Measurements and Uncertainties

Sources of errors:-

1- Choice of instruments
2- The Expirementer
3- The Environment
4- The way The expirement is done
5- The way The physical quantity is measured

- A measurement can never be take without any error. <u>But</u> it can be estimated when all errors are very small

errors
- → **Random errors**
  - → uncertainty in a measurement (we estimate it)
  - → ~ ~ ~ several ~ (The mean)

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$$: best estimation

$$\sigma_s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$: Sample standard deviation

$$\sigma_m = \frac{\sigma_s}{\sqrt{N}}$$: standard deviation of The mean

- → **Systematic errors**
  - • looking perpendicularly not in a wrong way — زاوية النظر
  - • عدم البدء من التدريج الصحيح
  - • uncalibrated Instrument — ان يكون الورنش مضبوط بطريقة خاطئة

Precision and Accuracy

Precision → **Random errors**
- • High → less precision
- • low → more precision

Exp: 9.82 ± 0.5 less
9.8 ± 0.1 More precise

Accuracy → **Systematic error**
- High → less Accuracy
- low → More Accuracy

Exp: True value: 9.86
X_A = 9.87 ± 0.01 → More Accurate
X_B = 10.1 ± 0.4 → less Accurate

Discrepancy test : accepted / not accepted

* True value X
* Result $\bar{X} \pm \Delta X$

- steps:
1- $D = |X - \bar{X}|$
2- $2 \times \Delta X$
3- if $D > 2\Delta X$ not accepted
if $D \leq 2\Delta X$ accepted

significant figures

• هي الأرقام المعنوية التي عبّرن عنها

Exp: 900 : 1 significant
900̇ : 3 ~
900.0 : 4 sig
0.020 : 2 sig

ΔX : should always be 1 sig figure unless the leading digit was one Then we keep the digit after Exp $0.123 \approx 0.12$
$0.16 \approx 2$
or 1.6

Rounding Rules :-

• any number less Than 5 we fix The sig. fig
• ~ ~ more ~ 5 we round The last sig fig up
• if it was 5 : Exp $35 \rightarrow 40$
بعطي الفردي
$45 \rightarrow 40$
بعطي الزوجي
$0.7251 \rightarrow 0.73$
بعطي الـ2

Values

* Addition and substraction

- The n° with the fewest decimal places limits the number of decimal places in the result
  المنازل العشرية

* Multiplication and division

- we find how much of sig fig Ther is in The numbers Multiplied : The less Controls The result
  كل المنازل

- $\sqrt{13} = 3.782 \approx 3.8$ (2 s.f)

  $\sqrt{2.4+10.2} = \sqrt{12.6} = 3.549$ تصغير قاعدة الجمع

- $\sin(24) = 0.406 \approx 0.41$

- $\cos(70) = 0.342 \approx 0.3$

Uncertainity

* Addition and substraction

$R = x \pm y \quad \Delta R = \Delta x + \Delta y$: general rule

* Constant Multipliers.

$R = ax \pm by \quad \Delta R = a\Delta x + b\Delta y$

But if a and b are not Const Then

$\Delta R = a\Delta x + x\Delta a + b\Delta y + \Delta b$

* Multiplication and division

$A = xy$

$\frac{\Delta A}{A} = \frac{y\Delta x + x\Delta y}{A}$ for 2 values

$\frac{\Delta A}{A} = \frac{y\Delta x}{xy} + \frac{x\Delta y}{xy}$ for more Than 2 values

$= \frac{\Delta x}{x} + \frac{\Delta y}{y}$

* Raising to powers

$$R = X^{l} y^{m} Z^{h}$$

$$\frac{\Delta R}{R} = |l| \frac{\Delta X}{X} + |m| \frac{\Delta y}{y} + |h| \frac{\Delta Z}{Z}$$

