Circuit

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والله ولي التوفيق

Ch 1 + Ch 2 + Ch 3 :-

ch1 :

① **Voltage** : it is the energy per unit charge created by the seperation

$$v = \frac{dw}{dq}$$, w: energy in joule
q: charge in coulomb
V: voltage in volts

② **Current** : it's the rate of charge flow.

$$I = \frac{dq}{dt} \text{ or } \frac{Nq_e}{t}$$, i: current in Ampere
t: time in seconds

③ **power** : it's the rate of change in energy

$$p = \frac{dw}{dt}, \quad v = \frac{dw}{dq} \Rightarrow dw = v\,dq$$

$$p = \frac{v\,dq}{dt} = vi$$, p: power in watts

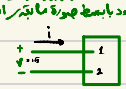
Note:

Some important prefixes:

pico	10^{-12}
Nano	10^{-9}
micro	10^{-6}
milli	10^{-3}
kilo	10^{3}
Mega	10^{6}
giga	10^{9}
tera	10^{12}

Some important terminology :-

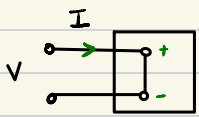
Basic element
- has two terminals
- we can find I-V relation for this element



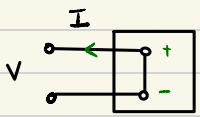
terminal 1 is greater than terminal 2 with V = (15V)
or terminal 2 is greater than terminal 1 with V = -15V
$V_1 > V_2$ by 15 V.

Passive Sign Convention:

* $p = IV$ or $p = -IV$

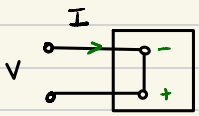


$p = +IV$

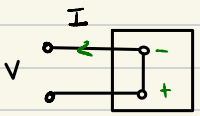


$p = -IV$

اذا التيار داخل على الموجب ← (+)
اذا التيار طالع من الموجب ← (-)



$p = -IV$



$p = +IV$

الجواب النهائي لكل الحالات محكوم أنه يكون
(+) أو (-) حسب قيمة I & V

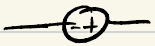
Power Signs :-

* power is positive ⇒ (absorbing) ⇒ $p > 0$ ((consuming, power is being delivered to the element)
  ↳ storing
* power is negative ⇒ (delivering) ⇒ $p < 0$ (suppling)

(Usually, current sources and voltage source are considerred to be suppliers (active elements)

passive elements : (storing) : capacitor, inductor & resistor

↳ Active elements : produce energy

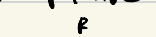


voltage source

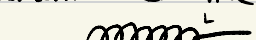
current source

& generators

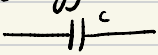
↳ Passive elements : Consume energy



resistor

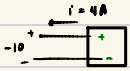


inductor



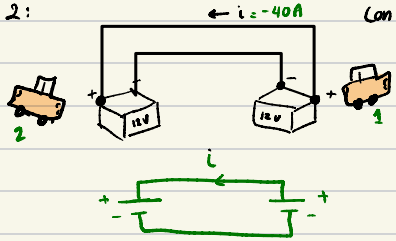
capacitor

Example 1: Suppose we have the following voltage & current:



$p = -IV = -4 \times -10 = 40$ watt $\Rightarrow$ absorbing

Example 2:



Consider the following car battery connection One battery is dead and the other one is charging it. If the current i is measured and found to be -40 A, which car had the dead battery.

$p_1 = -IV$

$= -(-40 \times 12) = 480$ watt $\Rightarrow$ consuming

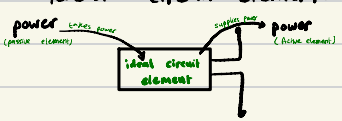
$p_2 = +IV$

$= -40 \times 12 = -480$ watt $\Rightarrow$ absorbing

(the first car has the dead battery)

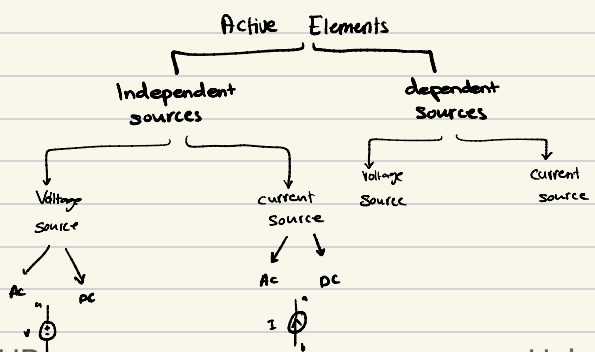
Ch2: Circuit elements

ideal circuit element: are two terminal device



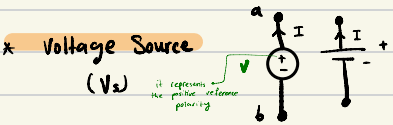
1 Active Elements (Sources):-

Convert Non-electrical energy → Electric Energy



① Independent Source:

* Voltage Source

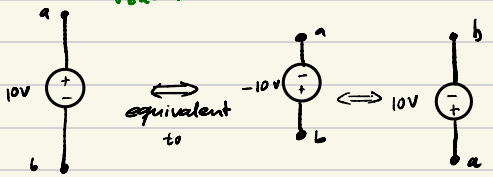


ideal independent voltage source
it maintain a specific voltage across its terminals, independent of the current that flows through it

(V → is constant
I → depends on the circuit's connection)

$V_{ab} = V$

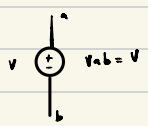
$V_{ba} = -V$



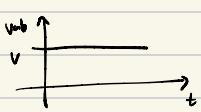
Voltage source

D : Direct
A : alternating

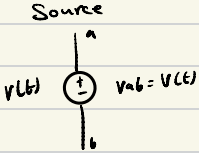
Dc Voltage Source



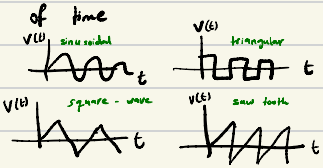
Where V is Const



AC Voltage Source



Where V(t) is a function of time

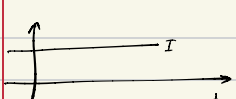


* Current Source:

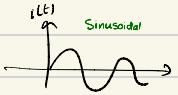
ideal independent current source
it maintains a specific current through it
independent of the voltage developed across its terminals

∴ I → constant
∴ V → depends on the circuit's connections

DC "constant"

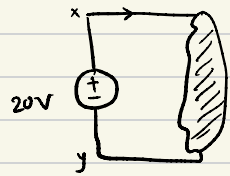


AC "function of time"



and the other graphs --

case 1

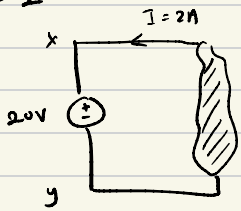


$V_{xy} = 20\ V$

$p_s = -IV = -40$ watt $\Rightarrow$ delivered to the Network

[Machine (generator) or Battery (Discharge mode)

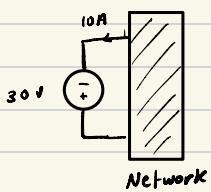
case 2



$p = +IV = 2 \times 20 = 40$ Watt $\Rightarrow$ (absorbed by the source

Machine (motor) or Battery (charging mode) or delivered to the source)

Example:

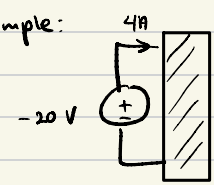


$p = -IV = -10 \times 30 = -300$ watt

then the battering is providing energy

$\Rightarrow$ generator

Example:



$p = -IV$

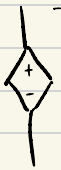
$= -4(-20)$

$= 80$ watt then the battery is absorbing

② Dependent Source: (controlled sources)

They generate a voltage or a current that's determined by a voltage or a current at a specified location in the circuit.

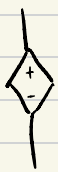
Voltage Source:



$V = \text{Const} \cdot V_x$

dep. Voltage source

Voltage - controlled



$r = \text{Const} \cdot i_x$

dep · Voltage Source

current - controlled

Curren Source:

$i = \text{Const} \cdot V_x$

dep · Current Source

voltage - controlled

$i = \text{const.} \; i_x$

dep · current source

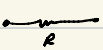
current - controlled

Examples (Transistors)

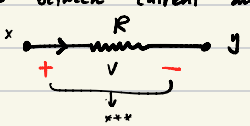
② passive Elements

* Resistors R: always absorbing power (consume it and convert it to heat)

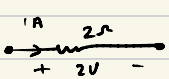
. unit: ohms [Ω]

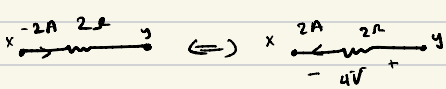
. symbol 

. The relationship between current and voltage across the resistor is linear (ohm's law)



Example:



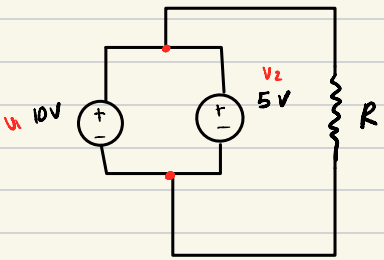


Conductance (G): it is the reciprocal of resistance and measured in [siemens or Ʊ (mho)]

$$G = \frac{1}{R}$$

Example : decide wether the connection is valid or not ?

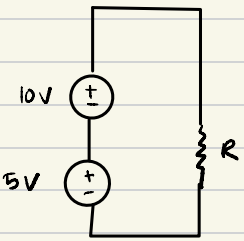
*(في الجهد مهم التي كيفي على حالة التوازي ← تساوي الجهد تماماً)



invalid connection → solution is to connect a resistor with one of the sources

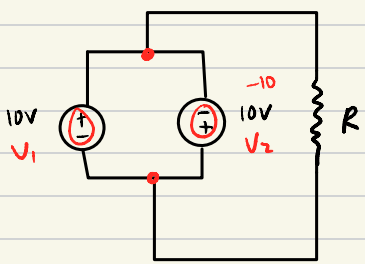
since $V_1 \neq V_2 \Rightarrow$ parallel connection

$\therefore$ must $(V_1 = V_2)$



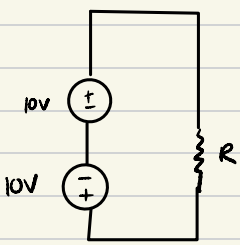
valid connection

(series connection ✓)

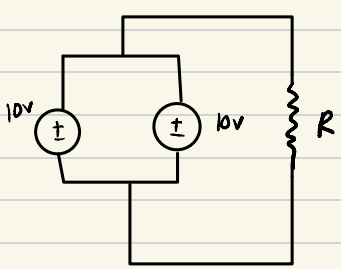


Invalid Connection

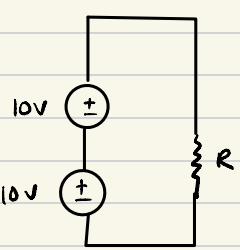
$V_1 \neq V_2 \Rightarrow 10 \neq -10$



Valid Connection



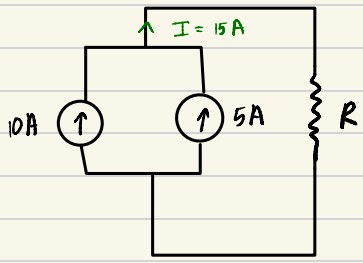
Valid Connection



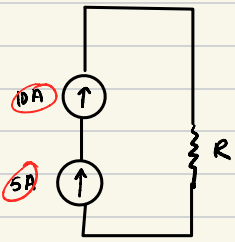
Valid Connection

Example:

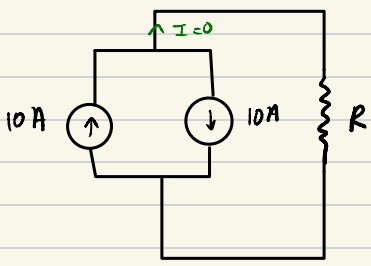
(في حالة التيار التي يكون على حالة التوالي ← عادي + لازم)



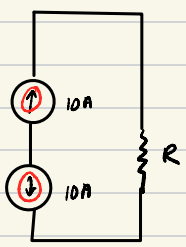
Valid ⇒ parallel connection



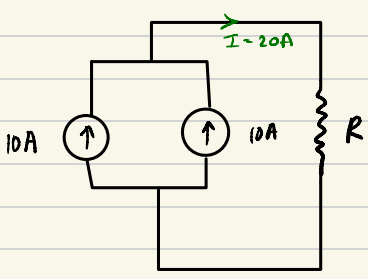
Invalid connection



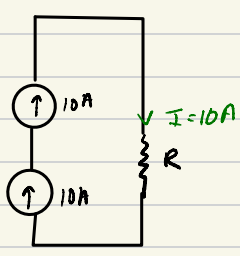
Valid connection



Invalid connection



Valid connection

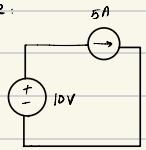


Valid

* node : النقاط او الكتر في نقطة two terminals

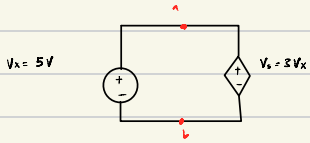
معينة

Example:



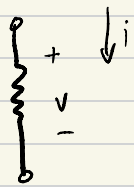
Connection is valid

Example

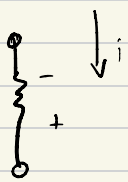


Invalid connection
Since Vo ≠ Vx
∵ 3Vx ≠ Vx

ohm's law

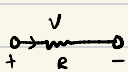


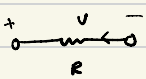
$V = +IR \Rightarrow$ voltage drop



$V = -IR \Rightarrow$ voltage rise

power for resistors is always (+) since it's always absorbing





$V = +IR$

$P = +IV$

$P = I(IR)$

$= I^2R$ (+) value

or $P = \frac{V^2}{R}$ (+) value

$V = -IR$

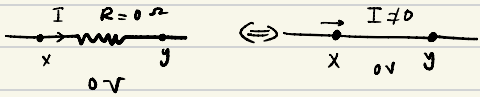
$P = -IV$

$= -I(-IR)$

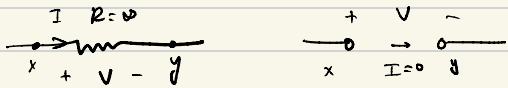
$= I^2R$ (+)

OR $= \frac{V^2}{R}$ (+)

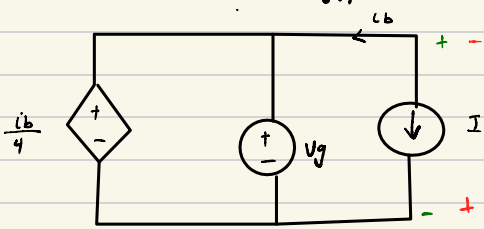
When $R = 0 \Rightarrow V = RI = 0 \Rightarrow$ The element is called Short Circuit



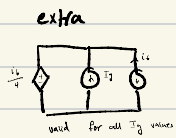
When $R = \infty \Rightarrow I = 0 \Rightarrow$ the element is called open circuit



Example: ① find the value of V_g that makes the connection valid (page 52) ② find P_{8A}



V = -2 V = 2



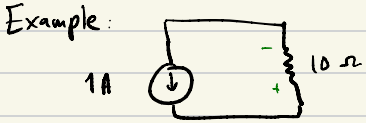
$i_b = -8A \Rightarrow \frac{i_b}{4} = \frac{-8}{4} = -2V$

$V_g = -2V$

$P_{8A} = +IV = (8)(-2) = -16$ watt

$P_{8A} = -IV = 8 \times 2 = -16$ watt

Example:



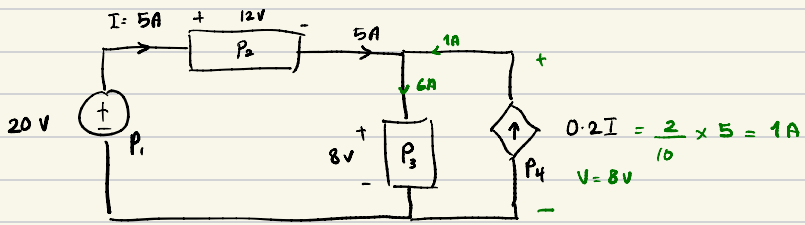
$P_{10\Omega} = ??$

$P_{10\Omega} = I^2 R = 1 \times 10 = 10$ watt

$\Sigma P(t) = 0$

* $\Sigma P_{del} = \Sigma P_{abs}$

Example:



$P_1 = -IV = -5 \times 20 = -100\ W$ (del)

$P_2 = +IV = +12 \times 5 = 60\ W$ (abs)

$P_3 = +IV = +6 \times 8 = 48\ W$ (abs)

$P_4 = -IV = -8 \times 1 = -8\ W$ (del)

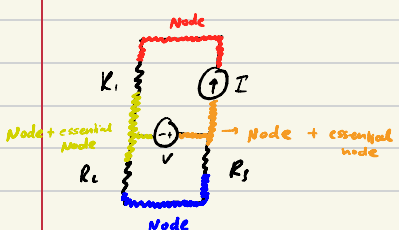
Kirchoff's Laws:-

Kirchhoff's voltage law (KVL)

Kirchhoff's current law (KCL)

Node: it's a point of connection of two or more circuit elements.

Essential node: it's a point of connection of three or more circuit elements

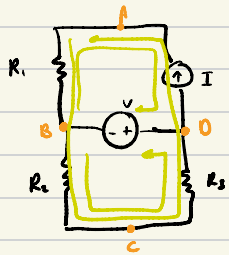


4 Nodes & 2 essential nodes

Loop: it is any closed path through the circuit such that no node is crossed more than once

Mesh: it is a loop with no loops inside it

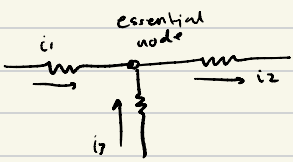
Example :



loops: ABDA, BCDB, ABCDA

Mesh: ABDA, BCDB

KCL & KVL:

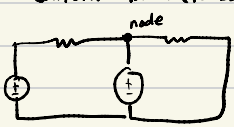


$I_1 + I_3 - I_2 = 0$

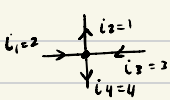
or $-I_1 - I_3 + I_2 = 0$

or $I_1 + I_3 = I_2$

Kirchhoff's Current law: (KCL)

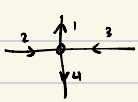


$\Sigma I_{in} = \Sigma I_{out}$



$i_3 + i_2 = i_2 + 4$

$5 = 5$ ✓



$\Sigma I_{out} = 0$

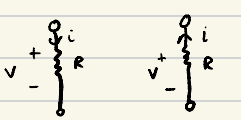
$1 + 4 - 2 - 3 = 0$

or $\Sigma I_{in} = 0$

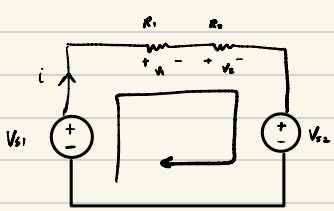
$2 + 3 - 1 - 4 = 0$

Kirchhoff's voltage law: (KVL)

$\Sigma V_{closed\ path} = 0$



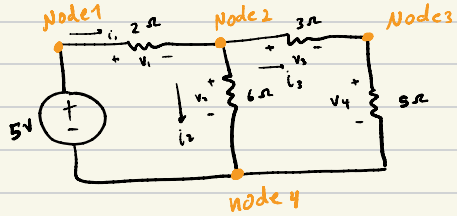
$V = +IR$ $V = -IR$



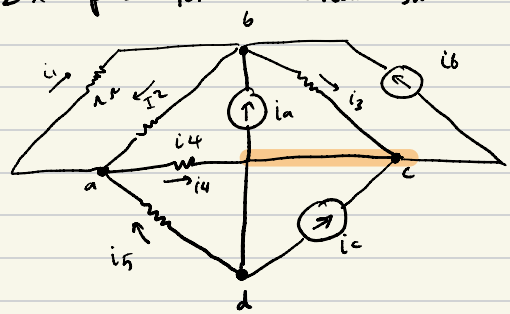
$-V_{s1} + V_1 + V_2 + V_{s2} = 0$

$-V_{s1} + (+IR_1) + (IR_2) + V_{s2} = 0$

Example



Example: for the circuit shown apply KCL to each Node a, b, c & d



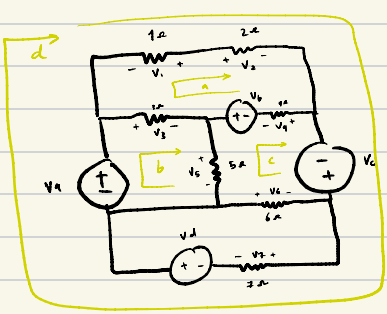
a ⇒ $i_5 + i_2 - i_4 - i_1 = 0 \Rightarrow i_5 + i_2 = i_4 + i_1$

b ⇒ $i_b + i_1 - i_2 - i_3 + i_a = 0 \Rightarrow i_a + i_b + i_1 = i_2 + i_3$

c ⇒ $i_3 + i_4 - i_b + i_c = 0 \Rightarrow i_3 + i_4 + i_c = i_b$

d ⇒ $-i_c - i_a - i_5 = 0$

Example



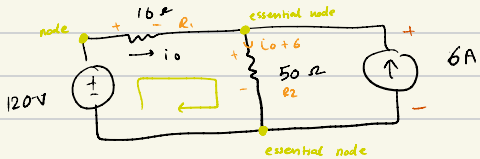
loop ⓐ ⇒ $+V_4 - V_b - V_3 - V_1 + V_2 = 0$

loop ⓑ ⇒ $-V_a + V_3 + V_5 = 0$

loop ⓒ ⇒ $-V_6 - V_5 + V_b - V_4 - V_c = 0$

loop ⓓ ⇒ $-V_1 + V_2 - V_c + V_3 - V_d - V_a = 0$

Example : find i_o ?



