MASS AND ENERGY ANALYSIS: CONTROL VOLUMES

We are going to extend the conservation of energy to systems that involve mass flow across their boundaries, i.e. *control volumes*.

Any arbitrary region in space can be selected as control volume. There are no concrete rules for the selection of control volumes. The boundary of control volume is called a control surface.

Conservation of Mass

Like energy, mass is a conserved property, and it cannot be created or destroyed. Mass and energy can be converted to each other according to Einstein's formula: $E = mc^2$, where c is the speed of light. However, except for nuclear reactions, the conservation of mass principle holds for all processes.

For a control volume undergoing a process, the conservation of mass can be stated as:

$$\begin{matrix} \textit{total mass} & - & \textit{total mass} & = & \textit{net change in} \\ \textit{entering CV} & & \textit{leaving CV} & & \textit{mass within CV} \end{matrix}$$

$$\sum m_i - \sum m_e = \Delta m_{CV}$$

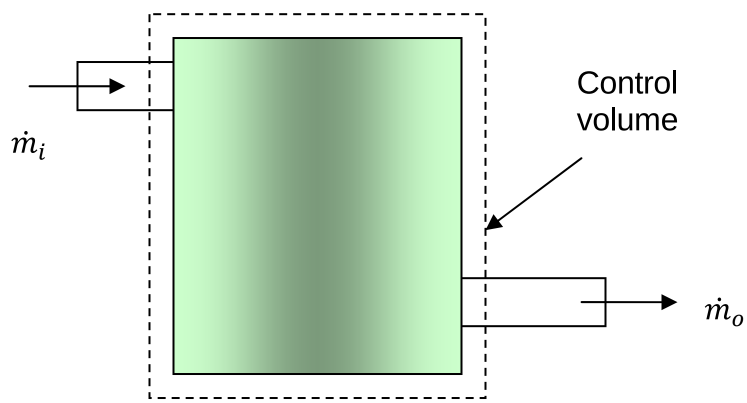


Fig. 1: Conservation of mass principle for a CV.

The conservation of mass can also be expressed in the rate form:

$$\sum \dot{m}_i - \sum \dot{m}_o = \frac{dm_{CV}}{dt}$$

The amount of mass flowing through a cross section per unit time is called the mass flow rate and is denoted by $\dot{m}$. The mass flow rate through a differential area dA is:

$$d\dot{m} = \rho V_n dA$$

where V_n is the velocity component normal to dA. Thus, the mass flow rate for the entire cross-section is obtained by:

$$\dot{m} = \int_A \rho V_n \, dA \qquad \left(\frac{kg}{s}\right)$$

Assuming one-dimensional flow, a uniform (averaged or bulk) velocity can be defined:

$$\dot{m} = \rho V_{ave} dA \quad \left(\frac{kg}{s}\right)$$

where V_{ave} or simply V (m/s) is the fluid velocity (average) normal to the cross sectional area. The volume of the fluid flowing through a cross-section per unit time is called the volumetric flow, $\dot{V}$:

$$\dot{V} = \int_A V_n \, dA = V.A \quad \left(\frac{m^3}{s}\right)$$

The mass and volume flow rates are related by:

$$\dot{m} = \rho \dot{V} = \dot{V}/v.$$

Conservation of Energy

For control volumes, an additional mechanism can change the energy of a system: mass flow in and out of the control volume. Therefore, the conservation of energy for a control volume undergoing a process can be expressed as

total energy crossing boundary as heat and work	+	*total energy of mass entering CV*	–	*total energy of mass leaving CV*	=	*net change in energy of CV*

$$Q - W + \sum E_{in,mass} + \sum E_{out,mass} = \Delta E_{CV}$$

This equation is applicable to any control volume undergoing any process. This equation can also be expressed in rate form:

$$\dot{Q} - \dot{W} + \sum dE_{in,mass}/dt + \sum dE_{out,mass}/dt = dE_{CV}/dt$$

