PHYS I

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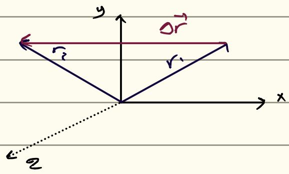
Chapter 4 :- "Motion in 2 & 3 dimensions."

« فكرة هاد السكشن بالزبط نفس فكرة سابقه 2 ونفس القوانين ولكن صارت الحركة ب اتجاهين او 3 مش اتجاه واحد »

1 .. Position & Displacement :

Let :- $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$



* Displacement $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

↳ change in Position.

$\therefore \Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

$= \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

2 .. Average & instantaneous velocity :

↳ في فترة زمنية ↳ في لحظة معينة.

* $\vec{V}_{avg} = \dfrac{\Delta\vec{r}}{\Delta t}$

$= \dfrac{\Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}}{\Delta t}$.

* $\vec{V}_{inst} = \dfrac{d\vec{r}}{dt}$

$= \dfrac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k})$.

$= v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

مهم :-

* ال Visnt هي المماس عند النقطة المطلوبة وطبعاً المماس ما بمثل موقع الجسم ولكن بمثل سرعته عند تلك النقطة.

* Average speed : $(S_{avg}) = \dfrac{\text{distance}}{\Delta t}$ (m/s).

3 .. Average & instantaneous Acceleration :

$\vec{a}_{avg} = \dfrac{\Delta\vec{v}}{\Delta t}$

$= \dfrac{\Delta v_x\hat{i} + \Delta\vec{v}_y\hat{j} + \Delta\vec{v}_z\hat{k}}{\Delta t}$.

$\vec{a}_{inst} = \dfrac{d\vec{v}}{dt}$

$= \dfrac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$

$= \vec{a}_x\hat{i} + \vec{a}_y\hat{j} + \vec{a}_z\hat{k}$.

Sample Problem 4.01 **Two-dimensional position vector, rabbit run**

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \quad (4\text{-}5)$$

and

$$y = 0.22t^2 - 9.1t + 30. \quad (4\text{-}6)$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?

$X = -0.31(15)^2 + 7.2(15) + 28$

$= 66.25$ m

$y = 0.22(15)^2 - 9.1(15) + 30$

$= -57$m

$\therefore \vec{r} = 66.25\hat{i} - 57\hat{j}$

* Find the Position at $t_1 = 0$, $t_2 = 15$ sec?

at $t = 0$

$X = -0.31(0) + 7.2(0) + 28$

$= 28$

$y = 0.22(0) - 9.1(0) + 30$

$= 30$

$\vec{r}_1 (t_1 = 0) = 28\hat{i} + 30\hat{j}$.

$\therefore \Delta\vec{r} = (66.25 - 28)\hat{i} + (-57 - 30)\hat{j}$

$= 38.25\hat{i} - 87\hat{j}$.

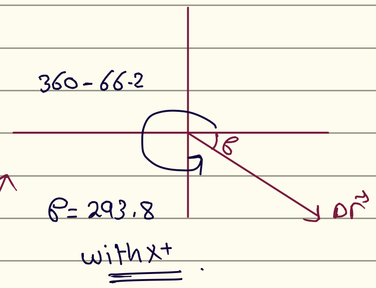
* Find the magnitud & direction of $\Delta\vec{r}$?

$|\Delta\vec{r}| = \sqrt{(38.25)^2 + (87)^2}$

$= 95$ m

$\tan\theta = \dfrac{\Delta r_y}{\Delta r_x} \longrightarrow \tan\theta = \dfrac{-87}{38.25} = 2.3$

$\theta = -66.2$



* Find $\vec{V}_{avg}$ ($t_1 = 0$, $t_2 = 15$ sec) ?

$\vec{V}_{avg} = \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{38.25\hat{i} - 87\hat{j}}{15}$

$= (2.5\hat{i} - 5.8\hat{j})$ m/s.

Find V_{inst} ($t = 2$ sec) ?

$\vec{r} = (-0.31t^2 + 7.2t + 28)\hat{i} + (0.22t^2 - 9.1t + 30)\hat{j}$

$\vec{V}_{inst} = (-0.62t + 7.2)\hat{i} + (0.44t - 9.1)\hat{j}$

$\vec{V}_{inst}(t = 2\text{sec}) = (-0.62(2) + 7.2)\hat{i} + (0.44(2) - 9.1)\hat{j}$

$= 5.96\hat{i} - 8.22\hat{j}$ m/s.

4 .. projectile Motion :

حركة المقذوفات في بعدين.

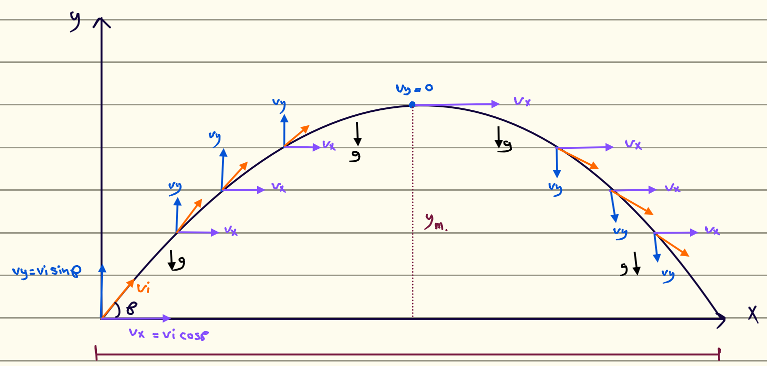
Note :-

١. الجسم يتسارع بتسارع ثابت وهو تسارع الجاذبية $a_y = -g$

2. لا يوجد تسارع على المحور السيني ∴ السرعة ثابتة.

