Fundamentals Physics

Tenth Edition

Halliday

Chapter 10_1

Rotation

10-1 Rotational Variables (5 of 15)

- We now look at motion of **rotation**
- We will find the same laws apply
- But we will need new quantities to express them
  - Torque
  - Rotational inertia
- A **rigid body** rotates as a unit, locked together
- We look at rotation about a **fixed axis**
- These requirements exclude from consideration:
  - The Sun, where layers of gas rotate separately
  - A rolling bowling ball, where rotation and translation occur

10-1 Rotational Variables (6 of 15)

- The fixed axis is called the **axis of rotation**
- Figs 10-2, 10-3 show a reference line
- The **angular position** of this line (and of the object) is taken relative to a fixed direction, the **zero angular position**

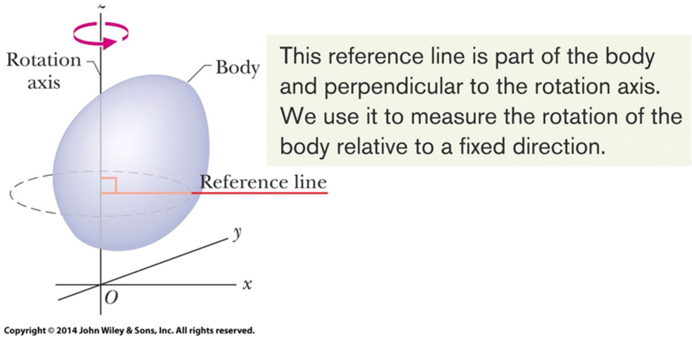


Figure 10-2

10-1 Rotational Variables (7 of 15)

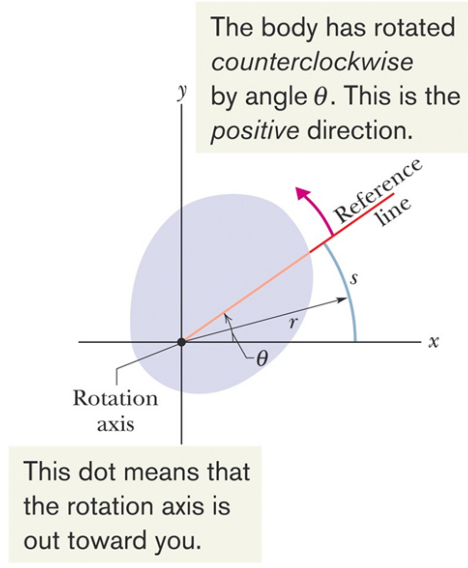


Figure 10-3

10-1 Rotational Variables (8 of 15)

- Measure using **radians** (rad): dimensionless

$$\theta = \frac{s}{r} \quad \text{(radian measure)}. \qquad \textbf{Equation (10-1)}$$

$$1 \text{ rev} = 360^\circ = \frac{2\pi r}{r} = 2\pi \text{ rad}, \qquad \textbf{Equation (10-2)}$$

- Do not reset θ to zero after a full rotation

10-1 Rotational Variables (9 of 15)

- We know all there is to know about the kinematics of rotation if we have $\theta(t)$ for an object
- Define angular displacement as:

$$\Delta\theta = \theta_2 - \theta_1. \qquad \textbf{Equation (10-4)}$$

10-1 Rotational Variables (10 of 15)

- "Clocks are negative":

  An angular displacement in the counterclockwise direction is positive, and one in the clockwise direction is negative.

10-1 Rotational Variables (11 of 15)

Checkpoint 1

A disk can rotate about its central axis like a merry-go-round. Which of the following pairs of values for its initial and final angular positions, respectively, give a **negative angular displacement:** (a) −3 rad, +5 rad, (b) −3 rad, −7 rad, (c) 7 rad, −3 rad?

