Bolts under fatigue loading

Bolts and screws have high stress concentration at the fillet under the bolt head and at the begging of the thread on the bolt shank. The following table (table 8-11, Shigly) gives K_f for *Rolled threads*, *Cut threads*, and the *Fillet*.

SAE Grade	Metric Grade	Rolled Thread	Cut Thread	Fillet
0-2	3.6-5.8	2.2	2.8	2.1
4-8	6.6-10.9	3.0	3.8	2.3

Table (8-11) K_f for Threaded elements.

If not specified use: **Rolled thread** with **machine finish**.

Fatigue Loading:

Most critical situation: Load varies between no load → max. load.
$P = (0 - P_{max})$

Bolt Load:
No external load → $P = 0$ → $P_b = F_{min} = F_i$
Max. external load → $P = P_{max}$ → $P_b = F_{max} = P_{b(max)}$

Bolt stress:

$$\sigma_a = \frac{F_a}{A_t} = \frac{F_a}{A_t} = \frac{F_{max} - F_{min}}{2A_t}$$

$$P_b = \frac{K_b}{K_b + K_m} P + F_i \quad \rightarrow \quad F_a = \frac{CP + F_i - F_i}{2} = \frac{CP}{2}$$

$$\rightarrow \sigma_a = \frac{Cp}{2A_t} \qquad (1)$$

$$\sigma_m = \frac{F_m}{A_t}, \quad F_m = \frac{F_{max} + F_{min}}{2} = \frac{CP + F_i + F_i}{2} = \frac{CP}{2} + F_i$$

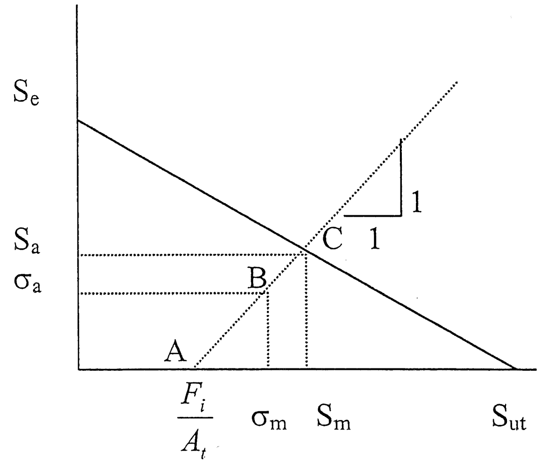
$$\rightarrow \sigma_m = \frac{Cp}{2A_t} + \frac{F_i}{A_t} \qquad (2)$$

but: $\sigma_a = \frac{Cp}{2A_t}$

$$\rightarrow \sigma_m = \sigma_a + \frac{F_i}{A_t}$$

When $\sigma_a = 0$ ➔ $\sigma_m = \dfrac{F_i}{A_t}$

Using the Goodman line:



$$➔\ n = \frac{AC}{AB} = \frac{S_a}{\sigma_a} = \frac{S_m - {F_i}/{A_t}}{\sigma_m - {F_i}/{A_{t\,m}}} \quad \text{where} \quad S_a = \frac{S_{ut} - \left({F_i}/{A_t}\right)}{1 + \left({S_{ut}}/{S_e}\right)} \quad \text{and} \quad \sigma_a = \frac{CP}{2A_t}$$

or $\dfrac{1}{n} = \dfrac{\sigma_m}{S_{ut}} + \dfrac{\sigma_a}{S_e}$

$$➔\ n = \frac{S_{ut} - \left({F_i}/{A_t}\right)}{\left[1 + \left({S_{ut}}/{S_e}\right)\right]\dfrac{CP}{2A_t}} \qquad (3)$$

Note:

1. In equation (3) K_f must be used as a strength reduction factor

Otherwise ➔ $\dfrac{\sigma_a}{\sigma_m} \neq \dfrac{1}{1}$

2. Table (8-12) ➔ $\mathbf{S_e}$ (fully corrected including K_f effect) for **SAE** and **ISO** bolts.

<u>Static Failure:</u>

$$n = \frac{S_y}{\sigma_{max}} \quad ➔ \quad \sigma_{max} = \sigma_m + \sigma_a = \frac{CP}{2A_t} + \frac{CP}{2A_t} + \frac{F_i}{A_t} = \frac{CP}{A_t} + \frac{F_i}{A_t}$$

$$n = \frac{S_y}{\dfrac{CP}{A_t} + \dfrac{F_i}{A_t}} = \frac{S_y A_t}{CP + F_i}$$

