Chapter 5:-

Dimensional Analysis and Similiraty

- Dimensional analysis is a method for reducing the number and complexity of expiremental variables

- Variables and constants:-

→ Dimensional variables :- quantities that has dimensions and theire numerical values may chang; ex, speeed, velocity, acc- ---

→ Dimensional constants:- equantities that has dimensions but its numerical values are Constant
ex: g, Plank's constant ---

→ Pure constants, Angles and revalutions are dimensionless

The Pi Theorem :-
The dimensionless groups founed from the theorem are power products denoted by π_1, π_2, π_3 ---

Steps of finding pi's :-

1- Find number of variables n

Ex: $P = f(F, A, \mu)$

① ② ③ ④

$\boxed{n=4}$ (P is included : Do NOT forget the first variable)

2- Find number of repeating variables j (P,F مثلاً Repeating)

هاد بكون معطى بالسؤال

3- Find number of Pi's : $k = n - j = 4 - 2 = 2$ (A, μ)

5- Determine which system to use:

MLTΘ   FLTΘ

M → mass, L → length, T → Time, Θ → Temperture, F → Force

هاي الخطوة قبل عشان تعرف تختار ال system

4- list the dimensions of each variable

6- Take the first Group :-

$\Pi_1 = (A)^1 P^a F^b = F^0 L^0 T^0$ or $M^0 L^0 T^0$

$\Pi_2 = (\mu)^1 P^a F^b = F^0 L^0 T^0$ or $M^0 L^0 T^0$

(Π_1, Π_2 → Dimensionless)

find a, b in each case

See Example 5.3

5.5 Modeling and Prototype

Model ⟶ Prototype

1. Geometrically similarity
2. Kinematic similarity
3. Dynamic similarity

1- The scale should be the same for all dimensions. Angles stay the same

2- Froude number must be equal

$$Fr_m = Fr_p$$

$$\frac{V_m^2}{g L_m} = \frac{V_p^2}{g L_p}$$

ممكن تعطاه طول ال prototype وال scale وانت بتوجد طول ال model

$$L_m = \alpha L_p$$

(α → scale)

$$\sqrt{\alpha} = \frac{V_m}{V_p} = \left(\frac{L_m}{L_p}\right)^{1/2} = \frac{T_m}{T_p}$$

3- Dynamic Similarity :- Re and Mash equality

- Compressible flow: Reynolds number and Mach number and specefic heat ratio
- Incompressible flow:
  - No free surface: Reynolds are equal
  - with a free surface:
    - Ray. equal
    - Froude num
    - Weber num
    - Cavitation num

- Explanation of Ma, Re, Fr

① Mash number

$$Ma = \frac{V}{a}$$ (V → Velocity)

$$a = \sqrt{KRT}$$ (T → Temp; R → Gas. Constant; K → $\frac{c_p}{c_v}$)

② Raynold's number:-

$$Re = \frac{\rho V L}{\mu}$$

ρ → Density; V → Velocity; L → length / Diameter of inlet; μ → Viscosity

③ Froude number $= \frac{V}{\sqrt{2gL}}$

Combining Pi's Theorem and modeling

If the Question is asking for a specific value of a property for a model or a prototype (with similarity : Dynamically and Geometrically)

- you can use Pi Theorem to find a law for the property and Then use relation

$$(\Pi)_{model} = (\Pi)_{prototype} = constant$$