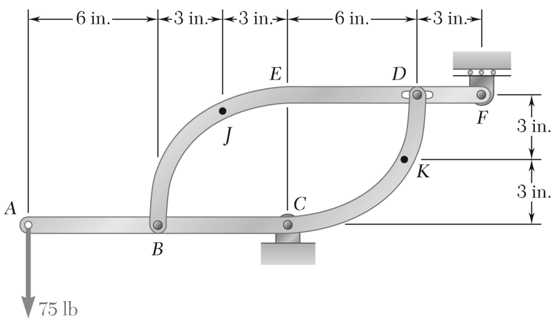
$$F = +750 \text{ N} \qquad \mathbf{F} = 750 \text{ N} \uparrow \blacktriangleleft$$

$$+\circlearrowright \Sigma M_J = 0: \quad M - \left(\frac{300}{340}\right)(850 \text{ N})(120 \text{ mm}) - \left(\frac{160}{340}\right)(850 \text{ N})(200 \text{ mm}) = 0$$

$$M = +170 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 170.0 \text{ N}\cdot\text{m} \circlearrowright$$

<u>Internal forces acting on *KCD*</u> are equal and opposite

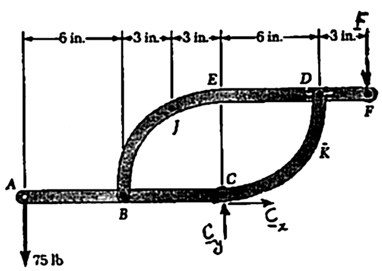
$$\mathbf{F} = 750 \text{ N} \downarrow, \quad \mathbf{V} = 400 \text{ N} \longrightarrow, \quad \mathbf{M} = 170.0 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$



PROBLEM 7.7

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at A. Determine the internal forces at Point J.

SOLUTION



Free body: Entire frame

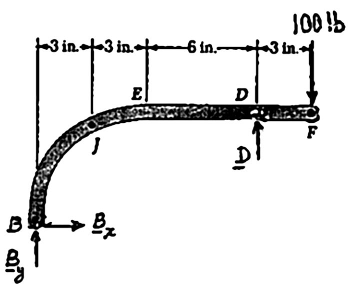
$$+\circlearrowleft \Sigma M_C = 0: \quad (75 \text{ lb})(12 \text{ in.}) - F(9 \text{ in.}) = 0$$

$$\mathbf{F} = 100 \text{ lb} \downarrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75 \text{ lb} - 100 \text{ lb} = 0$$

$$C_y = +175 \text{ lb} \qquad \mathbf{C} = 175 \text{ lb} \uparrow \triangleleft$$



Free body: Member *BEDF*

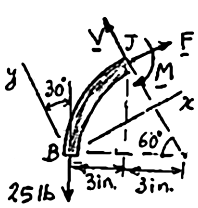
$$+\circlearrowleft \Sigma M_B = 0: \quad D(12 \text{ in.}) - (100 \text{ lb})(15 \text{ in.}) = 0$$

$$\mathbf{D} = 125 \text{ lb} \uparrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 125 \text{ lb} - 100 \text{ lb} = 0$$

$$B_y = -25 \text{ lb} \qquad \mathbf{B} = 25 \text{ lb} \downarrow \triangleleft$$



Free body: *BJ*

$$+\nearrow \Sigma F_x = 0: \quad F - (25 \text{ lb}) \sin 30° = 0$$

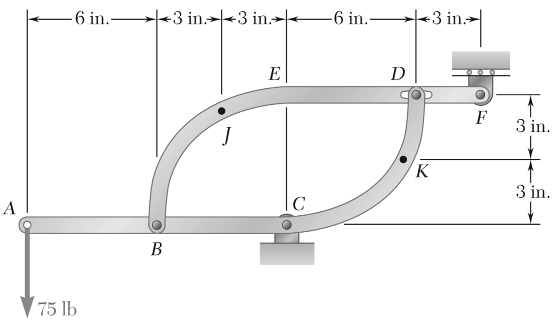
$$\mathbf{F} = 12.50 \text{ lb} \measuredangle\ 30.0° \blacktriangleleft$$

$$+\nwarrow \Sigma F_y = 0: \quad V - (25 \text{ lb}) \cos 30° = 0$$

$$\mathbf{V} = 21.7 \text{ lb} \searrow 60.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -M + (25 \text{ lb})(3 \text{ in.}) = 0$$

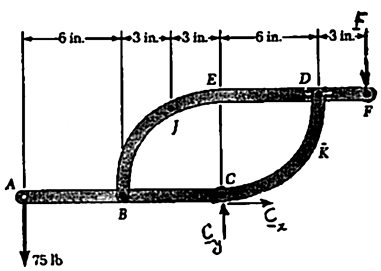
$$\mathbf{M} = 75.0 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$



PROBLEM 7.8

Two members, each consisting of a straight and a quarter-circular portion of rod, are connected as shown and support a 75-lb load at A. Determine the internal forces at Point K.

SOLUTION



Free body: Entire frame

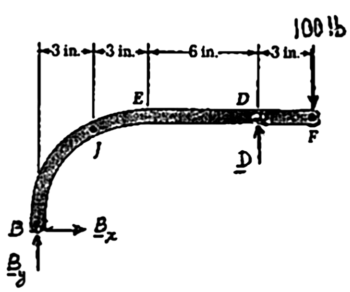
$$+\circlearrowleft \Sigma M_C = 0: \quad (75 \text{ lb})(12 \text{ in.}) - F(9 \text{ in.}) = 0$$

$$\mathbf{F} = 100 \text{ lb} \downarrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 75 \text{ lb} - 100 \text{ lb} = 0$$

$$C_y = +175 \text{ lb} \qquad \mathbf{C} = 175 \text{ lb} \uparrow \triangleleft$$



Free body: Member *BEDF*

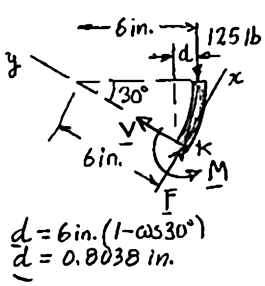
$$+\circlearrowleft \Sigma M_B = 0: \quad D(12 \text{ in.}) - (100 \text{ lb})(15 \text{ in.}) = 0$$

$$\mathbf{D} = 125 \text{ lb} \uparrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 125 \text{ lb} - 100 \text{ lb} = 0$$

$$B_y = -25 \text{ lb} \qquad \mathbf{B} = 25 \text{ lb} \downarrow \triangleleft$$



Free body: *DK*

We found in Problem 7.11 that

$$\mathbf{D} = 125 \text{ lb} \uparrow \text{ on } BEDF.$$

Thus $\qquad \mathbf{D} = 125 \text{ lb} \downarrow$ on *DK*. $\triangleleft$

$$+\nearrow \Sigma F_x = 0: \quad F - (125 \text{ lb})\cos 30° = 0$$

$$\mathbf{F} = 108.3 \text{ lb} \measuredangle 60.0° \blacktriangleleft$$

PROBLEM 7.8 (Continued)

$$+\Sigma F_y = 0: \quad V - (125 \text{ lb})\sin 30° = 0$$

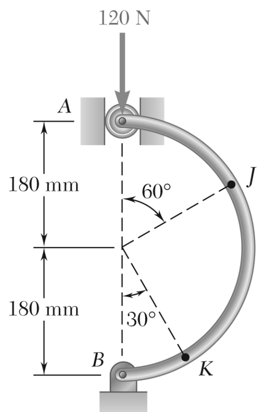
$$\mathbf{V} = 62.5 \text{ lb} \measuredangle 30.0° \blacktriangleleft$$

$$+\Sigma M_K = 0: \quad M - (125 \text{ lb})d = 0$$

$$M = (125 \text{ lb})d = (125 \text{ lb})(0.8038 \text{ in.})$$
$$= 100.5 \text{ lb} \cdot \text{in.}$$

$$\mathbf{M} = 100.5 \text{ lb} \cdot \text{in.} \blacktriangleleft$$

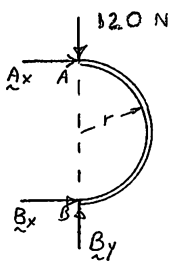
PROBLEM 7.9



A semicircular rod is loaded as shown. Determine the internal forces at Point J.

SOLUTION

FBD Rod:



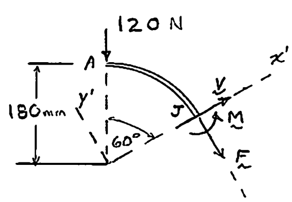
$$\Sigma M_B = 0: \quad A_x(2r) = 0$$

$$\mathbf{A}_x = 0$$

$$\nearrow \Sigma F_{x'} = 0: \quad V - (120 \text{ N})\cos 60° = 0$$

$$\mathbf{V} = 60.0 \text{ N} \nearrow \blacktriangleleft$$

FBD AJ:



$$\searrow \Sigma F_{y'} = 0: \quad F + (120 \text{ N})\sin 60° = 0$$

$$F = -103.923 \text{ N}$$

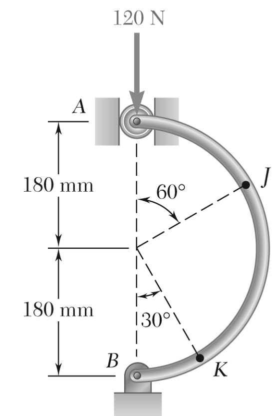
$$\mathbf{F} = 103.9 \text{ N} \nwarrow \blacktriangleleft$$

$$\Sigma M_J = 0: \quad M - [(0.180 \text{ m})\sin 60°](120 \text{ N}) = 0$$

$$M = 18.7061$$

$$\mathbf{M} = 18.71 \blacktriangleleft$$

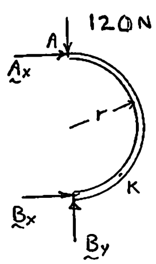
PROBLEM 7.10



A semicircular rod is loaded as shown. Determine the internal forces at Point *K*.

SOLUTION

FBD Rod:



$+\uparrow \Sigma F_y = 0\text{:}\quad B_y - 120\text{ N} = 0 \quad \mathbf{B}_y = 120\text{ N}\uparrow$

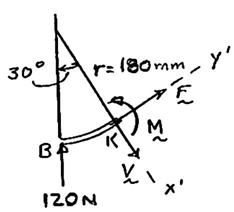
$+\circlearrowleft \Sigma M_A = 0\text{:}\quad 2rB_x = 0 \quad \mathbf{B}_x = 0$

$+\searrow \Sigma F_{x'} = 0\text{:}\quad V - (120\text{ N})\cos 30° = 0$

$V = 103.923\text{ N}$

$\mathbf{V} = 103.9\text{ N}\searrow$ ◀

FBD BK:



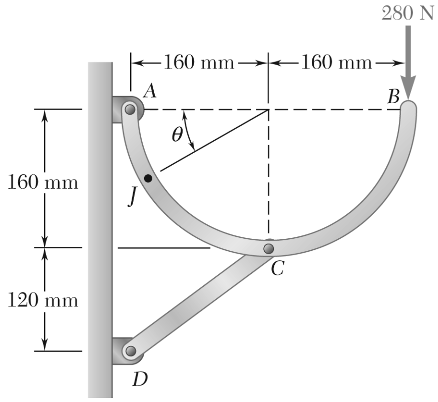
$+\nearrow \Sigma F_{y'} = 0\text{:}\quad F + (120\text{ N})\sin 30° = 0$

$F = -60\text{ N}$

$\mathbf{F} = 60.0\text{ N}\swarrow$ ◀

$+\circlearrowleft \Sigma M_K = 0\text{:}\quad M - [(0.180\text{ m})\sin 30°](120\text{ N}) = 0$

$\mathbf{M} = 10.80\text{ N}\cdot\text{m} \circlearrowright$ ◀

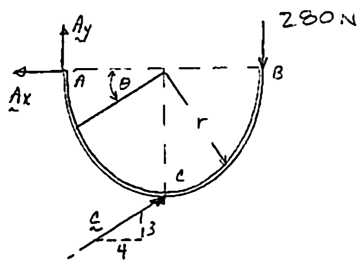


PROBLEM 7.11

A semicircular rod is loaded as shown. Determine the internal forces at Point J knowing that $\theta = 30°$.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(280\text{ N}) = 0$$

$$\mathbf{C} = 400\text{ N} \nearrow$$

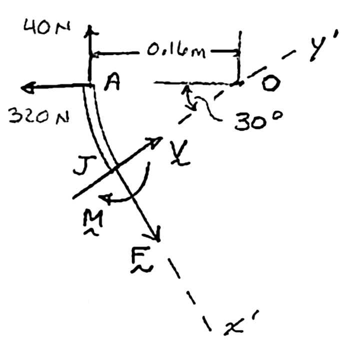
$$\xrightarrow{} \Sigma F_x = 0: \quad -A_x + \frac{4}{5}(400\text{ N}) = 0$$

$$\mathbf{A}_x = 320\text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + \frac{3}{5}(400\text{ N}) - 280\text{ N} = 0$$

$$\mathbf{A}_y = 40.0\text{ N} \uparrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: \quad F - (320\text{ N})\sin 30° - (40.0\text{ N})\cos 30° = 0$$

$$F = 194.641\text{ N}$$

$$\mathbf{F} = 194.6\text{ N} \searrow 60.0° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V - (320\text{ N})\cos 30° + (40\text{ N})\sin 30° = 0$$

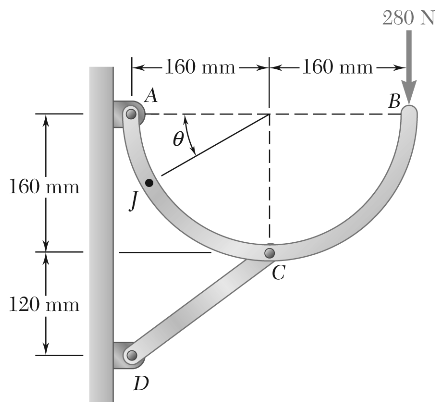
$$V = 257.13\text{ N}$$

$$\mathbf{V} = 257\text{ N} \measuredangle 30.0° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: \quad (0.160\text{ m})(194.641\text{ N}) - (0.160\text{ m})(40.0\text{ N}) - M = 0$$

$$M = 24.743$$

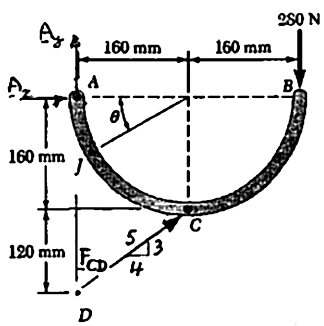
$$\mathbf{M} = 24.7\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 7.12

A semicircular rod is loaded as shown. Determine the magnitude and location of the maximum bending moment in the rod.

SOLUTION



<u>Free body: Rod *ACB*</u>

$$+\circlearrowleft \Sigma M_A = 0: \quad \left(\frac{4}{5}F_{CD}\right)(0.16\text{ m}) + \left(\frac{3}{5}F_{CD}\right)(0.16\text{ m})$$

$$-(280\text{ N})(0.32\text{ m}) = 0$$

$$\mathbf{F}_{CD} = 400\text{ N} \nearrow \triangleleft$$

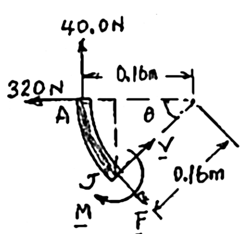
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + \frac{4}{5}(400\text{ N}) = 0$$

$$A_x = -320\text{ N} \qquad \mathbf{A}_x = 320\text{ N} \longleftarrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + \frac{3}{5}(400\text{ N}) - 280\text{ N} = 0$$

$$A_y = +40.0\text{ N} \qquad \mathbf{A}_y = 40.0\text{ N} \uparrow \triangleleft$$

<u>Free body: *AJ* (For $\theta < 90°$)</u>



$$+\circlearrowleft \Sigma M_J = 0: \quad (320\text{ N})(0.16\text{ m})\sin\theta - (40.0\text{ N})(0.16\text{ m})(1-\cos\theta) - M = 0$$

$$M = 51.2\sin\theta + 6.4\cos\theta - 6.4 \qquad (1)$$

For maximum value between *A* and *C*:

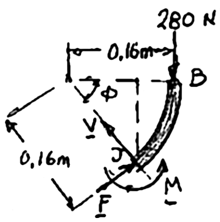
$$\frac{dM}{d\theta} = 0: \quad 51.2\cos\theta - 6.4\sin\theta = 0$$

$$\tan\theta = \frac{51.2}{6.4} = 8 \qquad \theta = 82.87° \blacktriangleleft$$

Carrying into (1):

$$M = 51.2\sin 82.87° + 6.4\cos 82.87° - 6.4 = +45.20\text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 7.12 (Continued)



Free body: BJ (For $\theta > 90°$)

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (280\text{ N})(0.16\text{ m})(1-\cos\phi) = 0$$

$$M = (44.8\text{ N}\cdot\text{m})(1-\cos\phi)$$

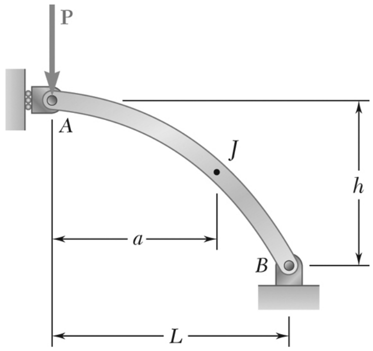
Largest value occurs for $\phi = 90°$, that is, at C, and is

$$M_C = 44.8\text{ N}\cdot\text{m} \triangleleft$$

We conclude that

$$M_{max} = 45.2\text{ N}\cdot\text{m} \quad \text{for} \quad \theta = 82.9° \blacktriangleleft$$

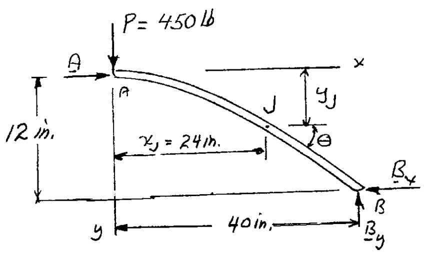
PROBLEM 7.13



The axis of the curved member AB is a parabola with vertex at A. If a vertical load **P** of magnitude 450 lb is applied at A, determine the internal forces at J when $h = 12$ in., $L = 40$ in., and $a = 24$ in.

SOLUTION

Free body AB



$$+\uparrow \Sigma F_y = 0: \quad -450 \text{ lb} + B_y = 0$$

$$\mathbf{B}_y = 450 \text{ lb} \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B_x(12 \text{ in.}) - (450 \text{ lb})(40 \text{ in.}) = 0$$

$$\mathbf{B}_x = 1500 \text{ lb} \longleftarrow$$

$$\Sigma F_x = 0: \quad \mathbf{A} = 1500 \text{ lb} \longrightarrow$$

Parabola: $\quad y = kx^2$

At B: $\quad 12 \text{ in.} = k(40 \text{ in.})^2 \qquad k = 0.0075$

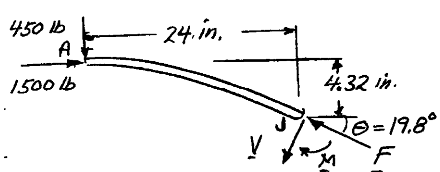
Equation of parabola: $\quad y = 0.0075x^2$

$$\text{slope} = \frac{dy}{dx} = 0.015x$$

At J: $\quad x_J = 24 \text{ in.} \qquad y_J = 0.0075(24)^2 = 4.32 \text{ in.}$

$$\text{slope} = 0.015(24) = 0.36, \quad \tan\theta = 0.36, \quad \theta = 19.8°$$

Free body AJ



$$+\circlearrowleft \Sigma M_J = 0: \quad (450 \text{ lb})(24 \text{ in.}) - (1500 \text{ lb})(4.32 \text{ in.}) - M = 0$$

$$M = 4320 \text{ lb} \cdot \text{in.} \qquad \mathbf{M} = 4320 \text{ lb} \cdot \text{in.} \circlearrowleft \blacktriangleleft$$

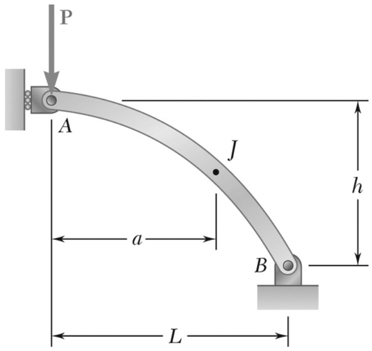
PROBLEM 7.13 (Continued)

$$+\nwarrow \Sigma F = 0: \quad F - (450 \text{ lb})\sin 19.8° - (1500 \text{ lb})\cos 19.8° = 0$$

$$F = +1563.8 \text{ lb} \qquad \mathbf{F} = 1564 \text{ lb} \searrow 19.8° \blacktriangleleft$$

$$+\nearrow \Sigma F = 0: \quad -V - (450 \text{ lb})\cos 19.8° + (1500 \text{ lb})\sin 19.8° = 0$$

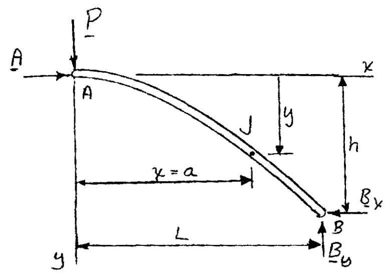
$$V = +84.71 \text{ lb} \qquad \mathbf{V} = 84.7 \text{ lb} \swarrow 70.2° \blacktriangleleft$$



PROBLEM 7.14

Knowing that the axis of the curved member AB is a parabola with vertex at A, determine the magnitude and location of the maximum bending moment.

SOLUTION



Parabola $\quad y = kx^2$

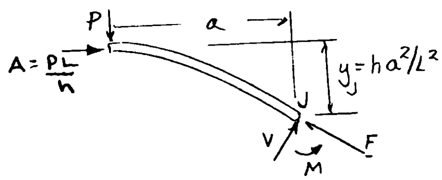
At B: $\quad h = kL^2$

$\quad k = h/L^2$

Equation of parabola $\quad y = hx^2/L^2$

$+\circlearrowleft \Sigma M_B = 0\text{:} \quad P(L) - A(h) = 0$ $\qquad$ $\mathbf{A} = PL/h \longrightarrow$

Free body AJ $\quad$ At J: $x_J = a \qquad y_J = ha^2/L^2$



$$+\circlearrowleft \Sigma M_J = 0\text{:} \quad P_a - (PL/h)(ha^2/L^2) + M = 0$$

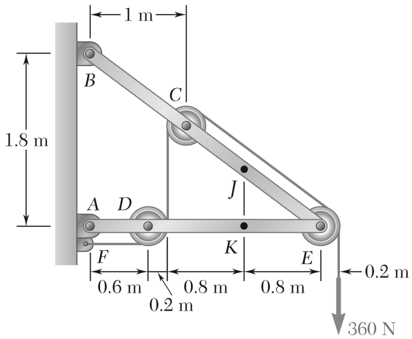
$$M = P\left(\frac{a^2}{L} - a\right)$$

For maximum: $\quad \dfrac{dM}{da} = P\left(\dfrac{2a}{L} - 1\right) = 0$

$M_{\max}$ occurs at: $\quad a = \dfrac{1}{2}L$ ◀

$$M_{\max} = P\left[\frac{(L/2)^2}{L} - \frac{L}{2}\right] = -\frac{PL}{4}$$

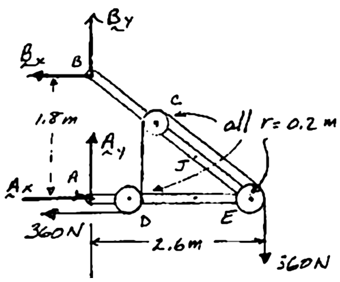
$|M|_{\max} = \dfrac{1}{4}PL$ ◀



PROBLEM 7.15

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point J of the frame shown.

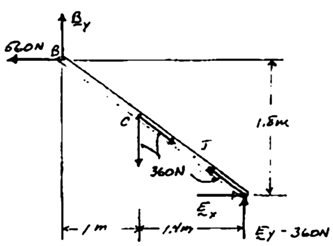
SOLUTION



FBD Frame with pulley and cord:

$$\circlearrowleft \Sigma M_A = 0: \quad (1.8\text{ m})B_x - (2.6\text{ m})(360\text{ N}) - (0.2\text{ m})(360\text{ N}) = 0$$

$$\mathbf{B}_x = 560\text{ N} \longleftarrow$$



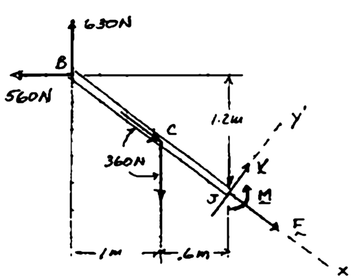
FBD BE:

Note: Cord forces have been moved to pulley hub as per Problem 6.91.

$$\circlearrowleft \Sigma M_E = 0: \quad (1.4\text{ m})(360\text{ N}) + (1.8\text{ m})(560\text{ N}) - (2.4\text{ m})B_y = 0$$

$$\mathbf{B}_y = 630\text{ N} \uparrow$$

FBD BJ:



$$\searrow \Sigma F_{x'} = 0: \quad F + 360\text{ N} - \frac{3}{5}(630\text{ N} - 360\text{ N}) - \frac{4}{5}(560\text{ N}) = 0$$

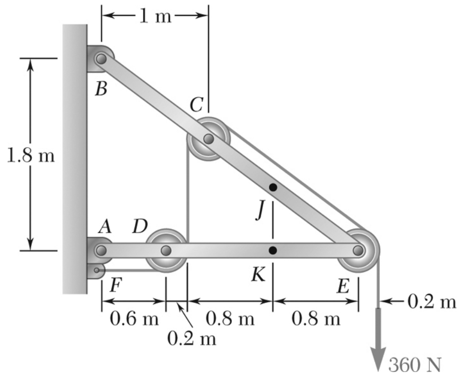
$$\mathbf{F} = 250\text{ N} \nwarrow 36.9° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: \quad V + \frac{4}{5}(630\text{ N} - 360\text{ N}) - \frac{3}{5}(560\text{ N}) = 0$$

$$\mathbf{V} = 120.0\text{ N} \measuredangle 53.1° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M + (0.6\text{ m})(360\text{ N}) + (1.2\text{ m})(560\text{ N}) - (1.6\text{ m})(630\text{ N}) = 0$$

$$\mathbf{M} = 120.0\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

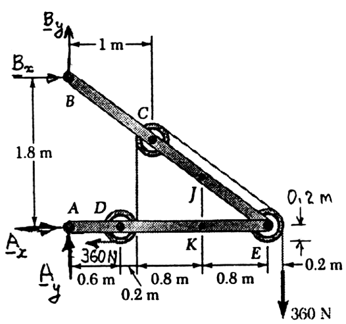


PROBLEM 7.16

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point K of the frame shown.

SOLUTION

Free body: frame and pulleys



$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(1.8 \text{ m}) - (360 \text{ N})(0.2 \text{ m}) - (360 \text{ N})(2.6 \text{ m}) = 0$$

$$B_x = -560 \text{ N} \qquad \mathbf{B}_x = 560 \text{ N} \longleftarrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 560 \text{ N} - 360 \text{ N} = 0$$

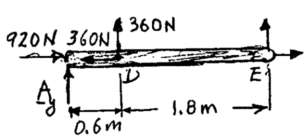
$$A_x = +920 \text{ N} \qquad \mathbf{A}_x = +920 \text{ N} \longrightarrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 360 \text{ N} = 0$$

$$A_y + B_y = 360 \text{ N} \qquad (1)$$

Free body: member AE

We recall that the forces applied to a pulley may be applied directly to the axle of the pulley.



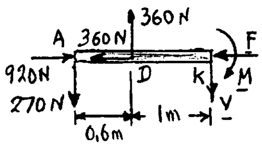
$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4 \text{ m}) - (360 \text{ N})(1.8 \text{ m}) = 0$$

$$A_y = -270 \text{ N} \qquad \mathbf{A}_y = 270 \text{ N} \downarrow \triangleleft$$

From (1): $\quad B_y = 360 \text{ N} + 270 \text{ N}$

$$B_y = 630 \text{ N} \qquad \mathbf{B}_y = 630 \text{ N} \uparrow \triangleleft$$

PROBLEM 7.16 (Continued)



Free body: *AK*

$\xrightarrow{+} \Sigma F_x = 0$: $920 \text{ N} - 360 \text{ N} - F = 0$

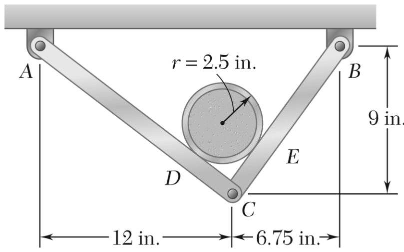
$F = +560 \text{ N}$ $\quad$ $\mathbf{F} = 560 \text{ N} \leftarrow$ ◀

$+\uparrow \Sigma F_y = 0$: $360 \text{ N} - 270 \text{ N} - V = 0$

$V = +90.0 \text{ N}$ $\quad$ $\mathbf{V} = 90.0 \text{ N} \downarrow$ ◀

$+\circlearrowright \Sigma M_K = 0$: $(270 \text{ N})(1.6 \text{ m}) - (360 \text{ N})(1 \text{ m}) - M = 0$

$M = +72.0 \text{ N} \cdot \text{m}$ $\quad$ $\mathbf{M} = 72.0 \text{ N} \cdot \text{m} \circlearrowright$ ◀

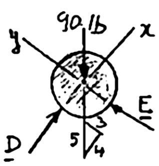


PROBLEM 7.17

A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member AC.

SOLUTION

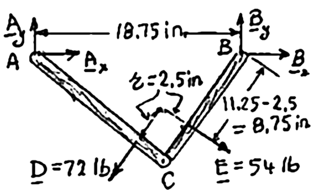
Free body: 10-ft section of pipe



$$+\nearrow \Sigma F_x = 0: \quad D - \frac{4}{5}(90 \text{ lb}) = 0 \qquad \mathbf{D} = 72 \text{ lb} \nearrow \triangleleft$$

$$+\nwarrow \Sigma F_y = 0: \quad E - \frac{3}{5}(90 \text{ lb}) = 0 \qquad \mathbf{E} = 54 \text{ lb} \nwarrow \triangleleft$$

Free body: Frame



$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(18.75 \text{ in.}) + (72 \text{ lb})(2.5 \text{ in.}) + (54 \text{ lb})(8.75 \text{ in.}) = 0$$

$$A_y = +34.8 \text{ lb} \qquad \mathbf{A}_y = 34.8 \text{ lb} \uparrow \triangleleft$$

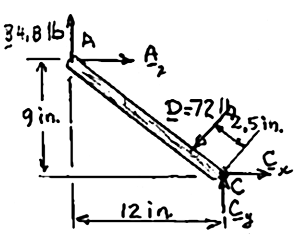
$$+\uparrow \Sigma F_y = 0: \quad B_y + 34.8 \text{ lb} - \frac{4}{5}(72 \text{ lb}) - \frac{3}{5}(54 \text{ lb}) = 0$$

$$B_y = +55.2 \text{ lb} \qquad \mathbf{B}_y = 55.2 \text{ lb} \uparrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x - \frac{3}{5}(72 \text{ lb}) + \frac{4}{5}(54 \text{ lb}) = 0$$

$$A_x + B_x = 0 \qquad (1)$$

Free body: Member AC



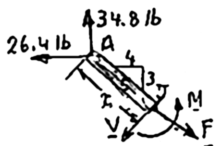
$$+\circlearrowleft \Sigma M_C = 0: \quad (72 \text{ lb})(2.5 \text{ in.}) - (34.8 \text{ lb})(12 \text{ in.}) - A_x(9 \text{ in.}) = 0$$

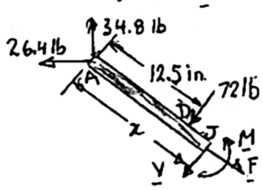
$$A_x = -26.4 \text{ lb} \qquad \mathbf{A}_x = 26.4 \text{ lb} \longleftarrow \triangleleft$$

From (1): $$B_x = -A_x = +26.4 \text{ lb}$$

$$\mathbf{B}_x = 26.4 \text{ lb} \longrightarrow \triangleleft$$

PROBLEM 7.17 (Continued)





Free body: Portion AJ

For $x \leq 12.5$ in. $(AJ \leq AD)$:

$$+\circlearrowleft \Sigma M_J = 0: \quad (26.4 \text{ lb})\frac{3}{5}x - (34.8 \text{ lb})\frac{4}{5}x + M = 0$$

$$M = 12x$$

$$M_{\max} = 150 \text{ lb} \cdot \text{in. for } x = 12.5 \text{ in.}$$

$$M_{\max} = 150.0 \text{ lb} \cdot \text{in. at } D \quad \triangleleft$$

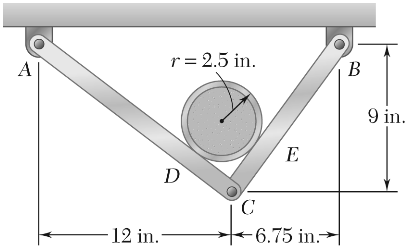
For $x > 12.5$ in.$(AJ > AD)$:

$$+\circlearrowleft \Sigma M_J = 0: \quad (26.4 \text{ lb})\frac{3}{5}x - (34.8 \text{ lb})\frac{4}{5}x + (72 \text{ lb})(x - 12.5) + M = 0$$

$$M = 900 - 60x$$

$$M_{\max} = 150 \text{ lb} \cdot \text{in. for } x = 12.5 \text{ in.}$$

Thus: $$M_{\max} = 150.0 \text{ lb} \cdot \text{in. at } D \quad \blacktriangleleft$$

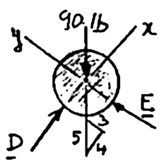


PROBLEM 7.18

For the frame of Problem 7.17, determine the magnitude and location of the maximum bending moment in member BC.

PROBLEM 7.17 A 5-in.-diameter pipe is supported every 9 ft by a small frame consisting of two members as shown. Knowing that the combined weight of the pipe and its contents is 10 lb/ft and neglecting the effect of friction, determine the magnitude and location of the maximum bending moment in member AC.

SOLUTION

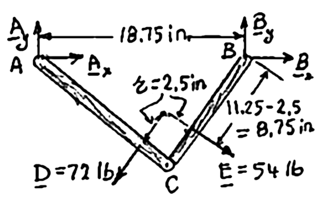


Free body: 10-ft section of pipe

$$+\nearrow \Sigma F_x = 0: \quad D - \frac{4}{5}(90 \text{ lb}) = 0 \qquad \mathbf{D} = 72 \text{ lb} \nearrow \;\triangleleft$$

$$+\nwarrow \Sigma F_y = 0: \quad E - \frac{3}{5}(90 \text{ lb}) = 0 \qquad \mathbf{E} = 54 \text{ lb} \nwarrow \;\triangleleft$$

Free body: Frame



$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(18.75 \text{ in.}) + (72 \text{ lb})(2.5 \text{ in.}) + (54 \text{ lb})(8.75 \text{ in.}) = 0$$

$$A_y = +34.8 \text{ lb} \qquad \mathbf{A}_y = 34.8 \text{ lb} \uparrow \;\triangleleft$$

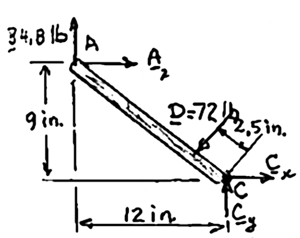
$$+\uparrow \Sigma F_y = 0: \quad B_y + 34.8 \text{ lb} - \frac{4}{5}(72 \text{ lb}) - \frac{3}{5}(54 \text{ lb}) = 0$$

$$B_y = +55.2 \text{ lb} \qquad \mathbf{B}_y = 55.2 \text{ lb} \uparrow \;\triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x - \frac{3}{5}(72 \text{ lb}) + \frac{4}{5}(54 \text{ lb}) = 0$$

$$A_x + B_x = 0 \qquad (1)$$

Free body: Member AC



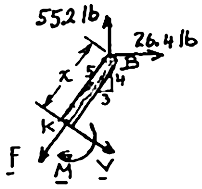
$$+\circlearrowleft \Sigma M_C = 0: \quad (72 \text{ lb})(2.5 \text{ in.}) - (34.8 \text{ lb})(12 \text{ in.}) - A_x(9 \text{ in.}) = 0$$

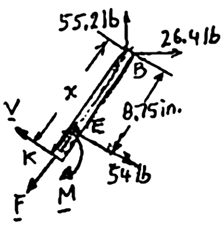
$$A_x = -26.4 \text{ lb} \qquad \mathbf{A}_x = 26.4 \text{ lb} \leftarrow \;\triangleleft$$

From (1): $\quad B_x = -A_x = +26.4 \text{ lb}$

$$\mathbf{B}_x = 26.4 \text{ lb} \rightarrow \;\triangleleft$$

PROBLEM 7.18 (Continued)





Free body: Portion BK

For $x \leq 8.75$ in. $(BK \leq BE)$:

$$+\circlearrowleft \Sigma M_K = 0: \quad (55.2 \text{ lb})\frac{3}{5}x - (26.4 \text{ lb})\frac{4}{5}x - M = 0$$

$$M = 12x$$

$$M_{max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{for} \quad x = 8.75 \text{ in.}$$

$$M_{max} = 105.0 \text{ lb} \cdot \text{in. at } E \ \triangleleft$$

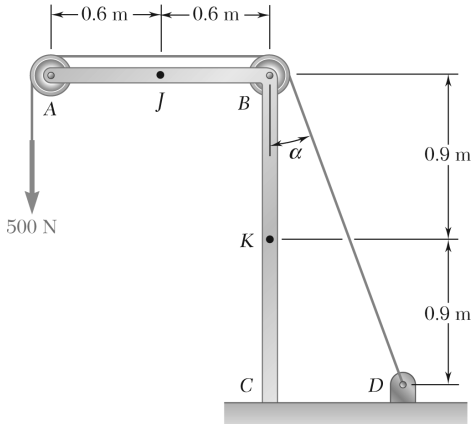
For $x > 8.75$ in. $(BK > BE)$:

$$+\circlearrowleft \Sigma M_K = 0: \quad (55.2 \text{ lb})\frac{3}{5}x - (26.4 \text{ lb})\frac{4}{5}x - (54 \text{ lb})(x - 8.75 \text{ in.}) - M = 0$$

$$M = 472.5 - 42x$$

$$M_{max} = 105.0 \text{ lb} \cdot \text{in.} \quad \text{for} \quad x = 8.75 \text{ in.}$$

Thus $$M_{max} = 105.0 \text{ lb} \cdot \text{in. at } E \ \blacktriangleleft$$



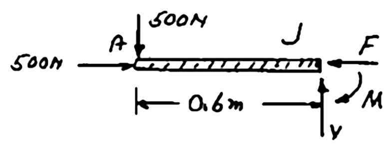
PROBLEM 7.19

Knowing that the radius of each pulley is 150 mm, that $\alpha = 20°$, and neglecting friction, determine the internal forces at (*a*) Point *J*, (*b*) Point *K*.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins *A* and *B*. Radius does not enter computations: (cf. Problem 6.90)

(*a*) Free body: *AJ*



$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - F = 0$

$F = 500\text{ N}$ $\qquad$ $\mathbf{F} = 500\text{ N} \leftarrow$ ◀

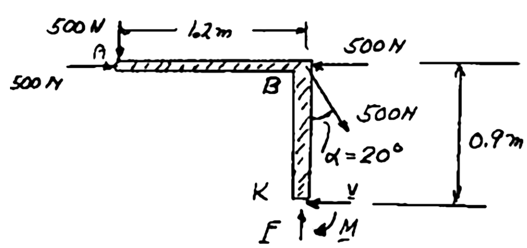
$+\uparrow \Sigma F_y = 0: \quad V - 500\text{ N} = 0$

$V = 500\text{ N}$ $\qquad$ $\mathbf{V} = 500\text{ N} \uparrow$ ◀

$+\circlearrowleft \Sigma M_J = 0: \quad (500\text{ N})(0.6\text{ m}) = 0$

$M = 300\text{ N} \cdot \text{m}$ $\qquad$ $\mathbf{M} = 300\text{ N} \cdot \text{m} \circlearrowright$ ◀

(*b*) Free body: *ABK*



$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - 500\text{ N} + (500\text{ N})\sin 20° - V = 0$

$V = 171.01\text{ N}$ $\qquad$ $\mathbf{V} = 171.0\text{ N} \leftarrow$ ◀

PROBLEM 7.19 (Continued)

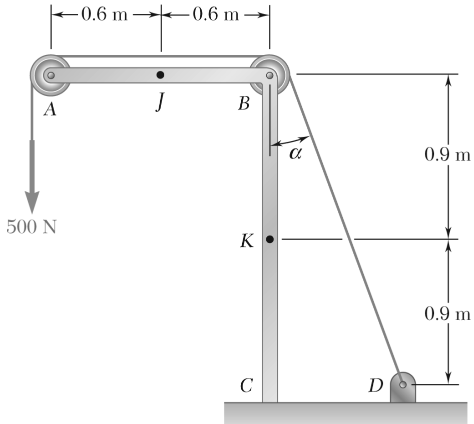
$$+\uparrow \Sigma F_y = 0: \quad -500\text{ N} - (500\text{ N})\cos 20° + F = 0$$

$$F = 969.8\text{ N} \qquad \mathbf{F} = 970\text{ N}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (500\text{ N})(1.2\text{ m}) - (500\text{ N})\sin 20°(0.9\text{ m}) - M = 0$$

$$M = 446.1\text{ N}\cdot\text{m}$$

$$\mathbf{M} = 446\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



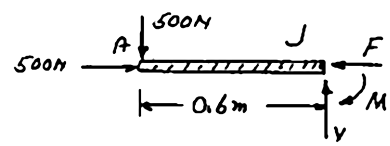
PROBLEM 7.20

Knowing that the radius of each pulley is 150 mm, that $\alpha = 30°$, and neglecting friction, determine the internal forces at (*a*) Point *J*, (*b*) Point *K*.

SOLUTION

Tension in cable = 500 N. Replace cable tension by forces at pins *A* and *B*. Radius does not enter computations: (cf. Problem 6.90)

(*a*) Free body: *AJ*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - F = 0$$

$$F = 500\text{ N} \qquad \mathbf{F} = 500\text{ N} \longleftarrow \blacktriangleleft$$

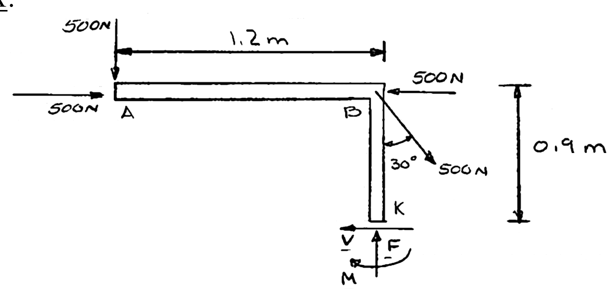
$$+\!\uparrow \Sigma F_y = 0: \quad V - 500\text{ N} = 0$$

$$V = 500\text{ N} \qquad \mathbf{V} = 500\text{ N}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (500\text{ N})(0.6\text{ m}) = 0$$

$$M = 300\text{ N}\cdot\text{m} \qquad \mathbf{M} = 300\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*) FBD: Portion *ABK*:



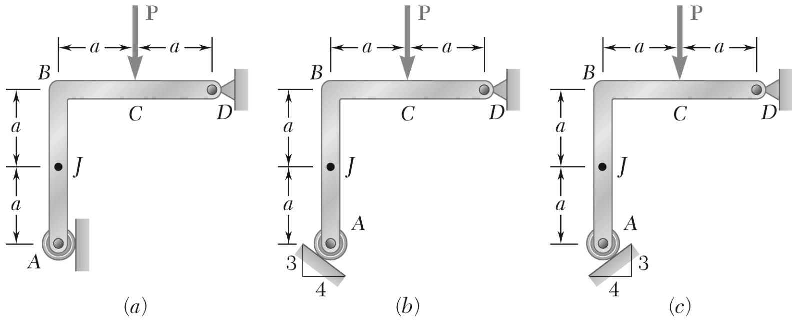
$$\xrightarrow{+} \Sigma F_x = 0: \quad 500\text{ N} - 500\text{ N} + (500\text{ N})\sin 30° - V \qquad \mathbf{V} = 250\text{ N} \longleftarrow \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -500\text{ N} - (500\text{ N})\cos 30° + F = 0 \qquad \mathbf{F} = 933\text{ N}\uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (500\text{ N})(1.2\text{ m}) - (500\text{ N})\sin 30°(0.9\text{ m}) - M = 0 \qquad \mathbf{M} = 375\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

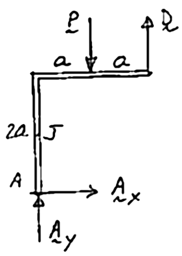
PROBLEM 7.21

A force **P** is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point *J*.



SOLUTION

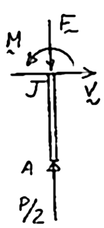
(*a*) **FBD Rod:**



$$\rightarrow \Sigma F_x = 0: \quad A_x = 0$$

$$\circlearrowleft \Sigma M_D = 0: \quad aP - 2aA_y = 0 \qquad A_y = \frac{P}{2}$$

FBD AJ:

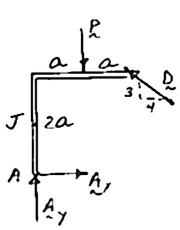


$$\rightarrow \Sigma F_x = 0: \quad \mathbf{V} = 0 \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{P}{2} - F = 0 \qquad \mathbf{F} = \frac{P}{2}\downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad \mathbf{M} = 0 \blacktriangleleft$$

(*b*) **FBD Rod:**



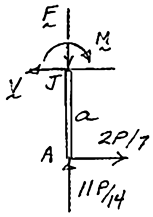
$$\circlearrowleft \Sigma M_A = 0: \quad 2a\left(\frac{4}{5}D\right) + 2a\left(\frac{3}{5}D\right) - aP = 0 \qquad D = \frac{5P}{14}$$

$$\rightarrow \Sigma F_x = 0: \quad A_x - \frac{4}{5}\frac{5}{14}P = 0 \qquad A_x = \frac{2P}{7}$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P + \frac{3}{5}\frac{5}{14}P = 0 \qquad A_y = \frac{11P}{14}$$

PROBLEM 7.21 (Continued)

FBD AJ:

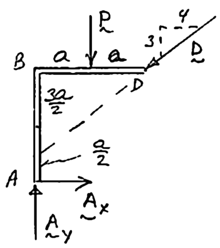


$$\xrightarrow{}\Sigma F_x = 0: \quad \frac{2}{7}P - V = 0 \qquad \mathbf{V} = \frac{2P}{7} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{11P}{14} - F = 0 \qquad \mathbf{F} = \frac{11P}{14} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad a\frac{2P}{7} - M = 0 \qquad \mathbf{M} = \frac{2}{7}aP \circlearrowright \blacktriangleleft$$

(*c*) **FBD Rod:**

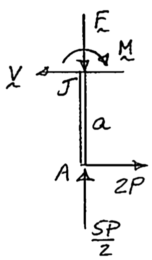


$$\circlearrowleft \Sigma M_A = 0: \quad \frac{a}{2}\left(\frac{4D}{5}\right) - aP = 0 \qquad D = \frac{5P}{2}$$

$$\xrightarrow{}\Sigma F_x = 0: \quad A_x - \frac{4}{5}\frac{5P}{2} = 0 \qquad A_x = 2P$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P - \frac{3}{5}\frac{5P}{2} = 0 \qquad A_y = \frac{5P}{2}$$

FBD AJ:



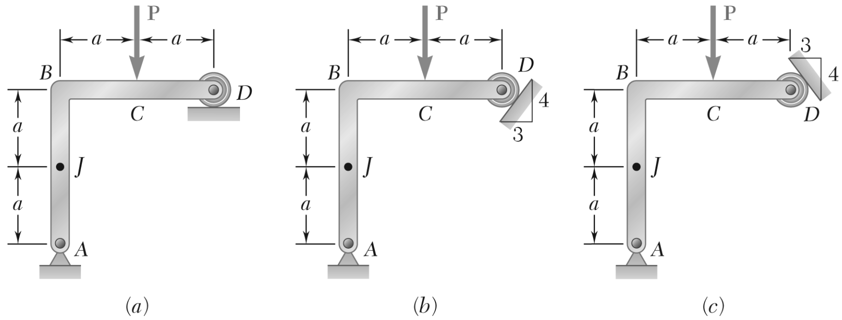
$$\xrightarrow{}\Sigma F_x = 0: \quad 2P - V = 0 \qquad \mathbf{V} = 2P \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad \frac{5P}{2} - F = 0 \qquad \mathbf{F} = \frac{5P}{2} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad a(2P) - M = 0 \qquad \mathbf{M} = 2aP \circlearrowright \blacktriangleleft$$

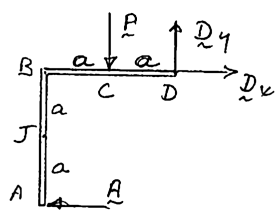
PROBLEM 7.22

A force **P** is applied to a bent rod that is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at Point *J*.