3 nodes

$$-120 + V_1 + V_2 = 0$$

$$10 i_o + 50(i_o + 6) = 120$$

$$60 i_o + 300 = 120$$

$$i_o = -3\ A$$

$$P_{120V} = IV = -(-3 \times 120)$$

$$= 360\ watt \quad (absorbing)$$

$$P_{10\Omega} = +IV = (-3)(+10 \times -3) = 90\ watt$$

$$P_{50\Omega} = +IV = (3)(+3 \times 50) = 450\ watt$$

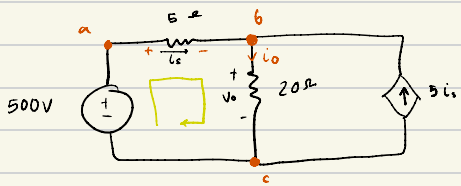
$$P_{6A} = -(360 + 90 + 450)$$

$$= -900\ watt$$

or $\Rightarrow P = -VI = -(150 \times 6) = -900\ watt$

$V = +IR = (-3+6) \times 50 = 150\ V$

Example



$$i_s + 5i_s = i_o \Rightarrow 6 i_s = i_o$$

$$-500 + V_s + V_o = 0$$

$$5 i_s + 6 i_s \times 20 = 500$$

$$125\ i_s = 500$$

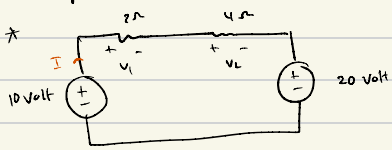
$$i_s = 4\ A$$

$\Rightarrow V_o = 6 i_s \times 20$

$= 6 \times 4 \times 20$

$= 480\ Volt$

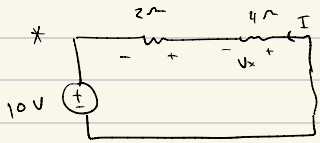
Example:



$-10 + 2I + 4I + 20 = 0$

$6I = -10$

$I = \frac{-10}{6}$ A

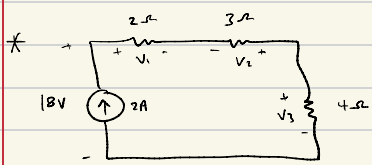


$Vx + 2I + 10 = 0$

$4I + 2I = -10$

$I = \frac{-10}{6}$ A

$Vx = +IR = \left(\frac{-10}{6} \times 4\right) = \frac{-20}{3}$ Volts

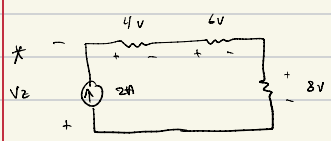


Find V_1, V_2 & V_3

$V_1 = +IR = 2 \times 2 = 4V$

$V_2 = -IR = -2 \times 3 = -6V$

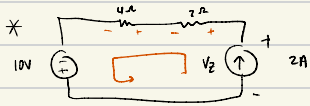
$V_3 = +IR = 2 \times 4 = 8V$



Find V_z?

$V_z + 4 + 6 + 8 = 0$

$V_z = -18$ V

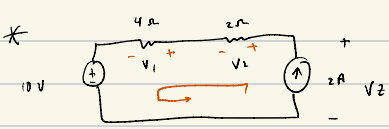


$-V_z + 4 + 8 - 10 = 0$

$V_z = 2$ Volt

$P_{2A} = -2 \times 2 = -4W$

$P_{10V} = -10 \times 2 = -20W$



Find V_z?

$-V_z + V_z + V_1 + 10 = 0$

$-V_z + 4 + 8 + 10 = 0$

$V_z = 22$ V

$P_{2\Omega} = 2 \times 4 = 8W$

$P_{4\Omega} = 2 \times 8 = 16W$

$P_{10V} = 2 \times 10 = 20W$

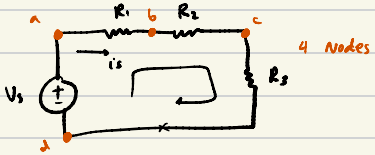
$P_{2A} = -2 \times 22 = -44W$

$\rightarrow \sum P = 0$

Chapter 3: Simple Resistive Circuits:

resistors in series & resistor in parallel

in Series:



Series elements have same current:

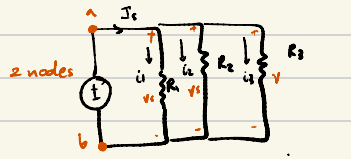
KVL:

$-V_s + iR_1 + R_2 i_s + R_3 i_s = 0$

$\underbrace{(R_1 + R_2 + R_3)}_{R_{eq}} i_s = V_s$

$R_{eq} = \sum_{i=1}^{N} R_i = R_1 + R_2 + \cdots + R_N$

in Parallel:



$i_1 = \frac{V_s}{R_1}, \; i_2 = \frac{V_s}{R_2}, \; i_3 = \frac{V_s}{R_3}$

KCL: $I_s = i_1 + i_2 + i_3$

$I_s = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3}$

$I_s = V_s \times \left(\frac{1}{R_{eq}}\right)$

$R_{eq} = \frac{V_s}{I_s}$

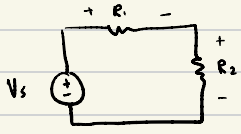
$\frac{1}{R_{eq}} = \sum_{i=1}^{N} \frac{1}{R_i}$

The Voltage Divider and Current-Divider Circuits

voltage Divider Rule: VDR

$V_1 \xrightarrow{?} V_s$

$V_2 \xrightarrow{?} V_s$



$R_{eq} = R_1 + R_2$

$I_s = \frac{V_s}{R_{eq}} = \frac{V_s}{R_1 + R_2}$

$V_1 = R_1 I_s \Rightarrow V_1 = R_1 \frac{V_s}{R_1 + R_2}$

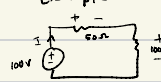
$V_2 = R_2 I_s \Rightarrow V_2 = R_2 \frac{V_s}{R_1 + R_2}$

$$V_1 = \frac{R_1}{R_1 + R_2} V_s \qquad V_2 = \frac{R_2}{R_1 + R_2} V_s$$

voltage divider rule is just for simple circuits

invalid ← current ↓ VS ↓ ↓ source

Example

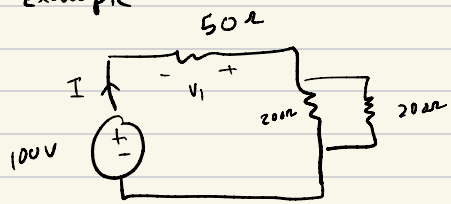


find V_1 & V_2?

$V_1 = \frac{50}{150} \times 100 = 33.33\,V$

$V_2 = 66.66$ Volt

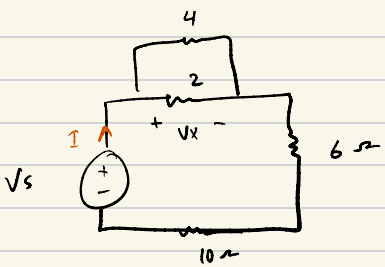
Example



Find V_1 ?

$$V_1 = -\frac{50}{\frac{200}{2}+50} \times 100$$

$$= -33.33\ V$$

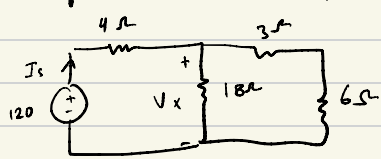


$V_X = ??$

$$\frac{4 \times 2}{2+4} = \frac{8}{6} = \frac{4}{3}$$

$$V_X = \frac{\frac{4}{3}}{\frac{4}{3}+16} \times V_S$$

Example : Find V_x??

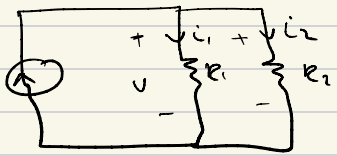


$3+6 = 7\Omega$

$$\frac{9 \times 18}{9+18} = 6\Omega$$

$$V_x = +\frac{6}{6+4} \times 120 = 72\ V$$

Current Divider Rule : (CDR):



$$V = i_1 R_1 = i_2 R_2$$

$$V = i_s R_{eq} = i_s \frac{R_1 R_2}{R_1 + R_2}$$

KCL : $i_s = i_1 + i_2$

$$R_1 i_1 = i_s \times \frac{R_1 R_2}{R_1 + R_2}$$

$$i_1 = \frac{R_2}{R_1 + R_2} \times i_s$$

$$i_2 = \frac{R_1}{R_2 + R_1} i_s$$

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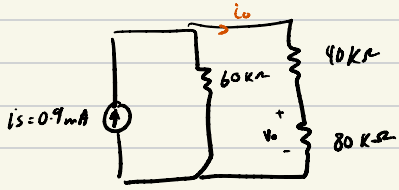
Example : Find v_o

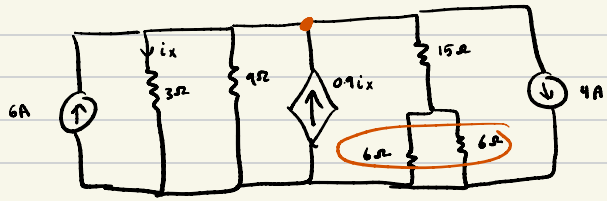


$$i_o = i_s \times \frac{60}{60 + 120} = 0.9 \times 10^{-3} \times \frac{60}{180}$$

$$= 3 \times 10^{-4} \text{ A}$$

$$V_o = i_o R = 3 \times 10^{-4} \times 80 \times 10^{3} = 24 \text{ Volt}$$

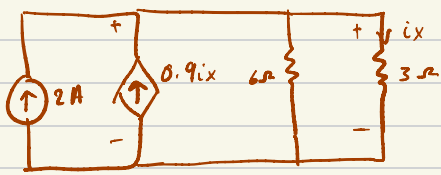
Example: Find $P_{0.9ix}$?



$\frac{6}{2} = 3\ \Omega$

$3 + 15 = 18\ \Omega$

$\frac{9 \times 18}{9+18} = 6\Omega$



$V = +IR = \frac{10}{3} \times 3 = 10V$

$$ix = \frac{6}{3+6} \times (2 + 0.9ix)$$

$$ix = \frac{2}{3}(2 + 0.9ix) \Rightarrow 3ix = 4 + 1.8ix$$

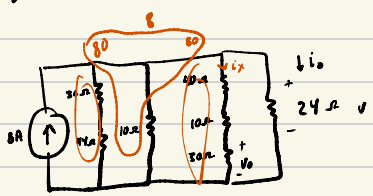
$$1.2ix = 4$$

$$ix = \frac{10}{3}$$

$P = -IV$ 0.3

$= -(0.9 \times \frac{10}{3} \times 10) = -30$ watt

Example: find i_o & V_o



$$i_o = \frac{R_{eq}}{R_{eq} + 24} \times 8$$

$$i_o = \frac{8}{8+24} \times 8$$

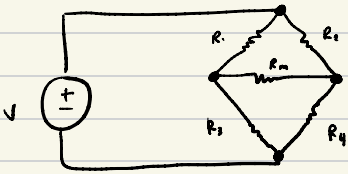
$= 2A$

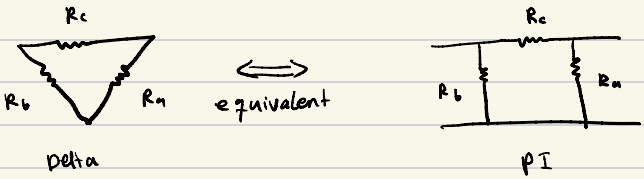
$$ix = \frac{6.48}{6.48+80} \times 8$$

$ix = 0.6\ A$

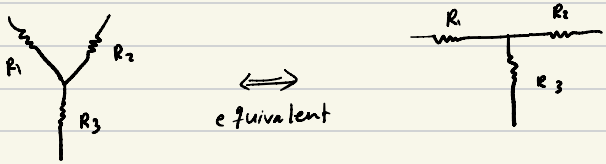
$V_o = +30 \times 0.6 = 18V$

Δ to Y & Y to Δ





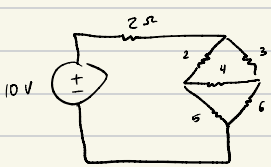
Delta PI

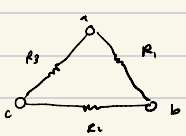


Δ → Y

Y → Δ

transformation

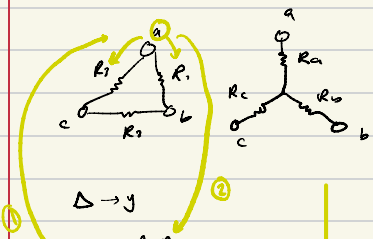




$$R_{ab} = R_1 \parallel (R_2 + R_3) = \frac{R_1 (R_2 + R_3)}{R_1 + R_2 + R_3} = R_a + R_b \quad \text{(1)}$$

$$R_{bc} = \frac{R_2 (R_3 + R_1)}{R_2 + R_1 + R_3} = R_b + R_c \quad \text{(2)}$$

$$R_{ca} = \frac{R_3 (R_1 + R_2)}{R_3 + R_2 + R_1} = R_c + R_a \quad \text{(3)}$$



if $R_1 = R_2 = R_3 = R_\Delta$

if $R_a = R_b = R_c = R_y$

$R_\Delta = 3R_y$

$R_y = \frac{1}{3} \cdot R_\Delta$

$\Delta \rightarrow y$

$y \rightarrow \Delta$

Final result :-

$$R_a = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

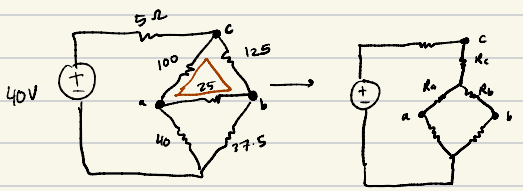
$$R_c = \frac{R_3 R_2}{R_1 + R_2 + R_3}$$

$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

Example:



$$R_a = \frac{25 \times 100}{100 + 125 + 25} = 10\ \Omega$$

$$R_b = \frac{125 \times 25}{100 + 125 + 25} = 12.5\ \Omega$$

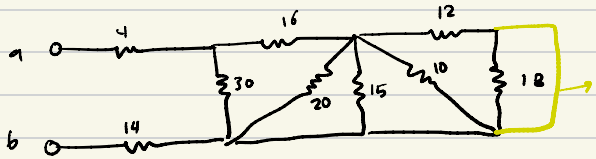
$$R_c = \frac{125 \times 100}{100 + 125 + 25} = 50\ \Omega$$

$$R_{eq} = 5 + 50 + (10 + 40) /\!/ (12.5 + 37.5)$$

$$= 80\ \Omega$$

$$P_{80\Omega} = \frac{V^2}{R_{eq}} = \frac{40^2}{80} \text{ (abs)} \qquad P_{40V} = -\frac{40^2}{80} \text{ (del.)}$$

Example:



$R_{ab} = ??$

parallel and series ⇒ Not Δ to Y

$12 + 18 = 30\,\Omega \Rightarrow \frac{30 \times 10}{40} = 7.5\,\Omega$

$\frac{7.5 \times 15}{7.5 + 15} = 5\,\Omega \Rightarrow \frac{5 \times 20}{20 + 5} = 4\,\Omega$

$4 + 16 = 20 \Rightarrow \frac{20 \times 30}{50} = 12\,\Omega$

$12 + 4 + 14 = 30\,\Omega$

$R_{ab} = 30\,\Omega$

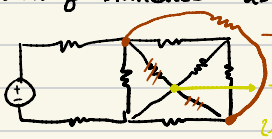
Ch4 : Techniques of Circuit Analysis

Nodal Method (Node Voltage Method)

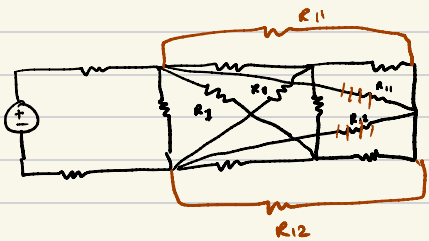
Terminology:

planner circuit: a circuit that can be drawn on a plane with no crossing branches as shown:

planner

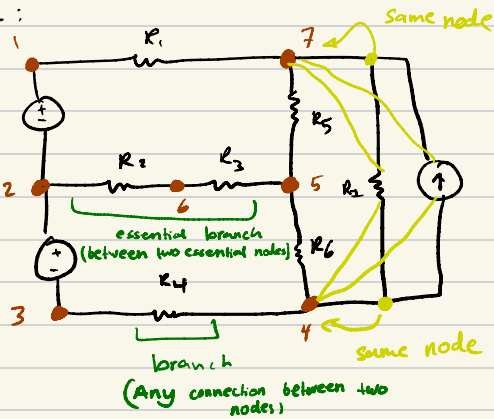


nonplanner



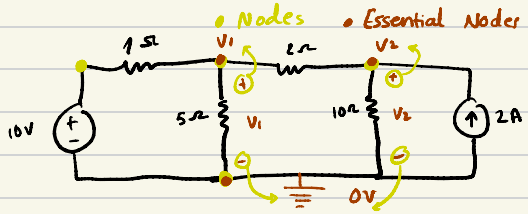
but it's hard to convertt R7 & R8 without any cross so it's nonplanner

Node :

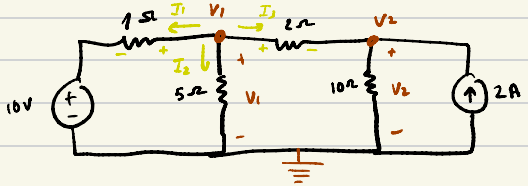


2 & 4 & 5 & 7 are essential nodes

Nodal Method: (for both planner & non planner)



1. identify all essential nodes.
2. Select one of the essential nodes as a reference node.
3. label all non reference essential nodes with alphabetical labels V_1, V_2..
4. Write KCl equation on all labels non reference nodes



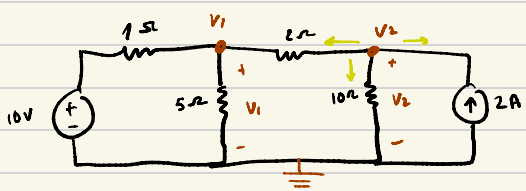
KCl $\Rightarrow$ $I_1 + I_2 + I_3 = 0$ $= \frac{(V_1 - 10)}{1} + \frac{V_1}{5} + \frac{(V_1 - V_2)}{2}$ $\Rightarrow$ Node Eq.1

