Chapter 6: Structures for Discrete-Time systems

To represent the discrete systems by structures consisting of an interconnections of the basic operations: Multiplication, addition, delay, memory

Examples:
$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a z^{-1}} \qquad |z| > a$$

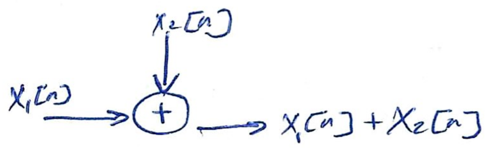
$$h[n] = b_0 a^n u[n] + b_1 a^{n-1} u[n-1]$$

$$y[n] - a y[n-1] = b_0 x[n] + b_1 x[n-1]$$

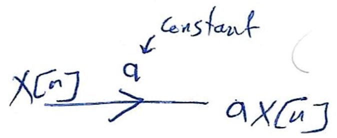
$$y[n] = a y[n-1] + b_0 x[n] + b_1 x[n-1]$$

The basic elements of implementing LTI system are:

① Adders:

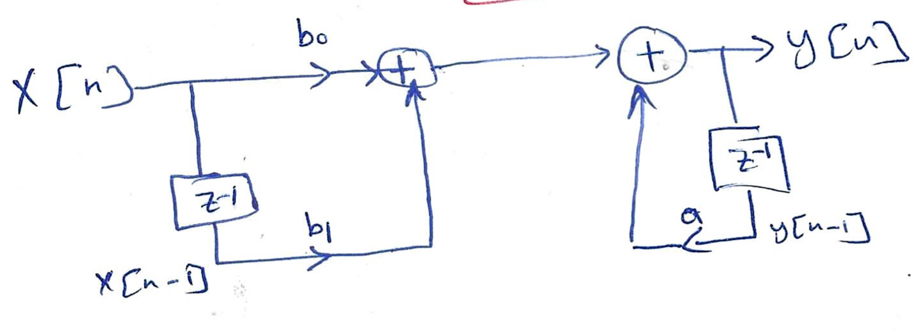


② Multiplier (by a constant):



③ Memory for storing delayed sequence values $\quad x[n] \rightarrow \boxed{z^{-1}} \rightarrow x[n-1]$

$x[n] \rightarrow \boxed{LTI} \rightarrow y[n]$

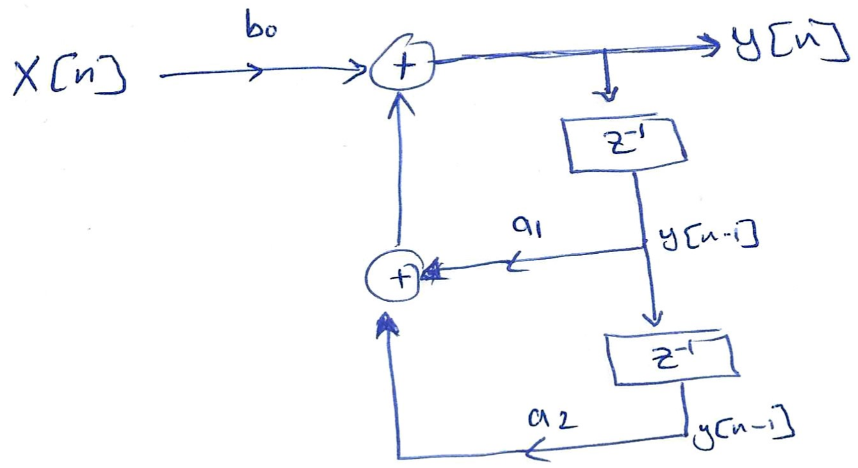


Example:- 2nd order difference equation:-

$$H(z) = \frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

$$y[n] = a_1 y[n-1] + a_2 y[n-2] + b_0 x[n]$$

Block diagram:- Direct form one



*general formula equation of difference equation:

