7.8 2+6+8

(1)

(2) e^x

$\boxed{a}$. ~~10x~~ $f(x) = 10x^4+30x+1$

$$\lim_{x\to\infty} \frac{10x^4+30x+1}{e^x} = \lim_{x\to\infty} \frac{40x^3+30}{e^x} = \lim_{x\to\infty} \frac{120x^2}{e^x} = \lim_{x\to\infty} \frac{240x}{e^x}$$

$$= \lim_{x\to\infty} \frac{240}{e^x} = 0$$

e^x is faster

$\boxed{b}$ $f(x) = x\ln x - x$

$$\lim_{x\to\infty} \frac{x\ln x - x}{e^x} = \lim_{x\to\infty} \frac{x(\ln x - 1)}{e^x} = \lim_{x\to\infty} \frac{x \cdot \frac{1}{x} + \ln x}{e^x}$$

$$= \lim_{x\to\infty} \frac{1+\ln x}{e^x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{e^x} = \lim_{x\to\infty} \frac{1}{x e^x} = 0$$

e^x is faster

$\boxed{k}$ $f(x) = \left(\frac{5}{2}\right)^x$

$$\lim_{x\to\infty} \frac{\left(\frac{5}{2}\right)^x}{e^x} = \lim_{x\to\infty} \left(\frac{5}{2e}\right)^x = 0$$

e^x is faster

$\boxed{d}$. $f(x) = \sqrt{1+x^4}$

$$\lim_{x\to\infty} \frac{\sqrt{1+x^4}}{e^x} = \lim_{x\to\infty} \sqrt{\frac{1+x^4}{e^{2x}}} = \sqrt{\lim_{x\to\infty} \frac{1+x^4}{e^{2x}}} = \sqrt{\lim_{x\to\infty} \frac{4x^3}{e^{2x}\cdot 2}}$$

$$= \sqrt{\lim_{x\to\infty} \frac{12x^2}{e^{2x}\cdot 4}} = \sqrt{\lim_{x\to\infty} \frac{24x}{e^{2x}\cdot 8}} = \sqrt{\lim_{x\to\infty} \frac{24}{e^{2x}\cdot 16}} = \sqrt{\lim_{x\to\infty} 0}$$

$= 0$

e^x is faster

$\boxed{e}$ $f(x) = e^{-x}$

$$\lim_{x\to\infty} \frac{e^{-x}}{e^x} = \lim_{x\to\infty} \frac{1}{e^{2x}} = 0$$, e^x is faster

$\boxed{f}$ $$\lim_{x\to\infty} \frac{x e^x}{e^x} = \lim_{x\to\infty} x = \infty$$

e^x is slower than xe^x.

$\boxed{g}$ $f(x) = e^{\cos x}$

$$\lim_{x\to\infty} \frac{e^{\cos x}}{e^x}$$, $-1 \le \cos x \le 1$

$$e^{-1} \le e^{\cos x} \le e^{1} \longrightarrow \underset{\downarrow\, 0}{\frac{e^{-1}}{e^x}} \le \underset{\downarrow\, 0}{\frac{e^{\cos x}}{e^x}} \le \underset{\downarrow\, 0}{\frac{e^{1}}{e^x}}$$

So $\underline{e^{\cos x}}$ is slower

(h) $F(x) = e^{x-1}$

$$\lim_{x\to\infty} \frac{e^{x-1}}{e^x} = \cancel{\lim_{x\to\infty}} \frac{e^{x-1}}{e^x} = \lim_{x\to\infty} e^{-1} = e^{-1}$$

Same

(6) [a] $f(x) = \log_2 x^2$

$$\lim_{x\to\infty} \frac{\log_2 x^2}{\ln x} = \lim_{x\to\infty} \frac{\frac{\ln x^2}{\ln 2}}{\ln x} = \frac{1}{\ln 2} \lim_{x\to\infty} \frac{2\ln x}{\ln x} = \frac{2}{\ln 2}$$ same

[b] $\log_{10} 10x$

$$\lim_{x\to\infty} \frac{\log_{10} 10x}{\ln x} = \lim_{x\to\infty} \frac{\frac{10}{10x\ln 10}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{x}{x\ln 10} = \frac{1}{\ln 10}$$ same

[c] $y = \frac{1}{\sqrt{x}}$

$$\lim_{x\to\infty} \frac{\frac{1}{\sqrt{x}}}{\ln x} = \lim_{x\to\infty} \frac{1}{\sqrt{x}\ln x} = 0$$

ln x is faster

(d) $y = \frac{1}{x2}$

$$\lim_{x\to\infty} \frac{\frac{1}{x2}}{\ln x} = \lim_{x\to\infty} \frac{1}{x^2 \ln x} = 0$$

$\ln x$ is ~~slower~~ faster.

(e) $y = x - 2\ln x$

$$\lim_{x\to\infty} \frac{x - 2\ln x}{\ln x} = \cancel{\lim \frac{1-\frac{2}{x}}{}}$$

$$= \lim_{x\to\infty} \frac{x}{\ln x} - 2$$

$$= \lim_{x\to\infty} \frac{1}{\frac{1}{x}} - 2 = \infty$$

so $x - 2\ln x$ is faster

(f) $$\lim_{x\to\infty} \frac{e^{-x}}{\ln x} = \lim_{x\to\infty} \frac{1}{e^{x} \ln x} = 0$$

$\ln x$ is faster

(g) $\ln(\ln x)$

$$\lim_{x\to\infty} \frac{\ln(\ln x)}{\ln x} = \lim_{x\to\infty} \frac{\frac{\frac{1}{x}}{\ln x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{1}{x \ln x} \cdot x = \lim_{x\to\infty} \frac{1}{\ln x} = 0$$

so $\ln x$ is faster.

(h) $\ln(2x+5)$

$$\lim_{x\to\infty} \frac{\ln(2x+5)}{\ln x} = \lim_{x\to\infty} \frac{\frac{2}{2x+5}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{2x}{2x+5} = 1$$

same

(8) order from slowest to fastest

$2^x, x^2, (\ln 2)^x, e^x$

e^x is faster than 2^x

2^x is faster than $(\ln 2)^x$

$$\lim_{x\to\infty} \frac{x^2}{(\ln 2)^x} = \lim_{x\to\infty} \frac{2x}{(\ln 2)^x \cdot \ln(\ln 2)} = \lim_{x\to\infty} \frac{2}{(\ln 2)^x \ln(\ln(\ln 2))} = \infty$$

so x^2 is faster than $(\ln 2)^x$

$(\ln 2)^x, x^2, 2^x, e^x$

$\longrightarrow$