Power transmission Elements.

Ch. 17.

flexible power transmission elements:

belts, chain :- connects shafts with considerale distance

⇒ provides flexibility between drive and driven machinary

Belts: quiet in operation, slippage between belts and pulleys → Inexact speed ratio.

flexibility and damping characteristics → reduce transmission shock and vibration.

flexible shafts: Transmitting small amount of torque

Application: Automotive speedometer.

Hydrodynamics action: Transmitt power between collinear shafts.

⇒ fluid clutch – hydrodynamic torque convertor.

Belts:

Types of belts: Table [17-1] common types of belts.

1- Flat belt — used with Crowned pulleys
2- Round ⎤
3- V-Belt ⎦ Crooved or sheave pulley
4- Timing — Toothed wheel or sprockets.

Belts characteristics:

1- Transmit power with shafts with high distance.
2- Because of slippage → Inexact speed ratio.
3- Idler or tension pulley is used to keep the belts tight, because the belts stretches slightly with time.

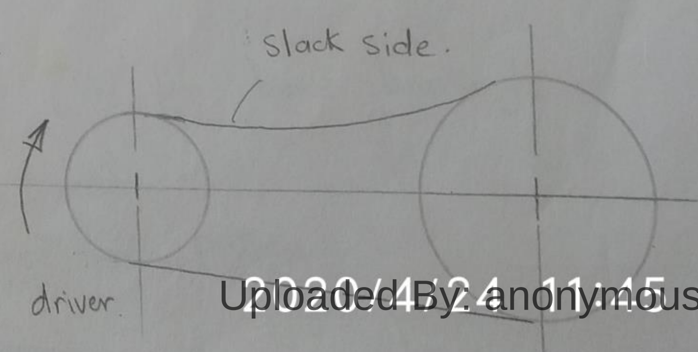
Belt drive:

1- open belt drive.

For flat belt: Slack (loose) side → Top

Others belts: either top or bottom

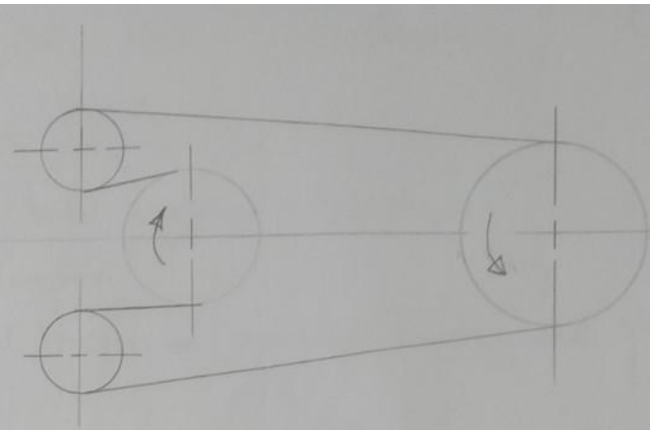
→ Installed with initial tension



2- Reversing drive.

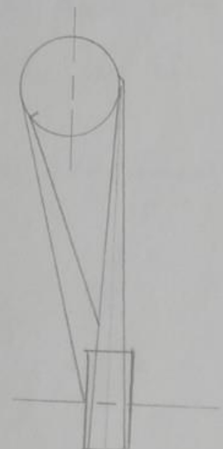
Cannot used for V-belt

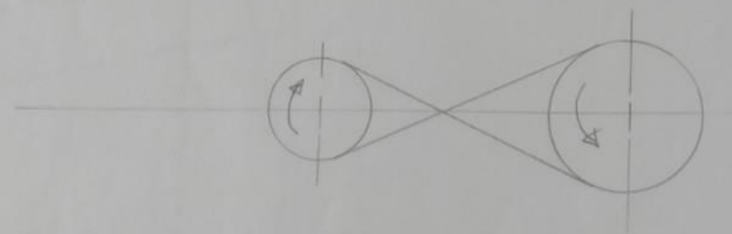
- Since both sides contact the pulleys.



3- Out of plane pulleys.

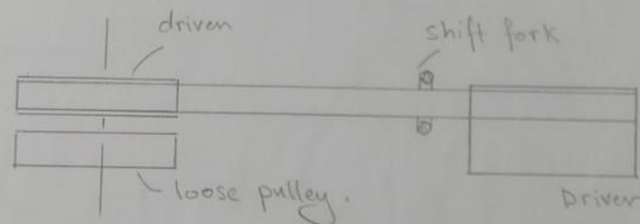
used with flat-belts.





Belt must leave each pulley at the midplane of other pulley face.

4 - Clutching action.



Flat belt material:

- Urethane
- rubber - Impregnated fabric reinforced with steel wire
- Fabricated as roll, cut and ends joined using special kits.

