Internal Forces and Moments

Chapter 7

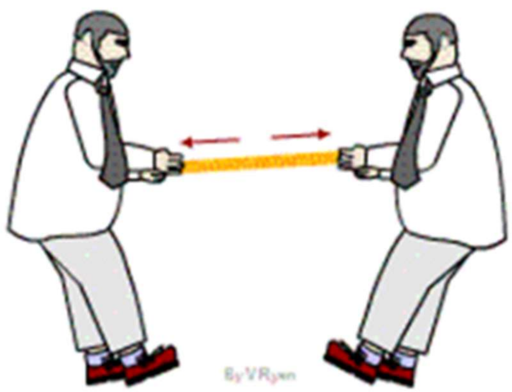
Outlines

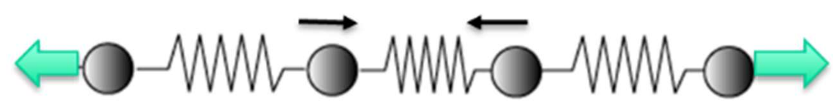
Objectives

- Consider the general state of internal member forces, which includes axial force, shearing force, and bending moment.
- Apply equilibrium analysis methods to obtain specific values, general expressions, and diagrams for shear and bending-moment in beams.
- Examine relations among load, shear, and bending moment, and use these to obtain shear and bending moment diagrams for beams.

7.1 Internal Forces in Members

Internal forces: the forces that induced by external forces and keeps the body together

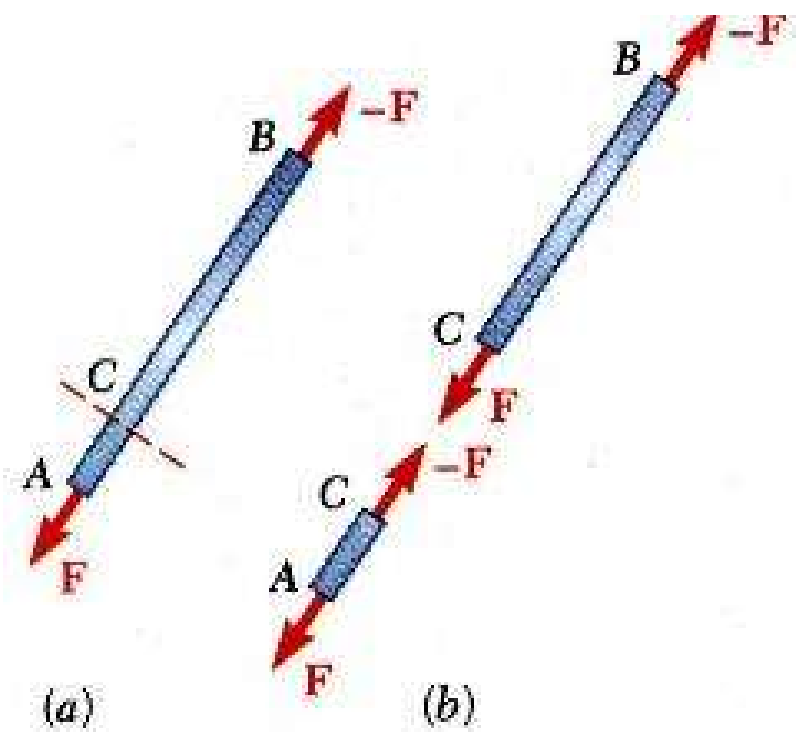




- Internal forces in a body can be determined using the concept of equilibrium. If a body is in equilibrium then any part of it is also in equilibrium.
- To determine the internal forces cut the body to two parts at specific location and consider the equilibrium of each part to identify the internal forces at this location.

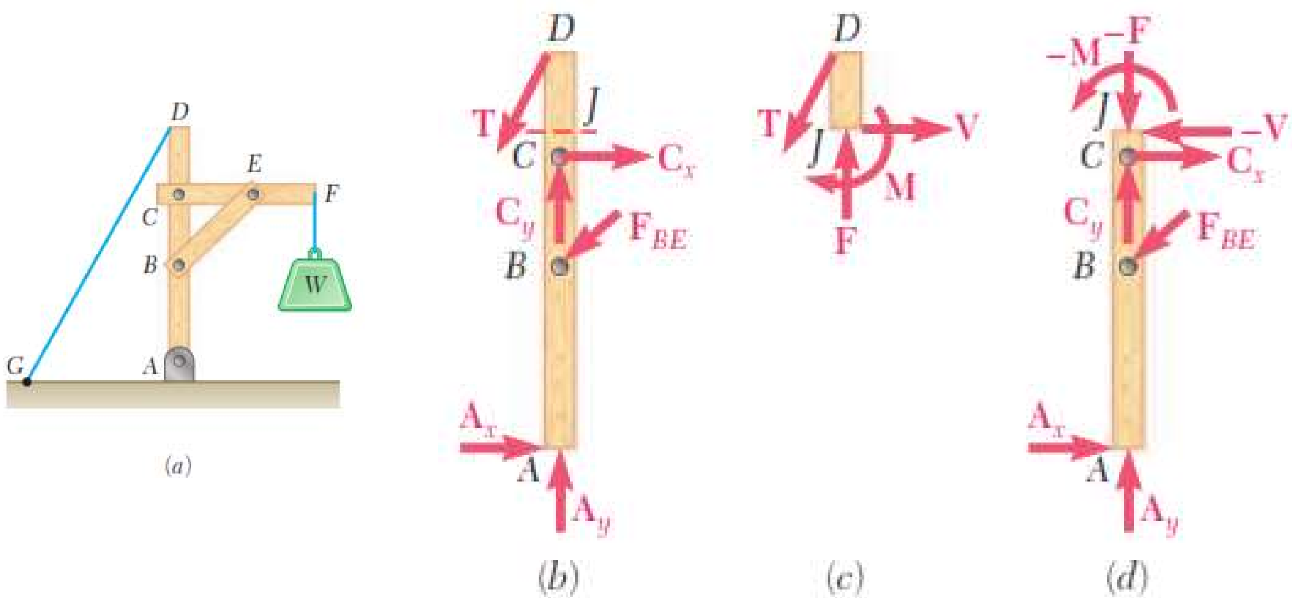
7.1 Internal Forces in Members

1. For a Straight two-force member AB that is in equilibrium under application of F and -F. Internal forces equivalent to F and -F are required for equilibrium of free-bodies AC and CB. In this case the internal force F is constant every where through the member length (wherever the cut)

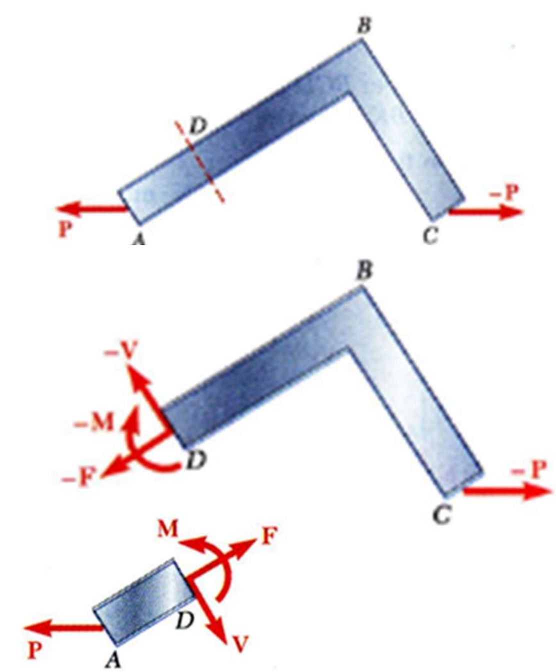


7.1 Internal Forces in Members

2. For a Multi-force member (ABCD) or not straight 2-force member. If the member is in equilibrium under application of external and member contact forces shown. Internal forces equivalent to a force-couple system are necessary for equilibrium of free-bodies JD and ABCJ. The force-couple system will have different values if the location of the cut changed.



