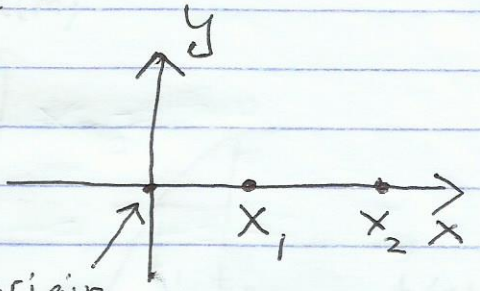
# Chapter 2

# Motion Along a Straight Line:

## Position and Displacement




initial position $= X_1$

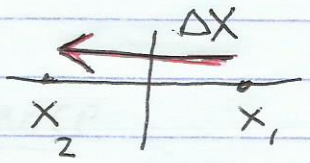
final position $= X_2$

Displacement = change in position

$\Delta X = X_2 - X_1$

example: suppose $X_1 = 5m$ at $t_1$

$X_2 = -5m$ at $t_2$




Displacement $= X_2 - X_1$

$\Delta X = -5 - 5 = -10m$

Displacement $\neq$ distance

example:- a particle moves

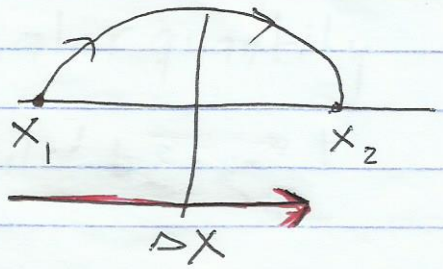
from $X_1 = -2m$ to

$X_2 = 2m$

along a semicle ( $\frac{1}{2}$ )

find 1) The displacement ?

2) The distance ?




1) Displacement = Change in position

$\Delta X = X_2 - X_1$

$= 2 - -2 = 4m$

distance $= 2\pi r/2 = \pi r$

$S = (3.14)(2) = 6.3m$

Solve checkpoint 1 Page 15



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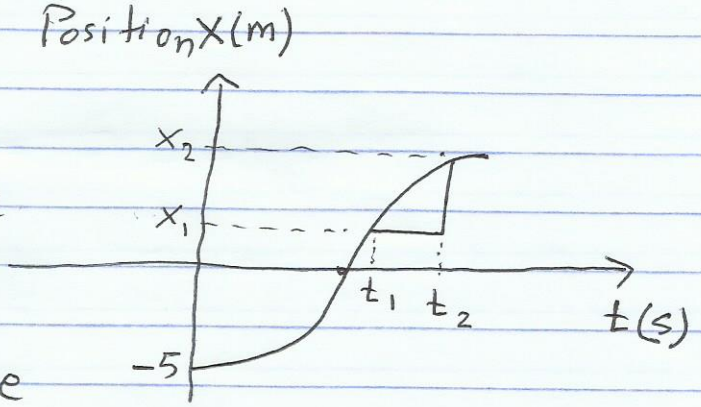
## Average velocity and Average speed:-

To find Average velocity you have to know $x(t)$ graphically or as equation




$$\text{average velocity} = \frac{\Delta x}{\Delta t}$$

$$\vec{V}_{avg} = \frac{x_2 - x_1}{t_2 - t_1}$$ from the graph

but the average speed can not be found from the graph, because

$$\text{average speed } (S_{avg}) = \frac{\text{total distance}}{\text{total time}}$$

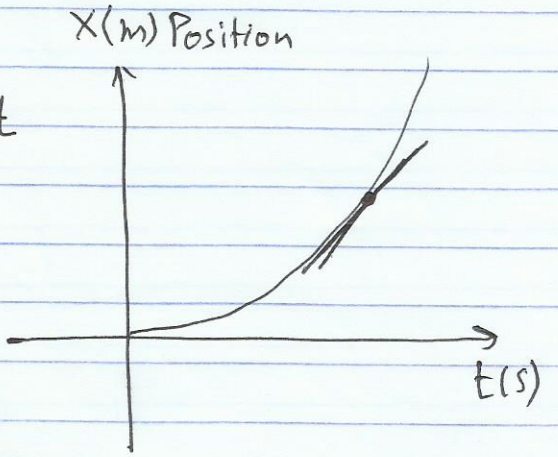
$$S_{avg} = \frac{\text{total distance}}{\Delta \text{time}} = \frac{\text{total distance}}{t_2 - t_1}$$

$S_{avg}$ always positive scalar quantity

$\vec{V}_{avg}$ is vector quantity could be $\nearrow +$ $\searrow -$

Instantaneous velocity:

$V_{inst}$ ① could be found from x Vs. t by plotting the tangent of the curve at a certain point, then find the slop.