$V_1 - 10 - I_1 = 0$

$I_1 = \frac{(V_1 - 10)}{1}$

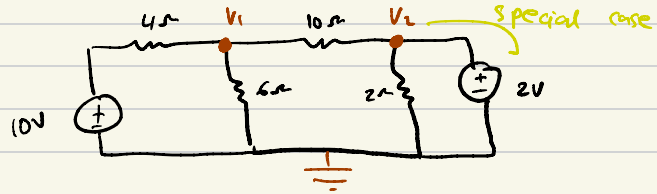
$I_2 = \frac{V_1}{5}$

$I_3 = \frac{V_1 - V_2}{2}$



$\frac{V_2 - V_1}{2} + \frac{V_2}{10} - 2 = 0$ $\Rightarrow$ Node Eq.2

Example

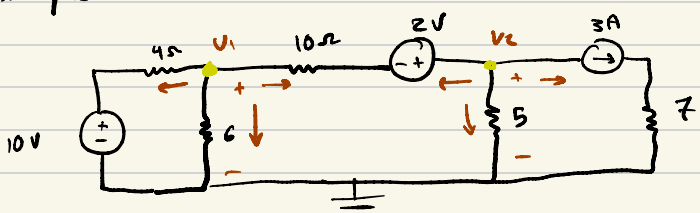


$V_2 = 2V$

$$\frac{V_1 - 10}{4} + \frac{V_1}{6} + \frac{V_1 - V_2}{10} = 0$$

$$\frac{V_1 - 10}{4} + \frac{V_1}{6} + \frac{V_1 - 2}{10} = 0 \Rightarrow V_1 = \_\_\_ V$$

Example

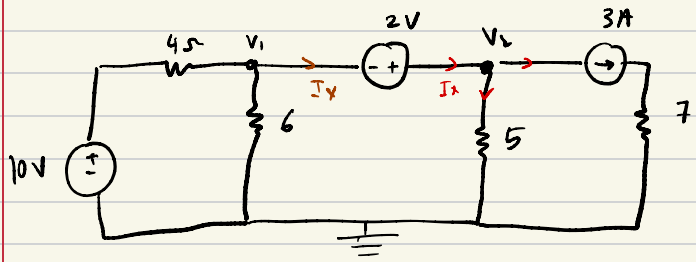


$$\frac{V_1}{6} + \frac{V_1 - 10}{4} + \frac{V_1 - V_2 + 2}{10} = 0$$

$$\frac{V_2 - V_1 - 2}{10} + \frac{V_2}{5} + 3 = 0$$

$V_1 = \_\_\_\_ V$, $V_2 = \_\_\_\_ V$

Special case:



$I_x = \frac{V_2}{5} + 3$

$$\frac{V_1 - 10}{4} + \frac{V_1}{6} + V_1 - V_2 + 2 = 0$$

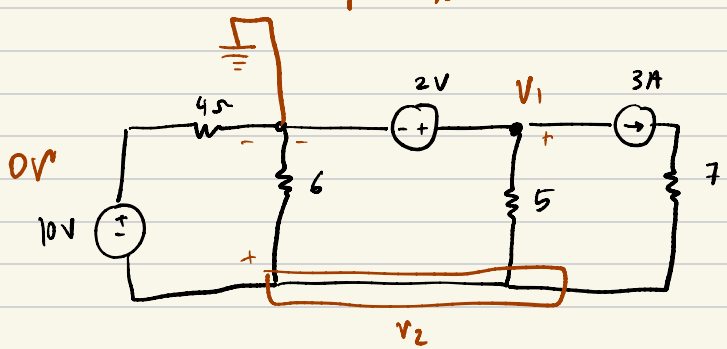
volt ✗ ⟹ "current pix"

$$\frac{V_1 - 10}{4} + \frac{V_1}{6} + I_x = 0$$

$$\frac{V_1 - 10}{4} + \frac{V_1}{6} + \boxed{\frac{V_2}{5} + 3} = 0$$

$$V_2 - V_1 = 2V$$

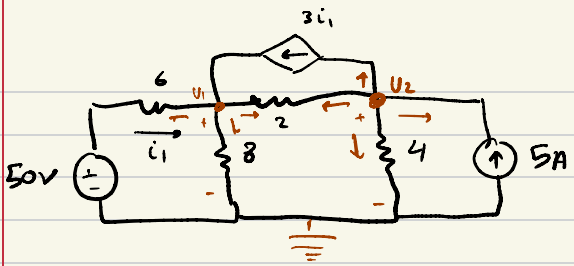
super Node



$V_1 = 2V$

$\frac{V_2 - V_1}{5}$ *

$$\frac{V_2 + 10}{4} + \frac{V_2}{6} + \frac{V_2 - 2}{5} - 3 = 0$$



(V1) $\frac{V_1 - 50}{6} + \frac{V_1}{8} + \frac{V_1 - V_2}{2} - 3i_1 = 0$

$i_1 = -\left(\frac{V_1 - 50}{6}\right)$

$4 \times \frac{(V_1 - 50)}{6} + \frac{V_1}{8} + \frac{V_1 - V_2}{2} = 0$

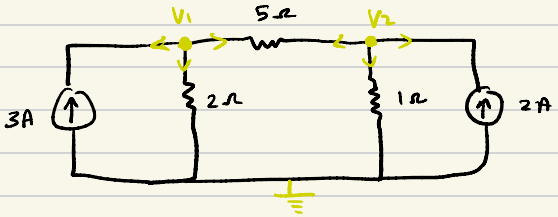
$\frac{31}{24} V_1 - 0.5 V_2 = \frac{100}{3}$ ---- (1)

(V2) $\frac{V_2 - V_1}{2} + 3i_1 + \frac{V_2}{4} - 5 = 0$

$\frac{V_2 - V_1}{2} + 3\left(\frac{50 - V_1}{6}\right) + \frac{V_2}{4} - 5 = 0$

$-V_1 + \frac{3}{4} V_2 = -20$ ---- (2)

Extra examples: with details

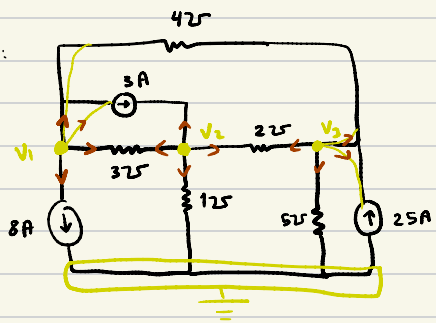
① 

$$-3 + \frac{V_1}{2} + \frac{V_1 - V_2}{5} = 0$$

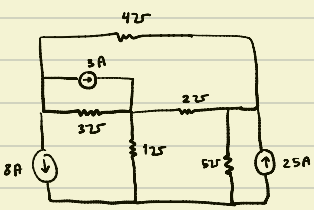
$$-2 + \frac{V_2}{1} + \frac{V_2 - V_1}{5} = 0$$

$$\boxed{\begin{aligned} V_1 &= 5V \\ V_2 &= 2.5V \end{aligned}}$$

② Example:



(Original circuit)



For $V_1 \Rightarrow 4(V_1 - V_3) + 3 + 3(V_1 - V_2) + 8 = 0$

$$4V_1 - 4V_3 + 3 + 3V_1 - 3V_2 + 8 = 0$$

$V_2 \Rightarrow 1V_2 + (V_2 - V_1) \times 3 + (V_2 - V_3) \times 2 - 3 = 0$

$$V_2 + 3V_2 - 3V_1 + 2V_2 - 2V_3 - 3 = 0$$

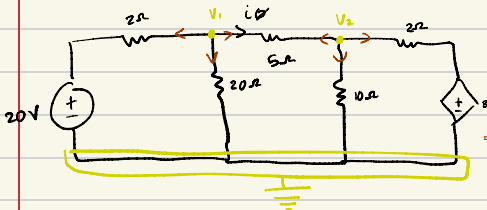
$V_3 \Rightarrow -25 + 5V_3 + 2(V_3 - V_2) + 4(V_3 - V_1) = 0$

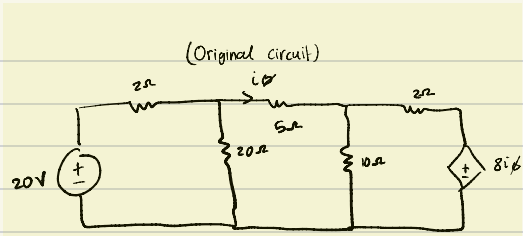
$$-25 + 5V_3 + 2V_3 - 2V_2 + 4V_3 - 4V_1 = 0$$

$$\begin{aligned} 7V_1 - 3V_2 - 4V_3 &= -11 \\ -3V_1 + 6V_2 - 2V_3 &= 3 \\ -4V_1 - 2V_2 + 11V_3 &= 25 \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} V_1 &= 1V \\ V_2 &= 2V \\ V_3 &= 3V \end{aligned}$$

(You can solve this using Cramers Rule)

(3) Example :





$$V_1 \Rightarrow \quad \frac{V_1-20}{2} + \frac{V_1}{20} + \frac{V_1-V_2}{5} = 0$$

$$i_\phi = \frac{V_1-V_2}{5}$$

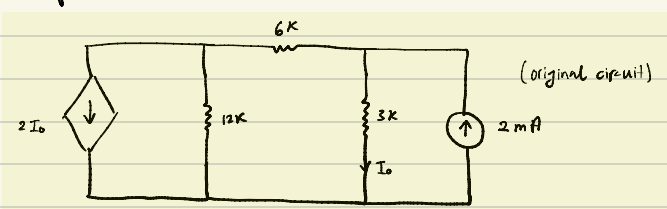
$$V_2 \Rightarrow \quad \frac{V_2-V_1}{5} + \frac{V_2}{10} + \frac{V_2 - 8(V_1-V_2)/5}{2} = 0$$

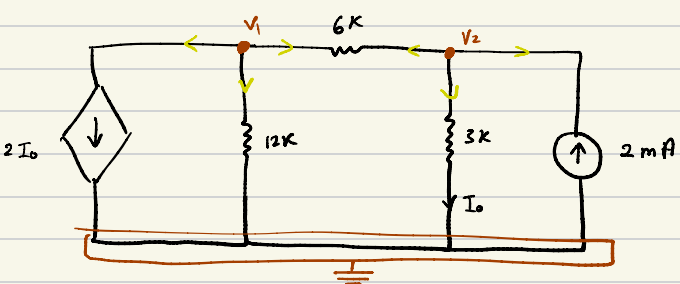
$$\frac{V_2-V_1}{5} + \frac{6V_2}{10} + \frac{-8(V_1-V_2)}{10} = 0$$

$\rightarrow V_1 = 16V$

$V_2 = 10V$

(3) Example :



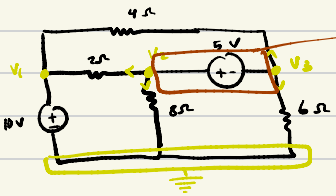


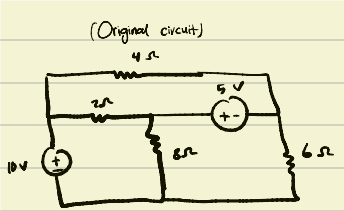
$$2I_o + \frac{V_1}{12K} + \frac{V_1-V_2}{6K} = 0 \qquad * \; I_o = \frac{V_2}{3K}$$

$$\frac{V_2-V_1}{6K} + \frac{V_2}{3K} - 2\times10^{-3} = 0$$

$$V_1 = -\frac{24}{5}V \;/\; V_2 = \frac{12}{5}V$$

Example :





$V_1 - 0 = 10 \Rightarrow V_1 = 10V$

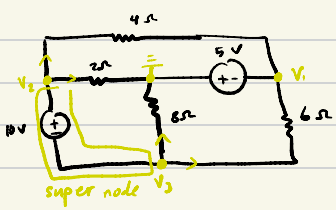
$V_2 - V_3 = 5V$

1 node بعتبرها
وبكتب معادلة كل التيارات

$$\frac{V_2 - V_1}{2} + \frac{V_2}{8} + \frac{V_3 - V_1}{4} + \frac{V_3}{6} = 0$$

$\therefore V_3 = 4.2\,V$ & $V_2 = 9.2\,V$

another way



$V_1 = -5\,V$

$(V_2 - V_3 = 10V)\ -\frac{3}{4}$

$$\frac{V_3 - V_1}{6} + \frac{V_3}{8} + \frac{V_2}{2} + \frac{V_2 - V_1}{4} = 0$$

$$\frac{V_3 + 5}{6} + \frac{V_3}{8} + \frac{V_2}{2} + \frac{V_2 + 5}{4} = 0$$

$$\left(\frac{V_3}{6} + \frac{V_3}{8}\right) + \left(\frac{V_2}{2} + \frac{V_2}{4}\right) = \left(\frac{-5}{6} + \frac{-5}{4}\right)$$

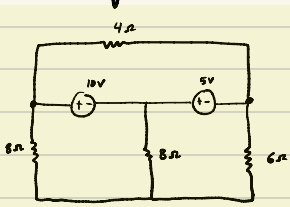
$$\frac{14\,V_3}{48} + \frac{3}{4}V_2 = -\frac{50}{24}$$

$$\frac{3}{4}V_3 - \frac{3}{4}V_2 = -\frac{30}{4}$$

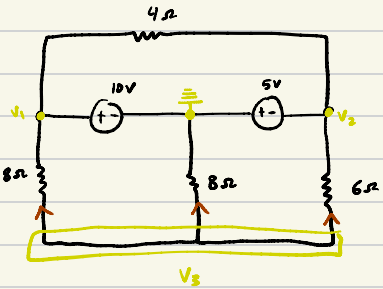
$V_3 = -9.2\,V$

$V_2 = 0.8V$

Example :



(original circuit)



$V_1 = 10V \quad / \quad V_2 = -5V$

$$\frac{V_3 - V_2}{6} + \frac{V_3}{8} + \frac{V_3 - V_1}{8} = 0$$

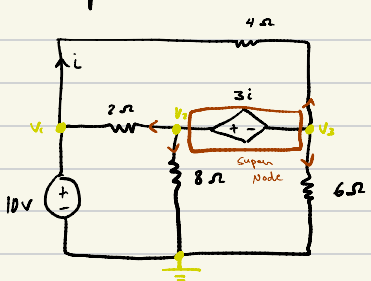
$$\frac{V_3 + 5}{6} + \frac{V_3}{8} + \frac{V_3 - 10}{8} = 0$$

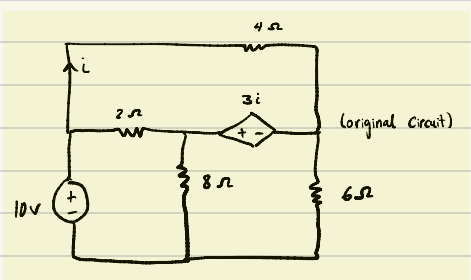
$$\frac{V_3}{6} + \frac{V_3}{4} = \frac{5}{4} - \frac{5}{6}$$

$$\frac{10 V_3}{24} = \frac{30 - 20}{24}$$

$$\boxed{V_3 = 1\ V}$$

Example :





$V_1 = 10V \quad / \quad V_2 - V_3 = 3i$

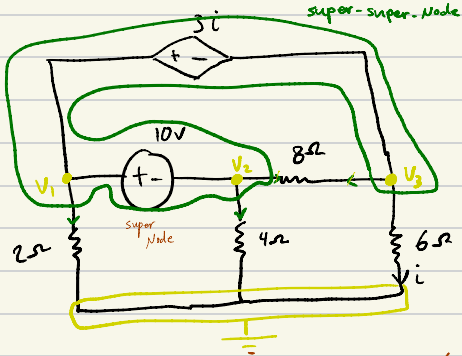
$$\frac{V_3 - V_1}{4} + \frac{V_3}{6} + \frac{V_2 - V_1}{2} + \frac{V_2}{8} = 0$$

$$\times \ i = \frac{V_1 - V_3}{4}$$

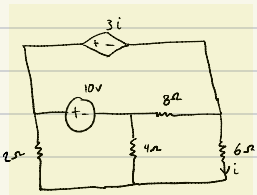
$$\Rightarrow \frac{V_3 - 10}{4} + \frac{V_3}{6} + \frac{V_2 - 10}{2} + \frac{V_2}{8} = 0$$

***

Example: (it's easier to take V_3 as a reference)



(original circuit)



$V_1 - V_3 = 3i$

$V_1 - V_2 = 10$

$i = \frac{V_3}{6}$

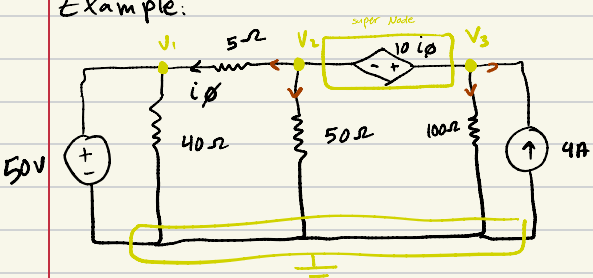
$V_1 - V_3 = \frac{8V_3}{6}$

$V_1 = 1.5\,V_3$

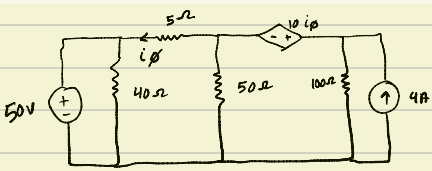
$$\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_2 - V_3}{8} + \frac{V_3 - V_2}{8} + \frac{V_3}{6} = 0$$

$$\frac{V_1}{2} + \frac{V_2}{4} + \frac{V_3}{6} = 0$$

Example:



(original circuit)



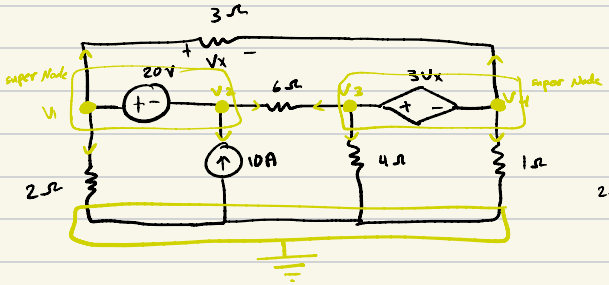
$V_1 = 50V$ $\qquad V_3 - V_2 = 10\,i\phi = 10 \times \frac{(V_2 - V_1)}{5} = 2(V_2 - V_1)$

$V_3 - V_2 = 2V_2 - 100 \quad \Rightarrow V_3 = 3V_2 - 100$

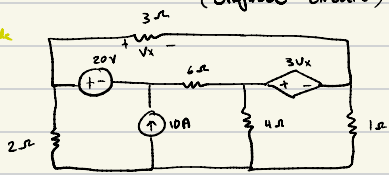
$$\frac{V_2 - V_1}{5} + \frac{V_2}{50} + \frac{V_3}{100} - 4 = 0$$

$$\frac{V_2 - 50}{5} + \frac{V_2}{50} + \frac{V_3}{100} = 4$$

Example:



(original circuit)



$V_1 - V_2 = 20$ ---①

$V_3 - V_4 = 3V_x$

$V_3 - V_4 = 3(V_1 - V_4)$

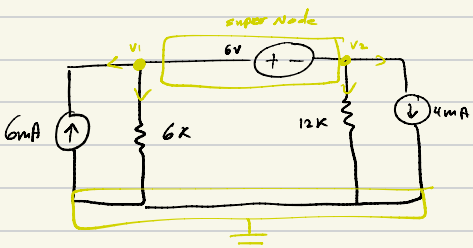
$V_3 - V_4 = 3V_1 - 3V_4$

$V_3 = 3V_1 - 2V_4$ ——— ②

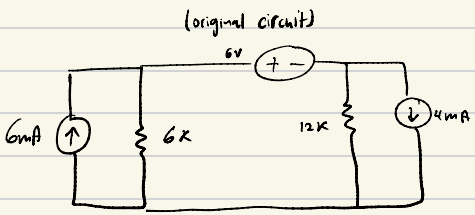
③ ---. $\frac{V_1 - V_4}{3} + \frac{V_1}{2} - 10 + \frac{V_2 - V_3}{6} = 0$

④ - - $\frac{V_3 - V_2}{6} + \frac{V_3}{4} + \frac{V_4}{1} + \frac{V_4 - V_1}{3} = 0$

Example:



