Sequences

10.1 :

(4) $a_n = 2 + (-1)^n$

$a_1 = 2 + (-1)^1 = 2 + -1 = 1 \qquad a_2 = 2 + (-1)^2 = 2 + 1 = 3$

$a_3 = 2 + (-1)^3 = 2 + -1 = 1 \qquad a_4 = 2 + (-1)^4 = 2 + 1 = 3$

(9) $a_1 = +2 \quad , \quad a_{n+1} = (-1)^{n+1}\, a_n/2$

$a_1 = 2 \qquad a_2 = \dfrac{(-1)^2 a_1}{2} = \dfrac{a_1}{2} = \dfrac{2}{2} = 1$

$a_3 = \dfrac{(-1)^3 a_2}{2} = \dfrac{-1}{2} \qquad a_4 = \dfrac{(-1)^4 a_3}{2} = \dfrac{-1}{4}$

$a_5 = \dfrac{(-1)^5 a_4}{2} = \dfrac{1}{8} \qquad a_6 = \dfrac{(-1)^6 a_5}{2} = \dfrac{1}{16}$

$a_7 = \dfrac{(-1)^7 a_6}{2} = \dfrac{-1}{32} \qquad a_8 = \dfrac{(-1)^8 a_7}{2} = \dfrac{-1}{64}$

$a_9 = \dfrac{(-1)^9 a_8}{2} = \dfrac{1}{128} \qquad a_{10} = \dfrac{(-1)^{10} a_9}{2} = \dfrac{1}{256}$

(16) $1, \dfrac{-1}{4}, \dfrac{1}{9}, \dfrac{-1}{16}, \dfrac{1}{25}, \ldots$

$a_n = \dfrac{(-1)^{n+1}}{n^2} \quad , n = 1, 2, 3, \ldots$

(30) $a_n = \dfrac{2n+1}{1 - 3\sqrt{n}}$

$\sqrt{n}$ قسمة

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{2n+1}{1-3\sqrt{n}} = \lim_{n\to\infty} \frac{2\sqrt{n} + \frac{1}{\sqrt{n}}}{\frac{1}{\sqrt{n}} - 3} = \mp\infty \quad \underline{\text{div}}$$

(47) $a_n = \dfrac{n}{2^n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n}{2^n} \overset{\text{Hopital}}{=} \lim_{n\to\infty} \frac{1}{2^n \ln 2} = \frac{1}{\infty} = 0 \quad \underline{\text{conv}}$$

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(62) $a_n = \sqrt[n]{3^{2n+1}}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt[n]{3^{2n+1}} = \lim_{n\to\infty} (3^{2n+1})^{\frac{1}{n}} = \lim_{n\to\infty} (3^{2n})^{\frac{1}{n}} \cdot (3)^{\frac{1}{n}}$$

$$= \lim_{n\to\infty} 3^2 \cdot 3^{\frac{1}{n}} = 9 \cdot 1 = 9 \quad \underline{\underline{\text{conv}}}$$

(69) $a_n = \left(\dfrac{3n+1}{3n-1}\right)^n$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(\frac{3n+1}{3n-1}\right)^n = \lim_{n\to\infty} \left(\frac{3n+1}{3n-1}\right)^n$$

$$y = \left(\frac{3n+1}{3n-1}\right)^n$$

$$\ln y = \ln\left(\frac{3n+1}{3n-1}\right)^n \Rightarrow \ln y = n \ln\left(\frac{3n+1}{3n-1}\right)$$

$$\lim_{n\to\infty} \ln y = \lim_{n\to\infty} n \ln\left(\frac{3n+1}{3n-1}\right) = \lim_{n\to\infty} \frac{\ln(3n+1) - \ln(3n-1)}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{\frac{3}{3n+1} - \frac{3}{3n-1}}{\frac{-1}{n^2}} = \lim_{n\to\infty} \frac{9n-3-9n-3}{(3n+1)(3n-1)} \cdot -n^2$$

$$= \lim_{n\to\infty} \frac{6n^2}{(3n+1)(3n-1)} = \lim_{n\to\infty} \frac{6n^2}{9n^2-1} = \frac{6}{9} = \frac{2}{3}$$

$$\Rightarrow \lim_{n\to\infty} a_n = e^{\frac{2}{3}} \quad \underline{\text{conv}}$$

