

19)

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{1}{2} \left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots \right]$$

$$= 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots$$

(10.8)

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

30)

$$f(x) = 2^x, \quad a=1$$

$$f'(x) = 2^x \ln 2$$

$$f''(x) = 2^x (\ln 2)^2$$

$$f'''(x) = 2^x (\ln 2)^3$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(1)(x-1)^n}{n!} = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \dots$$

$$= 2 + 2 \ln 2 (x-1) + \frac{2(\ln 2)^2}{2!}(x-1)^2 + \frac{2(\ln 2)^3}{3!}(x-1)^3 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{2(\ln 2)^n (x-1)^n}{n!}$$

3) $f(x) = \ln x, \quad a=1$

$$f'(x) = \frac{1}{x}$$

$$f''(x) = -\frac{1}{x^2}$$

$$f'''(x) = \frac{2}{x^3}$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(1)(x-1)^n}{n!} = 0 + (x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!} - \dots$$

$$P_0(x) = 0$$

$$P_1(x) = (x-1)$$

$$P_2(x) = (x-1) - \frac{1}{2}(x-1)^2$$

$$P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3!}(x-1)^3$$

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14) $f(x) = \frac{2+x}{1-x}$

$$f'(x) = \frac{(1-x) - (2+x) \cdot (-1)}{(1-x)^2} = \frac{1-x+2+x}{(1-x)^2} = \frac{3}{(1-x)^2}$$

$$f''(x) = \frac{6}{(1-x)^3}$$

$$f'''(x) = \frac{18}{(1-x)^4}$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$= 2 + 3x + \frac{6}{2!}x^2 + \frac{18}{3!}x^3 + \dots$$

$$= 2 + \sum_{n=1}^{\infty} 3x^n$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

20)

$$= \frac{1}{2} \left[1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots - 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots \right]$$

$$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

22) $f(x) = \frac{x^2}{x+1}$

$$f'(x) = \frac{2x(x+1) - x^2}{(x+1)^2} = \frac{2x^2 + 2x - x^2}{(x+1)^2} = \frac{x^2 + 2x}{(x+1)^2}$$

$$f''(x) = \frac{(2x+2)(x^2+2x) - (2x+2)(x^2+2x)}{(x+1)^4} = \frac{2}{(x+1)^3}$$

$$f'''(x) = \frac{-2 \cdot 3(x+1)^2}{(x+1)^6} = \frac{-6}{(x+1)^4}$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$= 0 + 0 + \frac{2}{2!}x^2 - \frac{6}{3!}x^3 + \dots$$

$$= \sum_{n=2}^{\infty} (-1)^n x^n$$

27) $f(x) = \frac{1}{x^2}, a=1$

$$f'(x) = -\frac{2}{x^3}$$

$$f''(x) = \frac{6}{x^4}$$

$$f'''(x) = -\frac{24}{x^5}$$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!} (x-1)^n = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \dots$$

$$= 1 - 2(x-1) + \frac{6}{2!}(x-1)^2 - \frac{24}{3!}(x-1)^3 + \dots$$

$$= \sum_{n=0}^{\infty} (-1)^n (x-1)^n (n+1)$$

32] $f(x) = \sqrt{x+1}$, $a=0$

$$f'(x) = \frac{1}{2\sqrt{x+1}}$$

$$f''(x) = \frac{-1}{4(x+1)^{3/2}}$$

$$f'''(x) = \frac{3}{8(x+1)^{5/2}}$$

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n &= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots \\ &= 1 + \frac{1}{2}x - \frac{1}{42!}x^2 + \frac{3}{83!}x^3 + \dots \\ &= \sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{2}\right)^n x^n \quad x \end{aligned}$$

37] $f(x) = e^x$ $x=a$

$$f'(x) = e^x$$

$$f''(x) = e^x$$

$$f'''(x) = e^x$$

$$\vdots$$

$$f^{(n)}(x) = e^x$$

$$\begin{aligned} \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \dots \\ &= e^a + e^a(x-a) + \frac{e^a}{2!}(x-a)^2 + \frac{e^a}{3!}(x-a)^3 + \frac{e^a}{4!}(x-a)^4 + \dots \\ &= e^a \left[1 + (x-a) + \frac{(x-a)^2}{2!} + \dots \right] \end{aligned}$$