Flat and round belt drive:

	flat belt	gear	V-belt.
Driving efficiency	98%	98%	70 - 96 %

Flat belt quieter and absorbes tortional vibration more than gear or V-belt. drives. Tody's It is limited high speed machines.

Torque transmitted by flat-belt:

$T = (P_1 - P_2)\, r$ where $\frac{P_1}{P_2} = e^{\mu \phi}$ [as for band brake].

P_1 = Tight side , μ = coefficient of friction.

P_2 = Slack side ϕ = angle of contact with pulley.

2020/4/24 11:46

Belt initial tension (P_i): depends on belts elastic characteristics.

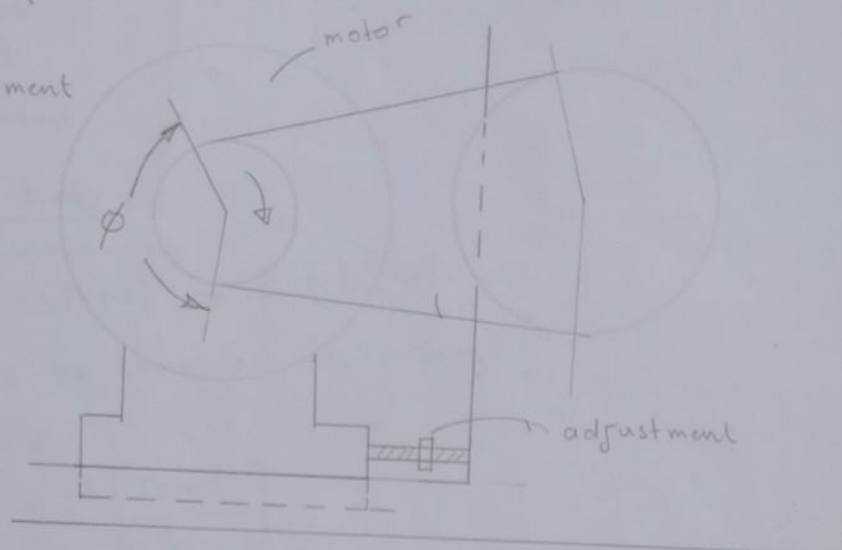
Satisfactory initial tension; $P_i = \frac{P_1 + P_2}{2}$

Initial tension lost with time → belt slightly stretches.

To provide a satisfactory initial tension:

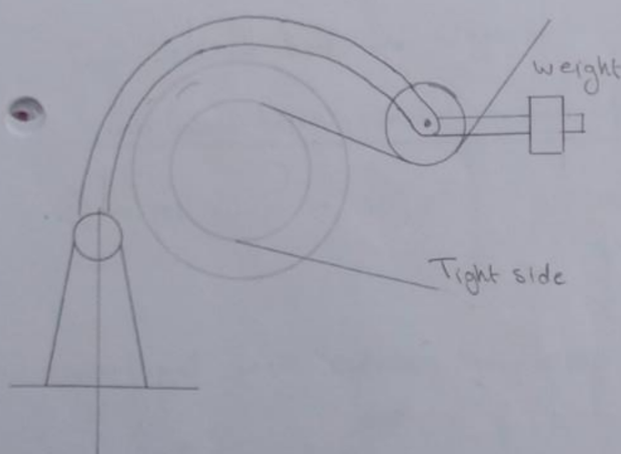
1- Install belt with excessive initial tension → overload bearing and shaft. Shorten the life of belt.

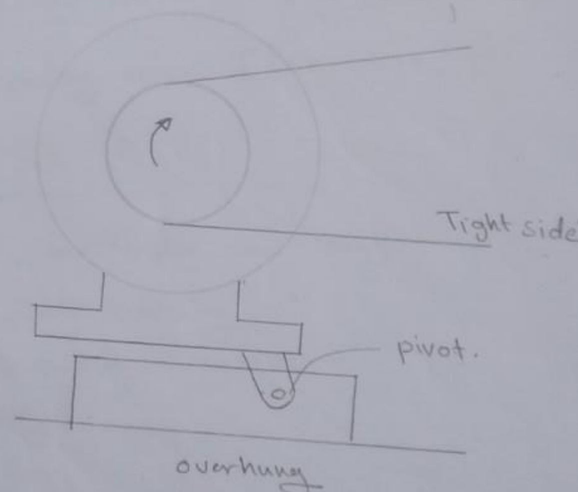
2- Manual adjustment



3- Pivoted, overhung motor.

4- weighted Idler pulley.





Slack side on top → tendency to sag acts to increase angle of wrap.