$V_{inst}$ = the slope of the tangent

② $V_{inst} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$ from equation $x(t)$



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## Average Acceleration and instanteous Acceleration:

$$a_{avg} = \frac{\Delta V}{\Delta t}$$

$$= \frac{V_2 - V_1}{\Delta t} \quad m/s/s = m/s^2$$

$$a_{inst} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}$$

$$a_{inst} = \frac{dv}{dt} \quad m/s^2$$



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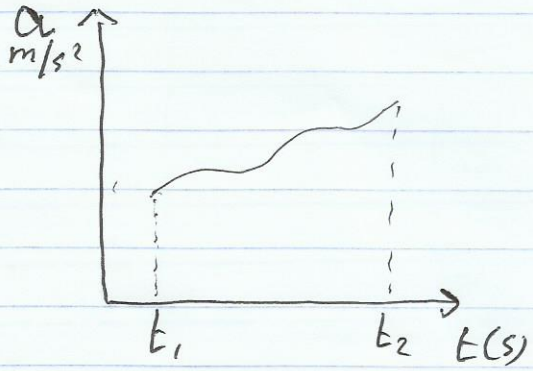
Finding (V) from (a)




$$a = \frac{dV}{dt} \Rightarrow dV = a\,dt$$

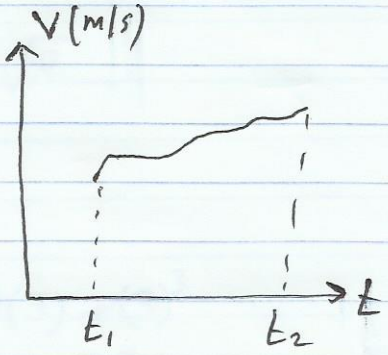
Integrate both sides

$$\int_{V_1}^{V_2} dV = \int_{t_1}^{t_2} a\,dt$$

$$V_2 - V_1 = \int_{t_1}^{t_2} a\,dt$$

= the area under the curve of (a) versus (t) from $t_1 \longrightarrow t_2$

Finding (x) from (V):




$$V = \frac{dx}{dt} \Rightarrow dx = V\,dt$$

Integrate both sides

$$\int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} V\,dt$$

$$X_2 - X_1 = \int_{t_1}^{t_2} V\,dt$$

= the area under the curve of (V) versus (t) between $t_1 \longrightarrow t_2$

Solve Sample Problem 2.01
Solve Sample Problem 2.02
Solve Sample Problem 2.06



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Sample Problem 2.03:-

$x = 4 - 27t + t^3$ , x in m ; t in s

a) Find $v(t)$ & $a(t)$ ?

$$v(t) = \frac{dx}{dt}$$

$$v(t) = -27 + 3t^2$$

$$a(t) = \frac{dv}{dt} = 6t$$

b) When $v = 0$ ? find $t$ ? at $v = 0$

$$0 = -27 + 3t^2$$

$$3t^2 = 27 \;,\; t^2 = 9 \;,\; t = \sqrt{9}$$

$$t = \pm 3 \text{ sec}$$

$$t = +3 \text{ sec}$$

Find $a$ and $x$ at $t = +3s$

$$a = +18 m/s^2 \;,\; x = 4 - 27(3) + (3)^3$$

$$= 4 - 81 + 27$$

$$= -50m$$

Note: at $t = 3s$ → $V_3 = 0$, $a_3 = +18 m/s^2$, $X_3 = -50m$

at $t = 0$ → $X_o = 4m$, $V_o = -27 m/s$, $a_o = 0$

at $t = 1s$ → $X_1 = 4 - 27 + 1 = -22m$, $V_1 = -27 + 3 = -24 m/s$, $a_1 = +6 m/s$