Multi-force member

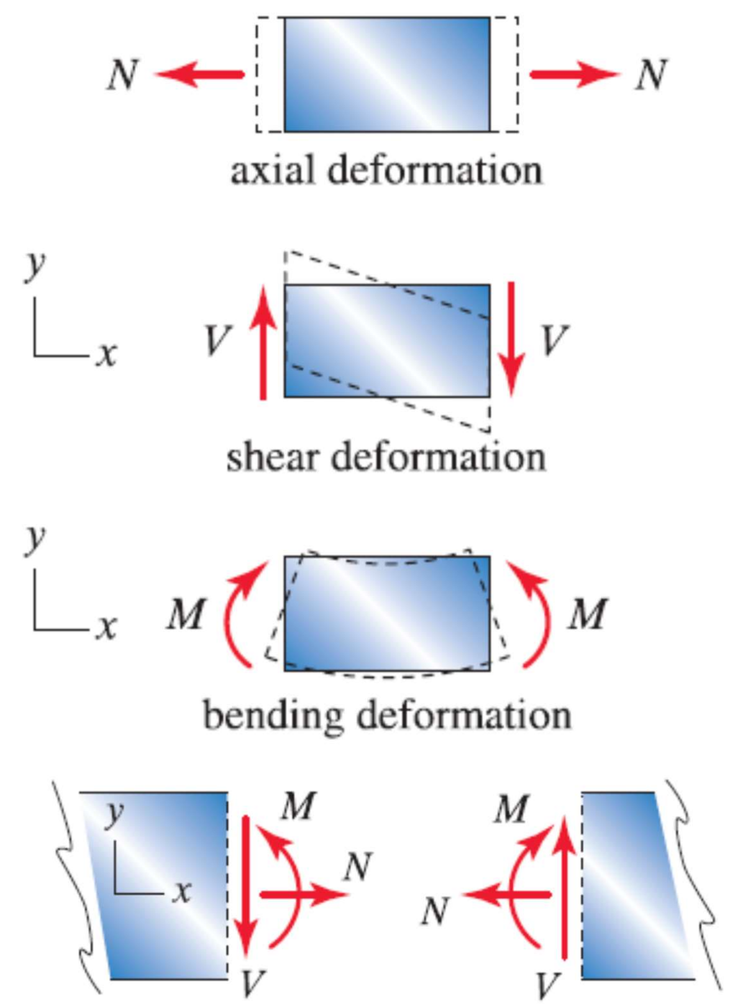


Not straight 2-force member

7.1 Internal Forces in Members

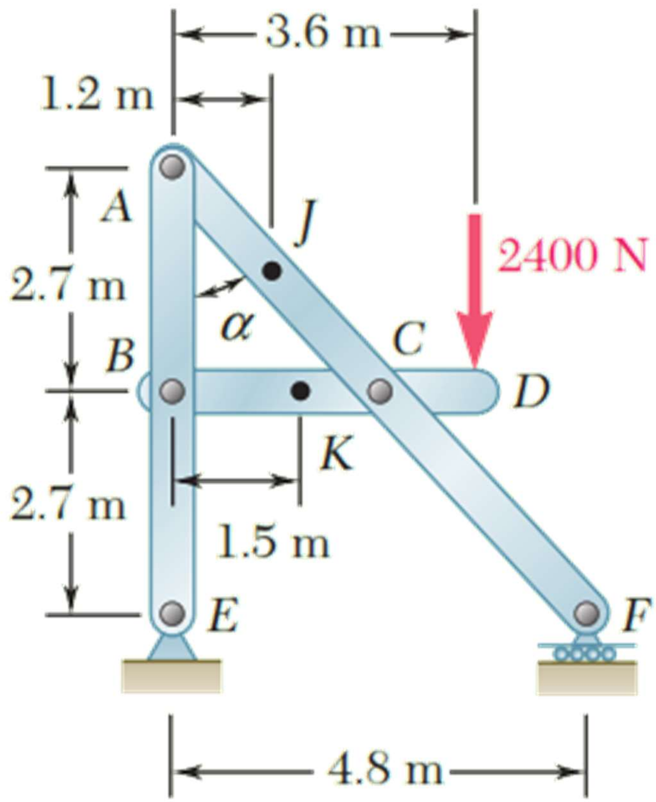
- To summarize, Internal forces that develop on a particular cross section of a structural member in two dimensions are :

1. The normal force or axial force (N), that gives rise to the axial deformation.
2. The shear force (V) that gives rise to the shear deformation.
3. The bending moment (M) that gives rise to the bending deformation.



- **Sign Convention.** We will usually follow the sign convention shown in the figures to indicate the positive internal forces.

Sample Problem 7.1

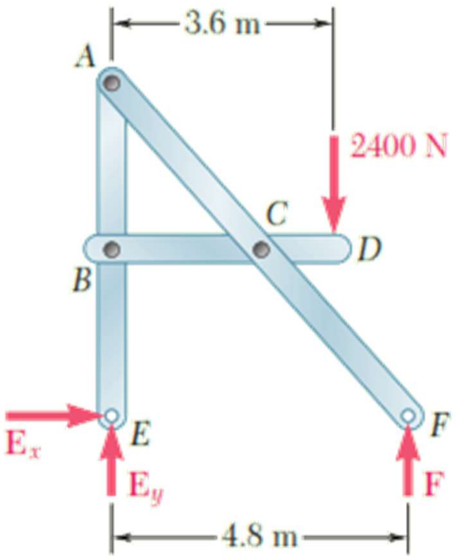


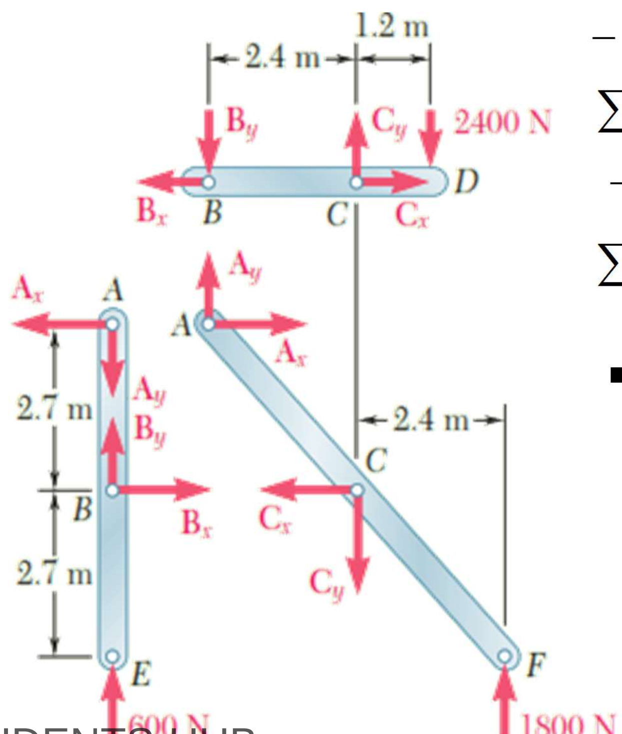
Determine the internal forces *(a)* in member *ACF* at point *J* and *(b)* in member *BCD* at *K*.

SOLUTION:

- Compute reactions and forces at connections for each member.
- Cut member ACF at J. The internal forces at J are represented by equivalent force-couple system which is determined by considering equilibrium of either part.
- Cut member BCD at K. Determine force-couple system equivalent to internal forces at K by applying equilibrium conditions to either part.

