

Chapter 5:-

Dimensional Analysis and Similarity

- **Dimensional analysis** is a **method** for reducing the number and complexity of experimental variables

- **Variables and constants:-**

→ **Dimensional variables** :- quantities that have dimensions and their numerical values may change. ex: speed, velocity, acc. ---

→ **Dimensional constants** :- quantities that have dimensions but their numerical values are constant. ex: g , Planck's constant ---

→ Pure constants, angles and revolutions are dimensionless

The Pi Theorem :-

The dimensionless groups found from the theorem are power products denoted by $\pi_1, \pi_2, \pi_3, \dots$

Steps of Finding pi's :-

1- Find number of variables n

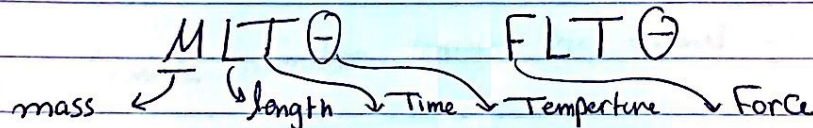
Ex: $P = f(E, A, \mu)$
 ① ② ③ ④

$n=4$ (P is included : Do NOT forget the first variable)

2- Find number of repeating variables j (P, F ^{is} Repeating)

3- Find number of pi's : $k = n - j = 4 - 2 = 2$
 A \leftarrow μ

5- Determine which System to use:



4- list the dimensions of each variable

6- Take the first Group:-

Dimensionless \rightarrow

$$\Pi_1 = (A)^1 P^a F^b = \text{FLT}^0 \text{ or } \text{MLT}^0$$

$$\Pi_2 = (M)^1 P^a F^b = \text{FLT}^0 \text{ or } \text{MLT}^0$$

find a, b in each case

See Example 5.3

5.5. Modeling and Prototype

Model $\xrightarrow{\hspace{2cm}}$ Prototype

1. Geometrically similarity

2. Kinematic similarity

3. Dynamic similarity

1- The scale should be the same for all dimensions
Angles stay the same

2- Froude number must be equal

$$Fr_m = Fr_p$$

$$\frac{V_m^2}{gL_m} = \frac{V_p^2}{gL_p}$$

prototype \rightarrow $\frac{V_p^2}{gL_p}$ \rightarrow scale \rightarrow model \rightarrow $\frac{V_m^2}{gL_m}$

$$L_m = \alpha L_p \quad \text{scale}$$

$$\sqrt{\alpha} = \frac{V_m}{V_p} = \left(\frac{L_m}{L_p} \right)^{\frac{1}{2}} = \frac{T_m}{T_p}$$

3- Dynamic Similarity :- Re and Mach equality

Compressible flow

Reynolds number and Mach number and specific heat ratio

Incompressible flow

No free surface

• Reynolds are equal

with a free surface

• Ray. equal
• Froude num
• Weber num
• Cavitation num

• Explanation of Ma , Re , Fr

① Mach number

$$Ma = \frac{V}{a}$$

Velocity

$$a = \sqrt{\frac{\gamma R T}{\frac{e p}{C_v}}}$$

Temp
Gas. constant

② Reynold's number:-

$$Re = \frac{\rho V L}{\mu}$$

Density

length / Diameter of inlet

Velocity

Viscosity

③ Froude number = $\frac{V}{\sqrt{2gL}}$

Combining Pi's Theorem and modeling

If the Question is asking for a specific value of a property for a model or a prototype (with similarity: Dynamically and Geometrically)

- you can use Pi Theorem to find a law for the property and then use relation

$$(\Pi)_{\text{model}} = (\Pi)_{\text{prototype}} = \text{constant}$$