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{k=0}^{M} a_k x[n-k]$$

$$y[n] - \sum_{k=1}^{N} a_k y[n-k] = \sum_{k=0}^{M} a_k x[n-k]$$

$$H(z) = \frac{\sum_{k=0}^{M} a_k x[n-k]}{1 - \sum_{k=1}^{N} a_k y[n-k]}$$

should be "1" $a_0 = 1$ or we need to make a division

then we can represent this equation using blocks as shown in the next figure

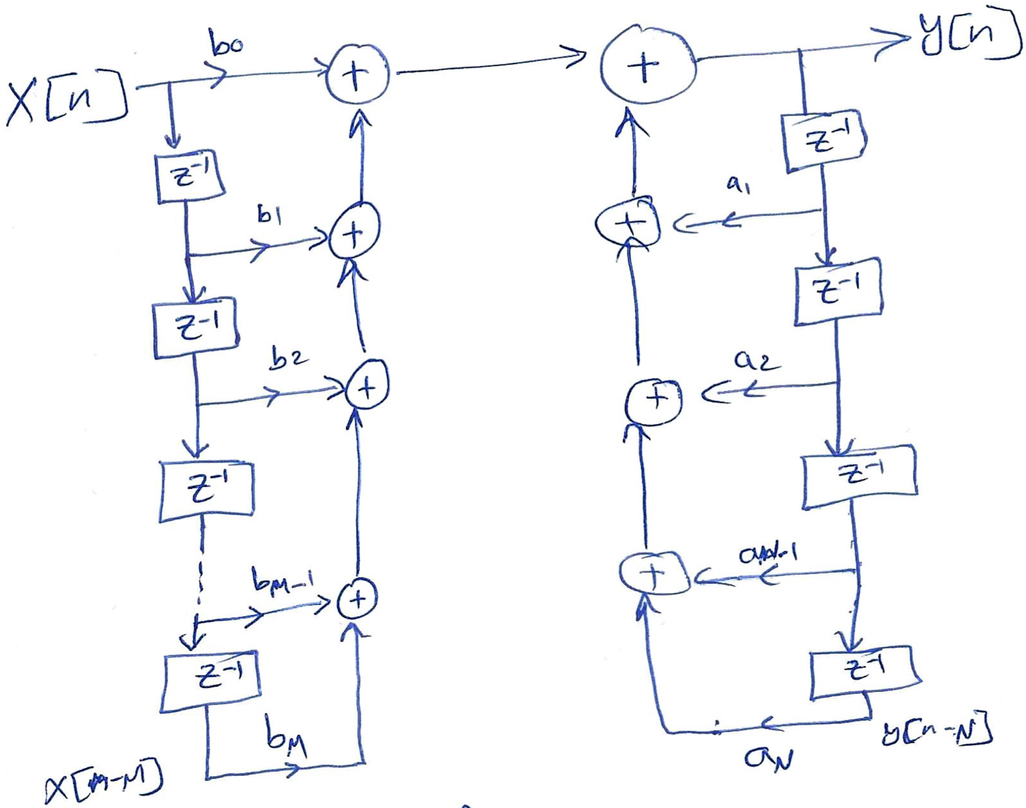
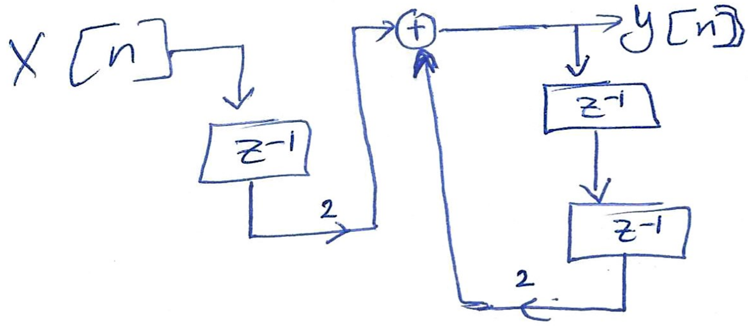
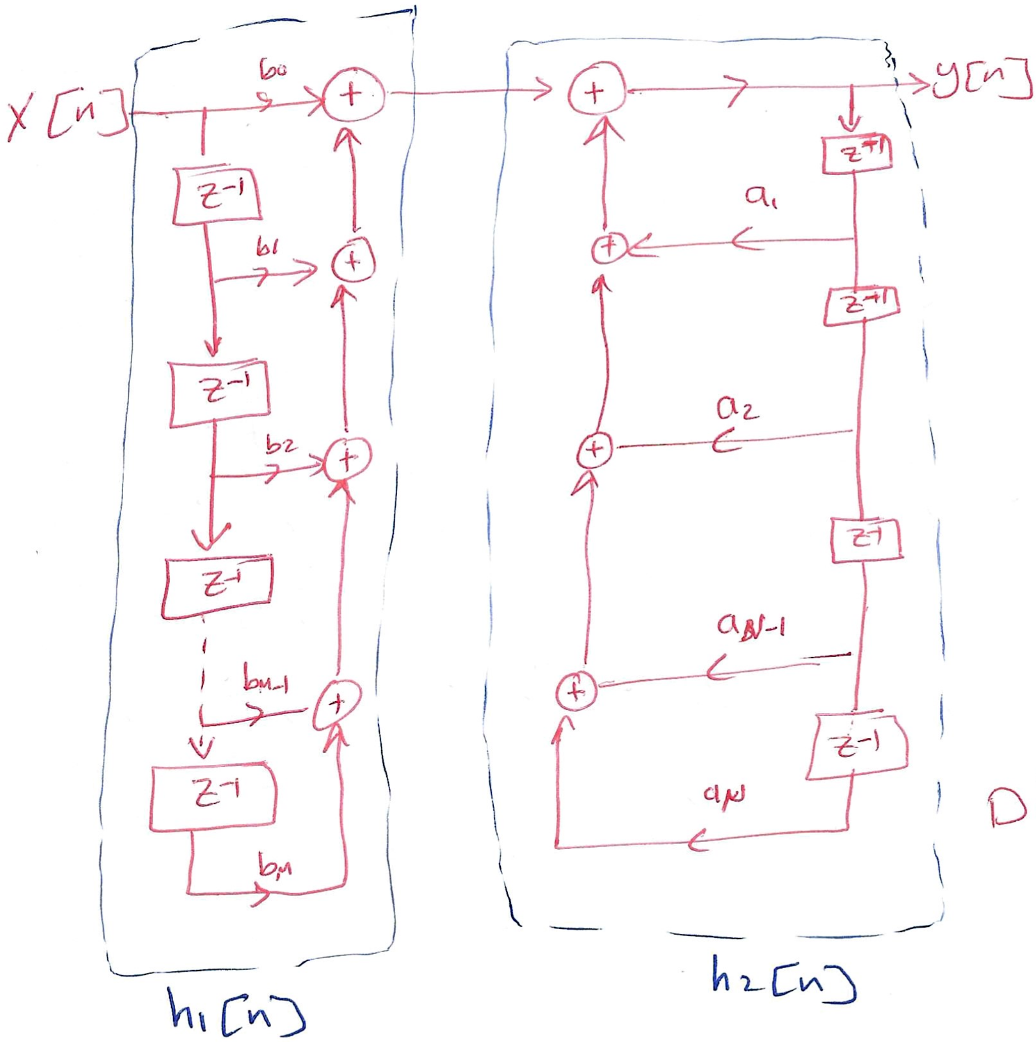