Sample Problem 7.1





- Compute reactions and connection forces. Consider entire frame as a free-body, and apply equilibrium conditions:

$$\sum M_E = 0: \quad -(2400\,\text{N})(3.6\,\text{m}) + F(4.8\,\text{m}) = 0 \qquad F = 1800\,\text{N}$$

$$\sum F_y = 0: \quad -2400\,\text{N} + 1800\,\text{N} + E_y = 0 \qquad E_y = 600\,N$$

$$\sum F_x = 0: \qquad E_x = 0$$

- Consider member *BCD* as free-body:

$$\sum M_B = 0:$$

$$-(2400\,\text{N})(3.6\,\text{m}) + C_y(2.4\,\text{m}) = 0 \qquad C_y = 3600\,\text{N}$$

$$\sum M_C = 0:$$

$$-(2400\,\text{N})(1.2\,\text{m}) + B_y(2.4\,\text{m}) = 0 \qquad B_y = 1200\,\text{N}$$

$$\sum F_x = 0: \quad -B_x + C_x = 0$$

- Consider member *ABE* as free-body:

$$\sum M_A = 0: \quad B_x(2.4\,\text{m}) = 0 \qquad B_x = 0$$

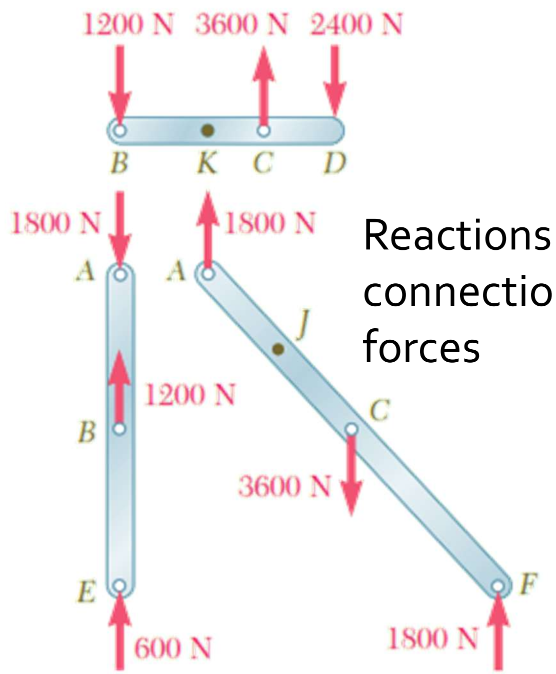
$$\sum F_x = 0: \quad B_x - A_x = 0 \qquad A_x = 0$$

$$\sum F_y = 0: \quad -A_y + B_y + 600\,\text{N} = 0 \qquad A_y = 1800\,\text{N}$$

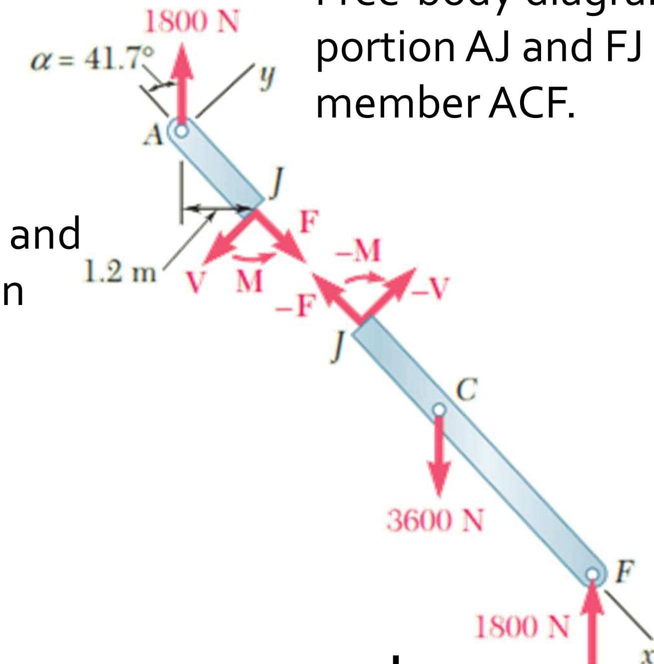
- From member *BCD*,

$$\sum F_x = 0: \quad -B_x + C_x = 0 \qquad C_x = 0$$

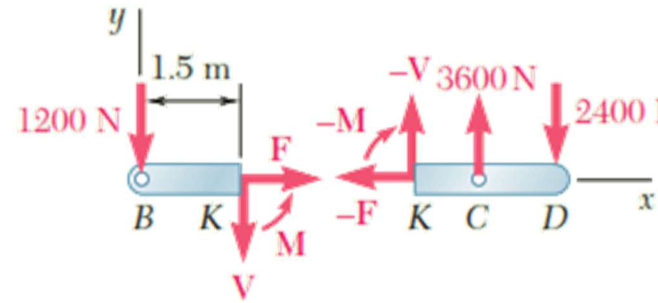
Sample Problem 7.1



Reactions and connection forces



Free-body diagrams of portion AJ and FJ of member ACF.



Free-body diagrams of portion BK and DK of member BCD.

a. Consider free-body AJ from member AGF:

$$\sum M_J = 0: \quad -(1800\,\text{N})(1.2\,\text{m}) + M = 0 \qquad \boxed{M = 2160\,\text{N}\cdot\text{m}}$$

$$\sum F_x = 0: \quad F - (1800\,\text{N})\cos 41.7° = 0 \qquad \boxed{F = 1344\,\text{N}}$$

$$\sum F_y = 0: \quad -V + (1800\,\text{N})\sin 41.7° = 0 \qquad \boxed{V = 1197\,\text{N}}$$

b. Consider free-body *BK* from member BCD:

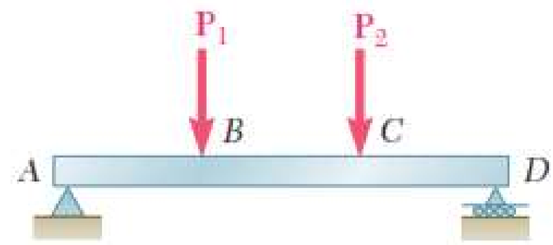
$$\sum M_K = 0: \quad (1200\,\text{N})(1.5\,\text{m}) + M = 0 \qquad \boxed{M = -1800\,\text{N}\cdot\text{m}}$$

$$\sum F_x = 0: \qquad \boxed{F = 0}$$

$$\sum F_y = 0: \quad -1200\,\text{N} - V = 0 \qquad \boxed{V = -1200\,\text{N}}$$

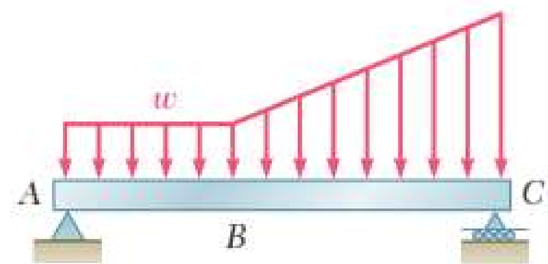
7.2A. Various Types of Beam Loading and Support

- Beam - are usually long, straight prismatic bars, designed to support loads applied at various points along its length.



(a) Concentrated loads

- Beam can be subjected to concentrated loads or distributed loads or combination of both.



(b) Distributed load

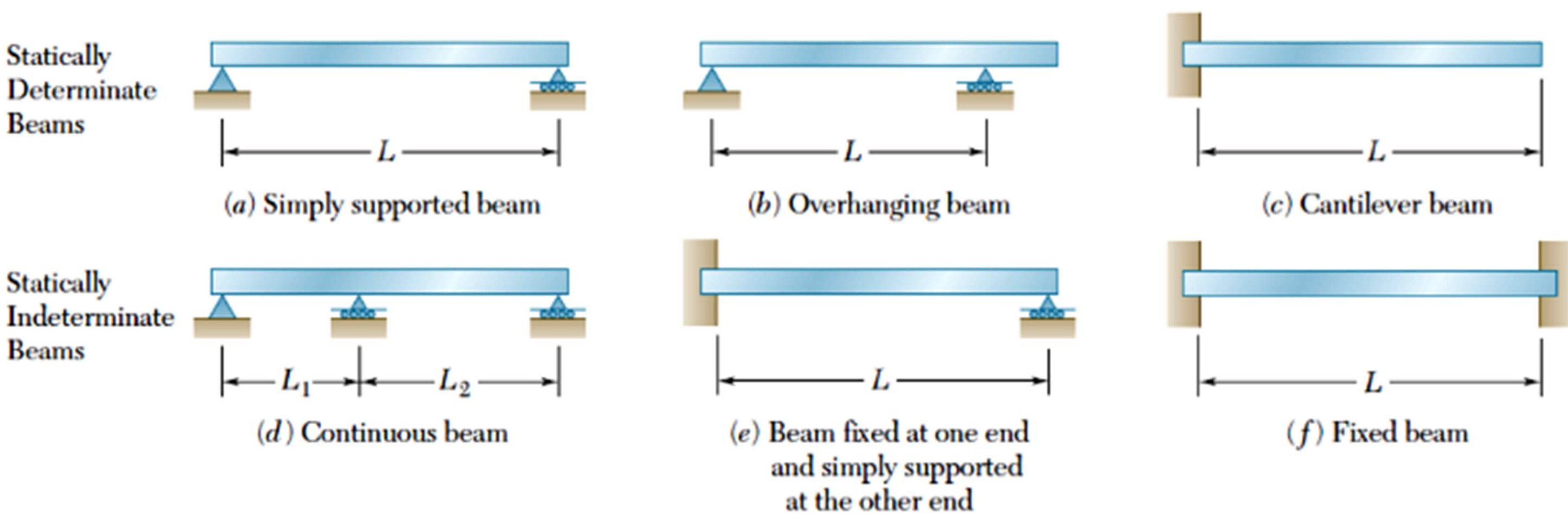
- Beam design is two-step process:

1) determine shearing forces and bending moments produced by applied loads

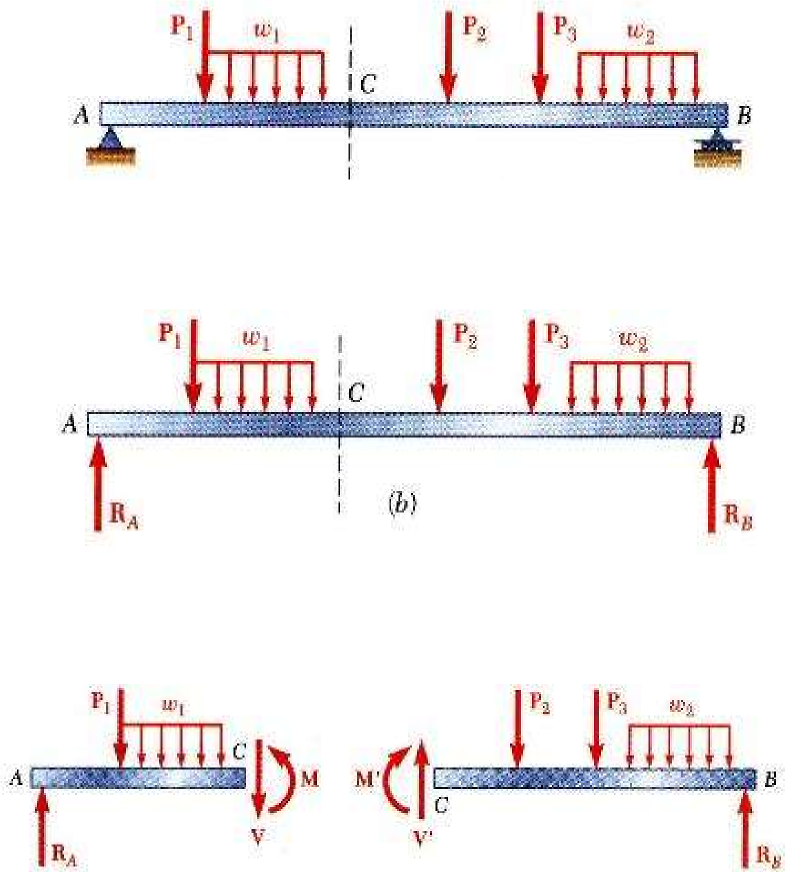
2) select cross-section best suited to resist shearing forces and bending moments

7.2A. Various Types of Beam Loading and Support

- Beams are classified according to the way in which they are supported.
- Reactions at beam supports are determinate if they involve only three unknowns. Otherwise, they are statically indeterminate.



7.2B Shear and Bending Moment in a Beam



- Wish to determine bending moment and shearing force at any point (for example, point C) in a beam subjected to concentrated and distributed loads then:

I. Determine reactions at supports by treating whole beam as free-body.

II. Cut beam at C and draw free-body diagrams for AC and CB. By definition, positive sense for internal force-couple systems are as shown for each beam section.

III. From equilibrium considerations, determine M and V or M’ and V’.

7.2C Shear and Bending Moment Diagrams

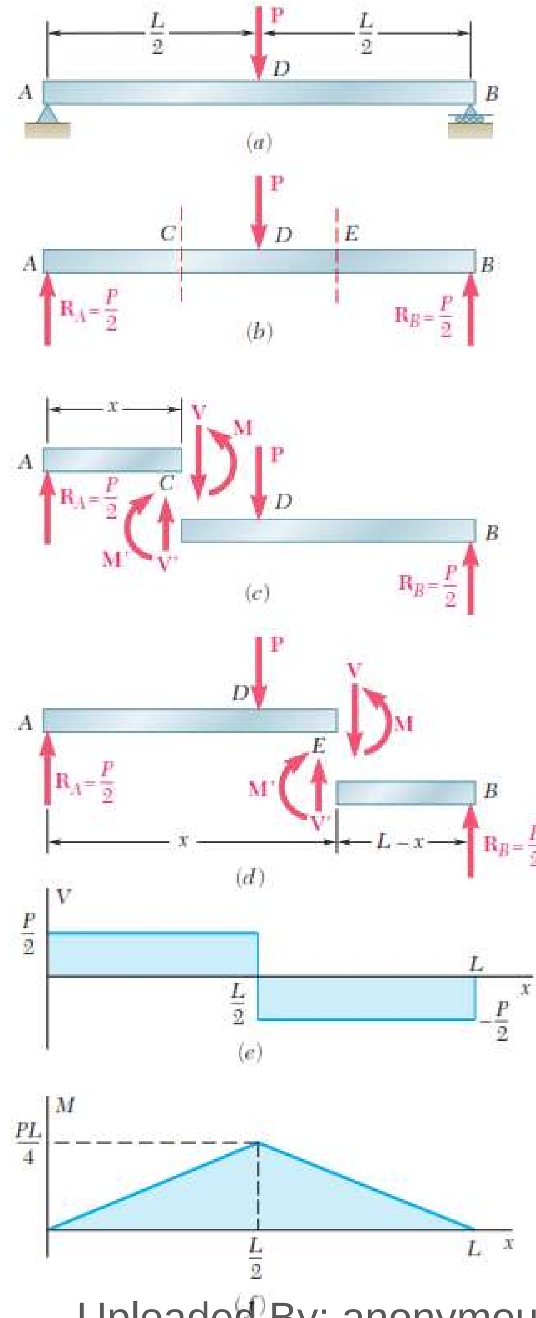
- Shear and moment diagrams are plots of the shear V and moment M as functions of position x.
  - Determine reactions at supports.
  - Cut beam at C and consider member AC,

$$V = +P/2 \quad M = +Px/2$$

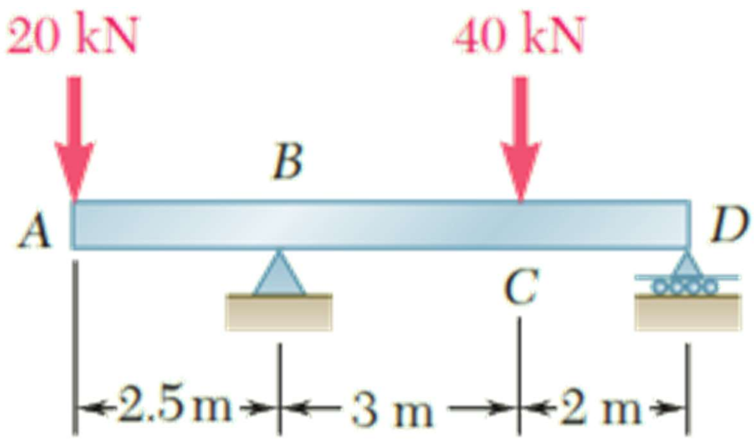
  - Cut beam at E and consider member EB,

$$V = -P/2 \quad M = +P(L-x)/2$$

  - Plot the equations. This gives shear and moment diagrams.
- For a beam subjected to concentrated loads, shear is constant between loading points and moment varies linearly. What else we can see?