3. عند أقصى ارتفاع $V_y = 0$ ولكنه $V_x \neq 0$ لأنها ثابتة.

4. نستطيع استخدام معادلات الحركة بتسارع ثابت



* Motion along x-axis :- (Horizontal Motion).

No acceleration , so V_x is constant.

$V_x = V_{ix} = V_i \cos\theta$.

To Find Range ?

$\Delta x = v_{ix}t + \cancel{\tfrac{1}{2}a_x t}$ (0)

$\Delta x = v_{ix}\, t \longrightarrow$ in general at any time.

$R = v_{ix}\, T_{(AB)}$ (T : time of flight). $\longrightarrow R = \dfrac{V_i^2 \sin(2\theta)}{g}$.

* Motion along y-axis:

$a_y = -g = -9.8\ m/s^2$ m/s^2.

$v_{iy} = v_i \sin\theta$.

* $v_{yf} = v_{iy} + at$

$v_{yf}^2 = v_{iy}^2 - 2g\,\Delta y$.

$\Delta y = v_{iy}t - \frac{1}{2}gt^2$.

* Flight time: زمن التحليق.

$\Delta y = v_{iy}t - \frac{1}{2}gt^2$

$0 = v_{iy}t - \frac{1}{2}gt^2$

$t_{AB} = \frac{2v_i \sin\theta}{g}$.

* The Maximum Height (H):

$v_{fy} = v_{iy} + at$

$0 = v_i \sin\theta - gt$

$t = \frac{v_i \sin\theta}{g}$

$\therefore \Delta y = v_{iy}t - \frac{1}{2}gt^2$

$\Delta y = v_{iy}\left(\frac{v_i \sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{v_i \sin\theta}{g}\right)^2$

$\therefore H = \frac{v_i^2 \sin^2\theta}{2g}$.

* to find the difference between R and H?

$R = \frac{v_i^2 \sin 2\theta}{g}$, $H = \frac{v_i^2 \sin^2\theta}{2g}$

$\frac{H}{R} = \left(\frac{\cancel{v_i^2}\sin^2\theta}{\cancel{2g}}\right)\left(\frac{\cancel{g}}{\cancel{v_i^2}\sin 2\theta}\right)$

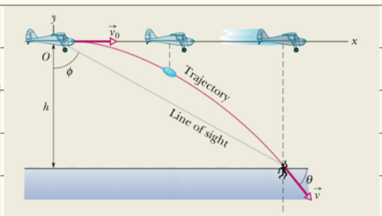
$\therefore \frac{H}{R} = \frac{\sin^2\theta}{4\sin\theta\cos\theta} \rightarrow \frac{1}{4}\tan\theta$.

$\therefore H = \frac{R\tan\theta}{4}$.

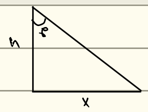
Sample Problem 4.04 Projectile dropped from airplane

In Fig. 4-14, a rescue plane flies at 198 km/h (= 55.0 m/s) and constant height h = 500 m toward a point directly over a victim, where a rescue capsule is to land.

(a) What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made?



$\tan\theta = \frac{x}{y}$



$\Delta x = x = v_{ix}t$, we need to find t.

$\Delta y = \Delta y = \cancel{v_{iy}t}^{\,0} - \frac{1}{2}gt^2$, $v_{iy} = 0 \rightarrow$ No y-comp of the velocity.

$-h = -\frac{1}{2}gt^2$

$500 = \frac{1}{2}(9.8)t^2$

$\therefore t = 10.1$ sec.

* $x = v_i \times t$

$= 55 \times 10.1$

$= 555.5\ m$

$\therefore \theta = \tan^{-1}\left(\frac{555.5}{500}\right)$

$= 48°$

(b) As the capsule reaches the water, what is its velocity $\vec{v}$?

$v_x = v_{ix} \longrightarrow$ const

$= 55\ m/s$.

$v_{yf} = v_{iy} - gt$

$= 0 - (9.8)(10.1)$

$= -99\ m/s$.

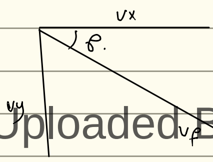
$\therefore \vec{v_f} = 55\hat{i} - 99\hat{j}$ m/s.

$|\vec{v_f}| = \sqrt{(55)^2 + (99)^2}$

$= 113\ m/s$.

$\theta = \tan^{-1}\left(\frac{-99}{55}\right)$

$\theta = -60.9°$



Sample Problem 4.05 Launched into the air from a water slide

One of the most dramatic videos on the web (but entirely fictitious) supposedly shows a man sliding along a long water slide and then being launched into the air to land in a water pool. Let's attach some reasonable numbers to such a flight to calculate the velocity with which the man would have hit the water. Figure 4-15*a* indicates the launch and landing sites and includes a superimposed coordinate system with its origin conveniently located at the launch site. From the video we take the horizontal flight distance as $D = 20.0$ m, the flight time as $t = 2.50$ s, and the launch angle as $\theta_0 = 40.0°$. Find the magnitude of the velocity at launch and at landing.

KEY IDEAS

(1) For projectile motion, we can apply the equations for constant acceleration along the horizontal and vertical axes *separately*. (2) Throughout the flight, the vertical acceleration is $a_y = -g = -9.8\ \text{m/s}^2$ and the horizontal acceleration is $a_x = 0$.

Calculations: In most projectile problems, the initial challenge is to figure out where to start. There is nothing wrong with trying out various equations, to see if we can somehow get to the velocities. But here is a clue. Because we are going to apply the constant-acceleration equations separately to the x and y motions, we should find the horizontal and vertical components of the velocities at launch and at landing. For each site, we can then combine the velocity components to get the velocity.