Figure: Direct form one

Example: $H(z) = \dfrac{2z^{-1}}{1 - 2z^{-2}}$

$$y[n] = 2y[n-2] + 2x[n-1]$$

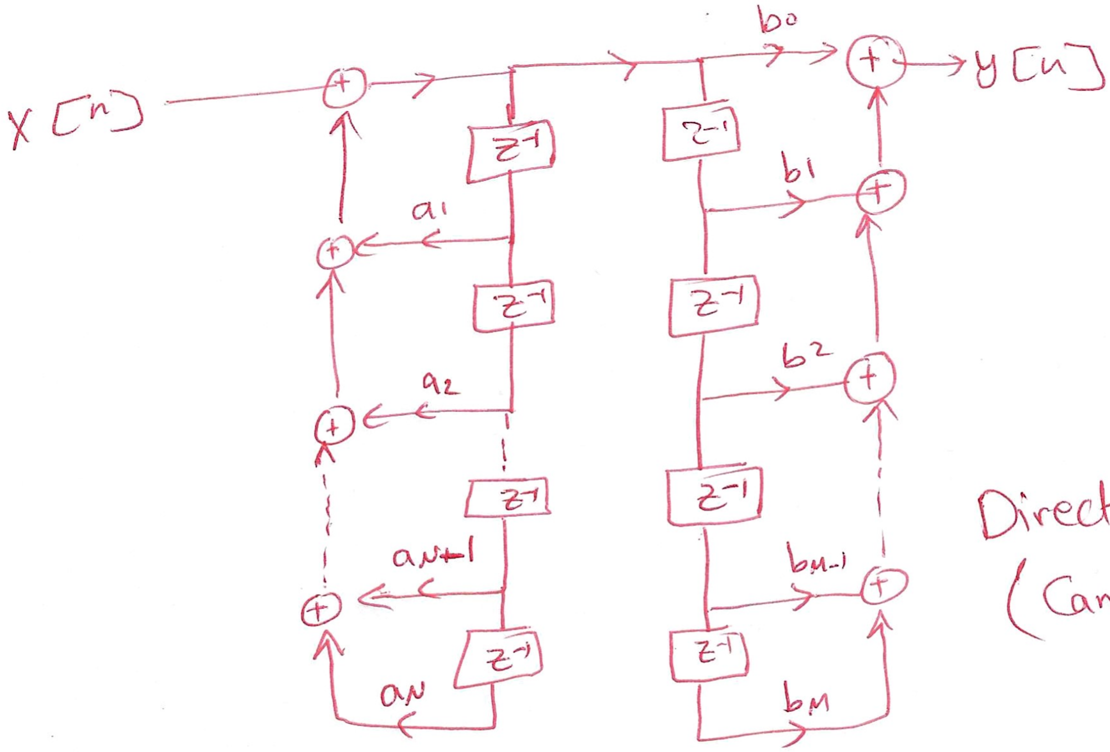




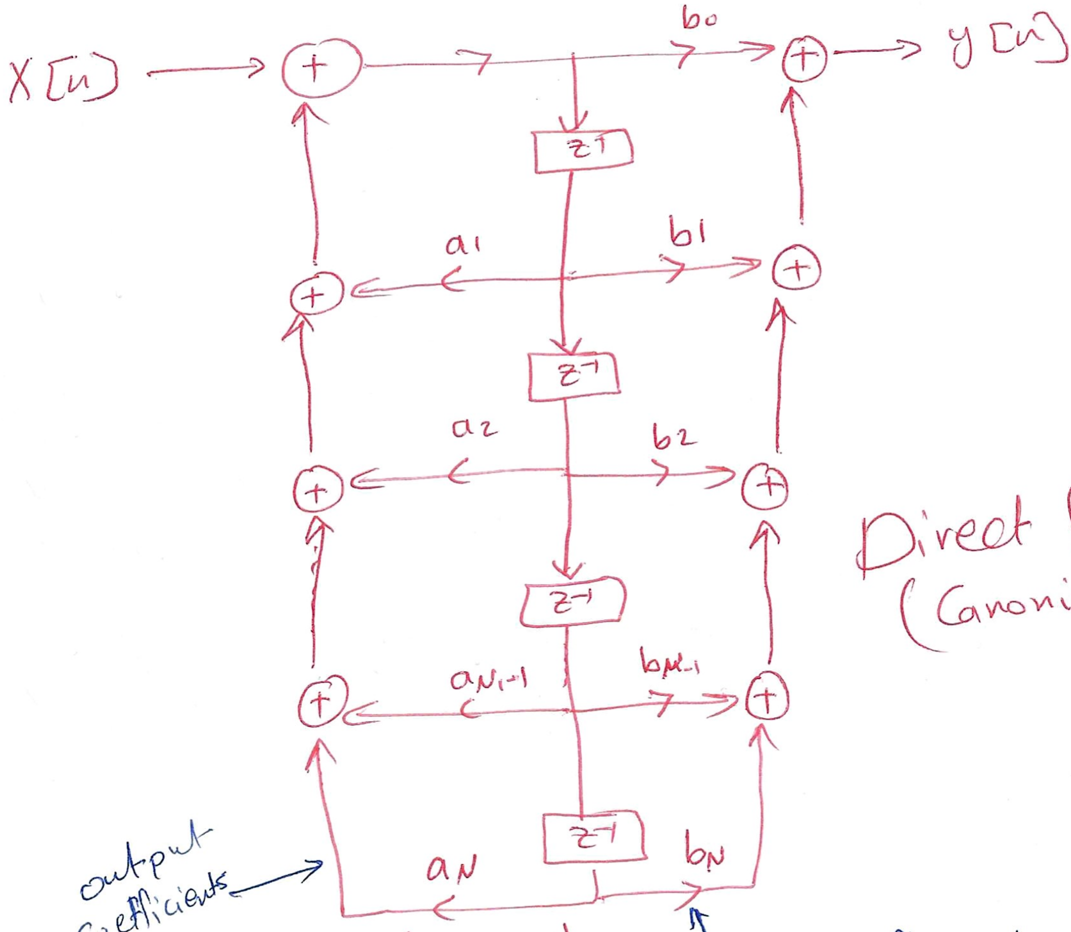
Direct form 1

$$h[n] = h_1[n] * h_2[n]$$
$$= h_2[n] * h_1[n]$$

} Commutative



Direct form 2
(Canonic form)



