

8.7

$$\int_1^{\infty} \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1} & \text{if } p > 1 \quad \text{conv} \\ \infty & \text{if } p \leq 1 \quad \text{div} \end{cases}$$

$$\int_0^1 \frac{dx}{x^p} = \begin{cases} \frac{1}{1-p} & \text{if } p < 1 \\ \infty & \text{if } p \geq 1 \end{cases}$$

$$\lim_{n \rightarrow \infty} \ln n = \infty \quad \lim_{n \rightarrow \infty} \sqrt[n]{n} = 1 \quad \lim_{n \rightarrow \infty} x^{\frac{1}{n}} = 1, x > 0 \quad \lim_{n \rightarrow \infty} x^n = 0, |x| < 1 \quad \lim_{n \rightarrow \infty} (1 + \frac{x}{n})^n = e^x \quad \lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0 \quad \text{"Taylor Series"}$$

- * A sequence $\{a_n\}$ is bounded from above if $\exists$ a number M st $a_n \leq M$ for all $n \Rightarrow$ least upper bound $\Leftarrow$ اقصى حد
- * A sequence $\{a_n\}$ is bounded from below if $\exists$ a number m st $a_n \geq m$ for all $n \Rightarrow$ greatest lower bound $\Leftarrow$ ادنى حد
- * A sequence $\{a_n\}$ is bounded if it is bounded from above **and** is bounded from below.
- * A sequence $\{a_n\}$ is not bounded if it is not bounded from above **and** is not bounded from below.
- * A sequence $\{a_n\}$ is non-decreasing if $a_n \leq a_{n+1} \quad \forall n \quad a_1 \leq a_2 \leq a_3 \leq \dots$
- * A sequence $\{a_n\}$ is non-increasing if $a_n \geq a_{n+1} \quad \forall n \quad a_1 \geq a_2 \geq a_3 \geq \dots$
- * A sequence $\{a_n\}$ is monotonic if it is **either** non-decreasing **or** non-increasing
- * If a sequence $\{a_n\}$ is **both** bounded **and** monotonic then $\{a_n\}$ converge

Test 1 (n^{th} partial sum test)

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \dots$$

$$\text{Find } S_n = \sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n$$

if $\lim_{n \rightarrow \infty} S_n = L$ then $\sum_{n=1}^{\infty} a_n$ converge to L

if $\lim_{n \rightarrow \infty} S_n = \text{div}$ then $\sum_{n=1}^{\infty} a_n$ diverge

* Telescoping series $\Rightarrow$ A series which terms cancel together to leave only two terms.

Test 2 n^{th} term test (for div)

$$\sum_{n=1}^{\infty} a_n$$

if $\lim_{n \rightarrow \infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ div

if $\lim_{n \rightarrow \infty} a_n = 0 \Rightarrow$ The test fail

* Harmonic Series div $\Rightarrow \sum_{n=1}^{\infty} \frac{1}{n}$

* Sequence a_n	series $\sum_{n=1}^{\infty} a_n$
$a_n = \frac{1}{n}$	$\sum \frac{1}{n}$ diverge
$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{n} = 0$	
a_n converge to zero	

Test 3 Geometric Series Test

if $|r| < 1 \Rightarrow$ converg to sum $\Rightarrow \text{sum} = \frac{a}{1-r}$

if $|r| \geq 1 \Rightarrow$ diverge

Integral Test (IT)

Consider $\sum_{n=k}^{\infty} a_n$, when:

إذا $(\dots, 2, 1, 0)$ conv بمعنى

• a_n positive terms

إذا $(\dots, 1, 0)$ div بمعنى

• $a_n = f(n)$ is cont, positive, decreasing on $[k, \infty)$

Then $\sum_{n=k}^{\infty} a_n$ and $\int_k^{\infty} f(x) dx$ both conv or both div

P-series Test:

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ conv if $p > 1$ by Integral test

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ div if $p \leq 1$ by Integral Test

Direct Comparison Test (DCT)

let $\sum a_n, \sum c_n, \sum d_n$ be series with nonnegative terms.

suppos that $d_n \leq a_n \leq c_n$ for all n large

• if $\sum c_n$ converges, then $\sum a_n$ also converge.

• if $\sum d_n$ diverge, then $\sum a_n$ also diverges.