(89) $a_n = \dfrac{1}{n}\displaystyle\int_1^n \frac{1}{x}\,dx$ $\qquad \displaystyle\int_1^n \frac{1}{x}\,dx = \ln x \Big|_1^n = \ln n - \cancel{\ln 1}$

$$\lim_{n\to\infty} \frac{\ln n}{n} = 0 \quad (\text{Theorem})$$

$\underline{\underline{\text{conv}}}$

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(91) $a_1 = 2$, $a_{n+1} = \dfrac{72}{1+a_n}$

$a_n \longrightarrow L$ (an converges).

$$\lim_{n\to\infty} a_{n+1} = L$$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{72}{1+a_n}$$

$$L = \frac{72}{1+L}$$

$$L(1+L) = 72$$

$$L + L^2 - 72 = 0 \Rightarrow L^2 + L - 72 = 0$$

$$(L+9)(L-8)$$

$L+9=0$ or $L-8=0$

$\underbrace{L=-9}_{\text{مرفوض}}$ $\quad \boxed{L=8}$

since $a_n > 0$ for $n \geq 1$

$\boxed{L=8}$

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Infinite Series

10.2 :

① $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots + \frac{2}{3^{n-1}} + \cdots$

$$S_n = \frac{a(1-r^n)}{(1-r)} = \frac{2\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}} \Rightarrow \lim_{n\to\infty} S_n = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = \boxed{3}$$

⑫ $\sum_{n=0}^{\infty} (-1)^n \frac{5}{4^n}$

$$(5-1) + \left(\frac{5}{2}-\frac{1}{3}\right) + \left(\frac{5}{4}-\frac{1}{9}\right) + \left(\frac{5}{8}-\frac{1}{27}\right) + \cdots$$

is the defference of two geometric series, the sum is

$$\frac{5}{1-\frac{1}{2}} - \frac{1}{1-\frac{1}{3}} = 10 - \frac{3}{2} = \frac{17}{2}$$

⑰ $\frac{1}{8} + \left(\frac{1}{8}\right)^2 + \left(\frac{1}{8}\right)^3 + \cdots$

* geometric series : $r = \frac{1}{8}$ $|r|<1$ ✓

converges to $\frac{a}{1-r} = \frac{\frac{1}{8}}{1-\frac{1}{8}} = \frac{\frac{1}{8}}{\frac{7}{8}} = \frac{1}{7}$

⑳ $0.\overline{234} = 0.234234234\cdots$

$$\frac{234}{1000} + \frac{234}{(1000)^2} + \frac{234}{(1000)^3} + \cdots$$

$$234\left(\frac{1}{1000} + \frac{1}{(1000)^2} + \frac{1}{(1000)^3} + \cdots\right)$$

$\Downarrow$ geometric

$r = \frac{1}{1000}$ $|r|<1 \Rightarrow$ conv to $\frac{a}{1-r} = \frac{\frac{1}{1000}}{1-\frac{1}{1000}} = \frac{1}{999}$

$$234\left(\frac{1}{999}\right) = \frac{234}{999}$$

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(34) $\sum_{n=0}^{\infty} \cos n\pi$

$\lim_{n\to\infty} \cos n\pi = DNE \Rightarrow$ diverges.

(48) $\sum_{n=1}^{\infty} (\tan^{-1}(n) - \tan^{-1}(n+1))$

$S_n = \tan^{-1}1 - \cancel{\tan^{-1}2} + \cancel{\tan^{-1}2} - \cancel{\tan^{-1}3} + \cancel{\tan^{-1}3} - \cancel{\tan^{-1}4} + \dots \cancel{\tan^{-1}n} - \tan^{-1}n+1 + \dots$

$S_n = \tan^{-1}1 - \tan^{-1}n+1$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \tan^{-1}1 - \tan^{-1}n+1 = \frac{\pi}{4} - \frac{\pi}{2} = \frac{-\pi}{4}$

(60) $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^n$

$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^n = \lim_{n\to\infty} \left(1+\frac{-1}{n}\right)^n = e^{-1} \neq 0$ div.

th.