<u>wrap angle ϕ</u>:

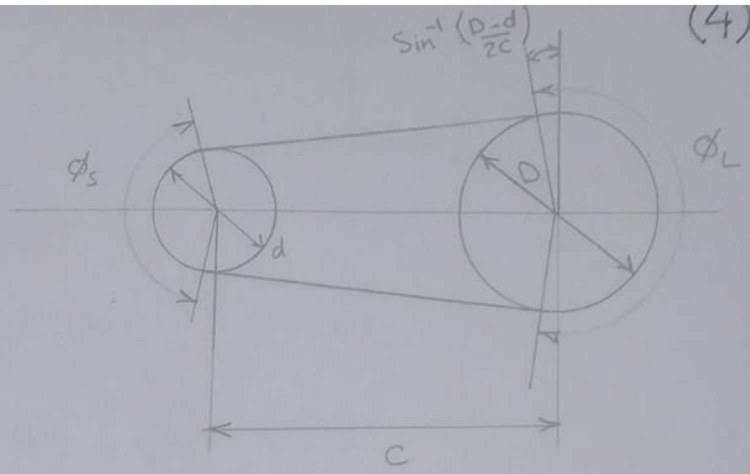
The capacity of belt drive is limited by angle of wrap (ϕ) on smaller pulley, this is more critical when driving pulleys of greatly differing size, spaced closely.

Open belt:

$$\phi_s = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\phi_L = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2-(D-d)^2} + \frac{1}{2}(D\phi_L + d\phi_s)$$



D = diameter of large pulley
d = dia of small pulley
C = center distance
L = Length of belt
ϕ = angle of contact.

For crossed belt:

- $$L = \sqrt{4C^2-(D+d)^2} + \frac{\phi}{2}(D+d)$$

wrap angle is the same for both pulleys:

$$\phi_L = \phi_s = \phi$$

$$\phi = \pi + 2\sin^{-1}\left[\frac{D+d}{2C}\right]$$

belt is subjected to bending stress when in contact with pulley

smaller pulley ⟶ greater bending stress ⇒ pulley size is limited.

For leather belt ⟶ tight side bending stress [250 – 400] psi.

- Centrifugal force creats tension of belt (P_c)

$P_c = mV^2 = m\,\omega^2 r^2$, m = mass per unit length.
V = belt velocity
r = pulley's radius.

Certirfugal force reduces the wrap angle.

$$\Rightarrow \frac{P_1 - P_c}{P_2 - P_c} = e^{\mu\phi}$$

Power transmitted:

$H = (P_1 - P_2)V$, H = watt , V = m/s , P = newtons

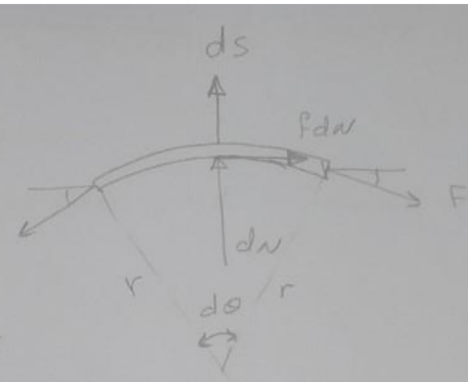
$H = \frac{(P_1 - P_2)V}{33000}$, H = hp , V = ft/min , P = lb

2020/4/24 11:53

Considering Centrifugal force.

$$ds = (m r\,d\theta)\, r\omega^2 = m r^2 \omega^2\, d\theta$$

$$= m V^2\, d\theta = F_c\, d\theta$$



V = belt speed

$$F_c = m r^2 \omega^2$$

$$V = \frac{\pi d n}{12}\ \text{fpm} \quad , \quad V = \frac{\pi d n}{60}\ m/s$$

w = weight of belt per unit length $\left(\frac{lbf}{ft}\right)$ $\left(\frac{N}{m}\right)$

$$w = 12 \rho\, b t \quad \frac{lbf}{ft} \quad , \quad w = \gamma b t \left(\frac{N}{m}\right) , \quad \gamma = \frac{N}{m^3}$$

ρ = desity $\frac{lbf}{in^3}$ b = belt width (in) , t = belt thickness (in)

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{w}{32.2}\left(\frac{V}{60}\right)^2 \qquad g = 32.2\ \frac{ft}{s^2}$$

$$\rightarrow \frac{F_1 - F_c}{F_2 - F_c} = e^{f\phi} \qquad F_c = \frac{w}{g} V^2 \quad (N)$$

$$g = 9.8\ m/s^2$$

Initial tension:

Flat belt is made of elastic member, belt loose tension with time.

at $t = 0 \rightarrow F_1 = F_2 = F_i$ = Initial tension.