Direct form 2
(Canonic form)

* minimum number of delay unit

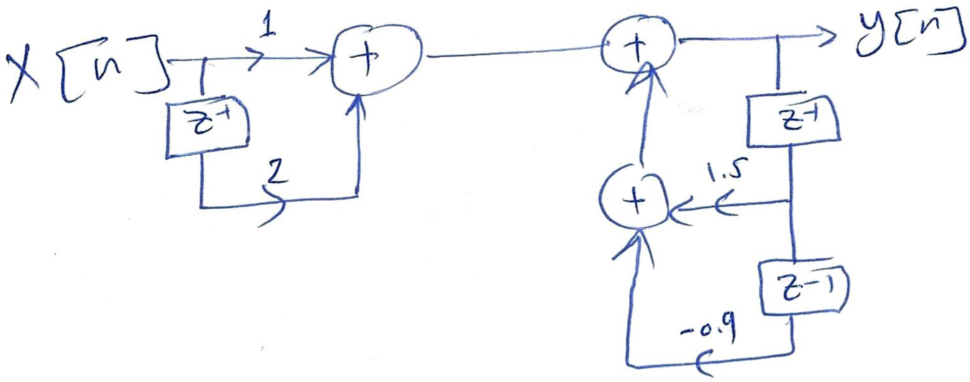
input coefficients

Examples-

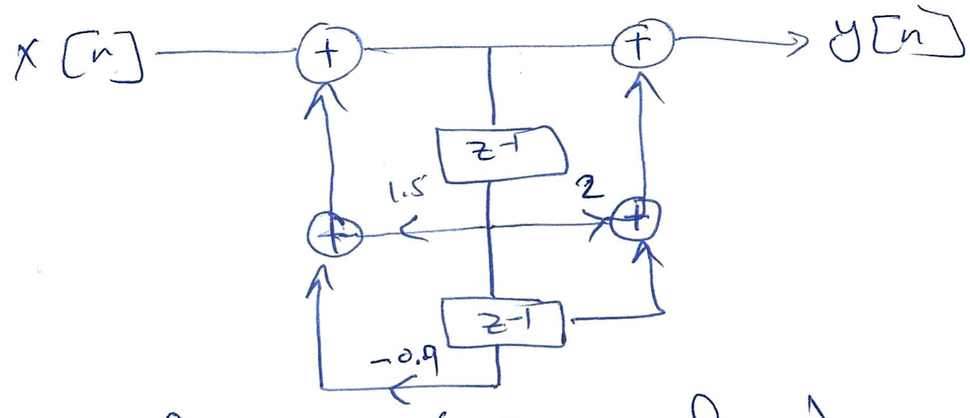
$$H(z) = \frac{1 + 2z^{-1}}{1 - 1.5z^{-1} + 0.9z^{-2}}$$

Draw direct form I ?

Draw direct form II ?

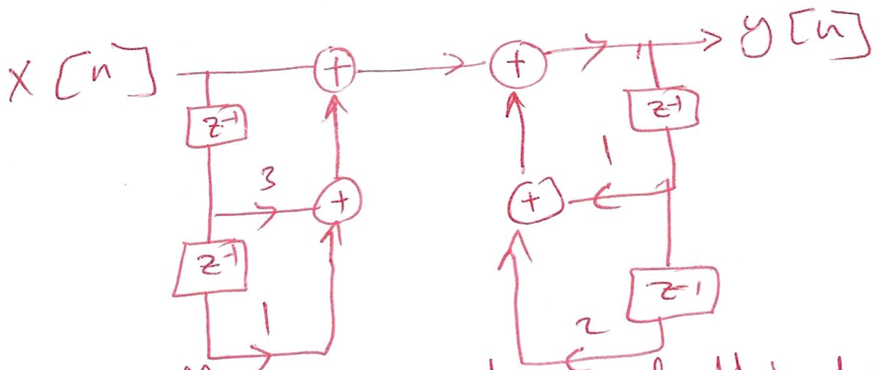


Direct form I



Direct form II (Canonic form)

Examples - LTI system



a) write the difference equation of this LTI system?

$$y[n] = y[n-1] + 2y[n-2] + x[n] + 3x[n-1] + x[n-2]$$

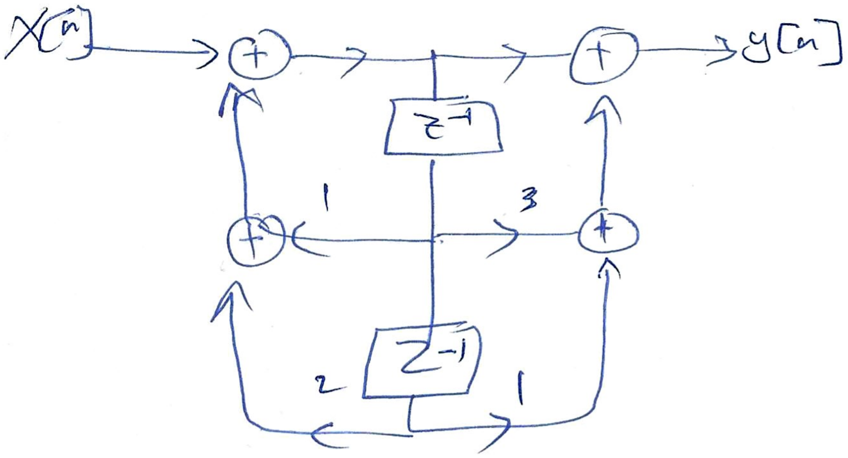
b) Find system function H(z)?

$$H(z) = \frac{1 + 3z^{-1} + z^{-2}}{1 - \cancel{z}z^{-1} - 2z^{-2}}$$

c) How many real multiplications and real additions are required to compute each sample of the output?