SOLUTION

(*a*) **FBD Rod:**

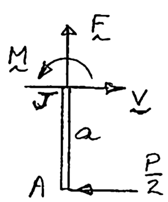


$$\circlearrowleft \Sigma M_D = 0: \quad aP - 2aA = 0$$

$$\mathbf{A} = \frac{P}{2} \longleftarrow$$

$$\longrightarrow \Sigma F_x = 0: \quad V - \frac{P}{2} = 0 \qquad \mathbf{V} = \frac{P}{2} \longrightarrow \blacktriangleleft$$

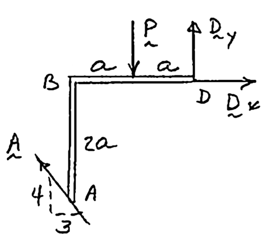
FBD AJ:



$$\uparrow \Sigma F_y = 0: \qquad \mathbf{F} = 0 \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - a\frac{P}{2} = 0 \qquad \mathbf{M} = \frac{aP}{2} \circlearrowright \blacktriangleleft$$

(*b*) **FBD Rod:**

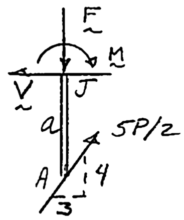


$$\circlearrowleft \Sigma M_D = 0: \quad aP - \frac{a}{2}\left(\frac{4}{5}A\right) = 0$$

$$\mathbf{A} = \frac{5P}{2} \nearrow$$

PROBLEM 7.22 (Continued)

FBD AJ:

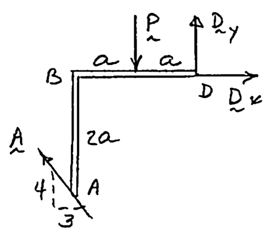


$\rightarrow \Sigma F_x = 0\colon\quad \dfrac{3}{5}\dfrac{5P}{2} - V = 0$ $\qquad\qquad \mathbf{V} = \dfrac{3P}{2} \leftarrow$ ◀

$\uparrow \Sigma F_y = 0\colon\quad \dfrac{4}{5}\dfrac{5P}{2} - F = 0$ $\qquad\qquad \mathbf{F} = 2P\downarrow$ ◀

$\mathbf{M} = \dfrac{3}{2}aP \circlearrowright$ ◀

(*c*) **FBD Rod:**



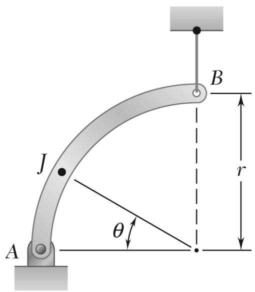
$\circlearrowleft \Sigma M_D = 0\colon\quad aP - 2a\left(\dfrac{3}{5}A\right) - 2a\left(\dfrac{4}{5}A\right) = 0$

$A = \dfrac{5P}{14}$

$\rightarrow \Sigma F_x = 0\colon\quad V - \left(\dfrac{3}{5}\dfrac{5P}{14}\right) = 0$ $\qquad\qquad \mathbf{V} = \dfrac{3P}{14} \rightarrow$ ◀

$\uparrow \Sigma F_y = 0\colon\quad \dfrac{4}{5}\dfrac{5P}{14} - F = 0$ $\qquad\qquad \mathbf{F} = \dfrac{2P}{7}\downarrow$ ◀

$\circlearrowleft \Sigma M_J = 0\colon\quad M - a\left(\dfrac{3}{5}\dfrac{5P}{14}\right) = 0$ $\qquad\qquad \mathbf{M} = \dfrac{3}{14}aP \circlearrowleft$ ◀

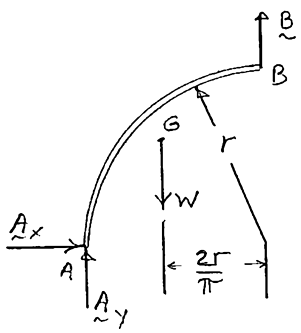


PROBLEM 7.23

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

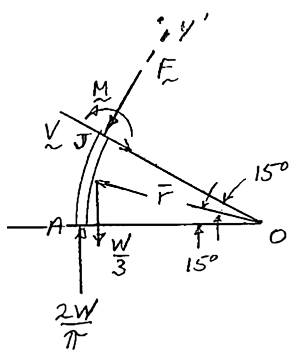
FBD Rod:



$$\xrightarrow{} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad \frac{2r}{\pi}W - rA_y = 0 \qquad \mathbf{A}_y = \frac{2W}{\pi}\uparrow$$

FBD AJ:



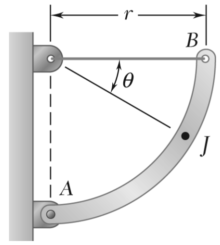
$$\alpha = 15°, \ \text{weight of segment} = W\frac{30°}{90°} = \frac{W}{3}$$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{r}{\frac{\pi}{12}}\sin 15° = 0.9886r$$

$$\nearrow \Sigma F_{y'} = 0: \quad \frac{2W}{\pi}\cos 30° - \frac{W}{3}\cos 30° - F = 0$$

$$\mathbf{F} = \frac{W\sqrt{3}}{2}\left(\frac{2}{\pi} - \frac{1}{3}\right) \swarrow$$

$$\circlearrowleft \Sigma M_0 = M + r\left(F - \frac{2W}{\pi}\right) + \bar{r}\cos 15°\frac{W}{3} = 0 \qquad \mathbf{M} = 0.0557Wr \circlearrowright \blacktriangleleft$$

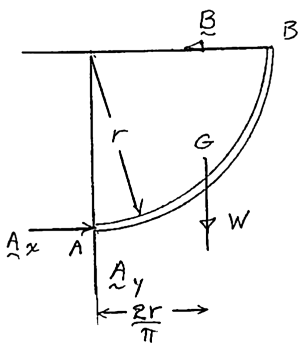


PROBLEM 7.24

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

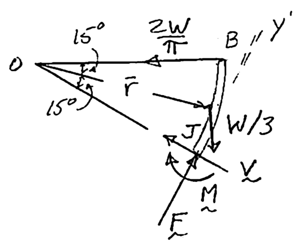
FBD Rod:



$$\circlearrowleft \Sigma M_A = 0: \quad rB - \frac{2r}{\pi}W = 0$$

$$\mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

FBD BJ:



$$\alpha = 15° = \frac{\pi}{12}$$

$$\bar{r} = \frac{r}{\frac{\pi}{12}}\sin 15° = 0.98862r$$

$$\text{Weight of segment } = W\frac{30°}{90°} = \frac{W}{3}$$

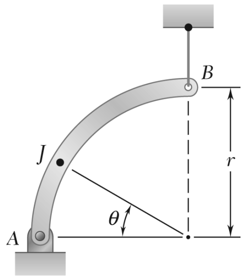
$$\nearrow \Sigma F_{y'} = 0: \quad F - \frac{W}{3}\cos 30° - \frac{2W}{\pi}\sin 30° = 0$$

$$\mathbf{F} = \left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right)W \nearrow$$

$$\circlearrowleft \Sigma M_0 = 0: \quad rF - (\bar{r}\cos 15°)\frac{W}{3} - M = 0$$

$$M = rW\left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right) - \left(0.98862\frac{\cos 15°}{3}\right)Wr \qquad \mathbf{M} = 0.289Wr \circlearrowright \blacktriangleleft$$

PROBLEM 7.25

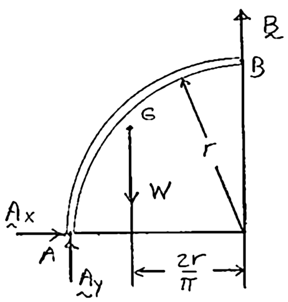


For the rod of Problem 7.23, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.23 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

FBD Rod:



$$\xrightarrow{} \Sigma F_x = 0: \quad A_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \quad \frac{2r}{\pi}W - rA_y = 0 \qquad A_y = \frac{2W}{\pi}$$

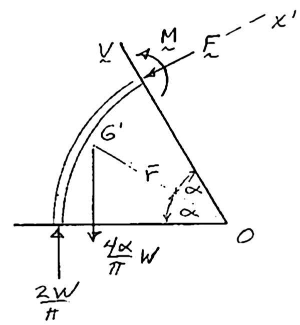
$$\alpha = \frac{\theta}{2}, \qquad \bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment} = W\frac{2\alpha}{\frac{\pi}{2}} = \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_{x'} = 0: \quad -F - \frac{4\alpha}{\pi}W\cos 2\alpha + \frac{2W}{\pi}\cos 2\alpha = 0$$

$$F = \frac{2W}{\pi}(1-2\alpha)\cos 2\alpha = \frac{2W}{\pi}(1-\theta)\cos\theta$$

FBD AJ: $\circlearrowleft \Sigma M_0 = 0: \quad M + \left(F - \dfrac{2W}{\pi}\right)r + (\bar{r}\cos\alpha)\dfrac{4\alpha}{\pi}W = 0$



$$M = \frac{2W}{\pi}(1+\theta\cos\theta - \cos\theta)r - \frac{4\alpha W}{\pi}\frac{r}{\alpha}\sin\alpha\cos\alpha$$

But, $\quad \sin\alpha\cos\alpha = \dfrac{1}{2}\sin 2\alpha = \dfrac{1}{2}\sin\theta$

so $\quad M = \dfrac{2r}{\pi}W(1-\cos\theta + \theta\cos\theta - \sin\theta)$

$$\frac{dM}{d\theta} = \frac{2rW}{\pi}(\sin\theta - \theta\sin\theta + \cos\theta - \cos\theta) = 0$$

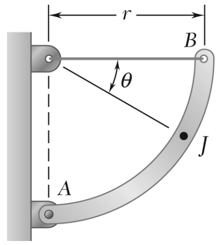
for $\quad (1-\theta)\sin\theta = 0$

$$\frac{dM}{d\theta} = 0 \quad \text{for} \quad \theta = 0, 1, n\pi\,(n = 1, 2, \cdots)$$

Only 0 and 1 in valid range

At $\quad \theta = 0 \quad M = 0, \quad$ at $\quad \theta = 1\text{ rad}$

at $\quad \theta = 57.3°$ $\qquad M = M_{max} = 0.1009Wr$ ◀



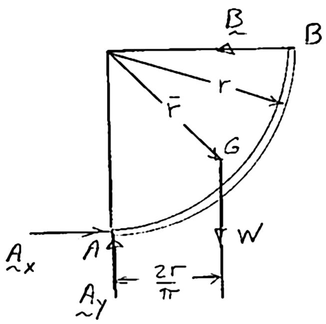
PROBLEM 7.26

For the rod of Problem 7.24, determine the magnitude and location of the maximum bending moment.

PROBLEM 7.24 A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at Point J when $\theta = 30°$.

SOLUTION

FBD Bar:



$$\circlearrowleft \Sigma M_A = 0: \quad rB - \frac{2r}{\pi}W = 0 \qquad \mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

$$\alpha = \frac{\theta}{2} \qquad \text{so} \qquad 0 \le \alpha \le \frac{\pi}{4}$$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment } = W\frac{2\alpha}{\frac{\pi}{2}}$$

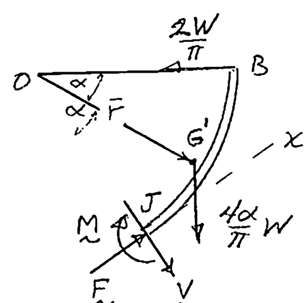
$$= \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_{x'} = 0: \quad F - \frac{4\alpha}{\pi}W\cos 2\alpha - \frac{2W}{\pi}\sin 2\alpha = 0$$

$$F = \frac{2W}{\pi}(\sin 2\alpha + 2\alpha\cos 2\alpha)$$

$$= \frac{2W}{\pi}(\sin\theta + \theta\cos\theta)$$

FBD BJ:



$$\circlearrowleft \Sigma M_0 = 0: \quad rF - (\bar{r}\cos\alpha)\frac{4\alpha}{\pi}W - M = 0$$

$$M = \frac{2}{\pi}Wr(\sin\theta + \theta\cos\theta) - \left(\frac{r}{\alpha}\sin\alpha\cos\alpha\right)\frac{4\alpha}{\pi}W$$

But, $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

PROBLEM 7.26 (Continued)

so $$M = \frac{2Wr}{\pi}(\sin\theta + \theta\cos\theta - \sin\theta)$$

or $$M = \frac{2}{\pi}Wr\theta\cos\theta$$

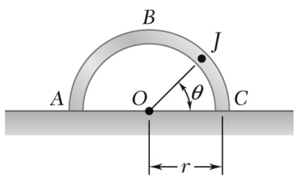
$$\frac{dM}{d\theta} = \frac{2}{\pi}Wr(\cos\theta - \theta\sin\theta) = 0 \quad \text{at } \theta\tan\theta = 1$$

Solving numerically $\theta = 0.8603 \text{ rad}$

and $\mathbf{M} = 0.357Wr$ ◀

at $\theta = 49.3°$ ◀

(Since $M = 0$ at both limits, this is the maximum)



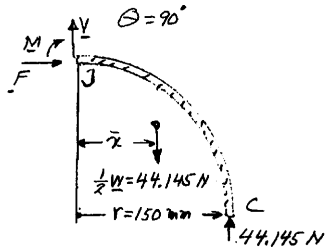
PROBLEM 7.27

A half section of pipe rests on a frictionless horizontal surface as shown. If the half section of pipe has a mass of 9 kg and a diameter of 300 mm, determine the bending moment at Point J when $\theta = 90°$.

SOLUTION

For half section $\quad m = 9\text{ kg}$

$$W = mg = (9)(9.81) = 88.29\text{ N}$$



Portion JC:

$$\text{Weight} = \frac{1}{2}W = 44.145\text{ N}$$

From Fig. 5.8B:

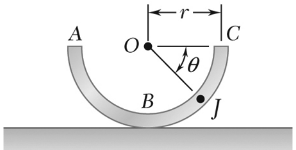
$$x = \frac{2r}{\pi} = \frac{2(150)}{\pi}$$

$$x = 95.49\text{ mm}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (44.145\text{ N})(0.15\text{ m}) - (44.145\text{ N})(0.09549\text{ m}) - M = 0$$

$$M = +2.406\text{ N}\cdot\text{m} \qquad \mathbf{M} = 2.41\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

PROBLEM 7.28

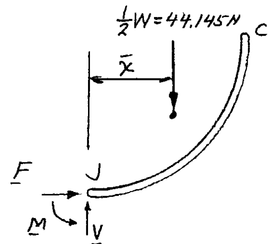


A half section of pipe rests on a frictionless horizontal surface as shown. If the half section of pipe has a mass of 9 kg and a diameter of 300 mm, determine the bending moment at point J when $\theta = 90°$.

SOLUTION

For half section $m = 9\text{ kg}$

$$W = mg = (9\text{ kg})(9.81\text{ m/s}^2) = 88.29\text{ N}$$



Free body JC

Weight of portion JC, $= \frac{1}{2}W = 44.145\text{ N}$

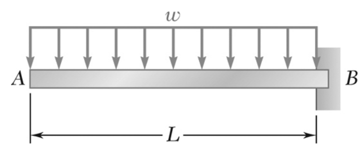
$$r = 150\text{ mm}$$

From Fig. 5.8B:

$$x = \frac{2r}{\pi} = \frac{2(150)}{\pi} = 95.49\text{ mm}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (44.145\text{ N})(0.09549\text{ m}) = 0$$

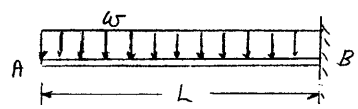
$$M = +4.2154\text{ N}\cdot\text{m} \qquad \mathbf{M} = 4.22\text{ N}\cdot\text{m} \circlearrowleft ◀$$

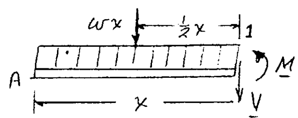


PROBLEM 7.29

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



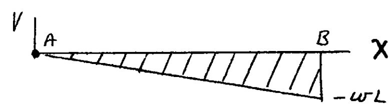


$$+\uparrow \Sigma F_y = 0: \quad -wx - V = 0$$

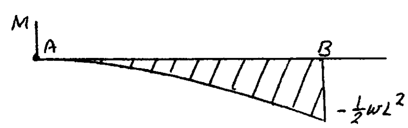
$$V = -wx$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad wx\left(\frac{x}{2}\right) + M = 0$$

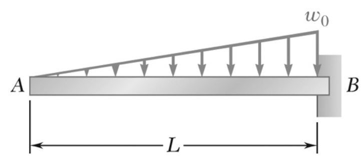
$$M = -\frac{1}{2}wx^2$$



$$|V|_{\max} = wL \blacktriangleleft$$



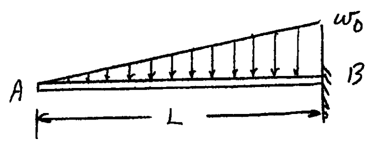
$$|M|_{\max} = \frac{1}{2}wL^2 \blacktriangleleft$$



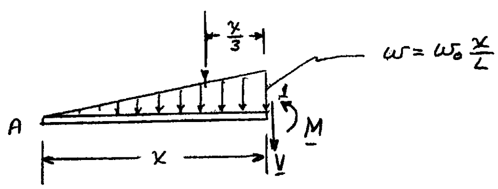
PROBLEM 7.30

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

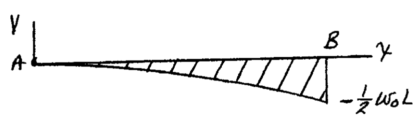


$$\frac{1}{2}wx = \frac{1}{2}\left(w_0\frac{x}{L}\right)x = \frac{1}{2}w_0\frac{x^2}{L} \qquad \text{By similar } \Delta\text{'s}$$

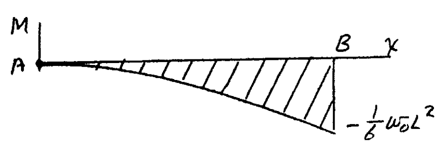


$$+\uparrow \Sigma F_y = 0: \quad -\frac{1}{2}w_0\frac{x^2}{L} - V = 0 \qquad V = -\frac{1}{2}w_0\frac{x^2}{L}$$

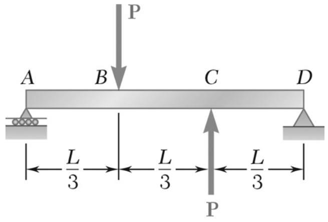
$$+\circlearrowleft \Sigma F_y = 0: \quad \left(\frac{1}{2}w_0\frac{x^2}{L}\right)\frac{x}{3} + M = 0 \qquad M = -\frac{1}{6}w_0\frac{x^3}{L}$$



$$|V|_{max} = \frac{1}{2}w_0L \blacktriangleleft$$



$$|M|_{max} = \frac{1}{6}w_0L^2 \blacktriangleleft$$

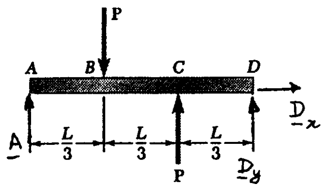


PROBLEM 7.31

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: entire beam



$$+\circlearrowleft \Sigma M_D = 0: \quad P\left(\frac{2L}{3}\right) - P\left(\frac{L}{3}\right) - AL = 0$$

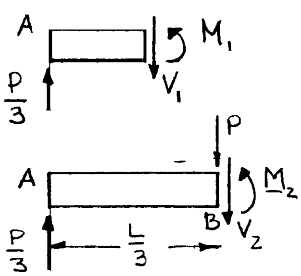
$$\mathbf{A} = P/3 \uparrow$$

$$\Sigma F_x = 0: \quad D_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{P}{3} - P + P + D_y = 0$$

$$D_y = -P/3 \qquad \mathbf{D} = P/3 \downarrow$$

(*a*) Shear and bending moment. Since the loading consists of concentrated loads, the shear diagram is made of horizontal straight-line segments and the B. M. diagram is made of oblique straight-line segments.



Just to the right of A:

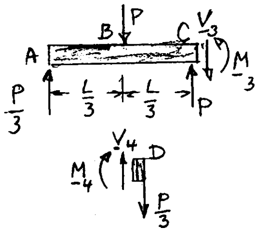
$$+\uparrow \Sigma F_y = 0: \quad -V_1 + \frac{P}{3} = 0 \qquad V_1 = +P/3 \ \triangleleft$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 - \frac{P}{3}(0) = 0 \qquad M_1 = 0 \ \triangleleft$$

Just to the right of B:

$$+\uparrow \Sigma F_y = 0: \quad -V_2 + \frac{P}{3} - P = 0, \qquad V_2 = -2P/3 \ \triangleleft$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - \frac{P}{3}\left(\frac{L}{3}\right) + P(0) = 0 \qquad M_2 = +PL/9 \ \triangleleft$$



Just to the right of C:

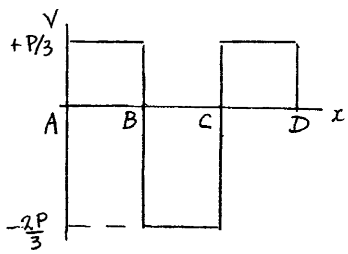
$$+\uparrow \Sigma F_y = 0: \quad \frac{P}{3} - P + P - V_3 = 0 \qquad V_3 = +P/3 \ \triangleleft$$

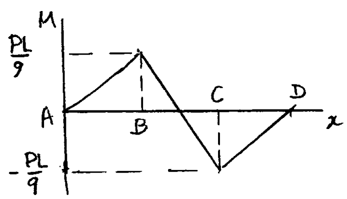
$$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 - \frac{P}{3}\left(\frac{2L}{3}\right) + P\frac{L}{3} - P(0) = 0 \qquad M_3 = -PL/9 \ \triangleleft$$

Just to the left of D:

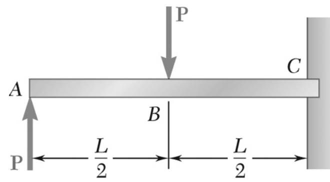
$$+\uparrow \Sigma F_y = 0: \quad V_4 - \frac{P}{3} = 0 \qquad V_4 = +\frac{P}{3} \ \triangleleft$$

$$+\circlearrowleft \Sigma M_4 = 0: \quad -M_4 - \frac{P}{3}(0) = 0 \qquad M_4 = 0 \ \triangleleft$$





(*b*) $|V|_{max} = 2P/3; \ |M|_{max} = PL/9$ ◀



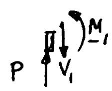
PROBLEM 7.32

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

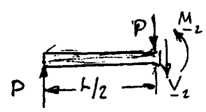
(*a*) Shear and bending moment.

Just to the right of *A*:



$V_1 = +P;$ $\qquad M_1 = 0 \triangleleft$

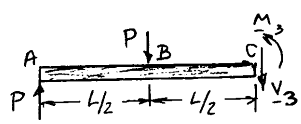
Just to the right of *B*:



$+\uparrow \Sigma F_y = 0: \quad P - P - V_2 = 0;$ $\qquad V_2 = 0 \triangleleft$

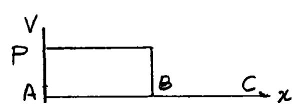
$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - P\left(\frac{L}{2}\right) = 0;$ $\qquad M_2 = +PL/2 \triangleleft$

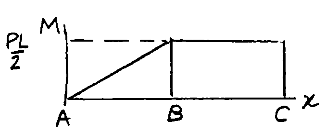
Just to the left of *C*:



$+\uparrow \Sigma F_y = 0: \quad P - P - V_3 = 0,$ $\qquad V_3 = 0 \triangleleft$

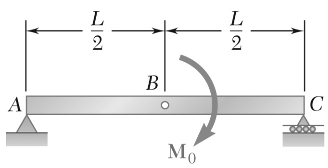
$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 + P\left(\frac{L}{2}\right) - PL = 0,$ $\qquad M_3 = +PL/2 \triangleleft$





(*b*) $|V|_{max} = P \blacktriangleleft$

$|M|_{max} = PL/2 \blacktriangleleft$

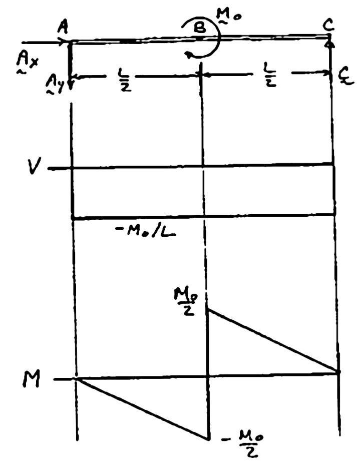


PROBLEM 7.33

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **FBD Beam:**



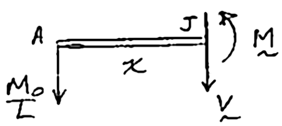
$$\circlearrowleft \Sigma M_C = 0: \quad LA_y - M_0 = 0$$

$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$$\uparrow \Sigma F_y = 0: \quad -A_y + C = 0$$

$$\mathbf{C} = \frac{M_0}{L} \uparrow$$

Along AB:



$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0$$

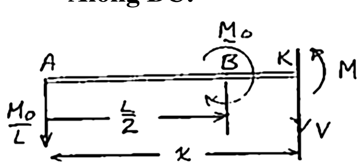
$$V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_J = 0: \quad x\frac{M_0}{L} + M = 0$$

$$M = -\frac{M_0}{L}x$$

Straight with $M = -\dfrac{M_0}{2}$ at B

Along BC:



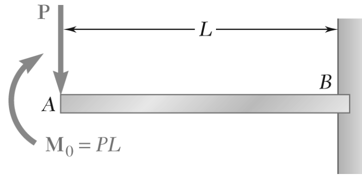
$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_K = 0: \quad M + x\frac{M_0}{L} - M_0 = 0 \qquad M = M_0\left(1 - \frac{x}{L}\right)$$

Straight with $M = \dfrac{M_0}{2}$ at B $\qquad M = 0$ at C

(*b*) From diagrams: $|V|_{max} = M_0/L$ ◀

$$|M|_{max} = \frac{M_0}{2} \text{ at } B \;◀$$

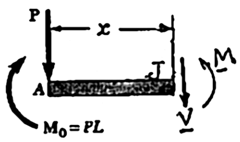


PROBLEM 7.34

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

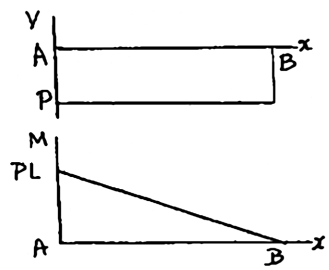
Free body: Portion *AJ*



$+\uparrow \Sigma F_y = 0: \quad -P - V = 0$ $\qquad V = -P \triangleleft$

$+\circlearrowleft \Sigma M_J = 0: \quad M + P_x - PL = 0$ $\qquad M = P(L - x) \triangleleft$

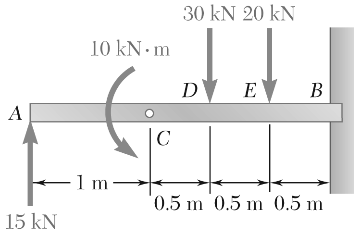
(*a*) The *V* and *M* diagrams are obtained by plotting the functions *V* and *M*.



(*b*)

$|V|_{\max} = P$ ◀

$|M|_{\max} = PL$ ◀

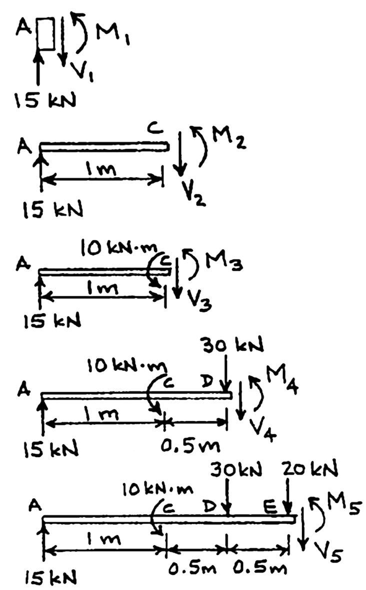


PROBLEM 7.35

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) Just to the right of *A*:



$$+\uparrow \Sigma F_y = 0 \quad V_1 = +15 \text{ kN} \quad M_1 = 0$$

Just to the left of *C*:

$$V_2 = +15 \text{ kN} \quad M_2 = +15 \text{ kN} \cdot \text{m}$$

Just to the right of *C*:

$$V_3 = +15 \text{ kN} \quad M_3 = +5 \text{ kN} \cdot \text{m}$$

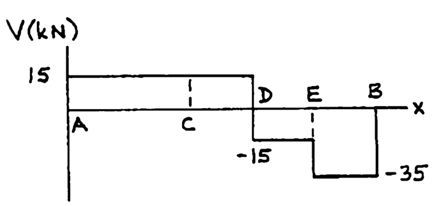
Just to the right of *D*:

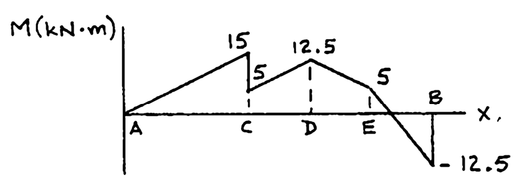
$$V_4 = -15 \text{ kN} \quad M_4 = +12.5 \text{ kN} \cdot \text{m}$$

Just to the right of *E*:

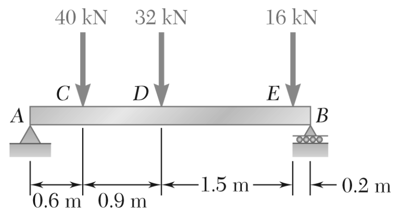
$$V_5 = -35 \text{ kN} \quad M_5 = +5 \text{ kN} \cdot \text{m}$$

At *B*: $M_B = -12.5 \text{ kN} \cdot \text{m}$





(*b*) $|V|_{max} = 35.0 \text{ kN}$ $|M|_{max} = 12.50 \text{ kN} \cdot \text{m}$ ◀

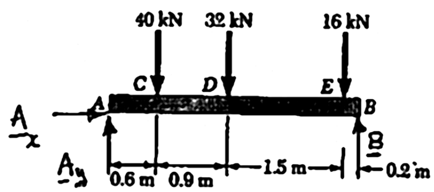


PROBLEM 7.36

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(3.2 \text{ m}) - (40 \text{ kN})(0.6 \text{ m}) - (32 \text{ kN})(1.5 \text{ m}) - (16 \text{ kN})(3 \text{ m}) = 0$$

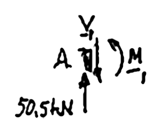
$B = +37.5$ kN $\qquad$ $\mathbf{B} = 37.5 \text{ kN} \uparrow$ ◁

$\Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y + 37.5 \text{ kN} - 40 \text{ kN} - 32 \text{ kN} - 16 \text{ kN} = 0$

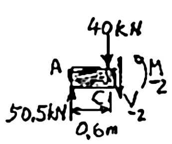
$A_y = +50.5$ kN $\qquad$ $\mathbf{A} = 50.5 \text{ kN} \uparrow$ ◁

(*a*) Shear and bending moment



Just to the right of *A*: $\qquad V_1 = 50.5$ kN $\qquad M_1 = 0$ ◁

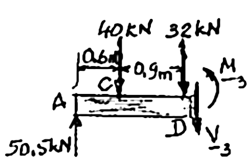
Just to the right of *C*:



$+\uparrow \Sigma F_y = 0: \quad 50.5 \text{ kN} - 40 \text{ kN} - V_2 = 0 \qquad V_2 = +10.5 \text{ kN}$ ◁

$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - (50.5 \text{ kN})(0.6 \text{ m}) = 0 \qquad M_2 = +30.3 \text{ kN} \cdot \text{m}$ ◁

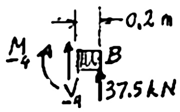
Just to the right of *D*:



$+\uparrow \Sigma F_y = 0: \quad 50.5 - 40 - 32 - V_3 = 0 \qquad V_3 = -21.5 \text{ kN}$ ◁

$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 - (50.5)(1.5) + (40)(0.9) = 0 \qquad M_3 = +39.8 \text{ kN} \cdot \text{m}$ ◁

Just to the right of E:

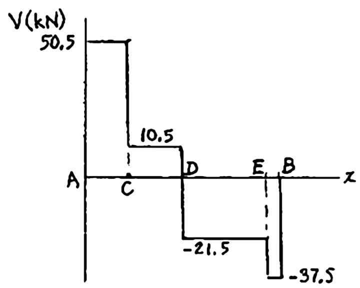


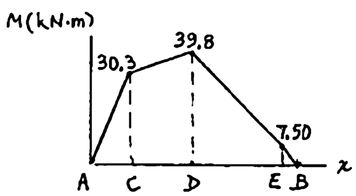
$+\uparrow \Sigma F_y = 0$: $V_4 + 37.5 = 0$ $\qquad V_4 = -37.5 \text{ kN}$ ◁

$+\circlearrowleft \Sigma M_4 = 0$: $-M_4 + (37.5)(0.2) = 0$ $\qquad M_4 = +7.50 \text{ kN} \cdot \text{m}$ ◁

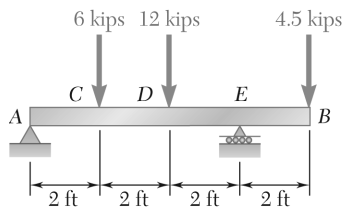
At B: $\qquad V_B = M_B = 0$ ◁

(*b*) $\qquad |V|_{\text{max}} = 50.5 \text{ kN}$ ◀





$|M|_{\text{max}} = 39.8 \text{ kN} \cdot \text{m}$ ◀

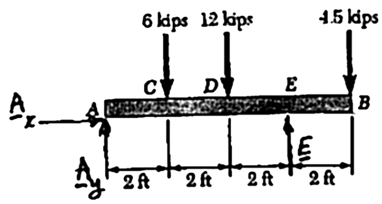


PROBLEM 7.37

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$+\circlearrowleft \Sigma M_A = 0: \quad E(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (12\text{ kips})(4\text{ ft}) - (4.5\text{ kips})(8\text{ ft}) = 0$

$E = +16\text{ kips}$ $\qquad\qquad$ $\mathbf{E} = 16\text{ kips}\uparrow \triangleleft$

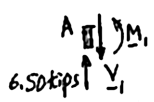
$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y + 16\text{ kips} - 6\text{ kips} - 12\text{ kips} - 4.5\text{ kips} = 0$

$A_y = +6.50\text{ kips}$ $\qquad\qquad$ $\mathbf{A} = 6.50\text{ kips}\uparrow \triangleleft$

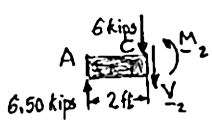
(*a*) Shear and bending moment

Just to the right of *A*:



$V_1 = +6.50\text{ kips} \quad M_1 = 0$ $\qquad$ $\triangleleft$

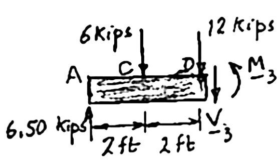
Just to the right of *C*:



$+\uparrow \Sigma F_y = 0: \quad 6.50\text{ kips} - 6\text{ kips} - V_2 = 0$ $\qquad$ $V_2 = +0.50\text{ kips} \triangleleft$

$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - (6.50\text{ kips})(2\text{ ft}) = 0$ $\qquad$ $M_2 = +13\text{ kip}\cdot\text{ft} \triangleleft$

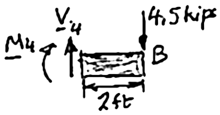
Just to the right of *D*:



$+\uparrow \Sigma F_y = 0: \quad 6.50 - 6 - 12 - V_3 = 0$ $\qquad$ $V_3 = +11.5\text{ kips} \triangleleft$

$+\circlearrowleft \Sigma M_3 = 0: \quad M_3 - (6.50)(4) - (6)(2) = 0$ $\qquad$ $M_3 = +14\text{ kip}\cdot\text{ft} \triangleleft$

Just to the right of E:

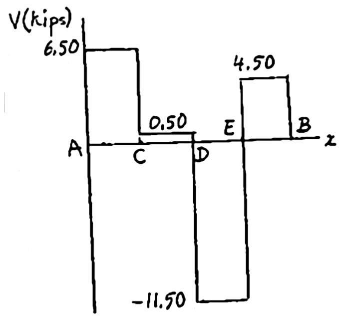


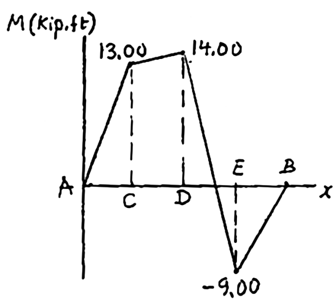
$+\uparrow \Sigma F_y = 0$: $V_4 - 4.5 = 0$ $\qquad V_4 = +4.5 \text{ kips}$ ◁

$+\circlearrowleft \Sigma M_4 = 0$: $-M_4 - (4.5)2 = 0$ $\qquad M_4 = -9 \text{ kip} \cdot \text{ft}$ ◁

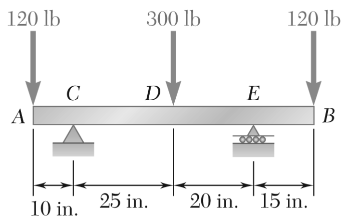
At B: $\qquad V_B = M_B = 0$ ◁

(*b*) $\qquad |V|_{\max} = 11.50 \text{ kips}$ ◀





$|M|_{\max} = 14.00 \text{ kip} \cdot \text{ft}$ ◀

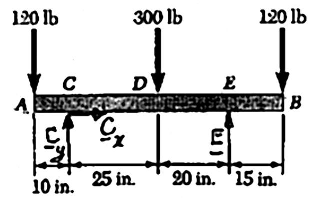


PROBLEM 7.38

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

<u>Free body: Entire beam</u>



$$+\circlearrowright \Sigma M_C = 0: \quad (120\text{ lb})(10\text{ in.}) - (300\text{ lb})(25\text{ in.}) + E(45\text{ in.}) - (120\text{ lb})(60\text{ in.}) = 0$$

$$E = +300\text{ lb} \qquad \mathbf{E} = 300\text{ lb}\uparrow \triangleleft$$

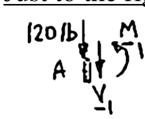
$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 300\text{ lb} - 120\text{ lb} - 300\text{ lb} - 120\text{ lb} = 0$$

$$C_y = +240\text{ lb} \qquad \mathbf{C} = 240\text{ lb}\uparrow \triangleleft$$

(*a*) <u>Shear and bending moment</u>

<u>Just to the right of *A*:</u>



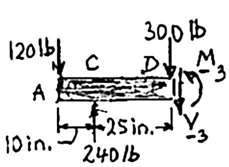
$$+\uparrow \Sigma F_y = 0: \quad -120\text{ lb} - V_1 = 0 \qquad V_1 = -120\text{ lb}, \quad M_1 = 0 \triangleleft$$

<u>Just to the right of *C*:</u>

$$+\uparrow \Sigma F_y = 0: \quad 240\text{ lb} - 120\text{ lb} - V_2 = 0 \qquad V_2 = +120\text{ lb} \triangleleft$$

$$+\circlearrowright \Sigma M_C = 0: \quad M_2 + (120\text{ lb})(10\text{ in.}) = 0 \qquad M_2 = -1200\text{ lb}\cdot\text{in.} \triangleleft$$

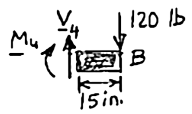
<u>Just to the right of *D*:</u>



$$+\uparrow \Sigma F_y = 0: \quad 240 - 120 - 300 - V_3 = 0 \qquad V_3 = -180\text{ lb} \triangleleft$$

$$+\circlearrowright \Sigma M_3 = 0: \quad M_3 + (120)(35) - (240)(25) = 0, \qquad M_3 = +1800\text{ lb}\cdot\text{in.} \triangleleft$$

Just to the right of E:

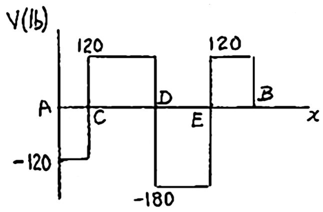


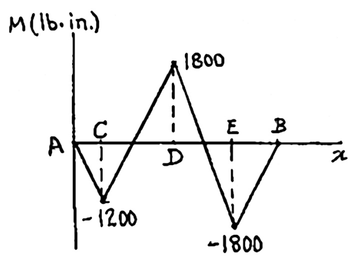
$+\Sigma F_y = 0$: $V_4 - 120 \text{ lb} = 0$ $\qquad V_4 = +120 \text{ lb}$ ◁

$+\circlearrowleft \Sigma M_4 = 0$: $-M_4 - (120 \text{ lb})(15 \text{ in.}) = 0$ $\qquad M_4 = -1800 \text{ lb} \cdot \text{in.}$ ◁

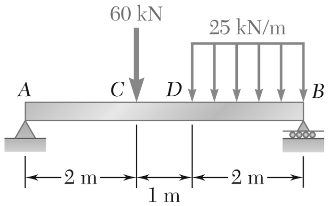
At B: $\qquad V_B = M_B = 0$ ◁

(*b*) $\qquad |V|_{max} = 180.0 \text{ lb}$ ◀





$|M|_{max} = 1800 \text{ lb} \cdot \text{in.}$ ◀

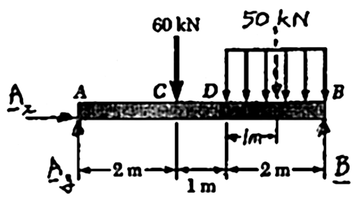


PROBLEM 7.39

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

<u>Free body: Entire beam</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad B(5\text{ m}) - (60\text{ kN})(2\text{ m}) - (50\text{ kN})(4\text{ m}) = 0$$

$B = +64.0\text{ kN}$ $\qquad$ $\mathbf{B} = 64.0\text{ kN}\uparrow$ ◁

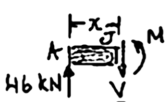
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 64.0\text{ kN} - 6.0\text{ kN} - 50\text{ kN} = 0$$

$A_y = +46.0\text{ kN}$ $\qquad$ $\mathbf{A} = 46.0\text{ kN}\uparrow$ ◁

(*a*) <u>Shear and bending-moment diagrams</u>.

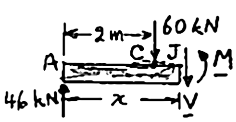
<u>From *A* to *C*:</u>



$+\uparrow \Sigma F_y = 0: \quad 46 - V = 0$ $\qquad$ $V = +46\text{ kN}$ ◁

$+\circlearrowleft \Sigma M_y = 0: \quad M - 46x = 0$ $\qquad$ $M = (46x)\text{kN}\cdot\text{m}$ ◁

<u>From *C* to *D*:</u>



$+\uparrow \Sigma F_y = 0: \quad 46 - 60 - V = 0$ $\qquad$ $V = -14\text{ kN}$ ◁

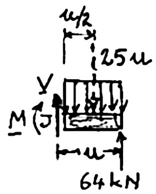
$$+\circlearrowleft \Sigma M_j = 0: \quad M - 46x + 60(x-2) = 0$$

$$M = (120 - 14x)\text{kN}\cdot\text{m}$$

For $\qquad x = 2\text{ m}: \quad M_C = +92.0\text{ kN}\cdot\text{m}$ ◁

For $\qquad x = 3\text{ m}: \quad M_D = +78.0\text{ kN}\cdot\text{m}$ ◁

From D to B:



$$+\uparrow \Sigma F_y = 0: \quad V + 64 - 25\mu = 0$$

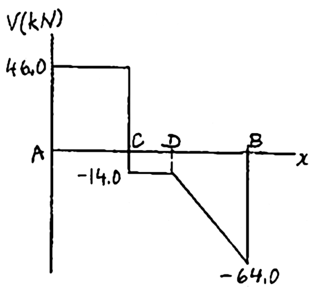
$$V = (25\mu - 64)\text{kN}$$

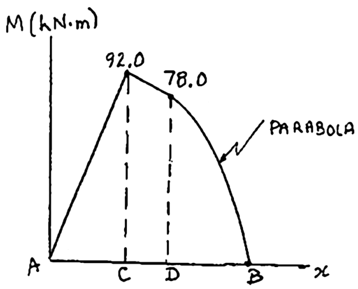
$$+\circlearrowleft \Sigma M_j = 0: \quad 64\mu - (25\mu)\left(\frac{\mu}{2}\right) - M = 0$$

$$M = (64\mu - 12.5\mu^2)\text{kN}\cdot\text{m}$$

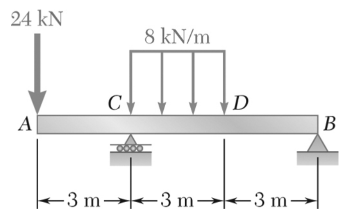
For $\quad \mu = 0: \quad V_B = -64 \text{ kN} \qquad M_B = 0$ ◁

(*b*) $\quad |V|_{max} = 64.0 \text{ kN}$ ◀





$|M|_{max} = 92.0 \text{ kN}\cdot\text{m}$ ◀

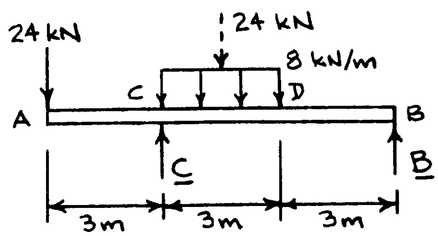


PROBLEM 7.40

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam

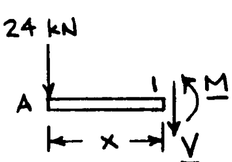


$$+\circlearrowleft \Sigma M_B = 0: \quad (24\text{ kN})(9\text{ m}) - C(6\text{ m}) + (24\text{ kN})(4.5\text{ m}) = 0 \qquad \mathbf{C} = 54\text{ kN}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 54 - 24 - 24 + B = 0$$

$$B = -6\text{ kN} \qquad \mathbf{B} = 6\text{ kN}\downarrow$$

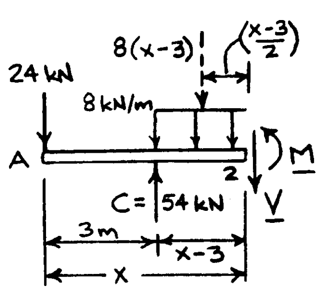
From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad -24 - V = 0 \qquad V = -24\text{ kN}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad (24)(x) + M = 0 \qquad M = (-24x)\text{kN}\cdot\text{m}$$

From *C* to *D*:



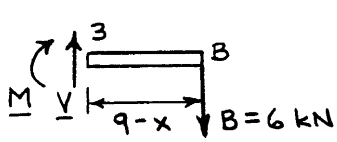
$$+\uparrow \Sigma F_y = 0: \quad -24 - 8(x-3) - V + 54 = 0$$

$$V = (-8x + 54)\text{kN}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad (24)(x) + 8(x-3)\left(\frac{x-3}{2}\right) - (54)(x-3) + M = 0$$

$$M = (-4x^2 + 54x - 198)\text{kN}\cdot\text{m}$$

From *D* to *B*:

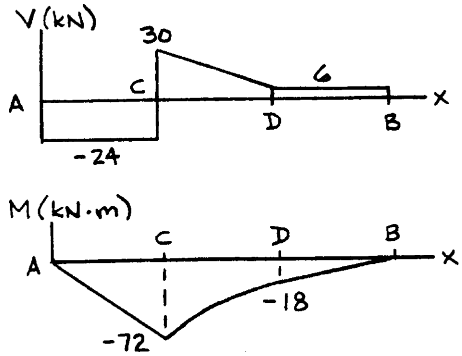


$$+\uparrow \Sigma F_y = 0: \quad V - 6 = 0 \qquad V = +6\text{ kN}$$

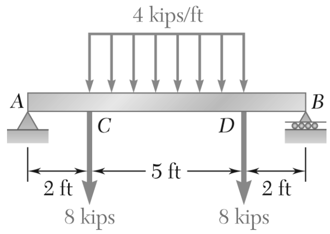
$$+\circlearrowleft \Sigma M_3 = 0: \quad -M - (6)(9-x) = 0 \qquad M = (6x - 54)\text{kN}\cdot\text{m}$$

PROBLEM 7.40 (Continued)

(*b*) $|V|_{max} = 30.0 \text{ kN}$ ◀



$|M|_{max} = 72.0 \text{ kN}\cdot\text{m}$ ◀

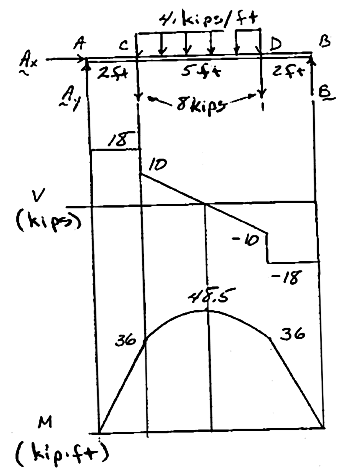


PROBLEM 7.41

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

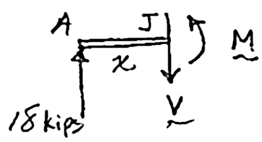
SOLUTION

(*a*) By symmetry:



$$A_y = B = 8\text{ kips} + \frac{1}{2}(4\text{ kips})(5\text{ ft}) \quad \mathbf{A}_y = \mathbf{B} = 18\text{ kips}\uparrow$$

Along AC:

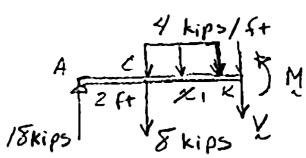


$$+\uparrow \Sigma F_y = 0: \quad 18\text{ kips} - V = 0 \quad V = 18\text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - x(18\text{ kips}) \quad M = (18\text{ kips})x$$

$$M = 36\text{ kip}\cdot\text{ft at } C(x = 2\text{ ft})$$

Along CD:



$$+\uparrow \Sigma F_y = 0: \quad 18\text{ kips} - 8\text{ kips} - (4\text{ kips/ft})x_1 - V = 0$$

$$V = 10\text{ kips} - (4\text{ kips/ft})x_1$$

$$V = 0 \text{ at } x_1 = 2.5\text{ ft (at center)}$$

$$+\circlearrowleft \Sigma M_K = 0: \quad M + \frac{x_1}{2}(4\text{ kips/ft})x_1 + (8\text{ kips})x_1 - (2\text{ ft} + x_1)(18\text{ kips}) = 0$$

$$M = 36\text{ kip}\cdot\text{ft} + (10\text{ kips/ft})x_1 - (2\text{ kips/ft})x_1^2$$

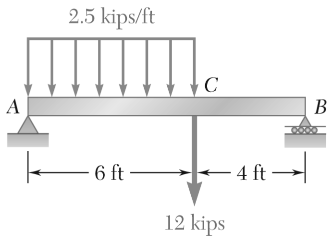
$$M = 48.5\text{ kip}\cdot\text{ft} \quad \text{at} \quad x_1 = 2.5\text{ ft}$$

Complete diagram by symmetry

(*b*) From diagrams:

$$|V|_{max} = 18.00\text{ kips} \blacktriangleleft$$

$$|M|_{max} = 48.5\text{ kip}\cdot\text{ft} \blacktriangleleft$$

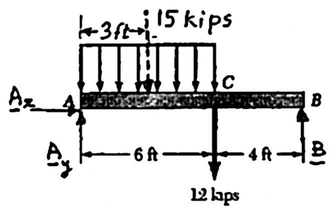


PROBLEM 7.42

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(10\text{ ft}) - (15\text{ kips})(3\text{ ft}) - (12\text{ kips})(6\text{ ft}) = 0$$

$$B = +11.70\text{ kips} \qquad \mathbf{B} = 11.70\text{ kips}\uparrow \triangleleft$$

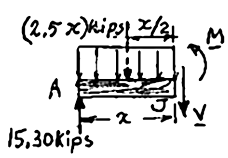
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 15 - 12 + 11.70 = 0$$

$$A_y = +15.30\text{ kips} \qquad \mathbf{A} = 15.30\text{ kips}\uparrow \triangleleft$$

(*a*) Shear and bending-moment diagrams

From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad 15.30 - 2.5x - V = 0$$

$$V = (15.30 - 2.5x)\text{ kips}$$

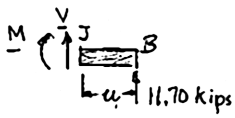
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (2.5x)\left(\frac{x}{2}\right) - 15.30x = 0$$

$$M = 15.30x - 1.25x^2$$

For $x = 0$: $\quad V_A = +15.30\text{ kips} \qquad M_A = 0 \triangleleft$

For $x = 6\text{ ft}$: $\quad V_C = +0.300\text{ kip} \qquad M_C = +46.8\text{ kip}\cdot\text{ft} \triangleleft$

From C to B:



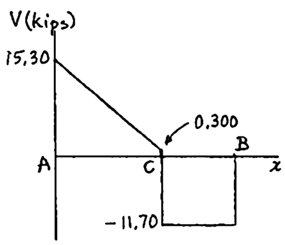
$+\uparrow \Sigma F_y = 0\colon \quad V + 11.70 = 0 \qquad V = -11.70 \text{ kips} \triangleleft$

$+\circlearrowleft \Sigma M_J = 0\colon \quad 11.70\mu - M = 0$

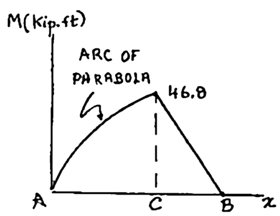
$$M = (11.70\mu) \text{ kip} \cdot \text{ft}$$

For $\mu = 4$ ft: $\qquad M_C = +46.8 \text{ kip} \cdot \text{ft} \triangleleft$

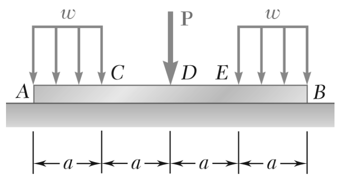
For $\mu = 0$: $\qquad M_B = 0 \triangleleft$



(*b*) $\qquad |V|_{\max} = 15.30 \text{ kips} \blacktriangleleft$



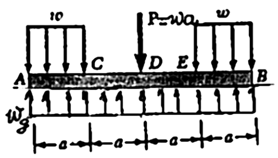
$|M|_{\max} = 46.8 \text{ kip} \cdot \text{ft} \blacktriangleleft$



PROBLEM 7.43

Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $P = wa$, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



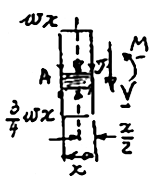
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - wa = 0$$

$$w_g = \frac{3}{4}w \quad \triangleleft$$

(a) Shear and bending-moment diagrams

From A to C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wx - V = 0$$

$$V = -\frac{1}{4}wx$$

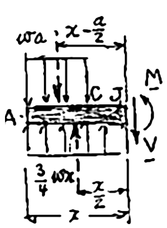
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (wx)\frac{x}{2} - \left(\frac{3}{4}wx\right)\frac{x}{2} = 0$$

$$M = -\frac{1}{8}wx^2$$

For $x = 0$: $\qquad V_A = M_A = 0 \quad \triangleleft$

For $x = a$: $\qquad V_C = -\frac{1}{4}wa \qquad M_C = -\frac{1}{8}wa^2 \quad \triangleleft$

From C to D:



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{4}wx - wa - V = 0$$

$$V = \left(\frac{3}{4}x - a\right)w$$

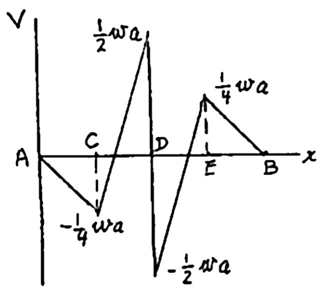
$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \frac{3}{4}wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{3}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \qquad (1)$$

For $x = a$: $\quad V_C = -\dfrac{1}{4}wa \qquad M_C = -\dfrac{1}{8}wa^2$ ◁

For $x = 2a$: $\quad V_D = +\dfrac{1}{2}wa \qquad M_D = 0$ ◁

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.