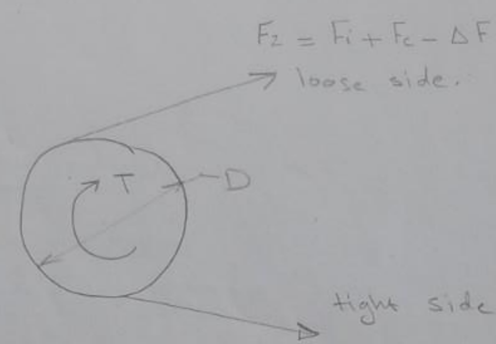
$$t\uparrow \rightarrow F_1\uparrow \rightarrow F_2\downarrow$$

$$F_1 = F_i + \Delta F$$
$$F_2 = F_i - \Delta F \qquad \text{solving} \rightarrow F_i = \frac{F_1 + F_2}{2}.$$

Including centrifugal force.

$$T = (F_1 - F_2)\, r$$

$$= 2\Delta F r \rightarrow \Delta F = \frac{T}{2r} = \frac{T}{D}$$



tight side:

$$F_1 = F_i + F_c + \Delta F$$
$$F_2 = F_i + F_c - \Delta F$$

$$\rightarrow F_1 + F_2 = 2F_i + 2F_c$$

$$F_i = \frac{F_1 + F_2}{2} - F_c$$

$$\rightarrow F_i = \frac{T}{D}\,\frac{e^{f\phi} + 1}{e^{f\phi} - 1} \quad , \quad F_1 = F_i + F_c + \frac{T}{D} = F_c + F_i\,\frac{2e^{f\phi}}{e^{f\phi} + 1} \qquad F_2 = F_i + F_c - \frac{T}{D}$$

$$= F_c + F_i\,\frac{2}{e^{f\phi} + 1}$$

Capacity of belt drive depend on ϕ = contact angle on smaller pulley and Initial tension = F_i

$T \propto F_i$ → For satisfactory belt drive.

∴ Satisfactory initial tension must be provided, and maintained with operation

Power transmitted

$$H = \frac{(F_1 - F_2) V}{33000} \quad (hp) \quad , \quad H = (F_1 - F_2) V \quad \text{watt}$$

Capacity of flat belt depend on allowable belt tension Force.

F_a = allowble belt tension force per unit width lb/in

Table [17-2] → F_a at $V = 600$ ft/min and min pulley dia.

life of belt depend on bending stress

$d \downarrow \rightarrow \sigma_b \uparrow$

allowable max. tension on belt.

$$(F_1)_a = C_p C_v (F_a) b$$

F_a = allowable belt tension at $V = 600$ ft/min

b = belt width.

C_p = pulley correction factor

C_v = velocity correction factor

C_p = account for bending stress on belt.

$C_p = 1.0$, Urethane belt.

C_p = Table [17-4] for lather and polyamide

C_v = Fig [17-9] for leather belt.

$C_v = 1.0$, Urethane and polyamide

let required power transmitted ⇒ $H = \frac{(F_1 - F_2)}{33000} V$

$$H_a = \frac{(F_{1a} - F_2) V}{33000 \; k_s \; n} = \frac{(F_{1a} - F_2) V}{33000 \; k_s \; n} = \frac{T n}{63025 \; k_s \; n}$$

k_s = service factor, Table [17-5]

n = factor of safety.

 H_a = allowable power transmitted → $H_a = n k_s H$

$$T = (F_1 - F_2)\,\frac{D}{2}$$

$$(F_1)_a = C_p C_v (F_a)\, b$$

$$\rightarrow F_2 = (F_1)_a - \frac{2T}{D}$$

$$\rightarrow F_i = \frac{F_1 + F_2}{2} - F_c.$$

$$\rightarrow H_a = \frac{(F_{1a} - F_2)\, V}{33000\, n\, k_s} = \frac{T\omega}{63025\, k_s\, n}$$

$$T = (F_{1a} - F_2)\,\frac{D}{2}.$$

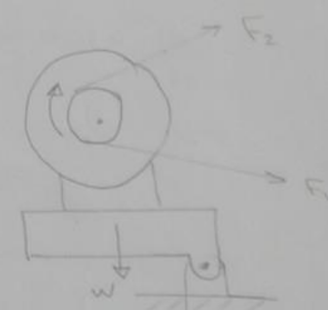
Based on allowable Torque.

$$\frac{F_{1a} - F_c}{F_2 - F_c} = e^{f'\phi} \rightarrow f' = \frac{1}{\phi}\ln\left[\frac{F_{1a} - F_c}{F_2 - F_c}\right]$$

$f' < f \rightarrow$ no belt slippage. $f =$ Table [17-2] for belt type.