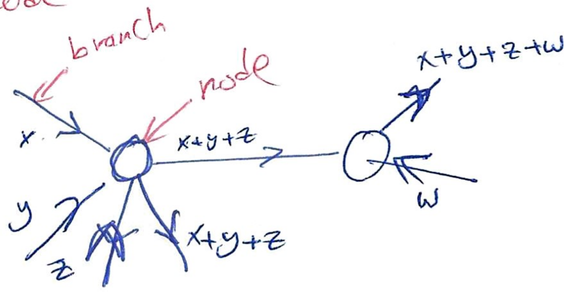
4 adders and 2 multipliers

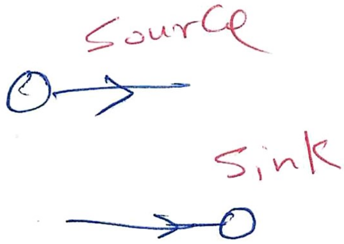
d) This realization requires four storage register (delay), is it possible to reduce no. of storage Registers? If so, draw the structure.

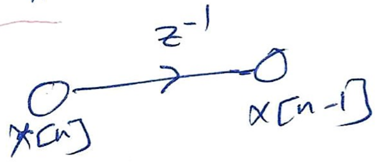


* Signal flow graph

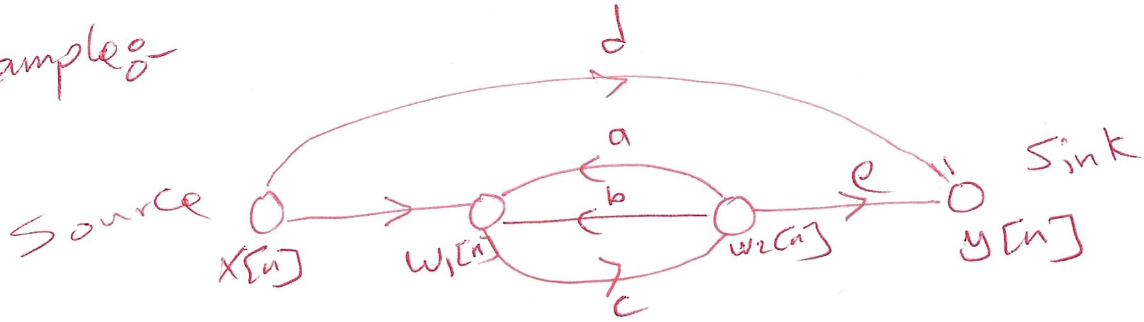
- branch
- node







Example:-



$$w_1[n] = x[n] + a\,w_2[n] + b\,w_2[n]$$

$$w_2[n] = c\,w_1[n]$$

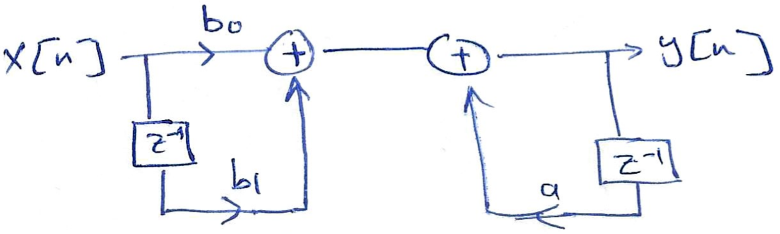
$$y[n] = e\,w_2[n] + d\,x[n]$$

Example: First order Digital Filter

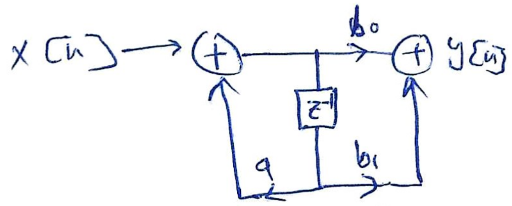
$$H(z) = \frac{b_0 + b_1 z^{-1}}{1 - a z^{-1}}$$

$$y[n] = a\,y[n-1] + b_0 x[n] + b\,x[n-1]$$

Block diagram

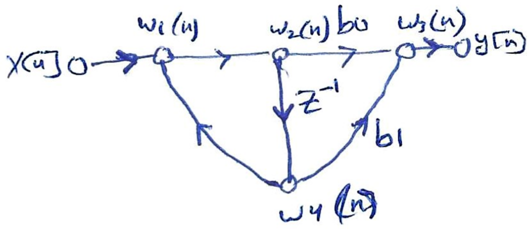


Direct form I



Direct form II

Signal flow graph



* Multi-step Algorithms to write the difference equation of a complex structure.