(67) $\sum_{n=0}^{\infty} \left(\frac{e}{\pi}\right)^n$

geometric series :- $a=1$ $\quad r=\frac{e}{\pi}$ $\quad |r|<1$ ✓ $\qquad \frac{e}{\pi} \quad \frac{2.1\dots}{3.14\dots} < 1$

converges to $\frac{a}{1-r} = \frac{1}{1-\frac{e}{\pi}} = \frac{\pi}{\pi - e}$

(76) $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n (x-3)^n = \sum_{n=0}^{\infty} \left(\frac{3-x}{2}\right)^n$

geometric series : $a = 1$, $r = \frac{3-x}{2}$

converges to $\frac{1}{1-\left(\frac{3-x}{2}\right)} = \frac{1}{\frac{2-3+x}{2}} = \frac{2}{x-1}$

for $|r|<1 \Rightarrow \left|\frac{3-x}{2}\right| < 1$

$-1 < \frac{3-x}{2} < 1$

$-2 < 3-x < 2$

$-5 < -x < -1 \Rightarrow 1 < x < 5$

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The Integral Test

10.3 :-

(9) $\sum_{n=1}^{\infty} \frac{n^2}{e^{n/3}}$

$f(x) = \frac{x^2}{e^{x/3}}$ is positive and continuos for $x \geq 1$

$$f'(x) = \frac{e^{\frac{x}{3}} \cdot 2x - x^2 e^{\frac{x}{3}} \cdot \frac{1}{3}}{(e^{\frac{x}{3}})^2} = \frac{x e^{\frac{x}{3}} (2 - \frac{x}{3})}{(e^{\frac{x}{3}})^2} = \frac{-x(6-x)}{3 e^{\frac{x}{3}}} < 0 \quad \text{for } x > 6$$

$\therefore f$ is decreasing for $x \geq 7$.

$$\int_7^{\infty} \frac{x^2}{e^{x/3}} dx = \lim_{b \to \infty} \int_7^{b} \frac{x^2}{e^{x/3}} dx$$

$\int \frac{x^2}{e^{\frac{x}{3}}}$ from parts

x^2	$+$	$e^{-\frac{x}{3}}$
$2x$	$-$	$-3 e^{-\frac{x}{3}}$
2	$+$	$9 e^{-\frac{x}{3}}$
0		$-27 e^{-\frac{x}{3}}$

$$\frac{-3x^2}{e^{x/3}} - \frac{18x}{e^{x/3}} - \frac{54}{e^{x/3}}$$

$$= \lim_{b \to \infty} \left[\frac{-3x^2}{e^{x/3}} - \frac{18x}{e^{x/3}} - \frac{54}{e^{x/3}} \right]_7^b$$

$$= \lim_{b \to \infty} \left(\frac{-3b^2 - 18b - 54}{e^{b/3}} + \frac{327}{e^{7/3}} \right)$$

$$= \lim_{b \to \infty} \left(\frac{3(-6b-18)}{e^{b/3}} \right) + \frac{327}{e^{7/3}}$$

$$= \lim_{b \to \infty} \left(\frac{-54}{e^{b/3}} \right) + \frac{327}{e^{7/3}} = \frac{327}{e^{7/3}}$$

$\int_7^{\infty} \frac{x^2}{e^{x/3}} dx$ converges $\Rightarrow \sum_{n=7}^{\infty} \frac{n^2}{e^{n/3}}$ converges $\Rightarrow$

$$\sum_{1}^{\infty} \frac{n^2}{e^{n/3}} = \frac{1}{e^{1/3}} + \frac{1}{e^{2/3}} + \frac{1}{e} + \frac{1}{e^{4/3}} + \frac{1}{e^{5/3}} + \frac{1}{e^{6/3}} + \sum_{n=7}^{\infty} \frac{n^2}{e^{n/3}} \text{ converges.}$$

(12) $\sum_{n=1}^{\infty} e^{-n}$

$\sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n \Rightarrow$ geometric series.

converges with $r = \left|\frac{1}{e}\right| < 1$ ✓

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(14) $\sum_{n=1}^{\infty} \frac{5}{n+1}$

$\int_1^{\infty} \frac{5}{x+1}\,dx$, $f(x) = \frac{5}{x+1}$ +ve, cont, decr.