Answer:

Choices (b) and (c)

10-1 Rotational Variables (12 of 15)

- **Average angular velocity**: angular displacement during a time interval

$$\omega_{\text{avg}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}, \qquad \textbf{Equation (10-5)}$$

- **Instantaneous angular velocity**: limit as $\Delta t \rightarrow 0$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}. \qquad \textbf{Equation (10-6)}$$

- If the body is rigid, these equations hold for all points on the body
- Magnitude of angular velocity = **angular speed**

10-1 Rotational Variables (13 of 15)

- Figure 10-4 shows the values for a calculation of average angular velocity

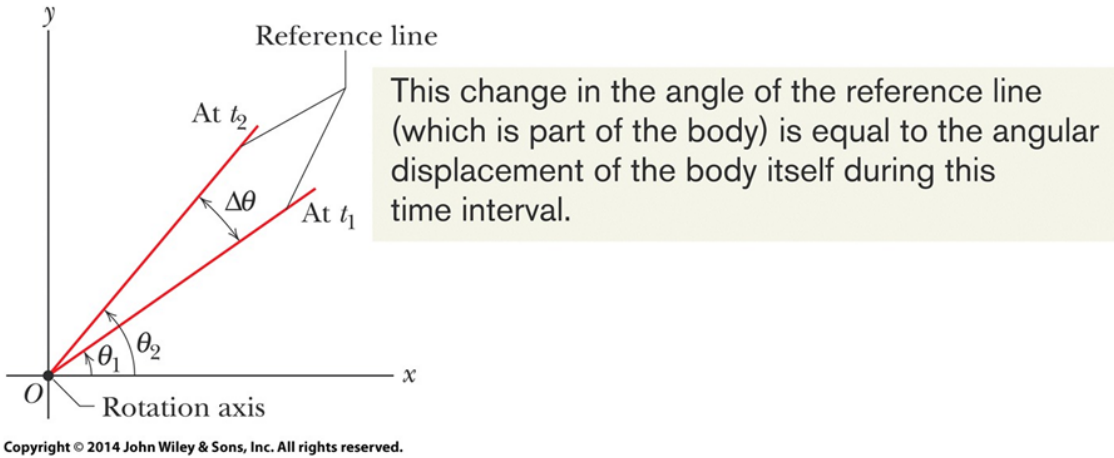


Figure 10-4

10-1 Rotational Variables (14 of 15)

- **Average angular acceleration:** angular velocity change during a time interval

$$\alpha_{\text{avg}} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t},$$

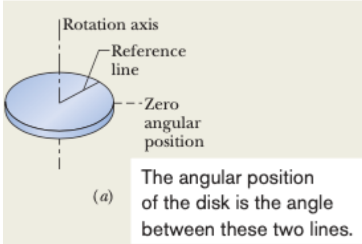
Equation (10-7)

- **Instantaneous angular velocity:** limit as $\Delta t \to 0$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}.$$

Equation (10-8)

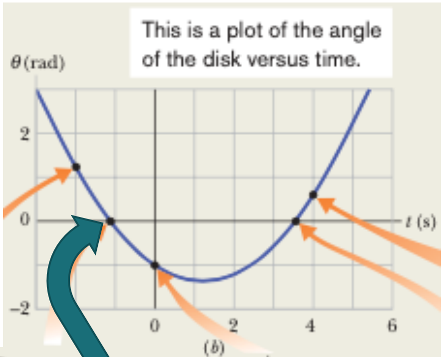
Sample Problem 10.01 Angular velocity derived from angular position



$$\theta = -1.00 - 0.600t + 0.250t^2, \qquad t \text{ in seconds, } \theta \text{ in radians}$$

(a) Graph the angular position of the disk versus time from $t = -3.0$ s to $t = 5.4$ s. Sketch the disk and its angular position reference line at $t = -2.0$ s, 0 s, and 4.0 s, and when the curve crosses the t axis.

so we graph the equation:



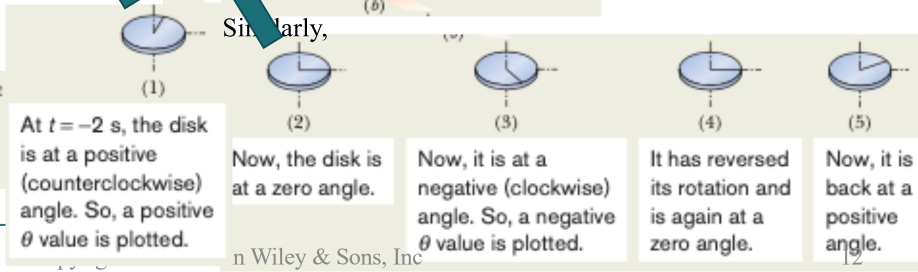
Sketch: For $t = -2.0$ s

$$\theta = -1.00 - (0.600)(-2.0) + (0.250)(-2.0)^2$$

$$= 1.2 \text{ rad} = 1.2 \text{ rad}\,\frac{360^\circ}{2\pi \text{ rad}} = 69^\circ.$$

rotated counter clockwise by 1.2rad=69˚

Similarly,



Sample Problem 10.01 Angular velocity derived from angular position

(b) At what time t_{min} does $\theta(t)$ reach the minimum value shown in Fig. 10-5*b*? What is that minimum value?

To find the extreme value (here the minimum) of a function, we take the first derivative of the function and set the result to zero.

$\frac{d\theta}{dt} = -0.600 + 0.500t.$ $=0$, $t_{min} = 1.20$ s, substitute into: $\theta = -1.00 - 0.600t + 0.250t^2$

$\theta = -1.36$ rad $= -77.9^\circ$. (*maximum clockwise* rotation of the disk)

(c) Graph the angular velocity v of the disk versus time from $t = -3.0$ s to $t = 6.0$ s. Sketch the disk and indicate the direction of turning and the sign of v at $t = -2.0$ s, 4.0 s, and t_{min}.

(d) Use the results in parts (a) through (c) to describe the motion of the disk from $t = -3.0$ s to $t = 6.0$ s.

Sample Problem 10.02 Angular velocity derived from angular acceleration

$\boldsymbol{\alpha}=5t^3-4t$, t in seconds and a in radians per second-squared, At $t = 0$, the top has angular velocity 5 rad/s, and a reference line on it is at angular position $\theta = 2$ rad.

(a) angular velocity $\omega(t)$?

$d\omega = \boldsymbol{\alpha}\, dt$,

$$\omega = \int (5t^3 - 4t)\, dt = \tfrac{5}{4}t^4 - \tfrac{4}{2}t^2 + C.$$

$\omega = 5$ rad/s at $t = 0$, 5 rad/s = 0 – 0 + C , so $C = 5$ rad/s. Then

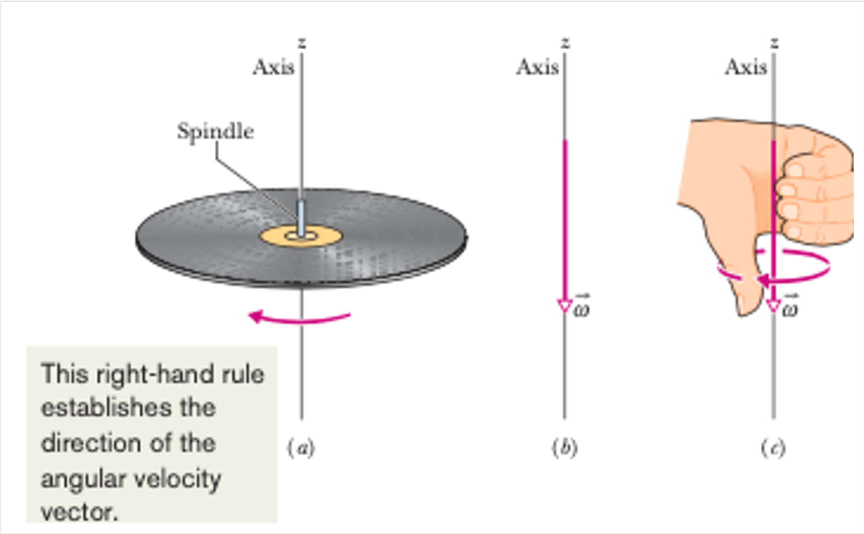
$$\omega = \tfrac{5}{4}t^4 - 2t^2 + 5.$$