$$w_1(n) = X(n) + a\,w_4(n)$$

$$w_2(n) = w_1(n)$$

$$w_3(n) = b_0 w_2(n) + b_1 w_4(n)$$

$$w_4(n) = w_2(n-1)$$

$$y[n] = w_3[n]$$

$$y[n] = b_0 w_2(n) + b_1 (w_4(n))$$
$$= b_0 w_2(n) + b_1 w_2(n-1)$$

$$w_1(z) = X(z) + a\,w_4(z)$$

$$w_2(z) = w_1(z)$$

$$w_2(z) = X(z) + a\,w_4(z)$$

$$w_4(z) = z^{-1} w_2(z)$$

$$w_2(z) = X(z) + a z^{-1} w_2(z)$$

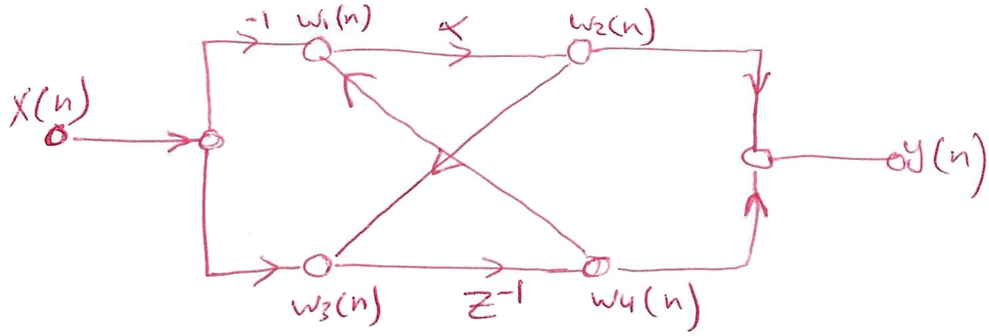
$$w_2(z) = \frac{X(z)}{1 - a z^{-1}}$$

$$Y(z) = b_0 W_2(z) + b_1 z^{-1} W_2(z)$$

$$Y(z) = b_0\left[\frac{X(z)}{1 - a z^{-1}}\right] + \frac{b_1 z^{-1} X(z)}{1 - a z^{-1}}$$

$$= \left(\frac{b_0 + b_1 z^{-1}}{1 - a z^{-1}}\right) X(z)$$

Example :-



Find the system Function $H(z)$?

$w_1(n) = -x(n) + w_4(n)$

$w_2(n) = \alpha w_1(n)$

$w_3(n) = x(n) + w_2(n)$

$w_4(n) = w_3(n-1)$

$y(n) = w_2(n) + w_4(n)$

$W_1(z) = -X(z) + W_4(z)$

$W_2(z) = \alpha W_1(z)$

$W_3(z) = X(z) + W_2(z)$

$W_4(z) = z^{-1} W_3(z)$

$Y(z) = W_2(z) + W_4(z)$

$$W_2(z) = \alpha W_1(z)$$
$$= \alpha(-X(z) + W_4(z))$$

$$W_4(z) = z^{-1} W_3(z)$$
$$= z^{-1}(X(z) + W_2(z))$$

$$W_4(z) = z^{-1}(X(z) + \alpha(-X(z) + W_4(z)))$$
$$= z^{-1}X(z) - z^{-1}\alpha X(z) + z^{-1}\alpha W_4(z)$$

$$W_4(z) - z^{-1}\alpha W_4(z) = z^{-1}X(z) - z^{-1}\alpha X(z)$$

$$W_4(z)(1 - z^{-1}\alpha) = X(z)(z^{-1} - z^{-1}\alpha)$$

$$W_4(z) = \frac{X(z)\, z^{-1}(1-\alpha)}{1 - z^{-1}\alpha}$$

$$W_2(z) = \alpha\left(W_4(z) - X(z)\right)$$

$$= \alpha\left(z^{-1}X(z) + z^{-1}W_2(z) - X(z)\right)$$

$$W_2(z) - \alpha z^{-1}W_2(z) = \alpha z^{-1}X(z) - \alpha X(z)$$

$$W_2(z) = \frac{\alpha(z^{-1}-1)}{1-\alpha z^{-1}} X(z)$$

$$Y(z) = W_2(z) + W_4(z)$$

$$= \frac{\alpha z^{-1} - \alpha}{1-\alpha z^{-1}} X(z) + \frac{z^{-1} - z^{-1}\alpha}{1-\alpha z^{-1}} X(z)$$

$$= \frac{\alpha z^{-1} - \alpha + z^{-1} - \alpha z^{-1}}{1 - \alpha z^{-1}} X(z)$$

$$Y(z) = \frac{z^{-1} - \alpha}{1-\alpha z^{-1}} X(z)$$

$$H(z) = \frac{z^{-1} - \alpha}{1-\alpha z^{-1}}$$

$$y[n] - \alpha y[n-1] = x(n-1) - \alpha x(n)$$

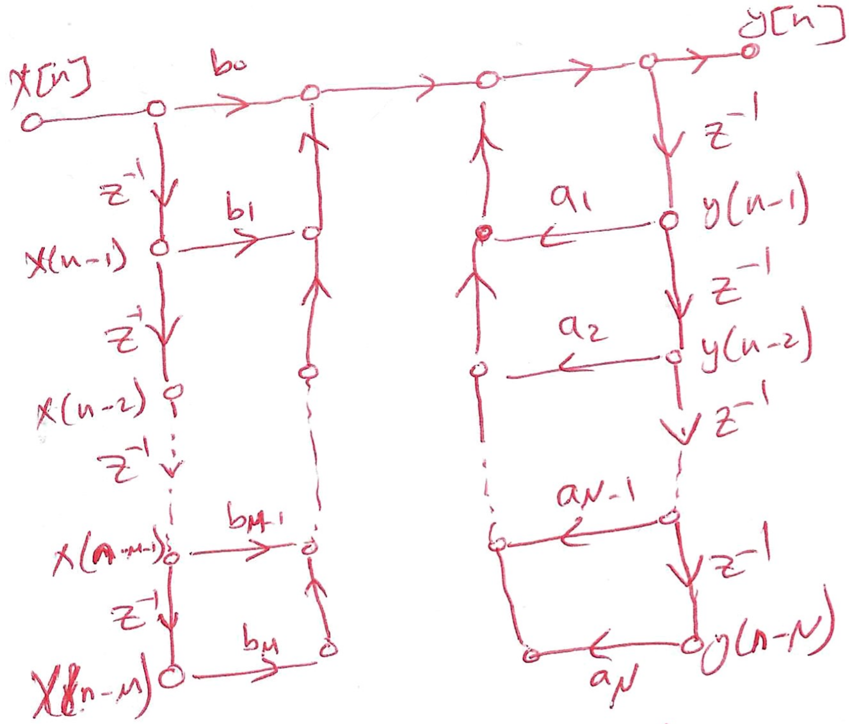
$$h(n) = (\alpha)^{n-1}u[n-1] - \alpha(\alpha)^n u[n]$$

