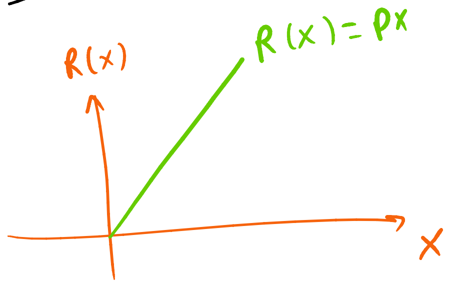
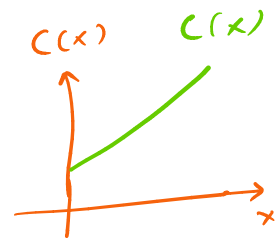
1.6 Recall that

$R(x) = p\,x$ line

$C(x) = mx + b$ line





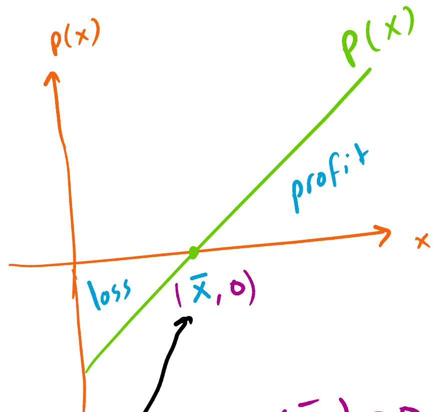
$P(x) = R(x) - C(x)$

$= px - (mx + b)$

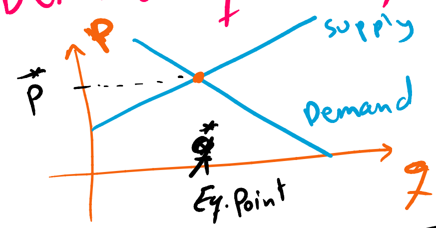
$= (p - m)x - b$ line

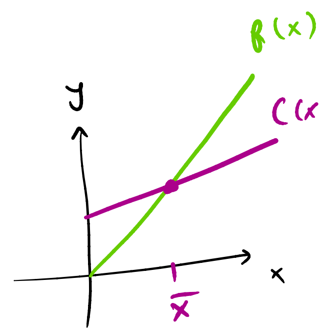
Supply $p = aq + b$ line

Demand $q = cq + d$ line



$P(\bar{x}) = 0$





Break-Even Point $\Rightarrow$ $C(x) = R(x)$

Q. What happen if $R(x)$ or $C(x)$ or demand or supply is not linear

A: we find Break-even by setting $C(x) = R(x)$
" " Eq. Point " " demand = Supply

Exp A monopoly market has a company whose total costs are $C(x) = 2000 + 40x + x^2$

total costs are $C(x) = 2000 + 40x + x$ and total revenue $R(x) = 130x$

quadratic not line

① Find the break-even point

$C(x) = R(x)$

$2000 + \cancel{40x} + x^2 = \cancel{130x}$

$-130x \qquad -130x$

$x^2 - 90x + 2000 = 0$

$(x - 40)(x - 50) = 0$

$a = 1, \quad b = -90, \quad c = 2000$

$D = \sqrt{b^2 - 4ac}$

$= \sqrt{(-90)^2 - 4(1)(2000)}$

$= \sqrt{8100 - 8000}$

$= \sqrt{100}$

$= 10$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-90) \pm 10}{2(1)} = \frac{90 \pm 10}{2}$$

$$x_1 = \frac{90+10}{2} = \frac{100}{2} = 50 \checkmark$$ مقبولة

$$x_2 = \frac{90-10}{2} = \frac{80}{2} = 40 \checkmark$$ //

$\left.\begin{array}{l} x_1 = 50 \\ x_2 = 40 \end{array}\right\}$ are break-even points

② Find Revenue at break even points

② Find Revenue at break even points

$R(x) = 130X \Rightarrow R(x_1) = R(50) = 130(50) = \underline{6500}$

$R(x_2) = R(40) = 130(40) = 5200$

③ Find cost at break-even points?

