Motion in 2 and 3 Dimensions

• Position and Displacement

If the position vector changes from $\vec{r_1}$ to $\vec{r_2}$ during a certain time interval Then the displacement $\Delta\vec{r}$ during that time interval is:-

$$\Delta\vec{r} = \vec{r_2} - \vec{r_1}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

($x_2 - x_1 \rightarrow \Delta x$, $y_2 - y_1 \rightarrow \Delta y$, $z_2 - z_1 \rightarrow \Delta z$)

In 2D: When an object reaches Max position in one direction Then it's velocity in this direction = 0 (⊥ component)

• Average velocity & Instantaneous velocity

If a particle moves through a displacement $\Delta\vec{r}$ in a time interval Δt then it's average velocity $\vec{V}_{avg}$ is

$$\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

V is the same direction as r

$$\vec{V}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}K$$

$$V_{Instant} = \frac{d\vec{r}}{dt}$$

V_{avg} takes the tangents (at r) direction

• Averag Accelaration & instantaneous Acceleration

$$a_{avg} = \frac{\vec{V_2} - \vec{V_1}}{\Delta t} = \frac{\Delta\vec{V}}{\Delta t}$$

$$\vec{a}_{Instant} = \frac{d\vec{v}}{dt}$$

If velocity changes in Magnitude or direction or Both The particle Must have $\vec{a}$

* if the velocity ⊥ Changes in direction ove magnitude → particle Must have an accelaration

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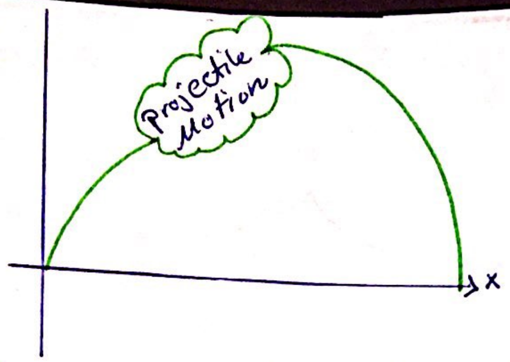
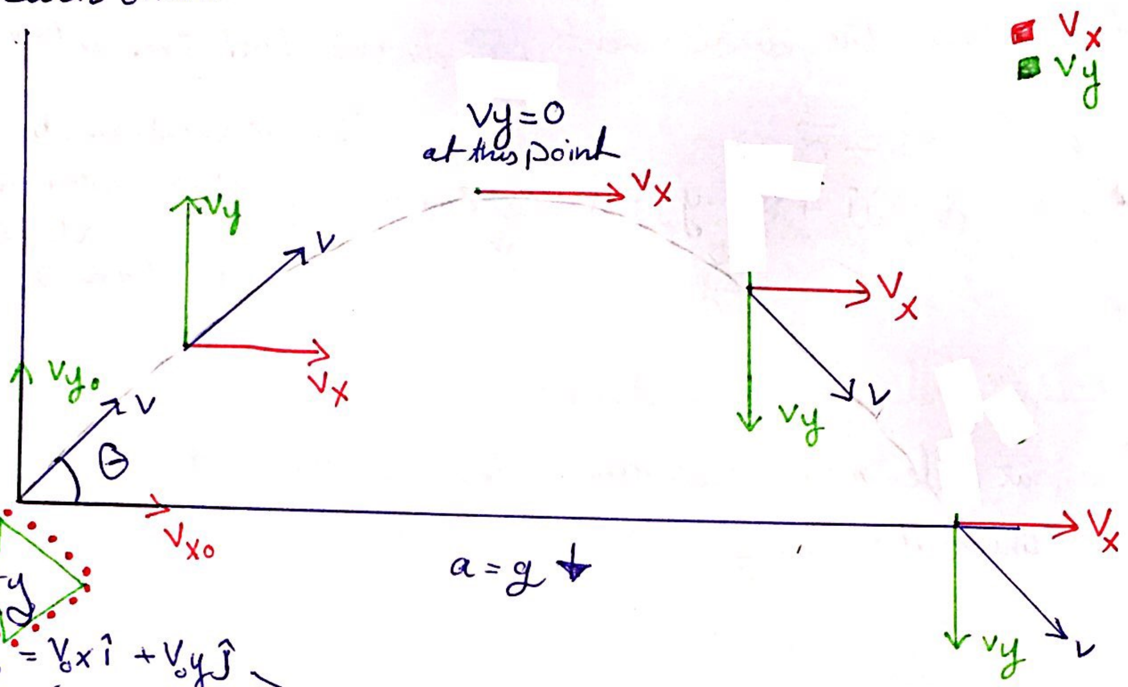


