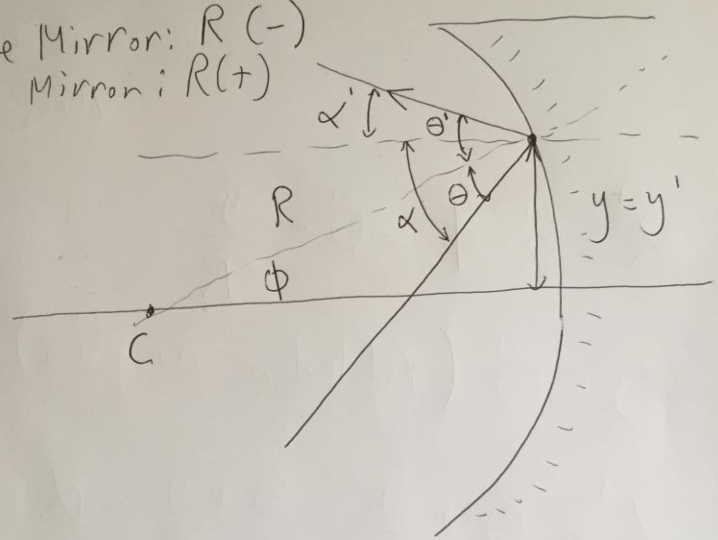
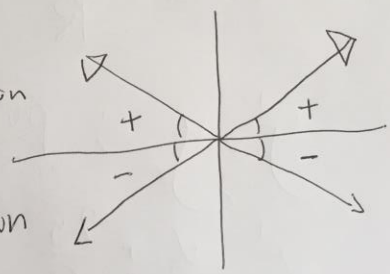
9.4 Reflection & Refraction at Curved Surfaces

Concave Mirror: R (-)
Convex Mirror: R(+)



Sign Convention



(+) for rays pointing up
(-) for rays pointing down

$\alpha' \rightarrow \text{up} \rightarrow (+)$

$\alpha \rightarrow \text{up} \rightarrow (+)$

2

$$\alpha = \Theta + \phi = \Theta + \frac{y}{-R}$$

$\sin\phi = \frac{y}{R}$

paraxial $\sin\phi = \phi$

$$\alpha' = \Theta' - \phi = \Theta' - \frac{y}{-R}$$

$\Theta = \Theta'$ (reflection angles).

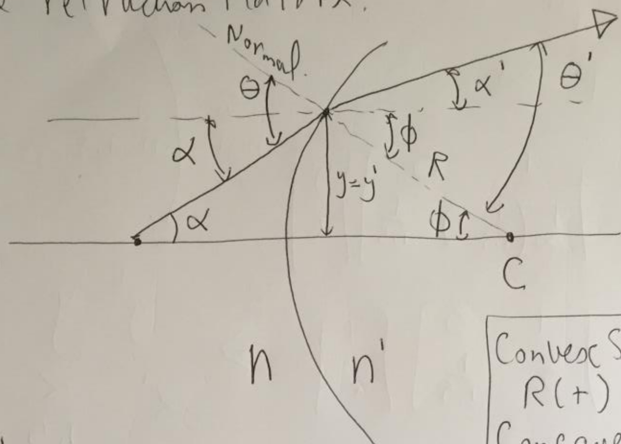
$$\alpha' = \Theta' + \frac{y}{R} = \Theta + \frac{y}{R} = \alpha + \frac{y}{R} + \frac{y}{R}$$

$y' = y$; point of reflection.

$$\alpha' = \alpha + \frac{2y}{R}$$

$\rightarrow$ ABCD matrix: $\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$

The refraction Matrix. 3



Convex Surface R(+)
Concave Surface R(−)

$$\alpha' = \theta' - \phi = \theta' - \frac{y}{R}$$

$$\alpha = \theta - \phi = \theta - \frac{y}{R}$$

using Snell's law: $n \sin\theta = n' \sin\theta'$

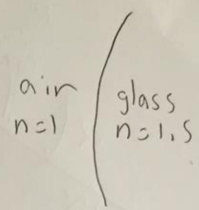
$$n\theta = n'\theta'$$

$$\alpha' = \left(\frac{n}{n'}\right)\theta - \frac{y}{R} = \frac{n}{n'}\left(\alpha + \frac{y}{R}\right) - \frac{y}{R}$$

$$\therefore \begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R}\left(\frac{n}{n'} - 1\right) & \frac{n}{n'} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

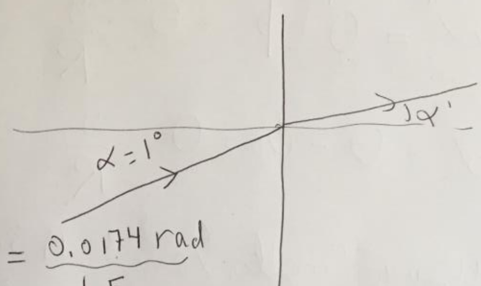
Example:

let $R = \infty$



$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{\infty}\left(\frac{1}{1.5}-1\right) & \frac{1}{1.5} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{1.5} \end{bmatrix} \begin{bmatrix} y \\ \alpha \end{bmatrix} \Rightarrow \begin{array}{l} y' = y \\ \alpha' = \dfrac{\alpha}{1.5} \end{array}$$