6.3

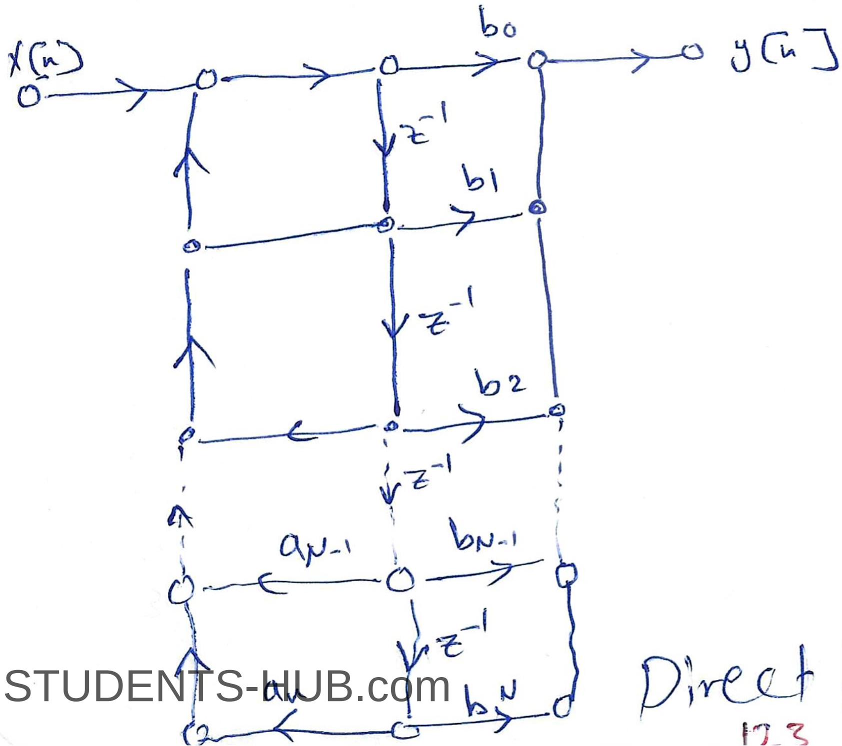
* Basic Structure for IIR Systems :-

* Direct forms :-

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$



signal flow graph of Direct form I

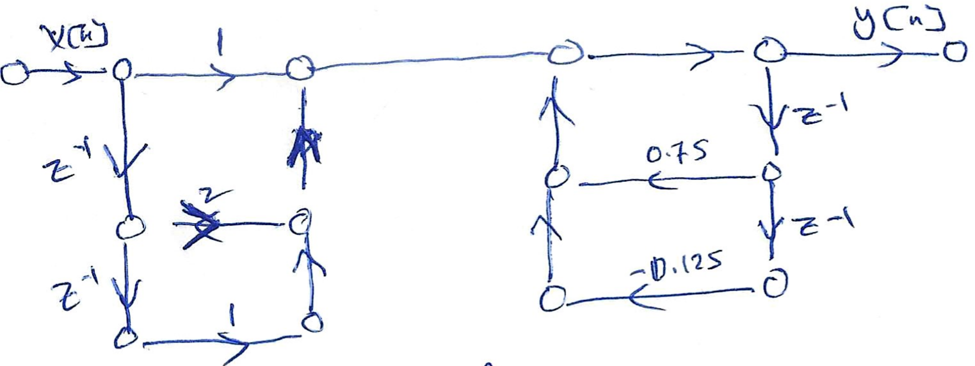


Direct form II

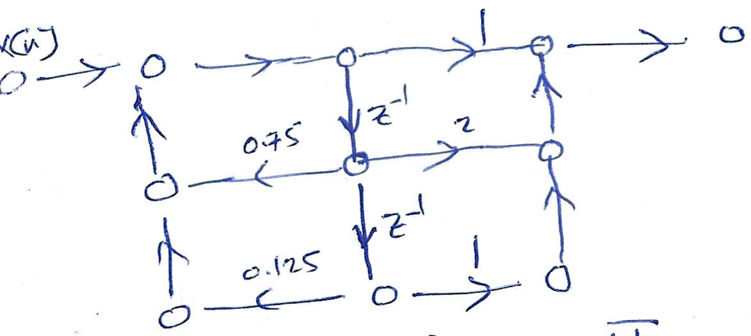
Example:- $H(z) = \dfrac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$