2D motion

» Horizontal and vertical motions are independent of each other



Velocity

$V_o = V_{ox}\,\hat{i} + V_{oy}\,\hat{j}$

$V_{ox} = V_o \cos\theta^\circ$ — Constant

$V_{oy} = V_o \sin\theta^\circ$ — Variable

$V = \sqrt{V_x^2 + V_y^2}$

$V_x = V_{ox}$

- $V_y = V_{oy} - gt$ or • $V_y^2 = V_{oy}^2 - 2g(y - y_o)$

$V_{oy} = V_o \sin\theta$

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Displacement

$X = V_0 t \cos\theta$

$y = V_0 t \sin\theta - \frac{1}{2} g t^2$

$\Delta r = \sqrt{X^2 + y^2}$

* throught the path $a_x = 0$, $a_y = g$
* path is parabolic
* air has a large effect on R or the path in general

• the Horizontal Range

$R = V_{0x} t_f$ --- ①

$\Delta y = V_{0y} t - \frac{1}{2} g t^2$

$\Delta y = 0$ لانه انطلق من مستوى للمستوى نفسه

$0 = V_{0y} t - \frac{1}{2} g t^2$

$t_f = 2 V_{0y} / g$

نعوضه في ① نستنتج

$$R = \frac{V_0^2 \sin 2\theta}{g}$$

$R_{Max} = V_0^2 / g$ when $\theta = 45°$

Range: تقطعها على نفس المستوى

• the height

$$h = \frac{V_0^2 \sin^2(\theta)}{2g}$$

• Relation between R and h

$\frac{h}{R} = \frac{\tan\theta}{4} \Rightarrow$

$$h = \frac{R \tan\theta}{4}$$

• The equation of the path

$$y = \tan\theta \cdot X - \frac{g \cdot X^2}{2 V_0^2 \cos^2\theta}$$

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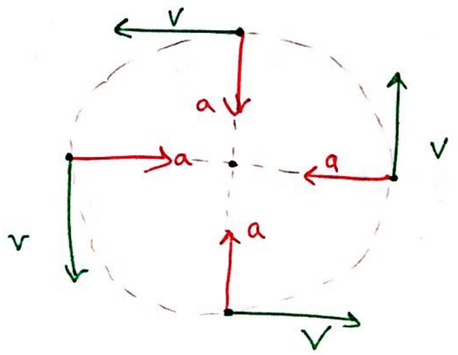
Uniform Circular Motion

- a particle travelling around a circle in a constant speed
- accelaration : Centripetal (uniform circular Motion)

$\vec{a}_r = \frac{V^2}{r}\hat{r}$ — V → speed of the particle, r → radius, $\hat{r}$: vector

$T = \frac{2\pi r}{V}$ T: period, $\pi = 3.14$



$a_t = 0$

$a_{tangent} = \frac{dv}{dt}$ (non-uniform circular Motion)

- V is not Constant
- so accelerating with a tangental accelaration

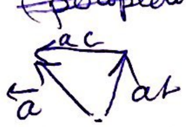
$a_t \neq 0$

$a_r = \frac{V^2}{2\pi r}$

- Net force : محصلة القوى

$\sum \vec{F} = m\vec{a}$

$\vec{a} = \vec{a}_t + \vec{a}_c$ (~~perpendicular~~)



$F = |\sum \vec{F}| = |m\vec{a}|$

$= m|\vec{a}|$

$= m\sqrt{at^2 + ac^2}$

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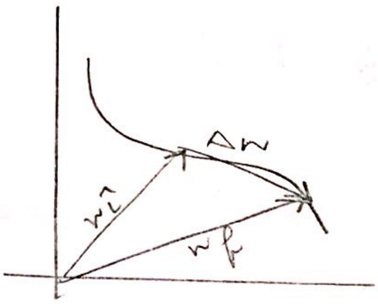
vectors

Displacement vector

$\Delta r = r_f - r_i$



velocity vectors

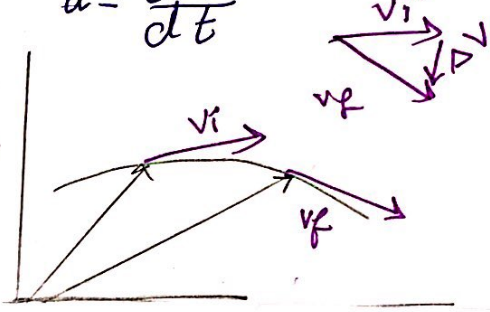
avg: $\bar{V} = \frac{\Delta r}{\Delta t}$

Ins: $V = \frac{dr}{dt}$

Accelaration vectors

$\bar{a} = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t_f - t_i}$

$a = \frac{dV}{dt}$



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Motion with Constant Accelaration

$r = xi + yj$

$v = v_x \hat{i} + v_y \hat{j}$

$v_f = v_{xf} \hat{i} + v_{yf} \hat{j} \longrightarrow \underline{v_f = v_i + at}$

$r_f = x_f \hat{i} + y_f \hat{j} \longrightarrow \underline{r_f = r_i + v_i t + \frac{1}{2} a t^2}$

• Difference between Motion in 1D, 2D and 3D

1D : The Motion is in a straight line

2D : The Motion is in a curved path but in a single plane

3D :- The Motion is throughout the space / not in a plane but in a complete space

Ex : a paper moving freely in the air

Note : • speed is the Magnitude of velocity

• if we have $\Delta \vec{V} \neq 0$ The There is a { Change in speed and / or Change in direction

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