Because we know the horizontal displacement $D = 20.0$ m, let's start with the horizontal motion. Since $a_x = 0$, we know that the horizontal velocity component v_x is constant during the flight and thus is always equal to the horizontal component v_{0x} at launch. We can relate that component, the displacement $x - x_0$, and the flight time $t = 2.50$ s with Eq. 2-15:

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2. \tag{4-32}$$

Substituting $a_x = 0$, this becomes Eq. 4-21. With $x - x_0 = D$, we then write

$$20\ \text{m} = v_{0x}(2.50\ \text{s}) + \tfrac{1}{2}(0)(2.50\ \text{s})^2$$

$$v_{0x} = 8.00\ \text{m/s}.$$

That is a component of the launch velocity, but we need the magnitude of the full vector, as shown in Fig. 4-15*b*, where the components form the legs of a right triangle and the full vector forms the hypotenuse. We can then apply a trig definition to find the magnitude of the full velocity at launch:

$$\cos\theta_0 = \frac{v_{0x}}{v_0},$$

and so

$$v_0 = \frac{v_{0x}}{\cos\theta_0} = \frac{8.00\ \text{m/s}}{\cos 40°}$$

$$= 10.44\ \text{m/s} \approx 10.4\ \text{m/s}. \quad \text{(Answer)}$$

Now let's go after the magnitude v of the landing velocity. We already know the horizontal component, which does not change from its initial value of 8.00 m/s. To find the vertical component v_y and because we know the elapsed time $t = 2.50$ s and the vertical acceleration $a_y = -9.8\ \text{m/s}^2$, let's rewrite Eq. 2-11 as

$$v_y = v_{0y} + a_y t$$

and then (from Fig. 4-15*b*) as

$$v_y = v_0 \sin\theta_0 + a_y t. \tag{4-33}$$

Substituting $a_y = -g$, this becomes Eq. 4-23. We can then write

$$v_y = (10.44\ \text{m/s})\sin(40.0°) - (9.8\ \text{m/s}^2)(2.50\ \text{s})$$

$$= -17.78\ \text{m/s}.$$

Now that we know both components of the landing velocity, we use Eq. 3-6 to find the velocity magnitude:

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(8.00\ \text{m/s})^2 + (-17.78\ \text{m/s})^2}$$

$$= 19.49\ \text{m/s} \approx 19.5\ \text{m/s}. \quad \text{(Answer)}$$

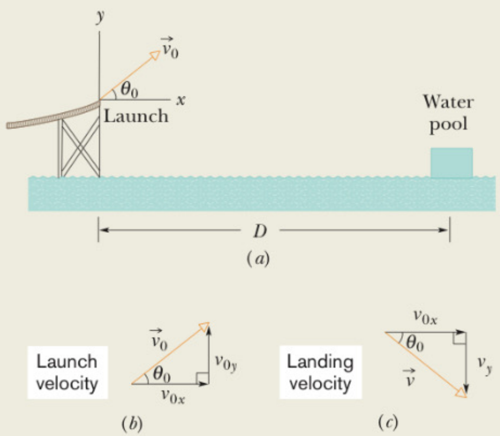


Figure 4-15 (*a*) Launch from a water slide, to land in a water pool. The velocity at (*b*) launch and (*c*) landing.

7 In a particle accelerator, the position vector of a particle is initially estimated as $\vec{r} = 6.0\hat{i} - 7.0\hat{j} + 3.0\hat{k}$ and after 10 s, it is estimated to be $\vec{r} = -3.0\hat{i} + 9.0\hat{j} - 3.0\hat{k}$, all in meters. In unit-vector notation, what is the average velocity of the particle?

$\vec{r}_1 = 6\hat{i} - 7\hat{j} + 3\hat{k}$

after 10 s :-

$\vec{r}_2 = -3\hat{i} + 9\hat{j} - 3\hat{k}$

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

$$= \frac{(-3\hat{i} + 9\hat{j} - 3\hat{k}) - (6\hat{i} - 7\hat{j} + 3\hat{k})}{10 - 0}$$

$$= \frac{-9\hat{i} + 16\hat{j} - 0.6}{10}$$

$$= (-0.9\hat{i} + 1.6\hat{j} - 0.6\hat{k})\ m/s.$$

$$* \ |\vec{v}_{avg}| = \sqrt{(0.9)^2 + (1.6)^2 + (0.6)^2}$$

$1.93\ m/s$

14 A proton initially has $\vec{v} = 4.0\hat{i} - 2.0\hat{j} + 3.0\hat{k}$ and then 4.0 s later has $\vec{v} = -2.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}$ (in meters per second). For that 4.0 s, what are (a) the proton's average acceleration $\vec{a}_{avg}$ in unit-vector notation, (b) the magnitude of $\vec{a}_{avg}$, and (c) the angle between $\vec{a}_{avg}$ and the positive direction of the x axis?

$\vec{v}_1 = 4\hat{i} - 2\hat{j} + 3\hat{k}$

after 4 sec :-

$\vec{v}_2 = -2\hat{i} - 2\hat{j} + 5\hat{k}$

a) $$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

$$= \frac{(-2\hat{i} - 2\hat{j} + 5\hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k})}{4 - 0}$$

$$= \frac{-6\hat{i} + 0\hat{j} + 2\hat{k}}{4}$$

$$= (-1.5\hat{i} + 0.5\hat{k})\ m/s^2$$

b) $$* \ |\bar{a}_{avg}| = \sqrt{(1.5)^2 + (0.5)^2}$$

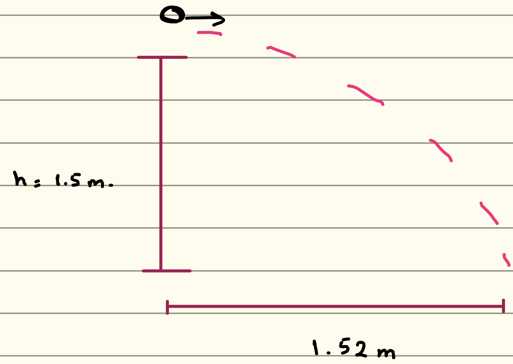
$$= 1.58\ m/s^2$$

c) $$\tan\theta = \frac{0.5}{1.5}$$

$= 0.3 \Rightarrow \theta = 18^\circ$ with $x-$

$180 - 18 \Rightarrow 162^\circ$ with $x+$

22 A small ball rolls horizontally off the edge of a tabletop that is 1.50 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. (a) How long is the ball in the air? (b) What is its speed at the instant it leaves the table?