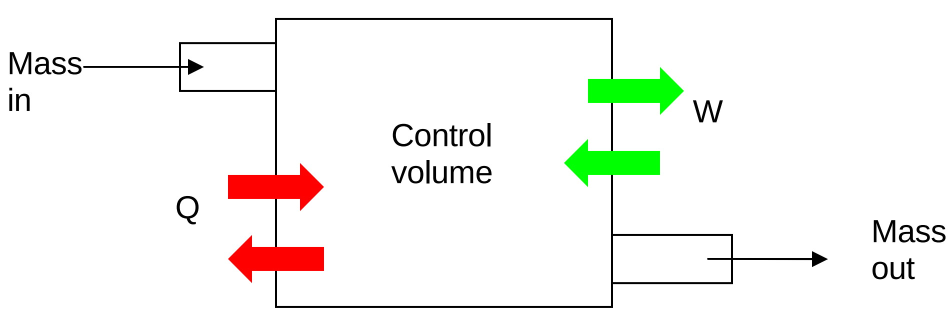


Fig. 2: Energy content of CV can be changed by mass flow in/out and heat and work interactions.

Work flow: is the energy that required to push fluid into or out of a control volume. Consider an imaginary piston (that pushes the fluid to CV) where the fluid pressure is P and the cross sectional area is A. The force acting on the piston is $F = PA$.

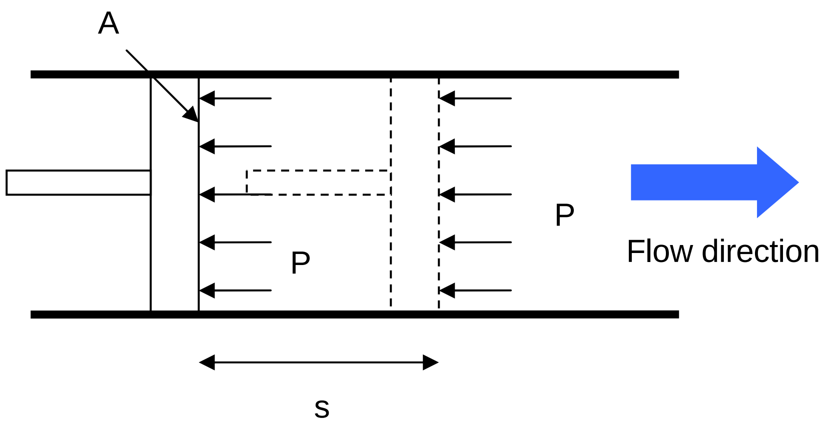


Fig. 3: Flow work.

The work done in pushing the fluid is:

$$W_{flow} = F.s = PA.s = PV \qquad (kJ)$$

or in a unit basis,

$$w_{flow} = W_{flow} / m = Pv \qquad (kJ/kg)$$

Note that the flow work is expressed in terms of properties.

The flow work can also be written as a rate equation.

The fluid entering or leaving a control volume possesses an additional form of energy (flow energy Pv). Therefore, the total energy of a flowing fluid on a unit-mass basis (denoted by θ) becomes:

$$\theta = Pv + e = Pv + (u + ke + pe) \qquad (kJ/kg)$$

Recall that enthalpy is defined as: $h = u + Pv$. Therefore, the above equation becomes:

$$\theta = h + ke + pe = h + V2 / 2 + gz \qquad (kJ/kg)$$

The property θ is called *methalpy*. By using enthalpy instead of internal energy, the energy associated with flow work into/out of control volume is automatically taken care of. This is the main reason that enthalpy is defined!

Steady-State Flow Process

A process during which a fluid flows through a control volume steadily is called steady-state process. A large number of devices such as turbines, compressors, and nozzles operates under the same conditions for a long time and can be modeled (classified) as steady-flow devices.

The term steady implies no change with time. The term uniform implies no change with location over a specified region.

A steady flow is characterized by the following:

1- No properties within the CV change with time. Thus, volume, mass, and energy of CV remain constant. As a result, the boundary work is zero. Also, total mass entering the CV must be equal to total mass leaving CV.

2- No properties change at the boundary of the CV with time. It means that the mass flow rate and the properties of the fluid at an opening must remain constant during a steady flow.