Draw Direct form I

Direct form II



Direct form I



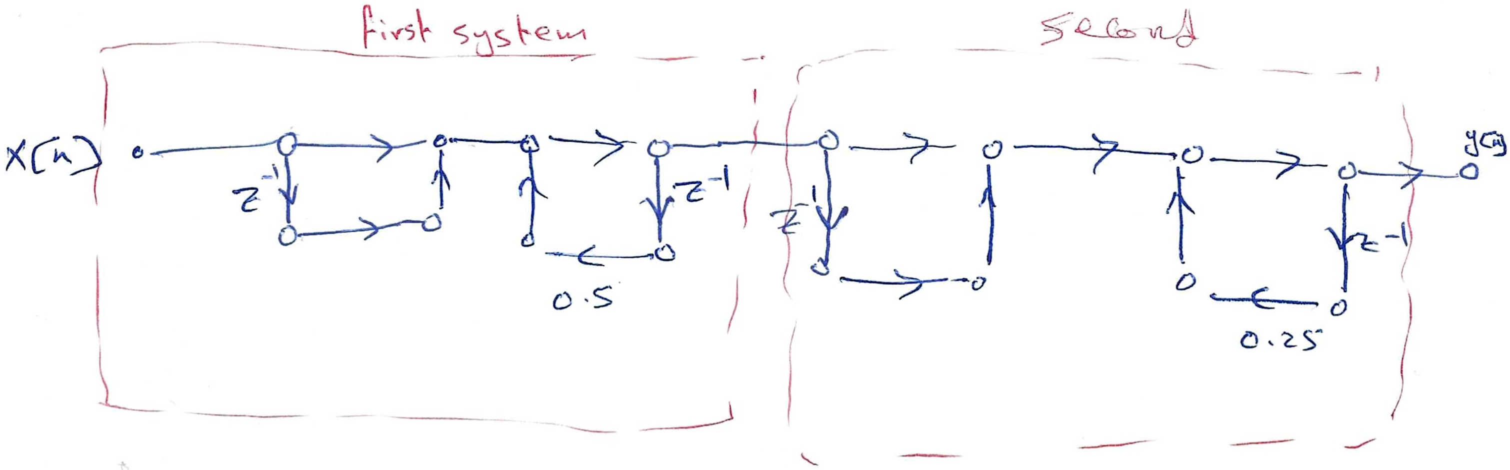
Direct form II

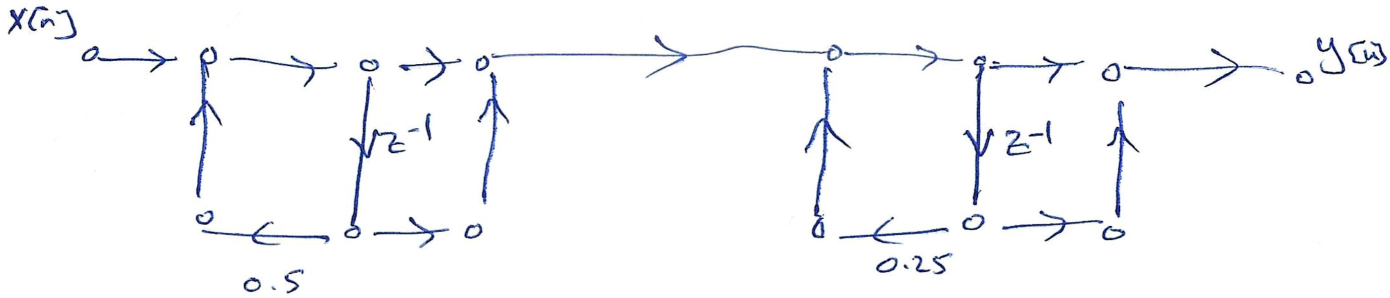
* Cascade form:-

$$H(z) = H_1(z) \cdot H_2(z) \cdot H_3(z) \cdots$$

Example:-

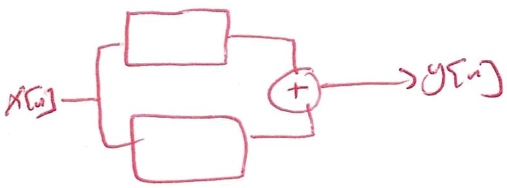
$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}} = \frac{(1+z^{-1})(1+z^{-1})}{(1-0.5z^{-1})(1-0.25z^{-1})}$$





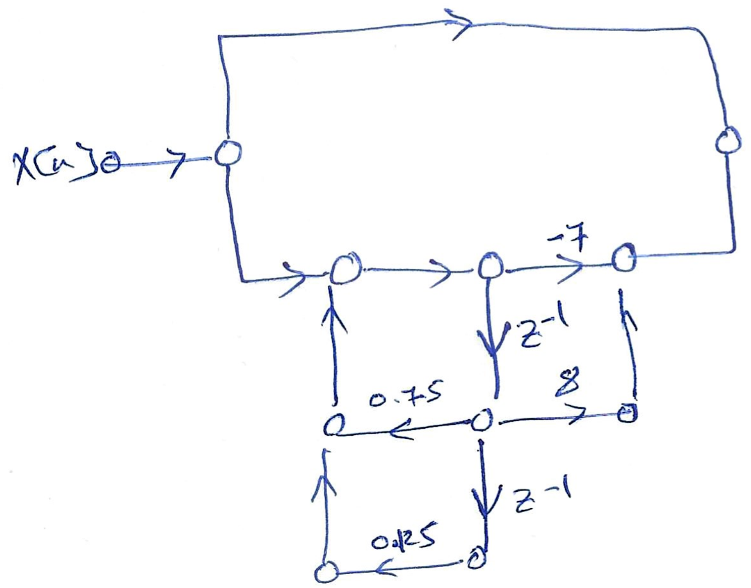
* Parallel form:-

$$H(z) = H_1(z) + H_2(z)$$



Example:- $H(z) = \dfrac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}}$

$$H(z) = 8 + \frac{-7 + 8z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$



$$H(z) = 8 + \frac{18}{1-0.5z^{-1}} - \frac{25}{1-0.25z^{-1}}$$