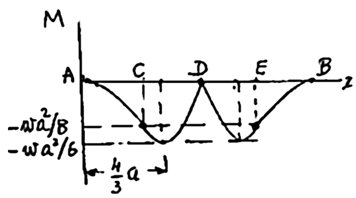


(*b*) $\qquad |V|_{\max} = \dfrac{1}{2}wa$ ◀

To find $|M|_{\max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

$$\frac{dM}{dx} = \frac{3}{4}wx - wa = 0, \quad x = \frac{4}{3}a$$

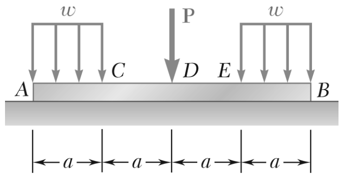
$$M = \frac{3}{8}w\left(\frac{4}{3}a\right)^2 - wa^2\left(\frac{4}{3} - \frac{1}{2}\right) = -\frac{wa^2}{6}$$



$$|M|_{\max} = \frac{1}{6}wa^2 \ ◀$$

Bending-moment diagram consists of four distinct arcs of parabola.

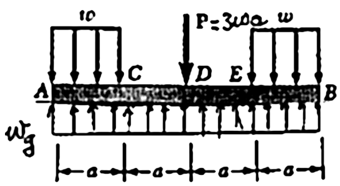
PROBLEM 7.44



Solve Problem 7.43 knowing that $P = 3wa$.

PROBLEM 7.43 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $P = wa$, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



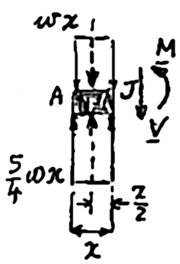
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - 3wa = 0$$

$$w_g = \frac{5}{4}w \quad \triangleleft$$

(*a*) Shear and bending-moment diagrams

From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{4}wx - wx - V = 0$$

$$V = +\frac{1}{4}wx$$

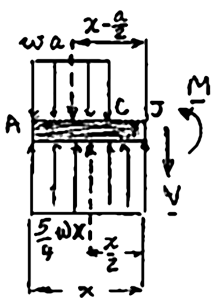
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (wx)\frac{x}{2} - \left(\frac{5}{4}wx\right)\frac{x}{2} = 0$$

$$M = +\frac{1}{8}wx^2$$

For $x = 0$: $\qquad V_A = M_A = 0 \quad \triangleleft$

For $x = a$: $\qquad V_C = +\frac{1}{4}wa \qquad M_C = +\frac{1}{8}wa^2 \quad \triangleleft$

From *C* to *D*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{4}wx - wa - V = 0$$

$$V = \left(\frac{5}{4}x - a\right)w$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \frac{5}{4}wx\left(\frac{x}{2}\right) = 0$$

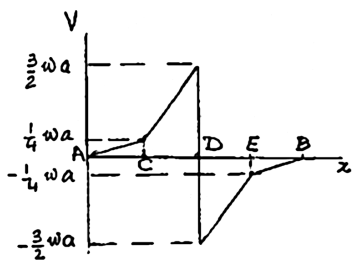
$$M = \frac{5}{8}wx^2 - wa\left(x - \frac{a}{2}\right) \qquad (1)$$

PROBLEM 7.44 (Continued)

For $x = a$: $V_C = +\frac{1}{4}wa,\ \ M_C = +\frac{1}{8}wa^2$ ◁

For $x = 2a$: $V_D = +\frac{3}{2}wa,\ \ M_D = +wa^2$ ◁

Because of the symmetry of the loading, we can deduce the values of V and M for the right-hand half of the beam from the values obtained for its left-hand half.

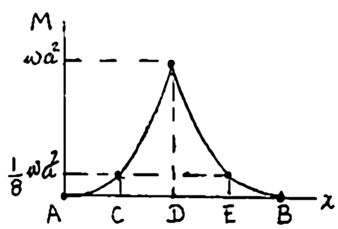


(*b*) $|V|_{max} = \frac{3}{2}wa$ ◀

To find $|M|_{max}$, we differentiate Eq. (1) and set $\frac{dM}{dx} = 0$:

$$\frac{dM}{dx} = \frac{5}{4}wx - wa = 0$$

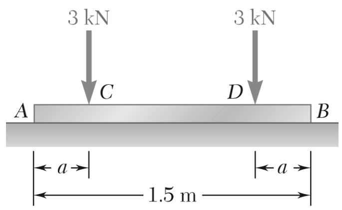
$$x = \frac{4}{5}a < a \quad \text{(outside portion } CD\text{)}$$



The maximum value of $|M|$ occurs at D:

$|M|_{max} = wa^2$ ◀

Bending-moment diagram consists of four distinct arcs of parabola.



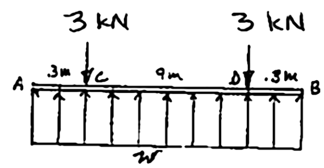
PROBLEM 7.45

Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.3$ m, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

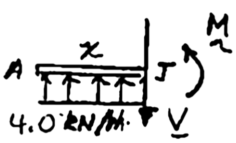
SOLUTION

(a) **FBD Beam:** $\uparrow \Sigma F_y = 0$: $w(1.5\text{ m}) - 2(3.0\text{ kN}) = 0$

$$w = 4.0\text{ kN/m}$$



Along AC:

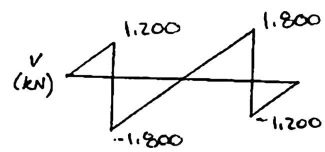


$\uparrow \Sigma F_y = 0$: $(4.0\text{ kN/m})x - V = 0$

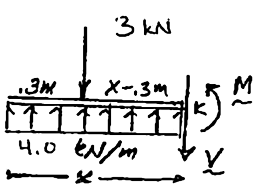
$$V = (4.0\text{ kN/m})x$$

$\circlearrowleft \Sigma M_J = 0$: $M - \dfrac{x}{2}(4.0\text{ kN/m})x = 0$

$$M = (2.0\text{ kN/m})x^2$$



Along CD:

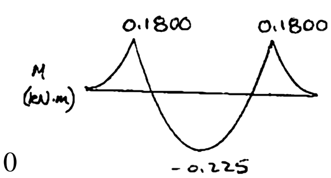


$\uparrow \Sigma F_y = 0$: $(4.0\text{ kN/m})x - 3.0\text{ kN} - V = 0$

$$V = (4.0\text{ kN/m})x - 3.0\text{ kN}$$

$\circlearrowleft \Sigma M_K = 0$: $M + (x - 0.3\text{ m})(3.0\text{ kN}) - \dfrac{x}{2}(4.0\text{ kN/m})x = 0$

$$M = 0.9\text{ kN}\cdot\text{m} - (3.0\text{ kN})x + (2.0\text{ kN/m})x^2$$



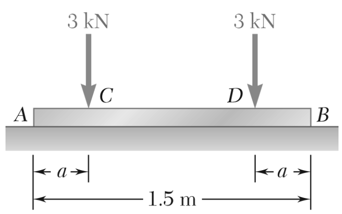
Note: $V = 0$ at $x = 0.75$ m, where $M = -0.225\text{ kN}\cdot\text{m}$

Complete diagrams using symmetry.

(b) $|V|_{max} = 1.800\text{ kN}$ ◀

$|M|_{max} = 0.225\text{ kN}\cdot\text{m}$ ◀

PROBLEM 7.46

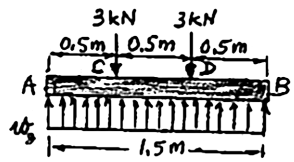


Solve Problem 7.45 knowing that $a = 0.5$ m.

PROBLEM 7.45 Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.3$ m, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

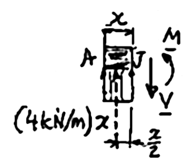
Free body: Entire beam



$$+\uparrow \Sigma F_y = 0: \quad w_g(1.5 \text{ m}) - 3 \text{ kN} - 3 \text{ kN} = 0 \qquad w_g = 4 \text{ kN/m} \ \triangleleft$$

(a) Shear and bending moment

From A to C:



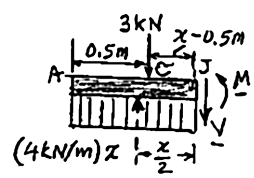
$$+\uparrow \Sigma F_y = 0: \quad 4x - V = 0 \quad V = (4x) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (4x)\frac{x}{2} = 0, \quad M = (2x^2) \text{ kN} \cdot \text{m}$$

For $x = 0$: $\qquad V_A = M_A = 0 \ \triangleleft$

For $x = 0.5$ m: $\qquad V_C = 2$ kN, $\qquad M_C = 0.500 \text{ kN} \cdot \text{m} \ \triangleleft$

From C to D: $\qquad +\uparrow \Sigma F_y = 0: \quad 4x - 3 \text{ kN} - V = 0$



$$V = (4x - 3) \text{ kN}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (3 \text{ kN})(x - 0.5) - (4x)\frac{x}{2} = 0$$

$$M = (2x^2 - 3x + 1.5) \text{ kN} \cdot \text{m}$$

For $x = 0.5$ m: $\qquad V_C = -1.00$ kN, $\qquad M_C = 0.500 \text{ kN} \cdot \text{m} \ \triangleleft$

For $x = 0.75$ m: $\qquad V_C = 0$, $\qquad M_C = 0.375 \text{ kN} \cdot \text{m} \ \triangleleft$

For $x = 1.0$ m: $\qquad V_C = 1.00$ kN, $\qquad M_C = 0.500 \text{ kN} \cdot \text{m} \ \triangleleft$

PROBLEM 7.46 (Continued)

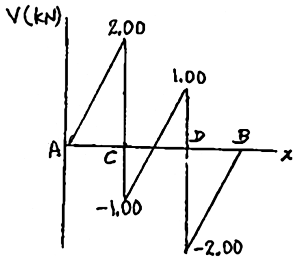
From D to B:

$+\uparrow \Sigma F_y = 0$: $V + 4\mu = 0 \quad V = -(4\mu)$ kN

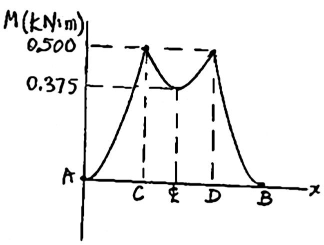
$+\circlearrowleft \Sigma M_J = 0$: $(4\mu)\dfrac{\mu}{2} - M = 0, \quad M = 2\mu^2$

For $\mu = 0$: $V_B = M_B = 0$ ◁

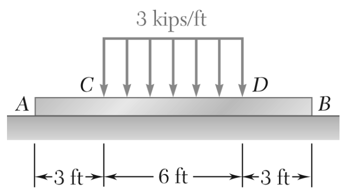
For $\mu = 0.5$ m: $V_D = -2$ kN, $M_D = 0.500$ kN·m ◁



(*b*) $|V|_{max} = 2.00$ kN ◀



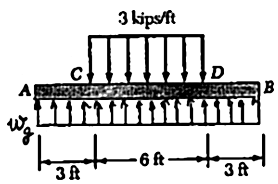
$|M|_{max} = 0.500$ kN·m ◀

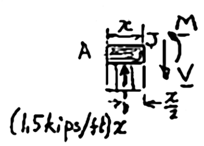


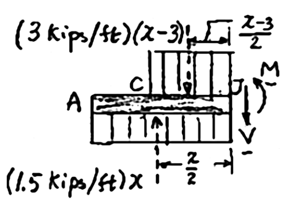
PROBLEM 7.47

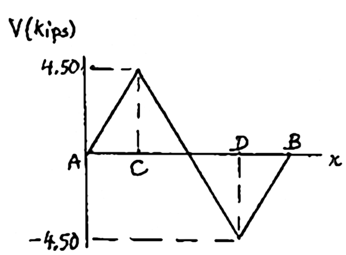
Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

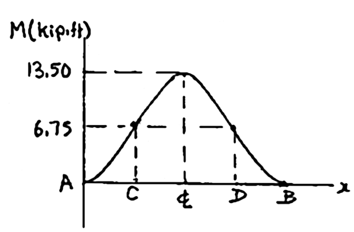
SOLUTION











Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad w_g(12 \text{ ft}) - (3 \text{ kips/ft})(6 \text{ ft}) = 0$$

$$w_g = 1.5 \text{ kips/ft} \quad \triangleleft$$

(a) Shear and bending-moment diagrams from A to C:

$$+\uparrow \Sigma F_y = 0: \quad 1.5x - V = 0 \qquad V = (1.5x)\text{kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M - (1.5x)\frac{x}{2} \qquad M = (0.75x^2)\text{kip}\cdot\text{ft}$$

For $x = 0$: $\qquad V_A = M_A = 0 \quad \triangleleft$

For $x = 3$ ft: $\quad V_C = 4.5$ kips, $\qquad M_C = 6.75 \text{ kip}\cdot\text{ft} \quad \triangleleft$

From C to D:

$$+\uparrow \Sigma F_y = 0: \quad 1.5x - 3(x-3) - V = 0$$

$$V = (9 - 1.5x)\text{kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + 3(x-3)\frac{x-3}{2} - (1.5x)\frac{x}{2} = 0$$

$$M = [0.75x^2 - 1.5(x-3)^2]\text{kip}\cdot\text{ft}$$

For $x = 3$ ft: $\quad V_C = 4.5$ kips, $\qquad M_C = 6.75 \text{ kip}\cdot\text{ft} \quad \triangleleft$

For $x = 6$ ft: $\quad V_{℄} = 0$, $\qquad M_{℄} = 13.50 \text{ kip}\cdot\text{ft} \quad \triangleleft$

For $x = 9$ ft: $\quad V_D = -4.5$ kips, $\qquad M_D = 6.75 \text{ kip}\cdot\text{ft} \quad \triangleleft$

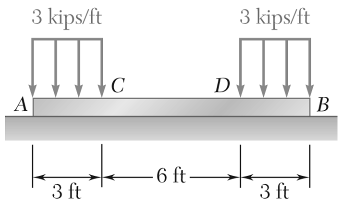
At B: $\qquad V_B = M_B = 0 \quad \triangleleft$

(b) $\qquad |V|_{max} = 4.50 \text{ kips} \quad \blacktriangleleft$

$$|M|_{max} = 13.50 \text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$

Bending-moment diagram consists or three distinct arcs of parabola, all located above the x axis.

Thus: $M \geq 0$ everywhere $\blacktriangleleft$

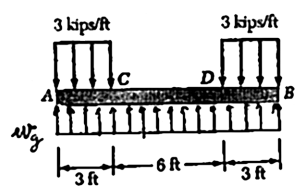


PROBLEM 7.48

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

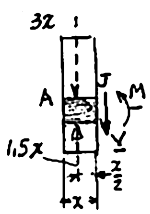
Free body: Entire beam



$$+\uparrow \Sigma F_y = 0: \quad w_g(12\text{ ft}) - (3\text{ kips/ft})(6\text{ ft}) = 0 \qquad w_g = 1.5\text{ kips/ft} \ \triangleleft$$

(*a*) Shear and bending-moment diagrams.

From *A* to *C*:



$$+\uparrow \Sigma F_y = 0: \quad 1.5x - 3x - V = 0$$

$$V = (-1.5x)\text{ kips}$$

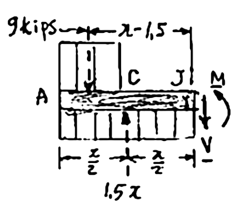
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (3x)\frac{x}{2} - (1.5x)\frac{x}{2} = 0$$

$$M = (-0.75x^2)\text{ kip}\cdot\text{ft}$$

For $x = 0$: $\quad V_A = M_A = 0 \ \triangleleft$

For $x = 3$ ft: $\quad V_C = -4.5\text{ kips} \quad M_C = -6.75\text{ kip}\cdot\text{ft} \ \triangleleft$

From *C* to *D*:



$$+\uparrow \Sigma F_y = 0: \quad 1.5x = 9 - V = 0, \quad V = (1.5x - 9)\text{ kips}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + 9(x - 1.5) - (1.5x)\frac{x}{2} = 0$$

$$M = 0.75x^2 - 9x + 13.5$$

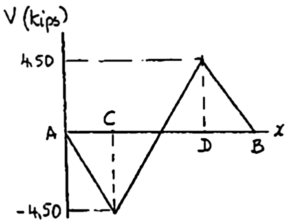
For $x = 3$ ft: $\quad V_C = -4.5\text{ kips}, \quad M_C = -6.75\text{ kip}\cdot\text{ft} \ \triangleleft$

For $x = 6$ ft: $\quad V_C = 0, \quad M_C = -13.50\text{ kip}\cdot\text{ft} \ \triangleleft$

PROBLEM 7.48 (Continued)

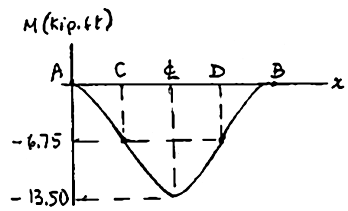
For $x = 9$ ft: $V_D = 4.5$ kips, $M_D = -6.75$ kip · ft ◁

At B: $V_B = M_B = 0$ ◁



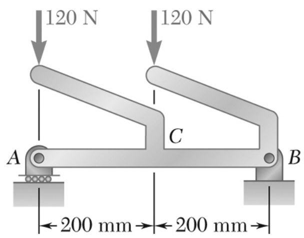
(b) $|V|_{max} = 4.50$ kips ◀

Bending-moment diagram consists of three distinct arcs of parabola.



$|M|_{max} = 13.50$ kip · ft ◀

Since entire diagram is below the x axis: $M \leq 0$ everywhere ◀

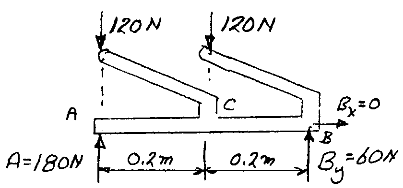


PROBLEM 7.49

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

<u>Reactions</u>:



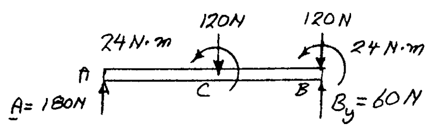
$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(0.4) - (120)(0.2) = 0$$

$$\mathbf{B}_y = 60 \text{ N} \uparrow$$

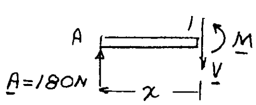
$$\Sigma F_x = 0: \qquad \mathbf{B}_x = 0$$

$$\Sigma F_y = 0: \qquad \mathbf{A} = 180 \text{ N} \uparrow$$

Equivalent loading on straight part of beam *AB*.



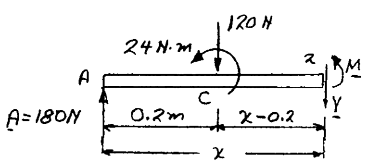
<u>From *A* to *C*</u>:



$$+\uparrow \Sigma F_y = 0: \quad V = +180 \text{ N}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad M = +180x$$

<u>From *C* to *B*</u>:

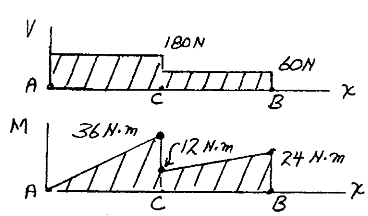


$$+\uparrow \Sigma F_y = 0: \quad 180 - 120 - V = 0$$

$$V = 60 \text{ N}$$

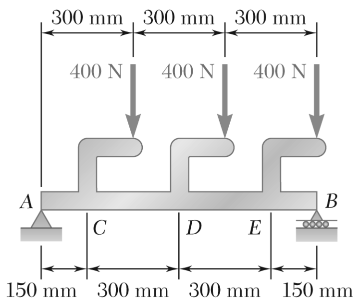
$$+\circlearrowleft \Sigma M_x = 0: \quad -(180 \text{ N})(x) + 24 \text{ N} \cdot \text{m} + (120 \text{ N})(x - 0.2) + M = 0$$

$$M = +60x$$



$$|V|_{\max} = 180.0 \text{ N} \blacktriangleleft$$

$$|M|_{\max} = 36.0 \text{ N} \cdot \text{m} \blacktriangleleft$$

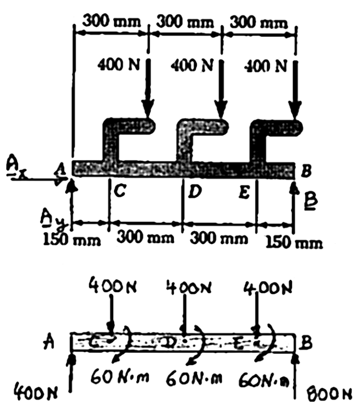


PROBLEM 7.50

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad B(0.9\text{ m}) - (400\text{ N})(0.3\text{ m}) - (400\text{ N})(0.6\text{ m}) - (400\text{ N})(0.9\text{ m}) = 0$$

$$B = +800\text{ N} \qquad \mathbf{B} = 800\text{ N}\uparrow \triangleleft$$

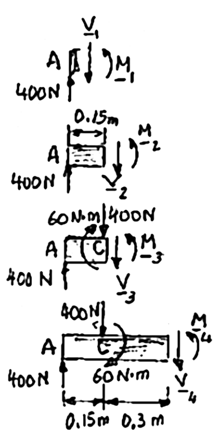
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 800\text{ N} - 3(400\text{ N}) = 0$$

$$A_y = +400\text{ N} \qquad \mathbf{A} = 400\text{ N}\uparrow \triangleleft$$

We replace the loads by equivalent force-couple systems at *C*, *D*, and *E*.

We consider successively the following *F-B* diagrams.



$V_1 = +400\text{ N}$ $\quad M_1 = 0$

$V_2 = +400\text{ N}$ $\quad M_2 = +60\text{ N}\cdot\text{m}$

$V_3 = 0$ $\quad M_3 = +120\text{ N}\cdot\text{m}$

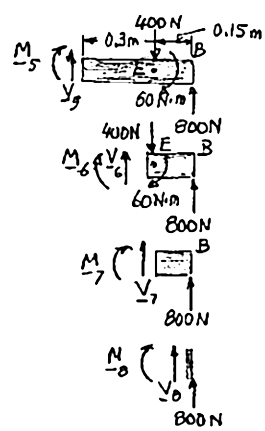
$V_4 = 0$ $\quad M_4 = +120\text{ N}\cdot\text{m}$

$V_5 = -400\text{ N}$ $\quad M_5 = +180\text{ N}\cdot\text{m}$

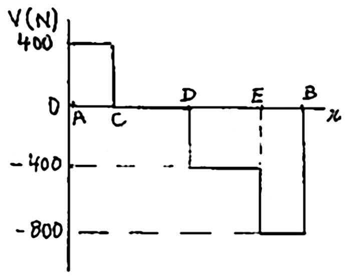
$V_6 = -400\text{ N}$ $\quad M_6 = +60\text{ N}\cdot\text{m}$

$V_7 = -800\text{ N}$ $\quad M_7 = +120\text{ N}\cdot\text{m}$

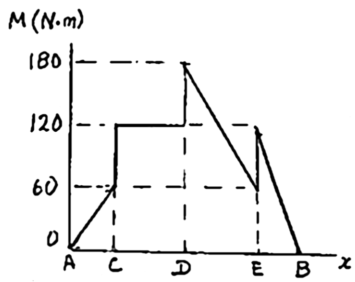
$V_8 = -800\text{ N}$ $\quad M_8 = 0$



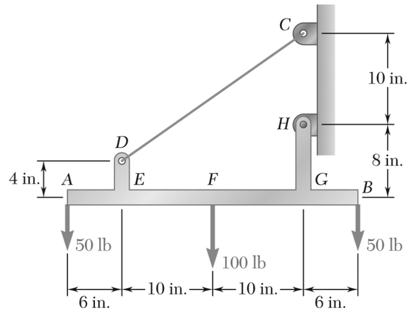
PROBLEM 7.50 (Continued)



(*b*) $|V|_{max} = 800\text{ N}$ ◀



$|M|_{max} = 180.0\text{ N}\cdot\text{m}$ ◀

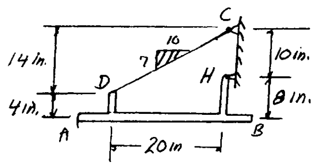


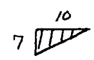
PROBLEM 7.51

Draw the shear and bending-moment diagrams for the beam AB, and determine the maximum absolute values of the shear and bending moment.

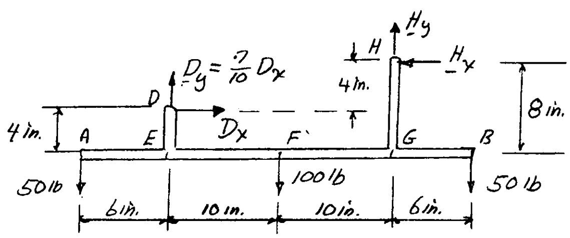
SOLUTION

Slope of cable CD is





$$\therefore \frac{D_y}{7} = \frac{D_x}{10}$$



$$+\circlearrowleft \Sigma M_H = 0: \quad (50 \text{ lb})(26 \text{ in.}) + D_x(4 \text{ in.}) - \left(\frac{7}{10}D_x\right)(20 \text{ in.}) + (100 \text{ lb})(10 \text{ in.}) - (50 \text{ lb})(6 \text{ in.}) = 0$$

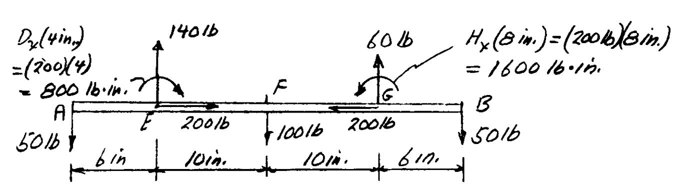
$$2000 + (4 - 14)D_x = 0 \qquad \mathbf{D}_x = 200 \text{ lb} \longrightarrow$$

$$D_y = \frac{7}{10}D_x = \frac{7}{10}(200) \qquad \mathbf{D}_y = 140 \text{ lb} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 200 \text{ lb} - H_x = 0 \qquad H_x = 200 \text{ lb} \longleftarrow$$

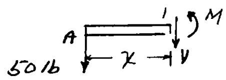
$$+\uparrow \Sigma F_y = 0: \quad 140 - 50 - 100 - 50 + H_y = 0 \qquad H_y = 60 \text{ lb} \uparrow$$

Equivalent loading on straight part of beam AB.



PROBLEM 7.51 (Continued)

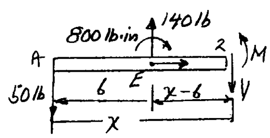
From A to E:



$$\Sigma F_y = 0: \quad V = -50 \text{ lb}$$

$$\Sigma M_1 = 0: \quad M = -50x$$

From E to F:



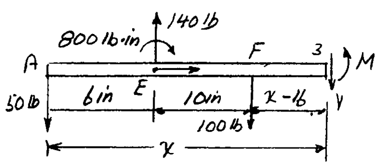
$$+\uparrow \Sigma F_y = 0: \quad -50 + 140 - V = 0$$

$$V = +90 \text{ lb}$$

$$+\circlearrowleft \Sigma M_2 = 0: \quad (50 \text{ lb})x - (140 \text{ lb})(x - 6) - 800 \text{ lb} \cdot \text{in.} + M = 0$$

$$M = -40 + 90x$$

From F to G:



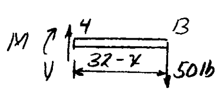
$$+\uparrow \Sigma F_y = 0: \quad -50 + 140 - 100 - V = 0$$

$$V = -10 \text{ lb}$$

$$+\circlearrowleft \Sigma M_3 = 0: \quad 50x - 140(x - 6) + 100(x - 16) - 800 + M = 0$$

$$M = 1560 - 10x$$

From G to B:

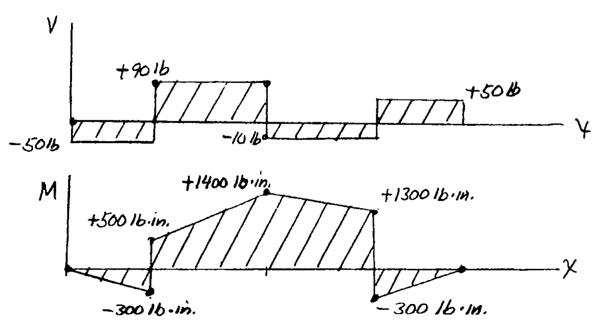


$$+\uparrow \Sigma F_y = 0: \quad V - 50 = 0$$

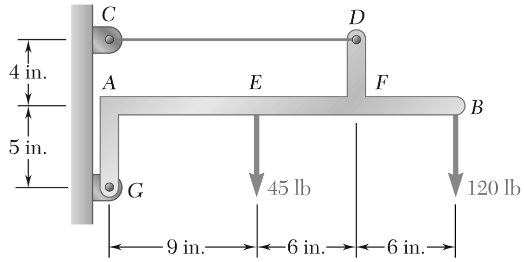
$$V = 50 \text{ lb}$$

$$+\circlearrowleft \Sigma M_4 = 0: \quad -M - (50)(32 - x) = 0$$

$$M = -1600 + 50x$$



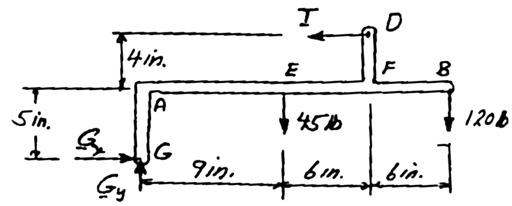
$$|V|_{max} = 90.0 \text{ lb}; \quad |M|_{max} = 1400 \text{ lb} \cdot \text{in.} \blacktriangleleft$$



PROBLEM 7.52

Draw the shear and bending-moment diagrams for the beam AB, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

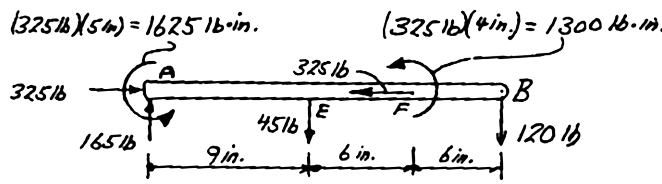


$$+\circlearrowleft \Sigma F_G = 0: \quad T(9\text{ in.}) - (45\text{ lb})(9\text{ in.}) - (120\text{ lb})(21\text{ in.}) = 0 \qquad T = 325\text{ lb}$$

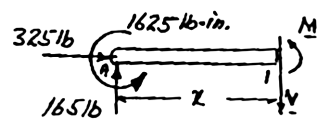
$$\xrightarrow{+} \Sigma F_x = 0: \quad -325\text{ lb} + G_x = 0 \qquad \mathbf{G}_x = 325\text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad G_y - 45\text{ lb} - 120\text{ lb} = 0 \qquad \mathbf{G}_y = 165\text{ lb}\uparrow$$

Equivalent loading on straight part of beam AB



From A to E:

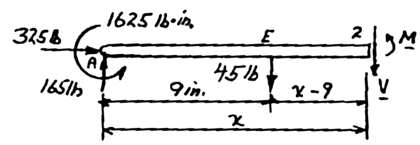


$$\Sigma F_y = 0: \quad V = +165\text{ lb}$$

$$+\circlearrowleft \Sigma M_1 = 0: \quad +1625\text{ lb}\cdot\text{in.} - (165\text{ lb})x + M = 0$$

$$M = -1625 + 165x$$

From E to F:



$$+\uparrow \Sigma F_y = 0: \quad 165 - 45 - V = 0$$

$$V = +120\text{ lb}$$

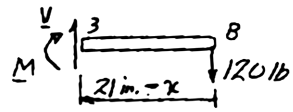
$$+\circlearrowleft \Sigma M_2 = 0: \quad +1625\text{ lb}\cdot\text{in.} - (165\text{ lb})x + (45\text{ lb})(x-9) + M = 0$$

$$M = -1220 + 120x$$

PROBLEM 7.52 (Continued)

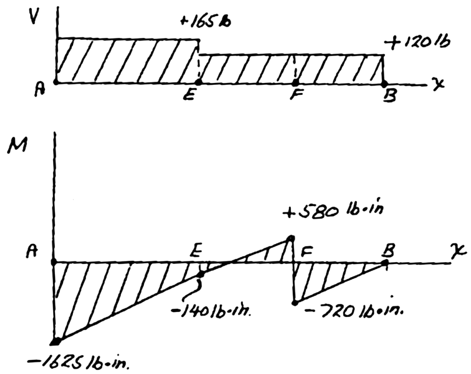
From F to B:

$$\Sigma F_y = 0 \quad V = +120\text{ lb}$$



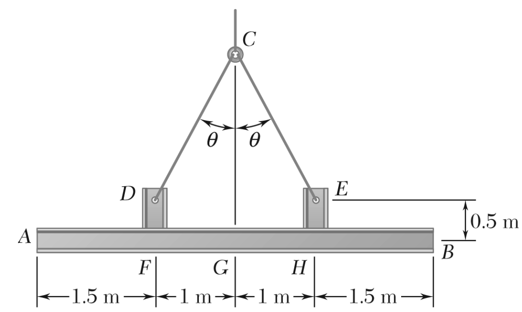
$$+\circlearrowleft \Sigma M_3 = 0: \quad -(120)(21-x) - M = 0$$

$$M = -2520 + 120x$$



$|V|_{max} = 165.0\text{ lb}$ ◀

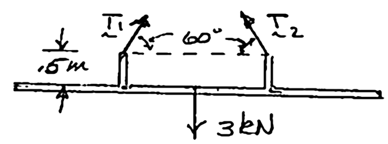
$|M|_{max} = 1625\text{ lb}\cdot\text{in.}$ ◀



PROBLEM 7.53

Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION



FBD Beam + channels:

(*a*) By symmetry: $T_1 = T_2 = T$

$$+\uparrow \Sigma F_y = 0: \quad 2T \sin 60° - 3 \text{ kN} = 0$$

$$T = \frac{3}{\sqrt{3}} \text{ kN}$$

$$T_{1x} = \frac{3}{2\sqrt{3}}$$

$$T_{1y} = \frac{3}{2} \text{ kN}$$

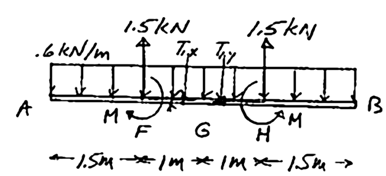
FBD Beam:

$$M = (0.5 \text{ m})\frac{3}{2\sqrt{3}} \text{ kN}$$
$$= 0.433 \text{ kN} \cdot \text{m}$$

With cable force replaced by equivalent force-couple system at *F* and *G*

Shear Diagram: *V* is piecewise linear

$$\left(\frac{dV}{dx} = -0.6 \text{ kN/m}\right) \text{with } 1.5 \text{ kN}$$



discontinuities at *F* and *H*.

$$V_{F^-} = -(0.6 \text{ kN/m})(1.5 \text{ m}) = 0.9 \text{ kN}$$

V increases by 1.5 kN to $+0.6$ kN at F^+

$$V_G = 0.6 \text{ kN} - (0.6 \text{ kN/m})(1 \text{ m}) = 0$$

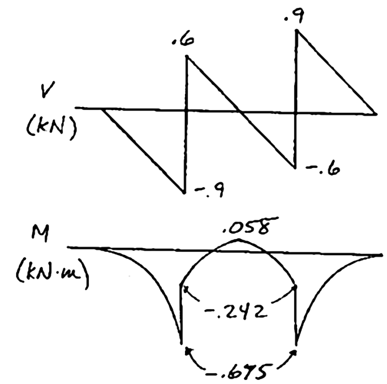
Finish by invoking symmetry

PROBLEM 7.53 (Continued)

Moment diagram: M is piecewise parabolic

$$\left(\frac{dM}{dx} \text{ decreasing with } V\right)$$

with discontinuities of .433 kN at F and H.



$$M_{F^-} = -\frac{1}{2}(0.9 \text{ kN})(1.5 \text{ m})$$
$$= -0.675 \text{ kN} \cdot \text{m}$$

M increases by 0.433 kN m to –0.242 kN · m at F^+

$$M_G = -0.242 \text{ kN} \cdot \text{m} + \frac{1}{2}(0.6 \text{ kN})(1 \text{ m})$$
$$= 0.058 \text{ kN} \cdot \text{m}$$

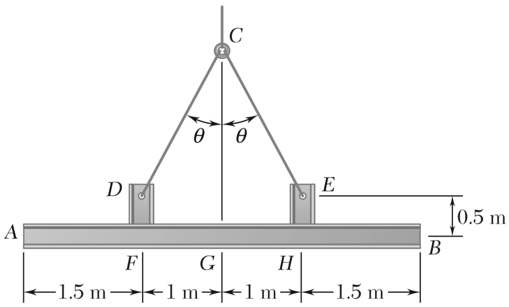
Finish by invoking symmetry

(*b*) $|V|_{max} = 900$ N ◀

at F^- and G^+

$|M|_{max} = 675$ N · m ◀

at F and G



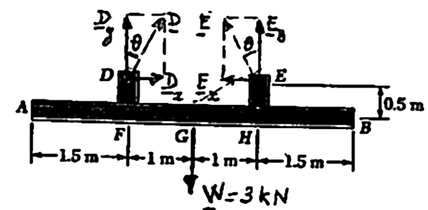
PROBLEM 7.54

Solve Problem 7.53 when $\theta = 60°$.

PROBLEM 7.53 Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

Free body: Beam and channels



From symmetry:

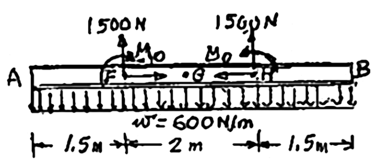
$$E_y = D_y$$

Thus: $$E_x = D_x = D_y \tan\theta \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + E_y - 3 \text{ kN} = 0 \qquad \mathbf{D}_y = \mathbf{E}_y = 1.5 \text{ kN}\uparrow \ \triangleleft$$

From (1): $$\mathbf{D}_x = (1.5 \text{ kN})\tan\theta \longrightarrow \qquad \mathbf{E} = (1.5 \text{ kN})\tan\theta \longleftarrow \ \triangleleft$$

We replace the forces at *D* and *E* by equivalent force-couple systems at *F* and *H*, where



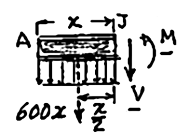
$$M_0 = (1.5 \text{ kN}\tan\theta)(0.5 \text{ m}) = (750 \text{ N}\cdot\text{m})\tan\theta \qquad (2)$$

We note that the weight of the beam per unit length is

$$w = \frac{W}{L} = \frac{3 \text{ kN}}{5 \text{ m}} = 0.6 \text{ kN/m} = 600 \text{ N/m}$$

(*a*) Shear and bending moment diagrams

From *A* to *F*:



$$+\uparrow \Sigma F_y = 0: \quad -V - 600x = 0 \quad V = (-600x) \text{ N}$$

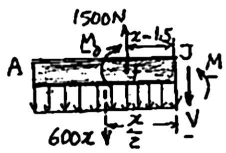
$$+\circlearrowleft \Sigma M_J = 0: \quad M + (600x)\frac{x}{2} = 0, \quad M = (-300x^2) \text{ N}\cdot\text{m}$$

For $x = 0$: $$V_A = M_A = 0 \ \triangleleft$$

For $x = 1.5$ m: $$V_F = -900 \text{ N}, \quad M_F = -675 \text{ N}\cdot\text{m} \ \triangleleft$$

PROBLEM 7.54 (Continued)

<u>From F to H:</u>



$$+\uparrow \Sigma F_y = 0: \quad 1500 - 600x - V = 0$$

$$V = (1500 - 600x)\text{ N}$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (600x)\frac{x}{2} - 1500(x - 1.5) - M_0 = 0$$

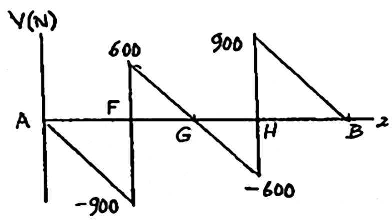
$$M = M_0 - 300x^2 + 1500(x - 1.5)\text{ N}\cdot\text{m}$$

For $x = 1.5$ m: $\quad V_F = +600$ N, $\quad M_F = (M_0 - 675)$ N·m ◁

For $x = 2.5$ m: $\quad V_G = 0, \quad M_G = (M_0 - 375)$ N·m ◁

<u>From G to B</u>, The V and M diagrams will be obtained by symmetry,

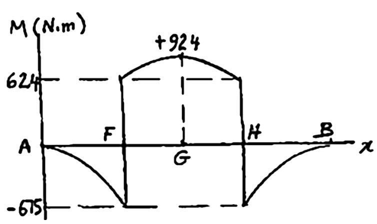
(*b*) $\quad |V|_{max} = 900$ N ◀



Making $\theta = 60°$ in Eq. (2): $\quad M_0 = 750\tan 60° = 1299$ N·m

Thus, just to the right of F: $\quad M = 1299 - 675 = 624$ N·m ◁

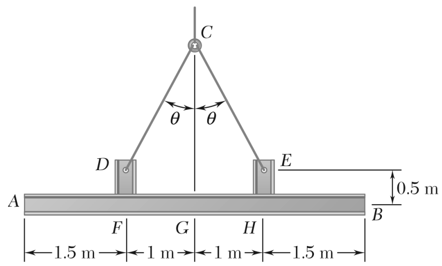
and $\quad M_G = 1299 - 375 = 924$ N·m ◁



(*b*) $\quad |V|_{max} = 900$ N ◀

$|M|_{max} = 924$ N·m ◀

PROBLEM 7.55

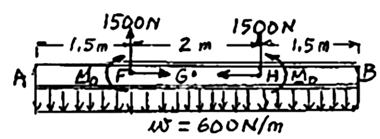


For the structural member of Problem 7.53, determine (*a*) the angle θ for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

PROBLEM 7.53 Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing that $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

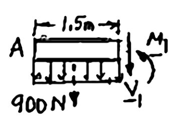
See solution of Problem 7.50 for reduction of loading or beam *AB* to the following:



where $$M_0 = (750\text{ N}\cdot\text{m})\tan\theta \quad \triangleleft$$

[Equation (2)]

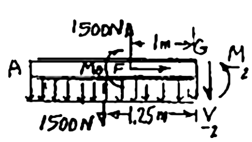
The largest negative bending moment occurs <u>Just to the left of *F*</u>:



$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 + (900\text{ N})\left(\frac{1.5\text{ m}}{2}\right) = 0 \qquad M_1 = -675\text{ N}\cdot\text{m} \quad \triangleleft$$

The largest positive bending moment occurs

<u>At *G*</u>:



$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 - M_0 + (1500\text{ N})(1.25\text{ m} - 1\text{ m}) = 0$$

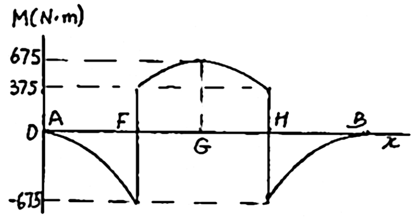
$$M_2 = M_0 - 375\text{ N}\cdot\text{m} \quad \triangleleft$$

Equating M_2 and $-M_1$:

$$M_0 - 375 = +675$$

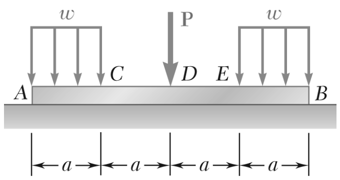
$$M_0 = 1050\text{ N}\cdot\text{m}$$

(*a*) From Equation (2): $\tan\theta = \dfrac{1050}{750} = 1.400$ $\theta = 54.5°$ ◀



(*b*) $|M|_{\max} = 675\ \text{N}\cdot\text{m}$ ◀

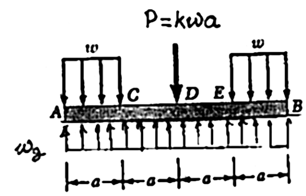
PROBLEM 7.56



For the beam of Problem 7.43, determine (*a*) the ratio $k = P/wa$ for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

PROBLEM 7.43 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that $P = wa$, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Entire beam

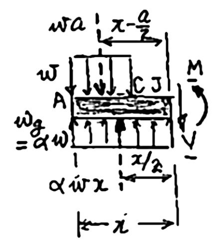
$$+\uparrow \Sigma F_y = 0: \quad w_g(4a) - 2wa - kwa = 0$$

$$w_g = \frac{w}{4}(2+k)$$

Setting $\qquad \dfrac{w_g}{w} = \alpha \qquad (1)$

We have $\qquad k = 4\alpha - 2 \qquad (2)$

Minimum value of B.M. For *M* to have negative values, we must have $w_g < w$. We verify that *M* will then be negative and keep decreasing in the portion *AC* of the beam. Therefore, M_{min} will occur between *C* and *D*.