* $R = e^{X}$ العدد النيبري

↳ $\Delta R = e^{X} \Delta X$

* $R = \ln X$

↳ $\Delta R = \frac{1}{X} \Delta X$

* $R = \sin \theta$

↳ $\Delta R = \cos\theta \; \Delta\theta$

* $R = \cos\theta$

↳ $\Delta R = |-| \sin\theta \; \Delta\theta$

Rad لـ لازم نحول الخطأ من الدرجات عن طريق

$2\pi \rightarrow 180$

→ الاشتقاق الجزئي

• $R = R(X, y, Z)$

↳ • $\Delta R = \left|\frac{\partial R}{\partial X}\right| \Delta X + \left|\frac{\partial R}{\partial y}\right| \Delta y + \left|\frac{\partial R}{\partial Z}\right| \Delta Z$

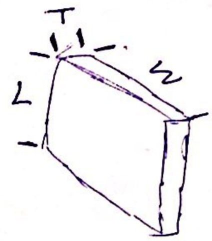
اشتقاق جزئي

Experiment 1

to identify the material

• Density & Distance between Atomes

Theory:



$$\text{Density} = \rho = \frac{M}{V} \quad V = L \times W \times T$$

(M → Mass, V → Volume)

• In Metal: atomes are almost spherical & identical = (lattice structure)

Total number of atomes:

$$N = n N_a = \frac{M}{A_w} N_A$$

(M → Mass, N_A → Avogadro's n°, A_w ← The Atomic mass of the Material)

or

$$N = \frac{M}{\rho a^3} \Rightarrow a = \sqrt[3]{\frac{A_w}{\rho N_a}}$$

now uncertainty in ρ:

$$\Delta\rho = \frac{\Delta M}{V} + \frac{M}{V^2} \Delta V$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{\Delta V}{V}$$

ΔM: estimated

$$\Delta V = WT\Delta L + \Delta WTL + W\Delta TL$$

$$\Rightarrow \frac{\Delta V}{V} = \frac{\Delta L}{L} + \frac{\Delta W}{W} + \frac{\Delta T}{T}$$

Exp#2: Conservation of linear Momentum

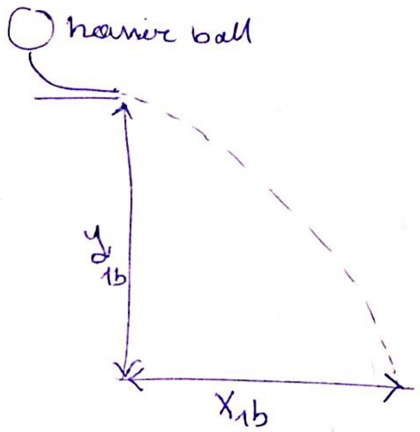
$\vec{P} = m\vec{v}$ → velocity

($\vec{P}$: linear Momentum, m: Mass)

- if there was N objects in an isolated system:- → No External resultant forces acts on it

$$\vec{P} = \sum_{i=1}^{N} m_i \vec{v_i}$$

$\vec{P}$: is conserved for an isolated system



*** Collision:-**

$\vec{P}$ before collision $= \vec{P}$ after collision

$$M_1 V_{1b} + \underbrace{M_2 V_{2b}}_{=0} = M_1 V_{1a} + M_2 V_{2a}$$

Theory.

$$\frac{P_a}{P_b} = \frac{M_1 v_{1a} + M_2 v_{2a}}{M_1 v_{1b}} = 1$$

$$y_{1b} = \frac{1}{2} g t^2$$

$$t = \sqrt{\frac{2y}{g}}$$

$$V_b = \frac{X_b}{t_b} \Rightarrow V = \frac{x}{\sqrt{2y/g}}$$

$$P_b = \frac{M_1 X_{1b}}{\sqrt{\frac{2y}{g}}}$$

- t is equal for the 2 balls before and after

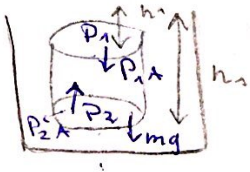
$$\frac{P_a}{P_b} = R = \frac{M_1 x_1 a + M_2 x_2 a}{M_1 X_{1b}} = \frac{A}{B}$$