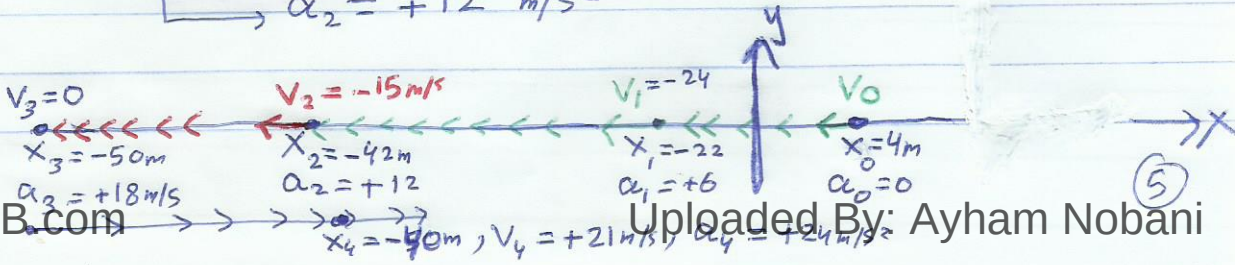
at $t = 2s$ → $X_2 = 4 - 27(2) + (2)^3 = -42m$, $V_2 = -27 + 3(2)^2 = -15 m/s$, $a_2 = +12 m/s^2$

At $t = 4s$

$X_4 = 4 - 27(4) + (4)^3 = -40m$

$V_4 = -27 + 3(4)^2 = -27 + 48 = +21 m/s$

$a_4 = +24 m/s^2$



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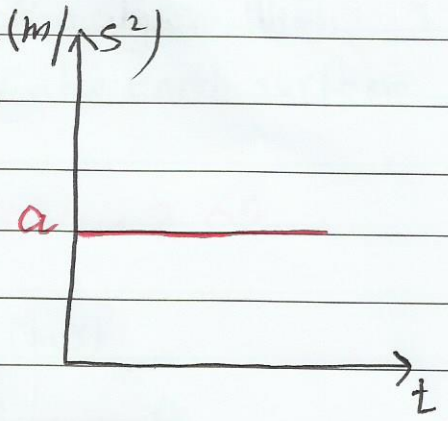
# Constant Acceleration:-

A Particle moves from $x_0$, with $V_0$ at constant acceleration




$$a = \frac{dV}{dt}$$

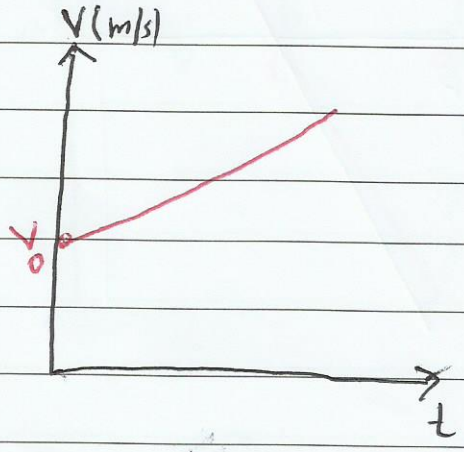
$$dV = a\,dt$$

$$\int_{V_0}^{V} dV = \int_0^t a\,dt = a\int_0^t dt$$

$$V - V_0 = at$$

$$V = V_0 + at \quad (1)$$ the velocity at time = t

find X at time = t




$$V = \frac{dx}{dt}$$

$$dx = V\,dt$$

$$\int_{x_0}^{x} dx = \int_0^t V\,dt$$

$$x - x_0 = \int_0^t (V_0 + at)\,dt = V_0 t + \frac{1}{2}at^2$$

$$x - x_0 = V_0 t + \frac{1}{2}at^2 \quad (2)$$

$$V_{avg} = \frac{V_0 + V}{2}$$

$$x - x_0 = V_{avg} \cdot t$$

$$x - x_0 = \left(\frac{V_0 + V}{2}\right)t \quad (3)$$ , from (1) $t = \frac{V - V_0}{a}$