$C(x) = 2000 + 40x + x^2 \Rightarrow C(x_1) = C(50) = 6500$

$\Rightarrow C(x_2) = C(40) = 5200$

$C(40) = 2000 + 40(40) + (40)^2$

$= 2000 + 1600 + 1600$

$= 5200$

④ sketch the total cost and total Revenue

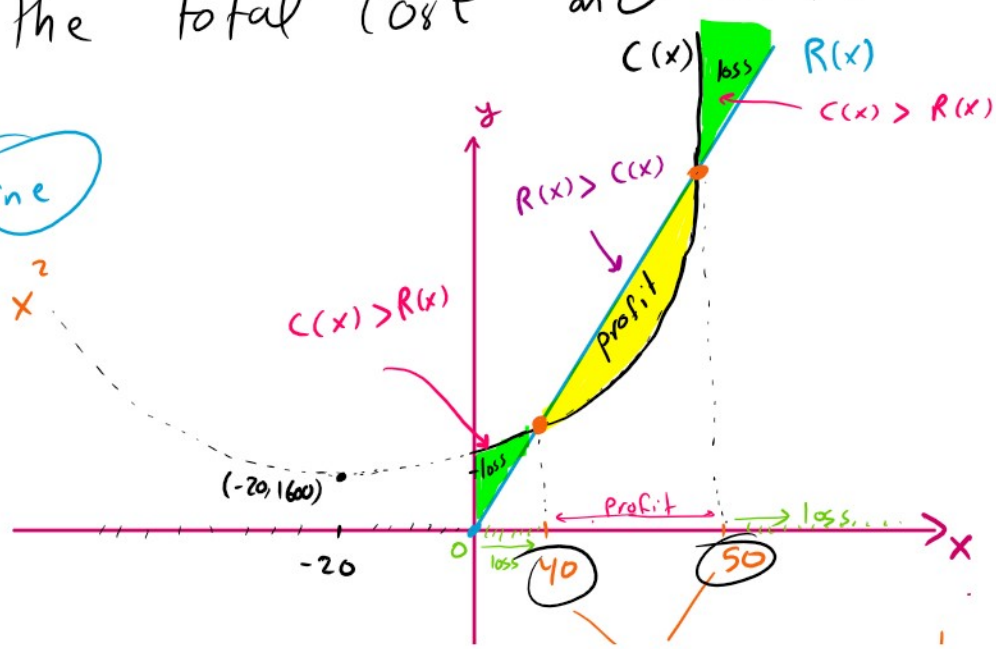
$R(x) = 130X$ line

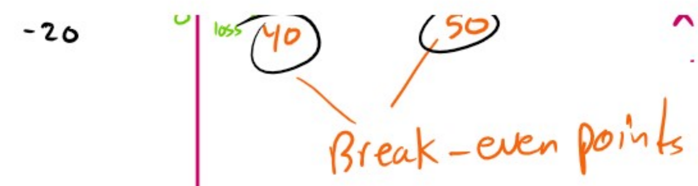
$C(x) = 2000 + 40x + x^2$

parabola

a>0 a<0

a-1>0 ⇒ opens up





$a = 1 > 0 \Rightarrow$ opens up

$b = 40$, $c = 2000$

Axis of symmetry $x = -\frac{b}{2a} = -\frac{40}{2(1)} = -20$

$C(-20) = 2000 + 40(-20) + (-20)^2 = 2000 - 800 + 400 = 1600$

vertex is $(-20, 1600)$

(5) Identify the regions where the company makes profit and where it has loss

loss region if it produces at level

$0 \leq x < 40$ or $x > 50$

profit region if it produces at level

$40 < x < 50$

(6) when the company makes zero profit

at $x = 40$ or at $x = 50$

(7) Find total profit

(7) Find

$$P(x) = R(x) - C(x)$$

فك الأقواس بعد اشارة -

$$= 130x - (2000 + 40x + x^2)$$

$$= 130x - 2000 - 40x - x^2$$

$$= 90x - 2000 - x^2$$