$$\frac{\Delta R}{R} = \frac{\Delta A}{A} + \frac{\Delta B}{B} = \frac{M_1 \Delta x_{1a} + x_{1a} \Delta M_1 + M_2 \Delta x_{2a} + x_{2a} \Delta M_2}{M_1 x_{1a} + M_2 x_{2a}} + \frac{M_1 \Delta x_{1b} + x_{1b} \Delta M_1}{M_1 x_{1b}}$$

Exp 3: Density of liquids

Fluids → gases, liquids

- exert forces on the walls of their containers (perpendicular to the surface) → pressure
- $P = \frac{F}{A}$ (F → force, A → area)
- pressure is larger at lower points
- a portion of liquid:



$$P_2A - mg - P_1A = 0$$

$$A(P_2 - P_1) = mg$$

$$(P_2 - P_1) = \frac{mg}{A}$$

$$\rho = \frac{m}{V}$$

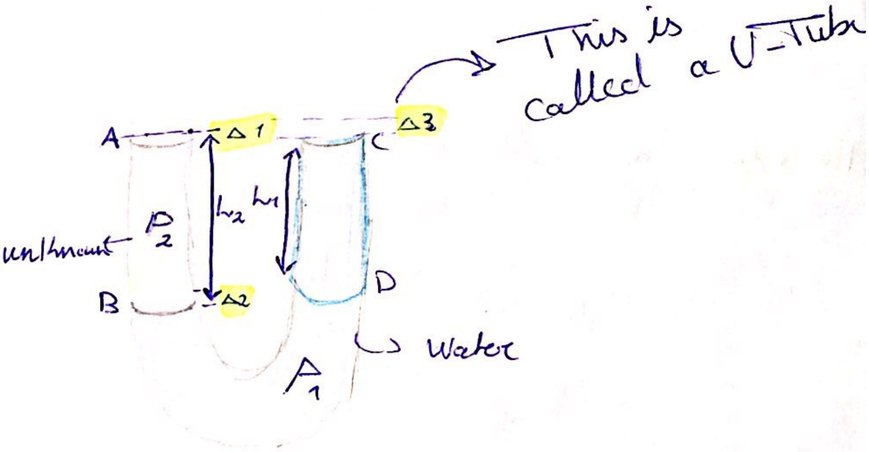
$$m = \rho V$$

$$P_2 - P_1 = \frac{\rho V g}{A}$$

$$P_2 - P_1 = \frac{\rho A (h_2 - h_1) g}{A}$$

$$P_2 - P_1 = (h_2 - h_1)\rho g$$

- U-Tube:-



1- $P_B - P_A = L_2 \times \rho_2 \times g$

2- $P_D - P_C = L_1 \times \rho_1 \times g$ (B = D, A = C)

