# 8.1 Integration by Parts



① $$\int u\,dv = uv - \int v\,du$$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

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Exp ① $\int x \cos x\,dx = \int u\,dv$

$u = x \qquad dv = \cos x\,dx$

$du = dx \Rightarrow v = \sin x$

$= uv - \int v\,du$

$= x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + c$

② $\int_0^{\pi} x\cos x\,dx = x\sin x\Big|_0^{\pi} - \int_0^{\pi} \sin x\,dx$

$= 0 + \cos x\Big|_0^{\pi} = -1 - 1 = -2$

③ $\int_0^{\pi} x\cos x\,dx$

| $f(x)$ and its derivatives | | $g(x)$ and its integrals |
|---|---|---|
| $f(x) = x$ | $(+)$ | $g(x) = \cos x$ |
| $1$ | $(-)$ | $\sin x$ |
| $0$ | | $-\cos x$ |

$= x\sin x + \cos x + c$




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Exp $\int \ln x\,dx = uv - \int v\,du$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}\,dx \Rightarrow v = x$

$= x\ln x - \int dx$

$= x\ln x - x + c$

**Exp** $\int x^2 e^x \, dx$

| $f(x)$ | | $g(x)$ |
|---|---|---|
| $f(x) = x^2$ | | $g(x) = e^x$ |
| $2x$ | $(+)$ | $e^x$ |
| $2$ | $(-)$ | $e^x$ |
| $0$ | $(+)$ | $e^x$ |

$$\int x^2 e^x \, dx = x^2 e^x - 2x e^x + 2e^x + C$$

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$\int x^2 e^x \, dx$ $\qquad u = x^2 \quad dv = e^x \, dx$, $\quad du = 2x \, dx \quad V = e^x$

$= x^2 e^x - \int 2x e^x \, dx$ $\qquad$ we need $\int x e^x \, dx$

$= x^2 e^x - 2 \int x e^x \, dx$ $\qquad u = x \quad dv = e^x \, dx$, $\quad du = dx \quad v = e^x$

$= x^2 e^x - 2\left[x e^x - \int e^x \, dx\right]$

$= x^2 e^x - 2x e^x + 2\int e^x \, dx = x^2 e^x - 2x e^x + 2e^x + C$

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**Exp*** $\int e^x \cos x \, dx$

| $f(x)$ | | $g(x)$ |
|---|---|---|
| $f(x) = e^x$ | | $g(x) = \cos x$ |
| $e^x$ | $(+)$ | $\sin x$ |
| $e^x$ | $(-)$ stop | $-\cos x$ |

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

$$2\int e^x \cos x \, dx = e^x \sin x + e^x \cos x$$





$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\sin x + \cos x) + C$$

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$\int e^x \cos x \, dx$ $\qquad u = e^x \quad dv = \cos x \, dx$, $\quad du = e^x \, dx \quad V = \sin x$

$\int e^x \cos x \, dx = e^x \sin x - \int e^x \sin x$ $\qquad u = e^x \quad dv = \sin x \, dx$, $\quad du = e^x \, dx \quad V = -\cos x$

$= e^x \sin x - \left[-e^x \cos x + \int e^x \cos x \, dx\right]$

$$\int e^x \cos x \, dx = e^x \sin x + e^x \cos x - \int e^x \cos x \, dx$$

$$2\int e^x \cos x \, dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x [\sin x + \cos x] + C$$

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**Exp** $\int_1^2 x \ln x \, dx$

$u = \ln x \qquad dv = x \, dx$

$du = \frac{dx}{x} \qquad v = \frac{x^2}{2}$

$$\int_1^2 x \ln x \, dx = \frac{x^2}{2} \ln x \Big|_1^2 - \int_1^2 \frac{1}{x} \frac{x^2}{2} \, dx$$

$$= \frac{x^2}{2} \ln x \Big|_1^2 - \frac{1}{2}\int_1^2 x \, dx$$

$$= 2\ln 2 - 0 - \frac{1}{2} \frac{x^2}{2} \Big|_1^2 = 2\ln 2 - 1 + \frac{1}{4}$$

$$= \ln 4 - \frac{3}{4}$$

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| | | |
|---|---|---|
| $f(x) = x$ | $(+)$ | $g(x) = \ln x$ |
| $1$ | $(-)$ | $x \ln x - x$ |
| $0$ | | $\int x \ln x \, dx - \frac{x^2}{2}$ |

$$\int x \ln x \, dx = x^2 \ln x - x^2 - \int x \ln x \, dx + \frac{x^2}{2}$$

$$2\int x \ln x \, dx = x^2 \ln x - \frac{x^2}{2}$$

$$\int_1^2 x \ln x \, dx = \left[\frac{x^2}{2} \ln x - \frac{x^2}{4}\right]_1^2$$

$$= \left[2\ln 2 - 1\right] - \left[0 - \frac{1}{4}\right]$$

$$= 2\ln 2 - \frac{3}{4} \quad \checkmark$$