$$= -x^2 + 90x - 2000$$

Note that $P(40) = P(50) = 0$

(8) Graph the profit function

$$P(x) = -x^2 + 90x - 2000$$

$\boxed{x = 40, \ x = 50}$ → Break-even points

$P(40) = 0$

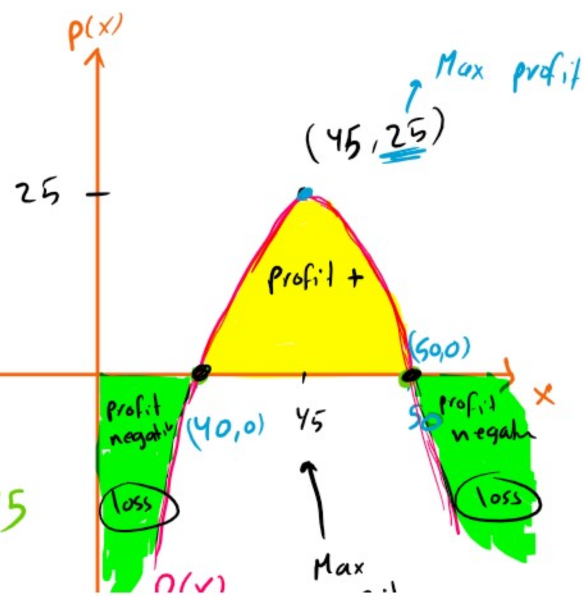
$P(50) = 0$

x-intercept

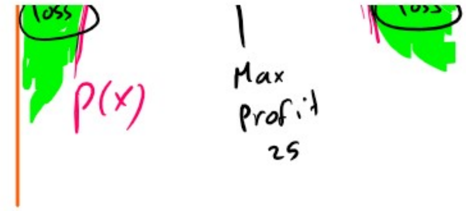
$a = -1 < 0 \Rightarrow$ opens down

$b = 90$

Axis of symmetry $x = -\frac{b}{2a} = -\frac{90}{2(-1)} = 45$



Axis of symmetry $x = -\frac{b}{2a} = -\frac{90}{2(-1)} = 45$



Vertex $\Rightarrow P(45) = -(45)^2 + 90(45) - 2000$

$\Downarrow$

$(45, 25)$

$= -2025 + 4050 - 2000$

$= 25$

⑨ At what level of production x the company makes maximum profit?

$x = 45 \Rightarrow$ company makes max profit

$P(45) = 25$

⑩ What does x-intercept for the profit function mean?

$\left.\begin{array}{l} x = 40 \\ x = 50 \end{array}\right\}$ are break-even points where profit = zero

⑪ For what values of x the profit is positive?

for $40 < x < 50$

⑫ For what values of x the profit is negative?

(12) For what values of x the profit is negative?