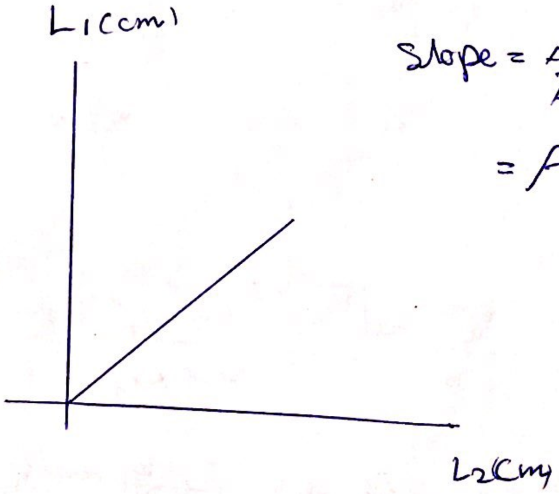
$$L_2 \rho_2 g = L_1 \rho_1 g$$

$$L_2 \rho_2 g = L_1 g$$

$$\rho_2 = \frac{L_1}{L_2}$$

$$\frac{\Delta\rho}{\rho} = \frac{\Delta L_1}{L_1} + \frac{\Delta L_2}{L_2}$$

$\Delta L_2 \to \Delta 1 + \Delta 2$, $\Delta L_1 \to \Delta 2 + \Delta 3$



$$\text{Slope} = \frac{\Delta L_1}{\Delta L_2} = \rho$$

Exp 4 :- Dc circuits

• $R = \frac{\text{Voltage}}{\text{Current}} = \frac{V}{I} = \frac{\text{فرق الجهد}}{\text{شدة التيار}}$ Potential difference / Current flowing

R → Resistance of a metalic conductor

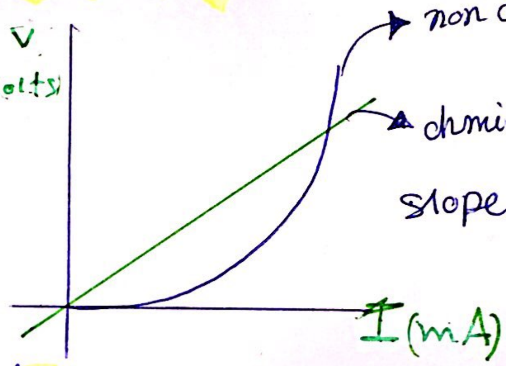
materials:
- ohmic : V depends linearly on I
- non-ohmic : V does not depend linearly on I

• Equivalent Resistance of 2 Resistors

I is the same for R_1 & R_2 → $R_{serie} = R_1 + R_2$

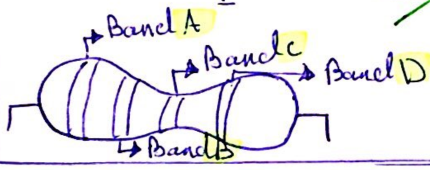
V is the same for R_1 & R_2 → $\frac{1}{R_{parallel}} = \frac{1}{R_1} + \frac{1}{R_2}$

$\Longrightarrow \frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$



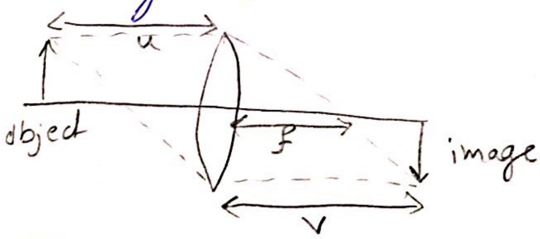
slope = $R = \frac{\Delta V}{\Delta I}$

Color code



R (Theoritically) = $AB \times 10^C \pm (D \times R)$

Exp 5:- focal length of a Convex lens



focal length:

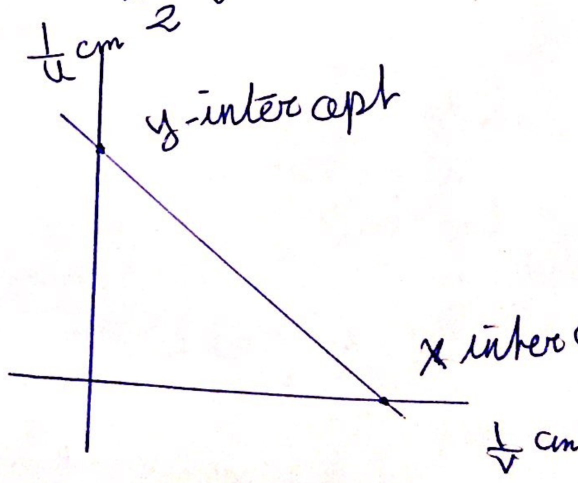
• The distance between the lens and the point of convergence of the light rays coming from the infinity

focal length ← $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ ($\frac{1}{u}$ → y-intercept, $\frac{1}{v}$ → x-intercept) $\Rightarrow \bar{f} = \frac{f_x + f_y}{2}$

$$\frac{\Delta f}{f^2} = \frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}$$