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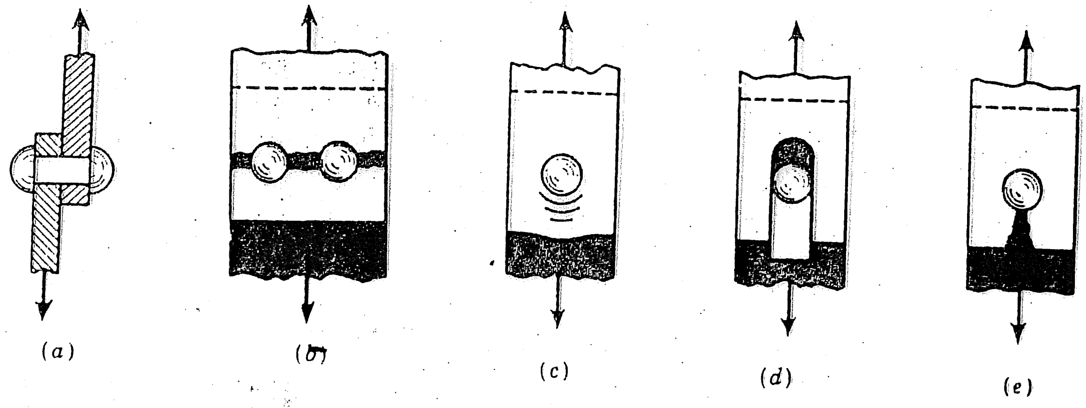
Bolted and Riveted Joints in SHEAR:



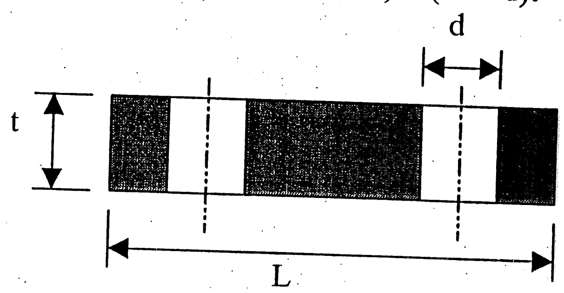
Shear Stress in the rivet: (Fig a)

$\tau = \dfrac{F}{A}$, where: A= cross-section area of the rivet (bolt), not hole diameter.

Tensile Failure of a revited member (Rupture): (Fig. b)

$\sigma = \dfrac{F}{A}$, A= net area of the plate

= (cross-sectional are of the plate – area of the rivet holes) = (L - 2d)t



Stress concentration factor must be included for fatigue loading and brittle materail .

Crushing (Bearing) of a riveted plate: (Fig. c)

Bearing stress: it is assumed that the stress is uniformly distributed over the projected are of the rivet or bolt hole.

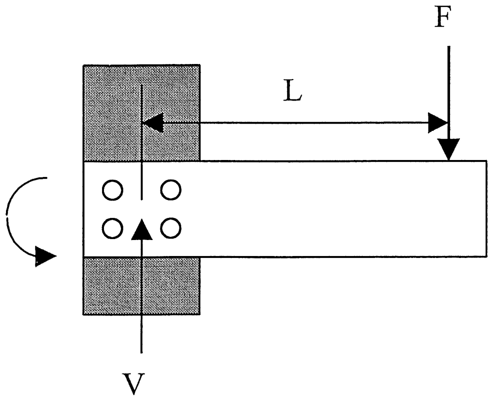
Bearing stress = $\sigma_B = \dfrac{F}{A}$, A= projected area of the rivet hole

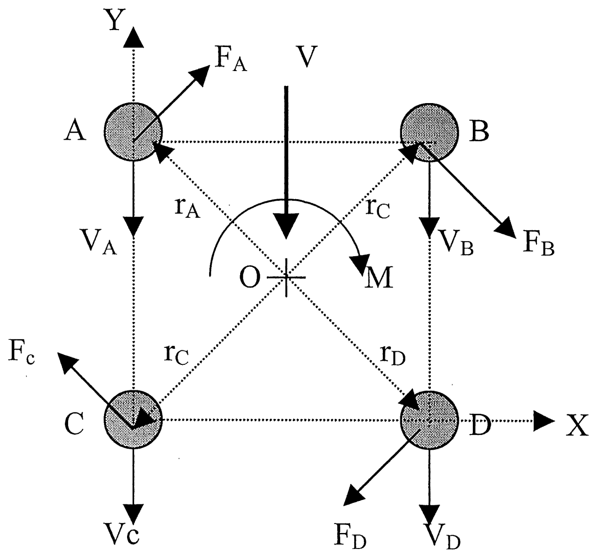
, for one rivet: A= td

Tearing (or shearing) the margin of the riveted member: (Fig. d and e)

It is avoided by placing the rivet ($1\frac{1}{2}d$) away from the margin.