3- The heat and mass interactions between the CV and its surroundings do not change with time.

Using the above observation, the conservation of energy principle for a general steady-flow system with multiple inlets and exits can be written as:

$$\dot{Q} - \dot{W} = \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e \right) - \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right)$$

$$\dot{Q} - \dot{W} = \sum \dot{m}_e \theta_e - \sum \dot{m}_i \theta_i$$

The mass balance for steady-state flow processes/devices becomes:

$$\sum \dot{m}_i - \sum \dot{m}_o = 0$$

For a single stream flow control volumes:

$$\dot{m}_i = \dot{m}_o \rightarrow \rho_i V_i A_i = \rho_o V_o A_o$$

Nozzles and Diffusers

A nozzle is a device that increases the velocity of a fluid at the expense of pressure. A diffuser is a device that increases the pressure of a fluid by slowing it down.

The cross sectional area of a nozzle decreases in the flow direction for subsonic flows and increase for supersonic flows.

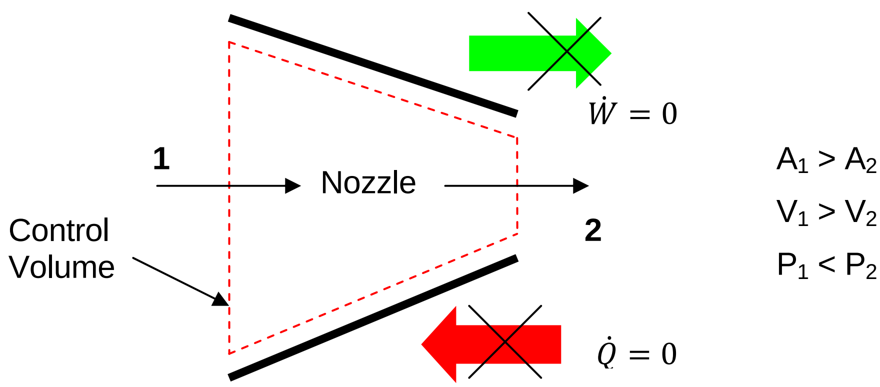


Fig. 4: Schematic of nozzle

For a nozzle, common assumptions and idealizations are:

$\dot{Q} = 0$, no time for heat transfer, due to high velocity

$\dot{W} = 0$, nozzles include no shaft or electric resistance wires

ΔPE = 0, no change in fluid elevation.

Mass equation for nozzles becomes:

$$\dot{m}_1 = \dot{m}_2 \qquad \text{or} \qquad \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

Energy balance:

$$\dot{Q} - \dot{W} = \dot{m}_1\theta_1 - \dot{m}_2\theta_2$$

$$\left(h + \frac{V_1^2}{2}\right)_{at\ inlet} = \left(h + \frac{V_2^2}{2}\right)_{at\ exit}$$

Diffusers are exactly the same device as nozzles; the only difference is the direction of the flow. Thus, all equations derived for nozzles hold for diffusers.

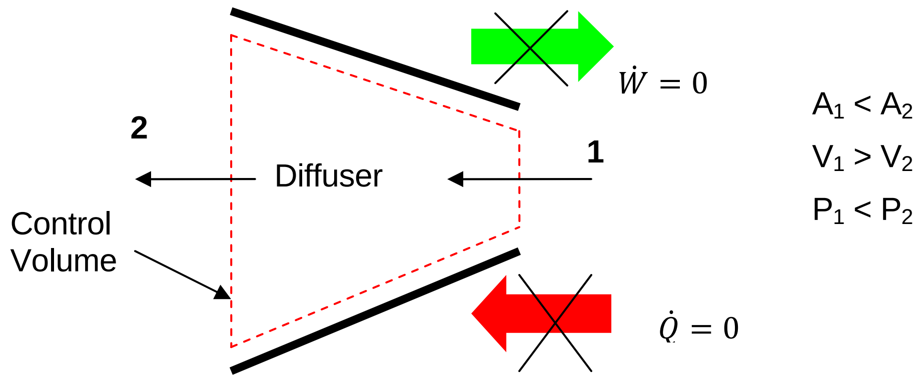


Fig. 5: Schematic for diffuser.