For $0 \leq x < 40$ or

$50 < x$

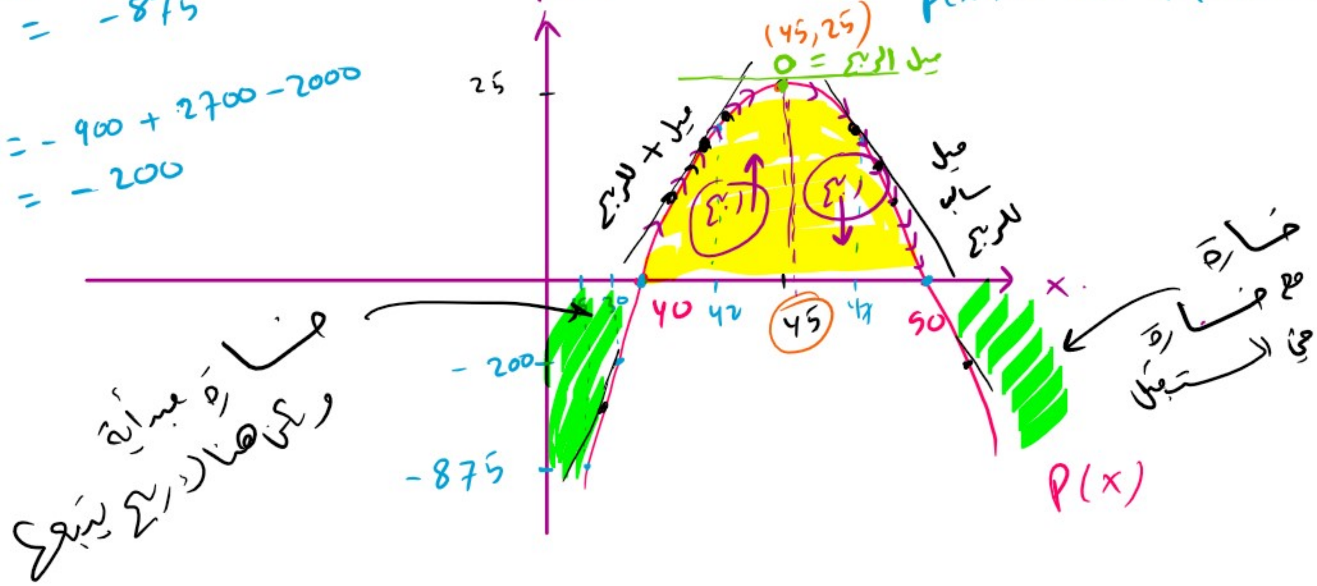
ميل

(13) Descrive the average rate of change of the profit before and closer and after $x = 45$

$p(15) = -225 + 1350 - 2000$
$= -875$

$P(x) = -x^2 + 90x - 2000$

$p(30) = -900 + 2700 - 2000$
$= -200$



Befor $x = 45 \Rightarrow$ the company's profil increases as $x \uparrow$

at $x = 45 \Rightarrow$ " " " is constat at 25

after $x = 45 \Rightarrow$ " " " is decreasing as $x \uparrow$

Before $x = 45 \Rightarrow$ slope of profit is $+$

at $x = 45 \Rightarrow$ " " " : 0

after $x = 45 \Rightarrow$ " " " " $-$

Exp If the supply and demand functions for a commodity are given by

supply: $p - q = 10 \Rightarrow \boxed{p = q + 10}$ ✓

demand: $q\,(2p - 10) = 2100$

① What is the Equilibrium quantity and price

$$\frac{q\,(2p-10)}{q} = \frac{2100}{q}$$

$$2p - \cancel{10} = \frac{2100}{q}$$
$$+10 \qquad\qquad +10$$

$$\frac{2p}{2} = \frac{2100}{q\,2} + \frac{10}{2}$$

$$\boxed{p = \frac{1050}{q} + 5}$$

Demand

Supply = Demand

$$q + \cancel{10} = \frac{1050}{q} + 5$$
$$-10 \qquad\qquad -10$$

$$q = \frac{1050}{q} - 5$$

multiply by q

$$q^2 = 1050 - 5q$$

$$q^2 = 1050 - 5q$$

$$q^2 + 5q - 1050 = 0$$

$a = 1$, $b = 5$, $c = -1050$

$$\Delta = \sqrt{b^2 - 4ac}$$
$$= \sqrt{(5)^2 - 4(1)(-1050)}$$
$$= \sqrt{25 + 4200}$$
$$= \sqrt{4225}$$
$$= 65$$

$$q = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-(5) \pm 65}{2(1)}$$

$$= \frac{-5 \pm 65}{2}$$

$$q^*_1 = \frac{-5 + 65}{2} = \frac{60}{2} = 30$$ مقبول $\Rightarrow p^* = q^* + 10$