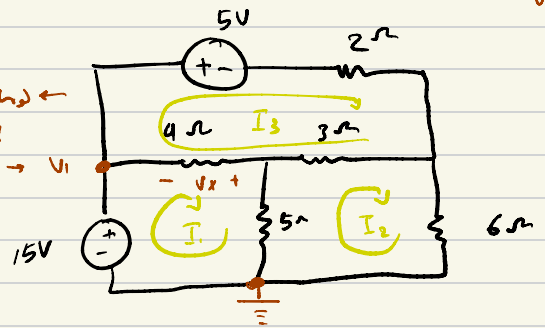
(original circuit)



$V_1 - V_2 = 6$

$-6mA + \frac{V_1}{6K} + \frac{V_2}{12K} + 4mA = 0$

Mesh Current Method:- (Only for planner circuit)



← توجد عقدة واحدة Nodal
← جهد → V_1

$V_x = -4(I_1 - I_3)$
$= 4(I_3 - I_1)$

- mesh currents
- all cw / ccw (clock wise) (counter clock wise)
- write kVL eq's

$-15 + 9I_1 - 4I_3 - 5I_2 = 0$

$14I_2 - 3I_3 - 5I_1 = 0$

$5 + 9I_3 - 4I_1 - 3I_2 = 0$

@ I_1 :

$$-15 + 4(I_1 - I_3) + 5(I_1 - I_2) = 0$$

$$9I_1 - 5I_2 - 4I_3 = 15 \quad \text{---} ①$$

@ I_2

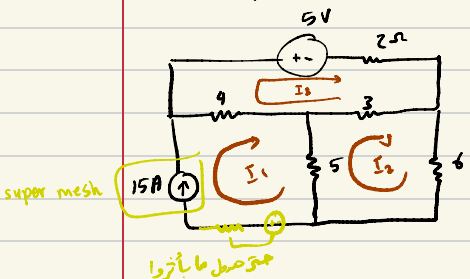
← تأخذ المقاومات المشتركة بين I_1 & I_2 وعكسها

$$-5I_1 + 14I_2 - 3I_3 = 0 \quad \text{---} ②$$

(3 + 5 + 6)

@ I_3 $\quad -4I_1 - 3I_2 + (2+3+4)I_3 = -5 \quad \text{---} ③$

Example:



موجود ما بأثر بـ

① $I_1 = 15$

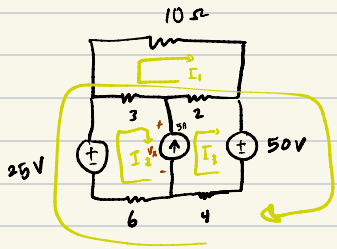
② $I_2 \Rightarrow \quad -5I_1 - 3I_3 + 14I_2 = 0$

$$14I_2 - 3I_3 = 5 \times 15$$

③ $I_3 \Rightarrow \quad -4I_1 - 3I_2 + 9I_3 = -5$

$$-4 \times 15 - 3I_2 + 9I_3 = -5$$

Example:



(a) $I_1 \quad -3I_2 + 15I - 2I_3 = 0 \quad ---①$

@ $I_2 \quad -25 + 3(I_2 - I_1) + V_x + 6I_2 = 0$

@ $I_3 \quad 50 + 4I_3 - V_x + 2(I_3 - I_1) = 0$

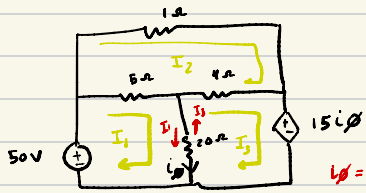
instead, take the large loop

$$-25 + 3(I_2 - I_1) + 2[I_3 - I_1) + 50 + 4I_3 + 6I_2 = 0$$

$$-5I_1 + 9I_2 + 6I_3 = -25 \quad ---②$$

$$5 = I_3 - I_2 \quad ---③$$

Extra Examples :-



$i_\phi = I_1 - I_3$ ⟹ حسب اتجاه التيار المعطى بكتب حسب I_1, I_2, I_3 بعضي الوضع التيار المعطى (+) والعكس صحيح (-)

$-50 + 25I_1 - 20I_3 - 5I_2 = 0$

$15i_\phi + 24I_3 - I_1 20 - 4I_2 = 0$

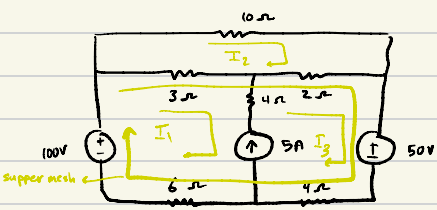
$10I_2 - 5I_1 - 4I_3 = 0$

$i_\phi = I_1 - I_3$

$I_1 = 29.6A$ / $I_2 = 26A$ / $I_3 = 28A$

Example :

Super mesh case



$I_3 - I_1 = 5A$ ----(1)

super mesh eq: $-100 + 3(I_1 - I_2) + 2(I_3 - I_2) + 50 + 4I_3 + 6I_1 = 0$

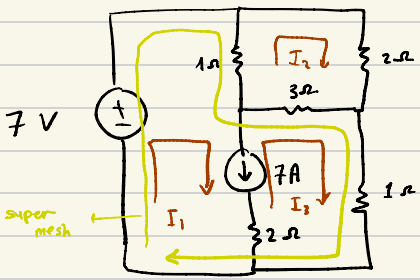
$-100 + 3I_1 - 3I_2 + 2I_3 - 2I_2 + 50 + 4I_3 + 6I_1 = 0$

$9I_1 - 5I_2 + 6I_3 = 50$ ----(2)

$I_2 \rightarrow$ $15I_2 - 3I_1 - 2I_3 = 0$ -----(3)

⟹ $I_1 = 1.75A$, $I_2 = 1.25$, $I_3 = 6.75A$

Example:



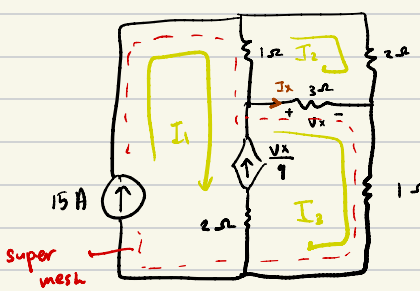
$I_1 - I_3 = 7$

@ $I_2 \Rightarrow \quad 6I_2 - 3I_3 - I_1 = 0$

@ super mesh $\Rightarrow \quad -7 + 1(I_1 - I_2) + 3(I_3 - I_2) + I_3 = 0$

$I_1 - 4I_2 + 4I_3 = 7$

Example:



$V_x = +I_x 3$

$= 3(I_3 - I_2)$

$V_x = 3I_3 - 3I_2$ ---

$I_3 - I_1 = \frac{V_x}{9} \Rightarrow \quad I_3 - I_1 = \frac{I_3 - I_2}{3}$ ---①

@ $I_2 \Rightarrow \quad 6I_2 - 3I_3 - I_1 = 0$ ---②

super mesh $\Rightarrow \quad I_1 = 15A$

$I_3 - 15 = \frac{I_3 - I_2}{3}$

$6I_2 - 3I_3 = 15$ → $I_3 = 17\,A$, $I_2 = 11\,A$, $I_1 = 15A$

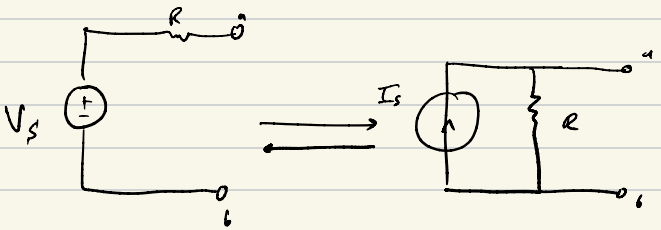
$3I_3 - 45 = I_3 - I_2$ $\qquad 6 \times 11 - 3I_3 = 15$

$2I_3 + I_2 = 45$

$-2I_3 + 4I_2 = 10$

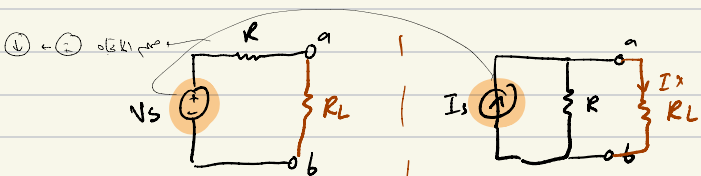
$5I_2 = 55 \Rightarrow I_2 = 11\,A \Rightarrow$

Source Transformation:



$V_{ab} = V_s$ $\qquad$ $V_{ab} = R I_s$

$V_s = I_s R$



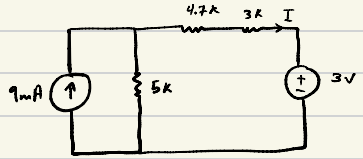
$V_{R_L} = V_{ab} = \frac{R_L}{R_L + R} V_s$

$V_{RL} = V_{ab} = R_L I_x$

$V_{RL} = R_L \times \frac{R}{R_L + R_L} \times I_s$

$V_{ab} = I_s R$

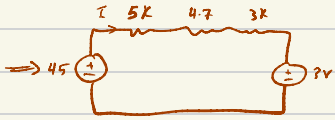
Example: Find I?



$V_s = I_s \times R$

$= 9 \times 5 = 45V$

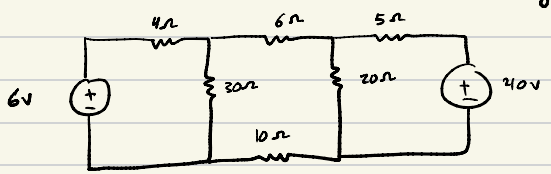
$\Rightarrow$



$$I = \frac{45 - 3}{5 + 4.7 + 3} = 3.31\ mA$$

Example: a) Find the power associated with the 6V source?

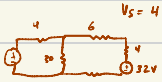
b) state wether the 6V source is absorbing or delivering power?



$40 = I_s \times 5 \Rightarrow I_s = 8A$

$\frac{20 \times 5}{25} = 4\Omega$

$V_s = 4 \times 8 = 32V$



$R = 20\Omega$

$I_s = \frac{32}{20} = 1.6A$

$R = \frac{20 \times 20}{50} = 12\ \Omega$

$V_s = 1.6 \times 12 = 19.2V$

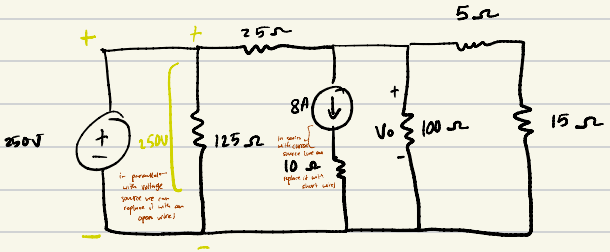
$\Rightarrow$

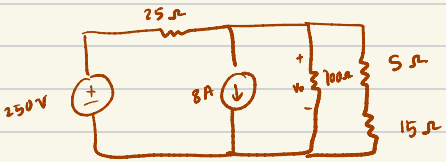
$$I = \frac{19.2 - 6}{16} = 0.825A$$

$P_{6V} = +IV = +0.825 \times 6 = 4.95\ W$ (absorbing power)

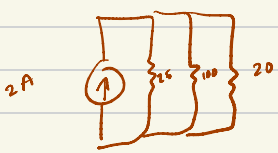
***

Example: find V_o using source transformation





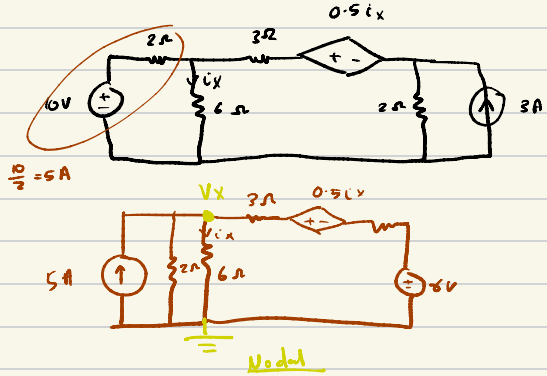
$I_s = \frac{250}{25} = 10A$



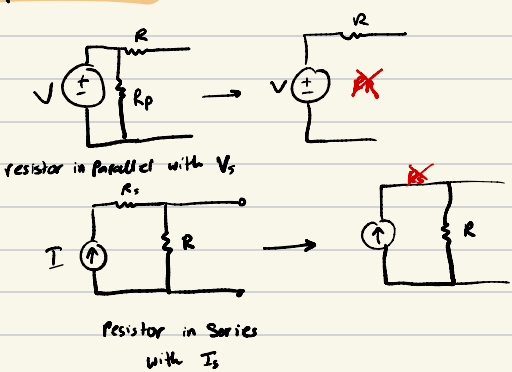
$20 // 100 // 25 = 10\Omega$

$V = IR = 2 \times 10 = 20V$

Example:



Special case:



Super Position:

you can calculate the voltage or current by finding the contribution of each individual indep. source acting alone dep sources → are left intact

steps: Kill all indep source except one

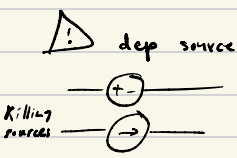
indep source & find the output V_o, i_o, ...

" " " V_o', i_o', ...

find the total contributions by adding $i_o = i_o' + i_o''$ / $V_o = V_o' + V_o''$ + ...

⚠ dep source are left intact

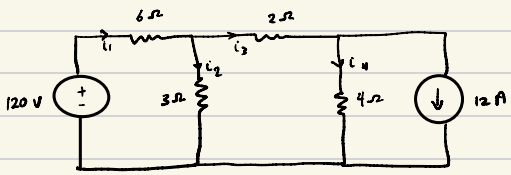
Killing sources



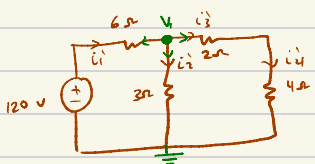
⇒ S.C

⇒ O.C

Example:



* to remove voltage sources ⇒ replace it with short wire ⇒ V=0
* to remove current sources ⇒ make it open circuit ⇒ I=0



$$\frac{V_1 - 120}{6} + \frac{V_1}{3} + \frac{V_1}{6} = 0$$

$$V_1 - 120 + 2V_1 + V_1 = 0$$

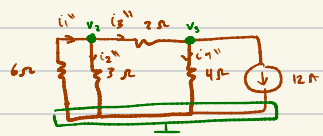
$$4V_1 = 120$$

$$\boxed{V_1 = 30\ V}$$

$$i_1' = \frac{120 - 30}{6} = 15\ A$$

$$i_2' = \frac{30}{3} = 10\ A$$

$$i_3' = i_4' = \frac{30}{6} = 5\ A$$



V_2 ⇒ $\frac{V_2}{6} + \frac{V_2}{3} + \frac{V_2 - V_3}{2} = 0$ ... (1)

V_3 ⇒ $\frac{V_3 - V_2}{2} + \frac{V_3}{4} + 12 = 0$ ..... (2)

$V_3 = -24$, $V_2 = -12$

$i_1'' = \frac{+12}{4} = +2A$ / $i_2'' = \frac{-12}{3} = -4A$ / $i_3'' = \frac{-12 + 24}{2} = 6A$

$i_4'' = \frac{-24}{4} = -6A$

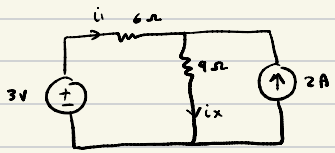
$i_1 = 15 + 2 = 17\ A$

$i_2 = 10 + (-4) = 6\ A$

$i_3 = 5 + 6 = 11\ A$

$i_4 = 5 + (-6) = -1\ A$

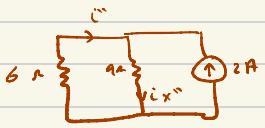
Example: Use SP to solve for i_x



$i_1' = 0.2\,A$

$i_x' = 0.2A$

$I = \frac{3}{15} = 0.2A$

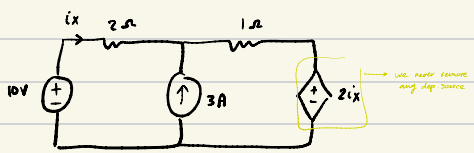


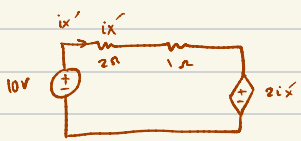
$i_x'' = \frac{2}{15} \times 2$

$= 0.8\,A$

$i_x = 0.2 + 0.2 = 1A$

Example:



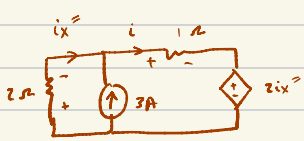


$i_x' = \frac{10 - 2i_x'}{3}$

$3i_x' = 10 - 2i_x'$

$i_x' = 2A$

$\therefore i_x = 2 - 0.6 = 1.4\,A$



$i = i_x'' + 3$

$2i_x'' + i + 2i_x'' = 0$

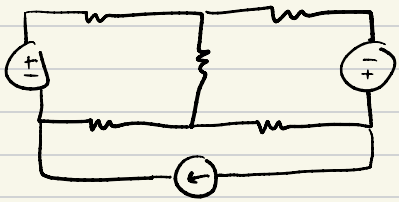
$2i_x'' + i_x'' + 3 + 2i_x'' = 0$

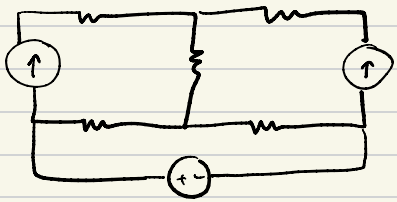
$5i_x'' = -3$

$i_x'' = -0.6\,A$

Problem 96: (solve)

Is Done (answ: V = 28V)

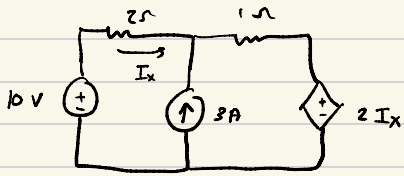


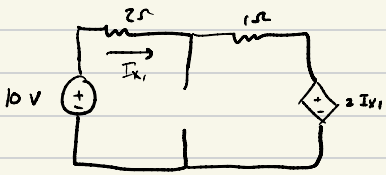


With mesh ⇒ easy

but super position ← Note

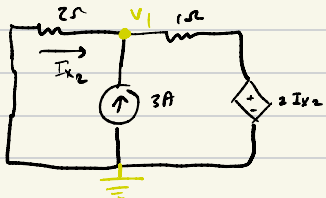
Example:





$-10 + 3i_{x_1} + 2I_{x_1} = 0$

$\boxed{I_{x_1} = 2A}$



$\frac{V_1}{2} - 3 + \frac{V_1 - 2I_{x_2}}{1} = 0$

But $I_{x_2} = -\frac{V_1}{2}$

$\frac{5V_x}{2} = 3 \Rightarrow V_x = 1.2V$

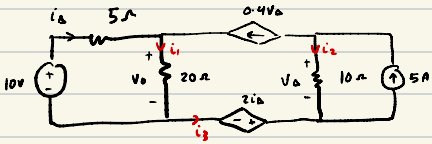
$i_{x_2} = -0.6A$

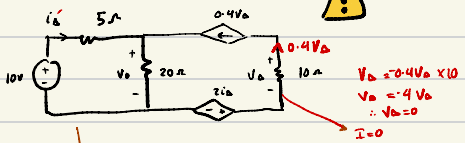
$i_x = 2 - 0.6 = 1.4A$

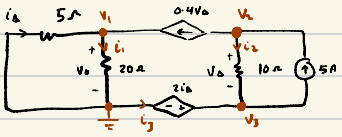
***

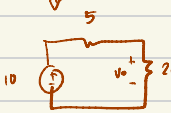
Example:

Find V_o?