Sample Problem 7.2

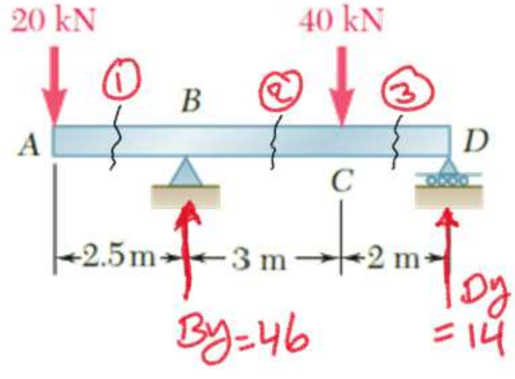


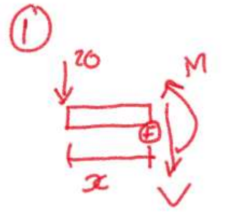
Draw the shear and bending-moment diagrams for the beam and loading shown.

SOLUTION:

- Taking entire beam as a free-body, calculate reactions at *B* and *D*.
- Find equivalent internal force-couple systems for free-bodies formed by cutting beam on either side of load application points.
- Plot results

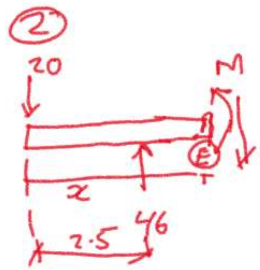
Sample Problem 7.2





$V = -20$

$M = -20x$

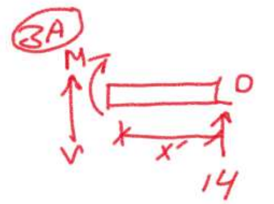


$46 - 20 = V \Rightarrow V = 26$

$\Sigma M_E = 0$

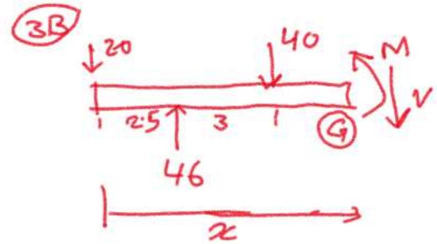
$M + 20(x) - 46(x - 2.5) = 0$

$M = 46x - 115 - 20x = 26x - 115$



$V = -14$

$M = 14x'$

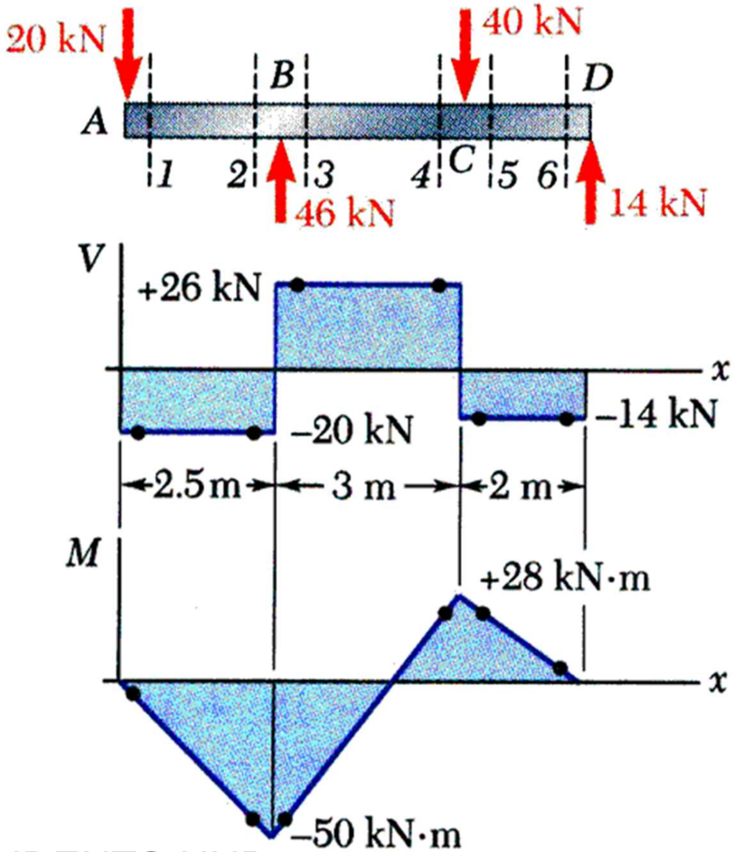


$-V - 20 - 40 + 46 = 0$

$V = -14$

$M + 40(x - 5.5) + 20(x) - 46(x - 2.5) = 0$

$M = -40x + 220 - 20x + 46x - 115 = -14x + 105$

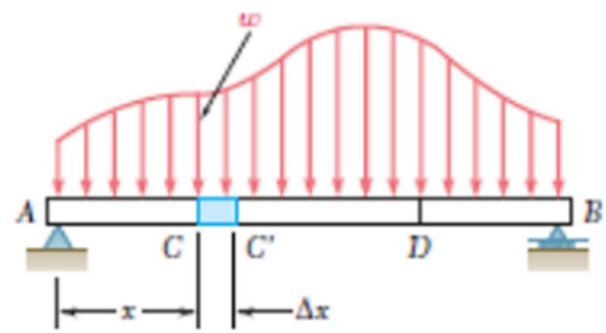


7.3 Relations Among Load, Shear, And Bending Moment

- Relations between load and shear: from b

$$V-(V+\Delta V)-w\Delta x=0$$
$$\frac{dV}{dx}=\lim_{\Delta x\to 0}\frac{\Delta V}{\Delta x}=-w$$

The slope of the shear diagram is equal to the distributed force's value.



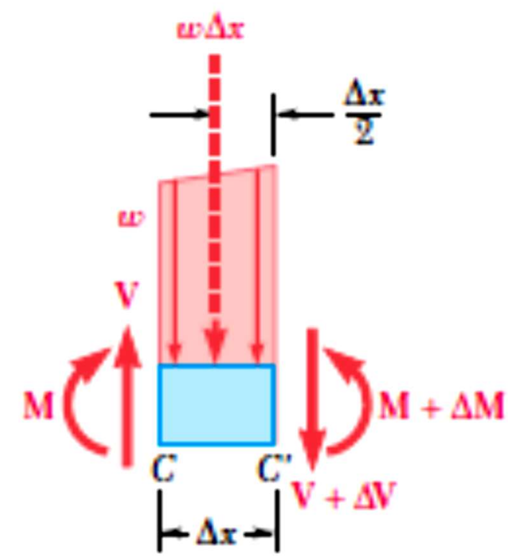
(a)

$$V_D-V_C=-\int_{x_C}^{x_D} w\,dx=-(\text{area under load curve})$$

- Relations between shear and bending moment:

$$(M+\Delta M)-M-V\Delta x+w\Delta x\frac{\Delta x}{2}=0$$
$$\frac{dM}{dx}=\lim_{\Delta x\to 0}\frac{\Delta M}{\Delta x}=\lim_{\Delta x\to 0}\left(V-\frac{1}{2}w\Delta x\right)=V$$

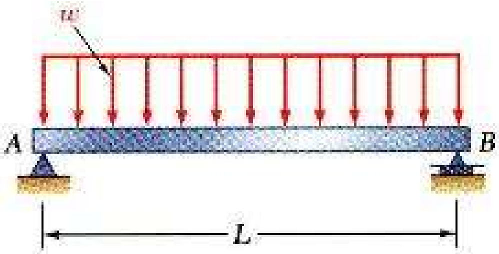
The slope of the moment diagram is equal to the value of the shear.