$f_x = f_y$ (Theoritically)

$\frac{1}{x\text{-intercept}} = \frac{1}{y\text{-intercept}}$



B

Exp5 :-

- if the object is placed At infinity Then the image will be formed at f

Exp4

- Voltmeter on parallel:

⇐ is a device used to measure The potential difference and it has a high ~~Resista~~ Resistance and if we connect

(2)

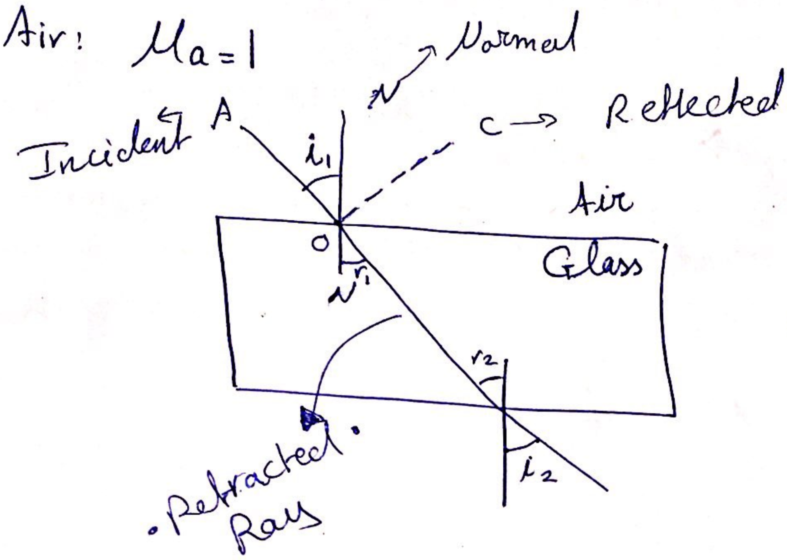
It on serie Then it will impede the current and no Reading will show and the Ammeter, If we connect it on parallel (it has a low resistance) a big amount of current would go through one branch and the Ammeter will burn out

Exp 6 :- Index of Refraction

$n = \frac{C}{V}$ → C: speed of light in vacuum, V: speed of light in medium

• The light bends when moving from a medium to another

for Air: $\mu_a = 1$



i: angle of incidence

r: angle of Refraction

Snell's law :- $\mu_a \sin(i) = \mu_g \sin(r)$

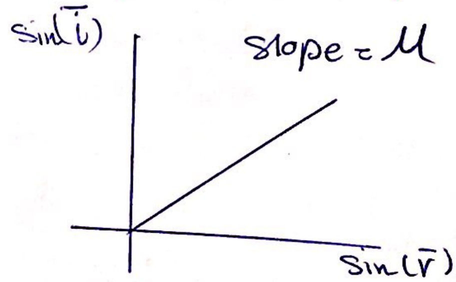
(i: Angle of incidence; r: angle of Refraction)

$\mu_a = 1$

$$\sin(i) = \mu_g \sin(r)$$

• μ_g is the slope

$$\mu_g = \frac{\sin(i)}{\sin(r)}$$



$$\frac{\Delta \mu_g}{\mu_g} = \frac{\Delta \sin(i)}{\sin i} + \frac{\Delta \sin(r)}{\sin r}$$

$$\frac{\Delta \mu_g}{\mu_g} = \frac{\cos i}{\sin i}\Delta i + \frac{\cos r}{\sin r}\Delta r$$