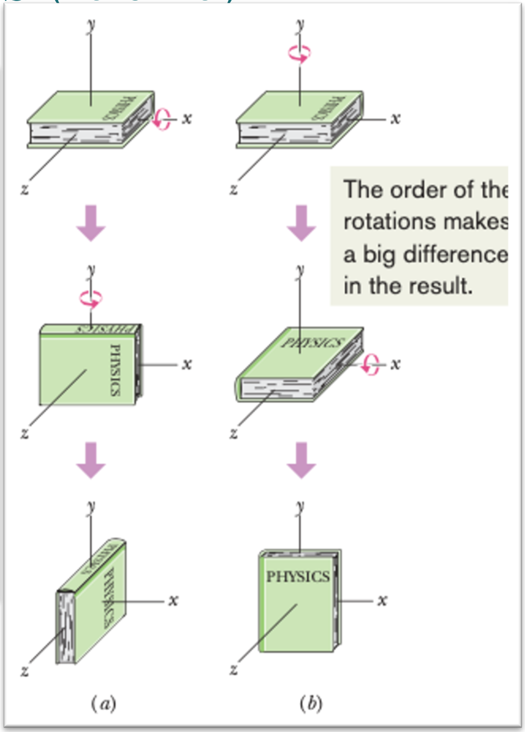
(b) angular position $\theta(t)$?

10-1 Rotational Variables (15 of 15)

- If the body is **rigid**, these equations **hold for all points** on the body
- With **right-hand rule** to determine direction, angular velocity & acceleration can be written as **vectors**
- If the body rotates around the vector, then **the vector points along the axis of rotation**
- Angular **displacements are not vectors**, because the order of rotation matters for rotations around different axes

10-1 Rotational Variables (15 of 15)





10-1 Rotation with Constant Angular Acceleration (2 of 4)

- **The same equations** hold as for constant linear acceleration, see Table 10-1
- We simply change **linear** quantities **to angular** ones
- Equations. **10-12 and 10-13 are the basic equations**: all others can be derived from them

10-2 Rotation with Constant Angular Acceleration (3 of 4)

Table 10-1 Equations of Motion for Constant Linear Acceleration and for Constant Angular Acceleration

Equation Number	Linear Equation	Missing Variable	Missing Variable	Angular Equation	Equation Number
(2-11)	$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$	(10-12)
(2-15)	$x - x_0 = v_0 t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$	(10-13)
(2-16)	$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$	(10-14)
(2-17)	$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$	(10-15)
(2-18)	$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$	(10-16)

10-2 Rotation with Constant Angular Acceleration (4 of 4)

Checkpoint 2

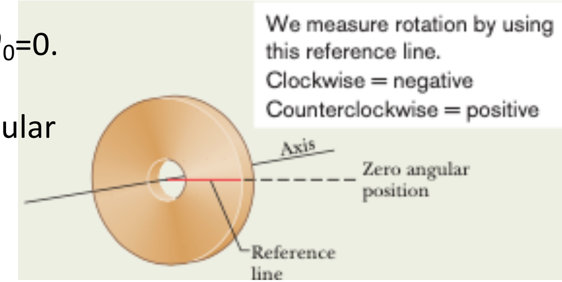
In four situations, a rotating body has angular position $\theta(t)$ given by (a) $\theta = 3t - 4$, (b) $\theta = -5t^3 + 4t^2 + 6$, (c) $\theta = \frac{2}{t^2} - \frac{4}{t}$, and (d) $\theta = 5t^2 - 3$. To which situations do the angular equations of Table 10-1 apply?