Substitute (1) in (3) $x - x_0 = \left(\frac{V_0 + V}{2}\right)\left(\frac{V - V_0}{a}\right) = \frac{V^2 - V_0^2}{2a}$

$$V^2 = V_0^2 + 2a(x - x_0) \quad (4)$$



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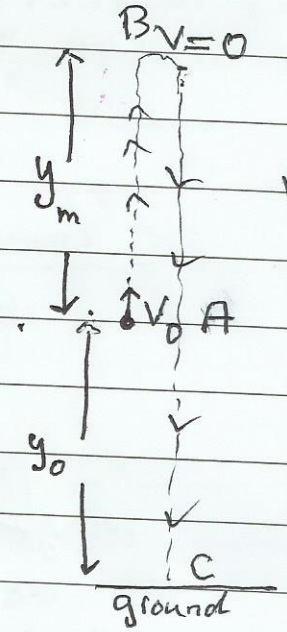
Free Fall : is the best example of motion object with constant acceleration near the earth surface where $a_y = -9.8\ m/s^2$

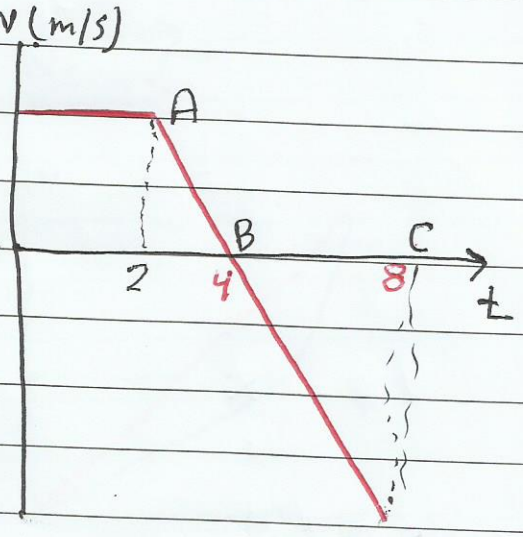
Solve sample Problem 2.05

(2-51)

a) $V_0 = 19.6$ m/s upward

at $t = 4s$, $V = 0$




from A → B

$V_y^2 = V_{oy}^2 + 2a_y \Delta y$

$0 = (19.6)^2 + 2(-9.8)y_m$

$y_m = \frac{(19.6)^2}{2(9.8)} = 19.6m$

$y_m = 19.6$ m above point A (the begining of free fall)

b) find $y_0$ ?

from A → C (ground)

- $t = 8 - 2 = 6s$
- $V_0 = +19.6$ m/s
- $\Delta y = -y_0$
- $a_y = -9.8\ m/s^2$

$\Delta y = V_{oy}t + \frac{1}{2}a_y t^2$

$-y_0 = (+19.6)(6) + \frac{1}{2}(-9.8)(6)^2$

$-y_0 = +117.6 - 176.4$

$-y_0 = -58.8$ m

the height of starting free fall = 58.8 m

Note: maximum height from the ground = 58.8 + 19.6

= 78.4 m

* find the velocity of the object when reached the ground

A → C $V_y = V_{oy} + a_y t \Rightarrow V_y = +19.6 + (-9.8)(6)$

$= -39.2$ m/s



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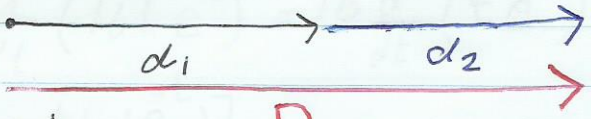
# Solving Lecture Problems (chapter 2)

(2-2) you walk $d_1 = 73.2\,m$ at $V_1 = 1.22\,m/s$

ⓐ then you run $d_2 = 73.2\,m$ at $V_2 = 3.05\,m/s$

Fing $V_{avg}$?




