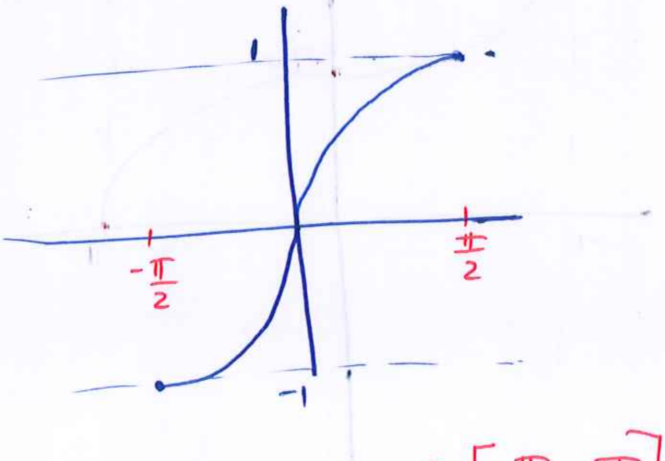
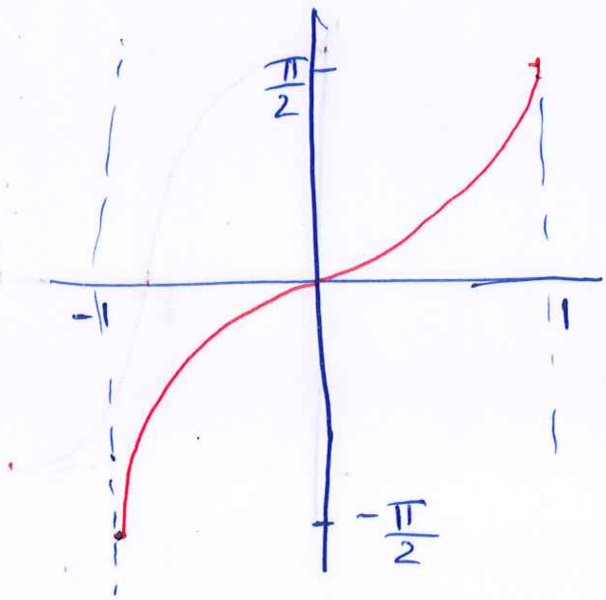
7.6 Inverse Trigonometric function

[1] $y = \sin x$

$y = \sin^{-1}(x) = \arc\sin(x)$





$y = \sin x$, $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

Some properties of $y = \sin^{-1}(x)$

① $Dom(\sin^{-1}(x)) = [-1, 1]$, $Range(\sin^{-1}(x)) = [-\frac{\pi}{2}, \frac{\pi}{2}]$

② $y = \sin^{-1}(x)$ is equivalent to $\sin y = x$, if $x \in Dom_{(\sin^{-1}(x))}$

③ $\sin^{-1}(\sin x) = x$ if $x \in Dom(\sin x)$

$x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

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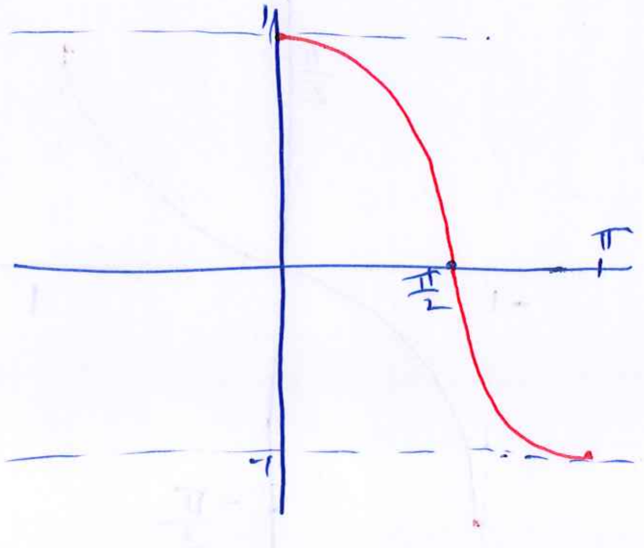
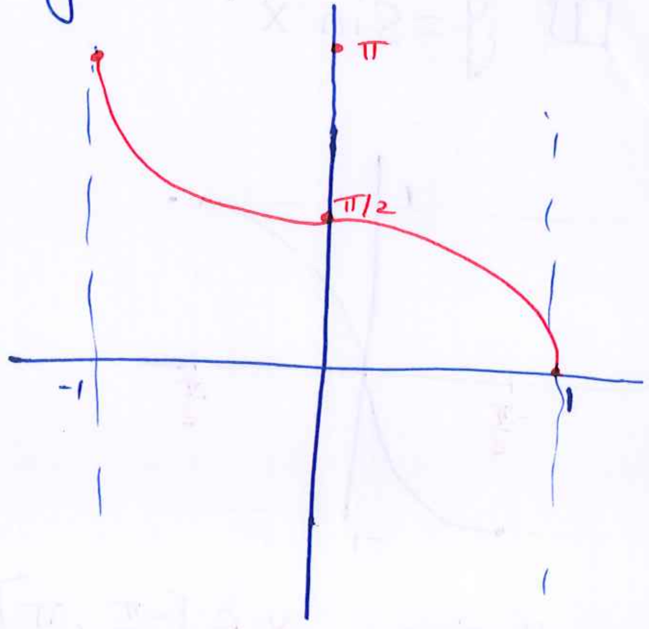
④ $\lim_{x \to 1^-} \sin^{-1}(x) = \frac{\pi}{2}$ $\lim_{x \to -1^+} \sin^{-1}(x) = -\frac{\pi}{2}$