$V_o = \frac{20}{25} \times 10 = 8V$

$\therefore V_o = V_{o_1} + V_{o_2}$

$= 8 + 16$

$= 24V$

$V_a = V_2 - V_3 \quad / V_3 = 2i_a = 2\times\frac{-V_1}{5} = \frac{-2V_1}{5}$

$V_1 \rightarrow \frac{V_1}{5} + \frac{V_1}{20} - 0.4V_a = 0$

$5V_1 - 8V_2 + 8V_3 = 0$

$5V_1 - 8V_2 + 8\left(-\frac{2}{5}V_1\right) = 0$

$25V_1 - 40V_2 - 16V_1 = 0$

$V_2 = \frac{9V_1}{40}$

$V_2 \rightarrow 0.4V_a + \frac{V_2 - V_3}{10} - 5 = 0$

$0.4(V_2 - V_3) + \frac{V_2 - V_3}{10} - 5 = 0$

$4V_2 - 4V_3 + V_2 - V_3 = 50$

$5V_2 - 5V_3 = 50$

$5V_2 - 5\left(\frac{-2V_1}{5}\right) = 50$

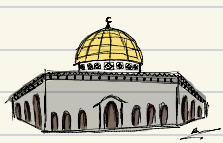
$5V_2 + 2V_1 = 50$

$5\times\frac{9V_1}{8\cdot40} + 2V_1 = 50$

$9V_1 + 16V_1 = 400$

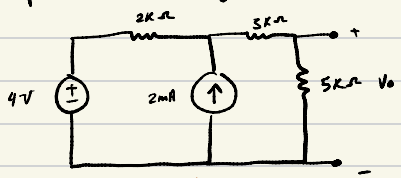
$\boxed{V_1 = 16V}$

$V_1 = V_o = 16V$

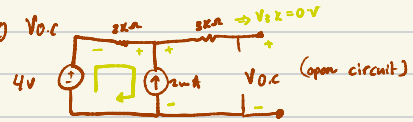


Thevenin and Norton Equivalents

Example: Find v_o using thevenin's theorem



Finding $V_{o.c}$

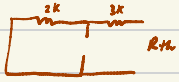


$V_{th} = V_{OC}$

$-4 - (2 \times 2) + V_{o.c} = 0$

$V_{o.c} = 8V = V_{th}$

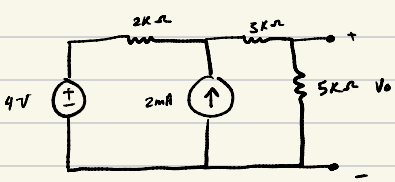
Finding R_{eq}



$R_{th} = 2 + 3 = 5K\Omega$

$V_o = \frac{5}{5+5} \times 8 = 4V$

Example: Find v_o using Norton's theorem



$I_2 = I_{sc} = I_N$

$I_2 - I_1 = 2$

$-4 + 2I_1 + 3I_2 = 0$

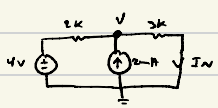
$2I_1 + 3I_2 = 4$

$-2I_1 + 2I_2 = 4$

$I_2 = 1.6\ mA$

$I_1 = -0.4\ mA$

or

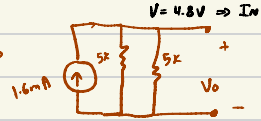


$\frac{V-4}{2} - 2 + \frac{V}{3} = 0$

$3V - 12 - 12 + 2V = 0$

$V = 4.8V \Rightarrow I_N = \frac{4.8}{3} = 1.6\ mA$

$\Rightarrow$

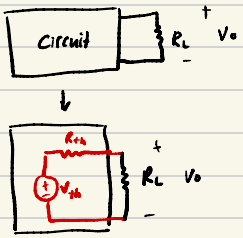


$V_o = 1.6 \times \frac{5 \times 5}{10}$

$= 4V$

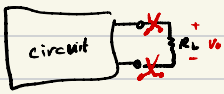
or $V_o = 0.8 \times 5 = 4V$

Lecture Notes:



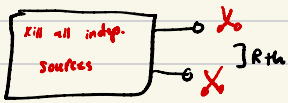
how to find V_{th} & R_{th} ?

$V_{th} = V_{o.c}$



② $R_{th} = ??$

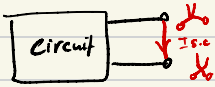
Case I No dep. source



R_{th} → series / parallel

Case II with dep. sources?

→ method A

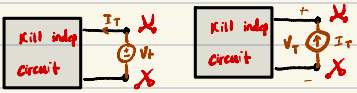


$$R_{th} = \frac{V_{o.c}}{I_{sc}}$$

→ method B : Test source Method

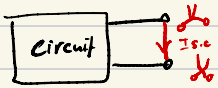
$$R_{th} = \frac{V_{Test}}{I_{test}}$$

all indep sources set to zero (dep ✗ on)

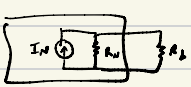


Norton eq. circuit

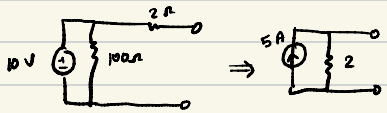
$I_{s.c} = I_N$



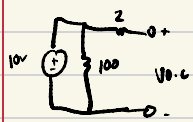
$R_N = R_{Th}$



Note: source transformation:

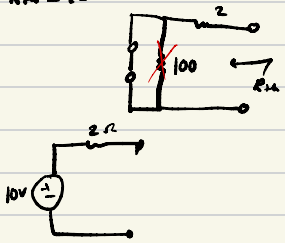


$V_{th} = V_{o.c}$

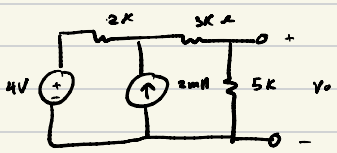


$V_{o.c} = V_{th} = 10$ Volts

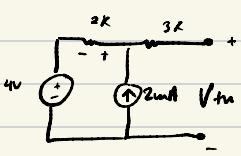
$R_{th} = 2\Omega$



Example:

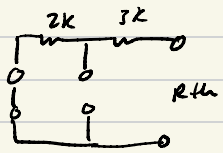


Sol:-

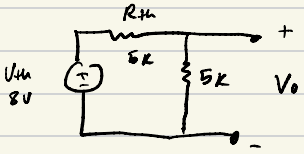


$V_{th} = 0 + 2 \times 2 + 4 = 8$ Volt

$R_{th} = ??$

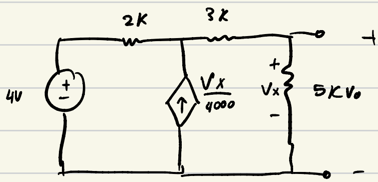


$R_{th} = 2 + 3 = 5k\Omega$

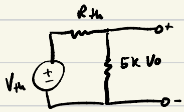


$V_o = \frac{5}{10} \times 8 = 4$ Volt

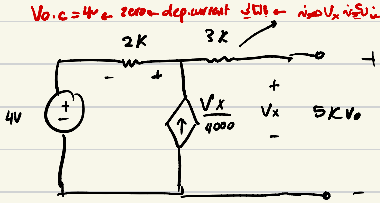
Example:



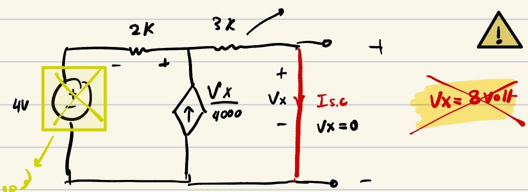
find V_o using thev eq. circuit



$V_{o.c} = 4v$ ← zero dep.current ← بالتالي ← صفر V_x علشان يتحقق



method A:



لوحده ال load بيظل dep source لحاله إلا قيمته صفر

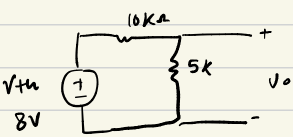
$$I_{s.c} = \frac{4}{2+3} = 0.8 \text{ mA}$$

$$Vx = \frac{\cancel{2}K\, Vx}{_2\cancel{4000}} + 4$$

$$\boxed{Vx = 8 \text{volt}}$$

$V_{th} = Vx = 8V$

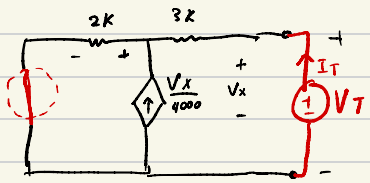
Note: ← القيم الي بيطلعها لا تكون open بتختلف عن * لا استخدمها *



$$V_o = \frac{5}{15} \times 8 = \frac{8}{3} V$$

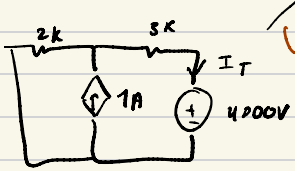
method b:

$$R_{th} = \frac{V_T}{I_T}\Big|_{\text{all indep. sources set to zero}}$$

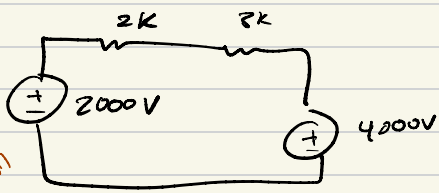


Let $V_T = 4000\ V$

$V_X = V_T = 4000V$



(source transformation)



$$I_T = \frac{4000 - 2000}{5K}$$

$$= 0.4\ A$$

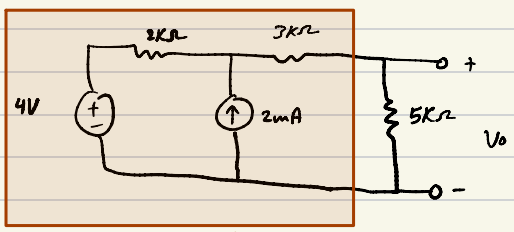
$$R_{th} = \frac{V_T}{I_T} = \frac{4000 \times 5}{2}$$

$$= 10\ K\Omega$$

Ch 4, Thevenin and Norton Equivalents

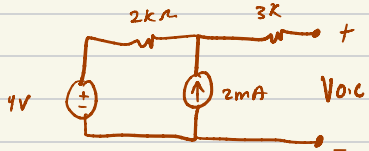
$V_{th} = I_N \times R_N$
$I_N = \frac{V_{th}}{R_{th}}$
$\Rightarrow R_{th} = R_N$ $\Rightarrow$ they're source transformation to each other.

Example: Find v_o using thevenin's theorem:



$V_o = \frac{V_{th} \times 5k}{5k + R_{th}}$

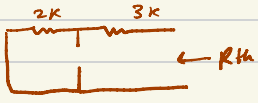
① find $V_{o.c} \Rightarrow R_{load} \Rightarrow$ open circuit $\Rightarrow V_{th}$



$V_{o.c} = 2 \times 2 + 4$

$V_{o.c} = 8V = V_{th}$

② to find R_{th} $\Rightarrow$ all sources are independent $\Rightarrow$ remove all independent sources



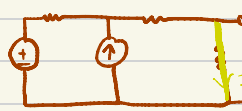
$R_{th} = 3 + 2 = 5k\Omega$

current source $\Rightarrow$ open

Voltage source $\Rightarrow$ short circuit

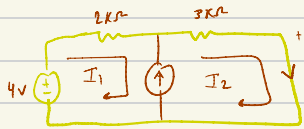
$V_o = \frac{8 \times 5}{5+5} = 4\,V$

Using Norton Techniques:



replace the load with short circuit

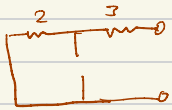
$\Rightarrow (V_o = 0)$ short circuit



IN (focus on direction)

$I_2 - I_1 = 2mA$

$\Rightarrow R_N \Rightarrow$



Super mesh $\Rightarrow$ $-4 + 2KI_1 + 3KI_2 = 0$

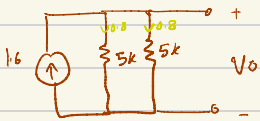
$-4 + 2KI_1 + 3K[2m + I_1] = 0$

$I_1 = -0.4mA$

$I_2 = 2 + 0.4 = 1.6\ mA$

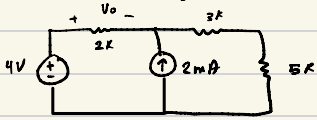
$I_N = I_{s.c} = 1.6\ mA$

$R_N = 3 + 2 = 5K\Omega$

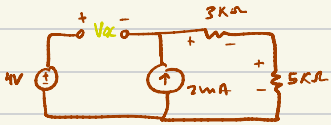


$V_o = 5 \times 1.6 = 4V$

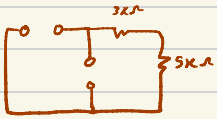
Example: Find Vo using thevenin's theorem:



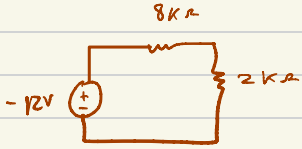
To find $V_{th} \Rightarrow V_{oc}$



$V_{oc} = 4 - 2(8) = -12V$

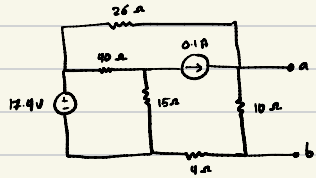


$\Rightarrow R_{th} = 3 + 5 = 8K\Omega$

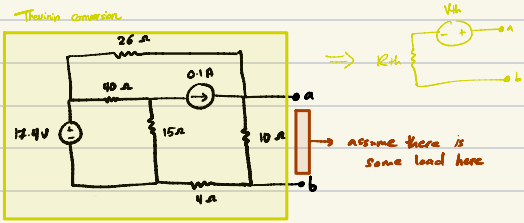


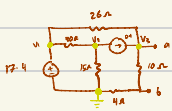
$V_o = \frac{-12 \times 2}{10} = -2.4V$

*** Example: for the circuit shown, find thevenin equivalent circuit with respect to the terminals a & b ?



Thevinin conversion





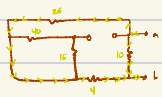
$V_{oc} = V_{ab} = V_{10\Omega} = \frac{1}{2} \times 10 = 5V$

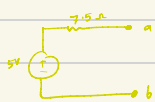
$V_1 = 17.4\ V$

$\frac{V_1 - 17.4}{40} + \frac{V_2}{15} + 0.1 = 0 \Rightarrow$ extra

$\frac{V_2 - 17.4}{26} - 0.1 + \frac{V_2}{14} = 0 \Rightarrow V_2 = 7V$

$I_{10\Omega} = \frac{7}{10+4} = \frac{1}{2} A$





$26 + 4 = 30$

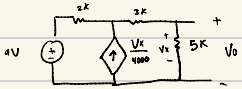
$R_{th} = \frac{15 \times 30}{45} = 7.5 \Omega$

⟹ Voltage for open circuit has a value, but the current is zero

⟹ Current for short circuit has a value but the voltage equals zero

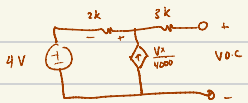
Me

Example: Find Vo using thevenin's theorem:



Two ways to calculate Rth:

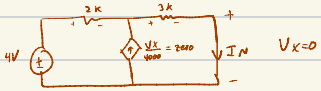
way 1 $R_{th} = \frac{V_{th}}{I_N = I_{sc}}$ (invalid when $V_{th}=0$ or $I_N = 0, \infty$)



$V_x = V_{oc}$

$V_{oc} = 0 + \frac{V_{oc}}{2} \times 2 \times 10^3 \times \frac{V_x}{4000} + 4$

$\frac{1}{2} V_{oc} = 4 \Rightarrow \boxed{V_{oc} = 8\ V}$



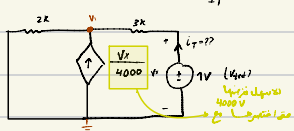
$-4 + I_N(5K) = 0$

$I_N = 0.8\ mA$

$R_{th} = \frac{V_{oc}}{I_N} = \frac{8}{0.8} = 10\ K\Omega$

Way 2 replace Rload with current source or voltage source & Kill every independent source

we get $R_{th} = \frac{V_T}{I_T}$



$\frac{V_1}{2K} - \frac{1}{4000} + \frac{V_1 - 1}{3K} = 0$

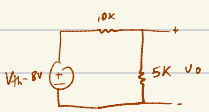
$\left(\frac{V_1}{2} + \frac{V_1 - 1}{3} = \frac{1}{4}\right) \times 12$

$6V_1 + 4V_1 - 4 = 3$

$V_1 = 0.7$

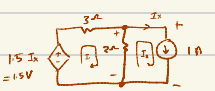
$i_T = -\left(\frac{0.7 - 1}{3K}\right) = \frac{3}{10} = 0.1\ mA$

$R_{th} = \frac{V_T}{I_T} = \frac{1}{0.1m} = 10K\Omega$



$V_o = \frac{8 \times 5}{15} = \frac{8}{3}\ V$

Example: Find Ix using thevenin's theorem:



$I_1 = 1A$

$I_1 \Rightarrow -1.5 + 5I_1 - 2 = 0$

$I_1 = 0.7A$

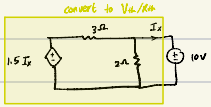
$I/_{2\Omega} = 1 - 0.7 = 0.3\ A$ ohm

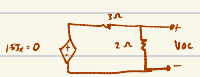
$V_T - 0.3 \times 2 = 0$

$\boxed{V_T = 0.6\ V}$

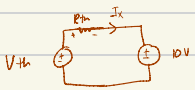
$R_{th} = \frac{V_T}{I_T} = \frac{0.6}{1} = 0.6\ \Omega$

convert to Vs/Rs





$V_{th} = 0 \Rightarrow R_{th} = \frac{V_{th}}{I_N} = 0$ (invalid)



$-V_{th} + I_x R_{th} + 10 = 0$

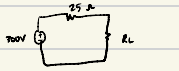
$I_x = \frac{-10}{\frac{6}{10}}$

$= -16.67\ A$

Maximum Power Transfer :-

Example : what is the value of R_L that will absorb the maximum power.

R_L	P(W)
15	843.75
20	888.88
25	900
30	892.56
35	875



for $R_L = R_{Th} = 25\ \Omega$

P will be the maximum

$$i = \frac{V_{Th}}{R_L + R_{Th}}$$

$$P = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L$$

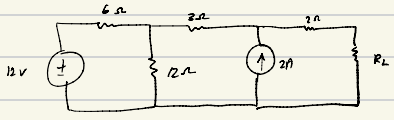
$$\frac{dP}{dR_L} = \frac{(V_{Th})^2\left[(R_{Th}+R_L)^2 - R_L\,2(R_{Th}+R_L)\right]}{(R_{Th}+R_L)^4}$$

$$(R_{Th}+R_L)\left[R_{Th}+R_L-2R_L\right]=0$$

$$R_{Th} = R_L$$

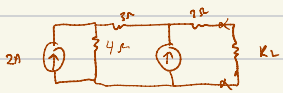
$$\Rightarrow P = \frac{V_{Th}^2}{4R_L^2} \times R_L = \frac{V_{Th}^2}{4R_L}$$

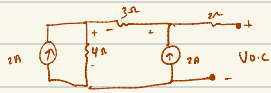
Example :- Find the value of R_L for maximum power transfer in the circuit shown - find the maximum power.



$12 = 6 \times I \Rightarrow I = 2$

$R' = \frac{12 \times 6}{18} = 4\ \Omega$





$V_{o.c} = V_{Th} = 0 + 2\times3 + 4\times4 = 22V$

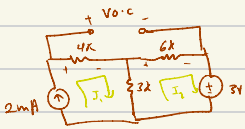
$R_{Th} = 2+3+4 = 9\ \Omega$

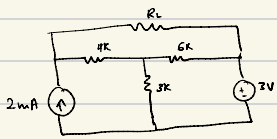
$R_L = R_{Th} = 9\ \Omega$

$P_{max} = \frac{22\times22}{4\times9} = \frac{121}{9}$

Example: Find the value of R_L for maximum power transfer in the circuit shown:

Find the maximum Power.