Shear of Bolts due to Eccentric Loading:





The beam in the figure is subjected to a eccentric load **F** at a distance L from the support. At the fixed support the beam is subjected to a Bending moment **M** and a shear force **V**. The rivet (bolt) group is subjected to the same load (i.e. **M** and **V**) but in the opposite direction. It is required to find the force taken by each rivet due to M and V.

1. Find the centroid of the bolt group

$$\bar{x} = \frac{A_A x_A + A_B x_B + A_C x_C + A_D x_D}{A_A + A_B + A_C + A_D} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum_{i=1}^{n} A_i}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum_{i=1}^{n} A_i}$$ where: n= number of bolts (rivets), A_i = Area of bolt *i*.

2. Shear force distribution

   a- Shear force due to direct shear force **V**

Force **V** ➔ divided equally for each bolt

➔ $v_i = \frac{V}{n}$, where: $\boldsymbol{v_i}$ = shear force on bolt ***i*** due to V.

b- Shear load due to moment **M** :

M=F×L

$M = M_A + M_B + M_C + M_D = r_A F_A + r_B F_B + r_C F_C + r_D F_D$ (1)

F_i= Force taken by bolt i due to the moment **M**.
r_i = radial distance from the cetriode to the bolt.

The magnitude of F_i depends on the radial distance of the bolt i from the centroide. The bolt with the largest distance form centriode will have the largest force

➔ $F_i \propto r_i$

➔ $$\frac{F_A}{r_A} = \frac{F_B}{r_B} = \frac{F_C}{r_C} \quad (2)$$

Substituting (2) in (1): ➔ $$\boxed{F_i = \frac{M r_i}{r_A^2 + r_B^2 + r_C^2 + \ldots}}$$

Bolts under shear and tensile loading:

Given a bolted joint thickness (grip) = L_G
Find:

1- Nut height H from table E-30 (or use, $H = \frac{7}{8} d$)
2- Washer thickness t_w from table provided in the notes. (App. E33)
3- Thread length L_T

Inch- series bolts:

$$L_T = \begin{cases} 2D + \frac{1}{4}, & L \le 6 \text{ in} \\ 2D + \frac{1}{2}, & L > 6 \text{ in} \end{cases}$$

Metric bolts:

$$L_T = \begin{cases} 2D + 6, & L \le 125 \quad D \le 48 \\ 2D + 12, & 125 < L \le 200 \\ 2D + 25, & L > 200 \end{cases}$$

4. Bolt length: $L > L_G + H$ (use Table A-17 for Standard length)
5. Length of unthreaded portion: $l_d = L - L_T$
6. Length of threaded portion within the grip: $l_T = L_G - L_d$
7. Area of unthreaded portion: $A_d = \dfrac{\pi d^2}{4}$
8. Area of threaded portion A_t (tensile area from tables 8-1, 8-2)