$\boxed{2}$ $y = \cos x$ $\qquad$ $y = \cos^{-1}(x) = \arccos x$





$y = \cos x \,, \quad x \in [0, \pi]$ $\qquad$ $y = \cos^{-1}(x) \,, \; x \in [-1, 1]$

Some properties $y = \cos^{-1}(x)$

① $\mathrm{Dom}\left(\cos^{-1} x\right) = [-1, 1]$, $\mathrm{Range}\left(\cos^{-1}(x)\right) = [0, \pi]$

② $y = \cos^{-1} x$ is equivalent to $\cos y = x$, if $x \in \mathrm{Dom}\left(\cos^{-1} x\right)$

③ $\cos^{-1}(\cos x) = x$, if $x \in [0, \pi]$

$\cos\left(\cos^{-1} x\right) = x$, if $x \in [-1, 1]$

④ $\lim\limits_{x \to -1^+} \cos^{-1}(x) = \pi$ $\qquad$ $\lim\limits_{x \to 1^-} \cos^{-1}(x) = 0$

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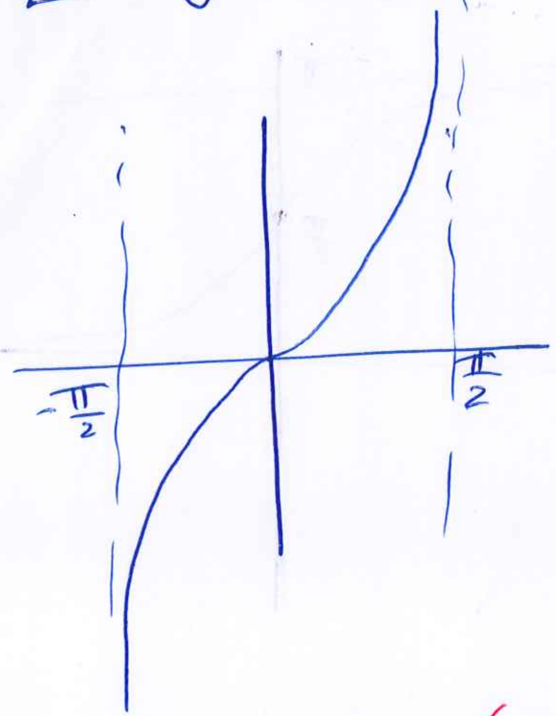
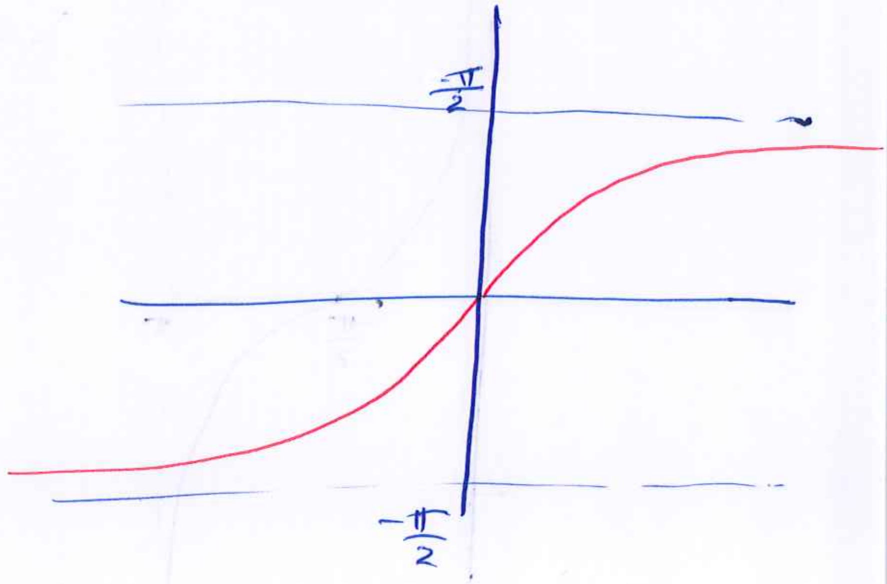
$\boxed{3}$ $y = \tan x$ $\qquad$ $y = \tan^{-1} x = \arctan x$





$y = \tan x \quad , \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Some properties of $y = \tan^{-1}(x)$

① $\text{Dom}(\tan^{-1} x) = (-\infty, \infty)$ $\qquad$ $\text{Range}(\tan^{-1} x) = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

② $y = \tan^{-1} x$ is equivalent to $\tan y = x$, if $x \in (-\infty, \infty)$

③ $\tan^{-1}(\tan x) = x$, if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\tan(\tan^{-1}(x)) = x$, if $x \in (-\infty, \infty)$

④ $\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$ $\qquad$ $\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

