

Experiment 4

①

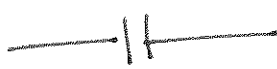
- Impedance of basic elements



$$Z_R = R$$



$$Z_L = j\omega L$$

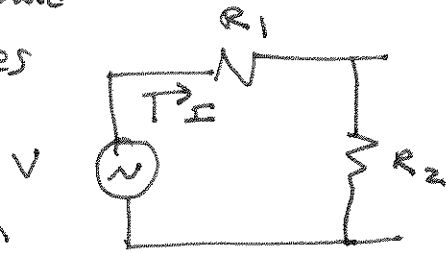


$$Z_C = 1/j\omega C$$

- Resistive circuit: current and voltages are all phasor quantities

$$I = \frac{V}{R}$$

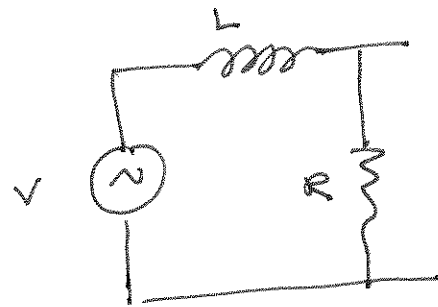
$\Rightarrow$ phase shift = 0 between V and I in a resistive circuit.



- Inductive circuit

$$I = \frac{V}{Z} = \frac{V}{R + j\omega L}$$

$$= \frac{V}{\sqrt{R^2 + (\omega L)^2}} \angle \tan^{-1} \frac{\omega L}{R}$$



$$I = \frac{V}{\sqrt{R^2 + (\omega L)^2}} \angle -\tan^{-1} \frac{\omega L}{R}$$

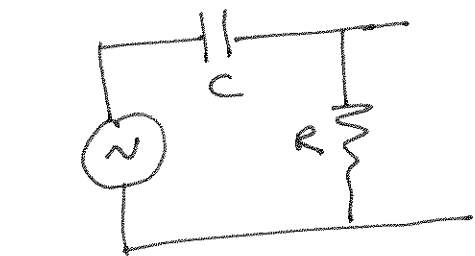
Here, I lags the voltage by $\tan^{-1} \frac{\omega L}{R}$

- Capacitive circuit

$$I = \frac{V}{Z} = \frac{V}{R + 1/j\omega C}$$

$$= \frac{V}{R - j \frac{1}{\omega C}}$$

$$= \frac{V}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} \angle \tan^{-1} \frac{1}{\omega RC}$$



$$\angle \tan^{-1} \frac{1}{\omega RC}$$

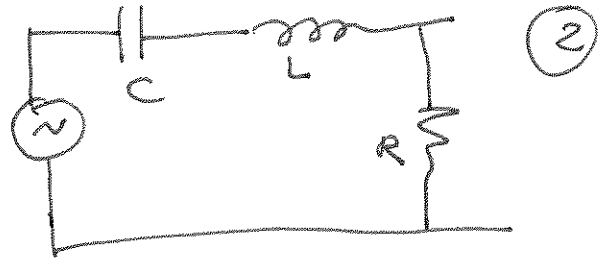
Here, I leads V by $\tan^{-1} \frac{1}{\omega RC}$

RLC circuit

$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\boxed{V = IZ} \text{ or } I = \frac{V}{Z}$$



Resonance frequency: frequency at which circuit becomes resistive, i.e.,

$$\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0^2 LC = 1 \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

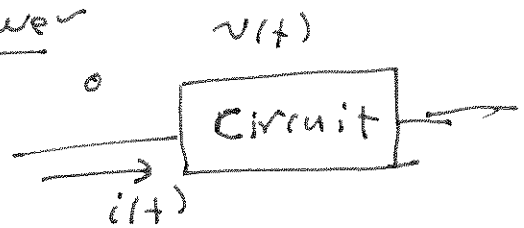
For $f < f_0$, $|Z_C| > |Z_L| \Rightarrow$ capacitive circuit.
For $f > f_0$, $|Z_L| > |Z_C| \Rightarrow$ inductive circuit

Sinusoidal steady-state power

consider a circuit for which the voltage and current are given by

$$v(t) = V_0 \cos \omega_0 t$$

$$i(t) = I_0 \cos(\omega_0 t + \phi)$$



The instantaneous power is

$$p(t) = V_0 I_0 \cos(\omega_0 t + \phi) \cos \omega_0 t$$

$$p(t) = \frac{V_0 I_0}{2} [\cos(2\omega_0 t + \phi) + \cos \phi]$$

The average power is

$$P_{av} = \frac{1}{T_0} \int_0^{T_0} p(t) dt = \frac{V_0 I_0}{2} \cos \phi$$

$$P_{av} = \frac{V_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}} \cos \phi = V_{rms} I_{rms} \cos \phi$$

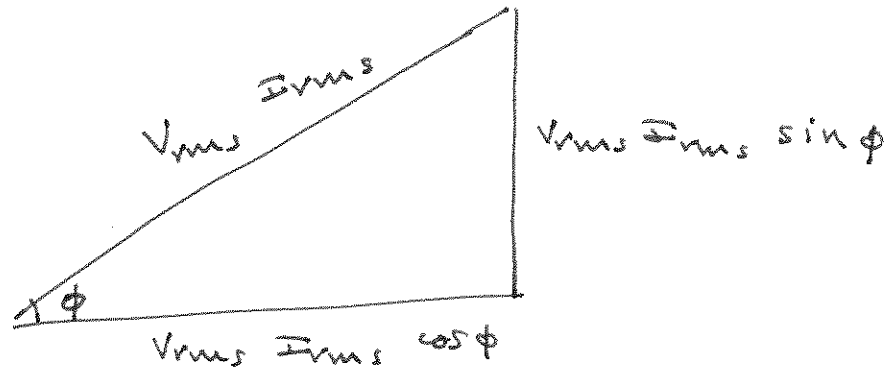
The power Triangle

3

$$\text{power factor} = \cos \phi$$

$$\text{active power} = V_{rms} I_{rms} \cos \phi$$

$$\text{reactive power} = V_{rms} I_{rms} \sin \phi$$



RMS Value

consider the voltage waveform

$$v(t) = V_0 (\cos \omega t + \phi)$$

The rms value of $v(t)$ is

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v(t)^2 dt} = \frac{V_0}{\sqrt{2}}$$

$$V_{rms} = \frac{\text{peak value}}{\sqrt{2}}$$

The phasor concept

voltages and currents are represented as complex quantities (no time dependence)

$$\text{examples: } i(t) = i_0 \cos(\omega t + 30^\circ) \Rightarrow I = \frac{i_0}{\sqrt{2}} \angle \frac{\pi}{6}$$

$$v(t) = 10 \cos(\omega t) \Rightarrow V = \frac{10}{\sqrt{2}} \angle 0$$