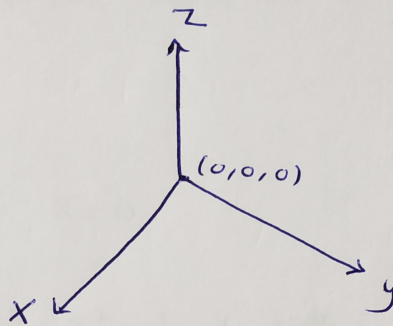


12.1 Three-Dimensional Coordinate Systems.



* The distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$

is $|P_1 P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

* The standard equation of the sphere of radius a and center (x_0, y_0, z_0) is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

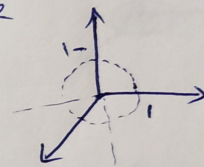
Questions: 8, 12, 14, 20, 26, 30, 36, 43

56, 64, 65.

Give a geometric description of the set of points in space whose coordinates satisfy the following:-

[8] $y^2 + z^2 = 1, x = 0$

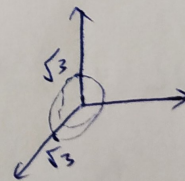
The circle $y^2 + z^2 = 1$ (with center $(0, 0, 0)$ and radius of 1) in the zy -plane



[12] $x^2 + (y-1)^2 + z^2 = 4, y = 0$

$$x^2 + (0-1)^2 + z^2 = 4 \rightarrow x^2 + z^2 = 3$$

The circle $x^2 + z^2 = 3$ (with center $(0, 0, 0)$ and radius of $\sqrt{3}$) in the xz -plane



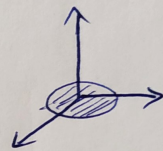
[14] $x^2 + y^2 + z^2 = 4, y = x$

a circle (with center $(0, 0, 0)$ and radius of 2) formed by the intersection of the sphere $x^2 + y^2 + z^2 = 4$ and the plane $y = x$

20 Describe the sets of points in space whose coordinates satisfy the following:-

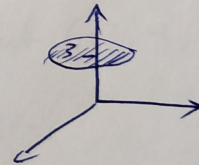
a) $x^2 + y^2 \leq 1$, $z = 0$

The disk (circumference and interior of the circle) $x^2 + y^2 \leq 1$ in the xy -plane.



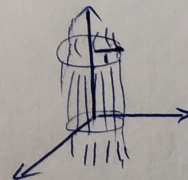
b) $x^2 + y^2 \leq 1$, $z = 3$

The disk (circumference and interior of $x^2 + y^2 = 1$) in the plane $z = 3$.



c) $x^2 + y^2 \leq 1$, no restriction on z

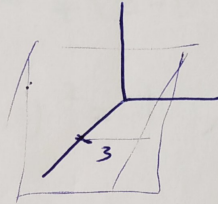
a solid cylindrical column of radius 1 whose axis is the z -axis



26 The plane through the point $(3, -1, 2)$
Perpendicular to the :-

a) x -axis

$$x = 3$$



b) y -axis

$$y = -1$$

c) z -axis :- $z = 2$

30 The circle of radius 1 centered at $(-3, 4,$
and lying in a plane parallel to the:-

a) xy -plane :- $(z = 1)$

$$(x+3)^2 + (y-4)^2 + (z-1)^2 = 1$$

$$(x+3)^2 + (y-4)^2 + (1-1)^2 = 1$$

$$(x+3)^2 + (y-4)^2 = 1, \quad z = 1$$

b) yz-plane $x = -3$

$$(x+3)^2 + (y-4)^2 + (z-1)^2 = 1$$

$$(-3+3)^2 + (y-4)^2 + (z-1)^2 = 1$$

$$(y-4)^2 + (z-1)^2 = 1, \quad x = -3$$

c) xz-plane:- $y = 4$

$$(x+3)^2 + (y-4)^2 + (z-1)^2 = 1$$

$$(x+3)^2 + (z-1)^2 = 1, \quad y = 4$$

36 Write inequality to describe :

The solid cube in the first octant bounded by the coordinate planes and the planes

$$x = 2, \quad y = 2 \quad \text{and} \quad z = 2$$

$$0 \leq x \leq 2, \quad 0 \leq y \leq 2, \quad 0 \leq z \leq 2.$$

43) Find the distance between points

$$P_1(1, 4, 5), P_2(4, -2, 7)$$

$$\begin{aligned} |P_1 P_2| &= \sqrt{(4-1)^2 + (-2-4)^2 + (5-7)^2} \\ &= \sqrt{9 + 36 + 4} = 7 \end{aligned}$$

56) Find the center and the radius of the sphere

$$x^2 + y^2 + z^2 - 6y + 8z = 0$$

$$x^2 + (y^2 - 6y) + (z^2 + 8z) = 0$$

$$x^2 + \left(y^2 - 6y + \left(-\frac{6}{2}\right)^2\right) + \left(z^2 + 8z + \left(\frac{8}{2}\right)^2\right) = 0 + \left(-\frac{6}{2}\right)^2 + \left(\frac{8}{2}\right)^2$$

$$x^2 + (y^2 - 6y + 9) + (z^2 + 8z + 16) = 9 + 16$$

$$x^2 + (y-3)^2 + (z+4)^2 = 25$$

Center : $(0, +3, -4)$, radius : 5

[64] Find the equation for the set of points equidistant from the point $P_1(0,0,2)$ and the xy -plane

We need to find the point $P_2(x,y,z)$

We take a point $P_3(x,y,0)$ on the xy -plane

$$|P_1 P_2| = |P_2 P_3|$$

$$\sqrt{(x-0)^2 + (y-0)^2 + (z-2)^2} = \sqrt{(x-x)^2 + (y-y)^2 + (z-0)^2}$$

$$\sqrt{x^2 + y^2 + z^2 - 4z + 4} = \sqrt{z^2}$$

$$x^2 + y^2 + z^2 - 4z + 4 = z^2$$

$$x^2 + y^2 - 4z + 4 = 0$$

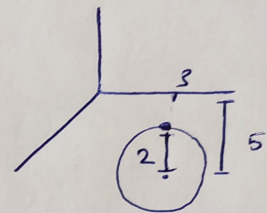
$$\frac{x^2}{4} + \frac{y^2}{4} + 1 = z$$

65 Find the point on the sphere $x^2 + (y-3)^2 + (z+5)^2 = 4$ nearest

a) the xy -plane

center $(0, 3, -5)$, radius 2

the nearest point is the point
at the top of the sphere
 $(0, 3, -3)$



b) the point $(0, 7, -5)$

$(0, 5, -5)$

