

Problem

Student A tries to define a function $g: \mathbf{Q} \rightarrow \mathbf{Z}$ by the rule $g\left(\frac{m}{n}\right) = m - n$ for all integers m and n with $n \neq 0$. Student B claims that g is not well defined. Justify student B's claim.

Step-by-step solution

Step 1 of 1

$g: \mathbf{Q} \rightarrow \mathbf{Z}$ defined by $g\left(\frac{m}{n}\right) = m - n, n \neq 0$

Suppose that g is well defined

Then, $x = y \Rightarrow g(x) = g(y)$, for every $x, y \in \mathbf{Q}$

Suppose that $\frac{m_1}{n_1} = \frac{m_2}{n_2}$, then $m_1 - n_1 \neq m_2 - n_2$

$$\Rightarrow g\left(\frac{m_1}{n_1}\right) \neq g\left(\frac{m_2}{n_2}\right) \left(\because \frac{2}{4} = \frac{3}{6} \text{ but } 2 - 4 \neq 3 - 6\right)$$

$$\therefore \frac{m_1}{n_1} = \frac{m_2}{n_2} \not\Rightarrow g\left(\frac{m_1}{n_1}\right) = g\left(\frac{m_2}{n_2}\right)$$

$\therefore g$ is not well defined