$I_1 = 2mA$

$9KI_2 - 3KI_1 + 3 = 0$

$3KI_2 - KI_1 + 1 = 0$

$3KI_2 = 2 - 1 = 1$

$I_2 = \frac{1}{3} mA$

$V_{oc} = 4X2 + 6X\frac{1}{3} = 10$

$R_{th} \Rightarrow \frac{6X3}{6+3} = 2\,K\Omega$

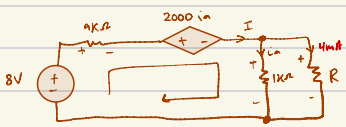
$R_{th} = 2 + 4 = 6\,K\Omega$

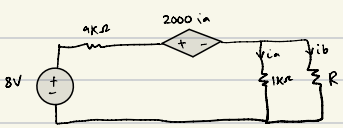
$R_L = R_{th} = 6K\Omega$

$P_{max} = \frac{(V_{th})^2}{4R_L} = \frac{(10)^2}{4X6K} = \frac{25}{6}$ m Watt

Example: For the circuit shown below, the current i_a has units of Amps. Determine:-

1. The resistance R that will cause the current is in the circuit to be 4mA.
2. The resistance R that will cause maximum power Transfer to it from the circuit





$I = i_a + 4m$

$-8 + 4KI + 2000i_a + i_a X 1K = 0$

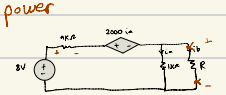
$4K[i_a + 4m] + 2K i_a + K i_a = 8$

$i_a = \frac{-28}{12} mA$

$R_m X R = 1X\cancel{K} X \frac{-28}{12}$

$R = -5833\,\Omega$ (negative resistor has a particular meaning ⇒ by increasing the voltage of the current decreases)

(2) $\Rightarrow R_{load} = R_{th} \Rightarrow$ to have maximum power



$R_{th} = \frac{V_{th}}{I_N} = \frac{\frac{X}{X}}{\frac{X}{X}} = 750\,\Omega$

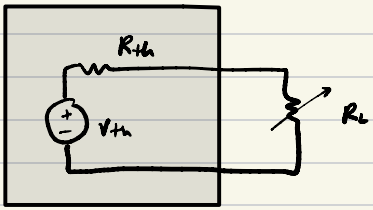
$-8 + 4I + 4K i_a + 2000 i_a = 0$

$i_a = \frac{8}{12} mA$

$V_{o.c} = V_{th} = \frac{8}{12} = \frac{2}{3} V$

$I_N \Rightarrow$

Maximum Power Transfer (revision)



↳ $V_L = \frac{R_L}{R_L + R_{th}} \times V_{th}$

$P_L = \frac{V_L^2}{R_L} = \frac{R_L}{(R_L + R_{th})^2} \times V_{th}^2$

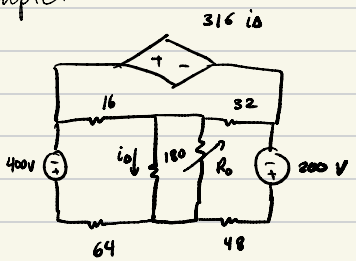
$\frac{\partial P_L}{\partial R_L} = \frac{(R_L + R_{th})^2 - 2R_L(R_L + R_{th})}{(R_L + R_{th})^4} V_{th}^2$

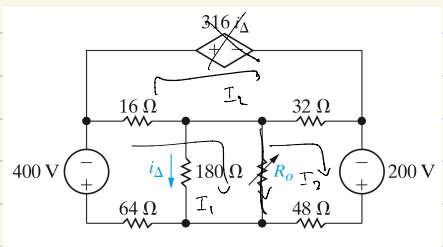
$\frac{\partial P_L}{\partial R_L} = 0 \Rightarrow (R_L + R_{th})^2 = 2R_L(R_L + R_{th})$

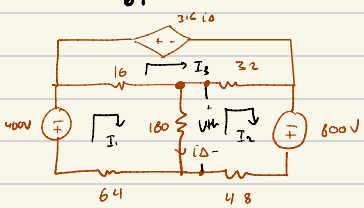
$\boxed{R_L = R_{th}}$

$\therefore$ max Power transfer $P_{max} = \frac{V_{th}^2}{4R_{th}}$

Example:







$i_\Delta = I_1 - I_2$

$316\, i_\Delta = 316 I_1 - 316 I_2$

$i_\Delta = 0$

$+400 + 80 I_1 - 16 I_2 = 0$

$80 I_3 - 200 - 32 I_2 = 0$

$48 I_2 - 16 I_1 - 32 I_3 = 0$

$I_1 = -5A$, $i_2 = 0$, $i_3 = 2.5A$

$I_N = I_1 - I_3 = -5 - 2.5$

$= -7.5A$

$R_{th} = \frac{-360}{-7.5} = 48\Omega$

① $I_1 \Rightarrow 400 + 260 I_1 - 180 I_2 - 16 I_3 = 0$

$260 I_1 - 180 I_2 - 16 I_3 = -400$ ---①

② $I_2 \Rightarrow -800 + 260 I_2 - 180 I_1 - 32 I_3 = 0$

$-180 I_1 + 260 I_2 - 32 I_3 = 800$ ---②

③ $I_3 \Rightarrow 316 I_1 - 316 I_2 + 48 I_2 - 32 I_2 - 16 I_1 = 0$

$300 I_1 - 348 I_2 + 48 I_3 = 0$ ---③

$I_1 = 3A$ / $I_2 = 5$ / $I_3 = 17.5A$

$i_\Delta = 3 - 5 = -2A$

$V_{th} = 180 \times -2 = -360V$

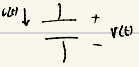
Ch 6 : Capacitor and Inductor Models

* Capacitor: (electric field)

$$C = \frac{q}{V} \Rightarrow q = CV \qquad \left(C = \frac{\epsilon A}{d} \right)$$

$$\frac{dq}{dt} = C \frac{dv}{dt}$$

$$i(t) = C \frac{dv}{dt} \Rightarrow$$

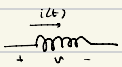


* Inductor: (magnetic field)

$$\text{induced voltage} = N \frac{d\Phi}{dt}$$

$$v(t) = N \frac{d}{dt}(NP i)$$

$$v(t) = N^2 P \frac{di}{dt}$$



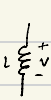
$$v(t) = L \frac{di}{dt}$$

In Summary

Capacitor :

$$i = C \frac{dv}{dt}$$

inductor



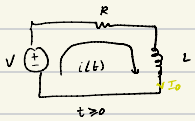
$$v(t) = L \frac{di}{dt}$$

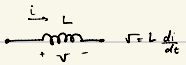
resistor

$$V = IR$$

Ch7 : First order RL + RC circuit

① first order RL circuit:





I_0: initial inductive current

i is continuous
(there is no instantaneous change for i(t))

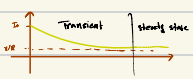
$i(t_i^-) = i(t_i^+) = i(t_i)$

KVL: $\Rightarrow \quad -V + Ri + L\frac{di}{dt} = 0$

$Ri + L\frac{di}{dt} = V$

$$\Rightarrow \boxed{i(t) = \frac{V}{R} + \left(I_0 - \frac{V}{R}\right) e^{-t/\tau} \quad , t \geq 0}$$

$\tau = \frac{L}{R}$ ⇒ time constant



$i(0) = I_0$ (initial) / $i(\infty) = \frac{V}{R}$ (final)

(when $t \to \infty$ ⇒ inductor acts like a short circuit)

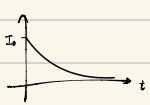
Note : when $t \to \infty$, $i = i_f$ = constant $\Rightarrow V_f = L\frac{di}{dt} = 0$

⇒ L appears as a short circuit.

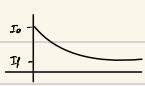
* Notes:

inductor $i(t) = i_f + (i_i - i_f) e^{-t/\tau}$, $t \geq 0$

① if $V = 0 \Rightarrow i_f = 0 \Rightarrow i = I_0 e^{-t/\tau}$ (Natural Response)

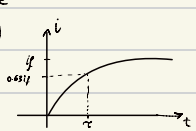


② if $V \neq 0 \Rightarrow i_f \neq 0 \Rightarrow i = i_f + (i_0 - i_f) e^{-t/\tau}$ (step Response)



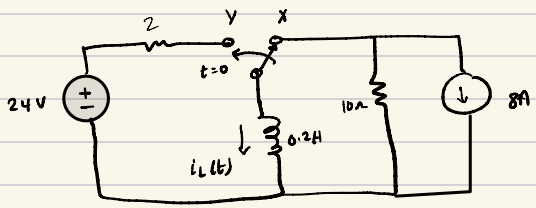
Note: assume that $i_i = I_0 = 0$

$$i(t) = i_f + (i_i - i_f) e^{-t/\tau}$$
$$= i_f \left(1 - e^{-t/\tau}\right)$$

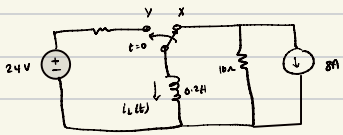


$i(\tau) = 0.63\, i_f$

Example



The switch is set in position X for a long time (reached the steady state), At $t=0$, the switch moves to position Y. find $i(t)$, $t \geq 0$

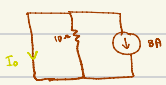


$i_L(0^-) = i_L(0^+) = I_o$

switch at position x — to switch at position y

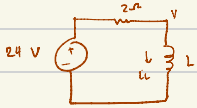
at $t=0^-$ ⇒ L is a short circuit

$t = 0^-$



$I_o = -8A$

for $t \geq 0$



$i_L(t) = i_f + (I_o - i_f)\, e^{-t/\tau}$

$i_i = I_o = -8A$

$t \to \infty$ (L is short circuit)

$i_f = \frac{24}{2} = 12A$

$\tau = L/R \quad t \geq 0$

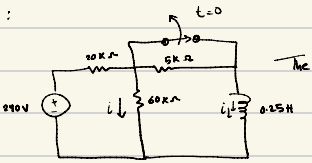
$R = 2\Omega$

$\tau = \frac{0.2}{2} = 0.1$ sec

$i(t) = 12 + (-8 - 12)\, e^{-10t}, \quad t \geq 0$

$i(t) = 12 - 20\, e^{-10t}$

Example :



The switch has been closed for a long time. The switch opens at t=0. find

$i_L(t)$ $t \geq 0$

$i(t)$ $t \geq 0^+$

at $t=0^- \Rightarrow$ L is short circuit

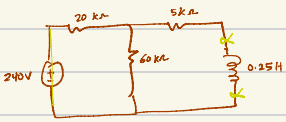
$I_o = \frac{240}{20K} = 12mA$

at $t \to \infty \Rightarrow$ L is short circuit

$\frac{V-240}{20K} + \frac{V}{60K} + \frac{V}{5K} = 0 \Rightarrow V = 45V$

$I_f = \frac{V}{5K} = 9mA$

$t \geq 0$ (to calculate R $\Rightarrow$ the same steps of R.w)



$\frac{20 \times 60}{20+60} = 15\ K\Omega$

$R_{eq} = 15 + 5 = 20K\Omega$

$\tau = \frac{L}{R} = \frac{0.25}{20K} = 12.5\ \mu s$

$i_L(t) = 9 \times 10^{-3} + \left[8 \times 10^{-3}\right] e^{-8\times10^4/t}$ A

$i_L(t) = 9 + 3e^{-8\times10^4 t}$ mA

to find $i(t) \Rightarrow t > 0^+$ (since it's not cont for resistors)

$i_L(t) = 9 + 3e^{-8\times10^4 t}$ mA

$L\frac{di(t)}{dt} = -60000\, e^{-8\times10^4}$ mV

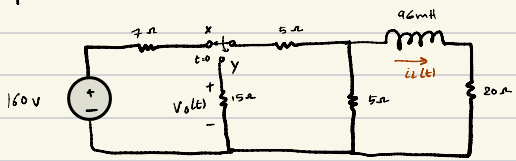
$V = 5 i_L(t) + V_L(t)$

$= 5 i_L(t) + L\frac{di}{dt}$

$i(t) = \frac{V}{R} = 45 + 15e^{-8\times10^4} - 6\times10^4\, e^{-8\times10^4}$

$i(t) = \frac{45 - 45e^{-8000t}}{60}$ mA, $t \geq 0^+$

Example:

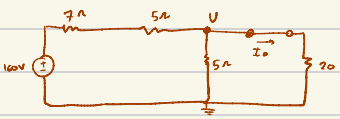


The switch has been in position x for a long time. At t=0, the switch moves to position y, find $V_o(t)$, $t \geq 0^+$

$i_L(t) = I_f + (I_i - I_f)e^{-t/\tau}$ $\Rightarrow$ $I_f = 0$ since there is no voltage source in this case

$i_L(t) = I_o e^{-t/\tau}$

Solution $\rightarrow$ $t = 0^-$ $\Rightarrow$ L is a short circuit



$$\frac{V-160}{12} + \frac{V}{5} + \frac{V}{20} = 0$$

$$100V - 16000 + 240V + 60V = 0$$

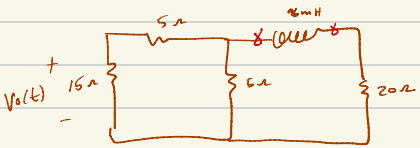
$$V = 40V$$

$$I_o = \frac{40}{20} = 2A$$

$$\tau = \frac{L}{R} = \frac{96m}{24} = 4ms$$

$$i_L(t) = 2e^{-250t}$$

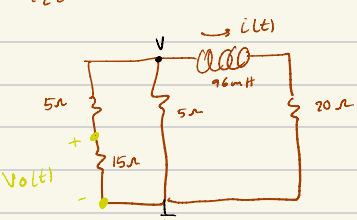
at $t > 0$



$5 + 15 = 20$

$\frac{20 \times 5}{25} = 4$

$20 + 4 = 24\ \Omega$



$$\frac{V}{20} + \frac{V}{5} + i_L = 0$$

$$5V = -20\,i_L$$

$$V = -4\,i_L$$

$$V_o(t) = i(t) \times R$$

$$= \frac{V}{20} \times 15 = \frac{-4 \times 2e^{-250t} \times 15}{20}$$

$$= -6e^{-250t}\ V$$

Summary for inductor cases.

① $i(t) = I_f + (I_0 - I_f)\, e^{-t/\tau}$

where $I_f \Rightarrow t \to \infty$ (L is sc) $\quad R_{th} \Rightarrow \tau = \frac{L}{R_{th}}$

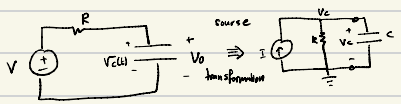
$I_0 \Rightarrow t = 0^-$ (L is sc)

② $i(t) = \frac{V_{th}}{R_{th}} + \left[I_0 - \frac{V_{th}}{R_{th}} \right] e^{-t/\tau}$

$I_0 \Rightarrow i = 0^- \to$ (L : s.c)

Thevenin equivalent circuit

② first order RC circuit :-



V_0 = initial capacitive voltage

$-I + \frac{V_c}{R} + C\frac{dV_c}{dt} = 0$

$I = \frac{V_c}{R} + C\frac{dV_c}{dt}$

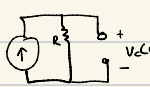
$\Rightarrow V_c(t) = RI + [V_0 - RI]\, e^{-t/\tau}$

$\tau = RC$

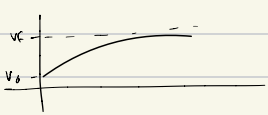
$$V_c(t) = RI + [V_0 - RI]e^{-t/\tau}$$

at $t = 0 \Rightarrow V_c$ = constant $= V_0$

$\Rightarrow i_c = C\frac{dv_c}{dt} = 0 \Rightarrow$ C appears as an open circuit



$V_c(t) = V_f + [V_0 - V_f]e^{-t/\tau}, \quad t \geq 0$



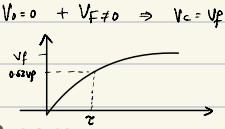
* If $I = 0 \Rightarrow V_f = IR = 0 \Rightarrow V_c = V_0 e^{-t/\tau}$, Natural Response

If $I \neq 0 \Rightarrow V_f \neq 0 \Rightarrow V_c = V_f + (V_0 - V_f)e^{-t/\tau}$, Step Response

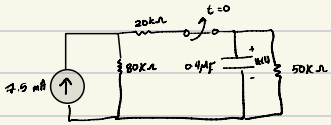
$V_c(0) = V_c(0^-) = V_c(0^+)$

$i_c = c\frac{dv_c}{dt} \Rightarrow$ continuous

* $V_0 = 0 + V_f \neq 0 \Rightarrow V_c = V_f(1 - e^{-t/\tau})$ $\quad V_c(\tau) = 0.63\, V_f$

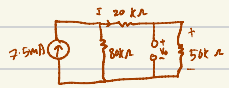


Example: calculate $v(t)$, $t \geq 0$, The switch has been closed for a long time, the switch opens at $t=0$, find $V_c(t)$



$V_c = V_f + (V_i - V_f)e^{-t/\gamma}$

for $t=0^-$ ⇒ C ⇒ open circuit



$I = \frac{80}{70+80} \times 7.5 = 4\ mA$

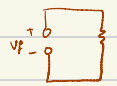
$V_o = 4 \times 50 = 200$

for $t \geq 0$

$\tau = RC = 0.4M \times 50K = 20ms$

$V_c = 200\, e^{-50t}$

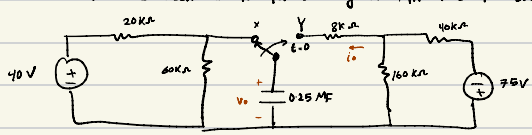
for $t \to \infty$ ⇒ (C ⇒ open circuit)



⇒ $V_f = IR = 0 \cdot V$

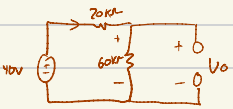
(Natural Response)

Example: The switch has been in X for a long time, At t=0 the switch moves to position Y find $V_o(t), t \geq 0$ & $i_o(t), t \geq 0^+$



$V_c(t) = V_f + [V_o - V_f]e^{-t/\tau}$, $\tau = RC$

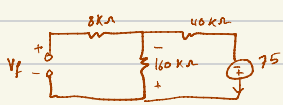
for $t=0^-$ ⇒ C is O.C



$I = \frac{40}{80K} = 0.5mA$

$V_o = 0.5 \times 60 = 30V$

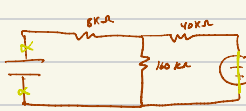
for $t \to \infty$ ⇒ C is O.C



$I = \frac{75}{200K} = 0.375mA$

$V_f = -160 \times 0.375 = -60V$

for $t > 0$ ⇒



$R_{th} = 8 + \frac{40 \times 160}{40+160}$

$= 40\ K\Omega$

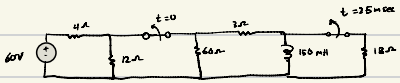
$\tau = RC = 40K \times 0.25\ M = 10 \times 10^{-3}\ s$

$V_c(t) = -60 + 90\, e^{-100t}$

$i_o(t) = C \times \frac{dv}{dt} = 90 \times -100 \times 0.25 \times 10^{-6} \times e^{-100t}$

$= -2.25 \times e^{-100t}\ mA$

Example: the two switches have been closed for a long time, find $i_L(t)$ for $t \geq 0$



solution

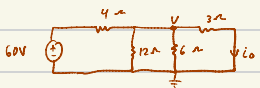
$$i_L = \begin{cases} i_{L1} & 0 \leq t \leq 35 \\ i_{L2} & t \geq 35m \end{cases}$$

i_{L1}, i_{L2} → for both cases ⇒ Natural response

$i_{L1} = I_{o1} e^{-t/\tau_1}$

$i_{L2} = I_{o2} e^{-(t-0.035)/\tau_2}$

case 1 ⇒ $t = 0^-$ (L is SC)



$$\frac{V-60}{4} + \frac{V}{12} + \frac{V}{6} + \frac{V}{3} = 0$$

$V = 18$ V

$i_o = \frac{18}{3} = 6$ A

⇒ to find R_{th} ⇒ $t > 0$

$(0 \leq t \leq 35)$

$\Rightarrow 3+6 = 9$

$R_{th} = \frac{9 \times 18}{18+9} = 6\Omega$

$\tau_1 = \frac{L}{R} = \frac{150}{6} = 25$ ms

$i_{L_1}(t) = 6e^{-40t}$

When $t \geq 35$ms

$R_{th} = 3+6 = 9\Omega$

$\tau = \frac{L}{R} = \frac{150}{9} = 16.67$ ms

case 2

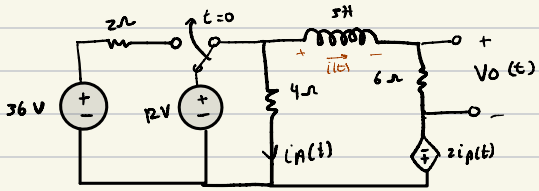
$I_{o2} = i_{L_2}(35m) = i_{L_1}(35m)$ since it's cont