Example: Nozzle

Steam enters a converging-diverging nozzle operating at steady state with $P_1 = 0.05$ *MPa*, $T_1 = 400°C$ and a velocity of *10 m/s*. The steam flows through the nozzle with negligible heat transfer and no significant change in potential energy. At the exit, $P_2 = 0.01$ *MPa*, and the velocity is *665 m/s*. The mass flow rate is *2 kg/s*. Determine the exit area of the nozzle, in m^2.

Assumptions:

1. Steady state operation of the nozzle
2. Work and heat transfer are negligible.

$$\dot{W} = \dot{Q} = 0$$

3. Change in potential energy from inlet to exit is negligible, ΔPE = 0.

The exit area can be calculated from the mass flow rate $\dot{m}$:

$$A_2 = \frac{\dot{m} v_2}{V_2}$$

We need the specific volume at state 2. So, state 2 must be found. The pressure at the exit is given; to fix the state 2 another property is required. From the energy equation enthalpy can be found:

$$h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2} \quad \Rightarrow \quad h_2 = h_1 + \frac{1}{2}\left(V_1^2 - V_2^2\right)$$

From Table A-6, $h_1 = 3279.3\ kJ/kg$ and velocities are given.

$$h_2 = 3279.3\ kJ/kg + \left[\frac{10^2 - 665^2}{2}\right]\left(\frac{m}{s}\right)^2\left[\frac{1\ N}{1 kg.m/s^2}\right]\left[\frac{1\ kJ}{10^3\ N.m}\right] = 3058.23\ kJ/kg$$

From Table A-6, with $P_2 = 0.01\ MPa$, and $h_2 = 3057.84\ kJ/kg$, (using interpolation) one finds $T_2 = 300°C$ (approximately) and $v_2 = 26.446\ m^3/kg$.

The exit area is:

$$A_2 = \frac{(2\ kg/s)(26.446\ m^3/kg)}{665\ m/s} = 0.07953\ m^2$$

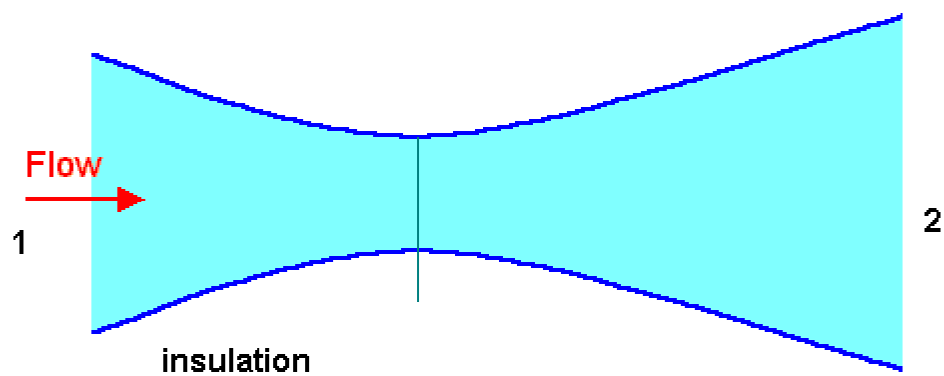


Fig. 6: Converging-diverging nozzle.

Turbines and Compressors

In steam, gas, or hydroelectric power plants, the device that derives the electric generator is turbine. The work of turbine is positive since it is done by the fluid.

Compressors, pumps, and fans are devices used to increase the pressure of the fluid. Work is supplied to these devices, thus the work term is negative.

Common assumptions for turbines and compressors:

$$\dot{Q} = 0.$$

$$\Delta PE = 0.$$

Example: Turbine

Steam enters a turbine at steady state with a mass flow rate of *4600 kg/h*. The turbine develops a power output of *1000 kW*. At the inlet the pressure is *0.05 MPa*, the temperature is *400 °C*, and the velocity is *10 m/s*. At the exit, the pressure is *10 kPa*, the quality is *0.9*, and the velocity is *50 m/s*. Calculate the rate of heat transfer between the turbine and surroundings, in *kW*.