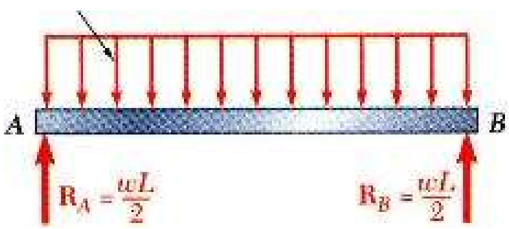


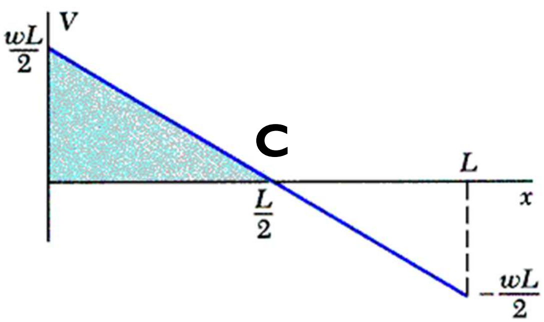
(b)

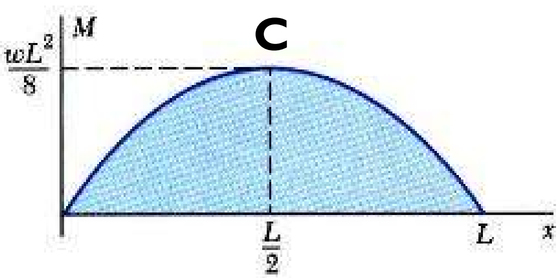
$$M_D-M_C=\int_{x_C}^{x_D} V\,dx=(\text{area under shear curve})$$

Example – Draw V&M Diagrams









- Reactions at supports, $R_A = R_B = \frac{wL}{2}$
- Shear curve,

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = \frac{wL}{2} - wx = w\left(\frac{L}{2} - x\right)$$

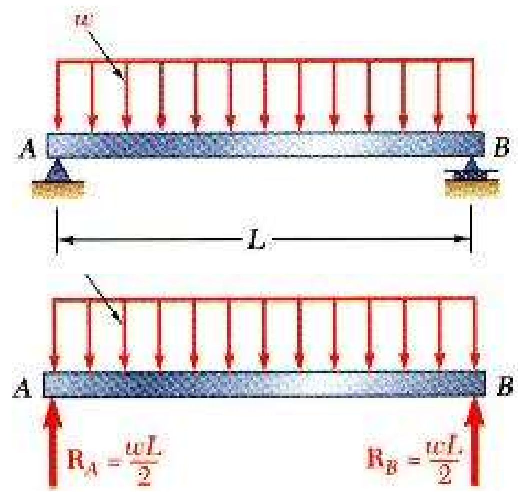
- Moment curve,

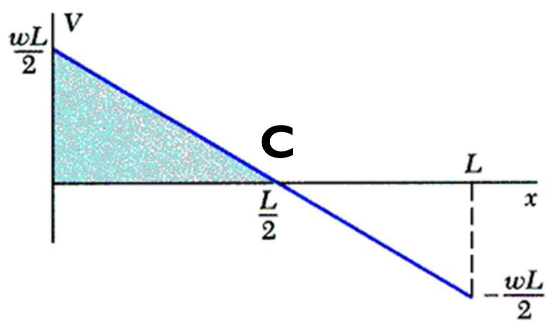
$$M - M_A = \int_0^x V\,dx$$

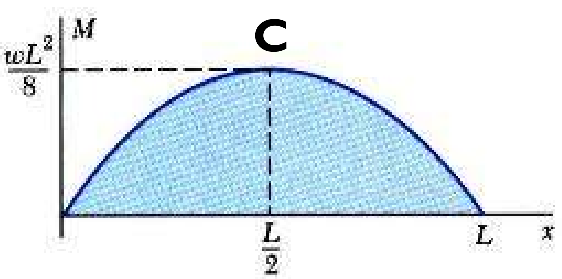
$$M = \int_0^x w\left(\frac{L}{2} - x\right)dx = \frac{w}{2}(Lx - x^2)$$

$$M_{max} = \frac{wL^2}{8}\left(M \text{ at } \frac{dM}{dx} = V = 0\right)$$

Example – Draw V&M Diagrams







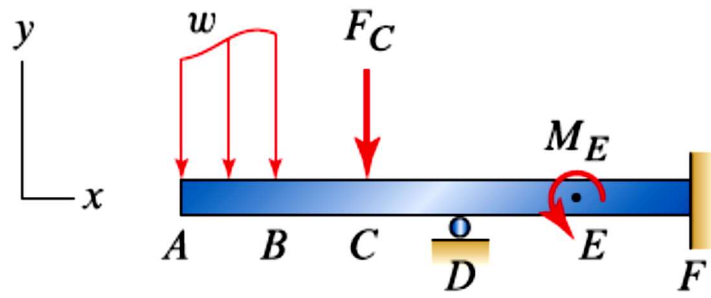
- **Reactions at supports** $R_A = R_B = \frac{wL}{2}$
- **Shear curve**
  1. Shear curve shall be straight line between A and B as the load is constant.
  2. Shear at A is Known = $\frac{wL}{2}$

$$V_B - V_A = -wL \rightarrow V_B = \frac{wL}{2} - wL = -\frac{wL}{2}$$

- **Moment curve**
  1. Moment curve shall be 2nd degree curve as shear curve is linear.
  2. M_A = 0 as A is pin support.
  3. The deference of the moment between A and B shall be zero as M_B = 0, and this is obvious from the shear curve.
  4. To draw the curve more points are needed. The mid span point C where V = 0 is the choice. The slope of M diagram is positive decreasing from A-C and negative increasing between C-B

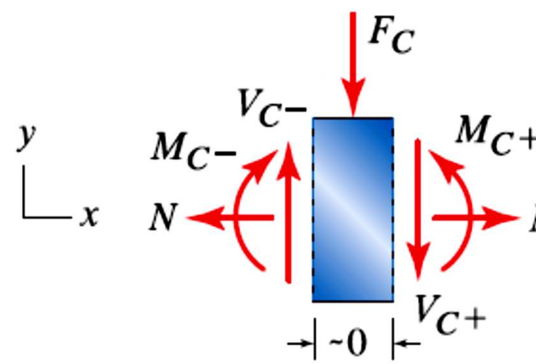
Tips for drawing shear and moment diagrams

- Point A is an unsupported end of a beam with no concentrated force and no moment applied. At A, the shear and moment are zero. This is true regardless of the presence of a distributed force w.



- At point B, a distributed force ends. The shear and moment just to the right of B are the same as those just to the left of B. The same comments apply to points where a distributed force begins.

- A concentrated force F_C acting in the negative y direction is applied at point C. The shear just to the right of C is lower than the shear just to the left of C by amount F_C. The moment just to the right of C is the same as that just to the left of C. The FBD and equilibrium equations shown in Fig. b justify the validity of these remarks.

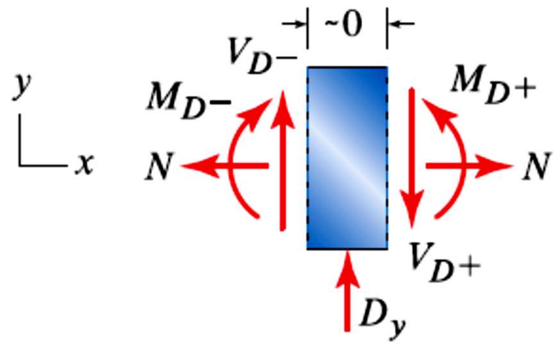


$$\sum F_y = 0: \quad (V_{C-}) - F_C - (V_{C+}) = 0$$
$$\Rightarrow \quad (V_{C+}) = (V_{C-}) - F_C$$

$$\sum M_C = 0: \quad -(M_{C-}) + (M_{C+}) = 0$$
$$\Rightarrow \quad (M_{C+}) = (M_{C-})$$

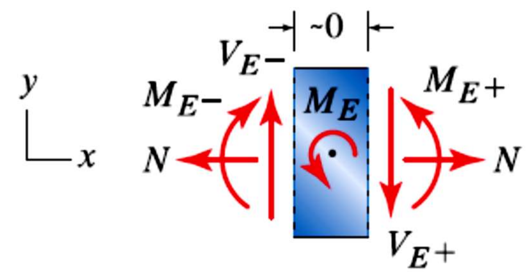
Tips for drawing shear and moment diagrams