$$V_{avg} = \frac{\text{displacement}}{\text{time}}$$

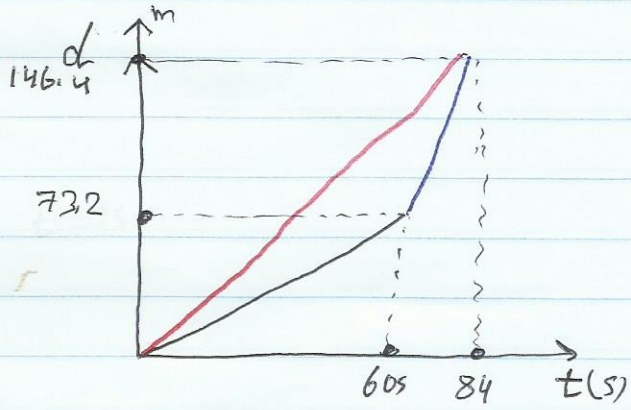
$$t_1 = \frac{d_1}{V_1} = 60\,s$$

$$t_2 = \frac{d_2}{V_2} = 24\,s$$

$$D = d_1 + d_2 = 146.4\,m$$

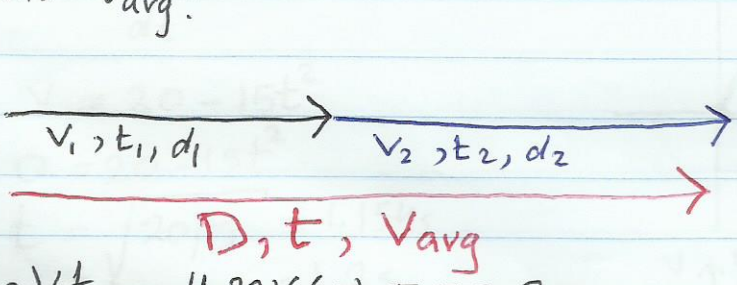
$$t_{tot} = t_1 + t_2 = 84\,s$$

$$V_{avg} = \frac{D}{t_{tot}} = \frac{146.4}{84} = \underline{1.74\,m/s}$$




b) You walk for $t_1 = 1\,min = 60\,s$ at $V_1 = 1.22\,m/s$, then you run for $t_2 = 1\,min = 60\,s$ at $V_2 = 3.05\,m/s$

Find $V_{avg}$?




$$d_1 = V_1 t_1 = (1.22)(60) = 73.2\,m$$

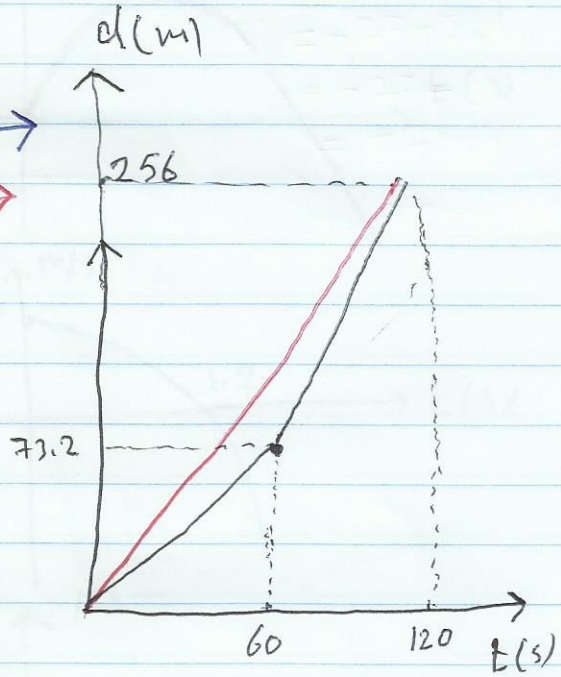
$$d_2 = V_2 t_2 = (3.05)(60) = 183\,m$$

$$t = t_1 + t_2 = 120\,s$$

$$D = d_1 + d_2 = 73.2 + 183$$

$$D = 256.2\,m$$

$$V_{avg} = \frac{D}{t} = \frac{256.2}{120} = 2.14\,m/s$$



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(2-14) $X = 16te^{-t}$

$x_0 = 0$

a) How far is the electron from the origin when it momentarily stop?