Assumptions:

Steady-state operation. The change in potential energy is negligible.

The energy balance for the turbine is:

$$\dot{Q}_{CV} - \dot{W}_{CV} = \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) - \dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right)$$

$$\dot{Q}_{CV} = \dot{W}_{CV} + \dot{m}\left[\left(h_2 - h_1\right) + \frac{1}{2}\left(V_2^2 - V_1^2\right)\right]$$

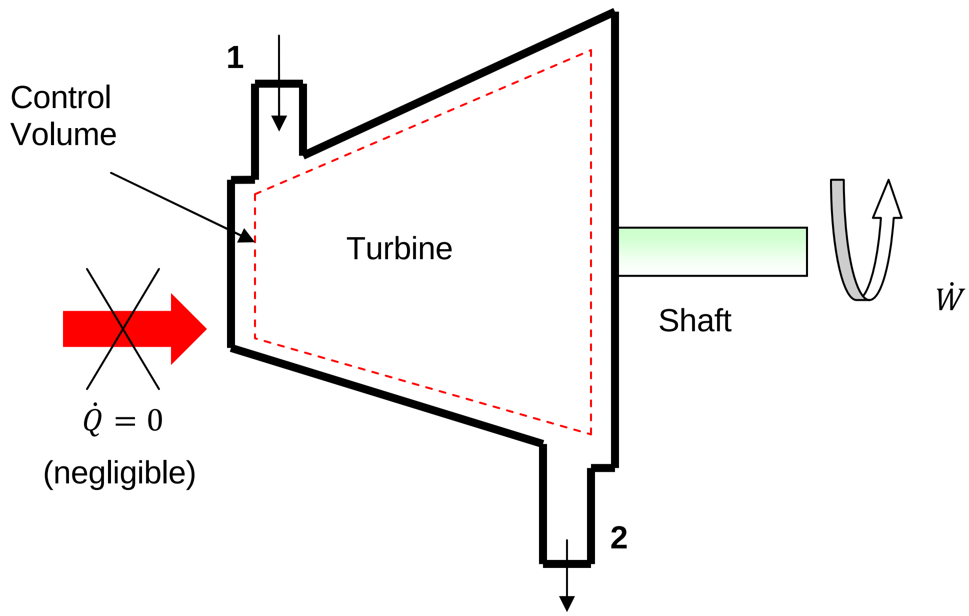


Fig. 7: Schematic for turbine.

Using Table A-6, $h_1 = 3279.3\ kJ/kg$, using Table A-5 with $x = 0.9$,

$$h_2 = h_f + x_2\, h_{fg} = 191.83\ kJ/kg + 0.9(2392.8\ kJ/kg) = 2345.35\ kJ/kg$$

Therefore,

$$h_2 - h_1 = 2345.35 - 3278.9 = -933.55\ kJ/kg$$

The specific kinetic energy change is:

$$\frac{1}{2}\left(V_2^2 - V_1^2\right) = 0.5\left(50^2 - 10^2\right)\left(\frac{m}{s}\right)^2 \frac{1\,N}{1\,kg.m/s^2}\frac{1\,kJ}{10^3\,N.m} = 1.2\,kJ/kg$$

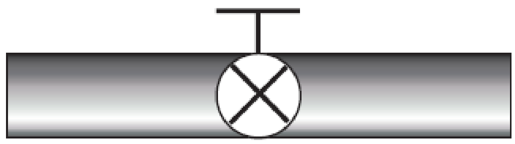
$\dot{Q}_{CV}$ = *(1000 kW) + 4600 kg/h (-831.8 + 1.2 kJ/kg) (1h / 3600 s) (1kW / 1 kJ/s)*

= -61.3 kW

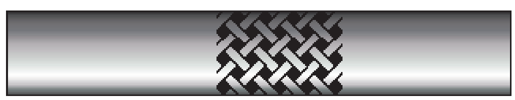
Note that change in specific kinetic energy is very small. Also $\dot{Q}_{CV}$ is small compared to $\dot{W}_{CV}$. The negative sign in $\dot{Q}_{CV}$ means that heat transfer is from CV to surroundings.