$= 6e^{-40(35\times10^{-3})} = 1.48$ A

$i_{L_2}(t) = 1.48\ e^{-60(t-35m)}$ A

$$i_L = \begin{cases} 60\ e^{-40t} \text{ A} & 0 \leq t \leq 35m \\ 1.48\ e^{-60(t-35m)} & t \geq 35m \end{cases}$$

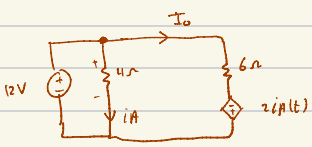
Example (with dependent sources)



$i(t) = i_f + (i_0 - i_f)\, e^{-t/\tau}$

$\tau = \frac{L}{R_{th}}$

solution at $t=0^-$ $\Rightarrow$ L is s.c

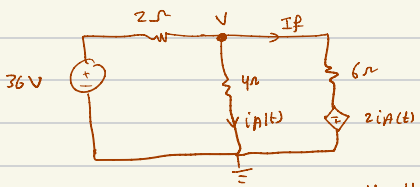


$12 = 4\, i_A \Rightarrow \boxed{i_A = 3A}$

$6I_0 - 2\times 3 - 4\times 3 = 0$

$\boxed{I_0 = 3A}$

$t \to \infty$ $\Rightarrow$ L is s.c



$V = 4\, i_A$

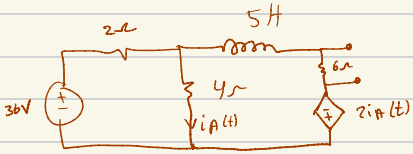
$\frac{V-36}{2} + \frac{V}{4} + \frac{V+2i_A}{6} = 0$

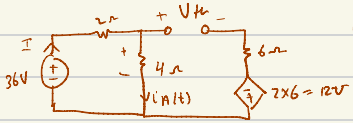
$\frac{4i_A}{2} - \frac{36}{2} + \frac{4i_A}{4} + \frac{6i_A}{6} = 0$

$\boxed{i_A = 4.5\ A}$

$\boxed{I_f = 4.5\ A}$

at $t>0$



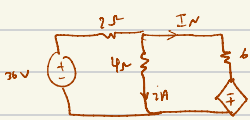


$R_{th} = \frac{V_{th}}{I_N} = \frac{36}{4.5} = 8\ \Omega$

$I = \frac{36}{6} = 6A$

$i_A = 6A$

$V_{th} = 4\times 6 + 12 = 36V$



$I_N = I_f = 4.5$

$i(t) = i_f + (I_0 - i_f)\, e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{3}{8}\ s$

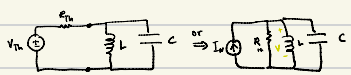
$i(t) = 4.5 - 1.5\, e^{-\frac{8}{3}t}\ A$

$V_o(t) = 6 \times i(t)$

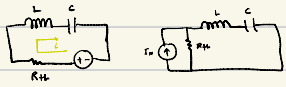
$= 27 - 9e^{-\frac{8}{3}t}\ V$

Chapter 8: second order RLC circuit

① parallel RLC circuit



② Series RLC circuit



① Parallel RLC circuit

I_o ⇒ initial inductive current

V_o ⇒ initial capacitive voltage

Norton equivalent circuit

KCL at node V

$$-I + \frac{V}{R} + \frac{1}{L}\int V dt + C\frac{dV}{dt} = 0$$

$$0 + \frac{1}{R}\frac{dV}{dt} + \frac{1}{L}V + C\frac{d^2V}{dt^2} = 0$$

$$\frac{d^2V}{dt^2} + \frac{1}{RC}\frac{dV}{dt} + \frac{1}{LC}V = 0 \quad \text{---> } 2^{nd} \text{ order differential equation}$$

$$V'' + \frac{1}{RC}V' + \frac{1}{LC}V = 0$$

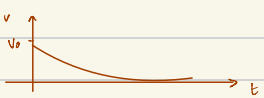
$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \Rightarrow \text{characteristic equation}$$

Roots of the equation

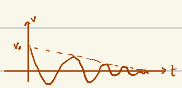
$$s_{1/2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

$$\alpha = \frac{1}{2RC} \text{ (Neper frequency)}, \quad \omega_o = \frac{1}{\sqrt{LC}} \text{ (resonant frequency)}$$

a) $\alpha > \omega_o \Rightarrow$ s_1 & s_2 diff, real roots ⇒ (over-damped Response)



b) $\alpha < \omega_o \Rightarrow$ s_1 & s_2 complex roots ⇒ (under-damped Response)



c) $\alpha = \omega_o \Rightarrow$ s_1 & s_2 real and equal roots ⇒ (Critically-damped Response)

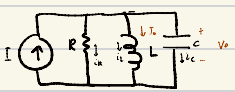
⇒ a Over-damped Response ($\alpha > w$)

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\alpha = \frac{1}{2RC} \qquad w = \frac{1}{\sqrt{LC}}$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - w^2}$$

$t \geq 0$



⇒ to find A_1 & A_2 ⇒ $v(0^+)$ & $v'(0^+)$

$$v(0^+) = v(0^-) = V_0$$

$$i_c(0^+) = C\frac{dv(0^+)}{dt} = Cv'(0^+) \qquad I = i_R + i_L + i_C \quad \text{at } t = 0^+$$

$$v'(0^+) = \frac{i_c(0^+)}{C} = \frac{I - i_R(0^+) - i_L(0^+)}{C}$$

$$v'(0^+) = \frac{I - \frac{V_0}{R} - I_0}{C}$$

$$v'(0^+) = \frac{1}{C}\left[I - \frac{V_0}{R} - I_0\right]$$

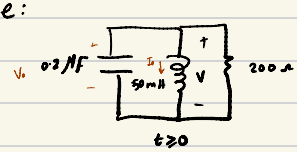
⇒ $v = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$$v(0^+) = V_0 = A_1 + A_2 \quad \text{---(2)}$$

$$v'(0^+) = \frac{1}{C}\left[I - I_0 - \frac{V_0}{R}\right] = s_1 A_1 + s_2 A_2 \quad \text{---(3)}$$

solve to find A_1 & A_2

Example:



Find $V \Rightarrow t \geq 0$

$V_0 = 12\,V$ / $I_0 = 30\,mA$

Solution:

$$\alpha_0 = \frac{1}{2RC} = \frac{1}{200 \times 2 \times 0.2\,M} = 1.25 \times 10^6 \text{ rad/s}$$

$$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \times 10^{-3} \times 0.2 \times 10^{-6}}} = 1 \times 10^6 \text{ rad/sec}$$

$\alpha > w_0$ ⇒ over-damped ⇒ two diff & real roots ⇒ $v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - w^2}$$

$s_1 = -5000$ rad/sec

$s_2 = -20\,000$ rad/sec

$$v(0^+) = v(0^-) = V_0$$

$$12 = A_1 + A_2 \quad \text{---(1)}$$

$$v'(0^+) = \frac{1}{C}\left[\overbrace{I}^{0\ (\text{no current source})} - \frac{V_0}{R} - I_0\right]$$

$$v'(0^+) = \frac{1}{0.2 \times 10^{-6}}\left[\frac{-12}{200} - 0.03\right] = -450 \times 10^3 \text{ V/sec}$$

$$-450\,000 = -5000 A_1 - 20\,000 A_2 \quad \text{---(2)}$$

$$A_1 = -14 \;/\; A_2 = 26$$

$$v(t) = -14e^{-5000t} + 26e^{-20000t} \text{ V}$$

⇒ B Under-damped Response ($\alpha < \omega_0$)

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

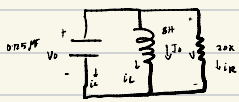
$$V = e^{-\alpha t}\left[B_1 \cos(\omega d\, t) + B_2 \sin(\omega d\, t)\right]$$

$$\omega d = \sqrt{\omega_0^2 - \alpha^2}$$

$$V(0^+) = B_1 = V_0$$

$$V'(0^+) = \omega d B_2 - \alpha B_1 = \frac{1}{C}\left[I - I_0 - \frac{V_0}{R}\right]$$ Solve for B_1 & B_2

Example:



$V_0 = 0$ / $I_0 = -12.25mA$ find V, iR, iL & iC

Solution: $\alpha = \frac{1}{2RC}$ $= 200$ rad/s $\qquad \omega = \frac{1}{\sqrt{LC}}$ $= 1000$ rad/s

$\omega > \alpha \Rightarrow$ then under-damped

general solution $\Rightarrow e^{-\alpha t}\left[B_1 \cos(\omega d\, t) + B_2 \sin(\omega d\, t)\right]$

$\omega d = \sqrt{\omega_0^2 - \alpha^2} = 980$ rad/s

$V_0 = B_1 = 0$

$V(t) = e^{-\alpha t} B_2 \sin(\omega d\, t)$

$V'(0^+) = \omega d B_2 - \cancel{B_1 \alpha}^{0}$

$V'(0^+) = 980 B_2 \Rightarrow V'(t) = \frac{1}{C}\left[\cancel{I}^{0} - I_0 - \cancel{\frac{V_0}{R}}^{0}\right]$

$\cancel{98000} = \cancel{980} B_2 \qquad = \frac{12.25m}{0.125\,M} = 98000\,V/sec$

$B_2 = 100$ V

$V(t) = e^{-200t}\left[100 \sin(980t)\right]$ Volt

$iR = \frac{V(t)}{R} = \frac{10V e^{-200t}}{20k} \sin(980t) = 5e^{-200t} \sin(980t)$ mA

$iC(t) = C \times \frac{dv}{dt} = 0.125M \times \left[e^{-200t} \cdot 100\cos(980t) \times 980 + 100\sin(980t) \times -200e^{-200t}\right]$

$= 12.25 e^{-200t} \cos(980t) - 2.5\, e^{-200t} \sin(980t)$ mA

$iL(t) = \frac{1}{L}\int V dt + \boxed{I_0}$ or $\overset{0}{\cancel{I}} = iR + iC + iL$

then $iL = -iR - iC$

$= -5e^{-200t}\sin(980t) - 12.25\, e^{-200t}\cos(980t) + 2.5e^{-200t}\sin(980t)$

$= -2.5e^{-200t}\sin(980t) - 12.25\, e^{-200t}\cos(980t)$ mA

⇒ c Critically - damped response

$\alpha = w_0 = \frac{1}{2RC}$ or $\frac{1}{\sqrt{LC}}$

$V = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

$V(0^+) = D_2$

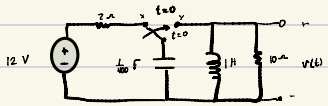
$V'(0^+) = D_1 - \alpha D_2 = \frac{1}{C}\left[I - I_0 - \frac{V_0}{R} \right]$

Summary

$\alpha = \frac{1}{2RC}$ $\quad w_0 = \frac{1}{\sqrt{LC}}$ $\quad V(0^+) = V_0$ $\quad V'(0^+) = \frac{1}{C}\left[I - \frac{V_0}{R} - I_0 \right]$

over-damped	under-damped	critically damped
$\alpha > w_0$	$\alpha < w_0$	$\alpha = w_0$
$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$	$v(t) = e^{-\alpha t}\left[B_1 \cos w_d t + B_2 \sin w_d t \right]$	$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$
$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - w_0^2}$	$w_d = \sqrt{w_0^2 - \alpha^2}$	
$V(0^+) = A_1 + A_2$	$V(0^+) = B_1$	$V(0^+) = D_2$
$V'(0^+) = A_1 s_1 + A_2 s_2 =$	$V'(0^+) = w_d B_2 - \alpha B_1$	$V'(0^+) = D_1 - \alpha D_2$

Example: The switch has been in position X for a long period of time. At $t=0$, the switch is moved to position Y. Find $v(t)$, $t \geq 0$

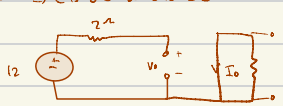


$\alpha = \frac{1}{2RC} = 20$ rad/sec (after switching) $\quad w_0 = \frac{1}{\sqrt{LC}} = 20$ rad/sec

$\alpha = w \Rightarrow$ critical - damped

$v(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

at $t=0^-$ ⇒ C is O.C & L is S.C



$V_0 = 12V \quad / \quad I_0 = 0$

$v(0^-) = v(0^+) = v(0) = \boxed{D_2 = 12V}$

$V(0^+) = D_1 - \alpha D_2$

$-480 = D_1 - 20 \times 12$

$D_1 = -240$

$V'(t) = \frac{1}{C}\left[I_s - I_0 - \frac{V_0}{R} \right]$ ← parallel resistor

$= \frac{480}{10} \times \frac{-12}{10}$

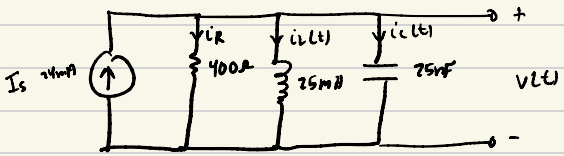
$= -480$ V/sec

$\therefore v(t) = -240 t e^{-20t} + 12 e^{-\alpha t}$

Example: $i_L(0^-)=0$ & $v_c(0^-)=0$

find $v(t)$ for $t>0$

find $i_L(t)$ for $t>0$



solution $\Rightarrow$ $\alpha = \frac{1}{2RC} = 5\times10^4$ rad/sec ; $\omega = \frac{1}{\sqrt{LC}} = 4\times10^4$ rad/sec

$\alpha > \omega \Rightarrow$ real, diff roots

$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s_1 = -5 + \sqrt{25-16} \times 10^4 = -2\times10^4$

$s_2 = -5 - \sqrt{25-16} \times 10^4 = -8\times10^4$

$v(0^-) = v(0^+) = 0 \Rightarrow A_1 + A_2 = 0$

$v'(0^+) = \frac{1}{C}\left[I_s - \frac{v_0}{R} - i_0\right]$

$v'(0^+) = \frac{24\times10^{-3}}{25n} = 96\times10^4$ V/s $\Rightarrow$ $96\times10^4 = -2\times10^4 A_1 - 8\times10^4 A_2$

$48 = -A_1 - 4A_2$

$0 = A_1 + A_2$

$A_2 = -16 \Rightarrow A_1 = 16$

① $v(t) = 16e^{-2\times10^4 t} - 16e^{-8\times10^4 t}$ V

② $I_s = i_R + i_L(t) + i_C(t)$

$i_C(t) = C\frac{dv}{dt}$

$i_C(t) = -8\times10^{-3}e^{-2\times10^4 t} + 32\times10^{-3}e^{-8\times10^4 t}$

$i_R = \frac{v}{R} = 40\times10^{-3}e^{-2\times10^4 t} - 40\times10^{-3}e^{-8\times10^4 t}$

$i_L(t) = (24 - 32e^{-2\times10^4 t} + 8e^{-8\times10^4 t})$ mA

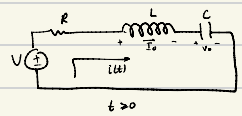
$= (I_s - i_C(t) - i_R(t))$

$i_L(t) = \frac{1}{L}\int v(t)\,dt + I_0$

$= 40\left[-8\times10^{-4}e^{-2\times10^4} + 2\times10^{-4}e^{-8\times10^4}\right]$

$= (8e^{-8\times10^4} - 32e^{-2\times10^4} + 24)$ mA

2 Series RLC circuit:



I_o: initial inductive current
V_o: initial capacitive voltage

KVL in the loop:

$$V = Ri + L\frac{di}{dt} + \left[\frac{1}{C}\int i(t)dt + V_o\right]$$

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i(t) + 0$$

$$i'' + \frac{R}{L}i' + \frac{1}{RC}i = 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{RC}s = 0$$

$$s_{1/2} = -\alpha \pm \sqrt{\alpha^2 - w_o}$$

$$\alpha = \frac{R}{2L}, \quad w_o = \frac{1}{\sqrt{LC}}$$

Case 1 $\Rightarrow$ $\alpha > w_o \Rightarrow$ over-damped

$$\Rightarrow i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

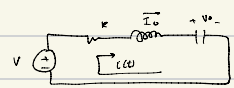
Case 2 $\Rightarrow$ $\alpha < w_o \Rightarrow$ under-damped

$$\Rightarrow i(t) = e^{-\alpha t}\left[B_1 \cos(w_d t) + B_2 \sin(w_d t)\right]$$

Case 3 $\Rightarrow$ $\alpha = w \Rightarrow$ critically damped

$$\Rightarrow i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$$

$\Rightarrow$ calculate $i(0^+)$ and $i'(0^+)$ to get coefficient values.



$$i(0^+) = I_o$$

$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L} = \frac{1}{L}(V_s - V_o - I_o R)$$

$$V = V_R(0^+) + V_L(0^+) + V_c(0^+)$$

$$V = RI_o + V_L(0^+) + V_o$$

$$V_L(0^+) = V - V_o - RI_o$$

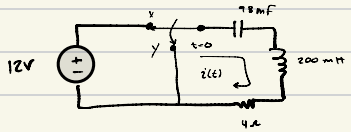
over Damped $\Rightarrow$ $i(0^+) = A_1 + A_2$

$$i'(0^+) = s_1 A_1 + s_2 A_2$$

under damped $\Rightarrow$ $i(0^+) = B_1$ / $i'(0^+) = w_d B_2 - \alpha B_1$

critical damped $\Rightarrow$ $i(0^+) = D_2$ / $i'(0^+) = D_1 - \alpha D_2$

Example: the switch has been in position X for a long time, at t=0 the switch moves to position Y, find i(t), t≥0, ② find voltage on R, C & L



Series connection

$\alpha = \frac{R}{2L}$ = 10 rad/sec $\omega = \frac{1}{\sqrt{LC}}$ = 7.1 rad/sec

$\alpha > \omega \Rightarrow$ over damped

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$

$s_1 = -17$ rad/s $s_2 = -3$ rad/s

at $t=0^-$

$I_o = 0$

$V_o = 12V$

$i(0^+) = 0 \Rightarrow A_1 + A_2 = 0$

$i'(0^+) = \frac{1}{L}(V - IR - V_o)$

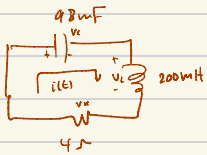
$= \frac{-12}{200m} = -60$ A/sec

$\Rightarrow -60 = -17A_1 - 3A_2$

$0 = 3A_1 + 3A_2$

$A_1 = 4.3$ $A_2 = -4.3$

$i(t) = 4.3e^{-17t} - 4.3e^{-3t}$ A, $t \geq 0$

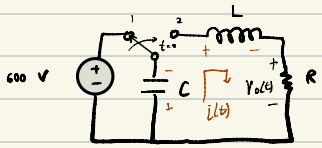


② $V_R = IR = 4\left[4.3e^{-17t} - 4.3e^{-3t}\right]$ V

$V_L = L\frac{di}{dt} = 200mH\left[4.3\times-17e^{-17t} - 4.3\times-3e^{-3t}\right]$

$V_C = -(V_R + V_L)$ or $\Rightarrow \frac{1}{C}\int i\,dt + V_o$

Example:

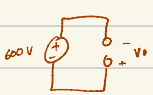


C = 8 MF, R = 50Ω, L = 50.3 mH

The switch has been in position 1 for a long time at t=0, the switch moves to position "2", find $v_o(t)$ for $t \geq 0^+$

Solution:

at $t = 0^-$



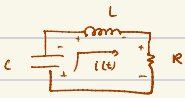
$V_o = -600\,V$

$I_o = 0$

$$i(0) = \frac{1}{L}\left[\overset{0}{V_s} - \overset{0}{I_o R} - V_o\right]$$

$$= \frac{600}{50.3\times10^{-3}} = 119.28\times10^3\ A/sec$$

When $t \geq 0$



Series connection

$$\alpha = \frac{R}{2L} \qquad \omega = \frac{1}{\sqrt{LC}}$$

$= 497$ rad/sec $= 800.8$ rad/sec

$\alpha < \omega \Rightarrow$ under damped Response

$$i(t) = e^{-497t}\left[B_1\cos(\omega_d t) + B_2 \sin(\omega_d t)\right]$$

$$i(0) = B_1 = 0 \qquad \omega_d = \sqrt{\omega^2 - \alpha^2} = 627\ rad/sec$$

$$\dot{i}(0) = \omega_d B_2 - \overset{0}{\alpha B_1}$$

$$119.28\times10^3 = 627\ B_2$$

$$B_2 = 190$$

$$i = (190 \sin(627t)\, e^{-497t})\ A, \quad t \geq 0$$

$$V_o = Ri = 9500 \sin(627t)\, e^{-497t}$$

Chapter 8: 2nd order RLC circuit (extra Notes)