$\lim_{b\to\infty} \int_1^b \frac{5}{x+1}\,dx = \lim_{b\to\infty} 5\ln(x+1)\Big|_1^b = 5\ln(b+1) - 5\ln 2 = \infty$

$\Rightarrow \int_1^{\infty} \frac{5}{x+1}\,dx$ diverges $\Rightarrow \sum_{n=1}^{\infty} \frac{5}{n+1}$ div.

(31) $\sum_{n=3}^{\infty} \frac{\frac{1}{n}}{(\ln n)\sqrt{\ln^2 n - 1}}$

$f(x) = \frac{\frac{1}{x}}{(\ln x)(\sqrt{\ln^2 x - 1})}$ +ve, cont, decr

$\int_3^{\infty} \frac{\left(\frac{1}{x}\right)}{(\ln x)(\sqrt{\ln^2 x - 1})}\,dx$

$u = \ln x$
$du = \frac{dx}{x}$

$\int_{\ln 3}^{\infty} \frac{1}{u\sqrt{u^2-1}}\,du$

$= \lim_{b\to\infty} \int_{\ln 3}^{b} \frac{du}{u\sqrt{u^2-1}} = \lim_{b\to\infty} \left[\sec^{-1} u\right]_{\ln 3}^{b} = \lim_{b\to\infty} \left[\sec^{-1} b - \sec^{-1} \ln 3\right]$

$= \lim_{b\to\infty} \cos^{-1}\left(\frac{1}{b}\right) - \sec^{-1}\ln 3 = \cos^{-1} 0 - \sec^{-1}\ln 3 = \frac{\pi}{2} - \sec^{-1}\ln 3 \approx 1.1439$

(36) $\sum_{n=1}^{\infty} \frac{2}{1+e^n}$

$f(x) = \frac{2}{1+e^x}$, +ve, cont, decr

$\int_1^{\infty} \frac{2}{1+e^x}\,dx$

$u = e^x$
$du = e^x\,dx$
$dx = \frac{du}{u}$

$\int_e^{\infty} \frac{2\,du}{u(1+u)}$

$= \int_e^{\infty} \left(\frac{2}{u} - \frac{2}{u+1}\right) du$

$\frac{2}{u(u+1)} = \frac{a}{u} + \frac{b}{u+1}$ partial

$2 = a(u+1) + b(u)$

$\boxed{a = 2}$, $\boxed{b = -2}$

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$$\int_e^{\infty} \frac{2}{u(u+1)}\,du = \int_e^{\infty} \left(\frac{2}{u} + \frac{-2}{u+1}\right) du$$

$$\lim_{b\to\infty} \int_e^{b} \left(\frac{2}{u} + \frac{-2}{u+1}\right) du = \lim_{b\to\infty} 2\ln\frac{u}{u+1}\Big|_e^{b}$$

$$= \lim_{b\to\infty} 2\ln\frac{b}{b+1} - 2\ln\frac{e}{e+1} = 2\ln 1 - 2\ln\left(\frac{e}{e+1}\right) = -2\ln\left(\frac{e}{e+1}\right) \Rightarrow \underline{\underline{\text{conv}}}$$

(39) $\sum_{n=1}^{\infty} \operatorname{sech} n$

$f(x) = \operatorname{sech} x$, +ve, cont, decr --

$$\int_1^{\infty} \operatorname{sech} x\,dx = 2\int_1^{\infty} \frac{e^x}{1+(e^x)^2}\,dx$$

$$= 2\lim_{b\to\infty} \int_1^{b} \frac{e^x}{1+(e^x)^2}\,dx = 2\lim_{b\to\infty} \tan^{-1} e^x \Big|_1^{b}$$

$$= 2\tan^{-1} e^b - 2\tan^{-1} e^1$$

$$= 2\left(\frac{\pi}{2}\right) - 2\,\cancel{\left(\frac{\pi}{4}\right)}\tan^{-1} e$$

$$= \cancel{\frac{\pi}{2}} = \pi - 2\tan^{-1} e \approx 0.71$$

$\sum_{n=1}^{\infty} \operatorname{sec} h n$ converges by integral test.