Throttling Valves

Any kind of flow restricting devices that causes a significant pressure drop in the fluid is called throttling valve.



a) an adjustable valve



b) a porous plug

c) a capillary tube

Fig. 8: Three kinds of throttling valves.

The pressure drop in fluid is often accompanied with a temperature drop in fluids. The magnitude of this temperature is governed by a property called the Joule-Thomson coefficient.

Common assumptions for throttling valves:

$$\dot{Q} = 0$$

$$\dot{W} = 0$$

$$\Delta KE = \Delta PE = 0$$

The conservation of energy becomes:

$$h_2 = h_1$$

$$u_1 + P_1 v_1 = u_2 + P_2 v_2$$

The flow energy increases during the process, $P_2 v_2 > P_1 v_1$, and it is done at the expense of the internal energy. Thus, internal energy decreases, which is usually accompanied by a drop in temperature.

For ideal gases $h = h(T)$ and since h remains constant the temperature has to remain constant too.

Mixing Chambers (Direct Contact Heat Exchangers)

Mixing chambers are used to mix two streams of fluids. Based on mass principle, the sum of incoming flow equals to sum of leaving fluid:

$$\sum \dot{m}_i = \sum \dot{m}_o$$

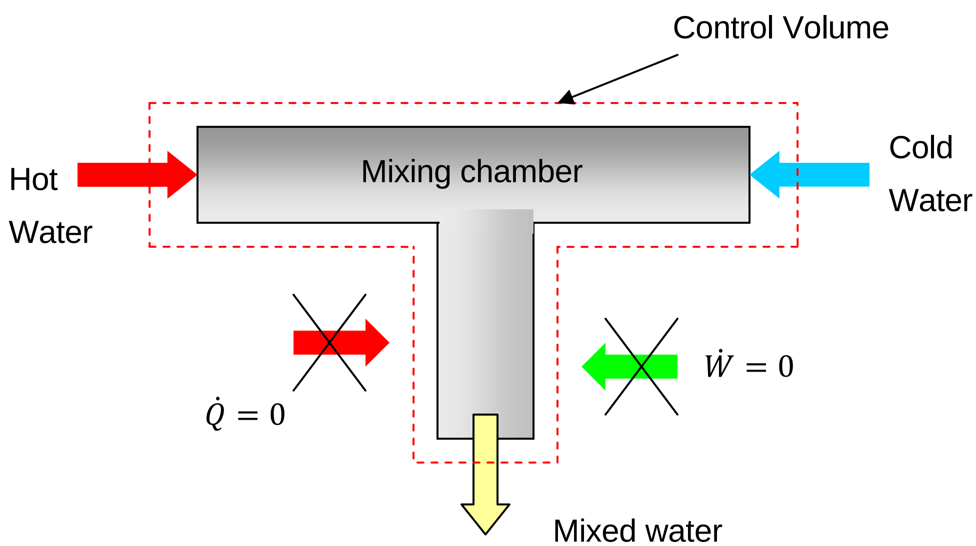


Fig. 9: T-elbow is a mixing chamber.

Common assumptions for mixing chamber:

$$\dot{Q} = 0$$

$$\dot{W} = 0$$

$$\Delta KE = \Delta PE = 0$$

The conservation of energy becomes:

$$\sum (\dot{m}h)_{inlet} = \sum (\dot{m}h)_{outlet}$$

Heat Exchangers

Heat exchangers are devices where two moving fluid streams exchange heat without mixing.

Common assumptions for heat exchangers:

$$\dot{W} = 0$$

$$\Delta KE = \Delta PE = 0$$

$\dot{Q}$ could be different depending on the selected CV, see example 3.