Answer:

Situations (a) and (d); the others do not have constant angular acceleration

Sample Problem 10.03 Constant angular acceleration, grindstone

Grindstone, $\boldsymbol{\alpha}$ = 0.35 rad/s, At time t = 0, ω = –4.6 rad/s, θ_0=0.



(a) At what time after t = 0 is the reference line at the angular position θ = 5.0 rev?

$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2,$$

$$\theta = 5.0 \text{ rev} = 10\pi \text{ rad}$$

$$10\pi \text{ rad} = (-4.6 \text{ rad/s})t + \tfrac{1}{2}(0.35 \text{ rad/s}^2)t^2.$$

Solve for t, t = 32 s.

(b) Describe the grindstone's rotation between t = 0 and t=32s.

(c) At what time t does the grindstone momentarily stop?

$$\omega = \omega_0 + \alpha t$$

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - (-4.6 \text{ rad/s})}{0.35 \text{ rad/s}^2} = 13 \text{ s}.$$

10-3 Relating the Linear and Angular Variables (2 of 5)

- Linear and angular variables are **related by *r***, perpendicular distance from the rotational axis
- Position (note θ must be in **radians**):

$$s = \theta r$$ **Equation (10-17)**

- Speed (note ω must be in **radian** measure):

$$v = \omega r$$ **Equation (10-18)**

- We can express the period in radian measure:

$$T = \frac{2\pi}{\omega}$$ **Equation (10-20)**

10-3 Relating the Linear and Angular Variables (3 of 5)

- Tangential acceleration (radians):

$$a_t = \alpha r \qquad \textbf{Equation (10-22)}$$

- We can write the radial acceleration in terms of angular velocity (radians):

$$a_r = \frac{v^2}{r} = \omega^2 r \qquad \textbf{Equation (10-23)}$$

10-3 Relating the Linear and Angular Variables (4 of 5)

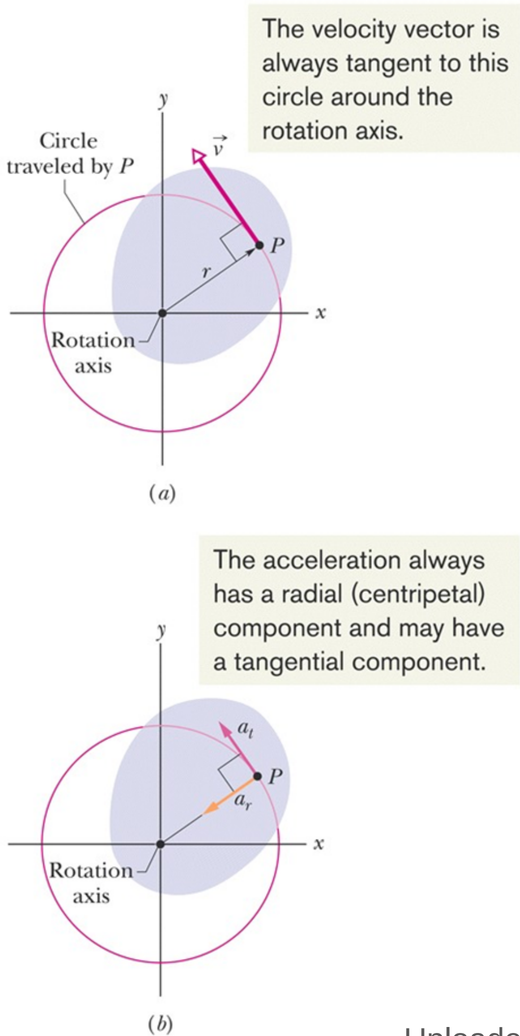


Figure 10-9

10-3 Relating the Linear and Angular Variables (5 of 5)

Checkpoint 3

A cockroach rides the rim of a rotating merry-go-round. If the angular speed of this system (merry-go-round + cockroach) is constant, does the cockroach have (a) radial acceleration and (b) tangential acceleration? If ω is decreasing, does the cockroach have (c) radial acceleration and (d) tangential acceleration?

Answer:

(a) yes

(b) no

(c) yes

(d) yes