$$v = \frac{dx}{dt} = \frac{d}{dt}(16te^{-t}) = 16\frac{d}{dt}(te^{-t})$$

$$= 16[(1)e^{-t} + t(-1e^{-t})]$$

$$v = 16e^{-t}(1-t)$$

$$v = 0 = 16e^{-t}(1-t)$$

$v=0 \Rightarrow t = 1s$,

$X = 16te^{-t}$, find x? at $t = 1s$

$= 16(1)e^{-1}$

$= 16(0.37)$

$= 5.89m$

[2-20] $X = 20t - 5t^3$

a) find t? at $v = 0$

$$v = \frac{dx}{dt}$$

$$v = 20 - 15t^2$$

$$0 = 20 - 15t^2$$

$$t = \sqrt{20/15} = 1.154s$$

$$= 1.2s$$

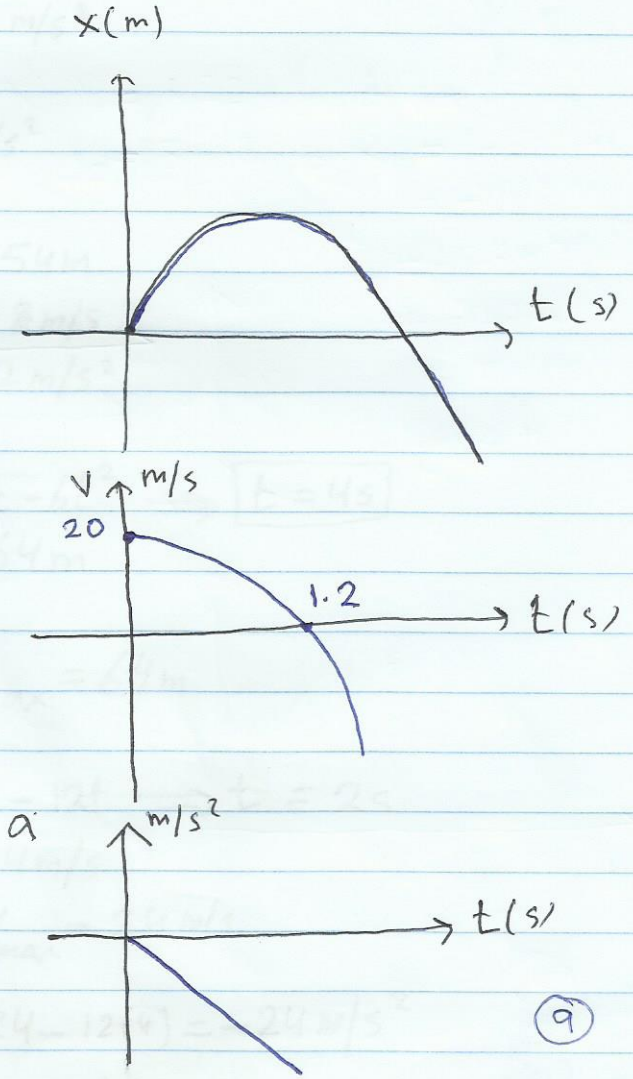
b) find t? at $a = 0$

$$a = \frac{dv}{dt} = -30t$$

$a = 0$, at $t = 0$

c) a is negative for $t > 0$

d) a is positive for $t < 0$



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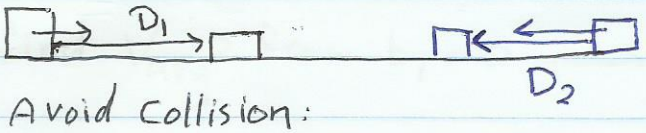
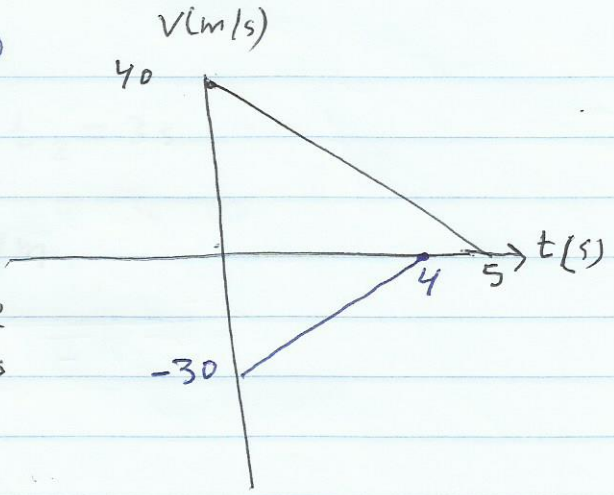
(2-41) ① $V_{01} = +40 m/s$ $V_{02} = -30 m/s$ ②




to Avoid collision:

train ① stopped after 5s

Displacement = Area und (V-t) curve

from $t_1 = 0 \longrightarrow t_2 = 5s$

$D_1 = \frac{1}{2}(5)(40) = +100 m$

train ② stopped after 4s

$D_2$ = Area under (V-t) curve from $t_1 = 0 \longrightarrow t_2 = 4s$

$D_2 = \frac{1}{2}(4)(-30) = -60 m$ (-) for direction

Distance between stopping train = 200 - 160 = 40m

(2-18) $X = 12t^2 - 2t^3$ m

$V = \frac{dx}{dt} = 24t - 6t^2$ m/s

$a = \frac{dv}{dt} = 24 - 12t$ $m/s^2$

at $t = 3s$

a) $X(3) = 12(3)^2 - 2(3)^3 = 54m$

b) $V(3) = 24(3) - 6(3)^2 = 18 m/s$

c) $a(3) = 24 - 12(3) = -12 m/s^2$

d) find $X_{max}$?