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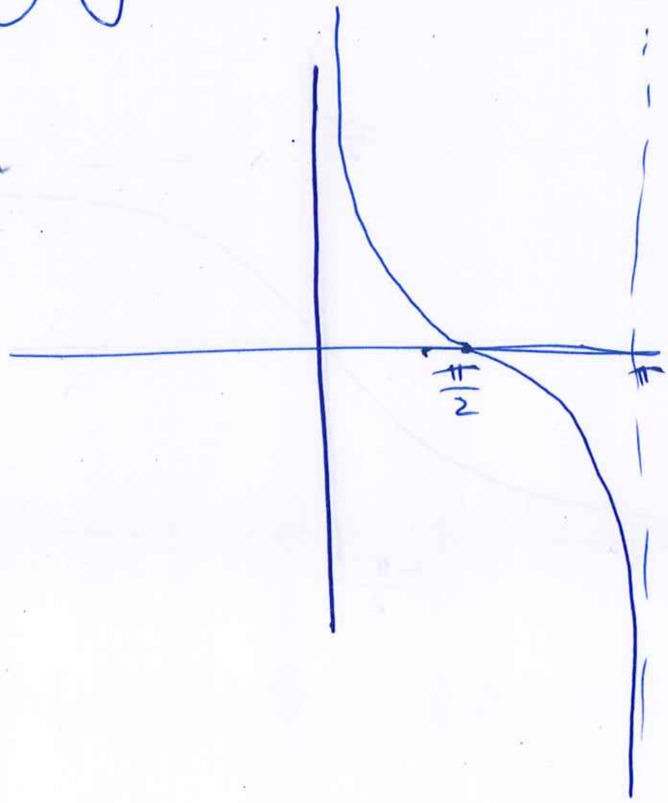
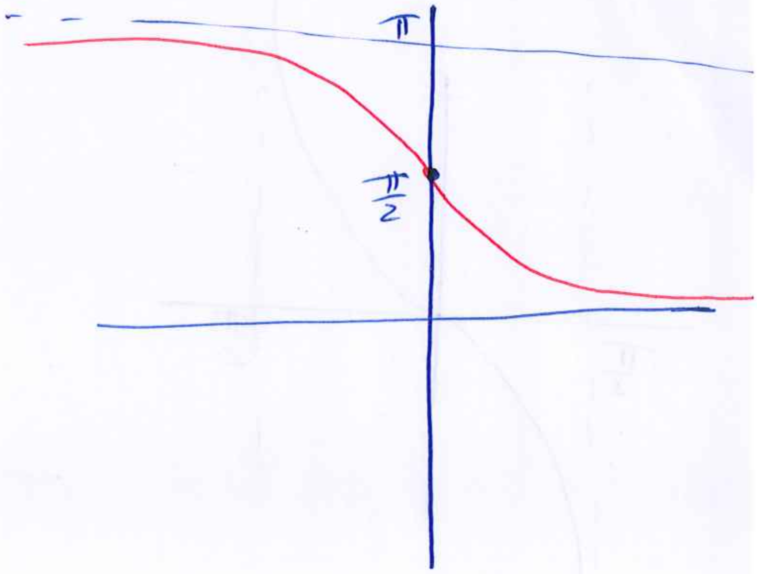
(4) $y = \cot x$ $\qquad$ $y = \cot^{-1}(x) = \text{arc} \cot x$





$y = \cot x$, $x \in (0, \pi)$ $\qquad$ $y = \cot^{-1}(x)$, $x \in (-\infty, \infty)$

Some properties of $y = \cot^{-1}(x)$

(1) $\text{Dom}(\cot^{-1} x) = (-\infty, \infty)$ $\qquad$ $\text{Range}(\cot^{-1} x) = (0, \pi)$

(2) $y = \cot^{-1} x$ is equivalent to $\cot y = x$, if $x \in (-\infty, \infty)$

(3) $\cot(\cot^{-1} x) = x$, if $x \in (-\infty, \infty)$

$\cot^{-1}(\cot x) = x$, if $x \in (0, \pi)$

(4) $\lim_{x \to \infty} \cot^{-1} x = 0$ $\qquad$ $\lim_{x \to -\infty} \cot^{-1} x = \pi$

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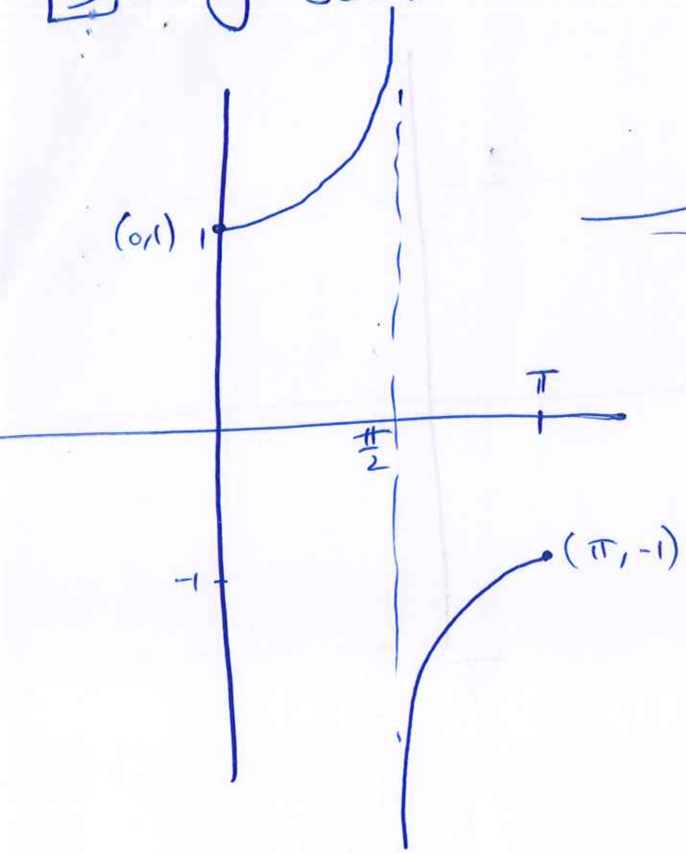
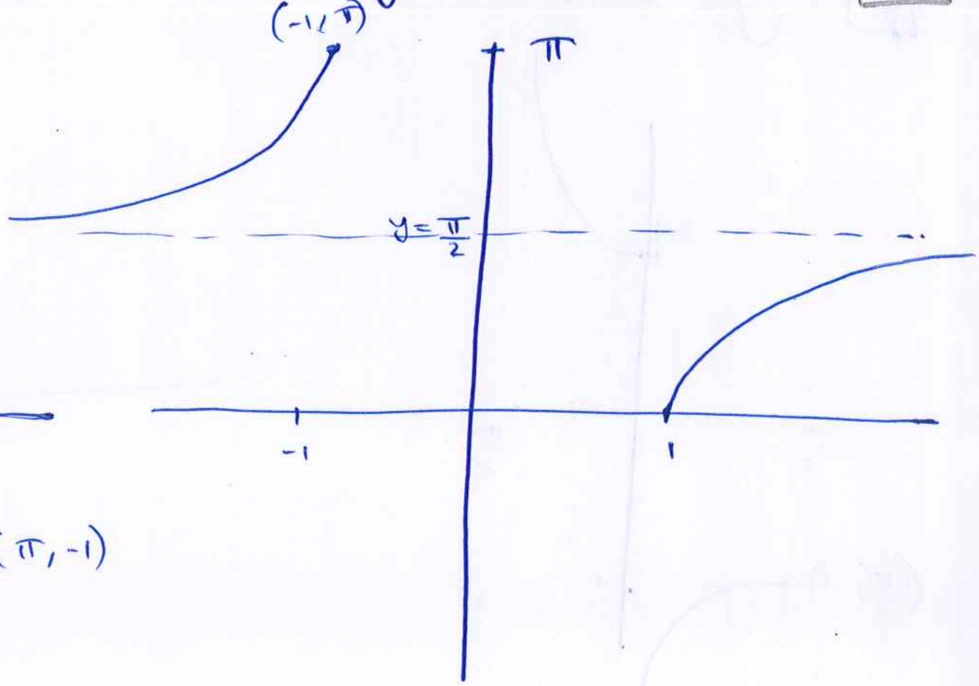
$\boxed{5}$ $y = \sec x$ $\qquad$ $y = \sec^{-1} x = \operatorname{arc\,sec} x$