a) $D_y = v_i t + \frac{1}{2} a t^2$

$-1.5 = 0 + \frac{1}{2}(-9.8) t^2$

$t^2 = 0.3 \rightarrow t = 0.55$ sec.

b) $D_x = v_i t + \frac{1}{2} a t^2$

$1.52 = v_i (0.55) + 0$

$v_i = 2.76$ m/s.

32 You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-26). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

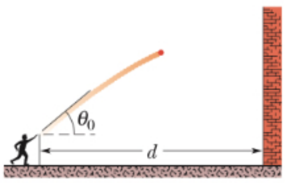


Figure 4-26 Problem 32.

a) $D_x = v_i t + \frac{1}{2} a t^2$

$22 = (25 \cos 40) t + 0$

$t = 1.15$ sec.

$D_y = v_i t + \frac{1}{2} a t^2$

$D_y = (25 \sin 40)(1.15) + \frac{1}{2}(-9.8)(1.15)^2$

$= 15.5$ m.

b) $v_{fy} = v_{iy} + at$

$= 25 \sin 40 + (-9.8)(1.15)$

$= 4.8$ m/s

$v_{fx} = v_{ix} + at$

$= 25 \cos 40$

$= 19.5$ m/s

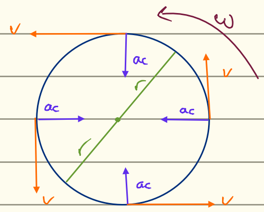
$\therefore v_f = \sqrt{(4.8)^2 + (19.5)^2}$

$= 19.7$ m/s.

d) no, V_y max $= 0$.

5 .. Uniform Circle Motion :

↲ الحركة الدائرية المنتظمة.



$V = \frac{2\pi r}{T}$ m/s (constant)

↲ ثابتة مقداراً

ومتغيرة اتجاهاً.

↳ since dir. of v is not constant.

* we have acceleration (a_c) . تسارع مركزي

$a_c = \frac{V^2}{r}$

$= \omega^2 r$

* frequancy (f) التردد.

$f = \frac{1}{T}$ rev/sec = Hz.

* angular freq or angular velocity (ω) . السرعة أو التردد الزاوي

$\omega = 2\pi f$ rad/sec

$= \frac{v}{r}$

* $v = \omega r$

$s = \theta r$

$a = \alpha r$

Sample Problem 4.06 **Top gun pilots in turns**

"Top gun" pilots have long worried about taking a turn too tightly. As a pilot's body undergoes centripetal acceleration, with the head toward the center of curvature, the blood pressure in the brain decreases, leading to loss of brain function.

There are several warning signs. When the centripetal acceleration is $2g$ or $3g$, the pilot feels heavy. At about $4g$, the pilot's vision switches to black and white and narrows to "tunnel vision." If that acceleration is sustained or increased, vision ceases and, soon after, the pilot is unconscious—a condition known as g-LOC for "g-induced loss of consciousness."

What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{i} + 500\hat{j})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{i} - 500\hat{j})$ m/s?

we want to find R.

↳ using the fact that $\vec{v}$ has opposite dir in half cycle $\Rightarrow$ $T = 2(24) = 48$ sec.

$v = \sqrt{(400)^2 + (500)^2}$

$= 640$ m/s

but $v = \frac{2\pi R}{T}$

$640 = \frac{2\pi R}{48}$ $\longrightarrow$ $R = 4889$ m.

$a_c = \frac{V^2}{R}$

$= \frac{(640)^2}{4889}$

$= 83.8\ m/s^2$

$= 8.5\ g.$

6 .. Relative Motion :

Measures of position and velocity depend on the reference frams of the measurer.

* How is observer moving?

our usual refrence frame is that of the ground.

* How to read subscripts?

1) PA $\longrightarrow$ P as measured by A.

2) PB $\longrightarrow$ P as measured by B.

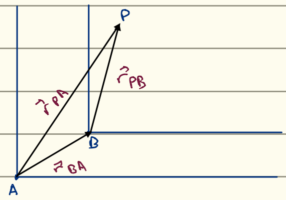
3) BA $\longrightarrow$ B as measured by A.

* if we have two frames (two observers).

from A at rest

from B moving at const.

* frames A and B are each watching the movement of object P.



* Position in different frames are related by :-

$x_{PA} = x_{PB} + x_{BA}$

* take $\frac{d}{dt}$ $\Rightarrow$ $\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$

drive 2 $\Rightarrow$ $\vec{a}_{PA} = \vec{a}_{PB}$.

Sample Problem 4.08 **Relative motion, two dimer**

In Fig. 4-20*a*, a plane moves due east while the pilot points the plane somewhat south of east, toward a steady wind that blows to the northeast. The plane has velocity $\vec{v}_{PW}$ relative to the wind, with an airspeed (speed relative to the wind) of 215 km/h, directed at angle θ south of east. The wind has velocity $\vec{v}_{WG}$ relative to the ground with speed 65.0 km/h, directed 20.0° east of north. What is the magnitude of the velocity $\vec{v}_{PG}$ of the plane relative to the ground, and what is θ?