$X_{max}$ is at $V = 0 = 24t - 6t^2 \Rightarrow \boxed{t = 4s}$

$X_{max} = 12(4)^2 - 2(4)^3 = 64m$

e) At $t = 4s \Rightarrow V = 0$, $X_{max} = 64m$

f) find $V_{max}$?

$V_{max}$ is at $a = 0 = 24 - 12t \Rightarrow t = 2s$

$V_{max} = 24(2) - 6(2)^2 = 24 m/s$

g) At $t = 2s \Rightarrow a = 0$, $V_{max} = 24 m/s$

h) find a? at V = 0

$V = 0$ at $t = 4s \Rightarrow a = 24 - 12(4) = -24 m/s^2$



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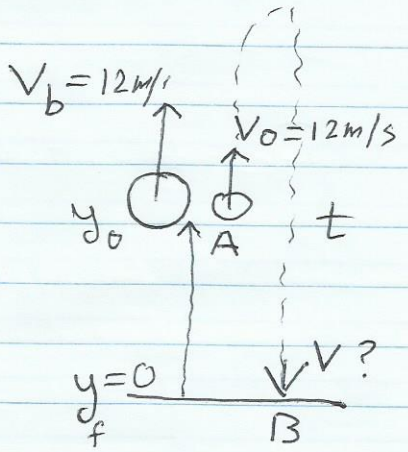
i) find $V_{avg}$? from $t_1 = 0 \longrightarrow t_2 = 3s$

$X(0) = 0$ , $X(3) = 54m$

$$V_{avg} = \frac{\Delta X}{\Delta t} = \frac{X(3) - X(0)}{\Delta t}$$

$$= \frac{54 - 0}{3}$$

$$= 18 m/s$$




(2-49) $y_0 = 80m$

$V_0 = +12 m/s$

a) Find the time for the package to reach ground?

from A $\longrightarrow$ B

$$\Delta y = V_{oy} t + \frac{1}{2} a_y t^2$$

$$y_f - y_0 = V_{oy} t + \frac{1}{2} a_y t^2$$

$$0 - 80 = +12t + \frac{1}{2}(-10)t^2$$

$$-80 = +12t - 5t^2$$

$$5t^2 - 12t - 80 = 0$$

$$t = \frac{-12 \mp \sqrt{(12)^2 - 4(5)(-80)}}{2(5)}$$

$$t = 5.4s$$

b) find the velocity when the package hit the ground?

$$V_y = V_{oy} + a_y t$$

$$= +12 + -10(5.4)$$

$$= -42 m/s$$ downward



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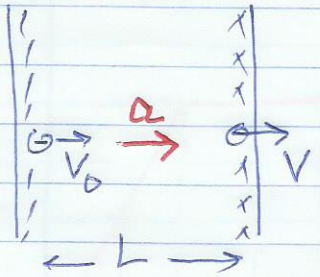
Problem 2.03 (Constant Acceleration)




$V_0 = 1.50 \times 10^5$ m/s

$L = 1.00$ cm

$V = 5.70 \times 10^6$ m/s

$a = ?$

$V^2 = V_0^2 + 2a\Delta x$

$(5.7\times10^6)^2 = (1.5\times10^5)^2 + 2a(1\times10^{-2})$

$$\frac{(5.7\times10^6)^2 - (1.5\times10^5)^2}{2(1\times10^{-2})} = a_x$$

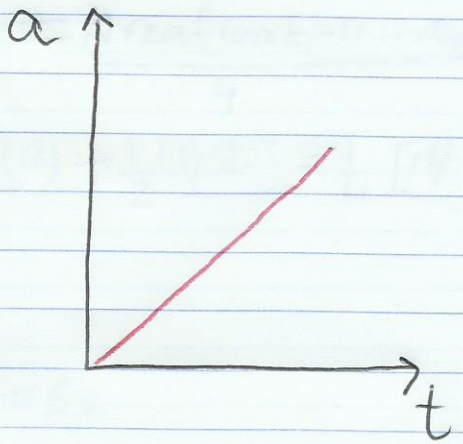
$a_x = 1.51 \times 10^{15}$ m/s$^2$

Problem (2.81) Variable Acceleration)

$a = 5t$

At $t = 2$s, $V_1 = +17$ m/s

At $t = 4$s, find $V_2$?