The mass and energy balance become:

$$\dot{m}_1 = \dot{m}_2 \qquad \text{and} \qquad \dot{m}_3 = \dot{m}_4$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_1 h_2 + \dot{m}_1 h_4$$

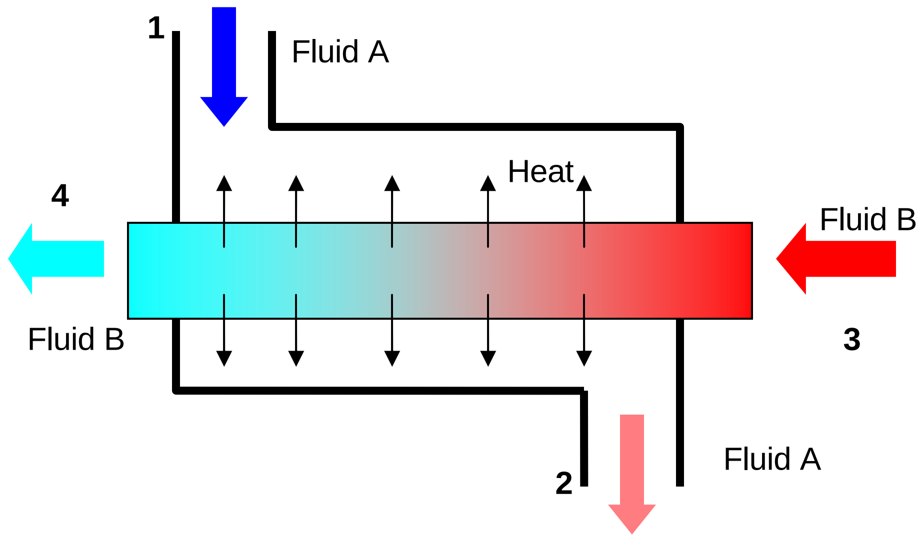
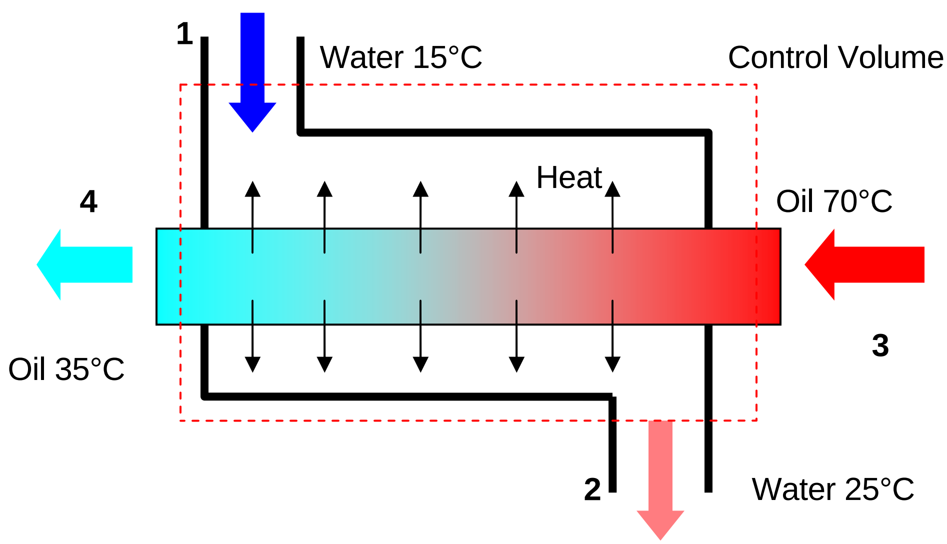


Fig. 10: Schematic for heat exchanger.

Example 3: Heat exchanger

Engine oil is to be cooled by water in a condenser. The engine oil enters the condenser with a mass flow rate of *6 kg/min* at *1 MPa* and *70°C* and leaves at *35°C*. The cooling water enters at *300 kPa* and *15°C* and leaves at *25°C*. Neglecting any pressure drops; determine a) the mass flow rate of the cooling water required, and b) the heat transfer rate from the engine oil to water.