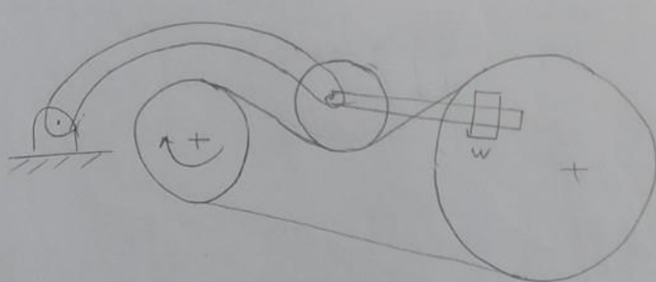
Initial tension control

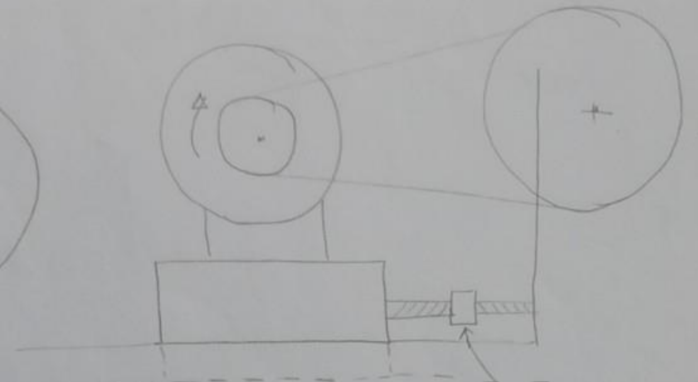
1- Pivoted Motor



2- Manual adjustment

3- Idler pulley.





Flat belt dip:

$$d = \frac{12\, C^2 w}{8 F_i} = \frac{3 C^2 w}{2 F_i}$$

$$d = \frac{C^2 w}{8 F_i} = m \qquad C = ft$$

F_i = initial tension, w = weight per width.

<u>Example</u>

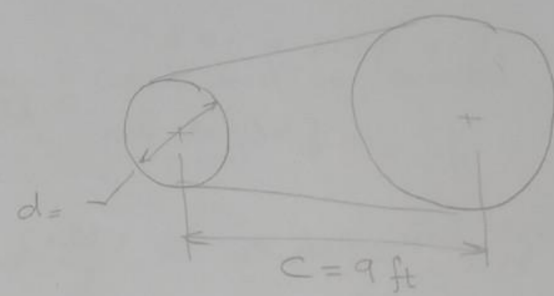
b = 6", polyamide (F-1) Flat belt.

$d_1 = 2"$, VR = 2.0, C = 9 ft

$\omega_1 = 1750$ rpm., hp = 2

$K_s = 1.25$

Find: F_c, F_i, F_{all}, F_2.
H_a, and dip



<u>Solution:</u>

$$\frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = 2$$

$$d_2 = 2d_1 = 4"$$

1- $F_c = \frac{w}{32.2}\left(\frac{V}{60}\right)^2$

From table [17-2] F-1, polyamide belt

$t = 0.05$, $F_a = 35$ lb/in, $\rho = 0.035$ lb/in³, $f = 0.5$

$$w = \rho bt = \left(0.035 \times 6 \times 0.05 \ \frac{lb}{in}\right) \times 12 = 0.126 \ \frac{lb}{ft}$$

$$F_c = \frac{w}{32.2}\left(\frac{V}{60}\right)^2$$

$$V = \frac{\pi d n}{12} = \frac{\pi(2)1750}{12} = 916.3 \text{ ft/min.}$$

$$F_c = 0.913 \text{ lb.}$$

$$H_{all} = \frac{T\omega}{63025 \, k_s n} \longrightarrow T = \frac{63025 \, k_s n H}{\omega} = \frac{63025 \times 1.25 H}{1750}$$

$$T = 90 \text{ lb.in}$$

$$F_{all} = C_p C_v F_{ta} b$$

$C_v = 1.0$ polyamide

$C_p = 0.7$ Table [17-4]

$$F_{all} = 0.7 \times 1 \times 35 \times 6 = 147 \text{ lb} \doteq (F_1)_{all}$$

$$r(F_1 - F_2) = T \rightarrow F_1 - F_2 = \frac{2T}{D} = 90 \rightarrow F_2 = 147 - 90 = 57 \text{ lb}$$

$$F_i = \frac{F_1 + F_2}{2} - F_c = \frac{147 + 57}{2} - 0.913 = 101.1 \text{ lb}$$

2020/4/24 11:56

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2020/4/24 11:56

$$f' = \frac{1}{\phi_s} \ln \frac{F_1 - F_c}{F_2 - F_c} =$$

$$\phi_s = \pi - \sin^{-1}\left(\frac{D-d}{2c}\right) = 3.13 \text{ rad.}$$

$C = 9 \times 12 = 108''$

$f' = 0.3 < 0.5$ No slipping

$H_a = K_s n H = 1.25 \times 2 = 2.5$

$$L = \sqrt{[4C^2 - (D-d)^2]} + \frac{D\phi_L + d\phi_s}{2}$$

$L = 225.3''$

$$\phi_L = \pi + 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$\phi_s = \pi - 2\sin^{-1}\left(\frac{D-d}{2C}\right)$$

$$dip = \frac{3}{2} \frac{C^2 w}{2F_i} = 3\frac{(9)^2 (0.126)}{2 \times (101.1)} = 0.151$$