From *C* to *D*:

$$+\circlearrowleft \Sigma M_J = 0: \quad M + wa\left(x - \frac{a}{2}\right) - \alpha wx\left(\frac{x}{2}\right) = 0$$

$$M = \frac{1}{2}w(\alpha x^2 - 2ax + a^2) \qquad (3)$$

We differentiate and set $\dfrac{dM}{dx} = 0$: $\qquad \alpha x - a = 0 \qquad x_{min} = \dfrac{a}{\alpha} \qquad (4)$

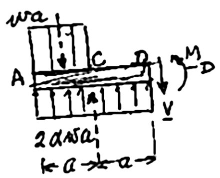
Substituting in (3):

$$M_{min} = \frac{1}{2}wa^2\left(\frac{1}{\alpha} - \frac{2}{\alpha} + 1\right)$$

$$M_{min} = -wa^2\frac{1-\alpha}{2\alpha} \qquad (5)$$

PROBLEM 7.56 (Continued)

Maximum value of bending moment occurs at D



$$+\circlearrowleft \Sigma M_D = 0: \quad M_D + wa\left(\frac{3a}{2}\right) - (2\alpha wa)a = 0$$

$$M_{\max} = M_D = wa^2\left(2\alpha - \frac{3}{2}\right) \qquad (6)$$

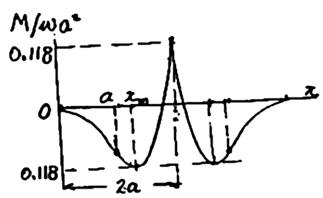
Equating $-M_{\min}$ and $M_{\max}$:

$$wa^2\frac{1-\alpha}{2\alpha} = wa^2\left(2\alpha - \frac{3}{2}\right)$$

$$4\alpha^2 - 2\alpha - 1 = 0$$

$$\alpha = \frac{2+\sqrt{20}}{8} \qquad \alpha = \frac{1+\sqrt{5}}{4} = 0.809$$

(*a*) Substitute in (2): $k = 4(0.809) - 2$ $\qquad k = 1.236$ ◀

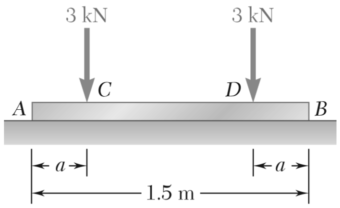


(*b*) Substitute for α in (5):

$$|M|_{\max} = -M_{\min} = -wa^2\frac{1-0.809}{2(0.809)} \qquad |M|_{\max} = 0.1180wa^2 \blacktriangleleft$$

Substitute for α in (4): $\qquad x_{\min} = \frac{a}{0.809}1.236a \triangleleft$

PROBLEM 7.57

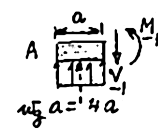


For the beam of Problem 7.45, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

PROBLEM 7.45 Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed and knowing that *a* = 0.3 m, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Force per unit length exerted by ground:

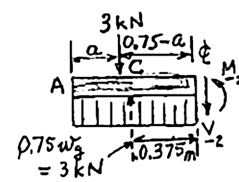


$$w_g = \frac{6\text{ kN}}{1.5\text{ m}} = 4\text{ kN/m}$$

The largest positive bending moment occurs Just to the left of *C*:

$$+\circlearrowleft \Sigma M_1 = 0: \quad M_1 = (4a)\frac{a}{2} \qquad M_1 = 2a^2 \;\triangleleft$$

The largest negative bending moment occurs



At the center line:

$$+\circlearrowleft \Sigma M_2 = 0: \quad M_2 + 3(0.75 - a) - 3(0.375) = 0 \qquad M_2 = -(1.125 - 3a) \;\triangleleft$$

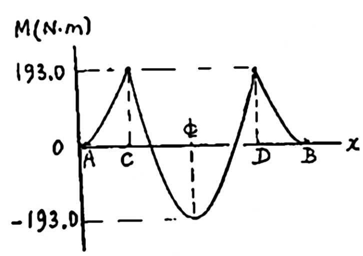
Equating M_1 and $-M_2$:

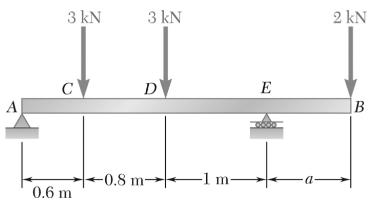
$$2a^2 = 1.125 - 3a$$

$$a^2 + 1.5a - 0.5625 = 0$$

(*a*) Solving the quadratic equation: $a = 0.31066,$ $\qquad a = 0.311\text{ m}$ ◀

(*b*) Substituting: $|M|_{max} = M_1 = 2(0.31066)^2$ $\qquad |M|_{max} = 193.0\text{ N}\cdot\text{m}$ ◀



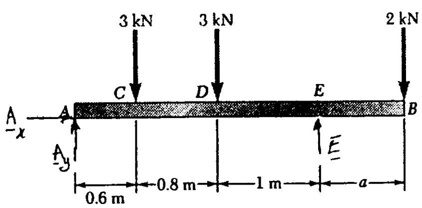


PROBLEM 7.58

For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

SOLUTION

<u>Free body: Entire beam</u>

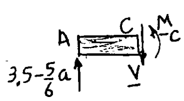


$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4) + (3)(1.8) + 3(1) - (2)a = 0$$

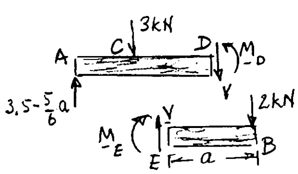
$$A_y = 3.5\text{ kN} - \frac{5}{6}a \qquad \mathbf{A} = 3.5\text{ kN} - \frac{5}{6}a\uparrow \;\triangleleft$$

<u>Free body: *AC*</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - \left(3.5 - \frac{5}{6}a\right)(0.6\text{ m}) = 0, \qquad M_C = +2.1 - \frac{a}{2} \;\triangleleft$$

<u>Free body: *AD*</u>



$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - \left(3.5 - \frac{5}{6}a\right)(1.4\text{ m}) + (3\text{ kN})(0.8\text{ m}) = 0$$

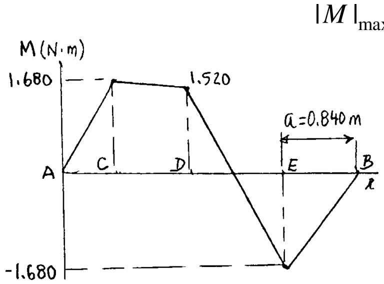
$$M_D = +2.5 - \frac{7}{6}a \;\triangleleft$$

<u>Free body: *EB*</u> $\quad +\circlearrowleft \Sigma M_E = 0: \quad -M_E - (2\text{ kN})a = 0 \qquad M_E = -2a \;\triangleleft$

We shall <u>assume</u> that $M_C > M_D$ and, thus, that $M_{max} = M_C$.

We set $M_{max} = |M_{min}|$ or $M_C = |M_E| = 2.1 - \dfrac{a}{2} = 2a \qquad a = 0.840\text{ m} \;\triangleleft$

$$|M|_{max} = M_C = |M_E| = 2a = 2(0.840) \qquad |M|_{max} = 1.680\text{ N}\cdot\text{m} \;\triangleleft$$



We must check our assumption.

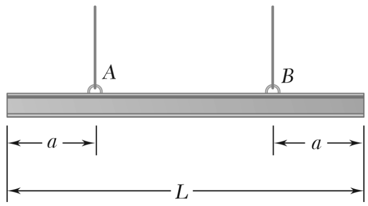
$$M_D = 2.5 - \frac{7}{6}(0.840) = 1.520\text{ N}\cdot\text{m}$$

Thus, $M_C > M_D$, O.K.

The answers are (*a*) $a = 0.840\text{ m}$ ◀

(*b*) $|M|_{max} = 1.680\text{ N}\cdot\text{m}$ ◀

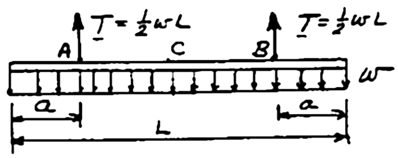
PROBLEM 7.59



A uniform beam is to be picked up by crane cables attached at A and B. Determine the distance a from the ends of the beam to the points where the cables should be attached if the maximum absolute value of the bending moment in the beam is to be as small as possible. (*Hint:* Draw the bending-moment diagram in terms of a, L, and the weight w per unit length, and then equate the absolute values of the largest positive and negative bending moments obtained.)

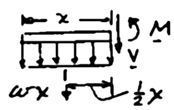
SOLUTION

w = weight per unit length



<u>To the left of A:</u>

$$+\circlearrowleft \Sigma M_1 = 0: \quad M + wx\left(\frac{x}{2}\right) = 0$$

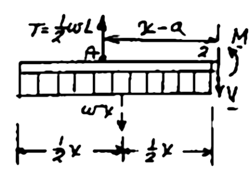


$$M = -\frac{1}{2}wx^2$$

$$M_A = -\frac{1}{2}wa^2$$

<u>Between A and B:</u>

$$+\circlearrowleft \Sigma M_2 = 0: \quad M - \left(\frac{1}{2}wL\right)(x-a) + (wx)\left(\frac{1}{2}x\right) = 0$$

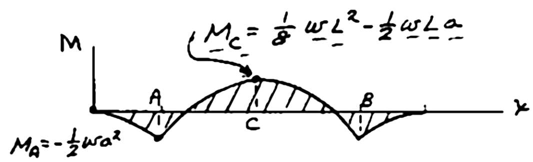


$$M = -\frac{1}{2}wx^2 + \frac{1}{2}wLx - \frac{1}{2}wLa$$

At center C:

$$x = \frac{L}{2}$$

$$M_C = -\frac{1}{2}w\left(\frac{L}{2}\right)^2 + \frac{1}{2}wL\left(\frac{L}{2}\right) - \frac{1}{2}wLa$$



PROBLEM 7.59 (Continued)

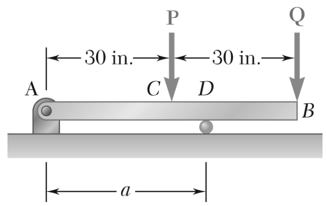
We set $|M_A| = |M_C|$: $\left|-\frac{1}{2}wa^2\right| = \left|\frac{1}{8}wL^2 - \frac{1}{2}wLa\right| + \frac{1}{2}wa^2 = \frac{1}{8}wL^2 - \frac{1}{2}wLa$

$$a^2 + La - 0.25L^2 = 0$$

$$a = \frac{1}{2}(L \pm \sqrt{L^2 + L^2}) = \frac{1}{2}(\sqrt{2} - 1)L$$

$$M_{\max} = \frac{1}{2}w(0.207L)^2 = 0.0214wL^2$$

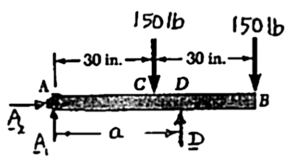
$a = 0.207L$ ◀



PROBLEM 7.60

Knowing that $P = Q = 150$ lb, determine (a) the distance a for which the maximum absolute value of the bending moment in beam AB is as small as possible, (b) the corresponding value of $|M|_{\max}$. (See hint for Problem 7.55.)

SOLUTION

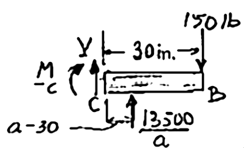


Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (150)(30) - (150)(60) = 0$$

$$D = \frac{13{,}500}{a} \qquad \triangleleft$$

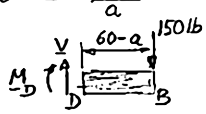
Free body: CB



$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{13{,}500}{a}(a-30) = 0$$

$$M_C = 9000\left(1 - \frac{45}{a}\right) \qquad \triangleleft$$

Free body: DB



$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60-a) = 0$$

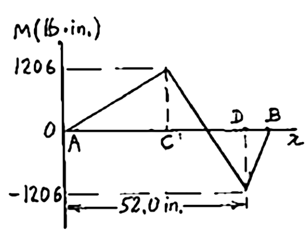
$$M_D = -150(60-a) \qquad \triangleleft$$

(a) We set

$$M_{\max} = |M_{\min}| \quad \text{or} \quad M_C = -M_D: \quad 9000\left(1 - \frac{45}{a}\right) = 150(60-a)$$

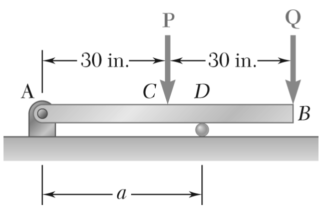
$$60 - \frac{2700}{a} = 60 - a$$

$$a^2 = 2700 \quad a = 51.96 \text{ in.} \qquad a = 52.0 \text{ in.} \blacktriangleleft$$



(b) $|M|_{\max} = -M_D = 150(60 - 51.96)$

$$|M|_{\max} = 1206 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

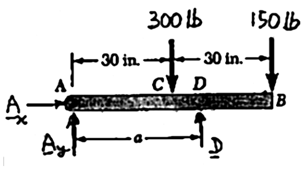


PROBLEM 7.61

Solve Problem 7.60 assuming that $P = 300$ lb and $Q = 150$ lb.

PROBLEM 7.60 Knowing that $P = Q = 150$ lb, determine (*a*) the distance a for which the maximum absolute value of the bending moment in beam AB is as small as possible, (*b*) the corresponding value of $|M|_{max}$. (See hint for Problem 7.55.)

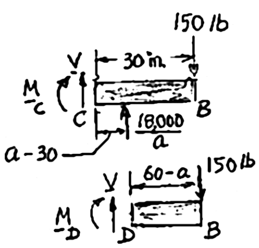
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_A = 0: \quad Da - (300)(30) - (150)(60) = 0$$

$$D = \frac{18{,}000}{a} \quad \triangleleft$$



Free body: *CB*

$$+\circlearrowleft \Sigma M_C = 0: \quad -M_C - (150)(30) + \frac{18{,}000}{a}(a-30) = 0$$

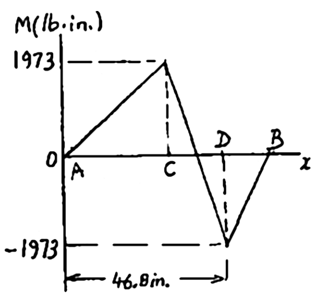
$$M_C = 13{,}500\left(1 - \frac{40}{a}\right) \quad \triangleleft$$

Free body: *DB*

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_D - (150)(60-a) = 0$$

$$M_D = -150(60-a) \quad \triangleleft$$

(*a*) We set



$$M_{max} = |M_{min}| \quad \text{or} \quad M_C = -M_D: \quad 13{,}500\left(1 - \frac{40}{a}\right) = 150(60-a)$$

$$90 - \frac{3600}{a} = 60 - a$$

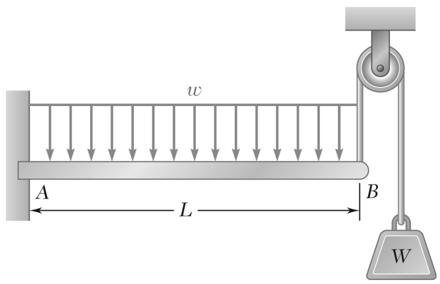
$$a^2 + 30a - 3600 = 0$$

$$a = \frac{-30 + \sqrt{15.300}}{2} = 46.847$$

$$a = 46.8 \text{ in.} \quad \blacktriangleleft$$

(*b*) $|M|_{max} = -M_D = 150(60 - 46.847)$

$$|M|_{max} = 1973 \text{ lb} \cdot \text{in.} \quad \blacktriangleleft$$

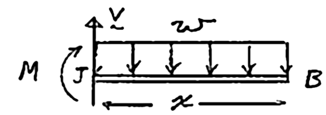


PROBLEM 7.62*

In order to reduce the bending moment in the cantilever beam *AB*, a cable and counterweight are permanently attached at end *B*. Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{max}$. Consider (*a*) the case when the distributed load is permanently applied to the beam, (*b*) the more general case when the distributed load may either be applied or removed.

SOLUTION

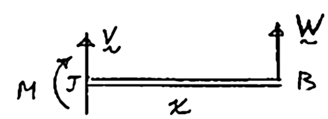
M due to distributed load:



$$\circlearrowleft \Sigma M_J = 0: \quad -M - \frac{x}{2}wx = 0$$

$$M = -\frac{1}{2}wx^2$$

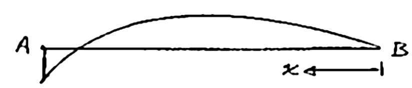
M due to counter weight:



$$\circlearrowleft \Sigma M_J = 0: \quad -M + xw = 0$$

$$M = wx$$

(*a*) **Both applied:**



$$M = W_x - \frac{w}{2}x^2$$

$$\frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$

And here $M = \frac{W^2}{2w} > 0$ so M_{max}; M_{min} must be at $x = L$

So $M_{min} = WL - \frac{1}{2}wL^2$. For minimum $|M|_{max}$ set $M_{max} = -M_{min}$,

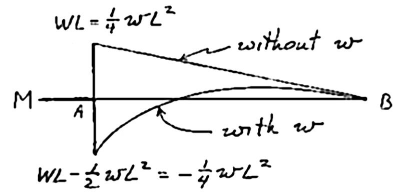
so $$\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \quad \text{or} \quad W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2} \text{ (need+)} \qquad W = (\sqrt{2} - 1)wL = 0.414\, wL \blacktriangleleft$$

PROBLEM 7.62* (Continued)

(*b*) ***w* may be removed**

$$M_{\max} = \frac{W^2}{2w} = \frac{(\sqrt{2}-1)^2}{2} wL^2 \qquad M_{\max} = 0.0858\, wL^2 \blacktriangleleft$$



Without w,

$$M = Wx$$

$$M_{\max} = WL \text{ at } A$$

With w (see Part a)

$$M = Wx - \frac{w}{2}x^2$$

$$M_{\max} = \frac{W^2}{2w} \text{ at } x = \frac{W}{w}$$

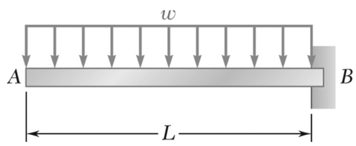
$$M_{\min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

For minimum $M_{\max}$, set $M_{\max}$ (no w) $= -M_{\min}$ (with w)

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow \qquad M_{\max} = \frac{1}{4}wL^2 \blacktriangleleft$$

With $$W = \frac{1}{4}wL \blacktriangleleft$$

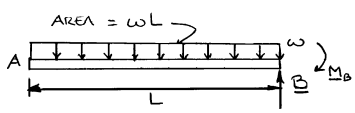
PROBLEM 7.63



Using the method of Section 7.6, solve Problem 7.29.

PROBLEM 7.29 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

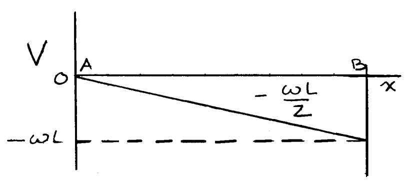
SOLUTION



Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad wL - B = 0$$

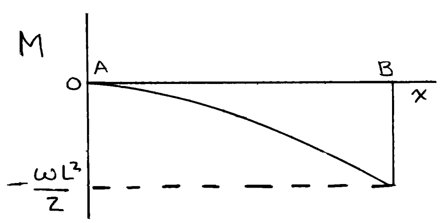
$$\mathbf{B} = wL \uparrow$$



Shear diagram

$$+\circlearrowleft \Sigma M_B = 0: \quad (wL)\left(\frac{L}{2}\right) - M_B = 0$$

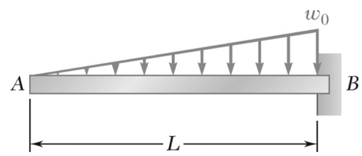
$$\mathbf{M}_B = \frac{wL^2}{2} \circlearrowright$$



Moment diagram

(*b*) $\quad |V|_{max} = wL; \qquad |M|_{max} = \dfrac{wL^2}{2}$ ◀

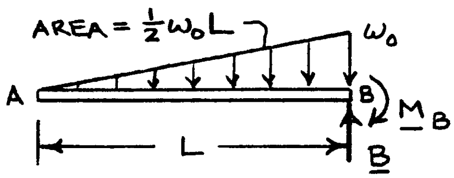
PROBLEM 7.64



Using the method of Section 7.6, solve Problem 7.30.

PROBLEM 7.30 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

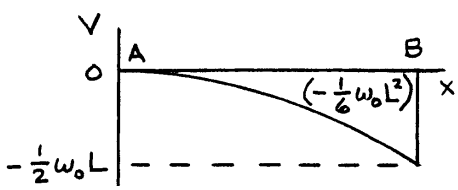
SOLUTION



Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad B - \frac{1}{2}(w_0)(L) = 0$$

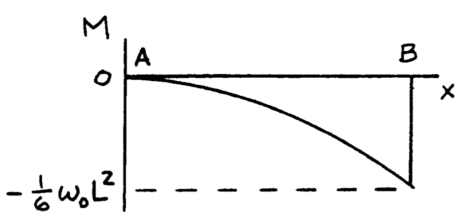
$$\mathbf{B} = \frac{1}{2}w_0L \uparrow$$



Shear diagram

$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{1}{2}(w_0)(L)\left(\frac{L}{3}\right) - M_B = 0$$

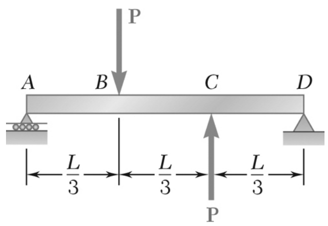
$$\mathbf{M}_B = \frac{1}{6}w_0L^2 \circlearrowright$$



Moment diagram

(*b*) $|V|_{max} = w_0L/2$ ◀

$|M|_{max} = w_0L^2/6$ ◀

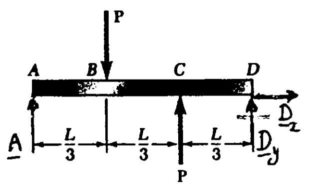


PROBLEM 7.65

Using the method of Section 7.6, solve Problem 7.31.

PROBLEM 7.31 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

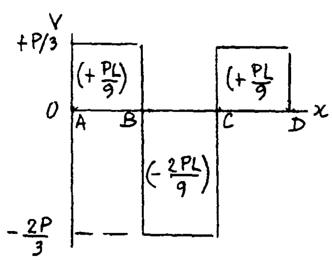
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_D = 0: \quad P\left(\frac{2L}{3}\right) - P\left(\frac{L}{3}\right) - AL = 0$$

$$\mathbf{A} = P/3 \uparrow$$

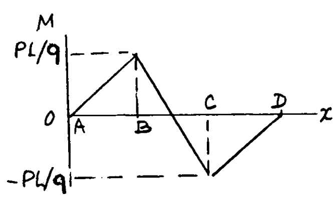


Shear diagram

We note that

$$V_A = A = +P/3$$

$$|V|_{max} = 2P/3 \blacktriangleleft$$

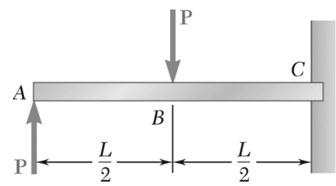


Bending diagram

We note that $M_A = 0$

$$|M|_{max} = PL/9 \blacktriangleleft$$

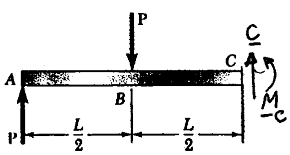
PROBLEM 7.66



Using the method of Section 7.6, solve Problem 7.32.

PROBLEM 7.32 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

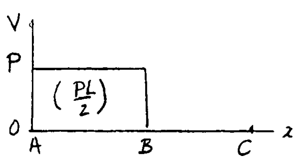
SOLUTION



Free body: Entire beam

$$\Sigma F_y = 0: \quad \mathbf{C} = 0$$

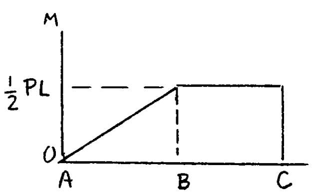
$$\Sigma M_C = 0: \quad \mathbf{M}_C = \frac{1}{2} PL$$



Shear diagram

At A: $\quad V_A = +P$

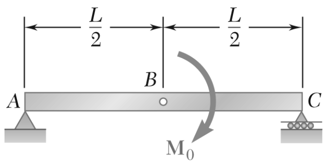
$$|V|_{max} = P \blacktriangleleft$$



Moment diagram

At A: $\quad M_A = 0$

$$|M|_{max} = \frac{1}{2} PL \blacktriangleleft$$

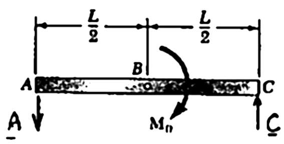


PROBLEM 7.67

Using the method of Section 7.6, solve Problem 7.33.

PROBLEM 7.33 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

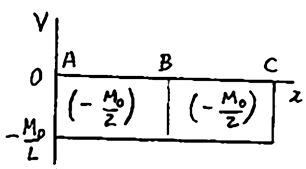


Free body: Entire beam

$$\Sigma F_y = 0: \quad A = C$$

$$+\circlearrowleft \Sigma M_C = 0: \quad Al - M_0 = 0$$

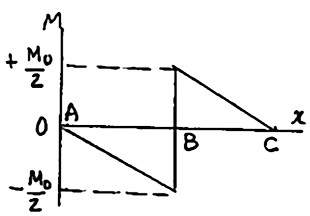
$$A = C = \frac{M_0}{L}$$



Shear diagram

At A: $\qquad V_A = -\frac{M_0}{L}$

$$|V|_{max} = \frac{M_0}{L} \blacktriangleleft$$

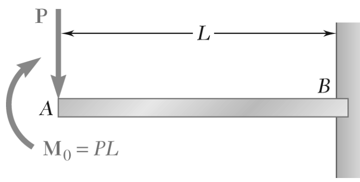


Bending-moment diagram

At A: $\qquad M_A = 0$

At B, M increases by M_0 on account of applied couple.

$$|M|_{max} = M_0/2 \blacktriangleleft$$

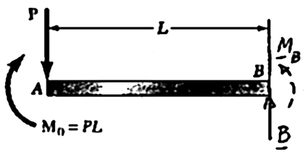


PROBLEM 7.68

Using the method of Section 7.6, solve Problem 7.34.

PROBLEM 7.34 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



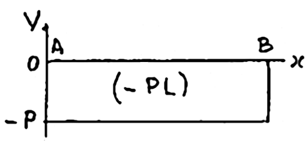
Free body: Entire beam

$$+\uparrow \Sigma F_y = 0: \quad B - P = 0$$

$$\mathbf{B} = P \uparrow$$

$$+\circlearrowleft \Sigma M_B = 0: \quad M_B - M_0 + PL = 0$$

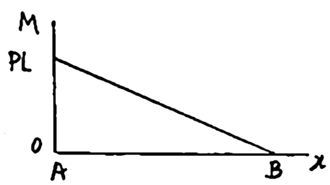
$$\mathbf{M}_B = 0$$



Shear diagram

At A: $\quad V_A = -P$

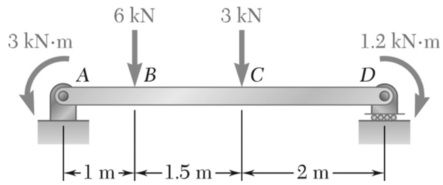
$$|V|_{max} = P \blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = M_0 = PL$

$$|M|_{max} = PL \blacktriangleleft$$



PROBLEM 7.69

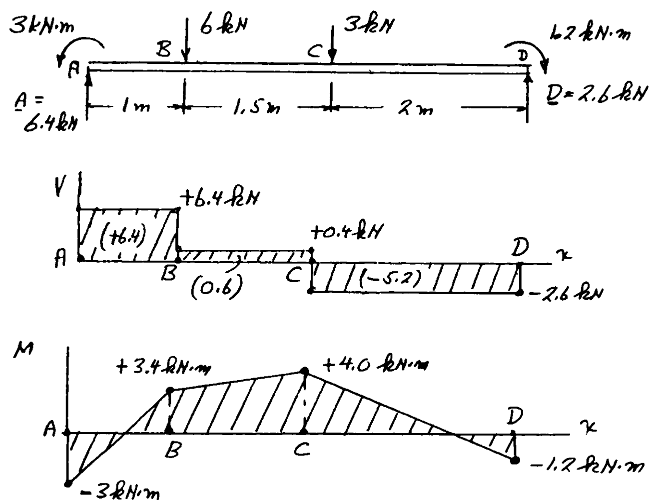
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

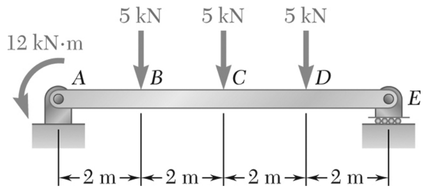
<u>Reactions</u> $\Sigma F_x = 0 \quad A_x = 0$

$$+\circlearrowleft \Sigma M_D = 0: \quad +3\text{ kN}\cdot\text{m} + (6\text{ kN})(3.5\text{ m}) + (3\text{ kN})(2\text{ m}) - 1.2\text{ kN}\cdot\text{m} - A_y(4.5\text{ m}) = 0$$

$$A_y = +6.4\text{ kN} \qquad A_y = +6.4\text{ kN}\uparrow$$



(*b*) $|V|_{max} = 6.40\text{ kN};$ $|M|_{max} = 4.00\text{ kN}\cdot\text{m}$ ◀



PROBLEM 7.70

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

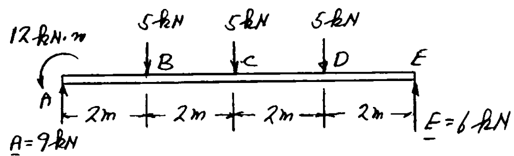
<u>Reactions</u>

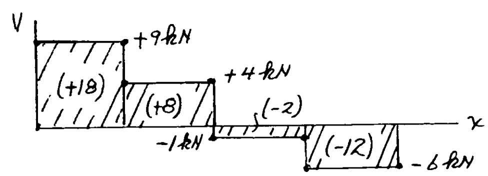
$$+\circlearrowleft \Sigma M_A = 0: \quad 12 \text{ kN}\cdot\text{m} - 3(5 \text{ kN})(4 \text{ m}) + E(8 \text{ m}) = 0$$

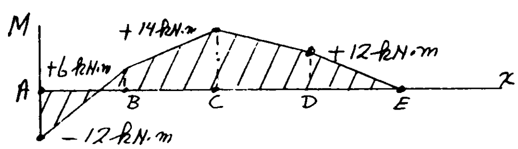
$$E = 6 \text{ kN}\uparrow$$

$$\Sigma F_y = 0$$

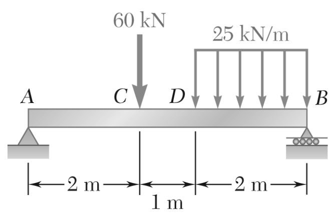
$$A = 9 \text{ kN}\uparrow$$







(*b*) $|V|_{max} = 9.00$ kN; $|M|_{max} = 14.00$ kN · m ◀

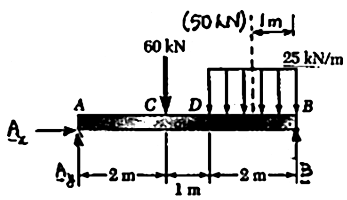


PROBLEM 7.71

Using the method of Section 7.6, solve Problem 7.39.

PROBLEM 7.39 For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



<u>Free body: Beam</u>

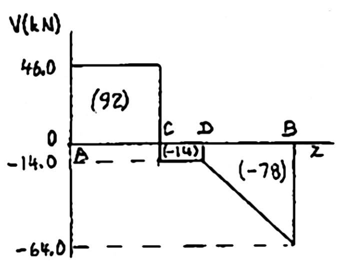
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (60 \text{ kN})(3 \text{ m}) + (50 \text{ kN})(1 \text{ m}) - A_y(5 \text{ m}) = 0$$

$$A_y = +46.0 \text{ kN} \quad \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B + 46.0 \text{ kN} - 60 \text{ kN} - 50 \text{ kN} = 0$$

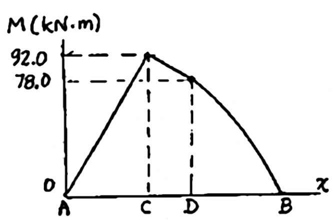
$$B = +64.0 \text{ kN} \quad \triangleleft$$



<u>Shear diagram</u>

At A: $\quad V_A = A_y = +46.0 \text{ kN}$

$$|V|_{\max} = 64.0 \text{ kN} \quad \blacktriangleleft$$

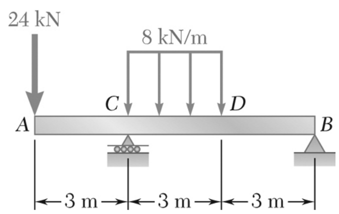


<u>Bending-moment diagram</u>

At A: $\quad M_A = 0$

$$|M|_{\max} = 92.0 \text{ kN} \cdot \text{m} \quad \blacktriangleleft$$

Parabola from D to B. Its slope at D is same as that of straight-line segment CD since V has no discontinuity at D.

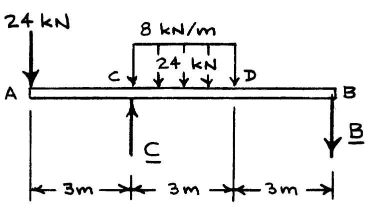


PROBLEM 7.72

Using the method of Section 7.6, solve Problem 7.40.

PROBLEM 7.40 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

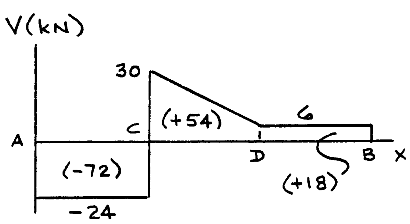
SOLUTION



Free body: Entire beam

$$+\circlearrowleft \Sigma M_B = 0: \quad (24\text{ kN})(9\text{ m}) - C(6\text{ m}) + (24\text{ kN})(4.5\text{ m}) = 0$$

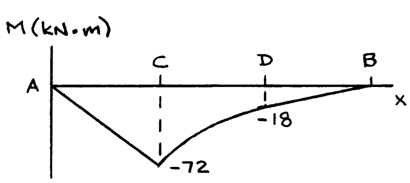
$$\mathbf{C} = 54\text{ kN}\uparrow$$



Shear diagram

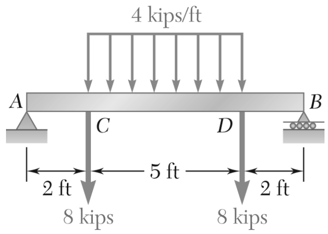
$$+\uparrow \Sigma F_y = 0: \quad 54 - 24 - 24 - B = 0$$

$$\mathbf{B} = 6\text{ kN}\downarrow$$



(*b*) $|V|_{max} = 30.0\text{ kN}; \qquad |M|_{max} = 72.0\text{ kN}\cdot\text{m}$ ◀

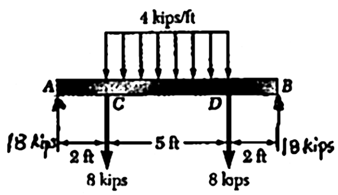
PROBLEM 7.73



Using the method of Section 7.6, solve Problem 7.41.

PROBLEM 7.41 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

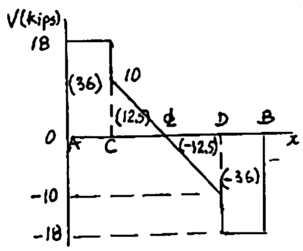


Reactions at supports.

Because of the symmetry:

$$A = B = \frac{1}{2}(8 + 8 + 4 \times 5)\,\text{kips}$$

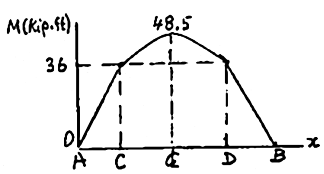
$$\mathbf{A} = \mathbf{B} = 18\ \text{kips}\uparrow \triangleleft$$



Shear diagram

At A: $V_A = +18$ kips

$$|V|_{\max} = 18.00\ \text{kips} \blacktriangleleft$$

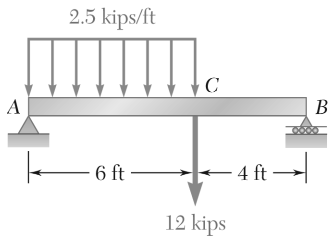


Bending-moment diagram

At A: $M_A = 0$

$$|M|_{\max} = 48.5\ \text{kip} \cdot \text{ft} \blacktriangleleft$$

Discontinuities in slope at C and D, due to the discontinuities of V.

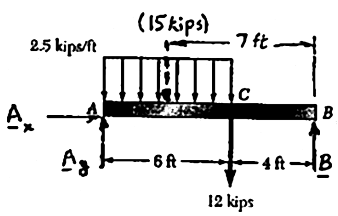


PROBLEM 7.74

Using the method of Section 7.6, solve Problem 7.42.

PROBLEM 7.42 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Free body: Beam

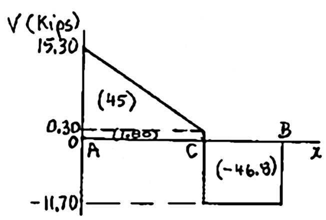
$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (12 \text{ kips})(4 \text{ ft}) + (15 \text{ kips})(7 \text{ ft}) - A_y(10 \text{ ft}) = 0$$

$$A_y = +15.3 \text{ kips} \quad \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B + 15.3 - 15 - 12 = 0$$

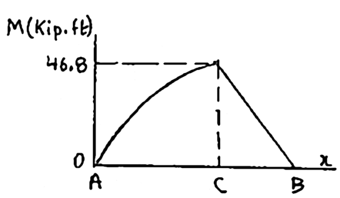
$$B = +11.7 \text{ kips} \quad \triangleleft$$



Shear diagram

At A: $\quad V_A = A_y = 15.3 \text{ kips}$

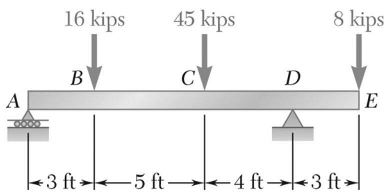
$$|V|_{max} = 15.30 \text{ kips} \quad \blacktriangleleft$$



Bending-moment diagram

At A: $\quad M_A = 0$

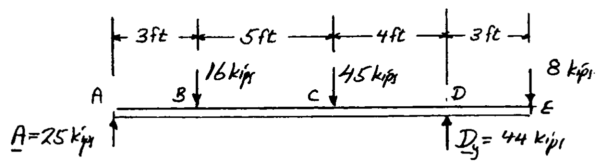
$$|M|_{max} = 46.8 \text{ kip} \cdot \text{ft} \quad \blacktriangleleft$$



PROBLEM 7.75

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



Reactions

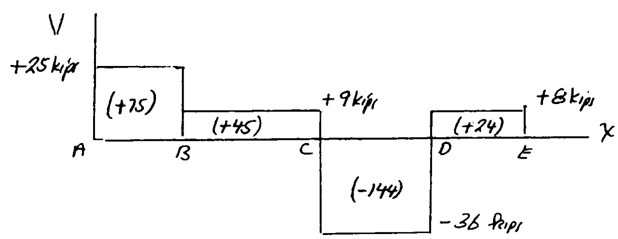
$$+\circlearrowleft \Sigma M_D = 0: \quad (16)(9) + (45)(4) - (8)(3) - A(12) = 0$$

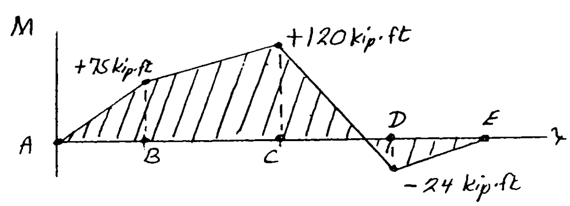
$A = +25$ kips $\qquad\qquad$ $\mathbf{A} = 25 \text{ kips}\uparrow$

$$+\uparrow \Sigma F_y = 0: \quad 25 - 16 - 45 - 8 + D_y = 0$$

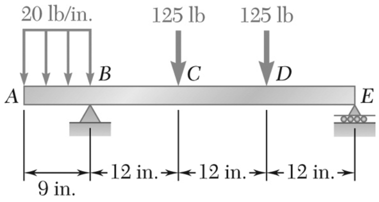
$D_y = +44,$ $\qquad\qquad$ $D_y = 44 \text{ kips}\uparrow$

$$\Sigma F_x = 0: \quad D_x = 0$$





(*b*) $\quad |V|_{\max} = 36.0$ kips; $\qquad\qquad$ $|M|_{\max} = 120.0 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 7.76

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

Reactions

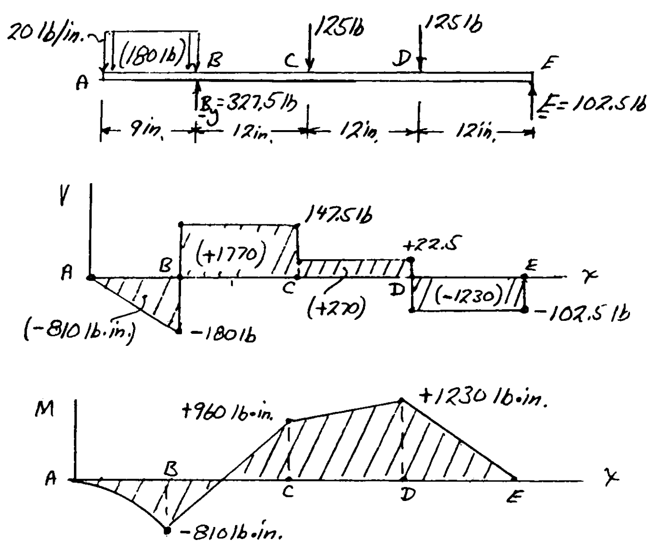
$$\Sigma F_x = 0: \quad B_x = 0$$

$$+\circlearrowleft \Sigma M_E = 0: \quad (20 \text{ lb/in.})(9 \text{ in.})(40.5 \text{ in.}) + (125 \text{ in.})(24 \text{ in.}) + (125 \text{ in.})(12 \text{ in.}) - B(36 \text{ in.}) = 0$$

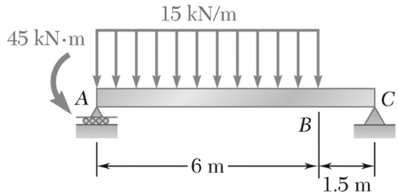
$$B_y = +327.5 \text{ lb} \qquad \mathbf{B}_y = 327.5 \text{ lb} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad -(20 \text{ lb/in.})(9 \text{ in.}) - 125 \text{ lb} - 125 \text{ lb} + 327.5 \text{ lb} + E = 0$$

$$E = +102.5 \text{ lb} \qquad \mathbf{E} = 102.5 \text{ lb} \uparrow$$



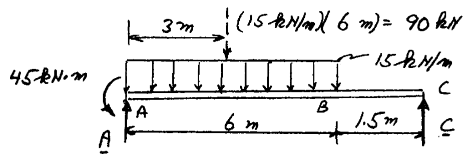
(*b*) $|V|_{max} = 180.0 \text{ lb};$ $|M|_{max} = 1230 \text{ lb} \cdot \text{in.}$ ◀



PROBLEM 7.77

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



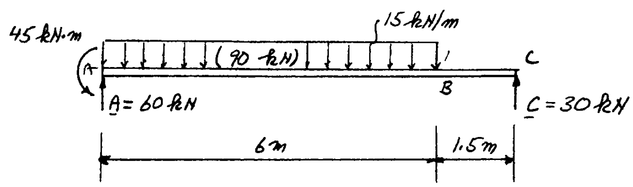
Reactions

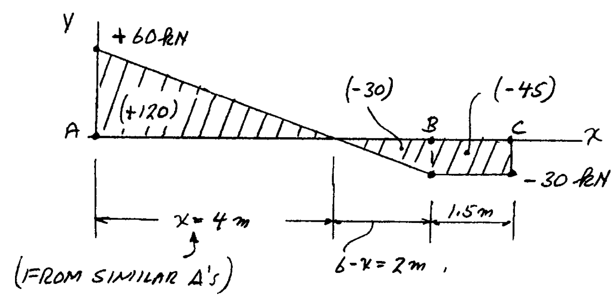
$$+\circlearrowleft \Sigma M_A = 0: \quad +45\text{ kN}\cdot\text{m} - (90\text{ kN})(3\text{ m}) + C(7.5\text{ m}) = 0$$

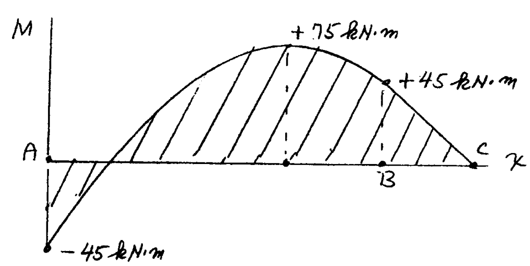
$$C = +30\text{ kN} \qquad \mathbf{C} = 30\text{ kN}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad A - 90\text{ kN} + 30\text{ kN} = 0$$

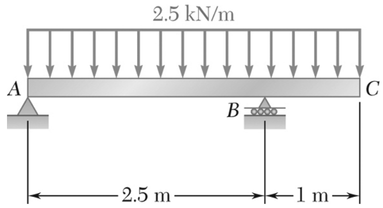
$$\mathbf{A} = 60\text{ kN}\uparrow$$







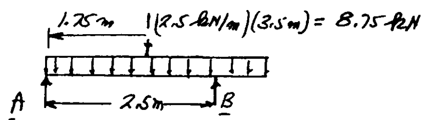
(*b*) 75.0 kN·m, 4.00 m from *A* ◀



PROBLEM 7.78

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

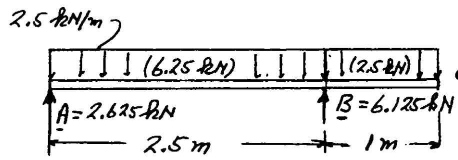


$$+\circlearrowleft \Sigma M_A = 0: \quad (8.75)(1.75) - B(2.5) = 0$$

$$\mathbf{B} = 6.125 \text{ kN}\uparrow$$

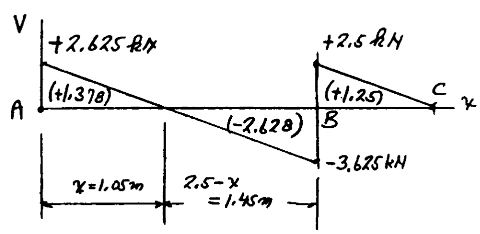
$$\Sigma F_y = 0:$$

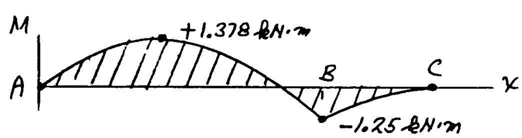
$$\mathbf{A} = 2.625 \text{ kN}\uparrow$$



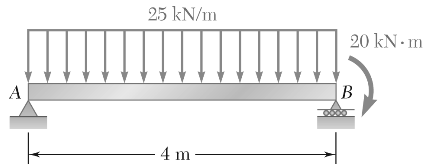
Similar Δ's

$$\underbrace{\frac{x}{2.625} = \frac{2.5-x}{3.625}}_{\substack{\text{Add numerators} \\ \text{and denominators} \\ x = 1.05\text{ m}}} = \frac{2.5}{6.25}$$





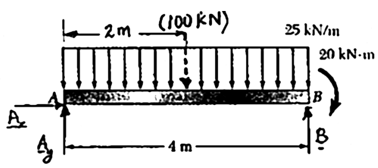
(*b*) $1.378 \text{ kN}\cdot\text{m}, 1.050 \text{ m from } A$ ◀



PROBLEM 7.79

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



<u>Free body: Beam</u>

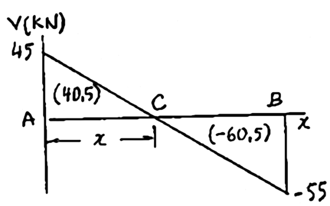
$$+\circlearrowleft \Sigma M_A = 0: \quad B(4\text{ m}) - (100\text{ kN})(2\text{ m}) - 20\text{ kN}\cdot\text{m} = 0$$

$$B = +55\text{ kN} \quad \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 55 - 100 = 0$$

$$A_y = +45\text{ kN} \quad \triangleleft$$



<u>Shear diagram</u>

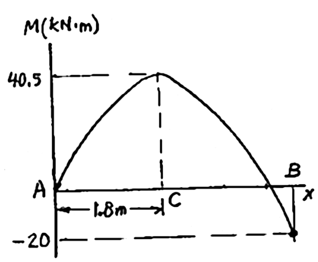
At A: $\quad V_A = A_y = +45\text{ kN}$

To determine Point C where $V = 0$:

$$V_C - V_A = -wx$$
$$0 - 45\text{ kN} = -(25\text{ kN}\cdot\text{m})x$$

$$x = 1.8\text{ m} \quad \triangleleft$$

We compute all areas bending-moment



<u>Bending-moment diagram</u>

At A: $\quad M_A = 0$

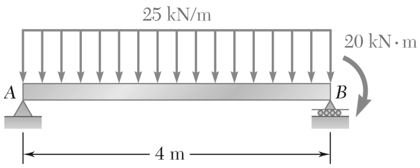
At B: $\quad M_B = -20\text{ kN}\cdot\text{m}$

$$|M|_{\max} = 40.5\text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$1.800\text{ m from } A \quad \blacktriangleleft$$

Single arc of parabola

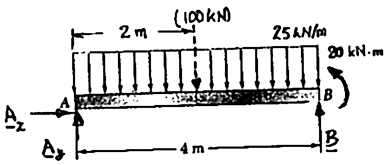
PROBLEM 7.80



Solve Problem 7.79 assuming that the 20-kN · m couple applied at B is counterclockwise.

PROBLEM 7.79 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION



Free body: Beam

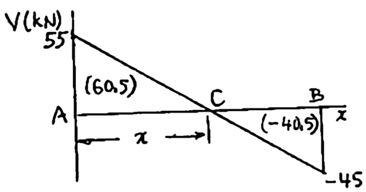
$$+\circlearrowleft \Sigma M_A = 0: \quad B(4\text{ m}) - (100\text{ kN})(2\text{ m}) - 20\text{ kN}\cdot\text{m} = 0$$

$$B = +45\text{ kN} \quad \triangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 45 - 100 = 0$$

$$A_y = +55\text{ kN} \quad \triangleleft$$



Shear diagram

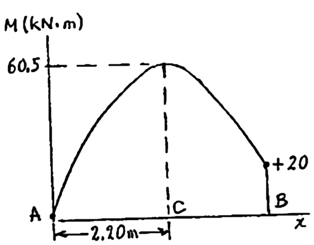
At A: $\quad V_A = A_y = +55\text{ kN}$

To determine Point C where $V = 0$:

$$V_C - V_A = -wx$$
$$0 - 55\text{ kN} = -(25\text{ kN/m})x$$

$$x = 2.20\text{ m} \quad \triangleleft$$

We compute all areas bending-moment



Bending-moment diagram

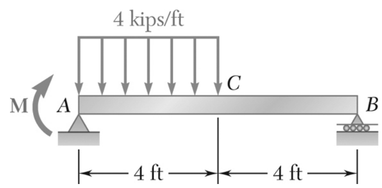
At A: $\quad M_A = 0$

At B: $\quad M_B = +20\text{ kN}\cdot\text{m}$

$$|M|_{\max} = 60.5\text{ kN}\cdot\text{m} \quad \blacktriangleleft$$

$$2.20\text{ m from } A \quad \blacktriangleleft$$

Single arc of parabola



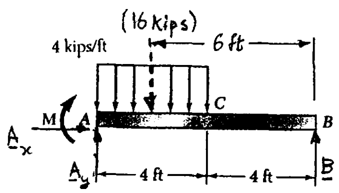
PROBLEM 7.81

For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment, knowing that (*a*) $M = 0$, (*b*) $M = 24$ kip · ft.