We choose the entire heat exchanger as our control volume, thus work transfer and heat transfer to the surroundings will be zero. From mass balance:

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_W \qquad \text{and} \qquad \dot{m}_3 = \dot{m}_4 = \dot{m}_{Oil}$$

The conservation of energy equation is:

$$\dot{Q} - \dot{W} = \sum \dot{m}_e \left(h_e + \frac{V_e^2}{2} + gz_e \right) - \sum \dot{m}_i \left(h_i + \frac{V_i^2}{2} + gz_i \right)$$

$$\sum \dot{m}_e h_e = \sum \dot{m}_e h_e$$

$$\dot{m}_W h_1 + \dot{m}_{Oil} h_3 = \dot{m}_W h_2 + \dot{m}_{Oil} h_4$$

$$\dot{m}_W = \frac{h_3 - h_4}{h_2 - h_1} \dot{m}_{Oil}$$

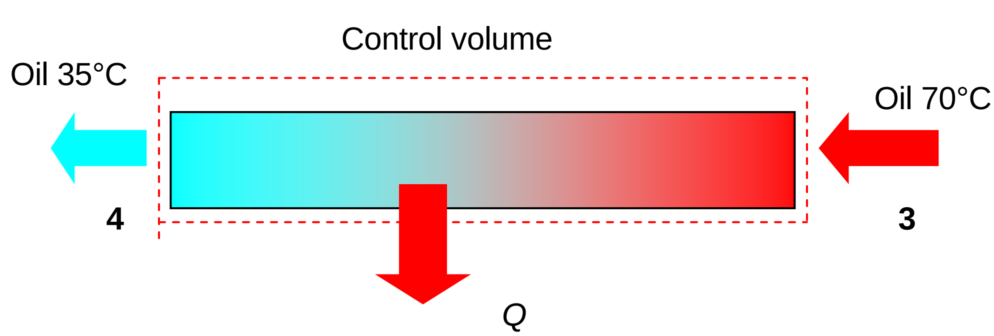
Assuming constant specific heat for both the oil and water at their average temperature,

$$\dot{m}_W = \frac{c_{P,Oil}(T_3 - T_4)}{c_{P,Water}(T_2 - T_1)} = \frac{\left(2.016 \frac{kJ}{kg.C}\right)(70-35)}{\left(4.18 \frac{kJ}{kg.C}\right)(25-15)} (6\ kg/\min) = 10.1\ kg/\min$$

b) To determine the heat transfer from the oil to water, choose the following CV. The energy equation becomes:

$$\dot{Q}_{Oil} - \dot{W} = \dot{m}_{Oil}(\Delta h) = \dot{m}_{Oil} c_{P,Oil}(T_4 - T_3)$$

$$\dot{Q}_{Oil} = (6\ kg/\min)\left(2.016 \frac{kJ}{kg.C}\right)(35-70) = -423.36\ kJ/kg$$



Energy Analysis of Unsteady-Flow Processes

Unsteady or transient processes involve changes within the control volume with time. Thus, it is important to keep track of the mass and energy contents of the control volume as well as the energy interactions across the boundary.

Note: transient (or unsteady) processes start and end over some finite time period.

The general form of mass balance for any processes can be written:

$$m_{in} - m_{out} = \Delta m_{system} = (m_2 - m_1)_{CV}$$

To analyze transient processes, we assume *uniform-flow process* which means: the fluid flow at any inlet or exit is uniform and steady; thus the fluid properties do not change with time or position over the cross-section of an inlet or exit.

The energy balance, for uniform-flow processes can be expressed as:

Net energy transfer by heat, work, and mass = *Changes in internal, kinetic, potential energies of the system*

$$E_{in} - E_{out} = \Delta E_{system}$$

$$Q - W - \sum m_{out}\theta_{out} + \sum m_{in}\theta_{in} = \left[m_2\left(u_2 + \frac{V_2^2}{2} + gz_2\right) - m_1\left(u_1 + \frac{V_1^2}{2} + gz_1\right)\right]_{CV}$$

When the kinetic and potential energy changes are negligible, the energy balance reduces to:

$$Q - W - \sum m_{out}h_{out} + \sum m_{in}h_{in} = [m_2u_2 - m_1u_1]_{CV}$$