$x(t) = x_n + x_f$

natural ↓ ↳ forced

$x_f = x(t \to \infty)$

The natural response

$$\frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x(t) = f(t)$$

$x(t)$ is either $v(t)$ or $i(t)$

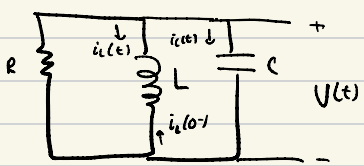
natural response $\Rightarrow f(t) = 0 \Rightarrow$ homogenous

* Natural Response of Parallel RLC circuit:

find $v(t)$ for $t>0$

Given $v_c(0^-) = 0$ volt $\Rightarrow v_c(0^-) = v_c(0^+) = 0$ volt

$i_L(0^-) = 10$ A



KCL:

$$i_R(t) + i_L(t) + i_C(t) = 0$$

$$\frac{v(t)}{R} + \frac{1}{L}\int_0^t v(t)\,dt - i(0^-) + C\frac{dv}{dt} = 0$$

(minus: since the initial condition has opp direction of $i_L(t)$)

$$\frac{1}{R}\frac{dv(t)}{dt} + \frac{1}{L}v(t) + C\frac{d^2v}{dt^2} = 0$$

we need to find 2nd IC

$$\frac{dv_c}{dt}\Big|_{t=0^+} = ??$$

$$\frac{v(0^+)}{R} + \frac{1}{L}\int_{0^-}^{0^+} v(t)\,dt + C\frac{dv(0^+)}{dt} = i_L(0^-)$$

$$\frac{dv(0^+)}{dt} = \frac{i_L(0^-)}{C} = \frac{10}{C} \quad \text{(for example)}$$

solutions of this equation

$$S_1, S_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

where $\alpha = \frac{1}{2RC}$ (parallel), $\omega = \frac{1}{\sqrt{LC}}$ (Resonant frequency)

(Damping coefficient)

Cases of (α & ω_o):

① $\alpha > \omega_o \Rightarrow$ over-damping $\rightarrow$ real, diff roots

$x_n(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, t > 0 \rightarrow s_1 \& s_2 < 0$

② $\alpha = \omega_o \Rightarrow$ critically-damped $\Rightarrow$ real, repeated roots

$x_n(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t}$

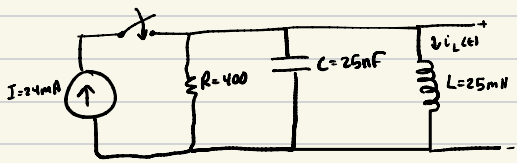
③ $\alpha < \omega_o \Rightarrow$ under-damped $\Rightarrow$ complex roots

$x_n(t) = e^{-\alpha t}[B_1 \cos \omega_d t + B_2 \sin \omega_d t]$

$\omega_d = \sqrt{\omega_o^2 - \alpha^2}$

step response $\Rightarrow$ parallel RLC

given: $v_c(0^-) = 0$ Volt, $i_L(0^-) = 0$ A → if they're not given find them before switching



Find $v(t)$ for $t>0$

Find $i_L(t)$ for $t>0$

$V = L\frac{di}{dt}$

① KCL

$I = I_R + i_L + i_C$

$$I = \frac{V}{R} + \frac{1}{L}\int_0^t V(t)\,dt + i_L(0^-) + C\frac{dV}{dt} \Rightarrow 0 = \frac{dV}{R dt} + C\frac{d^2V}{dt^2} + V(t) \rightarrow \text{Homogenous DE}$$

or $I = \frac{V(t)}{R} + i_L + C\frac{dv}{dt}$, $V_c = V_L = L\frac{di}{dt}$

$$I = \frac{L}{R}\frac{di}{dt} + i_L + LC\frac{d^2i}{dt^2}$$

↓ non Homogenous

$i_L(t) = i_n(t) + i_p(t)$

Natural response ← → forced Response $\Rightarrow i_p = K = I$

$i_L(t) = 24mA + A_1 e^{-20000t} + A_2 e^{-80000t}$, $t > 0$

To find A_1 & A_2, we need

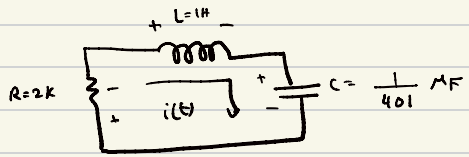
$i_L(0^+) = 0$ and $\frac{di_L(0^+)}{dt} = ??$

$$\frac{di(0^+)}{dt} = \frac{V_L(0^+)}{L}$$
$$= \frac{V_C(0^+)}{L}$$
$$= 0 \text{ (according to what is given)}$$

---

② Natural Response of series RLC circuit:

$V_C(0^-) = V_0$ / $i_L(0^-) = I_0$



KVL

$$i(t)R + V_L + V_C = 0$$
$$i(t)R + L\frac{di}{dt} + \frac{1}{C}\int_{0^+}^{t} i(t)\,dt + V_C(0^-) = 0 \quad \rightarrow \text{for IC}$$

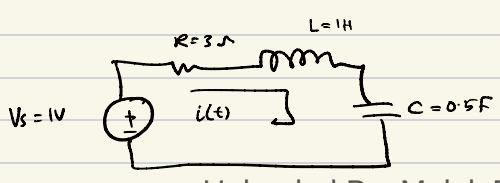
$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i(t)}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{1}{R}\frac{di}{dt} + \frac{i}{RC} = 0$$

$$\underbrace{\alpha = \frac{R}{2L}}_{\text{series}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Three diff solutions : " " "

Step response of series RLC circuit



KVL:

$$V_s = Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_{0^+}^{t} i\,dt + V_c(0^-) = 0$$

unknown $\Rightarrow i(t)$

we can find $i(t=0^+) = i_L(0^+) = i_L(0^-) = i_C(0^-) = 0$

from DE at $t=0^+$

$$V_s = RL(0^+) + L\frac{di}{dt}\Big|_{t=0^+} + V_C(0^-) + \frac{1}{C}\int_{0^-}^{0^+} V(t)\,dt$$

$$i(t) = \frac{1}{L}\left(V_s - V_0 - I_0 R\right)$$

then we find $i(t)$

$$\alpha = \frac{R}{2L} \quad / \quad \omega = \frac{1}{\sqrt{LC}}$$

(and we have to three case)

to find $V_c(t) \Rightarrow$ two ways

way 1

$$V_s = Ri + L\frac{di}{dt} + V_c(t)$$

$$i_c = C\frac{dV_c}{dt}$$

$$V_s = R \times C\frac{dv_c}{dt} + L \times C\frac{d^2v_c}{dt^2} + V_c(t)$$

non-Homogenous DE

$$V_c(t) = V_n(t) + V_f(t)$$

way 2

$V_c(t)$ & $i_c(t)$

$$i_c(t) = C\frac{dV_c(t)}{dt}$$

$$V_c(t) = \frac{1}{C}\int_0^t i(t)\,dt + V_c(0^-)$$

بتكامل من 0 لـ t

① find $V_{cf}(t) = K \Rightarrow$ substitute in DE

$$V_s = RC \times 0 + 0 + K$$

$$\boxed{K = V_s = 1}$$

② to find $V_{cn}(t) = A_1 e^{-t} + A_2 e^{-2t}$

& $\alpha = \frac{R}{2L}$, $\omega = \frac{1}{\sqrt{LC}}$

$$V(t) = V_n(t) + V_f(t)$$

$$= A_1 e^{-t} + A_2 e^{-2t} + 1$$

to find A_1 & A_2

$$V_L(0^-) = V_C(0^+) = 0 \Rightarrow 1 + A_1 + A_2 = 0$$

$$V_c'(0^+) = \frac{i_c(0^+)}{C}$$

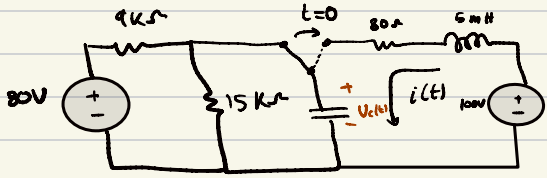
$$i_C(0^+) = i_L(0^-) = \frac{dV_c}{dt}$$

$$\Rightarrow \quad 0 = -A_1 - 2A_2$$

Example

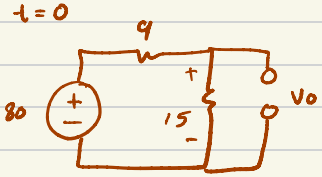
① find $i(t)$, $t>0$

② find $V_c(t)$, $t>0$



Solution:

at $t=0$

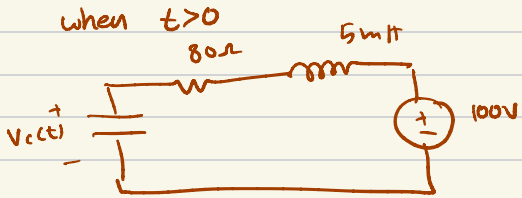


$I = 3.3$ mA

$V_o = 50$ V

$I_0 = 0$

when $t>0$



$V_c(t) = V_n(t) + V_f(t)$

$V_f = 100V$

$V_n(t) \Rightarrow \alpha = \frac{R}{2L}$ $\quad w = \frac{1}{\sqrt{LC}}$

$\alpha = 8\times10^3$ rad/s $\quad w_o = 10^4$ rad/s

$w > \alpha \Rightarrow$ complex roots

$w_d = \sqrt{w_o^2 - \alpha^2}$

$= 6\times10^3$ rad/s

$v(t) = e^{-\alpha t}[B_1 \cos w_d t + B_2 \sin w_d t]$

$= e^{-8\times10^3 t}[B_1 \cos(6\times10^3 t) + B_2 \sin(6\times10^3 t)]$

initial conditions ⇐

$V_c(0^-) = V_c(0^+) = 50$

$B_1 + 100 = 50$

$B_1 = -50$

$V_c'(0^+) \Rightarrow i_L(0^-) = i_L(0^+)$

$\frac{dv(t)}{dt} = \frac{i_L(0^+)}{C} = \frac{0}{C} = 0$

$0 = w_d B_2 - \alpha B_1$

$B_2 = -66.66$ rad/s

$v(t) = 100 + e^{-8\times10^3 t}[-50\cos(6\times10^3 t) - 66.66 \sin(6\times10^3 t)]$

$i(t) = C\frac{dv}{dt} =$

$$= 2\times10^{-6}\left[e^{-8\times10^{3}t}\times\left[-50\times-\sin(6\times10^{3}t)\times6\times10^{3}-66.66\times\cos(6\times10^{3}t)\times6\times10^{3}\right]\right.$$
$$\left.+\left[-50\cos(6\times10^{3}t)-66.66\sin(6\times10^{3}t)\times-8\times10^{3}e^{-8\times10^{3}t}\right]\right)$$

or $i(t) \Rightarrow$

$$i(t) = e^{-8\times10^{4}t}\left[B_1\cos(6\times10^{3}t)+B_2\sin(6\times10^{3}t)\right]$$

$$i(0^+) = i(0^-) = 0$$

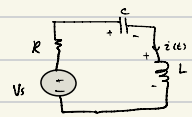
$$i'(0^+) = \frac{1}{L}\left[V_s - V_0 - \cancel{I_0 R}^{0}\right]$$

$$i'(0^+) = \frac{\overset{10}{\cancel{50}}}{\cancel{5}\times10^{-3}} = 10^4\ A$$

يتكمل

chapter 8 summary

Series



في حالة التوالي ← الانهيل إياد

i(t) ⇒ it has just one form, wether it was natural or forced response

$i(t) = i_n(t)$

① $\alpha = \frac{R}{2L}$ $\quad w_o = \frac{1}{\sqrt{LC}}$

↳ over damped $\alpha > w_o$

$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - w_o^2}$

↳ under damped $\alpha < w_o$

$w_d = \sqrt{w_o^2 - \alpha^2}$

$i(t) = e^{-\alpha t}\left[B_1 \cos(w_d t) + B_2 \sin(w_d t)\right]$

↳ critical damped $\alpha = w_o$

$i(t) = D_1 t e^{-\alpha t} + D_1 e^{-\alpha t}$

⟹ to find initial conditions

$i_L(0^-) = i_L(0^+) = I_o \Rightarrow$ at $t = 0^-$

I_o بشرط C: (O.C) / L: (S.C) ← بنرسم الدارة

$I_L'(0^+) = \frac{1}{L} \times \left[V_s - \underbrace{V_o}_{V_c(0^-)} - \underbrace{I_o R}_{i_L(0^-)}\right]$

بعرف $i_L(t) = i_c(t) = i_R(t)$

$V(t)$ ← في حال طلب التيار

$V(t) = V_n(t) + V_p(t)$

Voltage source لـ في حال وجود

$V_f(t) = V_s$

$V_n(t) \rightarrow$ same steps as $i'(t)$

at the end $\Rightarrow V(t) = V_n(t) + V_f(t)$

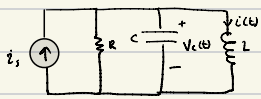
to find I.C:

$V_c(0^-) = V_c(0^+) = V_o$

$V_c'(0^+) = \frac{i_L(0^-)}{C}$

* Note ⟹ try to simplify the circuit to look like the general form: you may use thavenin and Norton techniques or by using source transformation

parallel



في حالة التوازي ← الانهيل إياد

V(t) ⇒ it has just one form, wether it was natural or forced response

$V(t) = V_n(t)$

① $\alpha = \frac{1}{2RC}$ $\quad w_o = \frac{1}{\sqrt{LC}}$

↳ over damped $\alpha > w_o$

$V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - w_o^2}$

↳ under damped $\alpha < w_o$

$w_d = \sqrt{w_o^2 - \alpha^2}$

$V(t) = e^{-\alpha t}\left[B_1 \cos(w_d t) + B_2 \sin(w_d t)\right]$

↳ critical damped $\alpha = w_o$

$V(t) = D_1 t e^{-\alpha t} + D_1 e^{-\alpha t}$

⟹ to find initial conditions

$V_c(0^-) = V_c(0^+) = V_o \Rightarrow$ at $t = 0^-$

V_o بشرط C: (O.C) / L: (S.C) ← بنرسم الدارة

$V_c'(0^+) = \frac{1}{C} \times \left[I_s - \underbrace{I_o}_{i_c(0^+)} - \underbrace{\frac{V_o}{R}}_{V_c(0^-)}\right]$

بعرف $V_c(t) = V_L(t) = V_R(t)$

$i(t)$ ← في حال طلب التيار

$i(t) = i_n(t) + i_p(t)$

current source لـ في حال وجود

$i_f(t) = I_s$

$i_n(t) \rightarrow$ same steps as $V(t)$

at the end $\Rightarrow i(t) = i_n(t) + i_f(t)$

to find I.C:

$i_L(0^-) = i_L(0^+) = I_o$

$i_L'(0^+) = \frac{V_c(0^-)}{L}$, $(V_c(0^+) = V_c(0^-))$

* Note ⟹ try to simplify the circuit to look like the general form: you may use thavenin and Norton techniques or by using source transformation

test yourself:

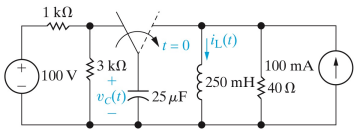
8.35 The switch in the circuit in Fig. P8.35 has been in the left position for a long time before moving to the right position at $t = 0$. Find

a) $i_L(t)$ for $t \geq 0$,

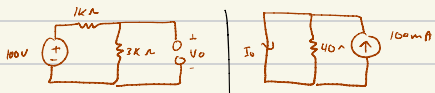
b) $v_C(t)$ for $t \geq 0$.

PSPICE MULTISIM

Figure P8.35



① at $t=0^-$



$I = \frac{100}{4} = 25mA$

$V_o = 3K \times 25m = 75V$

$V_c(0^-) = 75V$

$I_o = 100mA$

$I_L(0^-) = 100mA$

② at $t>0$

$i(t) = i_n(t) + i_f(t)$

$i_f(t) = I_s = 100\,mA$

$i_n(t) \Rightarrow \quad \alpha = \frac{1}{2RC} = 500\ rad/s \qquad \omega_0 = \frac{1}{\sqrt{LC}} = 400\ rad/s$

$\alpha > \omega_0 \Rightarrow$ real, diff roots (over damped)

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega^2} = -200\ rad/s$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega^2} = -800\ rad/s$

③ $i_n(t) = A_1 e^{-200t} + A_2 e^{-800t}$

$i(t) = 100m + A_1 e^{-200t} + A_2 e^{-800t}$

$i(0^-) = 100m$

$A_1 + A_2 = 0$

$i'(0) = \frac{V_c(0^+)}{L}$

$= 300\,A$

$300 = -200A_1 - 800A_2$

$3 = -2A_1 - 8A_2$

$0 = 2A_1 + 2A_2$

$3 = -6A_2$

$A_2 = -0.5$

$A_1 = 0.5$

$\therefore i(t) = 100m + 0.5e^{-200t} - 0.5e^{-800t}$

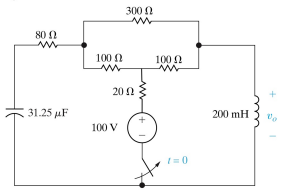
$\therefore V_c = V_L = L\frac{di}{dt}$

$= 250 \times 10^{-3}\left[0.5 \times -200e^{-200t} - 0.5 \times -800e^{-800t}\right]$

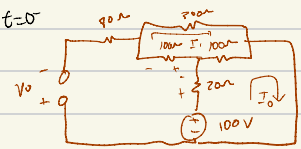
$= -25e^{-200t} + 100e^{-800t}$

8.47 The switch in the circuit shown in Fig. P8.47 has been closed for a long time. The switch opens at $t = 0$. Find $v_o(t)$ for $t \geq 0^+$.

Figure P8.47



(1) $t=0^-$



(A) $I_o \Rightarrow -100 + 120\,I_o - 100I = 0$

$12\,I_o - 10I = 10$

$12\,I_o - \frac{10 \times I_o}{5} = 10$

$10I_o = 10 \Rightarrow \boxed{I_o = 1A}$

(A) $I_1 \Rightarrow 500I_1 - 100I_o = 0$

$5I_1 = I_o$

$I = \frac{I_o}{5}$

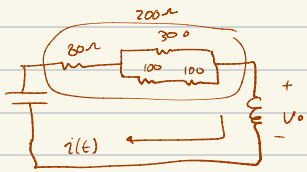
$I = \frac{1}{5} = 0.2\,A$

$-V_o - 100 + 20 \times 1 + 100 \times 0.2 = 0$

$\boxed{V_o = -60V}$

(2)

at $t > 0$



$i(t) \Rightarrow$

$\alpha = \frac{R}{2L}$ $\quad = 500\ rad/s$

$\omega_o = \frac{1}{\sqrt{LC}}$ $\quad = 400\ rad/s$

$\alpha > \omega$: over damped

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^2} = -200\ rad/s$

$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^2} = -800\ rad/s$

$i(t) = A_1 e^{-200t} + A_2 e^{-800t}$

$i(0^-) = i(0^+) = 1$

$1 = A_1 + A_2$

$i'(0^+) = \frac{1}{L}\left[v_s - V_o - I_o R\right]$

$= -700\ A/s$

$+700 = +200\,A_1 + 800\,A_2$

$-7 = -2A_1 - 8A_2$

$2 = 2A_1 + 2A_2$

$-5 = -6A_2$

$A_2 = 0.833$

$A_1 = 0.166$

$\star\ i(t) = 0.166 e^{-200t} + 0.833 e^{-800t}\ A$

$V(t) = L \frac{di}{dt}$

$= 200 \times 10^{-3}\left[-200 \times 0.166\, e^{-200t} + 0.833 \times 800\, e^{-800t}\right]$

$\star\ v(t) = -6.667\, e^{-200t} - 133.3\, e^{-800t}\ V$

الحمد لله

(ما مرونا في حال وجود خطأ في حكاية ما)

ما تنسونا من صالح دعائكم ❤