$$= 30 + 10$$

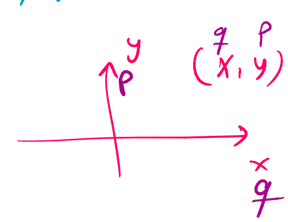
$$\boxed{p^* = 40}$$

$$q_2 = \frac{-5 - 65}{2} = \frac{-70}{2} = \cancel{-35}$$

Eq. quantity is $q^* = 30$

Eq. price is $p^* = 40$

Market Eq. is $(q^*, p^*) = (30, 40)$



② Draw the demand and supply functions

supply : $\boxed{p = q + 10}$ line

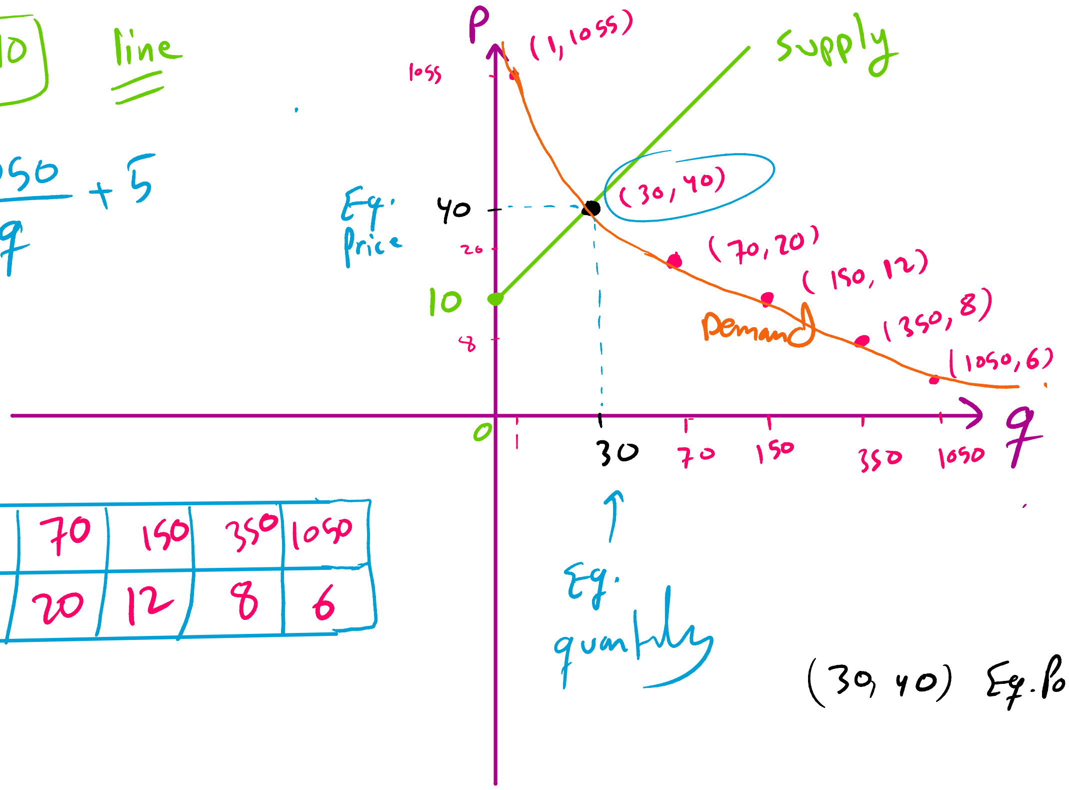
loss P (1, 1055)

supply

supply : $\boxed{p = q + 10}$ line

Demand : $p = \dfrac{1050}{q} + 5$

q	1	30	70	150	350	1050
p	1055	40	20	12	8	6



(30, 40) Eq. Point