$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

$$\int dv = \int (5t\,dt)$$

$$V = \frac{5t^2}{2} + C$$

$$+17 = \frac{5}{2}(2)^2 + C \Rightarrow C = 17 - 10 = 7 \text{ m/s}$$

$$V = \frac{5}{2}t^2 + 7$$

At $t = 4$s, $V = \frac{5}{2}(4)^2 + 7 = 47$ m/s



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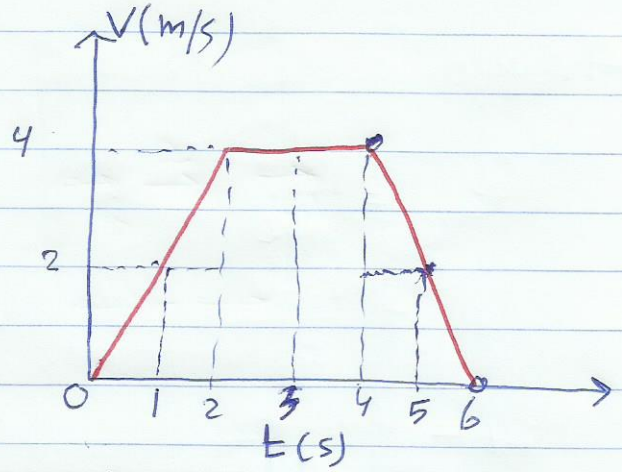
Problem (2.90)




At $t=0$, $V_0 = 0$

$x_0 = 0$

a) find $x$ ? at $t = 5s$

$\Delta x = \int v\,dt$

= Area under the curve V versus t, from $t_1 \rightarrow t_2$

$x_5 - x_0$ = Area from $t_1 = 0 \rightarrow t_2 = 5s$

$= \frac{1}{2}(2)(4) + (4-2)4 + \frac{1}{2}(1)(2) + 1(2) = 4 + 8 + 1 + 2 = 15$

$= 15\ m$ , $(x_5 = 15m)$ $x_5 = 15m$

b) At $t = 5s \rightarrow V_5 = 2\ m/s$

c) $\longrightarrow$ $a_5$ = slope $= \frac{\Delta V}{\Delta t} = \frac{0-4}{6-4} = \frac{-4}{2} = -2\ m/s^2$

$x_5 = 15\ m$

d) find $V_{avg}$ between $t_1 = 1s \rightarrow t_2 = 5s$ ?

$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_5 - x_1}{5-1}$ ~~=~~ $\frac{\text{Area from } t_1 = 1s \rightarrow t_2 = 5s}{4}$

$V_{avg} = \frac{1}{4}\left[1(2) + \frac{1}{2}(1)(2) + 2(4) + 1(2) + \frac{1}{2}(1)2\right] = \frac{1}{4}[14]$

$V_{avg} = \frac{14}{4} = 3.5\ m/s$

e) find $a_{avg}$ ? from $t_1 = 1s \rightarrow t_2 = 5s$

$a_{avg} = \frac{\Delta V}{\Delta t} = \frac{V_5 - V_1}{5-1} = \frac{2-2}{4} =$ zero

$a_{avg} = 0\ m/s^2$

(extra) find the avage velocity of the total motion from $t_1 = 0 \rightarrow t_2 = 6s$ ?

$V_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_6 - x_0}{6-0} = \frac{\text{the area from } t_1 = 0 \rightarrow t_2 = 6}{6}$

$V_{avg} = \frac{1}{6}\left[\frac{1}{2}(2)(4) + 2(4) + \frac{1}{2}(2)(4)\right] = \frac{1}{6}[4 + 8 + 4] = \frac{1}{6}[16]\ m/s$

$= \frac{8}{3} = 2.67\ m/s$

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