$$\alpha' = \frac{\alpha}{1.5} = \frac{\underbrace{0.0174 \text{ rad}}}{1.5}$$

$$\alpha' = 0.012 \text{ rad} = 0.69 \text{ deg.}$$

let us use Snell's law

$$n_i \sin\Theta_i = n_t \sin\Theta_t$$

$$(1)(\sin 1) = 1.5 \sin\Theta_t$$

$$\Theta_t = 0.67 \text{ deg.}$$

Same problem, but let $\alpha = 30^\circ$

$$\alpha' = \frac{\alpha}{1.5} = \frac{30}{1.5} = 20^\circ$$

Using Snell's law

$$(1) \sin 30 = 1.5 \sin \theta_t$$

$$\theta_t = 19.5^\circ$$

let $\alpha = 60^\circ$

$$\alpha' = \frac{60^\circ}{1.5} = 40^\circ$$

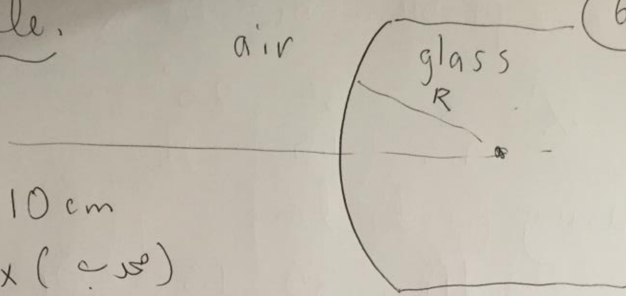
Using Snell's law

$$(1) \sin 60 = 1.5 \sin \theta_t$$

$$\theta_t = 35.3^\circ$$

The closer to paraxial approximatio range, the better.

Example:



$|R| = 10\text{ cm}$

convex (محدب)

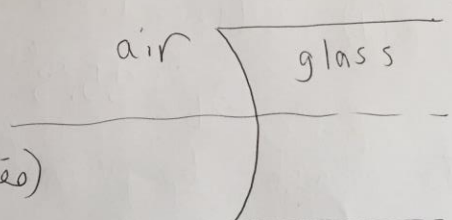
$R(+) = +10\text{ cm}$

$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{0.1}\left(\frac{1}{1.5}-1\right) & \frac{1}{1.5} \end{bmatrix}\begin{bmatrix} y \\ \alpha \end{bmatrix}$$

$$y' = y$$

$$\alpha' = 10\left(\frac{-1}{3}\right)y + \frac{2}{3}\alpha = \frac{-10}{3}y + \frac{2}{3}\alpha$$

Example:



$|R| = 10\text{cm}$

concave (مقعر)

$R(-) = -10$

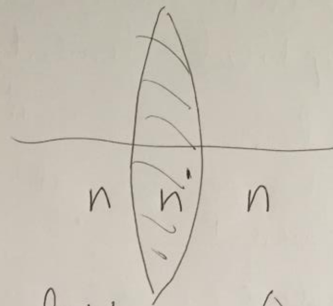
$$\begin{bmatrix} y' \\ \alpha' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{-0.1}\left(\frac{1}{1.5}-1\right) & \frac{1}{1.5} \end{bmatrix}\begin{bmatrix} y \\ \alpha \end{bmatrix} \Rightarrow \begin{aligned} y &= y' \\ \alpha' &= \frac{+10}{3}y + \frac{2}{3}\alpha \end{aligned}$$

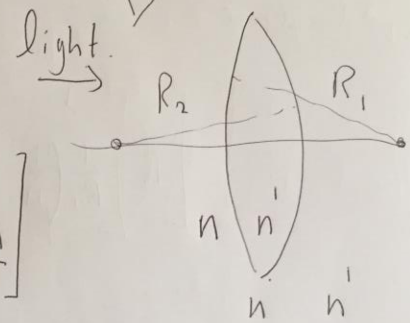
Examples:

1) Thin lens

R_1 : Convex $(+)$

R_2 : Concave $(-)$





$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2}\left(\frac{n}{1}-1\right) & \frac{n}{1} \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & 0 \\ \frac{1}{R_1}\left(\frac{1}{n}-1\right) & \frac{1}{n} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{R_2}(n-1) + \frac{n}{R_1}\left(\frac{1}{n}-1\right) & 1 \end{bmatrix}$$

$$\Longrightarrow \quad \begin{bmatrix} 1 & 0 \\ \frac{n}{R_2} - \frac{1}{R_2} + \frac{1}{R_1} - \frac{n}{R_1} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -n\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+1\left(\frac{1}{R_1}-\frac{1}{R_2}\right) & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) & +1 \end{bmatrix}$$

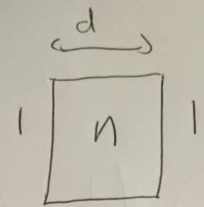
$$= \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

f = Focal length

= lens maker's formula

$$\boxed{= (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)}$$

Example:



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & n \end{bmatrix}\begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & n \end{bmatrix}\begin{bmatrix} 1 & \frac{d}{n} \\ 0 & \frac{1}{n} \end{bmatrix} = \begin{bmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{bmatrix}$$