$y = \sec x, \quad x \in [0, \pi] \setminus \{\frac{\pi}{2}\}$ $\qquad$ $y = \sec^{-1} x, \quad x \in (-\infty, -1] \cup [1, \infty)$

Some properties of $y = \sec^{-1} x$

① $\operatorname{Dom}(\sec^{-1} x) = (-\infty, -1] \cup [1, \infty)$

$\operatorname{Range}(\sec^{-1} x) = [0, \pi] \setminus \{\frac{\pi}{2}\}$.

② $y = \sec^{-1} x$ is equivalent to $\sec y = x$, if $x \in \operatorname{Dom}(\sec^{-1} x)$

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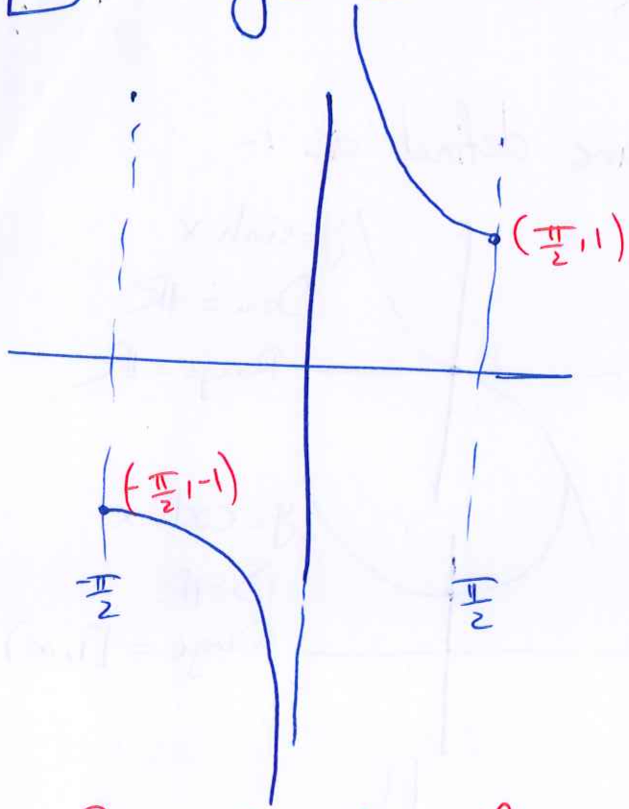
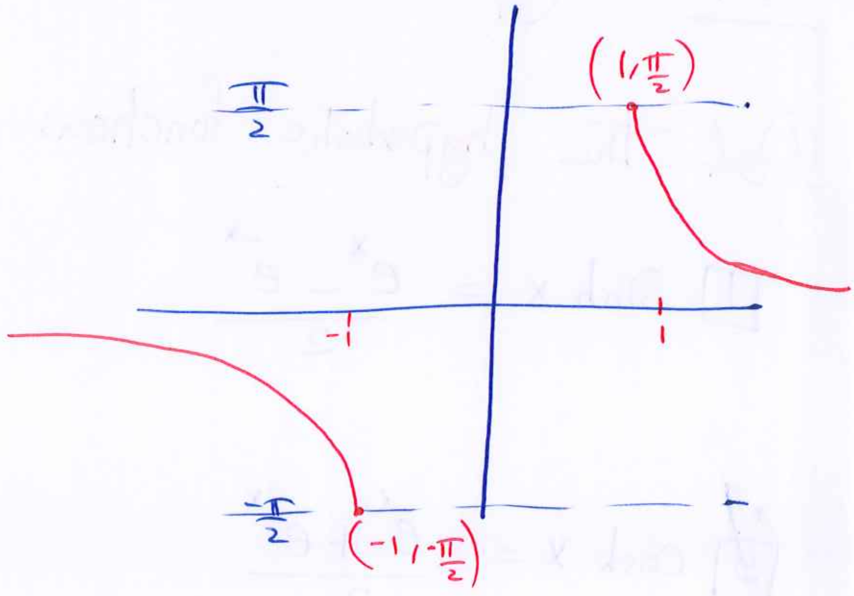
③ $\sec^{-1}(\sec x) = x$, if $x \in [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$