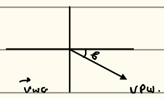
from A :- ground

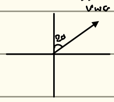
from B :- wind

P :- Pilot

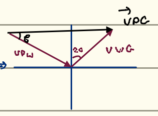
v_{PG} :- toward east.

$\vec{V}_{PW}$:- 215 km/h , θ South of east





$\vec{V}_{WG}$:- 65 km/h , 20° east of north.



$\therefore$ $\vec{V}_{PG}$ $\Rightarrow$

$\vec{V}_{PG} = \vec{V}_{PW} + \vec{V}_{WG}$

$\vec{V}_{PW} = 215 \cos\theta\, \hat{i} + 215 \sin\theta\, \hat{j}$

$\vec{V}_{WG} = 65 \sin 20\, \hat{i} + 65 \cos 20\, \hat{j}$.

$\vec{V}_{PG} = V_{PG}\, \hat{i} + 0\, \hat{j}$

$\therefore$ $\vec{V}_{PW} + \vec{V}_{WG} = (215 \cos\theta + 65 \sin 20)\, \hat{i} + (215 \sin\theta + 65 \cos 20)\, \hat{j}$

$\hookrightarrow$ but $\vec{V}_{PG} = \vec{V}_{PW} + \vec{V}_{WG}$

x- comp :-

$V_{PG} = 215 \cos\theta + 65 \sin 20$

y- comp :-

$0 = 215 \sin\theta + 65 \cos 20$

$\sin\theta = -\frac{65 \cos 20}{215}$

$\theta = 16.5°$

$\therefore$ $V_{PG} = 215 \cos 16.5° + 65 \sin 20$

$= 228$ Km/h.

$\therefore$ $\vec{V}_{PG} = 228\, \hat{i}$ Km/h (or due east).

Lecture problems:

7 In a particle accelerator, the position vector of a particle is initially estimated as $\vec{r} = 6.0\hat{i} - 7.0\hat{j} + 3.0\hat{k}$ and after 10 s, it is estimated to be $\vec{r} = -3.0\hat{i} + 9.0\hat{j} - 3.0\hat{k}$, all in meters. In unit vector notation, what is the average velocity of the particle?

$\vec{r}_1 = 6\hat{i} - 7\hat{j} + 3\hat{k}$

after 10 s :-

$\vec{r}_2 = -3\hat{i} + 9\hat{j} - 3\hat{k}$

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

$$= \frac{(-3\hat{i} + 9\hat{j} - 3\hat{k}) - (6\hat{i} - 7\hat{j} + 3\hat{k})}{10 - 0}$$

$$= \frac{-9\hat{i} + 16\hat{j} - 0.6}{10}$$

$$= (-0.9\hat{i} + 1.6\hat{j} - 0.6\hat{k})\ m/s.$$

$$* \ |\vec{v}_{avg}| = \sqrt{(0.9)^2 + (1.6)^2 + (0.6)^2}$$

$$= 1.93\ m/s$$

14 A proton initially has $\vec{v} = 4.0\hat{i} - 2.0\hat{j} + 3.0\hat{k}$ and then 4.0 s later has $\vec{v} = -2.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}$ (in meters per second). For that 4.0 s, what are (a) the proton's average acceleration $\vec{a}_{avg}$ in unit-vector notation, (b) the magnitude of $\vec{a}_{avg}$, and (c) the angle between $\vec{a}_{avg}$ and the positive direction of the x axis?

$\vec{v}_1 = 4\hat{i} - 2\hat{j} + 3\hat{k}$

after 4 sec :-

$\vec{v}_2 = -2\hat{i} - 2\hat{j} + 5\hat{k}$

a) $$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}$$

$$= \frac{(-2\hat{i} - 2\hat{j} + 5\hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k})}{4 - 0}$$

$$= \frac{-6\hat{i} + 0\hat{j} + 2\hat{k}}{4}$$

$$= (-1.5\hat{i} + 0.5\hat{k})\ m/s^2$$

b) $$* \ |\vec{a}_{avg}| = \sqrt{(1.5)^2 + (0.5)^2}$$

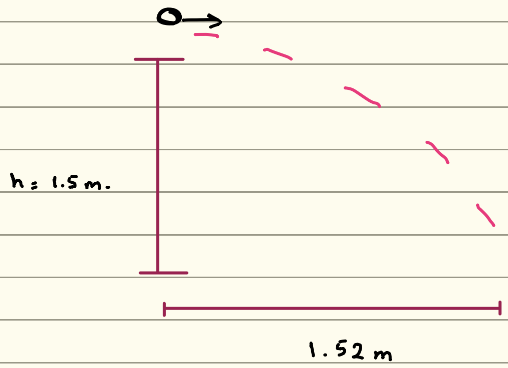
$$= 1.58\ m/s^2$$

c) $$\tan\theta = \frac{0.5}{1.5}$$

$$= 0.3 \Rightarrow \theta = 18^\circ \text{ with } x-$$

$$180 - 18 \Rightarrow 162^\circ \text{ with } x+$$

22 A small ball rolls horizontally off the edge of a tabletop that is 1.50 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. (a) How long is the ball in the air? (b) What is its speed at the instant it leaves the table?



a) $D_y = v_i t + \frac{1}{2} a t^2$

$-1.5 = 0 + \frac{1}{2}(-9.8)\, t^2$

$t^2 = 0.3 \rightarrow t = 0.55$ sec.

b) $D_x = v_i t + \frac{1}{2} a t^2$

$1.52 = v_i (0.55) + 0$

$v_i = 2.76$ m/s

32 You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-26). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall? What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall? (d) When it hits, has it passed the highest point on its trajectory?