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{f\phi_s} \qquad \frac{F_1 - F_c}{F_2 - F_c} = e^{0.5 \times 3.13} = 4.78$$

$$H = \frac{F_1 - F_2 \; V}{33000}$$

$$F_1 - F_2 = \frac{33000 \times hp}{V}$$

$F_2 = F_1 - 72$

$F_1 - F_c = 4.78 F_2 - 4.78 F_c$

$F_1 - 4.78 F_2 = -3.78 F_c$

$F_1 - 4.78 F_1 + 344.29 = -3.78 F_c$

$F_1(-3.78) = -347.75$

$F_1 = 92 \text{ lb}, \; F_2 = 20$

$$H_a = \frac{(F_{1a} - F_2)V}{33000}$$

$$H_a = \frac{147 - 20 \times 916.3}{33000} = 3.52 \text{ hp}$$

$$H_{nom} = \frac{(F_1 - F_2)V}{33000}$$

$H_a = k_s n H_{nom}$ $\quad n = \frac{3.52}{1.25 \times 2} = 1.41$

Flat belt pulleys should be crowned to keep belt from running out of pulley

Two pulley are in horizontal plane → The larger pulley is crowned.

If when of the pulleys is not in the horizontal plane → both pulleys must be crowned.

Table [17-5] → crown height of pulley [Crown must be rounded not angled]

F_a = allowable belt tension → Table [17-2] F_a at belt speed $V = 600$ fpm

For $V > 600$ ft/min → use velocity correction factor [C_v]

For leather belt — C_v → Fig. [17-6]

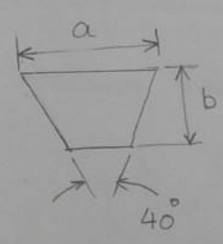
polyamide, Urethane — $C_v = 1.0$

K_s = Service factor, type of loading → Table [17-11]

Table [17-11] → K_s for flat and V-belt.

V-Belts :-

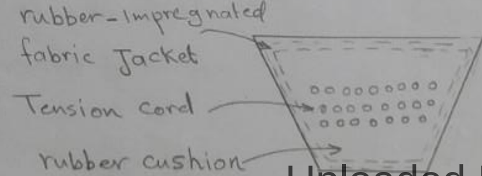
Are made to standard lengths, and cross-sectional sizes.



Standard sizes: A, B, C, D, E

Table [17-6] Standard V-belt sizes, Min. sheave dia. and hp range for one or more belts.

- pulley used with V-belt is called sheave
- V-belts works well with short center distance. because of the stretch resistance of their Interior tension cords.
- V-belt do not require frequent adjustment of Initial tension.
- Multiple V-belts can be used to increase the driving capacity up to 12-belts



- [illegible] equally.

V- belt Selection:

V- belt is specified by its cross-section area, pitch length.

1- cross-section size [A, B, C, D, E] is first selected.

2- To specify pitch length.

- First Interior circumference is obtained corresponding to section size From table [17-7]. [L_{in}] for standard V-belts. Table [17-10]
- pitch length is obtaind adding a quantity to Interior circumference.

$$\Rightarrow \quad L_p = L_{in} + \Delta L$$

$\Delta L \rightarrow$ Table [17-8] ΔL for various belts standard sizes.
Table [17-11]

L_p = pitch length, effective length of V-belt.

$$L_p = 2C + 1.57(D+d) + \frac{(D-d)^2}{4C}$$

C = Center distance

D = dia. of larger sheave.

d = dia. of Smaller sheave.

$$C = \frac{1}{4}\left\{\left[\left(L_p - \frac{\pi}{2}(D+d)\right]\right. + \sqrt{\left[L_p - \frac{\pi}{2}(D+d)\right]^2 - 2(D-d)^2}\right\}$$

L_p = pitch length of belt.

Center distance V-belt sheaves:

- Short center distance is recommended for V-belt. because vibration of slack side shorten belt life.

$$D < C < 3(D+d)$$

V- belt Rated capacity:-

$$H = K_1 K_2 H_R / K_s$$

H_R = rated horce power of V-belt based on $\phi = 180°$ and standard belt length (L_p).

Table [17-9] → horse power rating of standard V-belt. at differet speed. [H_R] — for $\phi = 180°$ contact angle.

for $\phi < 180°$ → use (K_1)

K_1 = angle of contact correction factor for $\phi > 180^\circ \rightarrow K_1 = 1.0$

$\phi < 180^\circ \rightarrow K_1 \rightarrow$ Fig. [17-7]. Table [17-13]

K_2 = belt length correction factor.

Longer belt is subject to load less times than shorter belt.