$\sec(\sec^{-1} x) = x$, if $x \in (-\infty, -1] \cup [1, \infty)$

④ $\lim_{x \to \infty} \sec^{-1} x = \lim_{x \to -\infty} \sec^{-1} x = \frac{\pi}{2}$

6 $y = \csc x$ $\qquad$ $y = \csc^{-1} x = \text{arc} \csc(x)$





Some properties of $y = \csc^{-1} x$

1. $\text{Dom}(\csc^{-1} x) = (-\infty, -1] \cup [1, \infty)$

   $\text{Range}(\csc^{-1} x) = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$

2. $y = \csc^{-1} x$ is equivalent to $\csc y = x$, if $x \in \text{Dom}(\csc^{-1} x)$

3. $\csc^{-1}(\csc x) = x$, if $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \setminus \{0\}$

   $\csc(\csc^{-1} x) = x$, if $x \in (-\infty, -1] \cup [1, \infty)$

4. $\lim_{x \to \infty} \csc^{-1} x = \lim_{x \to -\infty} \csc^{-1} x = 0$

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Examples:

Find the domain of the function $f(x) = \cos^{-1}\left(\frac{2^x - 5}{3}\right)$?

Sol. $Dom(f) = \{x : -1 \leq \frac{2^x - 5}{3} \leq 1\}$

$= \{x : -3 \leq 2^x - 5 \leq 3\}$

$\{x : 2 \leq 2^x \leq 8\}$

$\{x : \ln 2 \leq x \ln 2 \leq 3 \ln 2\}$

$\{x : \frac{\ln 2}{\ln 2} \leq x \leq \frac{3 \ln 2}{\ln 2}\}$

$\{x : 1 \leq x \leq 3\}$

Exp: Find the exact value of the following expression if it exist

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① $\cos\left(\frac{1}{2}\right)$ $\qquad \frac{1}{2} \in Dom_{\cos^{-1}(x)}$

Sol let $\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\cos\theta = \frac{1}{2} \longrightarrow \theta = \frac{\pi}{3} = \cos^{-1}\left(\frac{1}{2}\right)$

② $\sin^{-1}\left(\sin \frac{7\pi}{3}\right)$ $\qquad \frac{7\pi}{3} = 2\pi + \frac{\pi}{3}$ $\in Dom_{\sin x}$

$\sin^{-1}\left(\sin\left(2\pi + \frac{\pi}{3}\right)\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right) = \frac{\pi}{3}$

③ $\sin^{-1}\left(\sin\left(\frac{9\pi}{5}\right)\right) \rightarrow \frac{9\pi}{5} = 2\pi - \frac{\pi}{5}$

$= \sin^{-1}\left(\sin\left(2\pi - \frac{\pi}{5}\right)\right)$

$= \sin^{-1}\left(\sin\left(\frac{-\pi}{5}\right)\right) = -\frac{\pi}{5}$

$\downarrow$

$\frac{-\pi}{5} \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

④ $\cos\left(\cos^{-1}\left(\frac{-4}{3}\right)\right) =$ Does not exist

because $\frac{-4}{3} \notin Dom_{\cos^{-1}x}$

⑤ $\sin\left(\cos^{-1}\frac{\sqrt{2}}{2}\right) = \sin(\theta)$

$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\cos\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \longrightarrow \theta = \frac{\pi}{4} \longrightarrow \sin(\theta) = \frac{1}{\sqrt{2}}$

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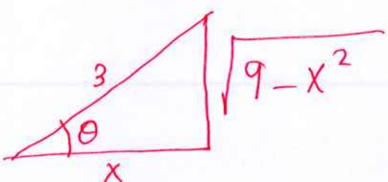
$\boxed{*}$ Simplify the following expression

① $\tan\left(\cos^{-1}\left(\frac{x}{3}\right)\right)$, $-3 \leq x \leq 3$

let $\theta = \cos^{-1}\left(\frac{x}{3}\right)$

$\cos\theta = \frac{x}{3}$



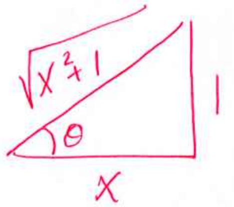
$\tan\theta = \frac{\sqrt{9-x^2}}{x}$

$\tan\left(\cos^{-1}\left(\frac{x}{3}\right)\right) = \frac{\sqrt{9-x^2}}{x}$

(2) $\sec^2\left(\cot^{-1} x\right)$

let $\theta = \cot^{-1} x$

$\cot\theta = x$

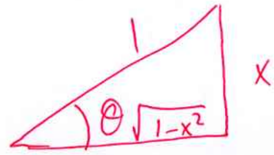


$$\sec^2\left(\cot^{-1} x\right) = \sec^2(\theta) = \left[\frac{\sqrt{x^2+1}}{x}\right]^2 = \frac{x^2+1}{x^2}$$