SOLUTION

Free body: Beam

$$\Sigma F_x = 0: \quad A_x = 0$$



$$+\circlearrowright \Sigma M_B = 0: \quad (16 \text{ kips})(6 \text{ ft}) - A_y(8 \text{ ft}) - M = 0$$

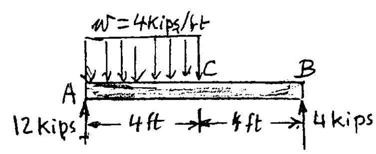
$$A_y = 12 \text{ kips} - \frac{1}{8}M \qquad (1) \;\triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B + 12 - \frac{1}{8}M - 16 = 0$$

$$B = 4 \text{ kips} + \frac{1}{8}M \qquad (2) \;\triangleleft$$

(*a*) $M = 0$:

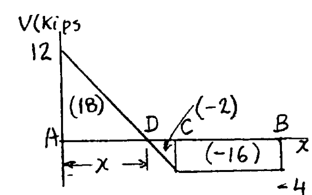
Load diagram



Making $M = 0$ in. (1) and (2).

$$A_y = +12 \text{ kips}$$
$$B = +4 \text{ kips}$$

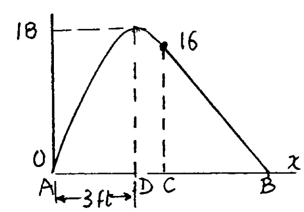
Shear diagram $\qquad V_A = A_y = +12$ kips



To determine Point *D* where $V = 0$:

$$V_D - V_A = -wx$$
$$0 - 12 \text{ kips} = -(4 \text{ kips/ft})x$$

$x = 3$ ft $\triangleleft$



We compute all areas

B. M. Diagram

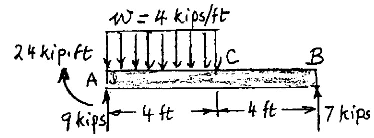
At *A*: $\qquad M_A = 0$

$|M|_{max} = 18.00$ kip·ft, ◀

3.00 ft from *A* ◀

Parabola from *A* to *C*

(*b*) $\underline{M = 24 \text{ kip} \cdot \text{ft}}$

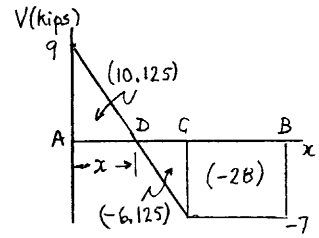


Load diagram

Making $M = 24$ kip · ft in (1) and (2)

$$A = 12 - \frac{1}{8}(24) = +9 \text{ kips}$$

$$B = 4 + \frac{1}{8}(24) = +7 \text{ kips}$$



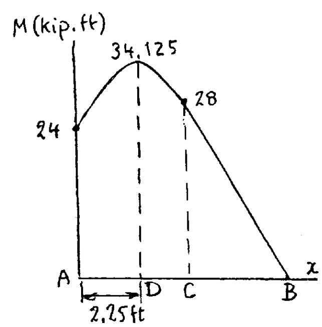
Shear diagram $\quad V_A = A_y = +9$ kips

To determine Point D where $V = 0$:

$$V_D = V_A = -wx$$

$$0 - 9 \text{ kips} = -(4 \text{ kips/ft})\, x$$

$x = 2.25$ ft ◁



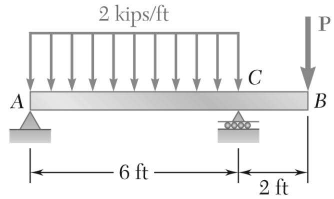
B. M. Diagram

At A: $\quad M_A = +24$ kip · ft

$|M|_{\max} = 34.1$ kip·ft, ◀

2.25 ft from A ◀

Parabola from A to C.

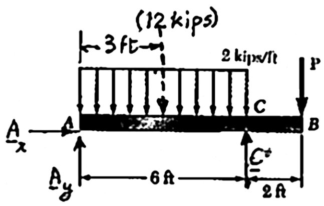


PROBLEM 7.82

For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment, knowing that (*a*) $P = 6$ kips, (*b*) $P = 3$ kips.

SOLUTION

Free body: Beam



$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad C(6\text{ ft}) - (12\text{ kips})(3\text{ ft}) - P(8\text{ ft}) = 0$$

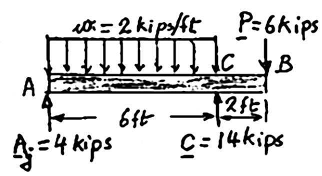
$$C = 6\text{ kips} + \frac{4}{3}P \qquad (1) \triangleleft$$

$$\Sigma F_y = 0: \quad A_y + \left(6 + \frac{4}{3}P\right) - 12 - P = 0$$

$$A_y = 6\text{ kips} - \frac{1}{3}P \qquad (2) \triangleleft$$

(*a*) $P = 6$ kips.

Load diagram

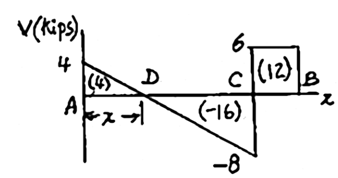


Substituting for P in Eqs. (2) and (1):

$$A_y = 6 - \frac{1}{3}(6) = 4\text{ kips}$$

$$C = 6 + \frac{4}{3}(6) = 14\text{ kips}$$

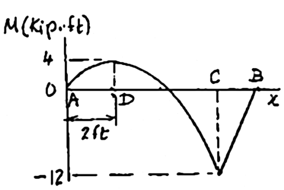
Shear diagram $\qquad V_A = A_y = +4$ kips



To determine Point D where $V = 0$:

$$V_D - V_A = -wx$$

$$0 - 4\text{ kips} = (2\text{ kips/ft})x \qquad x = 2\text{ ft} \triangleleft$$



We compute all areas

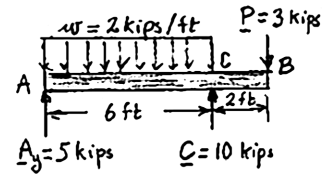
Bending-moment diagram

At A: $\qquad M_A = 0$

$$|M|_{max} = 12.00\text{ kip}\cdot\text{ft, at } C \blacktriangleleft$$

Parabola from A to C

(*b*) $\underline{P = 3 \text{ kips}}$

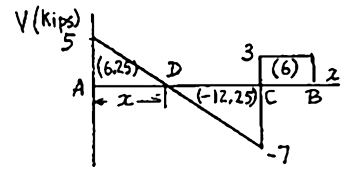


Load diagram

Substituting for P in Eqs. (2) and (1):

$$A = 6 - \frac{1}{3}(3) = 5 \text{ kips}$$

$$C = 6 + \frac{4}{3}(3) = 10 \text{ kips}$$



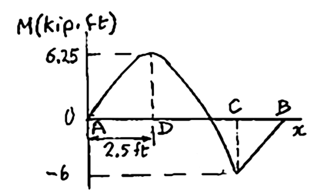
Shear diagram $\qquad V_A = A_y = +5 \text{ kips}$

To determine D where $V = 0$:

$$V_D - V_A = -wx$$

$$0 - (5 \text{ kips}) = -(2 \text{ kips/ft})\, x$$

$x = 2.5 \text{ ft}$ ◁



We compute all areas

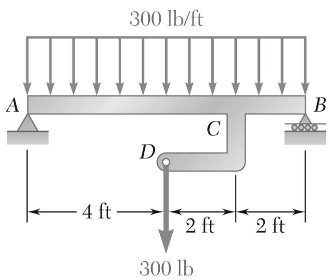
Bending-moment diagram

At A: $\qquad M_A = 0$

$|M|_{\max} = 6.25 \text{ kip} \cdot \text{ft}$ ◀

2.50 ft from A ◀

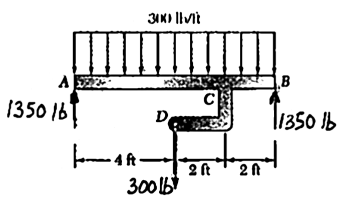
Parabola from A to C.



PROBLEM 7.83

(*a*) Draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

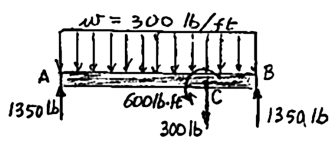


Reactions at supports

Because of symmetry of load

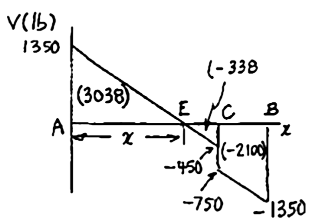
$$A = B = \frac{1}{2}(300 \times 8 + 300)$$

$$\mathbf{A} = \mathbf{B} = 1350 \text{ lb}\uparrow \triangleleft$$



Load diagram for *AB*

The 300-lb force at *D* is replaced by an equivalent force-couple system at *C*.



Shear diagram

At *A*: $V_A = A = 1350 \text{ lb}$

To determine Point *E* where $V = 0$:

$$V_E - V_A = -wx$$
$$0 - 1350 \text{ lb} = -(300 \text{ lb/ft})x$$

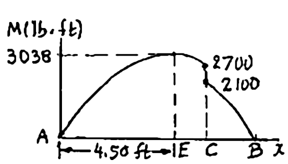
$$x = 4.50 \text{ ft} \triangleleft$$

We compute all areas

Bending-moment diagram

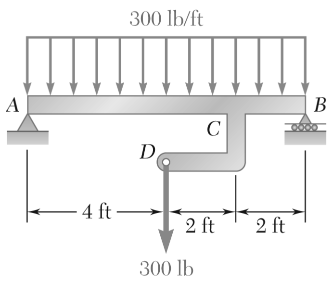
At *A*: $M_A = 0$

Note $600-\text{lb}\cdot\text{ft}$ drop at *C* due to couple



$$|M|_{\max} = 3040 \text{ lb}\cdot\text{ft} \blacktriangleleft$$

$$4.50 \text{ ft from } A \blacktriangleleft$$

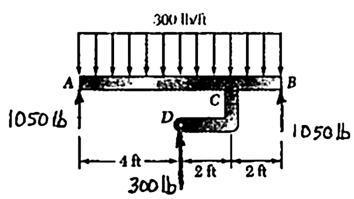


PROBLEM 7.84

Solve Problem 7.83 assuming that the 300-lb force applied at D is directed upward.

PROBLEM 7.83 (a) Draw the shear and bending-moment diagrams for beam AB, (b) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

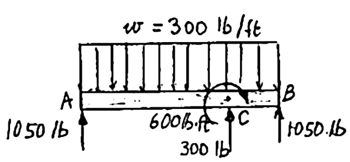


Reactions at supports

Because of symmetry of load:

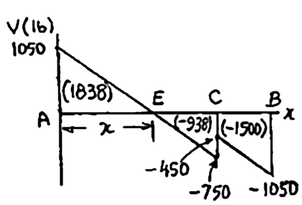
$$A = B = \frac{1}{2}(300 \times 8 - 300)$$

$$\mathbf{A} = \mathbf{B} = 1050 \text{ lb}\uparrow \triangleleft$$



Load diagram

The 300-lb force at D is replaced by an equivalent force-couple system at C.



Shear diagram

At A: $V_A = A = 1050 \text{ lb}$

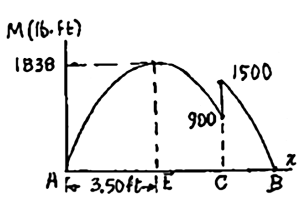
To determine Point E where $V = 0$:

$$V_E - V_A = -wx$$

$$0 - 1050 \text{ lb} = -(300 \text{ lb/ft})x$$

$$x = 3.50 \text{ ft} \triangleleft$$

We compute all areas



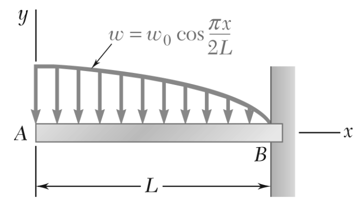
Bending-moment diagram

At A: $M_A = 0$

Note $600 - \text{lb} \cdot \text{ft}$ increase at C due to couple

$$|M|_{max} = 1838 \text{ lb} \cdot \text{ft} \blacktriangleleft$$

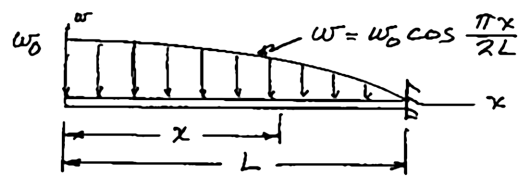
$$3.50 \text{ ft from } A \blacktriangleleft$$



PROBLEM 7.85

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



$$\frac{dv}{dx} = -w = w_0 \cos \frac{\pi}{2}\frac{x}{L}$$

$$V = -\int w\,dx = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L} + C_1 \qquad (1)$$

$$\frac{dM}{dx} = V = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L} + C_1$$

$$M = \int V\,dx = +w_0\left(\frac{2L}{\pi}\right)^2 \cos\frac{\pi x}{2L} + C_1 x + C_2 \qquad (2)$$

Boundary conditions

At $x = 0$: $\quad V = C_1 = 0 \quad C_1 = 0$

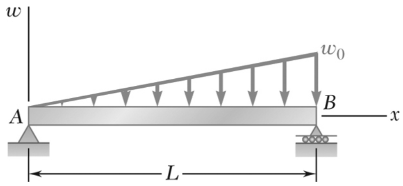
At $x = 0$: $\quad M = +w_0\left(\frac{2L}{\pi}\right)^2 \cos(0) + C_2 = 0$

$$C_2 = -w_0\left(\frac{2L}{\pi}\right)^2$$

Eq. (1) $\quad V = -w_0\left(\frac{2L}{\pi}\right)\sin\frac{\pi x}{2L}$ ◀

$$M = w_0\left(\frac{2L}{\pi}\right)^2\left(-1 + \cos\frac{\pi x}{2L}\right) ◀$$

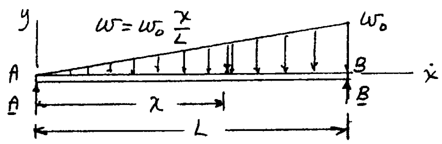
$M_{\max}$ at $x = L$: $\quad |M_{\max}| = w_0\left(\frac{2L}{\pi}\right)^2 |-1+0| = \frac{4}{\pi^2} w_0 L^2$ ◀



PROBLEM 7.86

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION



Eq. (7.1): $\dfrac{dV}{dx} = -w = -w_0 \dfrac{x}{L}$

$$V = \int -w_0 \frac{x}{L} dx = -\frac{1}{2} w_0 \frac{x^2}{L} + C_1 \qquad (1)$$

Eq. (7.3): $\dfrac{dM}{dx} = V = -\dfrac{1}{2} w_0 \dfrac{x^2}{L} + C_1$

$$M = \int \left(-\frac{1}{2} w_0 \frac{x^2}{L} + C_1 \right) dx = -\frac{1}{6} w_0 = \frac{x^3}{L} + C_1 x + C_2 \qquad (2)$$

(*a*) <u>Boundary conditions</u>

At $x = 0$: $M = 0 = 0 + 0 + C_2$ $\qquad C_2 = 0$

$x = L$: $M = 0 = -\dfrac{1}{6} w_0 L^2 + C_1 L$ $\qquad C_1 = \dfrac{1}{6} w_0 L$

Substituting C_1 and C_2 into (1) and (2):

$$V = -\frac{1}{2} w_0 \frac{x^2}{L} + \frac{1}{6} w_0 L \qquad V = \frac{1}{2} w_0 L \left(\frac{1}{3} - \frac{x^2}{L^2} \right) \blacktriangleleft$$

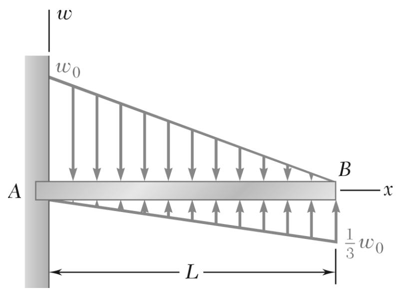
$$M = -\frac{1}{6} w_0 \frac{x^3}{L} + \frac{1}{6} w_0 L x \qquad M = \frac{1}{6} w_0 L^2 \left(\frac{x}{L} - \frac{x^3}{L^3} \right) \blacktriangleleft$$

(*b*) Max moment occurs when $V = 0$:

$$1 - 3\frac{x^2}{L^2} = 0 \qquad \frac{x}{L} = \frac{1}{\sqrt{3}} \qquad x = 0.577L \blacktriangleleft$$

$$M_{\max} = \frac{1}{6} w_0 L^2 \left[\frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}} \right)^3 \right] \qquad M_{\max} = 0.0642 w_0 L^2 \blacktriangleleft$$

Note: At $x = 0$, $\qquad A = V_A = \dfrac{1}{2} w_0 L \left(\dfrac{1}{3} \right) \qquad \mathbf{A} = \dfrac{1}{6} w_0 L \uparrow$



PROBLEM 7.87

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION

(*a*) We note that at $B(x = L)$: $V_B = 0$, $M_B = 0$ (1)

Load: $$w(x) = w_0\left(1 - \frac{x}{L}\right) - \frac{1}{3}w_0\left(\frac{x}{L}\right) = w_0\left(1 - \frac{4x}{3L}\right)$$

Shear: We use Eq. (7.2) between $C(x = x)$ and $B(x = L)$:

$$V_B - V_C = -\int_x^L w(x)dx \quad 0 - V(x) = -\int_x^L w(x)dx$$

$$V(x) = w_0\int_x^L \left(1 - \frac{4x}{3L}\right)dx$$

$$= w_0\left[x - \frac{2x^2}{3L}\right]_x^L = w_0\left(L - \frac{2L}{3} - x + \frac{2x^2}{3L}\right)$$

$$V(x) = \frac{w_0}{3L}(2x^2 - 3Lx + L^2) \qquad (2) \blacktriangleleft$$

Bending moment: We use to Eq. (7.4) between $C(x = x)$ and $B(x = L)$:

$$M_B - M_C = \int_x^L V(x)dx \quad 0 - M(x)$$

$$= \frac{w_0}{3L}\int_x^L (2x^2 - 3Lx + L^2)dx$$

$$M(x) = -\frac{w_0}{3L}\left[\frac{2}{3}x^3 - \frac{3}{2}Lx^2 + L^2x\right]^L$$

$$= -\frac{w_0}{18L}[4x^3 - 9Lx^2 + 6L^2x]_x^L$$

$$= -\frac{w_0}{18L}[(4L^3 - 9L^3 + 6L^3) - (4x^3 - 9Lx^2 + 6L^2x)]$$

$$M(x) = \frac{w_0}{18L}(4x^3 - 9Lx^2 + 6L^2x - L^3) \qquad (3) \blacktriangleleft$$

PROBLEM 7.87 (Continued)

(*b*) Maximum bending moment

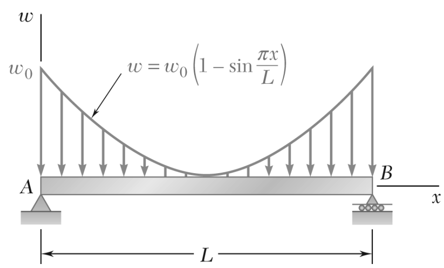
$$\frac{dM}{dx} = V = 0$$

Eq. (2): $$2x^2 - 3Lx + L^2 = 0$$

$$x = \frac{3 - \sqrt{9-8}}{4}L = \frac{L}{2}$$

Carrying into (3): $$M_{\max} = \frac{w_0 L^2}{72}, \quad \text{At} \quad x = \frac{L}{2}$$ ◀

Note: $$|M|_{\max} = \frac{w_0 L^2}{18} \quad \text{At} \quad x = 0$$



PROBLEM 7.88

For the beam and loading shown, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION

(*a*) Reactions at supports: $A = B = \frac{1}{2}W$, where $\frac{W}{L} = \text{Total load}$

$$W = \int_0^L w\,dx = w_0 \int_0^L \left(1 - \sin\frac{\pi x}{L}\right)dx$$

$$= w_0\left[x + \frac{L}{x}\cos\frac{\pi x}{L}\right]_0^L$$

$$= w_0 L\left(1 - \frac{2}{\pi}\right)$$

Thus $$V_A = A = \frac{1}{2}W = \frac{1}{2}w_0 L\left(1 - \frac{2}{\pi}\right)$$

$$M_A = 0 \qquad (1)$$

Load: $$w(x) = w_0\left(1 - \sin\frac{\pi x}{L}\right)$$

Shear: From Eq. (7.2):

$$V(x) - V_A = -\int_0^x w(x)\,dx$$

$$= -w_0 \int_0^x \left(1 - \sin\frac{\pi x}{L}\right)dx$$

Integrating and recalling first of Eqs. (1),

$$V(x) - \frac{1}{2}w_0 L\left(1 - \frac{2}{\pi}\right) = -w_0\left[x + \frac{L}{\pi}\cos\frac{\pi x}{L}\right]_0^x$$

$$V(x) = \frac{1}{2}w_0 L\left(1 - \frac{2}{\pi}\right) - w_0\left(2 + \frac{L}{\pi}\cos\frac{\pi x}{L}\right) + w_0\frac{L}{\pi}$$

$$V(x) = w_0\left(\frac{L}{2} - x - \frac{L}{\pi}\cos\frac{\pi x}{L}\right) \qquad (2) \blacktriangleleft$$

PROBLEM 7.88 (Continued)

Bending moment: From Eq. (7.4) and recalling that $M_A = 0$.

$$M(x) - M_A = \int_0^x V(x)dx$$

$$= w_0 \left[\frac{L}{2}x - \frac{1}{2}x^2 - \left(\frac{L}{\pi}\right)^2 \sin\frac{\pi x}{L} \right]_0^x$$

$$M(x) = \frac{1}{2}w_0 \left(Lx - x^2 - \frac{2L^2}{\pi^2}\sin\frac{\pi x}{L} \right) \quad (3) \blacktriangleleft$$

(*b*) Maximum bending moment

$$\frac{dM}{dx} = V = 0.$$

This occurs at $x = \frac{L}{2}$ as we may check from (2):

$$V\left(\frac{L}{2}\right) = w_0 \left(\frac{L}{2} - \frac{L}{2} - \frac{L}{\pi}\cos\frac{\pi}{2} \right) = 0$$

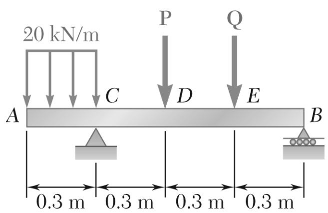
From (3):

$$M\left(\frac{L}{2}\right) = \frac{1}{2}w_0 \left(\frac{L^2}{2} - \frac{L^2}{4} - \frac{2L^2}{\pi^2}\sin\frac{\pi}{2} \right)$$

$$= \frac{1}{8}w_0 L^2 \left(1 - \frac{8}{\pi^2} \right)$$

$$= 0.0237 w_0 L^2$$

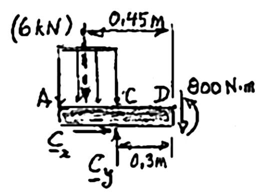
$$M_{max} = 0.0237 w_0 L^2, \quad \text{at} \quad x = \frac{L}{2} \quad \blacktriangleleft$$



PROBLEM 7.89

The beam AB is subjected to the uniformly distributed load shown and to two unknown forces $\mathbf{P}$ and $\mathbf{Q}$. Knowing that it has been experimentally determined that the bending moment is $+800 \text{ N}\cdot\text{m}$ at D and $+1300 \text{ N}\cdot\text{m}$ at E, (a) determine $\mathbf{P}$ and $\mathbf{Q}$, (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

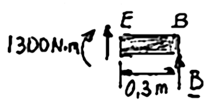


(a) Free body: Portion AD

$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -C_y(0.3 \text{ m}) + 0.800 \text{ kN}\cdot\text{m} + (6 \text{ kN})(0.45 \text{ m}) = 0$$

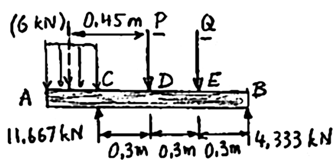
$$C_y = +11.667 \text{ kN} \qquad \mathbf{C} = 11.667 \text{ kN} \uparrow \ ◁$$



Free body: Portion EB

$$+\circlearrowleft \Sigma M_E = 0: \quad B(0.3 \text{ m}) - 1.300 \text{ kN}\cdot\text{m} = 0$$

$$\mathbf{B} = 4.333 \text{ kN} \uparrow \ ◁$$



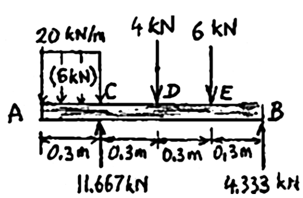
Free body: Entire beam

$$+\circlearrowleft \Sigma M_D = 0: \quad (6 \text{ kN})(0.45 \text{ m}) - (11.667 \text{ kN})(0.3 \text{ m}) - Q(0.3 \text{ m}) + (4.333 \text{ kN})(0.6 \text{ m}) = 0$$

$$\mathbf{Q} = 6.00 \text{ kN} \downarrow \ ◀$$

$$+\uparrow \Sigma M_y = 0: \quad 11.667 \text{ kN} + 4.333 \text{ kN} - 6 \text{ kN} - P - 6 \text{ kN} = 0$$

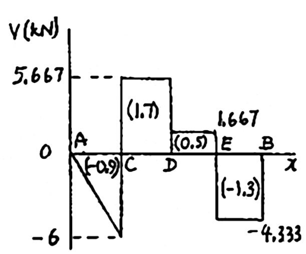
$$\mathbf{P} = 4.00 \text{ kN} \downarrow \ ◀$$



Load diagram

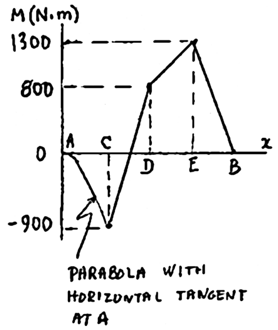
(b) Shear diagram

At A: $V_A = 0$



$$|V|_{max} = 6 \text{ kN} \ ◁$$

PROBLEM 7.89 (Continued)



<u>Bending-moment diagram</u>

At A: $M_A = 0$

$|M|_{\max} = 1300\ \text{N}\cdot\text{m}$ ◁

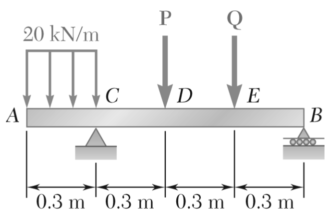
We check that

$$M_D = +800\ \text{N}\cdot\text{m} \quad \text{and} \quad M_E = +1300\ \text{N}\cdot\text{m}$$

As given:

At C: $M_C = -900\ \text{N}\cdot\text{m}$ ◀

PROBLEM 7.90

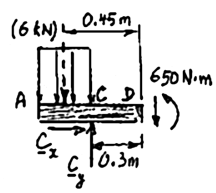


Solve Problem 7.89 assuming that the bending moment was found to be $+650\ \text{N}\cdot\text{m}$ at D and $+1450\ \text{N}\cdot\text{m}$ at E.

PROBLEM 7.89 The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+800\ \text{N}\cdot\text{m}$ at D and $+1300\ \text{N}\cdot\text{m}$ at E, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(*a*) Free body: Portion *AD*

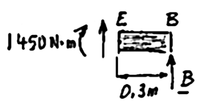


$$\Sigma F_x = 0: \quad C_x = 0$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -C(0.3\text{ m}) + 0.650\text{ kN}\cdot\text{m} + (6\text{ kN})(0.45\text{ m}) = 0$$

$$C_y = +11.167\text{ kN} \qquad \mathbf{C} = 11.167\text{ kN}\uparrow \ \triangleleft$$

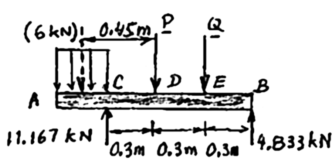
Free body: Portion *EB*



$$+\circlearrowleft \Sigma M_E = 0: \quad B(0.3\text{ m}) - 1.450\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{B} = 4.833\text{ kN}\uparrow \ \triangleleft$$

Free body: Entire beam



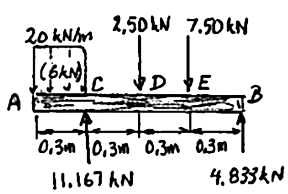
$$+\circlearrowleft \Sigma M_D = 0: \quad (6\text{ kN})(0.45\text{ m}) - (11.167\text{ kN})(0.3\text{ m})$$
$$-Q(0.3\text{ m}) + (4.833\text{ kN})(0.6\text{ m}) = 0$$

$$\mathbf{Q} = 7.50\text{ kN}\downarrow \ \blacktriangleleft$$

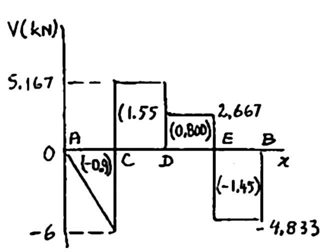
$$+\uparrow \Sigma M_y = 0: \quad 11.167\text{ kN} + 4.833\text{ kN}$$
$$-6\text{ kN} - P - 7.50\text{ kN} = 0$$

$$\mathbf{P} = 2.50\text{ kN}\downarrow \ \blacktriangleleft$$

Load diagram

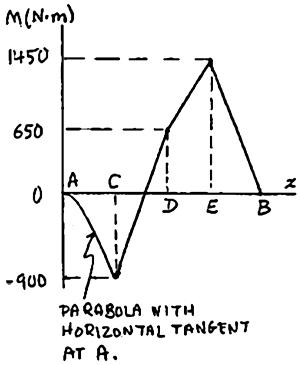


(*b*) Shear diagram



At *A*: $\quad V_A = 0$

$$|V|_{max} = 6\text{ kN} \ \triangleleft$$



Bending-moment diagram

At A: $M_A = 0$

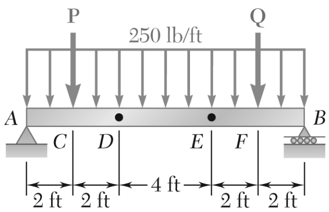
$$|M|_{\max} = 1450\ \text{N}\cdot\text{m} \ \triangleleft$$

We check that

$$M_D = +650\ \text{N}\cdot\text{m} \quad \text{and} \quad M_E = +1450\ \text{N}\cdot\text{m}$$

As given:

At C: $M_C = -900\ \text{N}\cdot\text{m}$ ◀

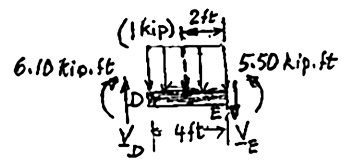


PROBLEM 7.91*

The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+6.10 \text{ kip}\cdot\text{ft}$ at D and $+5.50 \text{ kip}\cdot\text{ft}$ at E, (a) determine **P** and **Q**, (b) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(a) Free body: Portion DE



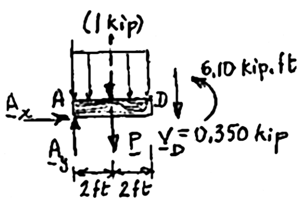
$$+\circlearrowleft \Sigma M_E = 0: \quad 5.50 \text{ kip}\cdot\text{ft} - 6.10 \text{ kip}\cdot\text{ft} + (1 \text{ kip})(2 \text{ ft}) - V_D(4 \text{ ft}) = 0$$

$$V_D = +0.350 \text{ kip}$$

$$+\uparrow \Sigma F_y = 0: \quad 0.350 \text{ kip} - 1 \text{ kip} - V_E = 0$$

$$V_E = -0.650 \text{ kip}$$

Free body: Portion AD



$$+\circlearrowleft \Sigma M_A = 0: \quad 6.10 \text{ kip}\cdot\text{ft} - P(2 \text{ ft}) - (1 \text{ kip})(2 \text{ ft}) - (0.350 \text{ kip})(4 \text{ ft}) = 0$$

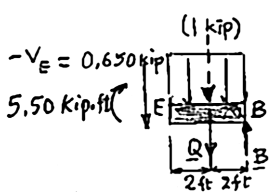
$$\mathbf{P} = 1.350 \text{ kips}\downarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1 \text{ kip} - 1.350 \text{ kip} - 0.350 \text{ kip} = 0$$

$$A_y = +2.70 \text{ kips} \qquad \mathbf{A} = 2.70 \text{ kips}\uparrow \blacktriangleleft$$

Free body: Portion EB



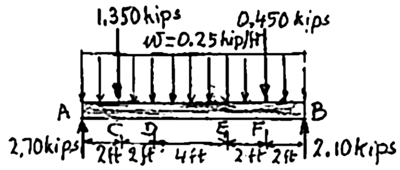
$$+\circlearrowleft \Sigma M_B = 0: \quad (0.650 \text{ kip})(4 \text{ ft}) + (1 \text{ kip})(2 \text{ ft}) + Q(2 \text{ ft}) - 5.50 \text{ kip}\cdot\text{ft} = 0$$

$$\mathbf{Q} = 0.450 \text{ kip}\downarrow \blacktriangleleft$$

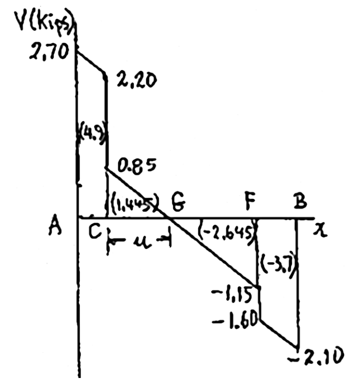
$$+\uparrow \Sigma F_y = 0: \quad B - 0.450 - 1 - 0.650 = 0$$

$$\mathbf{B} = 2.10 \text{ kips}\uparrow \blacktriangleleft$$

(*b*) Load diagram



Shear diagram



At A: $V_A = A = +2.70$ kips

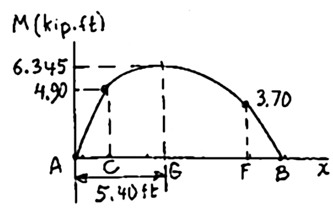
To determine Point G where $V = 0$, we write

$$V_G - V_C = -w\mu$$
$$0 - 0.85 \text{ kips} = -(0.25 \text{ kip/ft})\mu$$

$\mu = 3.40$ ft ◀

We next compute all areas

$|V|_{max} = 2.70$ kips at A ◀



Bending-moment diagram

At A: $M_A = 0$

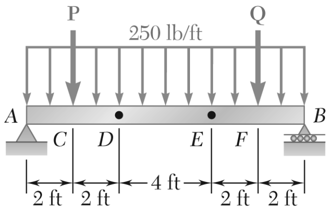
Largest value occurs at G with

$$AG = 2 + 3.40 = 5.40 \text{ ft}$$

$|M|_{max} = 6.345$ kip·ft ◀

5.40 ft from A ◀

Bending-moment diagram consists of 3 distinct arcs of parabolas.



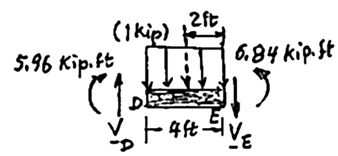
PROBLEM 7.92*

Solve Problem 7.91 assuming that the bending moment was found to be $+5.96\text{ kip}\cdot\text{ft}$ at D and $+6.84\text{ kip}\cdot\text{ft}$ at E.

PROBLEM 7.91* The beam AB is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+6.10\text{ kip}\cdot\text{ft}$ at D and $+5.50\text{ kip}\cdot\text{ft}$ at E, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

(*a*) Free body: Portion *DE*



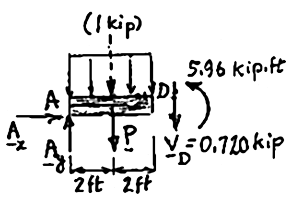
$$+\circlearrowleft \Sigma M_E = 0: \quad 6.84\text{ kip}\cdot\text{ft} - 5.96\text{ kip}\cdot\text{ft} + (1\text{ kip})(2\text{ ft}) - V_D(4\text{ ft}) = 0$$

$$V_D = +0.720\text{ kip}$$

$$+\uparrow \Sigma F_y = 0: \quad 0.720\text{ kip} - 1\text{kip} - V_E = 0$$

$$V_E = -0.280\text{ kip}$$

Free body: Portion *AD*



$$+\circlearrowleft \Sigma M_A = 0: \quad 5.96\text{ kip}\cdot\text{ft} - P(2\text{ ft}) - (1\text{ kip})(2\text{ ft}) - (0.720\text{ kip})(4\text{ ft}) = 0$$

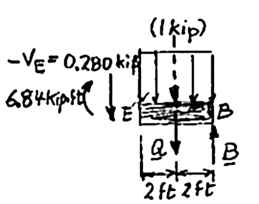
$$\mathbf{P} = 0.540\text{ kip}\downarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1\text{ kip} - 0.540\text{ kip} - 0.720\text{ kip} = 0$$

$$A_y = +2.26\text{ kips} \qquad \mathbf{A} = 2.26\text{ kips}\uparrow \triangleleft$$

Free body: Portion *EB*



$$+\circlearrowleft \Sigma M_B = 0: \quad (0.280\text{ kip})(4\text{ ft}) + (1\text{ kip})(2\text{ ft}) + Q(2\text{ ft}) - 6.84\text{ kip}\cdot\text{ft} = 0$$

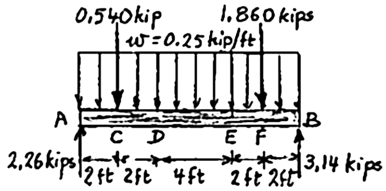
$$\mathbf{Q} = 1.860\text{ kips}\downarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad B - 1.860 - 1 - 0.280 = 0$$

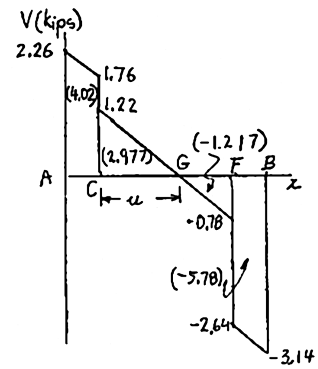
$$\mathbf{B} = 3.14\text{ kips}\uparrow \blacktriangleleft$$

PROBLEM 7.92* (Continued)

(*b*) Load diagram



Shear diagram



At A: $V_A = A = +2.26$ kips

To determine Point G where $V = 0$, we write

$$V_G - V_C = -w\mu$$

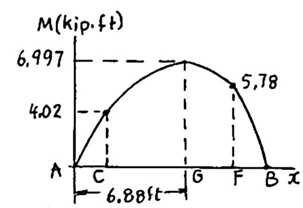
$$0 - (1.22 \text{ kips}) = -(0.25 \text{ kip/ft})\mu$$

$\mu = 4.88$ ft ◁

We next compute all areas

$|V|_{max} = 3.14$ kips at B ◀

Bending-moment diagram



At A: $M_A = 0$

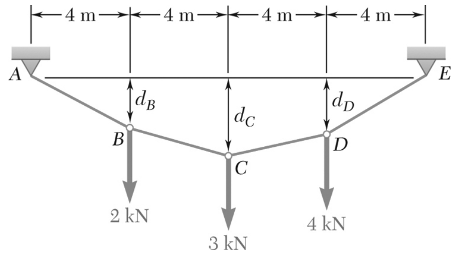
Largest value occurs at G with

$$AG = 2 + 4.88 = 6.88 \text{ ft}$$

$|M|_{max} = 6.997$ kip·ft ◀

6.88 ft from A ◀

Bending-moment diagram consists of 3 distinct arcs of parabolas.

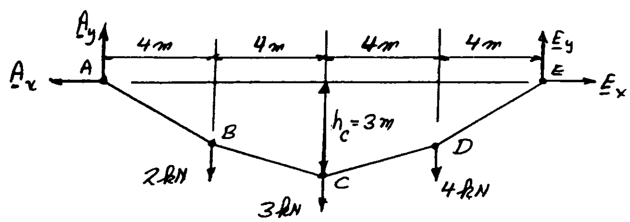


PROBLEM 7.93

Three loads are suspended as shown from the cable *ABCDE*. Knowing that $d_C = 3$ m, determine (*a*) the components of the reaction at *E*, (*b*) the maximum tension in the cable.

SOLUTION

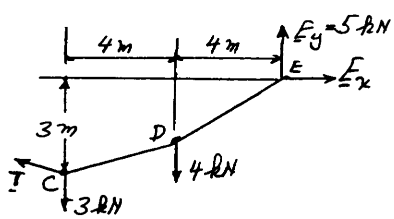
(*a*)



$$+\circlearrowleft \Sigma M_A = 0: \quad E_y(16\text{ m}) - (2\text{ kN})(4\text{ m}) - (3\text{ kN})(8\text{ m}) - (4\text{ kN})(12\text{ m}) = 0$$

$$E_y = +5\text{ kN} \qquad \mathbf{E}_y = 5.00\text{ kN}\uparrow \blacktriangleleft$$

Portion *CDE*:

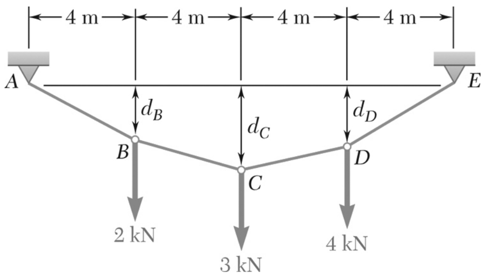


$$+\circlearrowleft \Sigma M_C = 0: \quad (5\text{ kN})(8\text{ m}) - (4\text{ kN})(4\text{ m}) - E_x(3\text{ m}) = 0$$

$$E_x = 8\text{ kN} \qquad \mathbf{E}_x = 8.00\text{ kN} \longrightarrow \blacktriangleleft$$

(*b*) Maximum tension occurs in *DE*:

$$T_m = \sqrt{E_x^2 + E_y^2} = \sqrt{8^2 + 5^2} \qquad T_m = 9.43\text{ kN} \blacktriangleleft$$

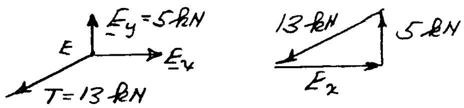


PROBLEM 7.94

Knowing that the maximum tension in cable *ABCDE* is 13 kN, determine the distance d_C.

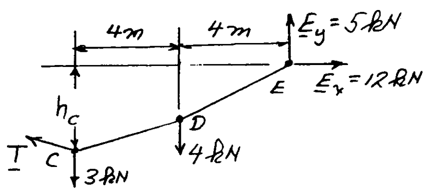
SOLUTION

Maximum tension of 13 kN occurs in *DE*. See solution of Problem 7.93 for the determination of $\mathbf{E}_y = 5.00 \text{ kN}\uparrow$



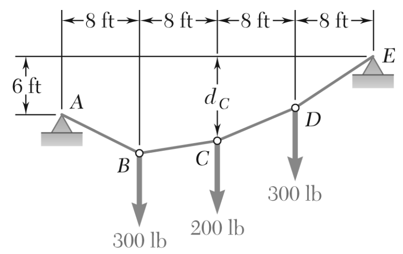
From force triangle $\quad E_x^2 + 5^2 = 13^2$

Portion CDE: $\quad E_x = 12 \text{ kN}$



$$+\circlearrowleft \Sigma M_C = 0: \quad (5 \text{ kN})(8 \text{ m}) - (12 \text{ kN})h_C - (4 \text{ kN})(4 \text{ m}) = 0$$

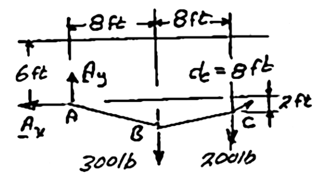
$$h_C = 2.00 \text{ m} \blacktriangleleft$$



PROBLEM 7.95

If $d_C = 8$ ft, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION



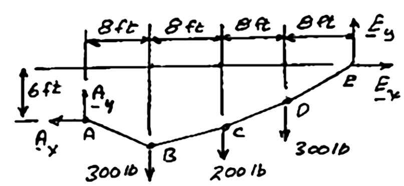
Free body: Portion *ABC*

$$+\circlearrowleft \Sigma M_C = 0$$

$$2A_x - 16A_y + 300(8) = 0$$

$$A_x = 8A_y - 1200 \qquad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_E = 0: \quad +6A_x + 32A_y - (300\text{ lb} + 200\text{ lb} + 300\text{ lb})16\text{ ft} = 0$$

$$3A_x + 16A_y - 6400 = 0$$

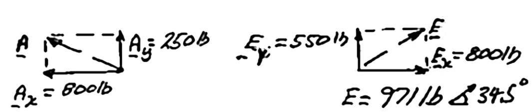
Substitute from Eq. (1):

$$3(8A_y - 1200) + 16A_y - 6400 = 0 \qquad \mathbf{A}_y = 250\text{ lb}\uparrow$$

Eq. (1) $\qquad A_x = 8(250) - 1200 \qquad \mathbf{A}_x = 800\text{ lb}\leftarrow$

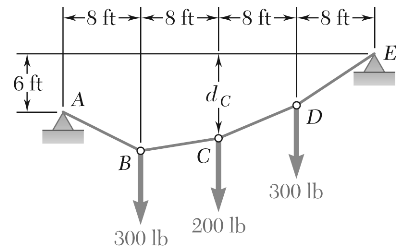
$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -800\text{ lb} + E_x = 0 \qquad \mathbf{E}_x = 800\text{ lb}\rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad 250 + E_y - 300 - 200 - 300 = 0 \qquad \mathbf{E}_y = 550\text{ lb}\uparrow$$



(*a*) $\qquad \mathbf{A} = 838\text{ lb} \searrow 17.35° \blacktriangleleft$

(*b*) $\qquad \mathbf{E} = 971\text{ lb} \nearrow 34.5° \blacktriangleleft$

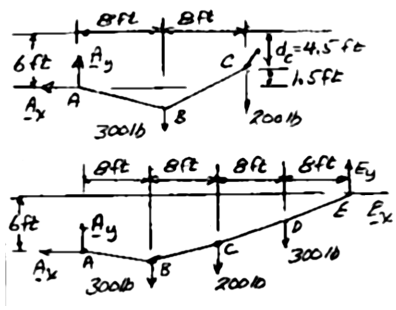


PROBLEM 7.96

If $d_C = 4.5$ ft, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION

Free body: Portion *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad -1.5A_x - 16A_y + 300 \times 8 = 0$$

$$A_x = \frac{(2400 - 16A_y)}{1.5} \qquad (1)$$

Free body: Entire cable

$$+\circlearrowleft \Sigma M_E = 0: \quad +6A_x + 32A_y - (300 \text{ lb} + 200 \text{ lb} + 300 \text{ lb})16 \text{ ft} = 0$$

$$3A_x + 16A_y - 6400 = 0$$

Substitute from Eq. (1):

$$\frac{3(2400 - 16A_y)}{1.5} + 16A_y - 6400 = 0$$

$$A_y = -100 \text{ lb}$$

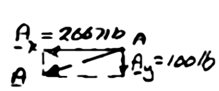
Thus A_y acts downward $\qquad \mathbf{A}_y = 100 \text{ lb} \downarrow$

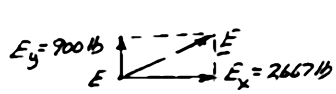
Eq. (1) $\qquad A_x = \dfrac{(2400 - 16(-100))}{1.5} = 2667 \text{ lb} \qquad \mathbf{A}_x = 2667 \text{ lb} \longleftarrow$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -2667 + E_x = 0 \qquad \mathbf{E}_x = 2667 \text{ lb} \longrightarrow$$

$$+\!\uparrow \Sigma F_y = 0: \quad A_y + E_y - 300 - 200 - 300 = 0$$

$$-100 \text{ lb} + E_y - 800 \text{ lb} = 0 \qquad \mathbf{E}_y = 900 \text{ lb} \uparrow$$

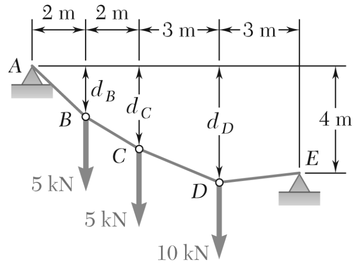




(*a*) $\qquad \mathbf{A} = 2670 \text{ lb} \measuredangle 2.10° \blacktriangleleft$

(*b*) $\qquad \mathbf{E} = 2810 \text{ lb} \measuredangle 18.65° \blacktriangleleft$

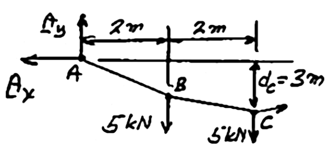
PROBLEM 7.97



Knowing that $d_C = 3$ m, determine (*a*) the distances d_B and d_D (*b*) the reaction at *E*.

SOLUTION

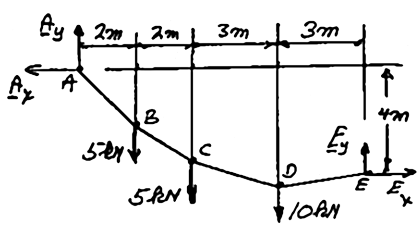
Free body: Portion *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad 3A_x - 4A_y + (5 \text{ kN})(2 \text{ m}) = 0$$

$$A_x = \frac{4}{3}A_y - \frac{10}{3} \qquad (1)$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_E = 0: \quad 4A_x - 10A_y + (5 \text{ kN})(8 \text{ m}) + (5 \text{ kN})(6 \text{ m}) + (10 \text{ kN})(3 \text{ m}) = 0$$

$$4A_x - 10A_y + 100 = 0$$

Substitute from Eq. (1):

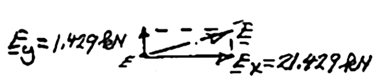
$$4\left(\frac{4}{3}A_y - \frac{10}{3}\right) - 10A_y + 100 = 0$$

$$A_y = +18.571 \text{ kN} \qquad \mathbf{A}_y = 18.571 \text{ kN}\uparrow$$

Eq. (1) $$A_x = \frac{4}{3}(18.511) - \frac{10}{3} = +21.429 \text{ kN} \qquad \mathbf{A}_x = 21.429 \text{ kN}\leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + E_x = 0 \quad -21.429 + E_x = 0 \qquad \mathbf{E}_x = 21.429 \text{ kN}\rightarrow$$

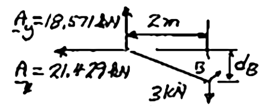
$$+\uparrow \Sigma F_y = 0: \quad 18.571 \text{ kN} + E_y + 5 \text{ kN} + 5 \text{ kN} + 10 \text{ kN} = 0 \qquad \mathbf{E}_y = 1.429 \text{ kN}\uparrow$$



(*b*) $\mathbf{E} = 21.5 \text{ kN} \measuredangle\ 3.81°$ ◀

PROBLEM 7.97 (Continued)

(*a*) Portion *AB*

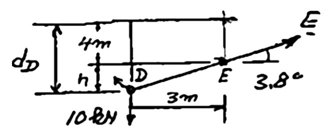


$$+\circlearrowleft \Sigma M_B = 0: \quad (18.571 \text{ kN})(2 \text{ m}) - (21.429 \text{ kN})d_B = 0$$

$d_B = 1.733$ m ◀

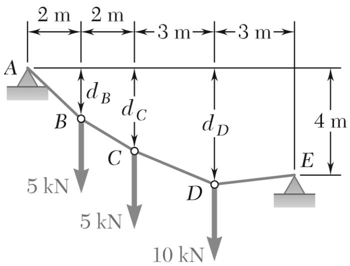
Portion *DE*

Geometry



$$h = (3 \text{ m}) \tan 3.8°$$
$$= 0.199 \text{ m}$$
$$d_D = 4 \text{ m} + 0.199 \text{ m}$$

$d_D = 4.20$ m ◀

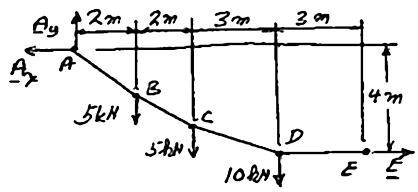


PROBLEM 7.98

Determine (*a*) distance d_C for which portion *DE* of the cable is horizontal, (*b*) the corresponding reactions at *A* and *E*.