The Limit Comparison Test (LCT)

Suppos that $a_n > 0$ and $b_n > 0$ for all $n > N$ (large n)

من الطرق لا يعاد b_n في اخذ النهاية في a_n حد

1) if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c > 0$, then $\sum a_n$ and $\sum b_n$ both converges or both diverges

$$\sum_{n=1}^{\infty} \frac{n!+2}{n!+n^{1000}} : b_n = \frac{1}{n!}$$

2) if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then $\sum a_n$ converges

3) if $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \infty$ and $\sum b_n$ diverges, then $\sum a_n$ diverges

Ratio test (RT)

عادةً يستخدم مع مبرهن المقارنة (DCT)

Consider the infinite series $\sum a_n$ with positive terms. Assume $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = p$. Then:

• if $p < 1$, Then the series converges.

• if $p > 1$, Then the series diverges.

• if $p = 1$, Then the test is inconclusive.

The Root test

Consider the infinite series $\sum a_n$ with $a_n > 0$ for $n \geq N$. Assume $\lim_{n \rightarrow \infty} \sqrt[n]{a_n} = p$ then:

• if $p < 1$, Then the series converges.

• if $p > 1$, Then the series diverges.

• if $p = 1$, Then the test is inconclusive.

Alternating series Test (AST)

$$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} u_n = u_1 - u_2 + u_3 - u_4 + \dots$$

AST: $\sum (-1)^{n+1} u_n$ conv if

* إذا تحققت جميع الشروط $\leftarrow$ conv

1) $u_n > 0 \forall n$, $u_n = |a_n|$

* إذا لم يتحقق الثالث $\leftarrow$ $\sum a_n$ div by n^{th} term test

2) $u_n \downarrow$ for large n ($u_{n+1} \leq u_n$)

* إذا لم يتحقق الثاني $\leftarrow$ we can't say $\sum a_n$ conv or div (Test fail)

3) $\lim_{n \rightarrow \infty} u_n = 0$ "if not $\sum (-1)^{n+1} u_n$ div by n^{th} term test"

Conv Abs

if $\lim_{n \rightarrow \infty} |a_n|$ conv, Then $\sum_{n=1}^{\infty} a_n$ conv Abs

Converge Conditionally

The infinite series $\sum a_n$ converge conditionally if it converge by AST but not Abs

Alternating P-series Test

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{n^p} = \begin{cases} \text{conv. Abs} & \text{if } p > 1 \\ \text{conv. condi} & \text{if } 0 < p \leq 1 \\ \text{div by } n^{\text{th}} \text{ term} & \text{if } p < 0 \end{cases}$$

Alternating Estimation Th

Assume $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^n u_n = u_1 - u_2 + u_3 - u_4 + \dots + (-1)^{n+1} u_n = L$ و S_n واجمعهم من خلال قاعدة ال Geometric

If we approximate L by $S_n = u_1 - u_2 + u_3 + \dots + (-1)^{n+1} u_n$ Then: $S_n = \frac{1}{a-r}$ و مبلغ يطلق ال L وبعنا بجمع الحدود بشكل طبيعي ويطالع S_n

1) The remainder $L - S_n$ has same sign as a_{n+1}

2) The error $|L - S_n| < u_{n+1} = |a_{n+1}|$ و اذا اعطاني ال Series وطلب اوجد ال sum ال يعطيني error اقل من فيه معينة. نحتاج بعد فاي الخطوات

3) $\min\{S_n, S_{n+1}\} < L < \max\{S_n, S_{n+1}\} \quad \forall n$ و النتيجة ان اعطاني اياها u_n و يوجد من خلالها فيه n ملا $n > 5$

و الة يكون $\frac{1}{a-r} = L$ فائدة الحد الى بعنا يعطيني ال Series ال حدودها متناهية S_n و يوجد S_n ال n التي طلعت مع S_n

$$\text{error} = |L - S_n| < u_{n+1} \quad \text{ع}$$

Power Series

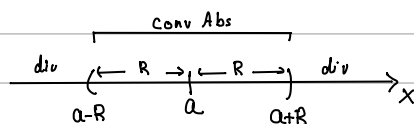
$$\sum_{n=0}^{\infty} a_n (x-a)^n = \frac{a_0}{a_0} + \frac{a_1}{a_1} (x-a) + \frac{a_2}{a_2} (x-a)^2 + \frac{a_3}{a_3} (x-a)^3 + \dots$$

a_i : Coefficients

a : center

R : Radius of convergence

I.C: Interval of convergence



$$I.C = |x-a| < R \Rightarrow -R < x-a < R \Rightarrow a-R < x < a+R$$

* To Find R and I.C $\Rightarrow$ we apply Ratio Test (الطريقة)