(3) $\cos\left(2\sin^{-1} x\right) + 2\left(4^{\log_2 x}\right)$

let $\theta = \sin^{-1} x$

$\sin\theta = x$



$$= \cos(2\theta) + 2\left(2^{\log_2 x^2}\right)$$

$$= \cos^2(\theta) - \sin^2(\theta) + 2x^2$$

$$= \cos^2\theta - \sin^2\theta + 2\sin^2\theta = \cos^2\theta + \sin^2\theta = 1$$

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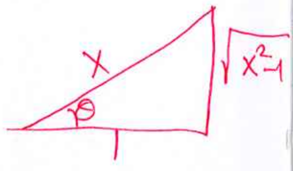
(4) Solve for x

$$\ln(x-e) = e^{-3\ln x}\, x^5 - \tan^2\left(\sec^{-1} x\right)$$

$\theta = \sec^{-1} x$

$\sec\theta = x$



$$\ln(x-e) = x^{-3}\, x^5 - \tan^2(\theta)$$

$$\ln(x-e) = x^2 - \left(x^2 - 1\right)$$

$$\ln(x-e) = 1$$

$$x - e = e \longrightarrow \boxed{x = 2e}$$

Some Identities Involving inverse Trigonometric function

$$* \quad \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \qquad |x| \leq 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \qquad \forall x$$

$$\sec^{-1} x + \csc^{-1} x = \frac{\pi}{2} \qquad |x| \geq 1$$

$$* \quad \sin^{-1}(-x) = -\sin^{-1}(x)$$

$$\tan^{-1}(-x) = -\tan^{-1}(x)$$

→ because the graph of $\sin^{-1}(x)$, $\tan^{-1}(x)$ symmetric about the origin

→ $\sin^{-1}(x)$, $\tan^{-1}(x)$ Odd functions

$$* \quad \sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right), \quad x \geq 1$$

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$$\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right), \quad x \geq 1$$

$$\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right), \quad x > 0$$

proof let $\theta = \cot^{-1}(x)$

$\cot(\theta) = x$

$\frac{1}{\tan\theta} = x \rightarrow \tan\theta = \frac{1}{x}$

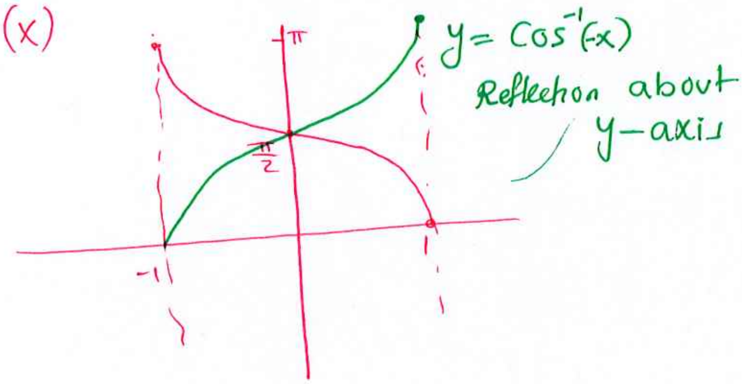
$\theta = \tan^{-1}\left(\frac{1}{x}\right)$

$$* \quad \cos^{-1}(-x) = \pi - \cos^{-1}(x), \quad |x| \leq 1$$

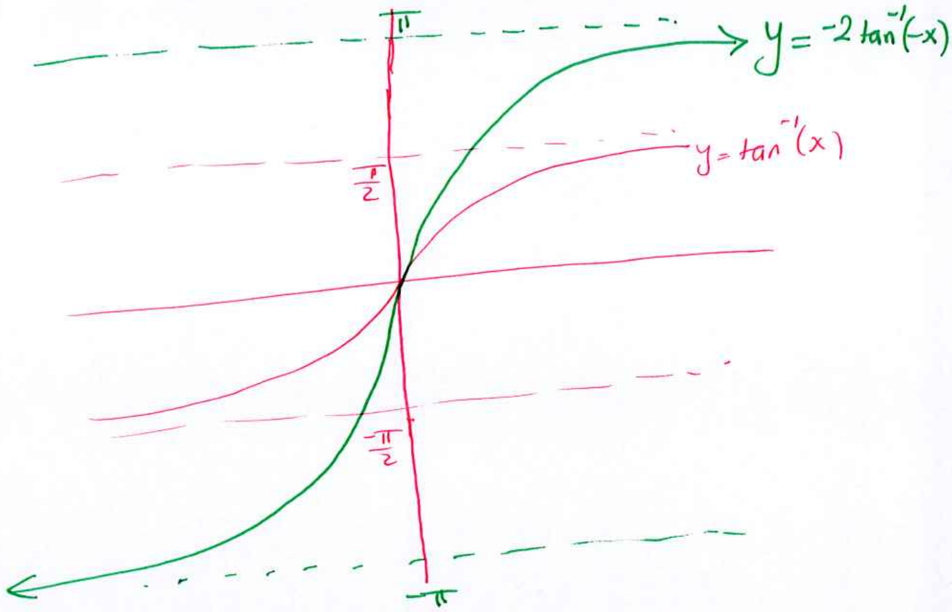
$$\sec^{-1}(-x) = \pi - \sec^{-1}(x), \quad |x| \geq 1$$

Exp: Sketch the graph of the following curve

① $y = \cos^{-1}(-x) = \pi - \cos^{-1}(x)$



② $y = -2\tan^{-1}(-x) = 2\tan^{-1}(x)$



Exp: Find the exact value for each of the following

① $\cos^{-1}\left(\frac{-1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

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② $\cot\left(\sin^{-1}\left(\frac{-1}{2}\right) - \sec^{-1}\left(\underset{\in D_{\sec^{-1}x}}{2}\right)\right)$

$\cot\left(-\sin^{-1}\left(\frac{1}{2}\right) - \cos^{-1}\left(\frac{1}{2}\right)\right) = \cot\left[-\left(\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)\right)\right]$

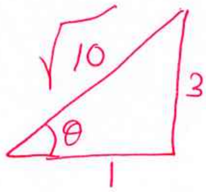
$= \cot\left[-\frac{\pi}{2}\right]$ $\quad$ $\cot(x)$ odd fn.