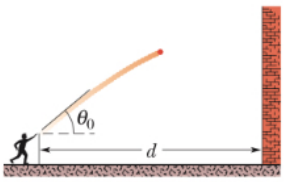


Figure 4-26 Problem 32.

a) $D_x = v_i t + \frac{1}{2} a t^2$

$22 = (25 \cos 40)\, t + 0$

$t = 1.15$ sec.

$D_y = v_i t + \frac{1}{2} a t^2$

$D_y = (25 \sin 40)(1.15) + \frac{1}{2}(-9.8)(1.15)^2$

$= 15.5$ m.

b) $v_{fy} = v_{iy} + at$

$= 25 \sin 40 + (-9.8)(1.15)$

$= 4.8$ m/s

$v_{fx} = v_{ix} + at$

$= 25 \cos 40$

$= 19.5$ m/s

$\therefore v_f = \sqrt{(4.8)^2 + (19.5)^2}$

19.7 m/s

d) no, v_y max $= 0$.

58 A rotating fan completes 1100 revolutions every minute. Consider the tip of a blade, at a radius of 0.15 m. (a) Through what distance does the tip move in one revolution? What are (b) the tip's speed and (c) the magnitude of its acceleration? (d) What is the period of the motion?

$f = \frac{1100 \text{ rev}}{\text{min}} \times \frac{\text{min}}{60 \text{ sec}} \rightarrow 18.33 \text{ rev/s}$

a) $D = 2\pi r$

$= 2(3.14)(0.15)$

$= 0.942 \text{ m}$

b) $v = \frac{2\pi r}{T}$

d) $T = \frac{1}{f} \rightarrow \frac{1}{18.33} = \boxed{0.054 \text{ sec}}$

$\therefore v = \frac{2(3.14)(0.15)}{0.054}$

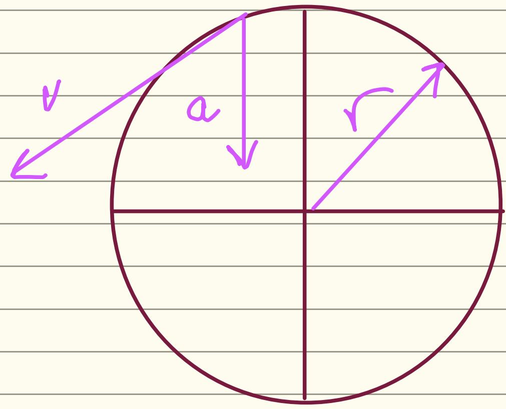
$= 17.44 \text{ m/s.}$

c) $a = \frac{v^2}{r}$

$= \frac{(17.44)^2}{0.15}$

$= 1949.4 \text{ m/s}^2$

60 A centripetal-acceleration addict rides in uniform circular motion with period $T = 2.0$ s and radius $r = 3.50$ m. At t_1 his acceleration is $\vec{a} = (6.00 \text{ m/s}^2)\hat{i} + (-4.00 \text{ m/s}^2)\hat{j}$. At that instant, what are the values of (a) $\vec{v} \cdot \vec{a}$ and (b) $\vec{r} \times \vec{a}$?



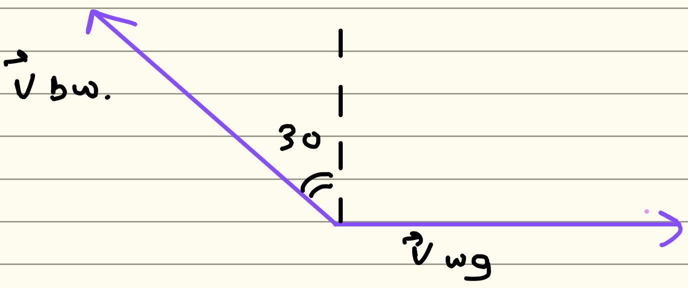
a) $\vec{v} \cdot \vec{a} = \vec{v} \cdot \vec{a} \cos 90$

$= 0$

b $\vec{r} \times \vec{a} = \vec{r} \times \vec{a} \sin 0$

$= 0$

*80 A 200 m wide river flows due east at a uniform speed of 2.5 m/s. A boat with a speed of 8.0 m/s relative to the water leaves the south bank pointed in a direction 30° west of north. What are the (a) magnitude and (b) direction of the boat's velocity relative to the ground? (c) How long does the boat take to cross the river?



- $\vec{V}_{wg} = 2.5\,\hat{i}$

$\vec{V}_{bw} = -8 \sin 30\,\hat{i} + 8 \cos 30\,\hat{j}$

$= -4\hat{i} + 6.9\,\hat{i}$

$\therefore \vec{V}_{bg} = \vec{V}_{wg} + \vec{V}_{bw}$

$= 2.5\hat{i} + (-4\hat{i} + 6.9)\hat{i}$

$= -1.5\,\hat{i} + 6.9\,\hat{i}$

$\vec{V}_{bg} = \sqrt{(1.5)^2 + (6.9)^2}$

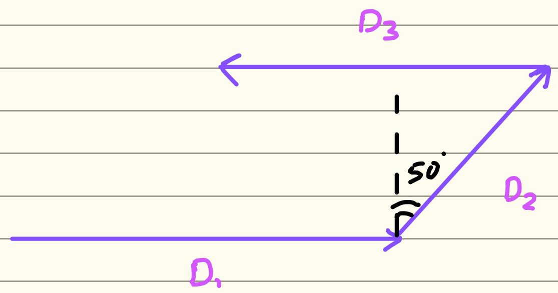
$= 7\ m/s$

$\tan\theta = \frac{-6.9}{1.5} \longrightarrow \theta = 78^\circ$

c) $V = \frac{D}{t}$

$7 = \frac{200}{t} \longrightarrow t = 29$ sec.