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Comparison Test

10.4 :

(2) $\sum_{n=1}^{\infty} \frac{n-1}{n^4+2}$, compare with $\underbrace{\sum_{n=1}^{\infty} \frac{1}{n^3}}$

$n^4 \le n^4+2$ conv. by p-test $p>1$

$\frac{1}{n^4} \ge \frac{1}{n^4+2}$

$\frac{n}{n^4} \ge \frac{n}{n^4+2} \Rightarrow \frac{1}{n^3} \ge \frac{n}{n^4+2} \ge \frac{n-1}{n^4+2}$

$\sum_{n=1}^{\infty} \frac{n-1}{n^4+2}$ conv. by D.C.T.

(14) $\sum_{n=1}^{\infty} \underbrace{\left(\frac{2n+3}{5n+4}\right)^n}_{a_n}$, compare with $\underbrace{\sum_{n=1}^{\infty} \left(\frac{2}{5}\right)^n}_{b_n}$

$\lim_{n\to\infty} \frac{a_n}{b_n}$ conv. geometric with $r = \left|\frac{2}{5}\right| < 1$

$\lim_{n\to\infty} \frac{\left(\frac{2n+3}{5n+4}\right)^n}{\left(\frac{2}{5}\right)^n} = \lim_{n\to\infty} \left(\frac{10n+15}{10n+8}\right)^n$

$\lim_{n\to\infty} n \ln\left(\frac{10n+15}{10n+8}\right) = \lim_{n\to\infty} \frac{\ln\left(\frac{10n+15}{10n+8}\right)}{\frac{1}{n}} \overset{L'H}{=} \lim_{n\to\infty} \frac{\frac{10}{10n+15} - \frac{10}{10n+8}}{-\frac{1}{n^2}}$

$\lim_{n\to\infty} \frac{70n^2}{(10n+5)(10n+8)} = \lim_{n\to\infty} \frac{70n^2}{100n^2+230n+120} = \frac{7}{10}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = e^{7/10} > 0$, $\sum_{n=1}^{\infty} \frac{2n+3}{5n+4}$ conv. by L.C.T.

(25) $\sum_{n=1}^{\infty} \cancel{\frac{(\ln n)^2}{n^3}} \left(\frac{n}{3n+1}\right)^n$, compare with $\left(\frac{n}{3n}\right)^n$

$3n < 3n+1$ $\qquad \sum_{1}^{\infty} \left(\frac{1}{3}\right)^n =$ conv. by geometric.

$\frac{1}{3n+1} < \frac{1}{3n}$

$\left(\frac{n}{3n+1}\right)^n < \left(\frac{n}{3n}\right)^n$

$\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$ converges by D.C.T.

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(30) $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{3/2}}$, compare with $\frac{1}{n^{5/4}}$

$\lim_{n\to\infty} \frac{\frac{(\ln n)^2}{n^{3/2}}}{\frac{1}{n^{5/4}}} = \lim_{n\to\infty} \frac{(\ln n)^2}{n^{\frac{1}{4}}}$ $\quad \sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$ conv. by P-test.

$\lim_{n\to\infty} = \frac{\frac{2\ln n}{n}}{\frac{1}{4n^{3/4}}} = 8 \lim_{n\to\infty} \frac{\ln n}{n^{1/4}} \doteq 8 \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{4n^{3/4}}} = 32 \lim_{n\to\infty} \frac{1}{n^{\frac{1}{4}}} = \boxed{0}$

$\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^{3/2}}$ conv. by L.C.T.