$= -\cot\left(\frac{\pi}{2}\right) = 0$

(3) $\sec\left[\tan^{-1}(-3)\right]$

$$\sec\left[-\tan^{-1}(3)\right] = \sec\left[\tan^{-1}(3)\right]$$

$$= \sec(\theta)$$



$$= \sqrt{10}$$

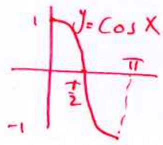
(4) $2^{\log_4 \pi^2} - \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$$4^{\frac{1}{2}\log_4 \pi^2} - \left[\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right]$$

$$4^{\log_4 \pi} - \pi + \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi - \pi + \frac{\pi}{4}$$

$\in [0, \pi]$

$$= \frac{\pi}{4}$$



Solve for x :-

$$\cos^{-1}(-x) - \ln\left(x e^{\sin^{-1} x}\right) = \frac{\pi}{2}$$

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$$\pi - \cos^{-1}(x) - \ln x - \ln e^{\sin^{-1} x} = \frac{\pi}{2}$$

$$\pi - \underline{\cos^{-1}(x)} - \ln x \underline{- \sin^{-1} x} = \frac{\pi}{2}$$

$$\pi - \frac{\pi}{2} - \ln x = \frac{\pi}{2}$$

$$\frac{\pi}{2} - \ln x = \frac{\pi}{2} \longrightarrow \ln x = 0$$

$$\boxed{x = 1}$$

* The derivative of Inverse Trigonometric Function

If U is a diff function of x, then

$\boxed{1}$ $$\frac{d}{dx}\left(\sin^{-1}(U)\right) = \frac{1}{\sqrt{1-U^2}}\frac{dU}{dx} \qquad |U|<1$$

$\boxed{2}$ $$\frac{d}{dx}\left(\cos^{-1}(U)\right) = \frac{-1}{\sqrt{1-U^2}}\frac{dU}{dx} \qquad |U|<1$$

$\boxed{3}$ $$\frac{d}{dx}\left(\tan^{-1}(U)\right) = \frac{1}{1+U^2}\frac{dU}{dx} \qquad \forall U$$

$\boxed{4}$ $$\frac{d}{dx}\left(\cot^{-1}(U)\right) = \frac{-1}{1+U^2}\frac{dU}{dx} \qquad \forall U$$

$\boxed{5}$ $$\frac{d}{dx}\left(\sec^{-1}(U)\right) = \frac{1}{|U|\sqrt{U^2-1}}\frac{dU}{dx} \qquad |U|>1$$

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$\boxed{6}$ $$\frac{d}{dx}\left(\csc^{-1}(U)\right) = \frac{-1}{|U|\sqrt{U^2-1}}\frac{dU}{dx} \qquad |U|>1$$

The following formula hold for any constant $a \neq 0$

$\boxed{1}$ $$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right) + c \quad , \quad u^2 < a^2$$

$\boxed{2}$ $$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right) + c \quad , \quad \forall u$$

$\boxed{3}$ $$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + c \quad , \quad |u| > a$$

Find the following limit

$\boxed{1}$ $$\lim_{x\to\infty} x \tan^{-1}\left(\frac{2}{x}\right) = 0 \cdot \infty$$

$$\lim_{x\to\infty} \frac{\tan^{-1}\left(\frac{2}{x}\right)}{\left(\frac{1}{x}\right)} \overset{\text{L'Hopital}}{=} \lim_{x\to\infty} \frac{\frac{1}{1+\left(\frac{2}{x}\right)^2} \cdot \frac{-2}{x^2}}{\frac{-1}{x^2}}$$

$$= 2 \lim_{x\to\infty} \frac{1}{1+\left(\frac{2}{x}\right)^2}$$

$$= 2$$

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$\boxed{2}$ $$\lim_{x \to 0^+} \frac{\sin^{-1}(x^2)}{(\sin^{-1} x)^2} = \frac{0}{0}$$

$$\lim_{x \to 0^+} \frac{\frac{1}{\sqrt{1-x^4}}(2x)}{2(\sin^{-1}(x)) \cdot \frac{1}{\sqrt{1-x^2}}}$$

$$= \lim_{x \to 0^+} \frac{\frac{2x}{\sqrt{(1-x^2)(1+x^2)}}}{\frac{2\sin^{-1}(x)}{\sqrt{1-x^2}}} = \lim_{x \to 0^+} \frac{\cancel{2}x}{\sqrt{1+x^2}} \cdot \frac{1}{\cancel{2}\sin^{-1}x} = \frac{0}{0}$$

L'Hopital

$$= \lim_{x \to 0^+} \frac{1}{\sqrt{1+x^2}\,\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x\,\frac{2x}{2\sqrt{1+x^2}}}$$

$$= \frac{1}{1+0} = 1$$

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Example $\boxed{2}$ Find $\frac{dy}{dx}$ for each of the following