<u>Shear Stress on Bolts:</u>

If the shear force will act at the unthreaded portion ➔ $\tau = \dfrac{F}{A_d}$

If the shear force acts at the threaded portion ➔ $\tau = \dfrac{F}{A_t}$

<u>Tesile Stress on Bolts:</u>

$$\sigma_b = \frac{Cp}{A_t} + \frac{F_i}{A_t}$$

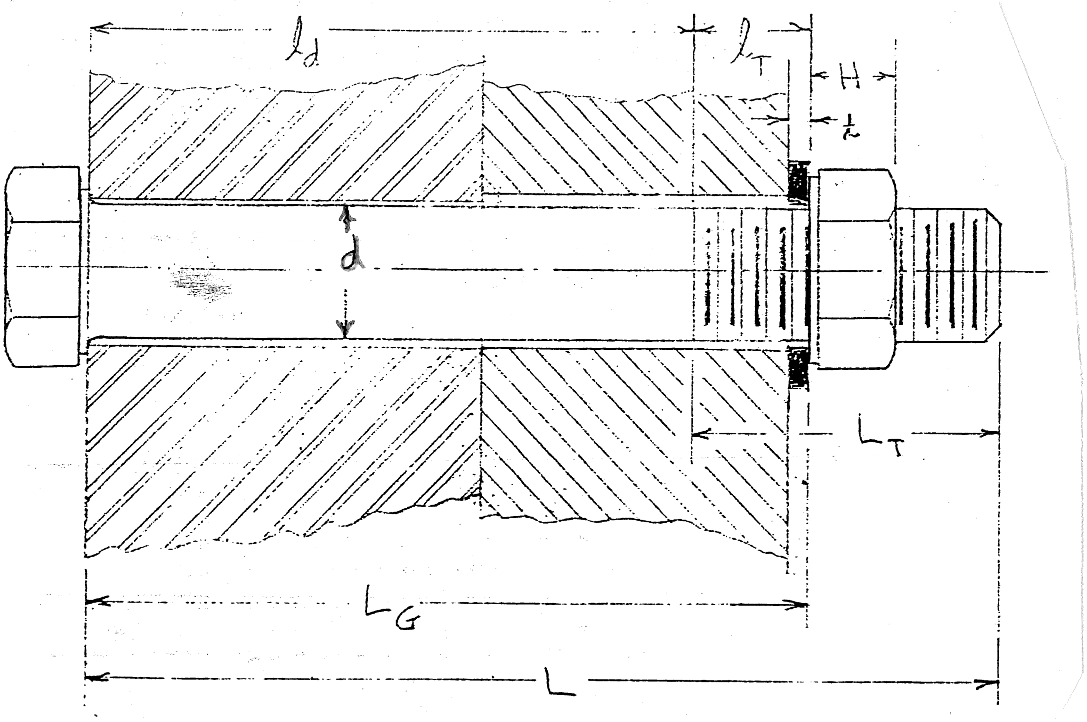
<u>Max combined Stress:</u>

$$\sigma' = \sqrt{\sigma_b^2 + \tau^2 \times 3}$$

<u>Bolt Design:</u>

Using the proof Strength: $n = \dfrac{S_p}{\sigma'}$

Using yield strength: $n = \dfrac{S_y}{\sigma'}$



Cap- Screw:

Find:

1. Washer thickness (provided tables) (E-33)
2. Grip : $l = g = \begin{cases} t_w + t_1 + \dfrac{t_2}{2}, \text{t}_2 < d \\ t_w + t_1 + \dfrac{d}{2}, \text{t}_2 \geq d \end{cases}$, where: tw= wahser thickness if any,

t1= thickness of unthreaded member, t2= thickness of threaded member

3. Thread length L_T; same as bolts
4. Length of unthreaded portion: $l_d = L - L_T$
5. Length of useful threaded portion: $l_T = L_G - L_d$
6. Area of Un threaded portion: $A_d = \dfrac{\pi d^2}{4}$
7. Area of threaded portion A_t (tensile area from tables 8-1, 8-2)

TABLE E-33 Dimensions of Metric Plain Washers. All dimensions in millimeters.

Washer size[1]	Min. ID	Max. OD	Max. thick.
1.6 N	1.95	4.00	0.70
1.6 R	1.95	5.00	0.70
1.6 W	1.95	6.00	0.90
2 N	2.50	5.00	0.90
2 R	2.50	6.00	0.90
2 W	2.50	8.00	0.90
2.5 N	3.00	6.00	0.90
2.5 R	3.00	8.00	0.90
2.5 W	3.00	10.00	1.20
3 N	3.50	7.00	0.90
[illegible] R	3.50	10.00	1.20
3 W	3.50	12.00	1.40
3.5 N	4.00	9.00	1.20
3.5 R	4.00	10.00	1.40
3.5 W	4.00	15.00	1.75
4 N	4.70	10.00	1.20
4 R	4.70	12.00	1.40
4 W	4.70	16.00	2.30
5 N	5.50	11.00	1.40
5 R	5.50	15.00	1.75
5 W	5.50	20.00	2.30
6 N	6.65	13.00	1.75
6 R	6.65	18.80	1.75
6 W	6.65	25.40	2.30
[illegible] N	8.90	18.80	2.30
8 R	8.90	25.40	2.30
8 W	8.90	32.00	2.80

Washer size[1]	Min. ID	Max. OD	Max. thick.
10 N	10.85	20.00	2.30
10 R	10.85	28.00	2.80
10 W	10.85	39.00	3.50
12 N	13.30	25.40	2.80
12 R	13.30	34.00	3.50
12 W	13.30	44.00	3.50
14 N	15.25	28.00	2.80
14 R	15.25	39.00	3.50
14 W	15.25	50.00	4.00
16 N	17.25	32.00	3.50
16 R	17.25	44.00	4.00
16 W	17.25	56.00	4.60
20 N	21.80	39.00	4.00
20 R	21.80	50.00	4.60
20 W	21.80	66.00	5.10
24 N	25.60	44.00	4.60
24 R	25.60	56.00	5.10
24 W	25.60	72.00	5.60
30 N	32.40	56.00	5.10
30 R	32.40	72.00	5.60
30 W	32.40	90.00	6.40
36 N	38.30	66.00	5.60
36 R	38.30	90.00	6.40
36 W	38.30	110.00	8.50

1 Same as screw or bolt size.
N = narrow; R = regular; W = wide

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