SOLUTION

Free body: Entire cable

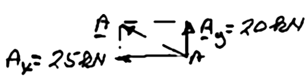


(*b*) $+\uparrow \Sigma F_y = 0$: $A_y - 5\text{ kN} - 5\text{ kN} - 10\text{ kN} = 0$ $\qquad$ $\mathbf{A}_y = 20\text{ kN}\uparrow$

$+\circlearrowleft \Sigma M_A = 0$: $E(4\text{ m}) - (5\text{ kN})(2\text{ m}) - (5\text{ kN})(4\text{ m}) - (10\text{ kN})(7\text{ m}) = 0$

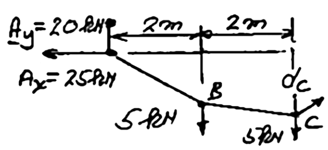
$E = +25\text{ kN}$ $\qquad$ $\mathbf{E} = 25.0\text{ kN} \longrightarrow$ ◀

$\Sigma F_x = 0$: $-A_x + 25\text{ kN} = 0$ $\qquad$ $\mathbf{A}_x = 25\text{ kN} \longleftarrow$



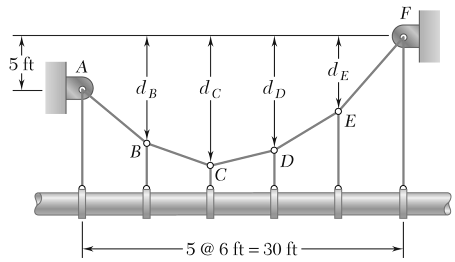
$\mathbf{A} = 32.0\text{ kN} \measuredangle\ 38.7°$ ◀

(*a*) Free body: Portion *ABC*



$+\circlearrowleft \Sigma M_C = 0$: $(25\text{ kN})d_C - (20\text{ kN})(4\text{ m}) + (5\text{ kN})(2\text{ m}) = 0$

$25d_C - 70 = 0$ $\qquad$ $d_C = 2.80\text{ m}$ ◀

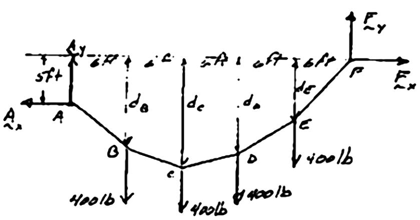


PROBLEM 7.99

An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD Cable: Hanger forces at *A* and *F* act on the supports, so $\mathbf{A}_y$ and $\mathbf{F}_y$ act on the cable.



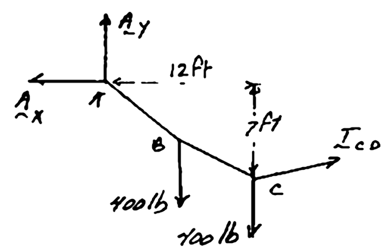
$$\circlearrowleft \Sigma M_F = 0: \quad (6\text{ ft} + 12\text{ ft} + 18\text{ ft} + 24\text{ ft})(400\text{ lb})$$

$$-(30\text{ ft})A_y - (5\text{ ft})A_x = 0$$

$$A_x + 6A_y = 4800\text{ lb} \qquad (1)$$

FBD ABC: $\circlearrowleft \Sigma M_C = 0: \quad (7\text{ ft})A_x - (12\text{ ft})A_y + (6\text{ ft})(400\text{ lb}) = 0 \qquad (2)$

Solving (1) and (2)



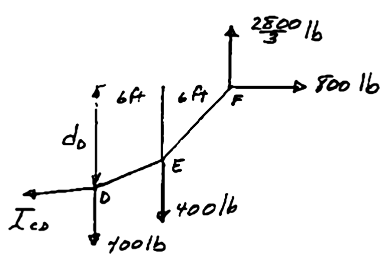
$$\mathbf{A}_x = 800\text{ lb} \longleftarrow$$

$$\mathbf{A}_y = \frac{2000}{3}\text{ lb}\uparrow$$

From FBD Cable: $\longrightarrow \Sigma F_x = 0: \quad -800\text{ lb} + F_x = 0$

$$\mathbf{F}_x = 800\text{ lb} \longrightarrow$$

FBD DEF:



$$\uparrow \Sigma F_y = 0: \quad \frac{200}{3}\text{ lb} - 4(400\text{ lb}) + F_y = 0$$

$$\mathbf{F}_y = \frac{2800}{3}\text{ lb}\uparrow$$

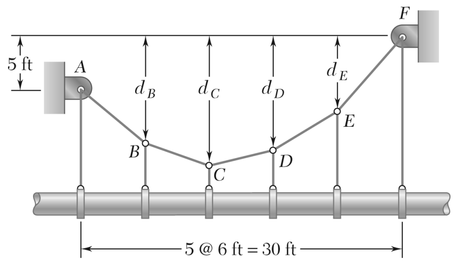
Since $A_x = F_x$ and $F_y > A_y$, $\qquad T_{max} = T_{EF} = \sqrt{(800\text{ lb})^2 + \left(\frac{2800}{3}\text{lb}\right)^2}$

PROBLEM 7.99 (Continued)

(*a*) $T_{\max} = 1229.27$ lb, $T_{\max} = 1229$ lb ◀

$$\circlearrowleft \Sigma M_D = 0: \quad (12\text{ ft})\left(\frac{2800}{3}\text{ lb}\right) - d_D(800\text{ lb}) - (6\text{ ft})(400\text{ lb}) = 0$$

(*b*) $d_D = 11.00$ ft ◀



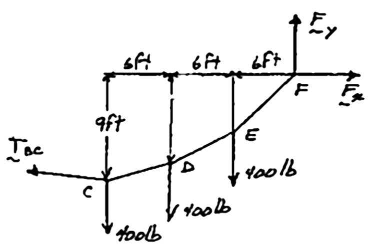
PROBLEM 7.100

Solve Problem 7.99 assuming that $d_C = 9$ ft.

PROBLEM 7.99 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

FBD CDEF:



$$\circlearrowleft \Sigma M_C = 0: \quad (18\text{ ft})F_y - (9\text{ ft})F_y - (6\text{ ft} + 12\text{ ft})(400\text{ lb}) = 0$$

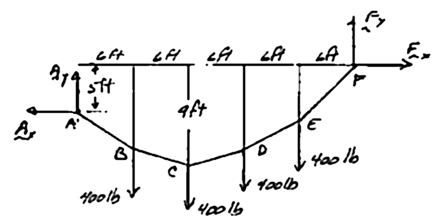
$$F_x - 2F_y = -800\text{ lb} \qquad (1)$$

FBD Cable:

$$\circlearrowleft EM_A = 0: \quad (30\text{ ft})F_y - (5\text{ ft})F_x$$
$$-(6\text{ ft})(1+2+3+4)(400\text{ lb}) = 0$$

$$F_x - 6F_y = -4800\text{ lb} \qquad (2)$$

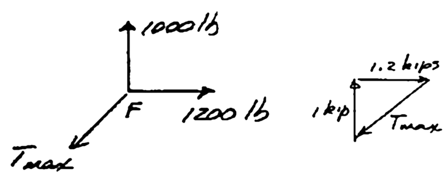
Solving (1) and (2),



$$\mathbf{F}_x = 1200\text{ lb} \longrightarrow, \quad \mathbf{F}_y = 1000\text{ lb}\uparrow$$

$$\longrightarrow \Sigma F_x = 0: \quad -A_x + 1200\text{ lb} = 0, \quad \mathbf{A}_x = 1200\text{ lb} \longrightarrow$$

Point F:



$$\uparrow \Sigma F_y = 0: \quad A_y + 1000\text{ lb} - 4(400\text{ lb}) = 0, \quad \mathbf{A}_y = 600\text{ lb}\uparrow$$

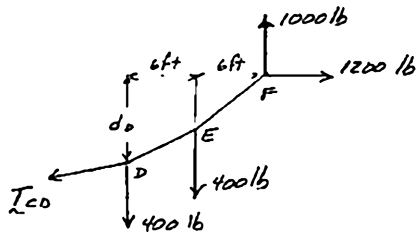
Since

$$A_x = A_y \quad \text{and} \quad F_y > A_y,\; T_{\max} = T_{EF}$$

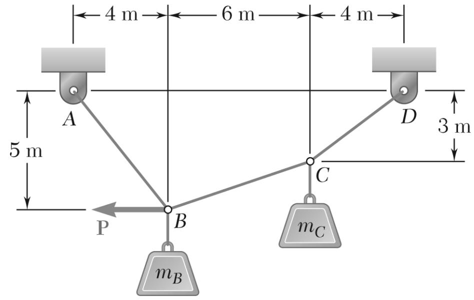
$$T_{\max} = \sqrt{(1\text{ kip})^2 + (1.2\text{ kips})^2}$$

(*a*) $T_{\max} = 1.562\text{ kips}$ ◀

FBD DEF: $\circlearrowleft \Sigma M_D = 0$: $(12\text{ ft})(1000\text{ lb}) - d_D(1200\text{ lb}) - (6\text{ ft})(400\text{ lb}) = 0$



(*b*) $d_D = 8.00\text{ ft}$ ◀

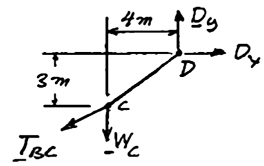


PROBLEM 7.101

Knowing that $m_B = 70$ kg and $m_C = 25$ kg, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

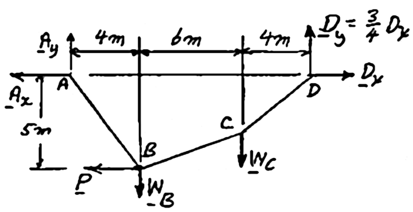
Free body: Portion *CD*



$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(4\text{ m}) - D_x(3\text{ m}) = 0$$

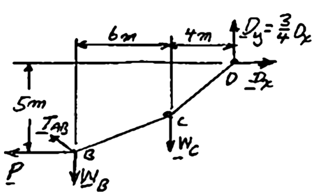
$$D_y = \frac{3}{4}D_x$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{4}D_x(14\text{ m}) - W_B(4\text{ m}) - W_C(10\text{ m}) - P(5\text{ m}) = 0 \tag{1}$$

Free body: Portion *BCD*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{4}D_x(10\text{ m}) - D_x(5\text{ m}) - W_C(6\text{ m}) = 0$$

$$D_x = 2.4W_C \tag{2}$$

For $\qquad m_B = 70\text{ kg} \quad m_C = 25\text{ kg}$

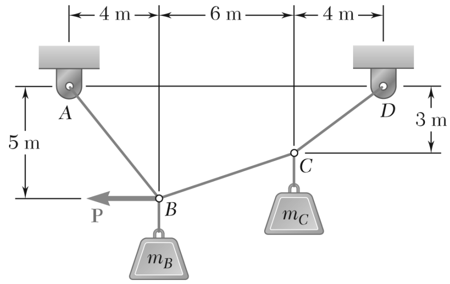
$g = 9.81\text{ m/s}^2$: $\qquad W_B = 70g \qquad W_C = 25g$

Eq. (2): $\qquad D_x = 2.4W_C = 2.4(25g) = 60g$

Eq. (1): $\qquad \frac{3}{4}60g(14) - 70g(4) - 25g(10) - 5P = 0$

$$100g - 5P = 0: \quad P = 20g$$

$$P = 20(9.81) = 196.2\text{ N} \qquad\qquad P = 196.2\text{ N} \blacktriangleleft$$

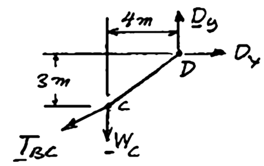


PROBLEM 7.102

Knowing that $m_B = 18$ kg and $m_C = 10$ kg, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

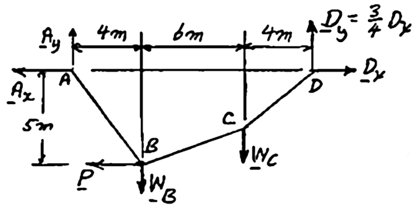
Free body: Portion *CD*



$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(4 \text{ m}) - D_x(3 \text{ m}) = 0$$

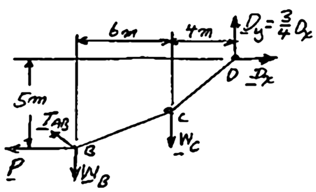
$$D_y = \frac{3}{4}D_x$$

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{4}D_x(14 \text{ m}) - W_B(4 \text{ m}) - W_C(10 \text{ m}) - P(5 \text{ m}) = 0 \qquad (1)$$

Free body: Portion *BCD*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{4}D_x(10 \text{ m}) - D_x(5 \text{ m}) - W_C(6 \text{ m}) = 0$$

$$D_x = 2.4W_C \qquad (2)$$

For $\quad m_B = 18 \text{ kg} \quad m_C = 10 \text{ kg}$

$g = 9.81 \text{ m/s}^2$: $\quad W_B = 18g \quad W_C = 10g$

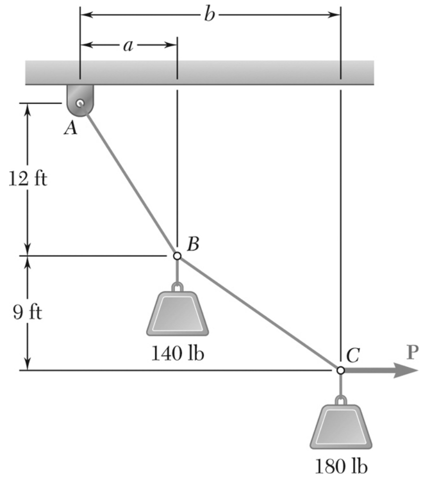
Eq. (2): $\quad D_x = 2.4W_C = 2.4(10g) = 24g$

Eq. (1): $\quad \frac{3}{4}24g(14) - (18g)(4) - (10g)(10) - 5P = 0$

$$80g - 5P: \quad P = 16g$$

$$P = 16(9.81) = 156.96 \text{ N}$$

$P = 157.0$ N ◀

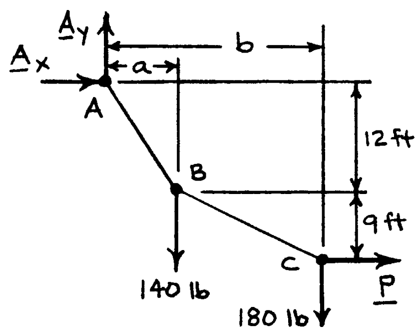


PROBLEM 7.103

Cable *ABC* supports two loads as shown. Knowing that $b = 21$ ft, determine (*a*) the required magnitude of the horizontal force **P**, (*b*) the corresponding distance *a*.

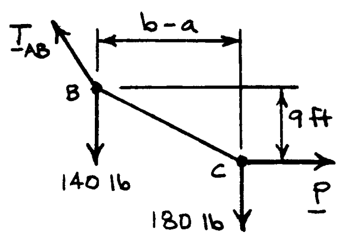
SOLUTION

Free body: *ABC*



$$+\circlearrowleft \Sigma M_A = 0: \quad P(12\text{ ft}) - (140\text{ lb})a - (180\text{ lb})b = 0 \qquad (1)$$

Free body: *BC*



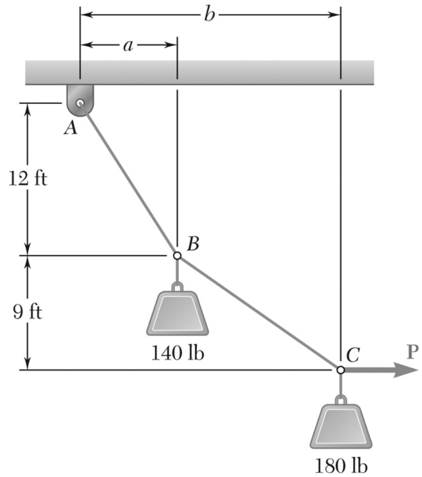
$$+\circlearrowleft \Sigma M_B = 0: \quad P(9\text{ ft}) - (180\text{ lb})(b - a) = 0 \qquad (2)$$

Data $\quad b = 21\text{ ft} \quad$ Eq. (1): $\quad 21P - 140a - 180(21) = 0 \qquad P = \frac{20}{3}a + 180 \qquad (3)$

Eq. (2): $\quad 9P - 180(21 - a) = 0 \qquad P = -20a + 420 \qquad (4)$

Equate (3) and (4) through P: $\quad \frac{20}{3}a + 180 = -20a + 420 \qquad$ (*b*) $\quad a = 9.00\text{ ft}$ ◀

Eq. (3): $\quad P = \frac{20}{3}(9.00) + 180 \qquad$ (*a*) $\quad P = 240\text{ lb}$ ◀

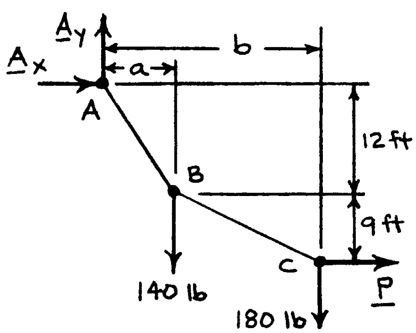


PROBLEM 7.104

Cable *ABC* supports two loads as shown. Determine the distances *a* and *b* when a horizontal force **P** of magnitude 200 lb is applied at *A*.

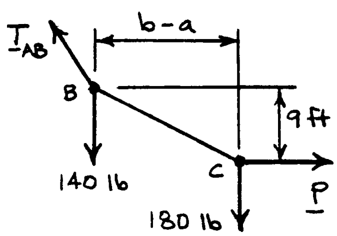
SOLUTION

<u>Free body: *ABC*</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad P(12\text{ ft}) - (140\text{ lb})a - (180\text{ lb})b = 0 \qquad (1)$$

<u>Free body: *BC*</u>



$$+\circlearrowleft \Sigma M_B = 0: \quad P(9\text{ ft}) - (180\text{ lb})(b-a) = 0 \qquad (2)$$

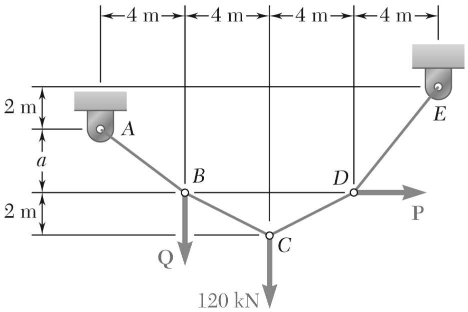
<u>Data</u> $\quad P = 200\text{ lb}$

$$\text{Eq. (1)}: \quad 200(21) - 140a - 180b = 0 \qquad (3)$$

$$\text{Eq. (2)}: \quad 200(9) + 180a - 180b = 0 \qquad (4)$$

$$(3)-(4): \quad 2400 - 320a = 0 \qquad a = 7.50\text{ ft} \blacktriangleleft$$

$$\text{Eq. (2)}: \quad (200\text{ lb})(9\text{ ft}) - (180\text{ lb})(b - 7.50\text{ ft}) = 0 \qquad b = 17.50\text{ ft} \blacktriangleleft$$

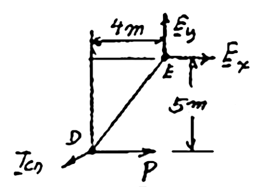


PROBLEM 7.105

If $a = 3$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

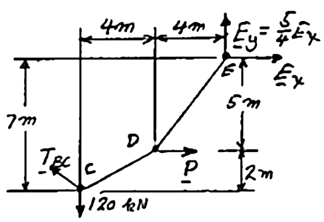
Free body: Portion *DE*



$$+\circlearrowleft \Sigma M_D = 0: \quad E_y(4 \text{ m}) - E_x(5 \text{ m}) = 0$$

$$E_y = \frac{5}{4}E_x$$

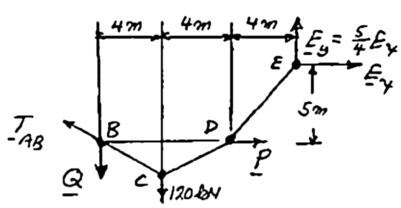
Free body: Portion *CDE*



$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{5}{4}E_x(8 \text{ m}) - E_x(7 \text{ m}) - P(2 \text{ m}) = 0$$

$$E_x = \frac{2}{3}P \qquad (1)$$

Free body: Portion *BCDE*



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{5}{4}E_x(12 \text{ m}) - E_x(5 \text{ m}) - (120 \text{ kN})(4 \text{ m}) = 0$$

$$10E_x - 480 = 0; \quad E_x = 48 \text{ kN}$$

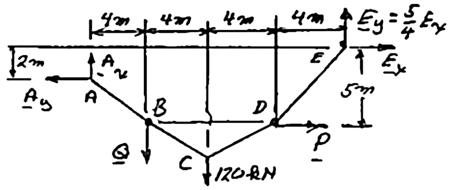
Eq. (1):

$$48 \text{ kN} = \frac{2}{3}P$$

$$P = 72.0 \text{ kN} \blacktriangleleft$$

PROBLEM 7.105 (Continued)

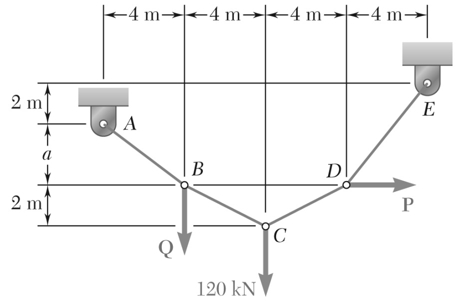
Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{5}{4}E_x(16\text{ m}) - E_x(2\text{ m}) + P(3\text{ m}) - Q(4\text{ m}) - (120\text{ kN})(8\text{ m}) = 0$$

$$(48\text{ kN})(20\text{ m} - 2\text{ m}) + (72\text{ kN})(3\text{ m}) - Q(4\text{ m}) - 960\text{ kN}\cdot\text{m} = 0$$

$$4Q = 120 \qquad Q = 30.0\text{ kN} \blacktriangleleft$$

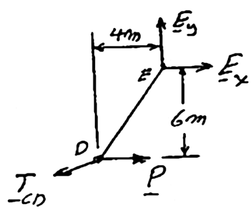


PROBLEM 7.106

If $a = 4$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

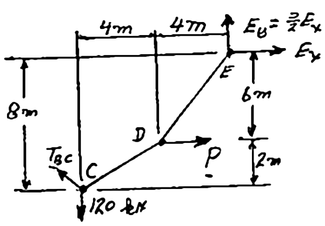
<u>Free body: Portion *DE*</u>



$$+\circlearrowleft \Sigma M_D = 0: \quad E_y(4 \text{ m}) - E_x(6 \text{ m}) = 0$$

$$E_y = \frac{3}{2} E_x$$

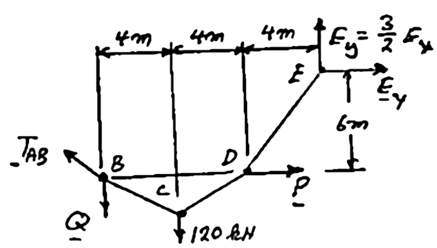
<u>Free body: Portion *CDE*</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{3}{2} E_x(8 \text{ m}) - E_x(8 \text{ m}) - P(2 \text{ m}) = 0$$

$$E_x = \frac{1}{2} P \qquad (1)$$

<u>Free body: Portion *BCDE*</u>



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{3}{2} E_x(12 \text{ m}) - E_x(6 \text{ m}) + (120 \text{ kN})(4 \text{ m}) = 0$$

$$12E_x = 480 \qquad E_x = 40 \text{ kN}$$

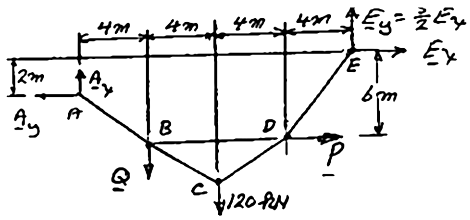
Eq (1):

$$E_x = \frac{1}{2} P; \qquad 40 \text{ kN} = \frac{1}{2} P$$

$$P = 80.0 \text{ kN} \blacktriangleleft$$

PROBLEM 7.106 (Continued)

Free body: Entire cable



$$+\circlearrowleft \Sigma M_A = 0: \quad \frac{3}{2}E_x(16 \text{ m}) - E_x(2 \text{ m}) + P(4 \text{ m}) - Q(4 \text{ m}) - (120 \text{ kN})(8 \text{ m}) = 0$$

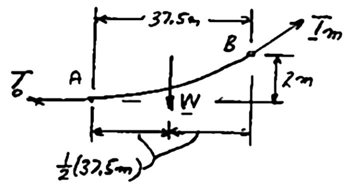
$$(40 \text{ kN})(24 \text{ m} - 2 \text{ m}) + (80 \text{ kN})(4 \text{ m}) - Q(4 \text{ m}) - 960 \text{ kN} \cdot \text{m} = 0$$

$$4Q = 240 \qquad Q = 60.0 \text{ kN} \blacktriangleleft$$

PROBLEM 7.107

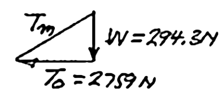
A transmission cable having a mass per unit length of 0.8 kg/m is strung between two insulators at the same elevation that are 75 m apart. Knowing that the sag of the cable is 2 m, determine (*a*) the maximum tension in the cable, (*b*) the length of the cable.

SOLUTION



$$w = (0.8\text{ kg/m})(9.81\text{ m/s}^2)$$
$$= 7.848\text{ N/m}$$
$$W = (7.848\text{ N/m})(37.5\text{ m})$$
$$W = 294.3\text{ N}$$

(*a*) $$+\circlearrowleft \Sigma M_B = 0: \quad T_0(2\text{ m}) - W\left(\frac{1}{2}37.5\text{ m}\right) = 0$$



$$T_0(2\text{ m}) - (294.3\text{ N})\frac{1}{2}(37.5\text{ m}) = 0$$

$$T_0 = 2759\text{ N}$$

$$T_m^2 = (294.3\text{ N})^2 + (2759\text{ N})^2 \qquad T_m = 2770\text{ N} \blacktriangleleft$$

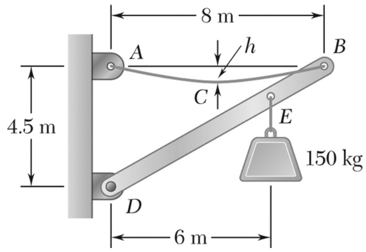
(*b*) $$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$$

$$= 37.5\text{ m}\left[1 + \frac{2}{3}\left(\frac{2\text{ m}}{37.5\text{ m}}\right)^2 + \cdots\right]$$

$$= 37.57\text{ m}$$

$$\text{Length} = 2s_B = 2(37.57\text{ m}) \qquad \text{Length} = 75.14\text{ m} \blacktriangleleft$$

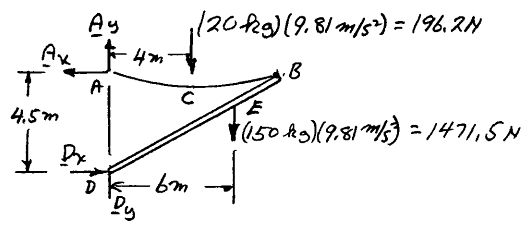
PROBLEM 7.108



The total mass of cable ACB is 20 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine (*a*) the sag h, (*b*) the slope of the cable at A.

SOLUTION

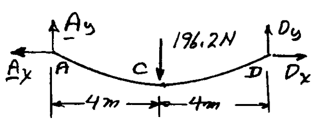
Free body: Entire frame



$$+\circlearrowleft \Sigma M_D = 0: \quad A_x(4.5\text{ m}) - (196.2\text{ N})(4\text{ m}) - (1471.5\text{ N})(6\text{ m}) = 0$$

$$A_x = 2136.4\text{ N}$$

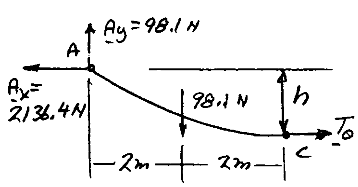
Free body: Entire cable



$$+\circlearrowleft \Sigma M_D = 0: \quad A_y(8\text{ m}) - (196.2\text{ N})(4\text{ m}) = 0$$

$$A_y = 98.1\text{ N}$$

(*a*) Free body: Portion AC



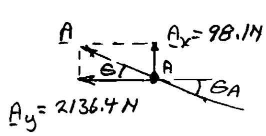
$$\Sigma F_x = 0: \quad T_0 = A_x = 2136.4\text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad T_0 h - (98.1\text{ N})(2\text{ m}) = 0$$

$$(2136.4\text{ N})h - 196.2\text{ N}\cdot\text{m} = 0$$

$$h = 0.09183\text{ m} \qquad h = 91.8\text{ mm} \blacktriangleleft$$

(*b*)



$$\tan\theta_A = \frac{A_x}{A_y} = \frac{98.1\text{ N}}{2136.4\text{ N}}$$

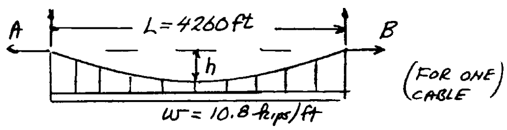
$$\tan\theta_A = 0.045918$$

$$\theta_A = 2.629° \qquad \theta_A = 2.63° \blacktriangleleft$$

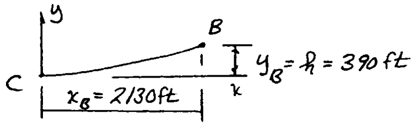
PROBLEM 7.109

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The uniform load supported by each cable is w = 10.8 kips/ft along the horizontal. Knowing that the span L is 4260 ft and that the sag h is 390 ft, determine (a) the maximum tension in each cable, (b) the length of each cable.

SOLUTION



(a)



At B:
$$y_B = \frac{wx_B^2}{2T_0}$$

$$390\text{ ft} = \frac{(10.8\text{ kips/ft})(2130\text{ ft})^2}{2T_0} \qquad T_0 = 62{,}819\text{ kips}$$

$$T_m = \sqrt{T_0^2 + w^2x_B^2}$$
$$= \sqrt{(62{,}819\text{ kips})^2 + (10.8\text{ kips/ft})^2(2130\text{ ft})^2}$$
$$= \sqrt{62{,}819^2 + (23{,}004)^2} = 66{,}898\text{ kips} \qquad T_m = 66{,}900\text{ kips} \blacktriangleleft$$

(b) Length of cable

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4\right]$$

$$= (2130\text{ ft})\left[1 + \frac{2}{3}\left(\frac{390\text{ ft}}{2130\text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{390\text{ ft}}{2130\text{ ft}}\right)^4\right] = 2176.65\text{ ft}$$

Total length: $2S_B = 2(2176.65\text{ ft}) = 4353.3\text{ ft}$ $\qquad L = 4353\text{ ft} \blacktriangleleft$

PROBLEM 7.110

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The design of the bridge allows for the effect of extreme temperature changes that cause the sag of the center span to vary from $h_w = 386$ ft in winter to $h_s = 394$ ft in summer. Knowing that the span is $L = 4260$ ft, determine the change in length of the cables due to extreme temperature changes.

SOLUTION

Eq. 7.10:

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right]$$

Winter:

$$y_B = h = 386 \text{ ft}, \quad x_B = \frac{1}{2}L = 2130 \text{ ft}$$

$$s_B = (2130)\left[1 + \frac{2}{3}\left(\frac{386}{2130}\right)^2 - \frac{2}{5}\left(\frac{386}{2130}\right)^4 + \cdots\right] = 2175.715 \text{ ft}$$

Summer:

$$y_B = h = 394 \text{ ft}, \quad x_B = \frac{1}{2}L = 2130 \text{ ft}$$

$$s_B = (2130)\left[1 + \frac{2}{3}\left(\frac{394}{2130}\right)^2 - \frac{2}{5}\left(\frac{394}{2130}\right)^4 + \cdots\right] = 2177.59 \text{ ft}$$

$$\Delta = 2(\Delta s_B) = 2(2177.59 \text{ ft} - 2175.715 \text{ ft}) = 2(1.875 \text{ ft})$$

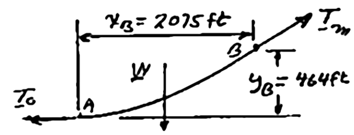
Change in length $= 3.75$ ft ◀

PROBLEM 7.111

Each cable of the Golden Gate Bridge supports a load $w = 11.1$ kips/ft along the horizontal. Knowing that the span L is 4150 ft and that the sag h is 464 ft, determine (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION

Eq. (7.8) Page 386:

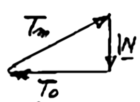


At B:

$$y_B = \frac{wx_B^2}{2T_0}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(11.1\text{ kip/ft})(2075\text{ ft})^2}{2(464\text{ ft})}$$

(*a*)



$$T_0 = 51.500\text{ kips}$$

$$W = wx_B = (11.1\text{ kips/ft})(2075\text{ ft}) = 23.033\text{ kips}$$

$$T_m = \sqrt{T_0^2 + W^2} = \sqrt{(51.500\text{ kips})^2 + (23.033\text{ kips})^2}$$

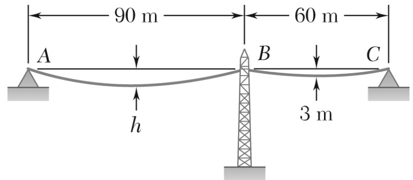
$$T_m = 56,400\text{ kips} \blacktriangleleft$$

(*b*)

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{y_B}\right)^4 + \cdots\right] \qquad \frac{y_B}{x_B} = \frac{464\text{ ft}}{2075\text{ ft}} = 0.22361$$

$$s_B = (2075\text{ ft})\left[1 + \frac{2}{3}(0.22361)^2 - \frac{2}{5}(0.22361)^4 + \cdots\right] = 2142.1\text{ ft}$$

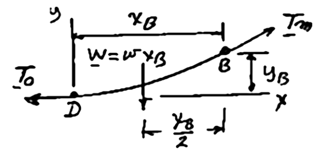
$$\text{Length} = 2s_B = 2(2142.1\text{ ft}) \qquad \underline{\text{Length}} = 4284\text{ ft} \blacktriangleleft$$



PROBLEM 7.112

Two cables of the same gauge are attached to a transmission tower at B. Since the tower is slender, the horizontal component of the resultant of the forces exerted by the cables at B is to be zero. Knowing that the mass per unit length of the cables is 0.4 kg/m, determine (*a*) the required sag h, (*b*) the maximum tension in each cable.

SOLUTION



$$W = wx_B$$

$$+\circlearrowleft \Sigma M_B = 0: \quad T_0 \; y_B - (wx_B)\frac{x_B}{2} = 0$$

(*a*) $\qquad$ Horiz. comp. $= T_0 = \dfrac{wx_B^2}{2y_B}$

<u>Cable *AB*</u>: $\qquad x_B = 45 \text{ m}$

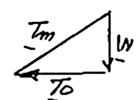
$$T_0 = \frac{w(45 \text{ m})^2}{2h}$$

<u>Cable *BC*</u>: $\qquad x_B = 30 \text{ m}, \quad y_B = 3 \text{ m}$

$$T_0 = \frac{w(30 \text{ m})^2}{2(3 \text{ m})}$$

Equate $T_0 = T_0$ $\qquad \dfrac{w(45 \text{ m})^2}{2h} = \dfrac{w(30 \text{ m})^2}{2(3 \text{ m})}$ $\qquad$ $h = 6.75 \text{ m}$ ◀

(*b*) $\qquad T_m^2 = T_0^2 + W^2$



<u>Cable *AB*</u>: $\qquad w = (0.4 \text{ kg/m})(9.81 \text{ m/s}) = 3.924 \text{ N/m}$

$$x_B = 45 \text{ m}, \quad y_B = h = 6.75 \text{ m}$$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924 \text{ N/m})(45 \text{ m})^2}{2(6.75 \text{ m})} = 588.6 \text{ N}$$

$$W = wx_B = (3.924 \text{ N/m})(45 \text{ m}) = 176.58 \text{ N}$$

$$T_m^2 = (588.6 \text{ N})^2 + (176.58 \text{ N})^2$$

<u>For *AB*</u>: $\qquad$ $T_m = 615 \text{ N}$ ◀

PROBLEM 7.112 (Continued)

Cable BC: $x_B = 30\text{ m}, \quad y_B = 3\text{ m}$

$$T_0 = \frac{wx_B^2}{2y_B} = \frac{(3.924\text{ N/m})(30\text{ m})^2}{2(3\text{ m})} = 588.6\text{ N} \quad \text{(Checks)}$$

$$W = wx_B = (3.924\text{ N/m})(30\text{ m}) = 117.72\text{ N}$$

$$T_m^2 = (588.6\text{ N})^2 + (117.72\text{ N})^2$$

For BC: $T_m = 600\text{ N}$ ◀

PROBLEM 7.113

A 50.5-m length of wire having a mass per unit length of 0.75 kg/m is used to span a horizontal distance of 50 m. Determine (*a*) the approximate sag of the wire, (*b*) the maximum tension in the wire. [*Hint:* Use only the first two terms of Eq. (7.10).]

SOLUTION

<u>First two terms of Eq. 7.10</u>

(*a*)

$$s_B = \frac{1}{2}(50.5 \text{ m}) = 25.25 \text{ m},$$

$$x_B = \frac{1}{2}(50 \text{ m}) = 25 \text{ m}$$

$$y_B = h$$

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$25.25 \text{ m} = 25 \text{ m}\left[1 - \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

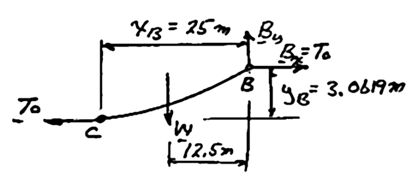
$$\left(\frac{y_B}{x_B}\right)^2 = 0.01\left(\frac{3}{2}\right)^2 = \sqrt{0.015}$$

$$\frac{y_B}{x_B} = 0.12247$$

$$\frac{h}{25\text{m}} = 0.12247$$

$$h = 3.0619 \text{ m} \qquad\qquad h = 3.06 \text{ m} ◀$$

(*b*) <u>Free body: Portion *CB*</u>



$$w = (0.75 \text{ kg/m})(9.81\text{m}) = 7.3575 \text{ N/m}$$

$$W = s_B\ w = (25.25 \text{ m})(7.3575 \text{ N/m})$$

$$W = 185.78 \text{ N}$$

$$+\circlearrowleft \Sigma M_0 = 0: \quad T_0\,(3.0619 \text{ m}) - (185.78 \text{ N})(12.5 \text{ m}) = 0$$

$$T_0 = 758.4 \text{ N}$$

$$B_x = T_0 = 758.4 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 185.78 \text{ N} = 0 \quad B_y = 185.78 \text{ N}$$

$$T_m = \sqrt{B_x^2 + B_y^2} = \sqrt{(758.4 \text{ N})^2 + (185.78 \text{ N})^2} \qquad\qquad T_m = 781 \text{ N} ◀$$

PROBLEM 7.114

A cable of length $L + \Delta$ is suspended between two points that are at the same elevation and a distance L apart. (*a*) Assuming that Δ is small compared to L and that the cable is parabolic, determine the approximate sag in terms of L and Δ. (*b*) If $L = 100$ ft and $\Delta = 4$ ft, determine the approximate sag. [*Hint:* Use only the first two terms of Eq. (7.10).

SOLUTION

Eq. 7.10 (First two terms)

(*a*)

$$s_B = x_B\left[1+\frac{2}{3}\left(\frac{y_B}{x_B}\right)^2\right]$$

$$x_B = L/2$$

$$s_B = \frac{1}{2}(L+\Delta)$$

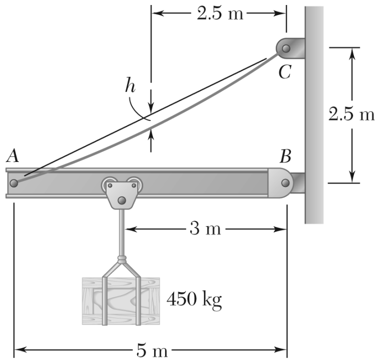
$$y_B = h$$

$$\frac{1}{2}(L+\Delta) = \frac{L}{2}\left[1+\frac{2}{3}\left(\frac{h}{\frac{L}{2}}\right)^2\right]$$

$$\frac{\Delta}{2} = \frac{4}{3}\frac{h^2}{L}; \quad h^2 = \frac{3}{8}L\Delta; \qquad h = \sqrt{\frac{3}{8}L\Delta} \blacktriangleleft$$

(*b*)

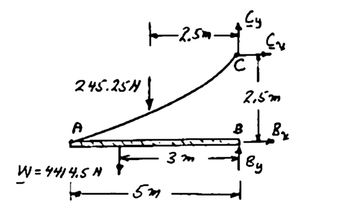
$$L = 100 \text{ ft}, \quad h = 4 \text{ ft}. \qquad h = \sqrt{\frac{3}{8}(100)(4)}; \qquad h = 12.25 \text{ ft} \blacktriangleleft$$

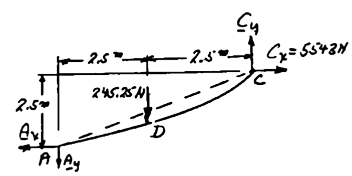


PROBLEM 7.115

The total mass of cable AC is 25 kg. Assuming that the mass of the cable is distributed uniformly along the horizontal, determine the sag h and the slope of the cable at A and C.

SOLUTION





Cable: $m = 25$ kg

$W = 25\,(9.81)$

$= 245.25$ N

Block: $m = 450$ kg

$W = 4414.5$ N

$$+\circlearrowleft \Sigma M_B = 0: \quad (245.25)(2.5) + (4414.5)(3) - C_x(2.5) = 0$$

$$C_x = 5543 \text{ N}$$

$$\Sigma F_x = 0: \quad A_x = C_x = 5543 \text{ N}$$

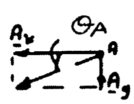
$$+\circlearrowleft \Sigma M_A = 0: \quad C_y(5) - (5543)(2.5) - (245.25)(2.5) = 0$$

$$\mathbf{C}_y = 2894 \text{ N} \uparrow$$

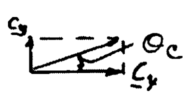
$$+\uparrow \Sigma F_y = 0: \quad C_y - A_y - 245.25 \text{ N} = 0$$

$$2894\ N - A_y - 245.25 \text{ N} = 0 \qquad \mathbf{A}_y = 2649 \text{ N} \downarrow$$

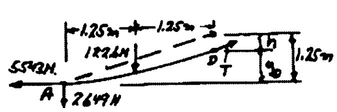
Point A: $\tan\theta_A = \dfrac{A_y}{A_x} = \dfrac{2649}{5543} = 0.4779;$ $\qquad \theta_A = 25.5° ◀$



Point C: $\tan\theta_C = \dfrac{C_y}{C_x} = \dfrac{2894}{5543} = 0.5221;$ $\qquad \theta_A = 27.6° ◀$



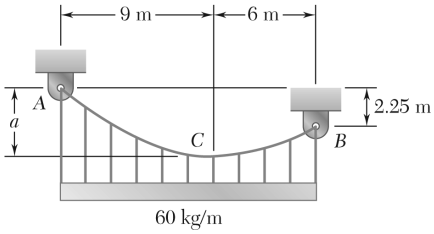
Free body: Half cable $\qquad W = (12.5 \text{ kg})\, g = 122.6$



$$+\circlearrowleft \Sigma M_0 = 0: \quad (122.6 \text{ N})(1.25 \text{ M}) + (2649 \text{ N})(2.5 \text{ m}) - (5543 \text{ N})\, y_d = 0$$

$$y_d = 1.2224 \text{ m; sag} = h = 1.25 \text{ m} - 1.2224 \text{ m}$$

$$h = 0.0276 \text{ m} = 27.6 \text{ mm} ◀$$

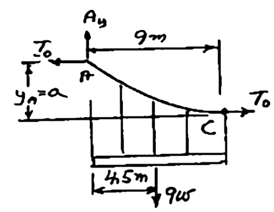


PROBLEM 7.116

Cable *ACB* supports a load uniformly distributed along the horizontal as shown. The lowest Point *C* is located 9 m to the right of *A*. Determine (*a*) the vertical distance *a*, (*b*) the length of the cable, (*c*) the components of the reaction at *A*.

SOLUTION

<u>Free body: Portion *AC*</u>

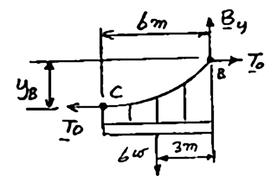


$$+\uparrow \Sigma F_y = 0: \quad A_y - 9w = 0$$

$$\mathbf{A}_y = 9w \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad T_0 a - (9w)(4.5 \text{ m}) = 0 \qquad (1)$$

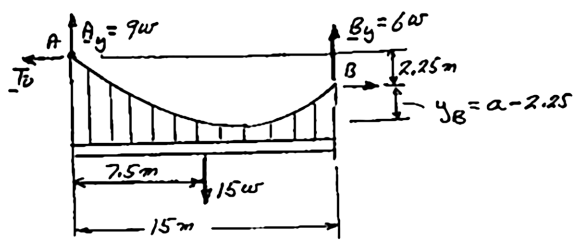
<u>Free body: Portion *CB*</u>



$$+\uparrow \Sigma F_y = 0: \quad B_y - 6w = 0$$

$$\mathbf{B}_y = 6w \uparrow$$

<u>Free body: Entire cable</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad 15w(7.5 \text{ m}) - 6w(15 \text{ m}) - T_0(2.25 \text{ m}) = 0$$

(*a*) $$T_0 = 10w$$

Eq. (1): $$T_0 a - (9w)(4.5 \text{ m}) = 0$$

$$10wa = (9w)(4.5) = 0 \qquad a = 4.05 \text{ m} \blacktriangleleft$$

PROBLEM 7.116 (Continued)

(*b*) <u>Length $= AC + CB$</u>

<u>Portion *AC*</u>: $x_A = 9 \text{ m}, \quad y_A = a = 4.05 \text{ m}; \quad \dfrac{y_A}{x_A} = \dfrac{4.05}{9} = 0.45$

$$s_{AC} = x_B\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_A}\right)^4 + \cdots\right]$$

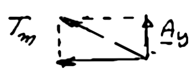
$$s_{AC} = 9 \text{ m}\left(1 + \frac{2}{3}0.45^2 - \frac{2}{5}0.45^4 + \cdots\right) = 10.067 \text{ m}$$

<u>Portion *CB*</u>: $x_B = 6 \text{ m}, \quad y_B = 4.05 - 2.25 = 1.8 \text{ m}; \quad \dfrac{y_B}{x_B} = 0.3$

$$s_{CB} = 6 \text{ m}\left(1 + \frac{2}{3}0.3^2 - \frac{2}{5}0.3^4 + \cdots\right) = 6.341 \text{ m}$$

Total length $= 10.067 \text{ m} + 6.341 \text{ m}$ Total length $= 16.41$ m ◀

(*c*) <u>Components of reaction at *A*.</u>

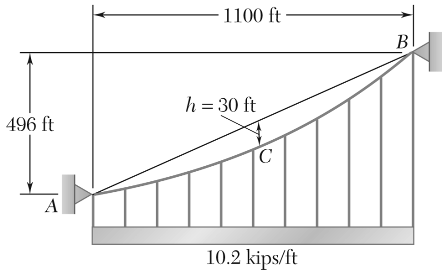


$$A_y = 9w = 9(60 \text{ kg/m})(9.81 \text{ m/s}^2)$$
$$= 5297.4 \text{ N}$$
$$A_x = T_0 = 10w = 10(60 \text{ kg/m})(9.81 \text{ m/s}^2)$$
$$= 5886 \text{ N}$$

$\mathbf{A}_x = 5890 \text{ N} \longleftarrow$ ◀

$\mathbf{A}_y = 5300 \text{ N} \uparrow$ ◀

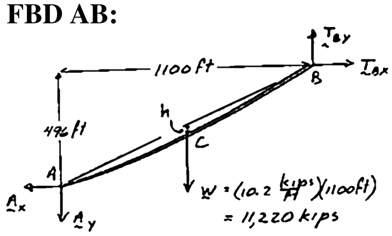
PROBLEM 7.117



Each cable of the side spans of the Golden Gate Bridge supports a load $w = 10.2$ kips/ft along the horizontal. Knowing that for the side spans the maximum vertical distance h from each cable to the chord AB is 30 ft and occurs at midspan, determine (*a*) the maximum tension in each cable, (*b*) the slope at B.

SOLUTION

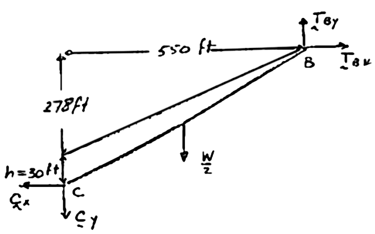
FBD AB:



$$\Sigma M_A = 0: \quad (1100 \text{ ft})T_{By} - (496 \text{ ft})T_{Bx} - (550 \text{ ft})W = 0$$

$$11T_{By} - 4.96T_{Bx} = 5.5W \qquad (1)$$

FBD CB:



$$\Sigma M_C = 0: \quad (550 \text{ ft})T_{By} - (278 \text{ ft})T_{Bx} - (275 \text{ ft})\frac{W}{2} = 0$$

$$11T_{By} - 5.56T_{Bx} = 2.75W \qquad (2)$$

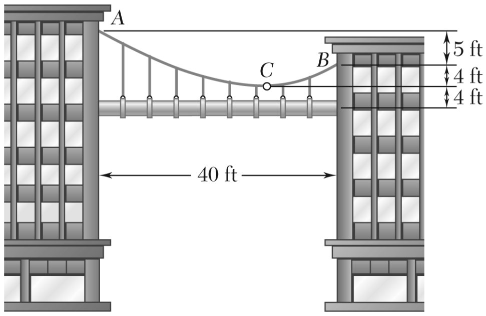
Solving (1) and (2) $\qquad T_{By} = 28,798 \text{ kips}$

$$T_{Bx} = 51,425 \text{ kips}$$

$$T_{\max} = T_B = \sqrt{T_{B_x}^2 + T_{B_y}^2} \qquad \tan\theta_B = \frac{T_{B_y}}{T_{B_x}}$$

So that $\qquad$ (*a*) $\qquad T_{\max} = 58,900 \text{ kips}$ ◀

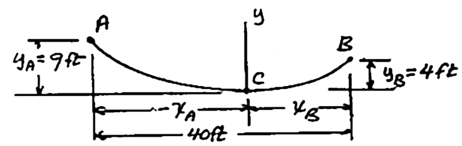
(*b*) $\qquad \theta_B = 29.2°$ ◀



PROBLEM 7.118

A steam pipe weighting 45 lb/ft that passes between two buildings 40 ft apart is supported by a system of cables as shown. Assuming that the weight of the cable system is equivalent to a uniformly distributed loading of 5 lb/ft, determine (*a*) the location of the lowest Point *C* of the cable, (*b*) the maximum tension in the cable.