(42) $\sum_{n=1}^{\infty} \ln\left(\frac{n}{n+1}\right)$

$S_n = (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + \text{-----} [(\ln n) - (\ln n+1)] + \text{---}$

$S_n = -\ln(n+1) \Rightarrow \lim_{n\to\infty} S_n = -\infty$, div. by def. of in. series.

(47) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^{1.1}}$, compare with $\sum_{n=1}^{\infty} \frac{\frac{\pi}{2}}{n^{1.1}}$

$\frac{\tan^{-1} n}{n^{1.1}} < \frac{\frac{\pi}{2}}{n^{1.1}}$ $\quad \frac{\pi}{2} \sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$ conv. by P-test

$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^{1.1}}$ converges by D.C.T.

(51) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$, compare with $\sum_{n=1}^{\infty} \frac{1}{n}$

div. (hemonic series)

$\lim_{n\to\infty} \frac{\frac{1}{n\sqrt[n]{n}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{n}} = 1$

$\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$ div. by L.C.T.

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Ratio & Root

10.5 :

(5) $\sum_{n=1}^{\infty} \frac{n^4}{4^n}$ +ve ✓

$\sum_{1}^{\infty} \frac{n^4}{4^n}$ $\boxed{\text{conv}}$ by Ratio Test

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^4}{4^{n+1}} \cdot \frac{4^n}{n^4} = \lim_{n\to\infty} \frac{n^4 + 4n^3 + 6n^2 + 4n + 1}{4n^4} = \frac{1}{4} < 1$

(13) $\sum_{n=1}^{\infty} \frac{8}{(3+(1/n))^{2n}}$ +ve ✓

$\lim_{n\to\infty} \sqrt[n]{\frac{8}{(3+1/n)^{2n}}} = \lim_{n\to\infty} \frac{\sqrt[n]{8}}{(3+\frac{1}{n})^2} = \frac{1}{9} < 1 \Rightarrow \sum_{n=1}^{\infty} \frac{8}{(3+1/n)^{2n}}$ conv by Root.T

(22) $\sum_{n=1}^{\infty} \left(\frac{n-2}{n}\right)^n$

$\lim_{n\to\infty} \left(\frac{n-2}{n}\right)^n = \lim_{n\to\infty} \left(1 + \frac{-2}{n}\right)^n = e^{-2} \neq 0$ div by n^{th} term test.

(32) $\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)\ln(n+1)}{2^{n+1}} \cdot \frac{2^n}{n \ln n} = \frac{1}{2} < 1$, $\sum_{n=1} \frac{n \ln n}{2}$ conv by Ratio.T

(37) $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$

$\sum_{1}^{\infty} \frac{n!}{(2n+1)!}$ conv by Ratio.T.

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)!}{(2n+3)!} \cdot \frac{(2n+1)!}{n!} = \lim_{n\to\infty} \frac{n+1}{(2n+3)(2n+2)} = 0 < 1$

(48) $a_1 = 3$, $a_{n+1} = \frac{n}{n+1} a_n$

* $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n}{n+1} = 1 \Rightarrow$ fail. R-test.

* $a_{n+1} = \frac{n}{n+1} a_n \Rightarrow a_{n+1} = \frac{n}{n+1} \left(\frac{n-1}{n} a_{n-1}\right) \Rightarrow a_{n+1} = \frac{n}{n+1} \cdot \frac{n-1}{n} \cdot \frac{n-2}{n-1} a_{n-2}$

$a_{n+1} = \frac{a_1}{n+1} = \frac{3}{n+1} \Rightarrow$ div.. which is the constant times the general term of div hemoric series.

(54) $a_1 = \frac{1}{2}$, $a_{n+1} = (a_n)^{n+1}$

$a_1 = \frac{1}{2}$ $a_2 = \left(\frac{1}{2}\right)^2$, $a_3 = \left(\left(\frac{1}{2}\right)^2\right)^3$ — $a_n = \left(\frac{1}{2}\right)^{n!} < \left(\frac{1}{2}\right)^n$ → Conv. geometric.

Conv. by D.C.T

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