(a) $y = \tan^{-1}(\ln x)$

$$\frac{dy}{dx} = \frac{1}{1+(\ln x)^2} \cdot \frac{1}{x}$$

(b) $y = \cot^{-1}\left(\sqrt{x^2-1}\right)$

$$\frac{dy}{dx} = \frac{-1}{1+\left(\sqrt{x^2-1}\right)^2} \cdot \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-1}}$$

$$= \frac{-x}{x^2\sqrt{x^2-1}}$$

$$= \frac{-1}{x\sqrt{x^2-1}}$$

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(c) $y = \sqrt{x}\left(\cos^{-1}\sqrt{x}\right)^4$

$$\frac{dy}{dx} = \cancel{\sqrt{x}}\ \overset{2}{\cancel{4}}\left(\cos^{-1}(\sqrt{x})\right)^3 \cdot \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{\cancel{2}\cancel{\sqrt{x}}} + \left(\cos^{-1}\sqrt{x}\right)^4 \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{-2\left(\cos^{-1}\sqrt{x}\right)^3}{\sqrt{1-x}} + \frac{\left(\cos^{-1}\sqrt{x}\right)^4}{2\sqrt{x}}$$

(d) $y = 3^{\tan^{-1} x} + \cot^{-1}(3^x)$

$$\frac{dy}{dx} = 3^{\tan^{-1}(x)}(\ln 3)\left(\frac{1}{1+x^2}\right) + \frac{-1}{1+(3^x)^2} \cdot 3^x \ln 3$$

$$= \frac{\ln 3 \; \cancel{\tan} \; 3^{\tan^{-1} x}}{1+x^2} + \frac{-3^x \ln 3}{1+9^x}$$

(e) $\sin^{-1}(\ln y) = \csc^{-1}(e^{xy})$

$$\frac{1}{\sqrt{1-\ln^2 y}} \cdot \frac{1}{y} \cdot y' = \frac{-1}{|\cancel{e^{xy}}| \sqrt{(e^{xy})^2 - 1}} \cdot (\cancel{e^{xy}})(xy' + y)$$

$$\frac{y'}{y\sqrt{1-(\ln y)^2}} = \frac{-xy' - y}{\sqrt{(e^{xy})^2 - 1}}$$

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--- - - - Cont. $y' =$

Evaluate the following integral

$\boxed{1}$ $\displaystyle\int \frac{dx}{9+3x^2}$

$$\frac{1}{3}\int \frac{dx}{3+x^2} = \frac{1}{3}\cdot\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+C$$

$\boxed{2}$ $\displaystyle\int \frac{\sec^2 x \quad dx}{\sqrt{4-\tan^2 x}}$

$u = \tan x$

$du = \sec^2 x\, dx$

$$\int \frac{du}{\sqrt{\underset{a^2}{\underline{4}}-u^2}} = \sin^{-1}\left(\frac{u}{2}\right)+C$$

$$= \sin^{-1}\left(\frac{\tan x}{2}\right)+C$$

$\boxed{3}$ $\displaystyle\int \frac{x^3+3x}{x^4+9}\,dx$

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$u = x^2$

$du = 2x\,dx$

$$\frac{1}{4}\int \frac{4x^3}{x^4+9}\,dx + \int \frac{3x}{x^4+9}\,dx$$

$$\frac{1}{4}\ln|x^4+9| + \frac{3}{2}\int \frac{du}{u^2+9}$$

$\frac{1}{4} \ln|x^4+9| + \frac{3}{2}\frac{1}{3}\tan^{-1}\left(\frac{u}{3}\right) + C$ $\boxed{155}$

$\frac{1}{4} \ln\left(x^4+9\right) + \frac{1}{2} \tan^{-1}\left(\frac{x^2}{3}\right) + C$

$\boxed{4}$ $\displaystyle\int \frac{1}{\sqrt{e^{2x}-16}}\,dx$

$u = e^x$

$du = e^x\,dx$

$$\int \frac{e^x}{e^x\sqrt{e^{2x}-16}}\,dx = \int \frac{e^x\,dx}{e^x\sqrt{e^{2x}-16}}$$

$$\int \frac{du}{u\sqrt{u^2-16}}$$

$$= \frac{1}{4}\sec^{-1}\left|\frac{u}{4}\right| + C$$

$u = e^x$ positive function

$$= \frac{1}{4}\sec^{-1}\left(\frac{e^x}{4}\right) + C$$

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$\boxed{5}$ $\displaystyle\int \frac{dx}{(x-2)\sqrt{x^2-4x+3}}$

$$\int \frac{dx}{(x-2)\sqrt{x^2-4x+4-1}} = \int \frac{dx}{(x-2)\sqrt{(x-2)^2-1}}$$

$$\int \frac{dx}{(x-2)\sqrt{(x-2)^2-1}}$$

$U = x-2$
$dU = dx$

$$\int \frac{dU}{U\sqrt{U^2-1}} = \sec^{-1}|U| + C$$

$$= \sec^{-1}|x-2| + C$$

$$\int_0^{\ln\sqrt{3}} \frac{e^x\,dx}{1+e^{2x}}$$

$U = e^x$
$dU = e^x\,dx$

$$\int_1^{\sqrt{3}} \frac{dU}{1+U^2} = \tan^{-1}(U)\Big]_1^{\sqrt{3}}$$

$$= \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$$

$$= \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

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$$\int \frac{x+4}{x^2+4}\,dx = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx + \int \frac{4}{x^2+4}\,dx$$

$$= \frac{1}{2}\ln|x^2+4| + 4\cdot\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$= \frac{1}{2}\ln|x^2+4| + 2\tan^{-1}\left(\frac{x}{2}\right) + C$$

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