$K_2 \rightarrow$ for various size and length $\rightarrow$ Table [17-10] [17-14]

K_S = Servise factor. $\rightarrow$ Table [17-11]. $\rightarrow$ Table [17-15]

V-belt sheave angle:

V-belt sheave angle < belt section angle $\Rightarrow$ wedging action.

wedging action $\rightarrow$ Increases normal force on belt element.

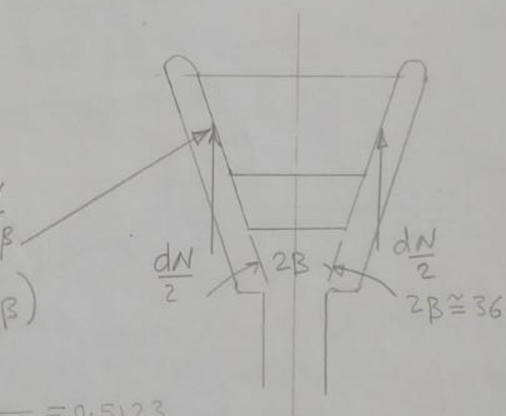
Normal force Increases from $dN \rightarrow \frac{dN}{\sin\beta}$

$\Rightarrow$ friction force $= \mu N \rightarrow$ Increases.

$\rightarrow$ Since torque transmission $\propto N$

$\Rightarrow$ Torque capacity increase $\approx$ 3 times $[F_f = 3.25\,\mu N]$. $(f\phi/\sin\beta)$

$$\Rightarrow \frac{P_1 - P_c}{P_2 - P_c} = e$$



$\frac{f}{\sin\beta} = 0.5123$

Example:

A 60 hp ICE drives brick-making machinery, 2 shifts / day

The drive and driven sheaves, d = 26 in, C ≥ 12 ft, sheave speed = 400 rpm.

Select V-belt arrangement.

Solution:

$$L_p = 2C + 1.57(D+d) + \frac{(D-d)^2}{4C} = 2 \times 12 \times 12 + 1.57(26+26) + 0 = 369.64''$$

From Table [17-7] select inside length Lin = 390 in of [C, D, E] V-belt.

From Table [17-11], $K_S = 1.4 \rightarrow$ light shock - non uniform torque.

Fig. [17-7], $K_1 = 1$, $\phi_s = \pi - 2\sin^{-1}\left(\frac{D-d}{2C}\right) = \pi$

at $\phi_s = 180$ — $K_1 = 1.0$

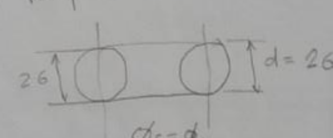


Table [17-10] C (belt) $\rightarrow K_2 = 1.2$, D (belt) $\rightarrow K_2 = 1.1$, E (belt) $\rightarrow K_2 = 1.05$

$$V = \frac{\pi d n_p}{12} = \frac{\pi(26)(400)}{12} = 2723 \text{ ft/min}$$

Centrifugal force:

$$F_c = K_c \left(\frac{V}{1000}\right)^2$$

$K_c \rightarrow$ Table [17-16] for standard belt size.

Horse power transmitted per belt.

$H_d = K_s H$, H = required Horse power.

$$H_d = \frac{K_s H n}{N}$$, n = factor of safety, N = No. of belts.

$$F_1 = F_c + \frac{\Delta \, e^{f\phi}}{e^{f\phi} - 1}$$

$$H_d = \frac{(F_1 - F_2) V}{33000}$$

$F_2 = F_1 - \Delta$

$$F_i = \frac{F_1 + F_2}{2} - F_c$$

$$\Delta = F_1 - F_2 = \frac{33000 \, H_d}{V}$$

Belt life:

Belt life is limited by bending force on small and large pulley

F_{b_1} , F_{b_2}

Max. Tension force in each pulley including tension by bending.

$$F_{t_1} = F_1 + F_{b_1} = F_1 + \frac{K_b}{d}$$

$$F_{t_2} = F_1 + F_{b_2} = F_1 + \frac{K_b}{D}$$

K_b = bending factor $\rightarrow$ Table [17-16]

Relation between tension force and no. of cycle.

$$F_t^{\,b} N_p = K^b$$

If $N_p > 10^9$ report that $N_p > 10^9$ and $t > \frac{N_p L_p}{720 V}$ hrs

N_p = No. of passes

K, b = constants $\rightarrow$ Table [17-17]

life time (hrs) $\Rightarrow t = \frac{N_p L_p}{720 V}$ (hrs)

2020/4/24 11:58

Rated power: H_R

Table [17-9]

For C (belt), $d = 26$, $H_R = 7.15 + \frac{723}{1000}(9.46 - 7.15) = 8.82$ hp

For D (belt), $d = 26$, $H_R = 13.9 + \frac{723}{1000}(18.1 - 13.9) = 16.93$ hp

For E (belt), $d = 26$, $H_R = 22.8 + \frac{723}{1000}(30.3 - 22.8) = 28.2$ hp

Required Horse power $= K_s H = 1.4 \times 60 = 84$ hp

but $H = \frac{K_1 K_2}{K_s} H_R \Rightarrow H_R = \frac{K_s H}{K_1 K_2} = \frac{84}{K_2}$