- A roller support is positioned at point D. The shear just to the right of D is higher than the shear just to the left of D by amount Dy, where Dy is the reaction the roller applies to the beam with positive Dy acting in the positive y direction. The moment just to the right of D is the same as that just to the left of D.



$$\sum F_y = 0: \quad (V_{D-}) + D_y - (V_{D+}) = 0$$
$$\Rightarrow \quad (V_{D+}) = (V_{D-}) + D_y$$
$$\sum M_C = 0: \quad -(M_{D-}) + (M_{D+}) = 0$$
$$\Rightarrow \quad (M_{D+}) = (M_{D-})$$

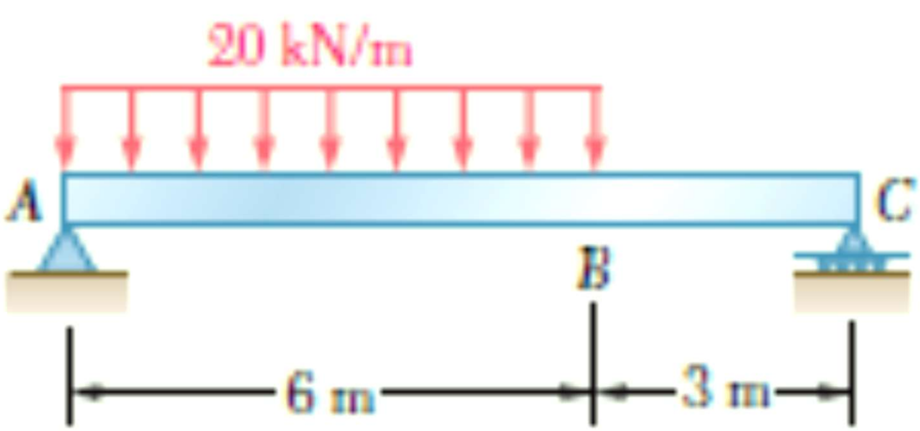
- A concentrated moment M_E acting counterclockwise is applied at point E. The shear just to the right of E is the same as that just to the left of E. The moment just to the right of E is lower than the moment just to the left of E by amount M_E. The FBD and equilibrium equations shown in Fig. justify the validity of these remarks.

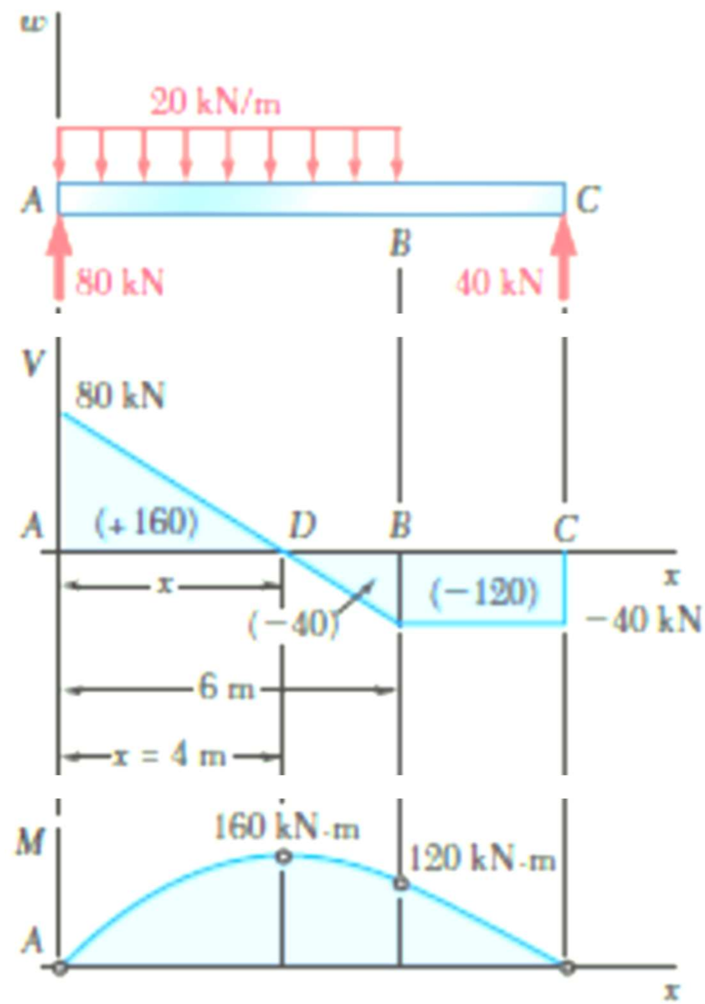


$$\sum F_y = 0: \quad (V_{E-}) - (V_{E+}) = 0$$
$$\Rightarrow \quad (V_{E+}) = (V_{E-})$$
$$\sum M_C = 0: \quad -(M_{E-}) + M_E + (M_{E+}) = 0$$
$$\Rightarrow \quad (M_{E+}) = (M_{E-}) - M_E$$

Sample Problem

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the location and magnitude of the maximum bending moment.



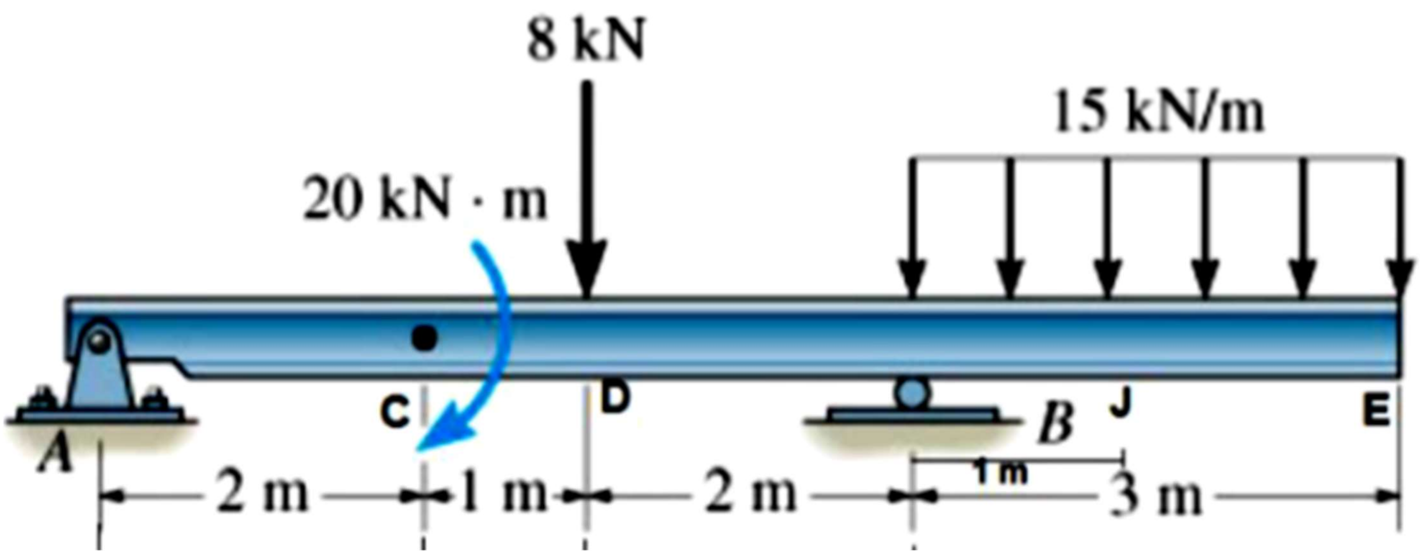


Sample Problem

An overhanging beam ABE is supported by a hinge at A and a roller at B. For the loading shown. Determine

1) The internal forces at J where J is 1 m to the right of B, and

2) Draw shear & bending moment diagrams.