Discussion problems:



5 A train at a constant 60.0 km/h moves east for 40.0 min, then in a direction 50.0° east of due north for 20.0 min, and then west for 50.0 min. What are the (a) magnitude and (b) angle of its average velocity during this trip?

a) $D_1 = v t_1$

$= \dfrac{(60)(40)}{60}$

$= 40$ m.

$D_2 = v t_2$

$= \dfrac{(60)(20)}{60}$

$= 20$ m

$D_3 = \dfrac{(60)(50)}{(60)}$

$= 50$ m.

$\vec{r}_1 = 40\,\hat{i}$

$\vec{r}_2 = 20 \sin 50\,\hat{i} + 20 \cos 50\,\hat{j}$

$\vec{r}_3 = -50\,\hat{i}$

$\therefore \vec{r}_f = \vec{r}_1 + \vec{r}_2 + \vec{r}_3$

$= 40\hat{i} + (20 \sin 50\,\hat{i} + 20 \cos 50\,\hat{j}) + (-50\hat{i})$

$= 5.3\hat{i} + 12.8\hat{j}$ m.

$\vec{v}_{avg} = \dfrac{\Delta \vec{r}}{\Delta t}$

$= \dfrac{5.3\hat{i} + 12.8\hat{j}}{1.83}$

$= (2.89\hat{i} + 7\hat{j})$ km/h.

b) $\tan^{-1}\left(\dfrac{7}{2.89}\right) = 67.5°$

11 A particle that is moving in an xy plane has a position vector given by $\vec{r} = (3.00t^3 - 6.00t)\hat{i} + (7.00 - 8.00t^4)\hat{j}$, where $\vec{r}$ is measured in meters and t is measured in seconds. For $t = 3.00$ s, in unit-vector notation, find (a) $\vec{r}$, (b) $\vec{v}$, and (c) $\vec{a}$. (d) Find the angle between the positive direction of the x axis and a line that is tangent to the path of the particle at $t = 3.00$ s.

a) $\vec{r} = (3t^3 - 6t)\hat{i} + (7 - 8t^4)\hat{j}$.

$\vec{r}(t=3) = (3(3)^3 - 6(3))\hat{i} + (7 - 8(3)^4)\hat{j}$

$= (63\hat{i} - 641\hat{j})$ m.

b) $\vec{v} = (9t^2 - 6)\hat{i} + (-32t^3)\hat{j}$

$\vec{v}(t=3) = (9(3)^2 - 6)\hat{i} + (-32(3)^3)\hat{j}$

$= (75\hat{i} - 864\hat{j})$ m/s.

c) $\vec{a} = (18t)\hat{i} + (-96t^2)\hat{j}$

$\vec{a}(t=3) = (18(3))\hat{i} + (-96(3)^2)\hat{j}$

$= (54\hat{i} - 864\hat{j})$ m/s^2

d) $\theta = \tan^{-1}\left(\dfrac{-864}{54}\right)$

$= -85°$

15 From the origin, a particle starts at $t = 0$ s with a velocity $\vec{v} = 7.0\hat{i}$ m/s and moves in the xy plane with a constant acceleration of $\vec{a} = (-9.0\hat{i} + 3.0\hat{j})$ m/s^2. At the time the particle reaches the maximum x coordinate, what is its (a) velocity and (b) position vector?

X-axis :-

$v_f^2 = v_i^2 + 2ad$

$0 = (7)^2 + (2)(-9)(d)$

$d = 2.72$ m.

$v_f = v_i + at$

$0 = 7 + (-9)t$

$t = 0.77$ sec.

y-axis :-

$v_f = v_i + at$

$v_f = 0 + (3)(0.77)$

$= 2.31$ m/s

$v_f^2 = v_i^2 + 2ad$

$(2.31)^2 = 0 + 2(3)d$

$d = 0.88$ m.

$\therefore$ a) $\vec{v}_f = (2.31\hat{j})$ m/s

b) $\vec{r} = (2.72\hat{i} + 0.88\hat{j})$ m.

20 In Fig. 4-23, particle A moves along the line $y = 30$ m with a constant velocity $\vec{v}$ of magnitude 3.0 m/s and parallel to the x axis. At the instant particle A passes the y axis, particle B leaves the origin with a zero initial speed and a constant acceleration $\vec{a}$ of magnitude 0.40 m/s^2. What angle θ between $\vec{a}$ and the positive direction of the y axis would result in a collision?

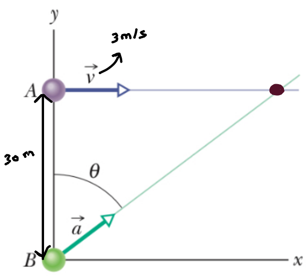


Figure 4-23 Problem 20.

Particle A :-

$y = 30$ m

$v = 3$ m/s $\longrightarrow$ const.

$a = 0 \rightarrow$ no acc.

$x_i = 0$

$x_f = ?$

- $\Delta x = v_i t + \frac{1}{2}at^2$

$x_f - x_i = v_i t + \frac{1}{2}at^2$

$x_f = v_i t \longrightarrow x_f = 3t$

Particle B :-

$v_i = 0$

$a = 0.4 \longrightarrow$ const

$x_i = 0$

$y_i = 0.$

- $\Delta y = v_i t + \frac{1}{2}at^2$

$y_f - y_i = v_i t + \frac{1}{2}(a\cos\theta)t^2$

$y_f = \frac{1}{2}(0.4\cos\theta)t^2$

$= 0.2\cos\theta\, t^2$

- $\Delta x = v_i t + \frac{1}{2}at^2$

$x_f = \frac{1}{2}(a\sin\theta)t^2$

$= 0.2\sin\theta\, t^2$

- Collision :-

$X_{Af} = X_{Bf}$

$3t = 0.2 \sin\theta\, t^2$

$t = \dfrac{15}{\sin\theta}$

$y_{Af} = y_{Bf}$

$30 = 0.2 \cos\theta\, t^2$

$30 = 0.2 \cos\theta \left(\dfrac{15}{\sin\theta}\right)^2$

$\dfrac{2}{3} = \dfrac{\cos\theta}{\sin^2\theta} \longrightarrow \dfrac{2}{3} = \dfrac{\cos\theta}{1 - \cos^2\theta} \longrightarrow \therefore 2\cos^2\theta + 3\cos\theta - 2 = 0$