For C belt → $H_R = 70$ hp

For D belt → $H_R = 76.36$ hp

For E belt → $H_R = 80$ hp

Number of belts : C → $N = \frac{70}{8.82} = 7.9$ → $N = 8$ belts

D → $N = 76.36/16.93 = 4.3$ → $N = 5$ belts

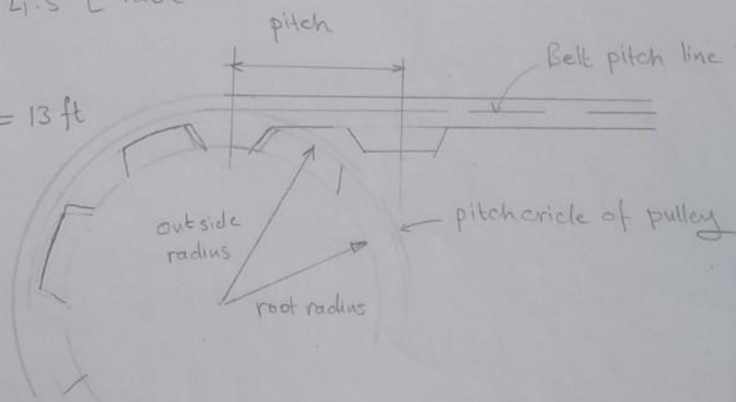
E → $N = 80/28.2 = 2.83$ → $N = 3$ belts

⇒ use **3 E390 belts** ⇒ $\Delta L = 4.5''$ [Table 17-8]

$L_p = 390 + 4.5 = 394.5''$

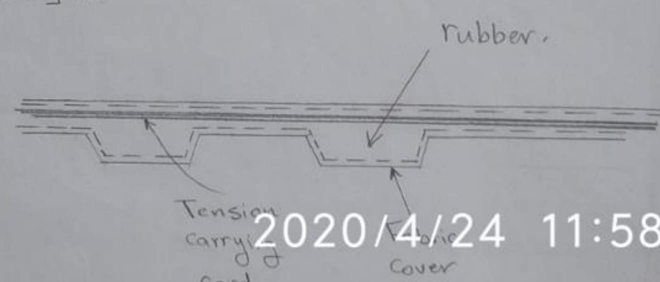
$L_p = 2C + 1.57(D+d) \Rightarrow C = 13$ ft

$C = 156.4''$



Timing Belts

- Drive is through teeth
  - → ther is no slippage
  - → Transmitt power at const. angular-Velocity ratio.
- No Initial tension is required
  No stretch because of initial cords.
- Operates at high ranges of speeds
  operates with effeciency 97-98 %.
- Quiet operation, No cordal speed Vibration → attractive for precision drive.
- Standard pitch listed in Table [17-12].
  Standard pitch length available [6-180] in
  pulley pitch dia. [0.6 - 35.8] in
  Number of grooves [10 - 120]



power per belt for E 390

$$H_d = \frac{K_s H}{N} = 1.4 \times 60 = 28 \text{ hp.}$$

$$H_d = \frac{(F_1 - F_2)V}{33000} \rightarrow F_1 - F_2 = \frac{33000 H_d}{V}$$

$$F_c = K_c \left(\frac{V}{1000}\right)^2$$

$K_c = 5.041$

$$F_c = 5.04 \left(\frac{2723}{1000}\right)^2 = 37.37 \text{ lb.}$$

$$\Delta = F_1 - F_2 = \frac{33000 \times 28}{2723} = 339.33 \text{ lb.}$$

$$F_1 = F_c + \Delta \frac{e^{f\phi}}{e^{f\phi} - 1} = 37.37 + \frac{339.33 \times 5}{5 - 1}$$

$\phi = \pi$

$F_1 = 461.53$ lb.

$F_2 = F_1 - \Delta = 122.2$ lb.

$$F_i = \frac{F_1 + F_2}{2} - F_c = 193.4 \text{ lb.}$$

Table [17-16] → $K_b = 10850$

$$F_{t_1} = F_1 + \frac{K_b}{d} = F_{t_2} = 775.1 \text{ lb.}$$

Table [17-17] $K = 6061$, $b = 11.1$

$$N_p = \left[\left(\frac{6061}{775.1}\right)^{-11.1} \times 2\right]^{-1} = 4.1 \times 10^9$$

No. of passes $N_p > 10^9 \rightarrow t > \frac{10^9 \times 394.5}{720 \times 2723} = 201 \times 10^3$ hr

2020/4/24 11:59