Sample Problem

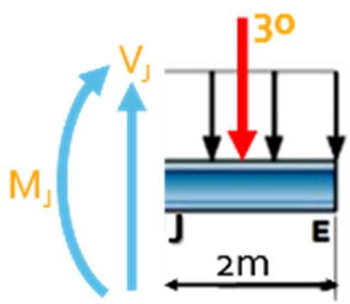
Reactions

$$\sum M_A = 0$$

$$45(6.5) + 8(3) + 20 = B_y(5)$$
$$\rightarrow B_y = 67.3\ KN$$

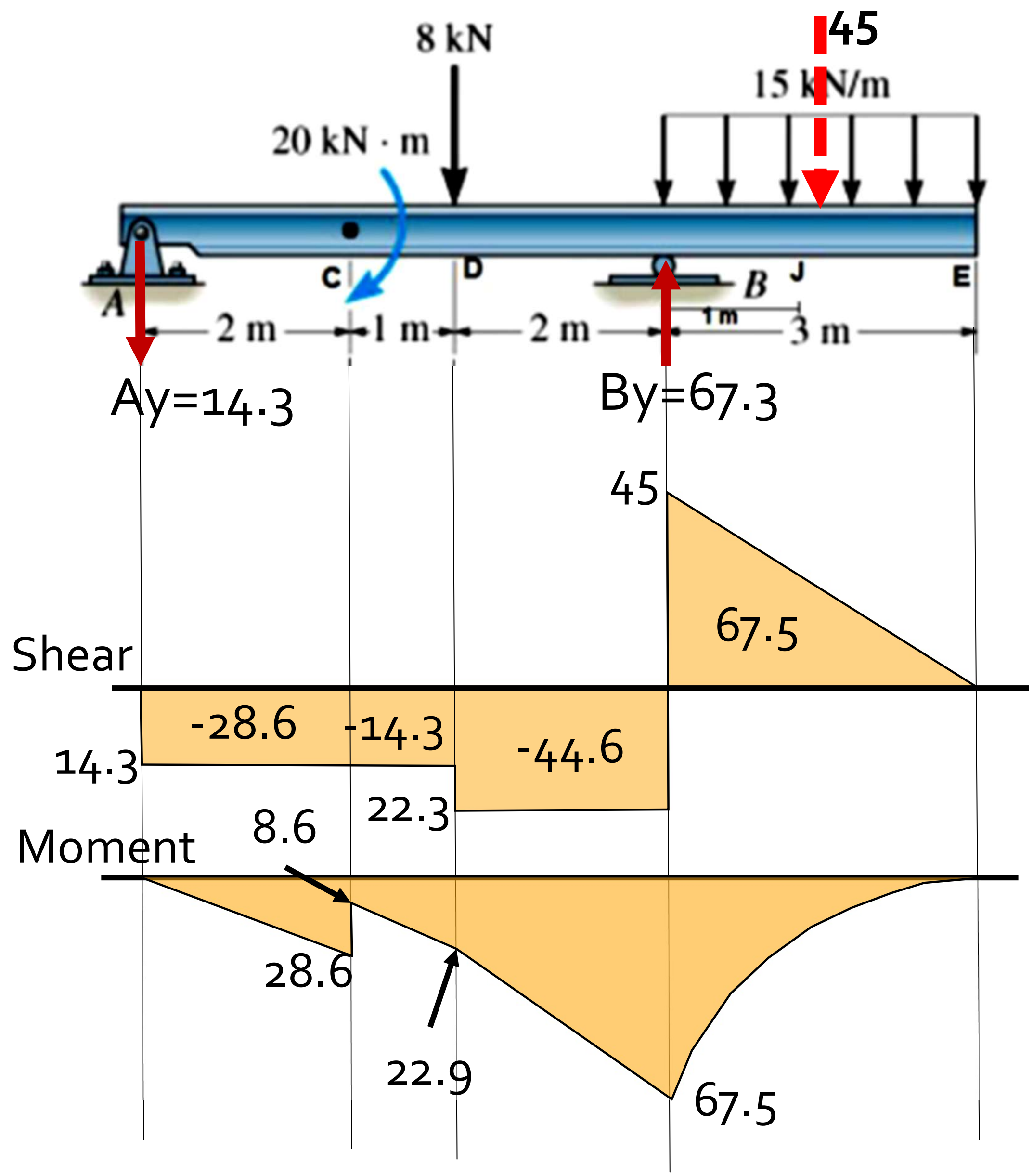
$$\sum F_y = 0\ ;\ \rightarrow A_y = 14.3 \downarrow\ KN$$

1) The internal forces at J



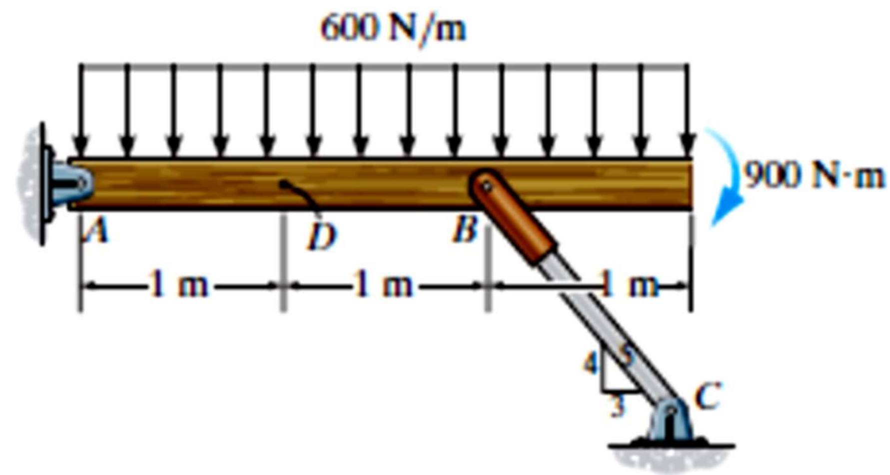
$$M_J = -30(1) = -30\ KN.m$$

$$V_J - 30 = 0 \rightarrow V_J = 30\ KN$$



Sample Problem

1. Draw the shear and bending-moment diagrams for the beam ADBE.
2. Determine the internal forces at D



Reactions

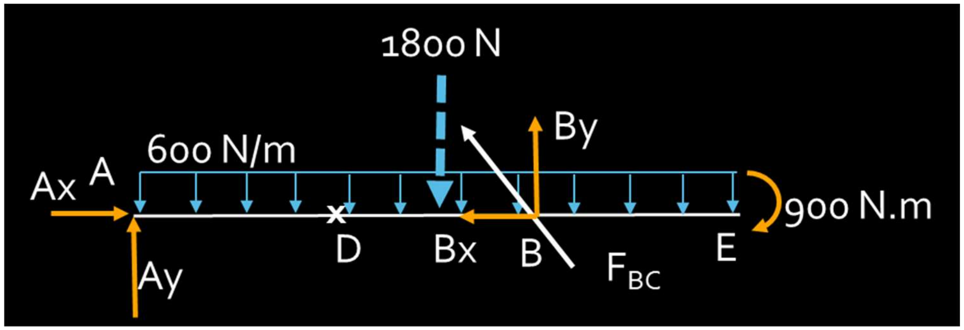
Ay = 0

Ax = 1350 N

By = 1800 N

Bx= 1350 N

FBD



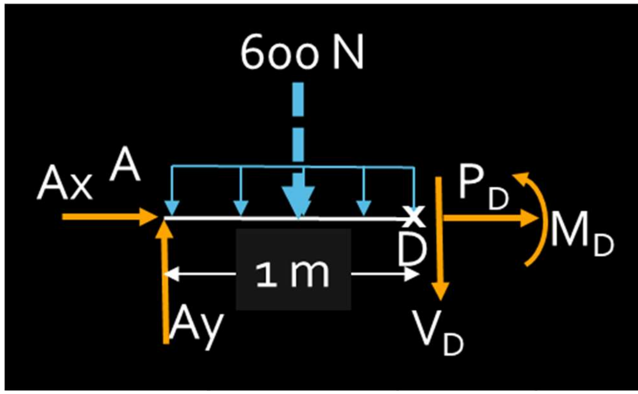
internal forces at D

P_D = -1350 N

V_D = -600 N

M_D = -300 N.m

Section Cut at D



Sample Problem