SOLUTION



Note: $$x_B - x_A = 40 \text{ ft}$$

or $$x_A = x_B - 40 \text{ ft}$$

(*a*) Use Eq. 7.8

Point *A*: $$y_A = \frac{wx_A^2}{2T_0}; \quad 9 = \frac{w(x_B - 40)^2}{2T_0} \tag{1}$$

Point *B*: $$y_B = \frac{wx_B^2}{2T_0}; \quad 4 = \frac{wx_B^2}{2T_0} \tag{2}$$

Dividing (1) by (2): $$\frac{9}{4} = \frac{(x_B - 40)^2}{x_B^2}; \quad x_B = 16 \text{ ft}$$

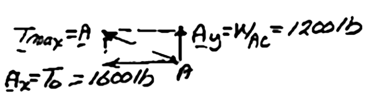
Point *C* is 16.00 ft to left of *B* ◀

(*b*) Maximum slope and thus $T_{\max}$ is at *A*

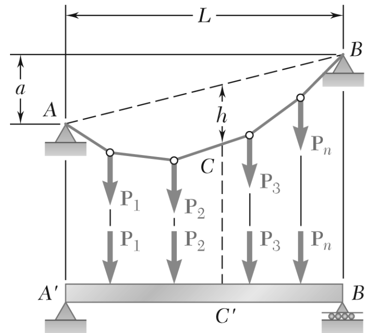
$$x_A = x_B - 40 = 16 - 40 = -24 \text{ ft}$$

$$y_A = \frac{wx_A^2}{2T_0}; \quad 9 \text{ ft} = \frac{(50 \text{ lb/ft})(-24 \text{ ft})^2}{2T_0}; \quad T_0 = 1600 \text{ lb}$$

$$W_{AC} = (50 \text{ lb/ft})(24 \text{ ft}) = 1200 \text{ lb}$$



$T_{\max} = 2000$ lb ◀



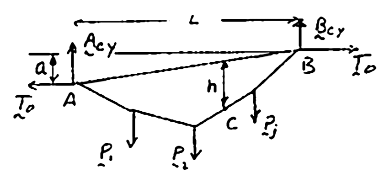
PROBLEM 7.119*

A cable AB of span L and a simple beam $A'B'$ of the same span are subjected to identical vertical loadings as shown. Show that the magnitude of the bending moment at a point C' in the beam is equal to the product T_0h, where T_0 is the magnitude of the horizontal component of the tension force in the cable and h is the vertical distance between Point C and the chord joining the points of support A and B.

SOLUTION

$$\circlearrowleft \Sigma M_B = 0: \quad LA_{Cy} + aT_0 - \Sigma M_{B\text{ loads}} = 0 \qquad (1)$$

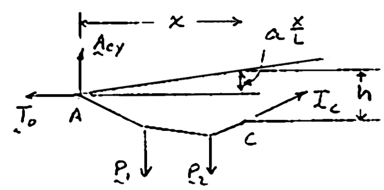
FBD Cable:



(Where $\Sigma M_{B\text{ loads}}$ includes all applied loads)

$$\circlearrowleft \Sigma M_C = 0: \quad xA_{Cy} - \left(h - a\frac{x}{L}\right)T_0 - \Sigma M_{C\text{ left}} = 0 \qquad (2)$$

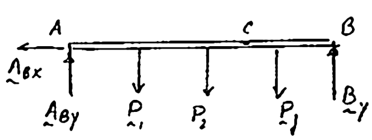
FBD AC:



(Where $\Sigma M_{C\text{ left}}$ includes all loads left of C)

$$\frac{x}{L}(1) - (2): \quad hT_0 - \frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} = 0 \qquad (3)$$

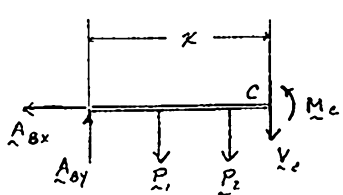
FBD Beam:



$$\circlearrowleft \Sigma M_B = 0: \quad LA_{By} - \Sigma M_{B\text{ loads}} = 0 \qquad (4)$$

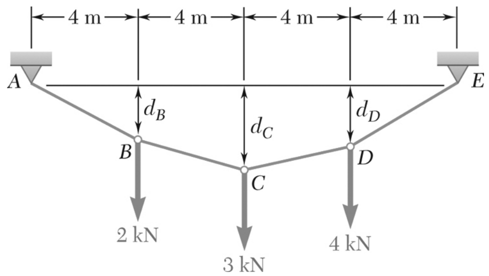
$$\circlearrowleft \Sigma M_C = 0: \quad xA_{By} - \Sigma M_{C\text{ left}} - M_C = 0 \qquad (5)$$

FBD AC:



$$\frac{x}{L}(4) - (5): \quad -\frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} + M_C = 0 \qquad (6)$$

Comparing (3) and (6) $\qquad M_C = hT_0 \qquad$ Q.E.D.



PROBLEM 7.120

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

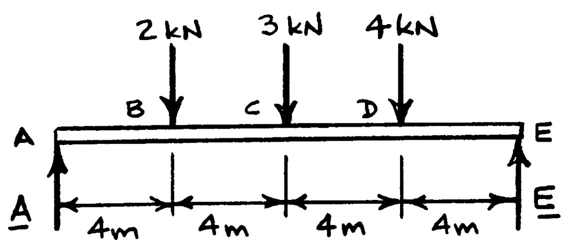
PROBLEM 7.94 Knowing that the maximum tension in cable *ABCDE* is 13 kN, determine the distance d_C.

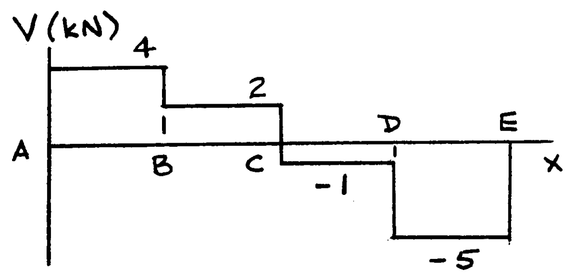
SOLUTION

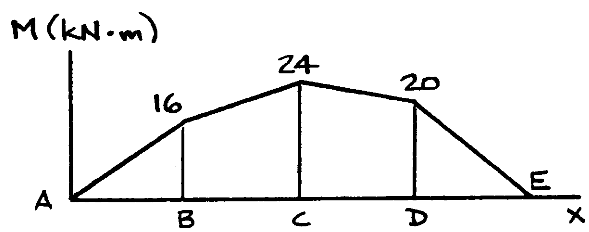
Free body: beam *AE*

$$+\circlearrowleft \Sigma M_E = 0: \quad -A(16) + 2(12) + 3(8) + 4(4) = 0 \qquad \mathbf{A} = 4 \text{ kN}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 4 - 2 - 3 - 4 + E = 0 \qquad \mathbf{E} = 5 \text{ kN}\uparrow$$



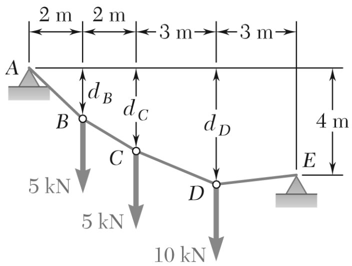




At *E*: $\quad T_m^2 = T_0^2 + E^2 \qquad 13^2 = T_0^2 + 5^2 \qquad T_0 = 12 \text{ kN}$

At *C*: $\quad M_C = T_0 h_C; \qquad 24 \text{ kN} \cdot \text{m} = (12 \text{ kN}) h_C$

$$h_C = d_C = 2.00 \text{ m} \qquad 2.00 \text{ m} \blacktriangleleft$$

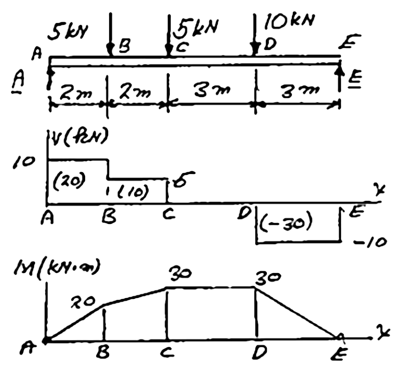


PROBLEM 7.121

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.97 (*a*) Knowing that $d_C = 3$ m, determine the distances d_B and d_D.

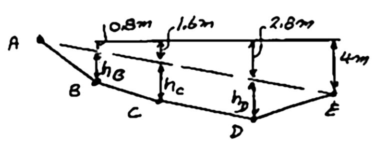
SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad A(10 \text{ m}) - (5 \text{ kN})(8 \text{ m}) - (5 \text{ kN})(6 \text{ m}) - (10 \text{ kN})(3 \text{ m}) = 0$$

$$A = 10 \text{ kN}$$

<u>Geometry</u>:



$$d_C = 1.6 \text{ m} + h_C$$
$$3 \text{ m} = 1.6 \text{ m} + h_C$$
$$h_C = 1.4 \text{ m}$$

Since $M = T_0 h$, h is proportional to M, thus

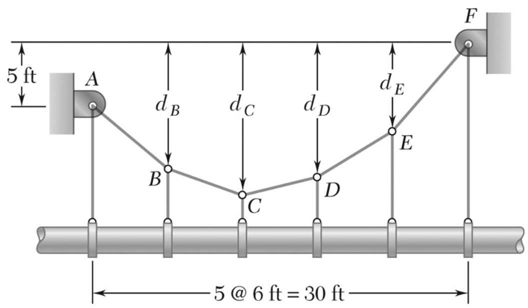
$$\frac{h_B}{M_B} = \frac{h_C}{M_C} = \frac{h_D}{M_D}; \quad \frac{h_B}{20 \text{ kN} \cdot \text{m}} = \frac{1.4 \text{ m}}{30 \text{ kN} \cdot \text{m}} = \frac{h_D}{30 \text{ kN} \cdot \text{m}}$$

$$h_B = 1.4\left(\frac{20}{30}\right) = 0.9333 \text{ m} \qquad h_D = 1.4\left(\frac{30}{30}\right) = 1.4 \text{ m}$$

$$d_B = 0.8 \text{ m} + 0.9333 \text{ m} \qquad d_D = 2.8 \text{ m} + 1.4 \text{ m}$$

$$d_B = 1.733 \text{ m} \blacktriangleleft \qquad d_D = 4.20 \text{ m} \blacktriangleleft$$

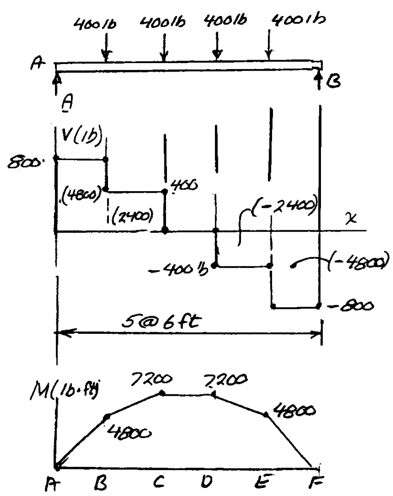
PROBLEM 7.122



Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

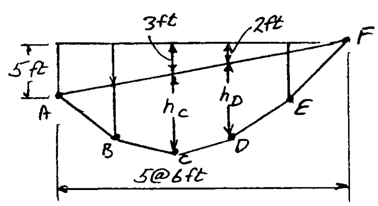
PROBLEM 7.99 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 12$ ft, determine (*a*) the maximum tension in the cable.

SOLUTION



$$A = B = \frac{1}{2}(4 \times 400) = 800 \text{ lb}$$

Geometry



$$d_C = h_C + 3 \text{ ft}$$

$$12 \text{ ft} = h_C + 3 \text{ ft}$$

$$h_C = 9 \text{ ft}$$

$$d_D = h_D + 2 \text{ ft} \qquad (1)$$

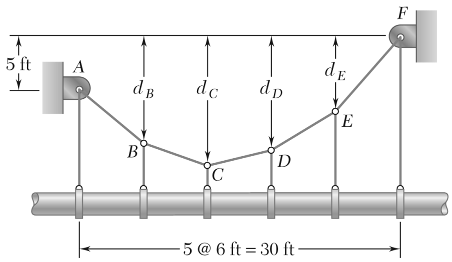
At *C*: $\quad M_C = T_0 h_C$

$$7200 \text{ lb} \cdot \text{ft} = T_0 (9 \text{ ft}) \qquad T_0 = 800 \text{ lb}$$

At *D*: $\quad M_D = T_0 h_D$

$$7200 \text{ lb} \cdot \text{ft} = (800 \text{ lb}) h_D \qquad h_0 = 9 \text{ ft}$$

Eq. (1): $\quad d_D = 9 \text{ ft} + 2 \text{ ft}$ $\qquad d_D = 11.00 \text{ ft}$ ◀

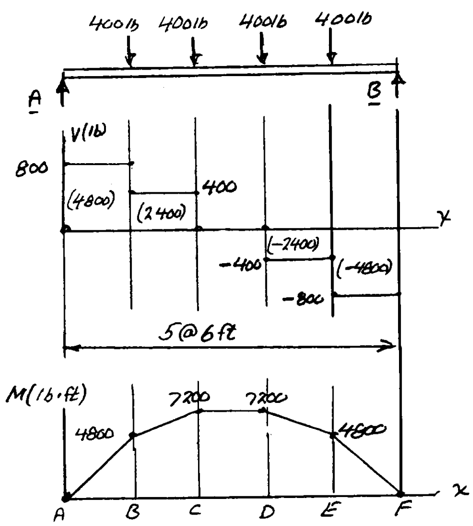


PROBLEM 7.123

Making use of the property established in Problem 7.119, solve the problem indicated by first solving the corresponding beam problem.

PROBLEM 7.100 An oil pipeline is supported at 6-ft intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents the tension in each hanger is 400 lb. Knowing that $d_C = 9$ ft, determine (*b*) the distance d_D.

SOLUTION



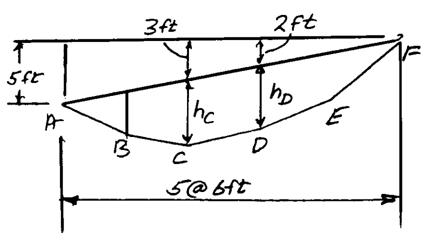
$$A = B = \frac{1}{2}(4 \times 400)$$

$$A = B = 800 \text{ lb}$$

At any point: $M = T_0 h$

We note that since $M_C = M_D$, we have $h_C = h_D$

<u>Geometry</u>



$$d_C = h_C + 3 \text{ ft}$$

$$9 \text{ ft} = h_C + 3 \text{ ft}$$

$$h_C = 6 \text{ ft}$$

and $h_D = 6$ ft

$$d_D = h_D + 2 \text{ ft} = 6 \text{ ft} + 2 \text{ ft}$$

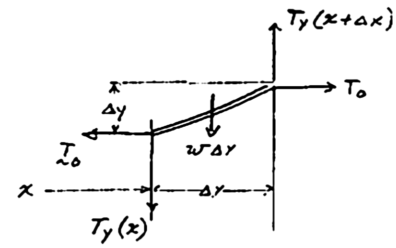
$$d_D = 8.00 \text{ ft} \blacktriangleleft$$

PROBLEM 7.124*

Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

FBD Elemental segment:



$$\uparrow \Sigma F_y = 0: \quad T_y(x+\Delta x) - T_y(x) - w(x)\Delta x = 0$$

So
$$\frac{T_y(x+\Delta x)}{T_0} - \frac{T_y(x)}{T_0} = \frac{w(x)}{T_0}\Delta x$$

But
$$\frac{T_y}{T_0} = \frac{dy}{dx}$$

So
$$\frac{\left.\dfrac{dy}{dx}\right|_{x+\Delta x} - \left.\dfrac{dy}{dx}\right|_{x}}{\Delta x} = \frac{w(x)}{T_0}$$

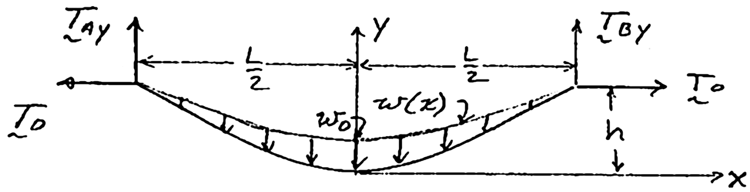
In $\lim\limits_{\Delta x \to 0}$:
$$\frac{d^2 y}{dx^2} = \frac{w(x)}{T_0} \qquad \text{Q.E.D.}$$

PROBLEM 7.125*

Using the property indicated in Problem 7.124, determine the curve assumed by a cable of span L and sag h carrying a distributed load $w = w_0 \cos(\pi x/L)$, where x is measured from mid-span. Also determine the maximum and minimum values of the tension in the cable.

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION



$$w(x) = w_0 \cos\frac{\pi x}{L}$$

From Problem 7.124

$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0}\cos\frac{\pi x}{L}$$

So $$\frac{dy}{dx} = \frac{W_0 L}{T_0 \pi}\sin\frac{\pi x}{L} \quad \left(\text{using } \left.\frac{dy}{dx}\right|_0 = 0\right)$$

$$y = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi x}{L}\right) \quad [\text{using } y(0) = 0] \blacktriangleleft$$

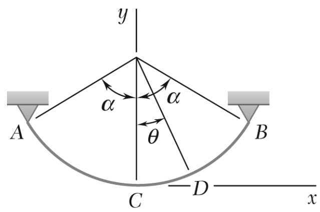
But $$y\left(\frac{L}{2}\right) = h = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi}{2}\right) \quad \text{so} \quad T_0 = \frac{w_0 L^2}{\pi^2 h}$$

And $$T_0 = T_{\min} \quad \text{so} \quad T_{\min} = \frac{w_0 L^2}{\pi^2 h} \blacktriangleleft$$

$$T_{\max} = T_A = T_B: \quad \frac{T_{By}}{T_0} = \left.\frac{dy}{dx}\right|_{x=L/2} = \frac{w_0 L}{T_0 \pi}$$

$$T_{By} = \frac{w_0 L}{\pi} \qquad T_B = \sqrt{T_{By}^2 + T_0^2} = \frac{w_0 L}{\pi}\sqrt{1 + \left(\frac{L}{\pi h}\right)^2} \blacktriangleleft$$

PROBLEM 7.126*

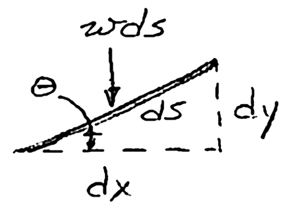


If the weight per unit length of the cable AB is $w_0/\cos^2\theta$, prove that the curve formed by the cable is a circular arc. (*Hint:* Use the property indicated in Problem 7.124.)

PROBLEM 7.124 Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

Elemental Segment:



Load on segment* $\qquad w(x)dx = \dfrac{w_0}{\cos^2\theta}ds$

But $\qquad dx = \cos\theta ds, \quad \text{so} \quad w(x) = \dfrac{w_0}{\cos^3\theta}$

From Problem 7.119 $\qquad \dfrac{d^2y}{dx^2} = \dfrac{w(x)}{T_0} = \dfrac{w_0}{T_0\cos^3\theta}$

In general $\qquad \dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = \dfrac{d}{dx}(\tan\theta) = \sec^2\theta\dfrac{d\theta}{dx}$

So $\qquad \dfrac{d\theta}{dx} = \dfrac{w_0}{T_0\cos^3\theta\sec^2\theta} = \dfrac{w_0}{T_0\cos\theta}$

or $\qquad \dfrac{T_0}{w_0}\cos\theta d\theta = dx = r d\theta\cos\theta$

Giving $r = \dfrac{T_0}{w_0} = \text{constant}$. So curve is circular arc $\qquad$ Q.E.D.

*For large sag, it is not appropriate to approximate ds by dx.

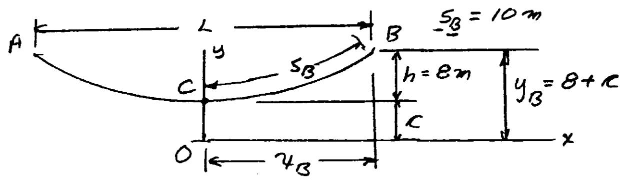
PROBLEM 7.127

A 20-m chain of mass 12 kg is suspended between two points at the same elevation. Knowing that the sag is 8 m, determine (*a*) the distance between the supports, (*b*) the maximum tension in the chain.

SOLUTION

$$\text{mass/meter} = (12\text{ kg})/(20\text{ m}) = 0.6\text{ kg/m}$$

$$w = (0.6\text{ kg/m})(9.81\text{ m/s}^2) = 5.886\text{ N/m}$$



Eq. 7.17: $\quad y_B^2 - s_B^2 = c^2; \quad (8+c)^2 - 10^2 = c^2$

$$64 + 16c + c^2 - 100 = c^2$$

$$16c = 36 \quad c = 2.25\text{ m}$$

Eq. 7.18: $\quad T_m = wy_B = (5.886\text{ N/m})(8\text{ m} + 2.25\text{ m})$

$$T_m = 60.33\text{ N}$$

$T_m = 60.3\text{ N}$ ◀

Eq. 7.15: $\quad s_B = c\sinh\dfrac{x_B}{c}; \quad 10 = (2.25\text{ m})\sinh\dfrac{x_B}{c}$

$$\sinh\frac{x_B}{c} = 4.444; \quad \frac{x_B}{c} = 2.197$$

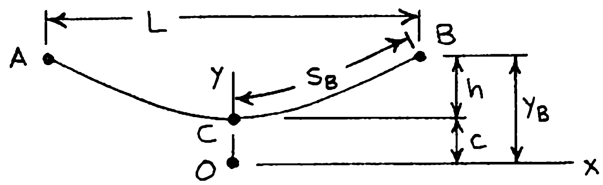
$$x_B = 2.197(2.25\text{ m}) = 4.944\text{ m}; \quad L = 2x_B = 2(4.944\text{ m}) = 9.888\text{ m}$$

$L = 9.89\text{ m}$ ◀

PROBLEM 7.128

A 600-ft-long aerial tramway cable having a weight per unit length of 3.0 lb/ft is suspended between two points at the same elevation. Knowing that the sag is 150 ft, find (*a*) the horizontal distance between the supports, (*b*) the maximum tension in the cable.

SOLUTION



Given:

$$\text{Length} = 600 \text{ ft}$$
$$\text{Unit mass} = 3.0 \text{ lb/ft}$$
$$h = 150 \text{ ft}$$

Then,

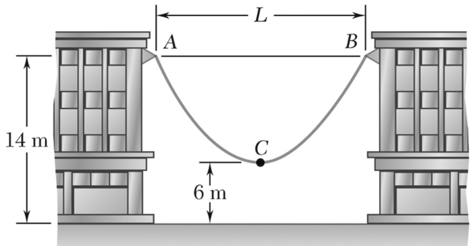
$$s_B = 300 \text{ ft}$$
$$y_B = h + c = 150 \text{ ft} + c$$
$$y_B^2 - s_B^2 = c^2; \quad (150 + c)^2 - (300)^2 = c^2$$
$$150^2 + 300c + c^2 - 300^2 = c^2$$
$$c = 225 \text{ ft}$$

$$s_B = c \sinh \frac{x_B}{c}; \quad 300 = 225 \sinh \frac{x_B}{225}$$

$$x_B = 247.28 \text{ ft}$$

$$\text{span} = L = 2x_B = 2(247.28 \text{ ft}) \qquad L = 495 \text{ ft} \blacktriangleleft$$

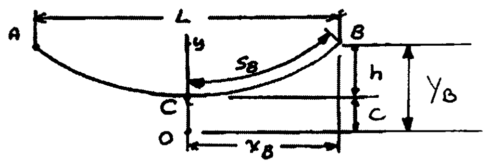
$$T_m = wy_B = (3 \text{ lb/ft})(150 \text{ ft} + 225 \text{ ft}) \qquad T_m = 1125 \text{ lb} \blacktriangleleft$$



PROBLEM 7.129

A 40-m cable is strung as shown between two buildings. The maximum tension is found to be 350 N, and the lowest point of the cable is observed to be 6 m above the ground. Determine (*a*) the horizontal distance between the buildings, (*b*) the total mass of the cable.

SOLUTION



$$s_B = 20 \text{ m}$$
$$T_m = 350 \text{ N}$$
$$h = 14 \text{ m} - 6 \text{ m} = 8 \text{ m}$$
$$y_B = h + c = 8 \text{ m} + c$$

Eq. 7.17: $y_B^2 - s_B^2 = c^2; \quad (8+c)^2 - 20^2 = c^2$

$$64 + 16c + c^2 - 400 = c^2$$
$$c = 21.0 \text{ m}$$

Eq. 7.15: $s_B = c\sinh\dfrac{x_B}{c}; \quad 20 = (21.0)\sinh\dfrac{x_B}{21.0}$

(*a*) $x_B = 17.7933 \text{ m}; \quad L = 2x_B$ $\qquad L = 35.6 \text{ m}$ ◀

(*b*) Eq. 7.18: $T_m = wy_B; \quad 350 \text{ N} = w(8 + 21.0)$

$$w = 12.0690 \text{ N/m}$$

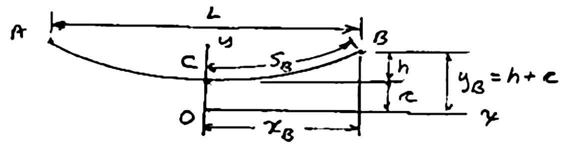
$$W = 2s_B w = (40 \text{ m})(12.0690 \text{ N/m})$$
$$= 482.76 \text{ N}$$
$$m = \frac{W}{g} = \frac{482.76 \text{ N}}{9.81 \text{ m/s}^2}$$

Total mass = 49.2 kg ◀

PROBLEM 7.130

A 200-ft steel surveying tape weighs 4 lb. If the tape is stretched between two points at the same elevation and pulled until the tension at each end is 16 lb, determine the horizontal distance between the ends of the tape. Neglect the elongation of the tape due to the tension.

SOLUTION



$$s_B = 100 \text{ ft}$$

$$w = \left(\frac{4 \text{ lb}}{200 \text{ ft}}\right) = 0.02 \text{ lb/ft} \quad T_m = 16 \text{ m}$$

Eq. 7.18: $T_m = wy_B; \quad 16 \text{ lb} = (0.02 \text{ lb/ft})y_B; \quad y_B = 800 \text{ ft}$

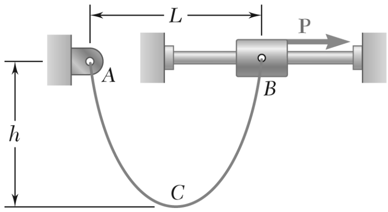
Eq. 7.17: $y_B^2 - s_B^2 = c^2; \quad (800)^2 - (100)^2 = c^2; \quad c = 793.73 \text{ ft}$

Eq. 7.15: $s_B = c \sinh \frac{x_B}{c}; \quad 100 = 793.73 \sinh \frac{x_B}{c}$

$$\frac{x_B}{c} = 0.12566; \quad x_B = 99.737 \text{ ft}$$

$$L = 2x_B = 2(99.737 \text{ ft})$$

$L = 199.5 \text{ ft}$ ◀

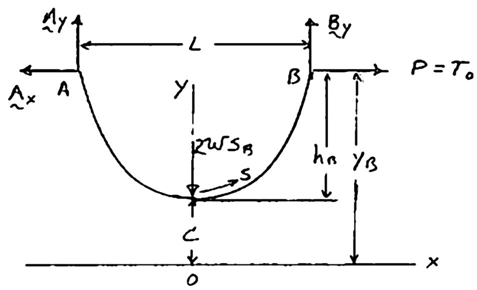


PROBLEM 7.131

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Neglecting the effect of friction, determine (a) the force **P** for which $h = 8$ m, (b) the corresponding span L.

SOLUTION

FBD Cable:



$$s_T = 20\text{ m} \quad \left(\text{so } s_B = \frac{20\text{ m}}{2} = 10\text{ m}\right)$$

$$w = (0.2\text{ kg/m})(9.81\text{ m/s}^2)$$
$$= 1.96200\text{ N/m}$$

$$h_B = 8\text{ m}$$

$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

So $$c = \frac{s_B^2 - h_B^2}{2h_B}$$

$$c = \frac{(10\text{ m})^2 - (8\text{ m})^2}{2(8\text{ m})}$$
$$= 2.250\text{ m}$$

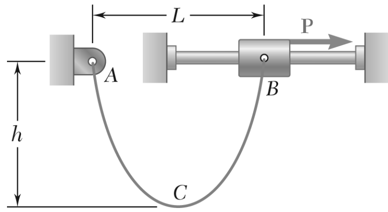
Now $$s_B = c \sinh\frac{x_B}{c} \rightarrow x_B = c \sinh^{-1}\frac{s_B}{c}$$

$$= (2.250\text{ m})\sinh^{-1}\left(\frac{10\text{ m}}{2.250\text{ m}}\right)$$

$$x_B = 4.9438\text{ m}$$

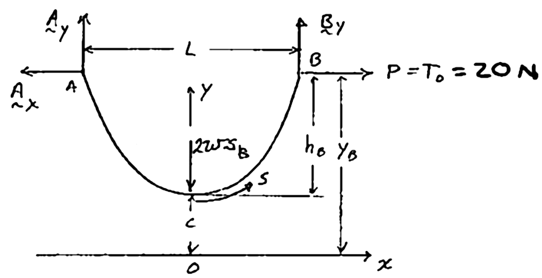
$P = T_0 = wc = (1.96200\text{ N/m})(2.250\text{ m})$ (a) $\mathbf{P} = 4.41\text{ N} \longrightarrow$ ◀

$L = 2x_B = 2(4.9438\text{ m})$ (b) $L = 9.89\text{ m}$ ◀

	**PROBLEM 7.132** A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Knowing that the magnitude of the horizontal force applied to the collar is $P = 20$ N, determine (a) the sag h, (b) the span L.

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m}, \quad w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2) = 1.96200 \text{ N/m}$$

$$P = T_0 = wc \quad c = \frac{P}{w}$$

$$c = \frac{20 \text{ N}}{1.9620 \text{ N/m}} = 10.1937 \text{ m}$$

$$y_B^2 = (h_B + c)^2 = c^2 + s_B^2$$

$$h^2 + 2ch - s_B^2 = 0 \quad s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}$$

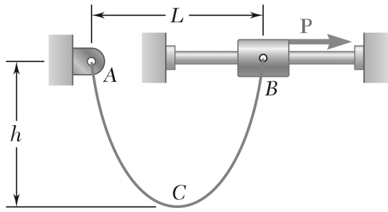
$$h^2 + 2(10.1937 \text{ m})h - 100 \text{ m}^2 = 0$$

$$h = 4.0861 \text{ m} \qquad (a) \qquad h = 4.09 \text{ m} \blacktriangleleft$$

$$s_B = c \sinh \frac{x_A}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c} = (10.1937 \text{ m}) \sinh^{-1}\left(\frac{10 \text{ m}}{10.1937 \text{ m}}\right)$$

$$= 8.8468 \text{ m}$$

$$L = 2x_B = 2(8.8468 \text{ m}) \qquad (b) \qquad L = 17.69 \text{ m} \blacktriangleleft$$

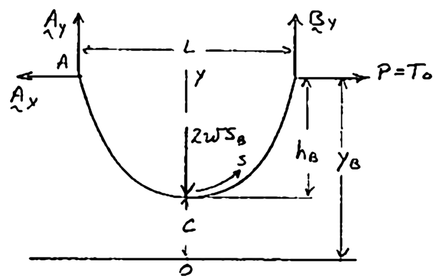


PROBLEM 7.133

A 20-m length of wire having a mass per unit length of 0.2 kg/m is attached to a fixed support at A and to a collar at B. Neglecting the effect of friction, determine (a) the sag h for which $L = 15$ m, (b) the corresponding force **P**.

SOLUTION

FBD Cable:



$$s_T = 20 \text{ m} \rightarrow s_B = \frac{20 \text{ m}}{2} = 10 \text{ m}$$

$$w = (0.2 \text{ kg/m})(9.81 \text{ m/s}^2) = 1.96200 \text{ N/m}$$

$$L = 15 \text{ m}$$

$$s_B = c \sinh \frac{x_B}{c} = c \sinh \frac{\frac{L}{2}}{c}$$

$$10 \text{ m} = c \sinh \frac{7.5 \text{ m}}{c}$$

Solving numerically: $c = 5.5504$ m

$$y_B = c \cosh\left(\frac{x_B}{c}\right) = (5.5504) \cosh\left(\frac{7.5}{5.5504}\right)$$

$$y_B = 11.4371 \text{ m}$$

$$h_B = y_B - c = 11.4371 \text{ m} - 5.5504 \text{ m}$$

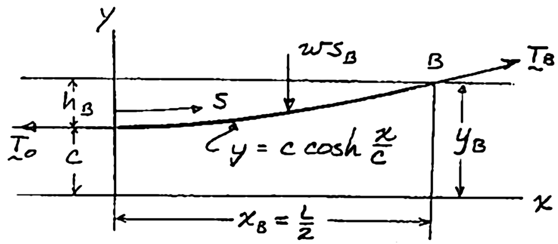
(a) $h_B = 5.89$ m ◀

$P = wc = (1.96200 \text{ N/m})(5.5504 \text{ m})$ (b) $\mathbf{P} = 10.89 \text{ N} \longrightarrow$ ◀

PROBLEM 7.134

Determine the sag of a 30-ft chain that is attached to two points at the same elevation that are 20 ft apart.

SOLUTION



$$s_B = \frac{30\text{ ft}}{2} = 15\text{ ft} \quad L = 20\text{ ft}$$

$$x_B = \frac{L}{2} = 10\text{ ft}$$

$$s_B = c\sinh\frac{x_B}{c}$$

$$15\text{ ft} = c\sinh\frac{10\text{ ft}}{c}$$

Solving numerically: $c = 6.1647\text{ ft}$

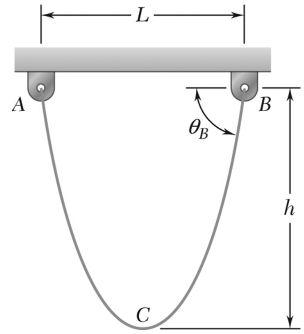
$$y_B = c\cosh\frac{x_B}{c}$$

$$= (6.1647\text{ ft})\cosh\frac{10\text{ ft}}{6.1647\text{ ft}}$$

$$= 16.2174\text{ ft}$$

$$h_B = y_B - c = 16.2174\text{ ft} - 6.1647\text{ ft}$$

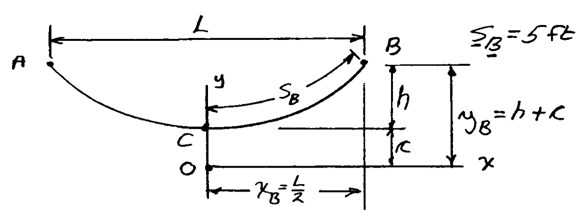
$h_B = 10.05\text{ ft}$ ◀



PROBLEM 7.135

A 10-ft rope is attached to two supports A and B as shown. Determine (a) the span of the rope for which the span is equal to the sag, (b) the corresponding angle θ_B.

SOLUTION



Note: Since $L = h$,

$$x_B = \frac{L}{2} = \frac{h}{2}$$

Eq. 7.16:

$$y_B = \cosh\frac{x_B}{c}$$

$$h + c = c\cosh\frac{h/2}{c}$$

$$\frac{h}{c} + 1 = \cosh\left(\frac{1}{2}\frac{h}{c}\right)$$

Solve for h/c:

$$\frac{h}{c} = 4.933$$

Eq. 7.16:

$$y = c\cosh\frac{x}{c}$$

$$\frac{dy}{dx} = \sinh\frac{x}{c}$$

At B:

$$\tan\theta_B = \left.\frac{dy}{dx}\right|_B = \sinh\frac{x_B}{c}$$

Substitute

$$x_B = \frac{h}{2}: \quad \tan\theta_B = \sinh\left(\frac{1}{2}\frac{h}{c}\right) = \sinh\left(\frac{1}{2}\times 4.933\right)$$

$$\tan\theta_B = 5.848$$

$$\theta_B = 80.3° \blacktriangleleft$$

PROBLEM 7.135 (Continued)

Eq. 7.17:

$$s_B = c\sinh\frac{x_B}{c} = c\sinh\left(\frac{1}{2}\frac{h}{c}\right)$$

$$5\text{ ft} = c\sinh\left(\frac{1}{2}\times 4.933\right)$$

$$5\text{ ft} = c(5.848)$$

$$c = 0.855$$

Recall that

$$\frac{h}{c} = 4.933$$

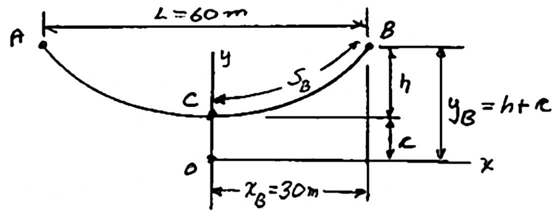
$$h = 4.933(0.855) = 4.218$$

$h = 4.22\text{ ft}$ ◀

PROBLEM 7.136

A 90-m wire is suspended between two points at the same elevation that are 60 m apart. Knowing that the maximum tension is 300 N, determine (*a*) the sag of the wire, (*b*) the total mass of the wire.

SOLUTION



$$s_B = 45 \text{ m}$$

Eq. 7.17:

$$s_B = c\sinh\frac{x_B}{c}$$

$$45 = c\sinh\frac{30}{c}; \quad c = 18.494 \text{ m}$$

Eq. 7.16:

$$y_B = c\cosh\frac{x_B}{c}$$

$$y_B = (18.494)\cosh\frac{30}{18.494}$$

$$y_B = 48.652 \text{ m}$$

$$y_B = h + c$$

$$48.652 = h + 18.494$$

$$h = 30.158 \text{ m} \qquad h = 30.2 \text{ m} \blacktriangleleft$$

Eq. 7.18:

$$T_m = wy_B$$

$$300 \text{ N} = w(48.652 \text{ m})$$

$$w = 6.166 \text{ N/m}$$

Total weight of cable

$$W = w(\text{Length})$$

$$= (6.166 \text{ N/m})(90 \text{ m})$$

$$= 554.96 \text{ N}$$

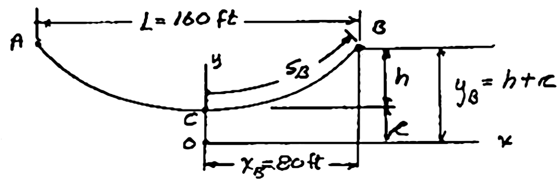
Total mass of cable

$$m = \frac{W}{g} = \frac{554.96 \text{ N}}{9.81 \text{ m/s}} = 56.57 \text{ kg} \qquad m = 56.6 \text{ kg} \blacktriangleleft$$

PROBLEM 7.137

A cable weighing 2 lb/ft is suspended between two points at the same elevation that are 160 ft apart. Determine the smallest allowable sag of the cable if the maximum tension is not to exceed 400 lb.

SOLUTION



Eq. 7.18: $T_m = wy_B; \quad 400 \text{ lb} = (2 \text{ lb/ft})\, y_B; \quad y_B = 200 \text{ ft}$

Eq. 7.16: $y_B = c\cosh\dfrac{x_B}{c}$

$$200 \text{ ft} = c\cosh\frac{80 \text{ ft}}{c}$$

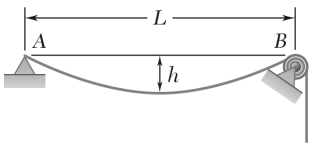
Solve for c: $c = 182.148$ ft and $c = 31.592$ ft

$$y_B = h + c; \quad h = y_B - c$$

For $c = 182.148$ ft; $h = 200 - 182.147 = 17.852$ ft ◁

For $c = 31.592$ ft; $h = 200 - 31.592 = 168.408$ ft ◁

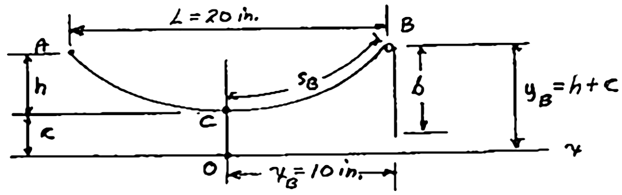
For $T_m \leq 400$ lb: smallest $h = 17.85$ ft ◀



PROBLEM 7.138

A uniform cord 50 in. long passes over a pulley at B and is attached to a pin support at A. Knowing that $L = 20$ in. and neglecting the effect of friction, determine the smaller of the two values of h for which the cord is in equilibrium.

SOLUTION



Length of overhang: $$b = 50\text{ in.} - 2s_B$$

Weight of overhang equals max. tension

$$T_m = T_B = wb = w(50\text{ in.} - 2s_B)$$

Eq. 7.15: $$s_B = c\sinh\frac{x_B}{c}$$

Eq. 7.16: $$y_B = c\cosh\frac{x_B}{c}$$

Eq. 7.18: $$T_m = wy_B$$

$$w(50\text{ in.} - 2s_B) = wy_B$$

$$w\left(50\text{ in.} - 2c\sinh\frac{x_B}{c}\right) = wc\cosh\frac{x_B}{c}$$

$$x_B = 10{:}\quad 50 - 2c\sinh\frac{10}{c} = c\cosh\frac{10}{c}$$

Solve by trial and error: $c = 5.549$ in. and $c = 27.742$ in.

For $c = 5.549$ in.

$$y_B = (5.549\text{ in.})\cosh\frac{10\text{ in.}}{5.549\text{ in.}} = 17.277\text{ in.}$$

$$y_B = h + c;\quad 17.277\text{ in.} = h + 5.549\text{ in.}$$

$$h = 11.728\text{ in.}$$

$h = 11.73$ in. ◀

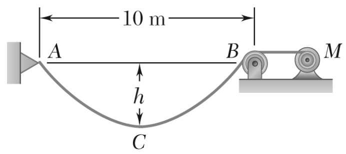
For $c = 27.742$ in.

$$y_B = (27.742\text{ in.})\cosh\frac{10\text{ in.}}{27.742\text{ in.}} = 29.564\text{ in.}$$

$$y_B = h + c;\quad 29.564\text{ in.} = h + 27.742\text{ in.}$$

$$h = 1.8219\text{ in.}$$

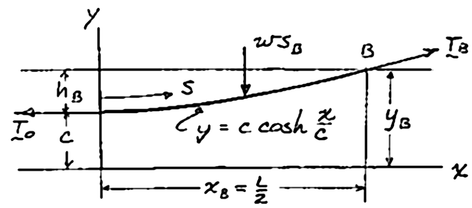
$h = 1.822$ in. ◀



PROBLEM 7.139

A motor M is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m, determine the maximum tension in the cable when $h = 5$ m.

SOLUTION



$$w = 0.4 \text{ kg/m} \quad L = 10 \text{ m} \quad h_B = 5 \text{ m}$$

$$y_B = c \cosh \frac{x_B}{c}$$

$$h_B + c = c \cosh \frac{L}{2c}$$

$$5 \text{ m} = c\left(\cosh \frac{5 \text{ m}}{c} - 1\right)$$

Solving numerically: $c = 3.0938$ m

$$y_B = h_B + c = 5 \text{ m} + 3.0938 \text{ m}$$

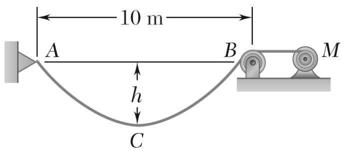
$$= 8.0938 \text{ m}$$

$$T_{\max} = T_B = w y_B$$

$$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(8.0938 \text{ m})$$

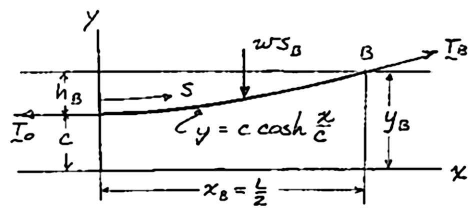
$T_{\max} = 31.8$ N ◀

PROBLEM 7.140



A motor M is used to slowly reel in the cable shown. Knowing that the mass per unit length of the cable is 0.4 kg/m, determine the maximum tension in the cable when $h = 3$ m.

SOLUTION



$$w = 0.4 \text{ kg/m}, \quad L = 10 \text{ m}, \quad h_B = 3 \text{ m}$$

$$y_B = h_B + c = c\cosh\frac{x_B}{c} = c\cosh\frac{L}{2c}$$

$$3 \text{ m} = c\left(c\cosh\frac{5 \text{ m}}{c} - 1\right)$$

Solving numerically:

$$c = 4.5945 \text{ m}$$

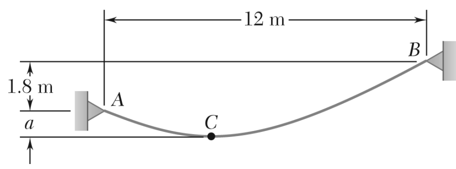
$$y_B = h_B + c = 3 \text{ m} + 4.5945 \text{ m}$$

$$= 7.5945 \text{ m}$$

$$T_{\max} = T_B = wy_B$$

$$= (0.4 \text{ kg/m})(9.81 \text{ m/s}^2)(7.5945 \text{ m})$$

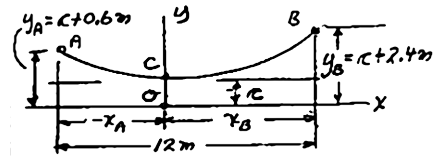
$$T_{\max} = 29.8 \text{ N} \blacktriangleleft$$



PROBLEM 7.141

The cable ACB has a mass per unit length of 0.45 kg/m. Knowing that the lowest point of the cable is located at a distance $a = 0.6$ m below the support A, determine (a) the location of the lowest Point C, (b) the maximum tension in the cable.

SOLUTION



Note: $$x_B - x_A = 12 \text{ m}$$

or, $$-x_A = 12 \text{ m} - x_B$$

Point A: $$y_A = c\cosh\frac{-x_A}{c}; \quad c + 0.6 = c\cosh\frac{12 - x_B}{c} \tag{1}$$

Point B: $$y_B = c\cosh\frac{x_B}{c}; \quad c + 2.4 = c\cosh\frac{x_B}{c} \tag{2}$$

From (1): $$\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1}\left(\frac{c+0.6}{c}\right) \tag{3}$$

From (2): $$\frac{x_B}{c} = \cosh^{-1}\left(\frac{c+2.4}{c}\right) \tag{4}$$

Add (3) + (4): $$\frac{12}{c} = \cosh^{-1}\left(\frac{c+0.6}{c}\right) + \cosh^{-1}\left(\frac{c+2.4}{c}\right)$$

Solve by trial and error: $$c = 13.6214 \text{ m}$$

Eq. (2): $$13.6214 + 2.4 = 13.6214\cosh\frac{x_B}{c}$$

$$\cosh\frac{x_B}{c} = 1.1762; \quad \frac{x_B}{c} = 0.58523$$

$$x_B = 0.58523(13.6214 \text{ m}) = 7.9717 \text{ m}$$

Point C is 7.97 m to left of B ◀

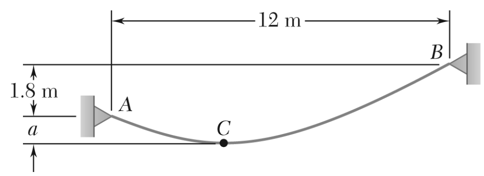
$$y_B = c + 2.4 = 13.6214 + 2.4 = 16.0214 \text{ m}$$

Eq. 7.18: $$T_m = wy_B = (0.45 \text{ kg/m})(9.81 \text{ m/s}^2)(16.0214 \text{ m})$$

$$T_m = 70.726 \text{ N}$$

$T_m = 70.7$ N ◀

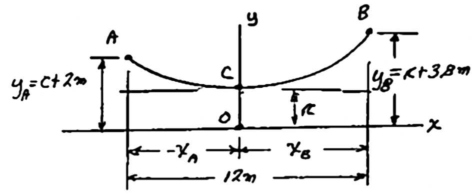
PROBLEM 7.142



The cable ACB has a mass per unit length of 0.45 kg/m. Knowing that the lowest point of the cable is located at a distance a = 2 m below the support A, determine (*a*) the location of the lowest Point C, (*b*) the maximum tension in the cable.

SOLUTION

Note:



$$x_B - x_A = 12 \text{ m}$$

or

$$-x_A = 12 \text{ m} - x_B$$

<u>Point A</u>: $$y_A = c\cosh\frac{-x_A}{c}; \quad c + 2 = c\cosh\frac{12 - x_B}{c} \qquad (1)$$

<u>Point B</u>: $$y_B = c\cosh\frac{x_B}{c}; \quad c + 3.8 = c\cosh\frac{x_B}{c} \qquad (2)$$

From (1): $$\frac{12}{c} - \frac{x_B}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right) \qquad (3)$$

From (2): $$\frac{x_B}{c} = \cosh^{-1}\left(\frac{c+3.8}{c}\right) \qquad (4)$$

Add (3) + (4): $$\frac{12}{c} = \cosh^{-1}\left(\frac{c+2}{c}\right) + \cosh^{-1}\left(\frac{c+3.8}{c}\right)$$

Solve by trial and error: $c = 6.8154$ m

Eq. (2): $$6.8154 \text{ m} + 3.8 \text{ m} = (6.8154 \text{ m})\cosh\frac{x_B}{c}$$

$$\cosh\frac{x_B}{c} = 1.5576 \qquad \frac{x_B}{c} = 1.0122$$

$$x_B = 1.0122(6.8154 \text{ m}) = 6.899 \text{ m}$$

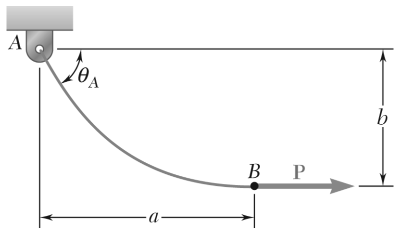
Point C is 6.90 m to left of B ◀

$$y_B = c + 3.8 = 6.8154 + 3.8 = 10.6154 \text{ m}$$

Eq. (7.18): $$T_m = wy_B = (0.45 \text{ kg/m})(9.81 \text{ m/s}^2)(10.6154 \text{ m})$$

$$T_m = 46.86 \text{ N}$$

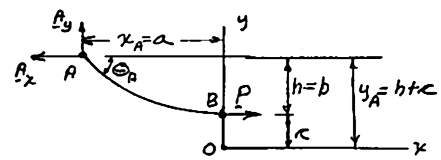
$T_m = 46.9$ N ◀



PROBLEM 7.143

A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force **P** applied at *B*. Knowing that $P = 180$ lb and $\theta_A = 60°$, determine (*a*) the location of Point *B*, (*b*) the length of the cable.