$\cos\theta = \frac{1}{2} \rightarrow \theta = 60^\circ$

$= 80.6$ m/s

27 A certain airplane has a speed of 290.0 km/h and is diving at an angle of $\theta = 30.0°$ below the horizontal when the pilot releases a radar decoy (Fig. 4-24). The horizontal distance between the release point and the point where the decoy strikes the ground is $d = 700$ m. (a) How long is the decoy in the air? (b) How high was the release point?

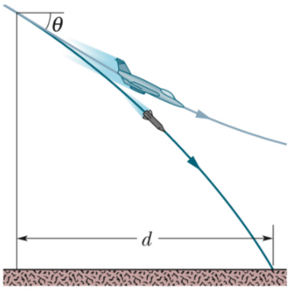


Figure 4-24 Problem 27.

a) $d = v_i t + \frac{1}{2} a t^2$

$700 = 80.6 \cos 30\, t + 0$

$t = 10$ sec.

b) $d_y = v_i t + \frac{1}{2} a t^2$

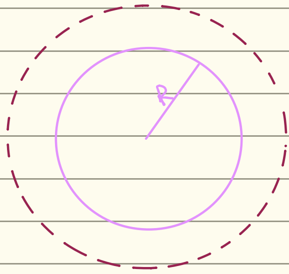
$-d_y = 80.6 \sin 30\, (10) + \frac{1}{2} (-9.8)(10)^2$

$-d = -903$ m

$d = 903$ m.

1.6h.

56 An Earth satellite moves in a circular orbit 750 km above Earth's surface with a period of 98.0 min. What are the (a) speed and (b) magnitude of the centripetal acceleration of the satellite?



نصف قطر القمر الصناعي = نق الكرة الأرضية + بعده عن الكرة الأرضية.

من جوجل = 6.38×10^3 km

a) $R_u = R + h$

$= 6.38 \times 10^3 + 750$

$= 7.02 \times 10^3$ km.

$\therefore V = \dfrac{2\pi R_u}{T}$

$= \dfrac{2(3.14)(7.02 \times 10^3)}{1.6}$

$= 27.6 \times 10^3$ km/h.

b) $a = \dfrac{V^2}{R}$

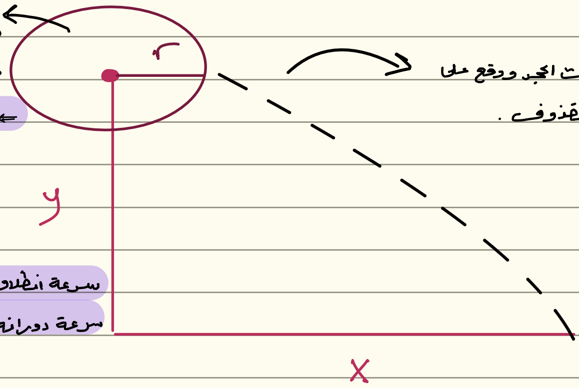
$= \dfrac{(27.6 \times 10^3)^2}{6.38 \times 10^3}$

$= 3.9$ km/h^2.

67 A boy whirls a stone in a horizontal circle of radius 1.5 m and at height 2.0 m above level ground. The string breaks, and the stone flies off horizontally and strikes the ground after traveling a horizontal distance of 10 m. What is the magnitude of the centripetal acceleration of the stone during the circular motion?

ولد ماسك حجر ويلف فيه بشكل دائري.

← الدائرة بشكل افقي



فجأة فلت الحجر ووقع على شكل مقذوف.

سرعة انطلاق الجسم نفس سرعة دورانه.

$X = v_i t + \frac{1}{2} a t^2$

$10 = v_i t + 0 \rightarrow 10 = v_i t$

$-y = v_i t + \frac{1}{2} a t^2$

$-2 = 0 + \frac{1}{2}(-10) t^2$

$\therefore t = 0.63$ sec

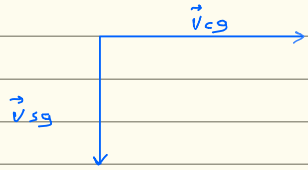
* $10 = v_i (0.63) \longrightarrow V_i = 15.9$ m/s.

- $a = \dfrac{v^2}{R}$

$= \dfrac{(15.9)^2}{1.5} \Rightarrow a = 168$ m/s^2

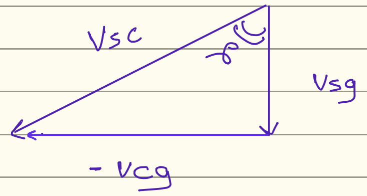
B

77 Snow is falling vertically at a constant speed of 8.0 m/s. At what angle from the vertical do the snowflakes appear to be falling as viewed by the driver of a car traveling on a straight, level road with a speed of 50 km/h?



$V_{sg} = 8\ m/s \longrightarrow$ (vertically downward) $\downarrow$

$V_{cg} = 13.9\ m/s \longrightarrow$ (to the right) $\longrightarrow$



* هو طالب الزاوية الي بنزل فيها الثلج بالنسبة للسيارة

* $V_{sg} = V_{sc} + V_{cg}$

$V_{sc} = V_{cg} - V_{sg}$

لأ الثلج بنزل بعكس اتجاه حركة السيارة .

$\tan\theta = \dfrac{13.9}{8} \longrightarrow \theta = 60^{\circ}$