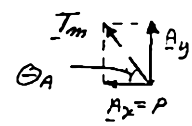
SOLUTION



Eq. 7.18: $T_0 = P = cw$

$$c = \frac{P}{w} = \frac{180 \text{ lb}}{3 \text{ lb/ft}} \quad c = 60 \text{ ft}$$

At *A*: $T_m = \dfrac{P}{\cos 60°}$



$$= \frac{cw}{0.5} = 2cw$$

(*a*) Eq. 7.18: $T_m = w(h+c)$

$$2cw = w(h+c)$$

$$2c = h + c \quad h = b = c$$

$b = 60.0$ ft ◀

Eq. 7.16: $y_A = c\cosh\dfrac{x_A}{c}$

$$h + c = c\cosh\frac{x_A}{c}$$

$$(60 \text{ ft} + 60 \text{ ft}) = (60 \text{ ft}) \quad \cosh\frac{x_A}{60}$$

$$\cosh\frac{x_A}{60 \text{ m}} = 2 \quad \frac{x_A}{60 \text{ m}} = 1.3170$$

$$x_A = 79.02 \text{ ft}$$

$a = 79.0$ ft ◀

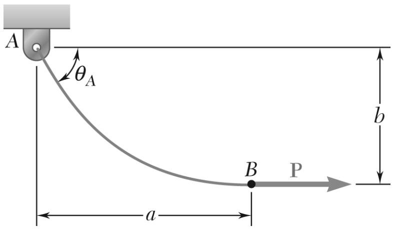
(*b*) Eq. 7.15: $s_A = c\sinh\dfrac{x_B}{c} = (60 \text{ ft})\sinh\dfrac{79.02 \text{ ft}}{60 \text{ ft}}$

$$s_A = 103.92 \text{ ft}$$

$$\text{length} = s_A$$

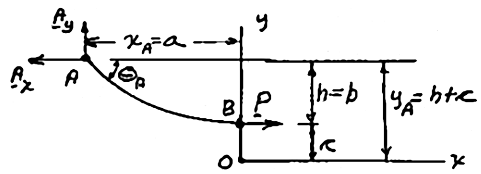
$s_A = 103.9$ ft ◀

PROBLEM 7.144



A uniform cable weighing 3 lb/ft is held in the position shown by a horizontal force **P** applied at *B*. Knowing that $P = 150$ lb and $\theta_A = 60°$, determine (*a*) the location of Point *B*, (*b*) the length of the cable.

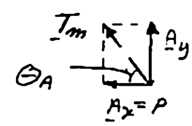
SOLUTION



Eq. 7.18: $T_0 = P = cw$

$$c = \frac{P}{w} = \frac{150 \text{ lb}}{3 \text{ lb/ft}} = 50 \text{ ft}$$

At *A*: $T_m = \dfrac{P}{\cos 60°}$



$$= \frac{cw}{0.5} = 2\,cw$$

(*a*) Eq. 7.18:

$$T_m = w(h+c)$$

$$2cw = w(h+c)$$

$$2c = h + c \quad h = c = b$$

$b = 50.0$ ft ◀

Eq. 7.16:

$$y_A = c\cosh\frac{x_A}{c}$$

$$h + c = c\cosh\frac{x_A}{c}$$

$$(50 \text{ ft} + 50 \text{ ft}) = (50 \text{ ft})\cosh\frac{x_A}{c}$$

$$\cosh\frac{x_A}{c} = 2 \quad \frac{x_A}{c} = 1.3170$$

$$x_A = 1.3170(50 \text{ ft}) = 65.85 \text{ ft}$$

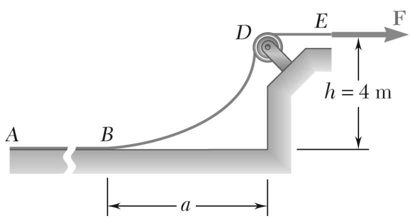
$a = 65.8$ ft ◀

(*b*) Eq. 7.15:

$$s_A = c\sinh\frac{x_A}{c} = (50 \text{ ft})\sinh\frac{65.85 \text{ ft}}{50 \text{ ft}}$$

$$s_A = 86.6 \text{ ft}$$

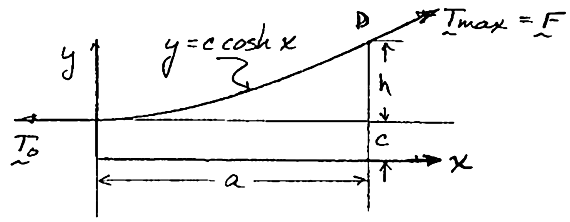
length $= s_A = 86.6$ ft ◀



PROBLEM 7.145

To the left of Point B the long cable $ABDE$ rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m, determine the force **F** when $a = 3.6$ m.

SOLUTION



$$x_D = a = 3.6 \text{ m} \quad h = 4 \text{ m}$$

$$y_D = c \cosh \frac{x_D}{c}$$

$$h + c = c \cosh \frac{a}{c}$$

$$4 \text{ m} = c\left(\cosh \frac{3.6 \text{ m}}{c} - 1\right)$$

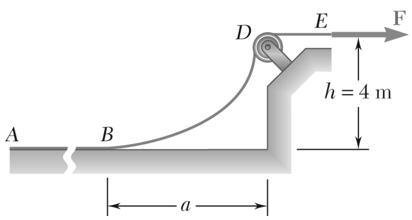
Solving numerically: $c = 2.0712$ m

Then $y_B = h + c = 4 \text{ m} + 2.0712 \text{ m} = 6.0712 \text{ m}$

$$F = T_{\max} = wy_B = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(6.0712 \text{ m})$$

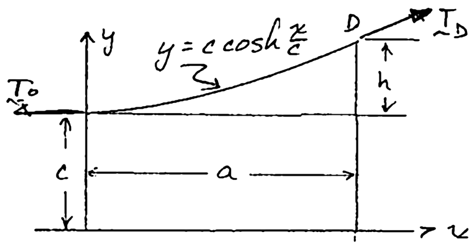
$\mathbf{F} = 119.1 \text{ N} \longrightarrow$ ◀

PROBLEM 7.146



To the left of Point B the long cable $ABDE$ rests on the rough horizontal surface shown. Knowing that the mass per unit length of the cable is 2 kg/m, determine the force **F** when $a = 6$ m.

SOLUTION



$$x_D = a = 6 \text{ m} \quad h = 4 \text{ m}$$

$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

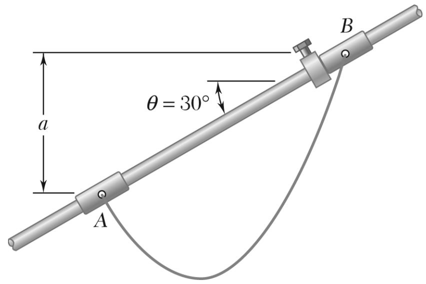
$$4 \text{ m} = c\left(\cosh\frac{6 \text{ m}}{c} - 1\right)$$

Solving numerically: $c = 5.054$ m

$$y_B = h + c = 4 \text{ m} + 5.054 \text{ m} = 9.054 \text{ m}$$

$$F = T_D = wy_D = (2 \text{ kg/m})(9.81 \text{ m/s}^2)(9.054 \text{ m})$$

$\mathbf{F} = 177.6 \text{ N} \longrightarrow$ ◀

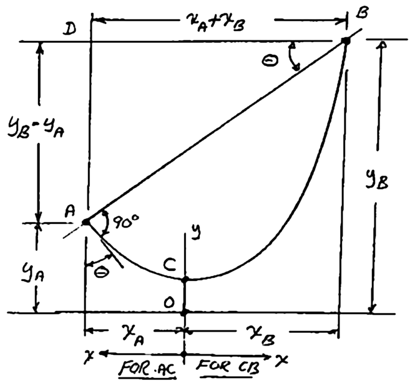


PROBLEM 7.147*

The 10-ft cable AB is attached to two collars as shown. The collar at A can slide freely along the rod; a stop attached to the rod prevents the collar at B from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance a.

SOLUTION

Collar at A: Since $\mu = 0$, cable $\perp$ rod



Point A:
$$y = c\cosh\frac{x}{c}; \quad \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = \left|\frac{dy}{dx}\right|_A = \sinh\frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90° - \theta))$$

$$x_A = c\sinh(\tan(90° - \theta)) \quad (1)$$

Length of cable = 10 ft
$$10 \text{ ft} = AC + CB$$

$$10 = c\sinh\frac{x_A}{c} + c\sinh\frac{x_B}{c}$$

$$\sinh\frac{x_B}{c} = \frac{10}{c} - \sinh\frac{x_A}{c}$$

$$x_B = c\sinh^{-1}\left[\frac{10}{c} - \sinh\frac{x_A}{c}\right] \quad (2)$$

$$y_A = c\cosh\frac{x_A}{c} \quad y_B = c\cosh\frac{x_B}{c} \quad (3)$$

In $\Delta\, ABD$:
$$\tan\theta = \frac{y_B - y_A}{x_B + x_A} \quad (4)$$

PROBLEM 7.147* (Continued)

Method of solution:

For given value of θ, choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c, calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A, x_B, y_A, y_B into Eq. (4) and calculate θ

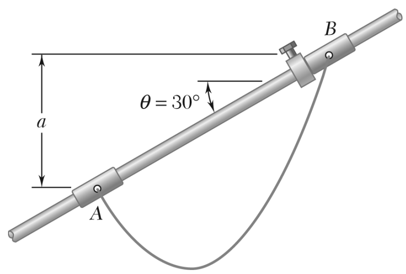
Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ.

For $\theta = 30°$

Result of trial and error procedure:

$$c = 1.803 \text{ ft}$$
$$x_A = 2.3745 \text{ ft}$$
$$x_B = 3.6937 \text{ ft}$$
$$y_A = 3.606 \text{ ft}$$
$$y_B = 7.109 \text{ ft}$$
$$\begin{aligned} a &= y_B - y_A \\ &= 7.109 \text{ ft} - 3.606 \text{ ft} \\ &= 3.503 \text{ ft} \end{aligned}$$

$a = 3.50 \text{ ft}$ ◀



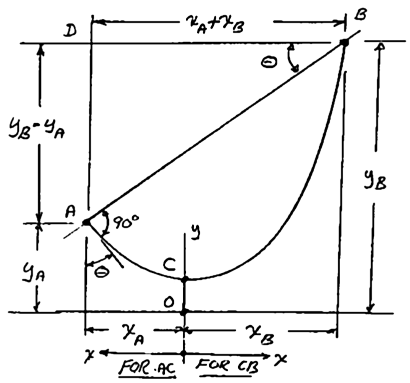
PROBLEM 7.148*

Solve Problem 7.147 assuming that the angle θ formed by the rod and the horizontal is 45°.

PROBLEM 7.147 The 10-ft cable AB is attached to two collars as shown. The collar at A can slide freely along the rod; a stop attached to the rod prevents the collar at B from moving on the rod. Neglecting the effect of friction and the weight of the collars, determine the distance a.

SOLUTION

Collar at A: Since $\mu = 0$, cable $\perp$ rod



Point A:

$$y = c\cosh\frac{x}{c}; \quad \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = \left|\frac{dy}{dx}\right|_A = \sinh\frac{x_A}{c}$$

$$\frac{x_A}{c} = \sinh(\tan(90° - \theta))$$

$$x_A = c\sinh(\tan(90° - \theta)) \qquad (1)$$

Length of cable = 10 ft

$$10 \text{ ft} = AC + CB$$

$$10 = c\sinh\frac{x_A}{c} + c\sinh\frac{x_B}{c}$$

$$\sinh\frac{x_B}{c} = \frac{10}{c} - \sinh\frac{x_A}{c}$$

$$x_B = c\sinh^{-1}\left[\frac{10}{c} - \sinh\frac{x_A}{c}\right] \qquad (2)$$

$$y_A = c\cosh\frac{x_A}{c} \quad y_B = c\cosh\frac{x_B}{c} \qquad (3)$$

PROBLEM 7.148* (Continued)

In $\Delta\, ABD$: $$\tan\theta = \frac{y_B - y_A}{x_B + x_A} \qquad (4)$$

Method of solution:

For given value of θ, choose trial value of c and calculate:

From Eq. (1): x_A

Using value of x_A and c, calculate:

From Eq. (2): x_B

From Eq. (3): y_A and y_B

Substitute values obtained for x_A, x_B, y_A, y_B into Eq. (4) and calculate θ

Choose new trial value of θ and repeat above procedure until calculated value of θ is equal to given value of θ.

For $\theta = 45°$

Result of trial and error procedure:

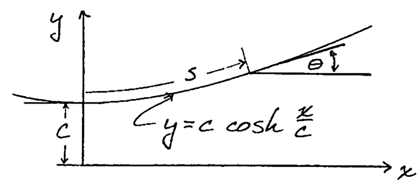
$$c = 1.8652 \text{ ft}$$
$$x_A = 1.644 \text{ ft}$$
$$x_B = 4.064 \text{ ft}$$
$$y_A = 2.638 \text{ ft}$$
$$y_B = 8.346 \text{ ft}$$
$$a = y_B - y_A$$
$$= 8.346 \text{ ft} - 2.638 \text{ ft}$$
$$= 5.708 \text{ ft}$$

$a = 5.71$ ft ◀

PROBLEM 7.149

Denoting by θ the angle formed by a uniform cable and the horizontal, show that at any point (*a*) $s = c \tan \theta$, (*b*) $y = c \sec \theta$.

SOLUTION



(*a*)
$$\tan\theta = \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$s = c\sinh\frac{x}{c} = c\tan\theta \qquad \text{Q.E.D.}$$

(*b*) Also
$$y^2 = s^2 + c^2(\cosh^2 x = \sinh^2 x + 1)$$

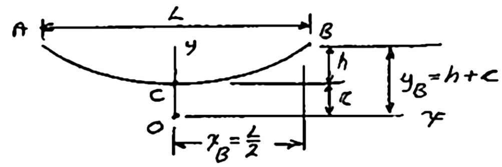
so
$$y^2 = c^2(\tan^2\theta + 1)c^2\sec^2\theta$$

and
$$y = c\sec\theta \qquad \text{Q.E.D.}$$

PROBLEM 7.150*

(*a*) Determine the maximum allowable horizontal span for a uniform cable of weight per unit length w if the tension in the cable is not to exceed a given value T_m. (*b*) Using the result of part *a*, determine the maximum span of a steel wire for which $w = 0.25$ lb/ft and $T_m = 8000$ lb.

SOLUTION



(*a*)

$$T_m = wy_B$$

$$= wc\cosh\frac{x_B}{c}$$

$$= wx_B\left(\frac{1}{\frac{x_B}{c}}\right)\cosh\frac{x_B}{c}$$

We shall find ratio $\left(\frac{x_B}{c}\right)$ for when T_m is minimum

$$\frac{dT_m}{d\left(\frac{x_B}{c}\right)} = wx_B\left[\frac{1}{\frac{x_B}{c}}\sinh\frac{x_B}{c} - \left(\frac{1}{\frac{x_B}{c}}\right)^2\cosh\frac{x_B}{c}\right] = 0$$

$$\frac{\sinh\frac{x_B}{c}}{\cosh\frac{x_B}{c}} = \frac{1}{\frac{x_B}{c}}$$

$$\tanh\frac{x_B}{c} = \frac{c}{x_B}$$

Solve by trial and error for: $\qquad \dfrac{x_B}{c} = 1.200 \qquad$ (1)

$s_B = c\sinh\dfrac{x_B}{c} = c\sinh(1.200)$: $\qquad \dfrac{s_B}{c} = 1.509$

<u>Eq. 7.17</u>: $\qquad y_B^2 - s_B^2 = c^2$

$$y_B^2 = c^2\left[1 + \left(\frac{s_B}{c}\right)^2\right] = c^2(1 + 1.509^2)$$

$$y_B = 1.810c$$

PROBLEM 7.150* (Continued)

Eq. 7.18:

$$T_m = wy_B$$
$$= 1.810\,wc$$
$$c = \frac{T_m}{1.810\,w}$$

Eq. (1):

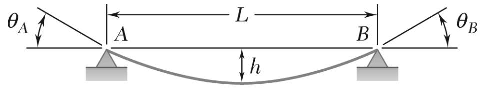
$$x_B = 1.200c = 1.200\frac{T_m}{1.810w} = 0.6630\frac{T_m}{w}$$

Span:

$$L = 2x_B = 2(0.6630)\frac{T_m}{w} \qquad L = 1.326\frac{T_m}{w} \blacktriangleleft$$

(*b*) For $w = 0.25\,\text{lb/ft}$ and $T_m = 8000\,\text{lb}$

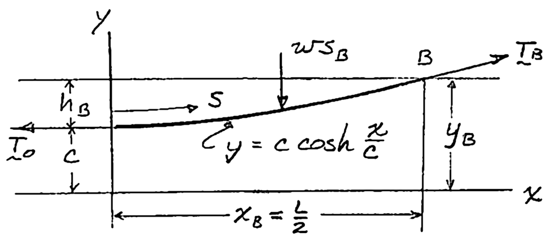
$$L = 1.326\frac{8000\text{ lb}}{0.25\,\text{lb/ft}}$$
$$= 42{,}432\,\text{ft} \qquad L = 8.04\text{ miles} \blacktriangleleft$$



PROBLEM 7.151*

A cable has a mass per unit length of 3 kg/m and is supported as shown. Knowing that the span L is 6 m, determine the *two* values of the sag h for which the maximum tension is 350 N.

SOLUTION



$$y_{max} = c\cosh\frac{L}{2c} = h + c$$

$$w = (3\text{ kg/m})(9.81\text{ m/s}^2) = 29.43\text{ N/m}$$

$$T_{max} = wy_{max}$$

$$y_{max} = \frac{T_{max}}{w}$$

$$y_{max} = \frac{350\text{ N}}{29.43\text{ N/m}} = 11.893\text{ m}$$

$$c\cosh\frac{3\text{ m}}{c} = 11.893\text{ m}$$

Solving numerically:

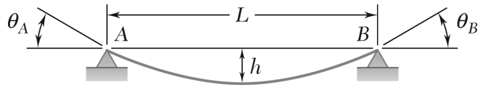
$$c_1 = 0.9241\text{ m}$$

$$c_2 = 11.499\text{ m}$$

$$h = y_{max} - c$$

$$h_1 = 11.893\text{ m} - 0.9241\text{ m} \qquad h_1 = 10.97\text{ m} ◀$$

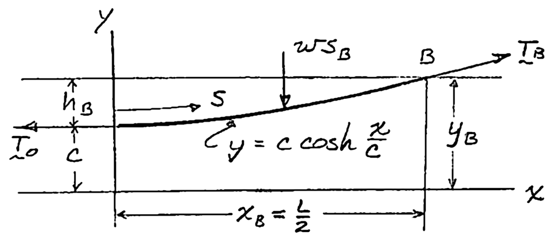
$$h_2 = 11.893\text{ m} - 11.499\text{ m} \qquad h_2 = 0.394\text{ m} ◀$$



PROBLEM 7.152*

Determine the sag-to-span ratio for which the maximum tension in the cable is equal to the total weight of the entire cable AB.

SOLUTION



$$T_{\max} = wy_B = 2ws_B$$

$$y_B = 2s_B$$

$$c\cosh\frac{L}{2c} = 2c\sinh\frac{L}{2c}$$

$$\tanh\frac{L}{2c} = \frac{1}{2}$$

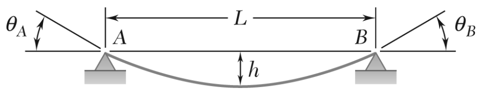
$$\frac{L}{2c} = \tanh^{-1}\frac{1}{2} = 0.549306$$

$$\frac{h_B}{c} = \frac{y_B - c}{c} = \cosh\frac{L}{2c} - 1 = 0.154701$$

$$\frac{h_B}{L} = \frac{\frac{h_B}{c}}{2\left(\frac{L}{2c}\right)} = \frac{0.5(0.154701)}{0.549306} = 0.14081$$

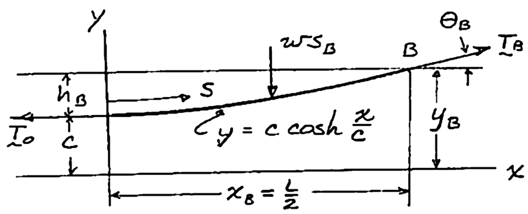
$$\frac{h_B}{L} = 0.1408 \blacktriangleleft$$

PROBLEM 7.153*



A cable of weight per unit length w is suspended between two points at the same elevation that are a distance L apart. Determine (*a*) the sag-to-span ratio for which the maximum tension is as small as possible, (*b*) the corresponding values of θ_B and T_m.

SOLUTION



(*a*) $$T_{\max} = wy_B = wc\cosh\frac{L}{2c}$$

$$\frac{dT_{\max}}{dc} = w\left(\cosh\frac{L}{2c} - \frac{L}{2c}\sinh\frac{L}{2c}\right)$$

For $\min T_{\max}$, $\dfrac{dT_{\max}}{dc} = 0$

$$\tanh\frac{L}{2c} = \frac{2c}{L} \rightarrow \frac{L}{2c} = 1.1997$$

$$\frac{y_B}{c} = \cosh\frac{L}{2c} = 1.8102$$

$$\frac{h}{c} = \frac{y_B}{c} - 1 = 0.8102$$

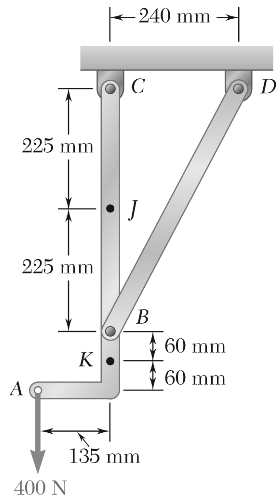
$$\frac{h}{L} = \left[\frac{1}{2}\frac{h}{c}\left(\frac{2c}{L}\right)\right] = \frac{0.8102}{2(1.1997)} = 0.3375 \qquad \frac{h}{L} = 0.338 \blacktriangleleft$$

(*b*) $$T_0 = wc \quad T_{\max} = wc\cosh\frac{L}{2c} \quad \frac{T_{\max}}{T_0} = \cosh\frac{L}{2c} = \frac{y_B}{c}$$

But $$T_0 = T_{\max}\cos\theta_B \quad \frac{T_{\max}}{T_0} = \sec\theta_B$$

So $$\theta_B = \sec^{-1}\left(\frac{y_B}{c}\right) = \sec^{-1}(1.8102) = 56.46° \qquad \theta_B = 56.5° \blacktriangleleft$$

$$T_{\max} = wy_B = w\frac{y_B}{c}\left(\frac{2c}{L}\right)\left(\frac{L}{2}\right) = w(1.8102)\frac{L}{2(1.1997)} \qquad T_{\max} = 0.755wL \blacktriangleleft$$

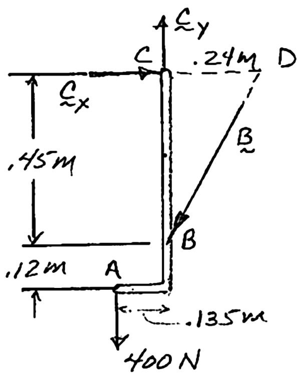


PROBLEM 7.154

Determine the internal forces at Point J of the structure shown.

SOLUTION

FBD ABC:



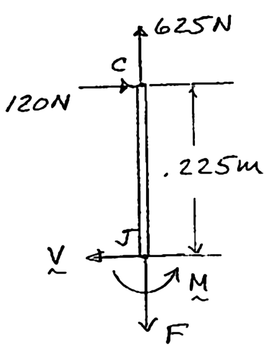
$$\circlearrowleft \Sigma M_D = 0: \quad (0.375 \text{ m})(400 \text{ N}) - (0.24 \text{ m})C_y = 0$$

$$\mathbf{C}_y = 625 \text{ N}\uparrow$$

$$\circlearrowleft \Sigma M_B = 0: \quad -(0.45 \text{ m})C_x + (0.135 \text{ m})(400 \text{ N}) = 0$$

$$\mathbf{C}_x = 120 \text{ N} \longrightarrow$$

FBD CJ:

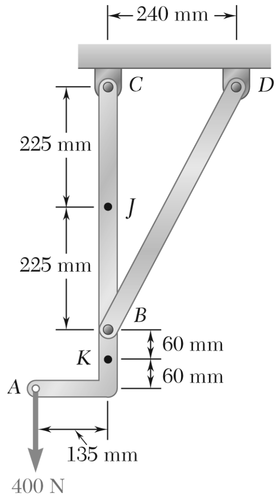


$$\uparrow \Sigma F_y = 0: \quad 625 \text{ N} - F = 0 \qquad \mathbf{F} = 625 \text{ N}\downarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: \quad 120 \text{ N} - V = 0 \qquad \mathbf{V} = 120.0 \text{ N} \longleftarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \quad M - (0.225 \text{ m})(120 \text{ N}) = 0$$

$$\mathbf{M} = 27.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

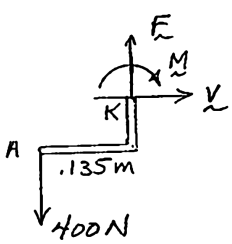


PROBLEM 7.155

Determine the internal forces at Point K of the structure shown.

SOLUTION

FBD AK:



$\xrightarrow{} \Sigma F_x = 0: \quad V = 0$

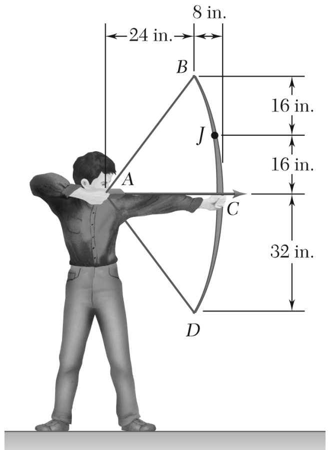
$\mathbf{V} = 0$ ◀

$\uparrow \Sigma F_y = 0: \quad F - 400 \text{ N} = 0$

$\mathbf{F} = 400 \text{ N} \uparrow$ ◀

$\circlearrowleft \Sigma M_K = 0: \quad (0.135 \text{ m})(400 \text{ N}) - M = 0$

$\mathbf{M} = 54.0 \text{ N} \cdot \text{m} \circlearrowright$ ◀

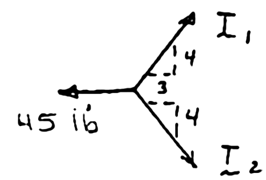


PROBLEM 7.156

An archer aiming at a target is pulling with a 45-lb force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at Point J.

SOLUTION

FBD Point A:

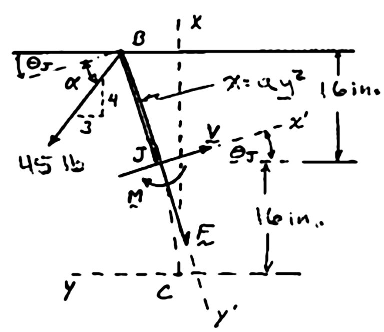


By symmetry $\quad T_1 = T_2$

$$\xrightarrow{\quad} \Sigma F_x = 0: \quad 2\left(\frac{3}{5}T_1\right) - 45\text{ lb} = 0 \quad T_1 = T_2 = 37.5\text{ lb}$$

Curve CJB is parabolic: $\quad x = ay^2$

FBD BJ:



At B: $\quad x = 8$ in. $\qquad y = 32$ in.

$$a = \frac{8\text{ in.}}{(32\text{ in.})^2} = \frac{1}{128\text{ in.}}$$

$$x = \frac{y^2}{128}$$

$$\text{Slope of parabola} = \tan\theta = \frac{dx}{dy} = \frac{2y}{128} = \frac{y}{64}$$

At J: $$\theta_J = \tan^{-1}\left[\frac{16}{64}\right] = 14.036°$$

So $$\alpha = \tan^{-1}\frac{4}{3} - 14.036° = 39.094°$$

$$\nearrow \Sigma F_{x'} = 0: \quad V - (37.5\text{ lb})\cos(39.094°) = 0$$

$\mathbf{V} = 29.1$ lb $\measuredangle$ 14.04° ◀

PROBLEM 7.156 (Continued)

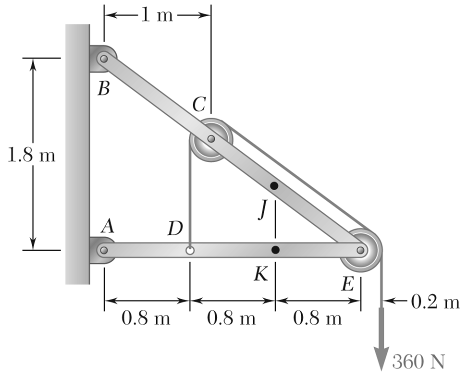
$$\searrow \Sigma F_{y'} = 0: \quad F + (37.5\text{ lb})\sin(39.094°) = 0$$

$$F = -23.647$$

$$\mathbf{F} = 23.6\text{ lb} \searrow 76.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad M + (16\text{ in.})\left[\frac{3}{5}(37.5\text{ lb})\right] + [(8-2)\text{ in.}]\left[\frac{4}{5}(37.5\text{ lb})\right] = 0$$

$$\mathbf{M} = 540\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

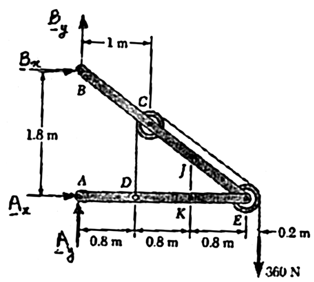


PROBLEM 7.157

Knowing that the radius of each pulley is 200 mm and neglecting friction, determine the internal forces at Point J of the frame shown.

SOLUTION

Free body: Frame and pulleys



$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(1.8 \text{ m}) - (360 \text{ N})(2.6 \text{ m}) = 0$$

$$B_x = -520 \text{ N} \qquad \mathbf{B}_x = 520 \text{ N} \longleftarrow \triangleleft$$

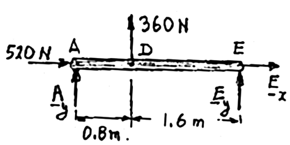
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 520 \text{ N} = 0$$

$$A_x = +520 \text{ N} \qquad \mathbf{A}_x = 520 \text{ N} \longrightarrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B_y - 360 \text{ N} = 0$$

$$A_y + B_y = 360 \text{ N} \qquad (1)$$

Free body: Member AE



$$+\circlearrowleft \Sigma M_E = 0: \quad -A_y(2.4 \text{ m}) - (360 \text{ N})(1.6 \text{ m}) = 0$$

$$A_y = -240 \text{ N} \qquad \mathbf{A}_y = 240 \text{ N} \downarrow \triangleleft$$

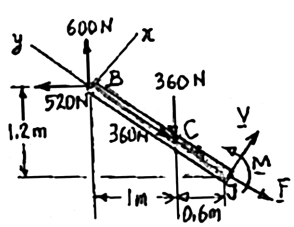
From (1):

$$B_y = 360 \text{ N} + 240 \text{ N}$$

$$B_y = +600 \text{ N} \qquad \mathbf{B}_y = 600 \text{ N} \uparrow \triangleleft$$

Free body: BJ

We recall that the forces applied to a pulley may be applied directly to its axle.



$$\nwarrow + \Sigma F_y = 0: \quad \frac{3}{5}(600 \text{ N}) + \frac{4}{5}(520 \text{ N})$$

$$-360 \text{ N} - \frac{3}{5}(360 \text{ N}) - F = 0$$

$$F = +200 \text{ N} \qquad \mathbf{F} = 200 \text{ N} \nwarrow 36.9° \blacktriangleleft$$

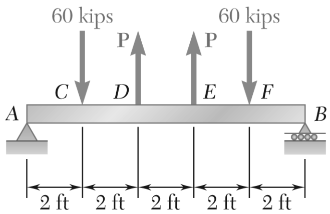
PROBLEM 7.157 (Continued)

$$+\nearrow \Sigma F_x = 0: \quad \frac{4}{5}(600\text{ N}) - \frac{3}{5}(520\text{ N}) - \frac{4}{5}(360\text{ N}) + V = 0$$

$$V = +120.0\text{ N} \qquad \mathbf{V} = 120.0\text{ N} \measuredangle\ 53.1° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad (520\text{ N})(1.2\text{ m}) - (600\text{ N})(1.6\text{ m}) + (360\text{ N})(0.6\text{ m}) + M = 0$$

$$M = +120.0\text{ N}\cdot\text{m} \qquad \mathbf{M} = 120.0\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

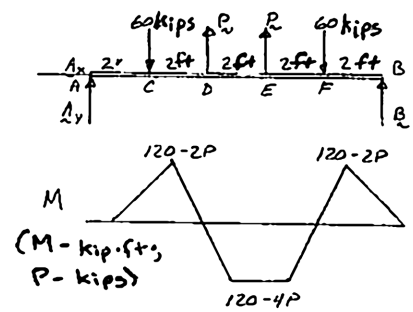


PROBLEM 7.158

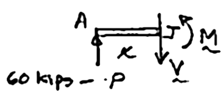
For the beam shown, determine (*a*) the magnitude P of the two upward forces for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$.

SOLUTION

By symmetry: $A_y = B = 60 \text{ kips} - P$

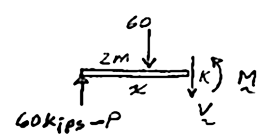


Along AC:



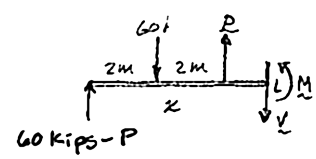
$$\circlearrowleft \Sigma M_J = 0: \quad M - x(60 \text{ kips} - P) = 0$$
$$M = (60 \text{ kips} - P)x$$
$$M = 120 \text{ kips} \cdot \text{ft} - (2 \text{ ft})P \quad \text{at} \quad x = 2 \text{ ft}$$

Along CD:



$$\circlearrowleft \Sigma M_K = 0: \quad M + (x - 2 \text{ ft})(60 \text{ kips}) - x(60 \text{ kips} - P) = 0$$
$$M = 120 \text{ kip} \cdot \text{ft} - Px$$
$$M = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P \quad \text{at} \quad x = 4 \text{ ft}$$

Along DE:



$$\circlearrowleft \Sigma M_L = 0: \quad M - (x - 4 \text{ ft})P + (x - 2 \text{ ft})(60 \text{ kips})$$
$$-x(60 \text{ kips} - P) = 0$$
$$M = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P \quad \text{(const)}$$

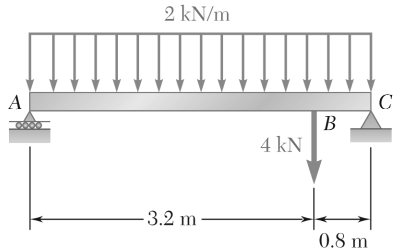
Complete diagram by symmetry

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$

$$120 \text{ kip} \cdot \text{ft} - (2 \text{ ft})P = -[120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P]$$

(*a*) $P = 40.0 \text{ kips}$ ◀

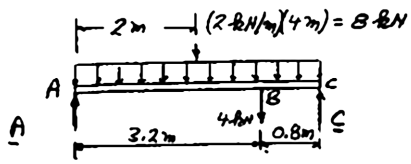
$M_{\min} = 120 \text{ kip} \cdot \text{ft} - (4 \text{ ft})P$ (*b*) $|M|_{\max} = 40.0 \text{ kip} \cdot \text{ft}$ ◀



PROBLEM 7.159

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

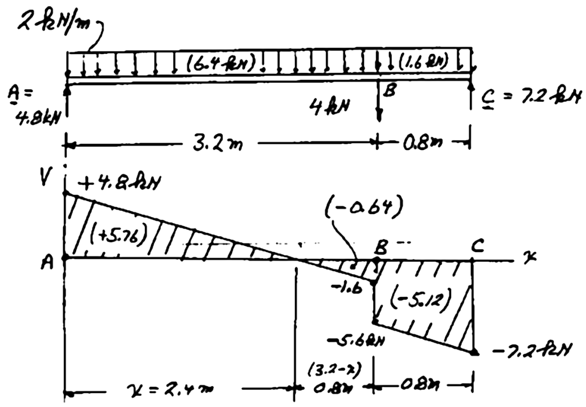


$$+\circlearrowleft \Sigma M_A = 0: \quad (8)(2) + (4)(3.2) - 4C = 0$$

$$\mathbf{C} = 7.2 \text{ kN} \uparrow$$

$$\Sigma F_y = 0: \quad \mathbf{A} = 4.8 \text{ kN} \uparrow$$

(*a*) Shear diagram

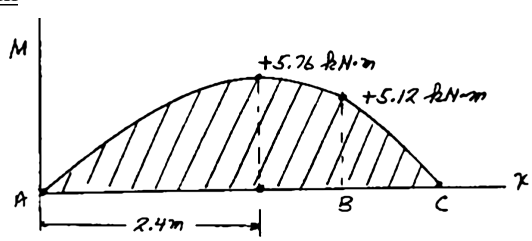


Similar Triangles: $\dfrac{x}{4.8} = \dfrac{3.2 - x}{1.6} = \dfrac{3.2}{6.4}; \quad x = 2.4 \text{ m}$

↑

Add num. & den.

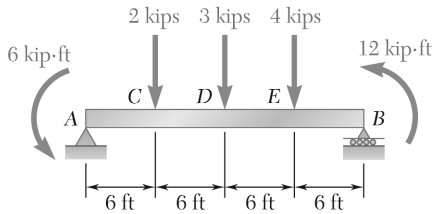
Bending-moment diagram



(*b*)

$$|V|_{max} = 7.20 \text{ kN} \blacktriangleleft$$

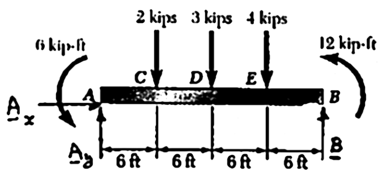
$$|M|_{max} = 5.76 \text{ kN} \cdot \text{m} \blacktriangleleft$$



PROBLEM 7.160

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

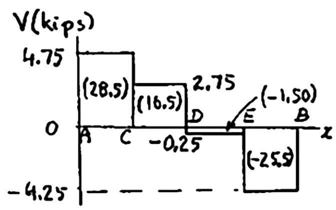


Free body: Beam

$$+\circlearrowleft \Sigma M_B = 0: \quad 6\text{ kip}\cdot\text{ft} + 12\text{ kip}\cdot\text{ft} + (2\text{ kips})(18\text{ ft}) + (3\text{ kips})(12\text{ ft}) + (4\text{ kips})(6\text{ ft}) - A_y(24\text{ ft}) = 0$$

$$A_y = +4.75\text{ kips} \quad \triangleleft$$

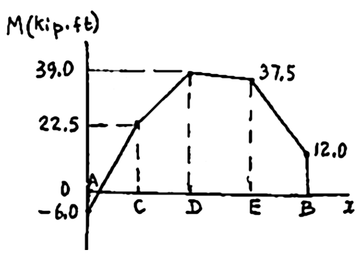
$$\Sigma F_x = 0: \quad A_x = 0$$



Shear diagram

At A: $\qquad V_A = A_y = +4.75\text{ kips}$

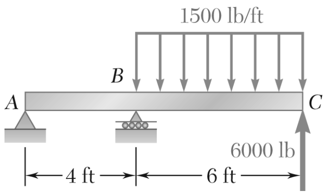
$$|V|_{max} = 4.75\text{ kips} \quad \blacktriangleleft$$



Bending-moment diagram

At A: $\qquad M_A = -6\text{ kip}\cdot\text{ft}$

$$|M|_{max} = 39.0\text{ kip}\cdot\text{ft} \quad \blacktriangleleft$$

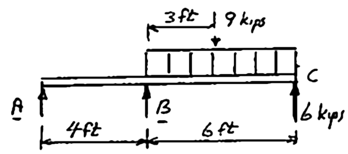


PROBLEM 7.161

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

Free body: Entire beam



$$+\circlearrowleft \Sigma M_A = 0: \quad (6\text{ kips})(10\text{ ft}) - (9\text{ kips})(7\text{ ft}) + 8(4\text{ ft}) = 0$$

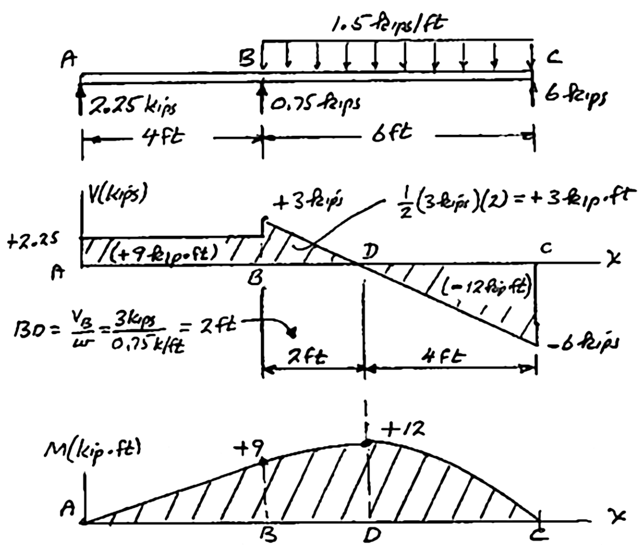
$$B = +0.75\text{ kips}$$

$$\mathbf{B} = 0.75\text{ kips}\uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad A + 0.75\text{ kips} - 9\text{ kips} + 6\text{ kips} = 0$$

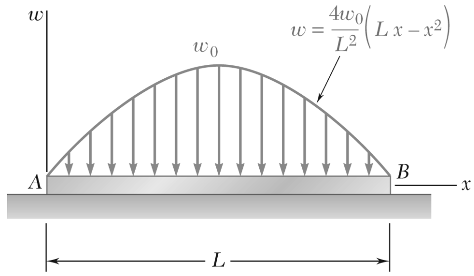
$$A = +2.25\text{ kips}$$

$$\mathbf{A} = 2.25\text{ kips}\uparrow$$



$M_{\max} = 12.00\text{ kip}\cdot\text{ft}$ ◀

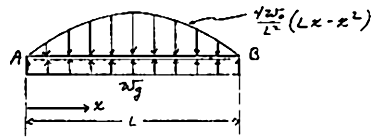
6.00 ft from A



PROBLEM 7.162

The beam AB, which lies on the ground, supports the parabolic load shown. Assuming the upward reaction of the ground to be uniformly distributed, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION



(*a*) $$+\uparrow\Sigma F_y = 0: \quad w_g L - \int_0^L \frac{4w_0}{L^2}(Lx - x^2)dx = 0$$

$$w_g L = \frac{4w_0}{L^2}\left(\frac{1}{2}LL^2 - \frac{1}{3}L^3\right) = \frac{2}{3}w_0 L$$

$$w_g = \frac{2w_0}{3}$$

Define $$\xi = \frac{x}{L} \text{ so } d\xi = \frac{dx}{L} \longrightarrow \text{net load } w = 4w_0\left[\frac{x}{L} - \left(\frac{x}{L}\right)^2\right] - \frac{2}{3}w_0$$

or $$w = 4w_0\left(-\frac{1}{6} + \xi - \xi^2\right)$$

$$V = V(0) - \int_0^\xi 4w_0 L\left(-\frac{1}{6} + \xi - \xi^2\right)d\xi$$

$$= 0 + 4w_0 L\left(\frac{1}{6}\xi + \frac{1}{2}\xi^2 - \frac{1}{3}\xi^3\right)$$

$$V = \frac{2}{3}w_0 L\,(\xi - 3\xi^2 + 2\xi^3) \blacktriangleleft$$

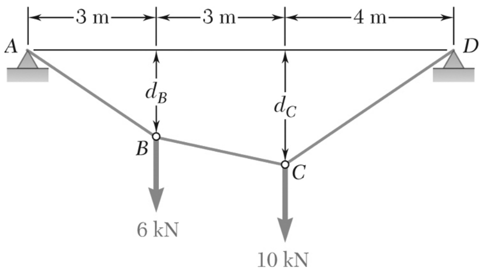
$$M = M_0 + \int_0^x V dx = 0 + \frac{2}{3}w_0 L^2 \int_0^\xi (\xi - 3\xi^2 + 2\xi^3)d\xi$$

$$= \frac{2}{3}w_0 L^2\left(\frac{1}{2}\xi^2 - \xi^3 + \frac{1}{2}\xi^4\right) = \frac{1}{3}w_0 L^2(\xi^2 - 2\xi^3 + \xi^4) \blacktriangleleft$$

(*b*) Max M occurs where $$V = 0 \longrightarrow 1 - 3\xi + 2\xi^2 = 0 \longrightarrow \xi = \frac{1}{2}$$

$$M\left(\xi = \frac{1}{2}\right) = \frac{1}{3}w_0 L^2\left(\frac{1}{4} - \frac{2}{8} + \frac{1}{16}\right) = \frac{w_0 L^2}{48}$$

$$M_{\max} = \frac{w_0 L^2}{48} \text{ at center of beam} \blacktriangleleft$$

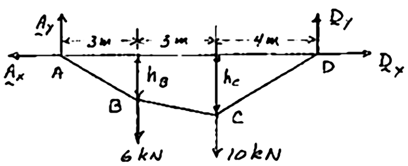


PROBLEM 7.163

Two loads are suspended as shown from the cable *ABCD*. Knowing that $d_B = 1.8$ m, determine (*a*) the distance d_C, (*b*) the components of the reaction at *D*, (*c*) the maximum tension in the cable.

SOLUTION

FBD Cable:



$$\xrightarrow{+} \Sigma F_x = 0: \quad -A_x + D_x = 0 \quad A_x = D_x$$

$$\circlearrowleft \Sigma M_A = 0: \quad (10 \text{ m})D_y - (6 \text{ m})(10 \text{ kN}) - (3 \text{ m})(6 \text{ kN}) = 0$$

$$\mathbf{D}_y = 7.8 \text{ kN} \uparrow$$

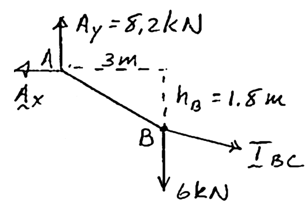
$$+\uparrow \Sigma F_y = 0: \quad A_y - 6 \text{ kN} - 10 \text{ kN} + 7.8 \text{ kN} = 0$$

$$\mathbf{A}_y = 8.2 \text{ kN} \uparrow$$

FDB AB:

$$\circlearrowleft \Sigma M_B = 0: \quad (1.8 \text{ m})A_x - (3 \text{ m})(8.2 \text{ kN}) = 0$$

$$\mathbf{A}_x = \frac{41}{3} \text{ kN} \longleftarrow$$

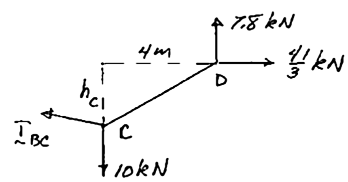


From above $\quad D_x = A_x = \dfrac{41}{3}$ kN

FBD CD:

$$\circlearrowleft \Sigma M_C = 0: \quad (4 \text{ m})(7.8 \text{ kN}) - d_C\left(\frac{41}{3} \text{ kN}\right) = 0$$

$$d_C = 2.283 \text{ m}$$



(*a*) $\quad d_C = 2.28 \text{ m}$ ◀

(*b*) $\quad \mathbf{D}_x = 13.67 \text{ kN} \longrightarrow$ ◀

$\mathbf{D}_y = 7.80 \text{ kN} \uparrow$ ◀

Since $A_x = B_x$ and $A_y > B_y$, max T is T_{AB}

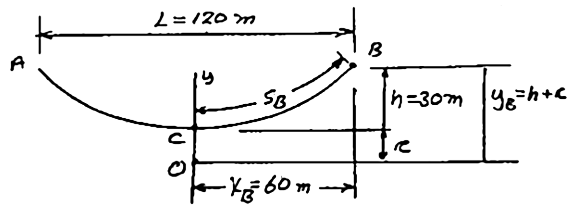
$$T_{AB} = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(\frac{41}{3} \text{ kN}\right)^2 + (8.2 \text{ kN})^2}$$

(*c*) $\quad T_{\max} = 15.94 \text{ kN}$ ◀

PROBLEM 7.164

A wire having a mass per unit length of 0.65 kg/m is suspended from two supports at the same elevation that are 120 m apart. If the sag is 30 m, determine (*a*) the total length of the wire, (*b*) the maximum tension in the wire.

SOLUTION



Eq. 7.16: $y_B = c\cosh\dfrac{x_B}{c}$

$$30m + c = c\cosh\frac{60}{c}$$

Solve by trial and error: $c = 64.459\text{ m}$

Eq. 7.15: $s_B = c\sin h\dfrac{x_B}{c}$

$$s_B = (64.456\text{ m})\sinh\frac{60\text{ m}}{64.459\text{ m}}$$

$$s_B = 69.0478\text{ m}$$

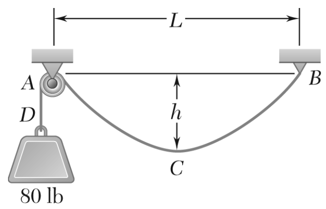
Length $= 2s_B = 2(69.0478\text{ m}) = 138.0956\text{ m}$ $\quad L = 138.1\text{ m}$ ◀

Eq. 7.18: $T_m = wy_B = w(h+c)$

$$= (0.65\text{ kg/m})(9.81\text{ m/s}^2)(30\text{ m} + 64.459\text{ m})$$

$$T_m = 602.32\text{ N}$$

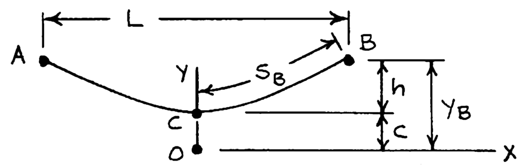
$T_m = 602\text{ N}$ ◀



PROBLEM 7.165

A counterweight D is attached to a cable that passes over a small pulley at A and is attached to a support at B. Knowing that L = 45 ft and h = 15 ft, determine (*a*) the length of the cable from A to B, (*b*) the weight per unit length of the cable. Neglect the weight of the cable from A to D.

SOLUTION



Given: $L = 45$ ft

$h = 15$ ft

$T_A = 80$ lb

$x_B = 22.5$ ft

By symmetry: $T_B = T_A = T_m = 80$ lb

We have $$y_B = c\cosh\frac{x_B}{c} = c\cosh\frac{22.5}{c}$$

and $$y_B = h + c = 15 + c$$

Then $$c\cosh\frac{22.5}{c} = 15 + c$$

or $$\cosh\frac{22.5}{c} = \frac{15}{c} + 1$$

Solve by trial for c: $c = 18.9525$ ft

(*a*) $$s_B = c\sinh\frac{x_B}{c}$$
$$= (18.9525 \text{ ft})\sinh\frac{22.5}{18.9525}$$
$$= 28.170 \text{ ft}$$

Length $= 2s_B = 2(28.170 \text{ ft}) = 56.3$ ft ◀

(*b*) $$T_m = wy_B = w(h + c)$$

$80 \text{ lb} = w(15 \text{ ft} + 18.9525 \text{ ft})$ $w = 2.36$ lb/ft ◀