Δi and Δr in radians

↓ estimation

Exp 7 Measuring g at BZU using least fit square method

$L = S + \frac{d}{2}$

T (period) → time required by the penelulum to finish one oscillation

$T\,(\text{period}) = 2\pi\sqrt{\frac{l}{g}}$ • only when θ is small < 15

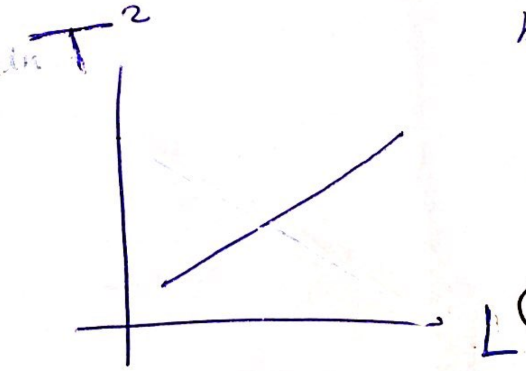
$T^2 = 4\pi^2\left(\frac{L}{g}\right)$ *

y ... m

$m = (\text{best slope}) = \frac{4\pi^2}{g}$

$\frac{\Delta g}{g} = \frac{\Delta m}{m}$

- finding the value of the slope m, the y-intercept b and their uncertanties using the least square fit method



best slope $= \frac{4\pi^2}{g}$

$m = \frac{4\pi^2}{g}$

$g = \frac{4\pi^2}{m}$

Exp 8: half-life of a draining water column

$$\frac{dh}{dt} = -\lambda\, h(t)$$

$$\Rightarrow \int_{h_0}^{h(t)} \frac{dh}{h} = \int_0^t -\lambda\, dt$$

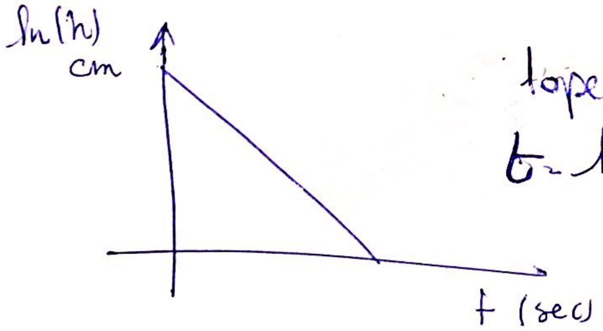
$h_0 = 50$ units $+D$ (in burette units)

$$h(t) = h_0 e^{-\lambda t}$$

~~best slope~~

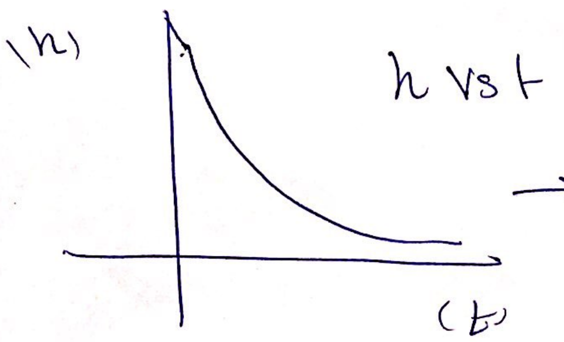
$\Rightarrow$ when $t_{1/2} \Rightarrow \frac{h_0}{2} = h_0 e^{-\lambda t_{1/2}}$

$$\Rightarrow t_{1/2} = \frac{\ln 2}{\lambda}$$



slope $= -\lambda$

$t = \frac{\ln 2}{\lambda}$



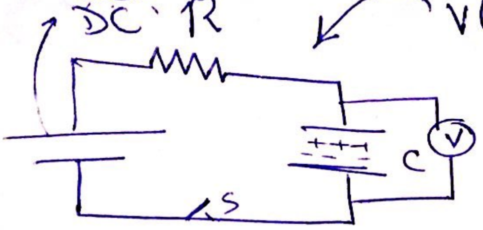
h vs t

$\rightarrow$ to obtain 6 measurements of $t_{1/2}$

Exp 9 : RC circuit

charging

$V(t) = \mathcal{E}(1 - e^{-1}) = 0.63\mathcal{E}$

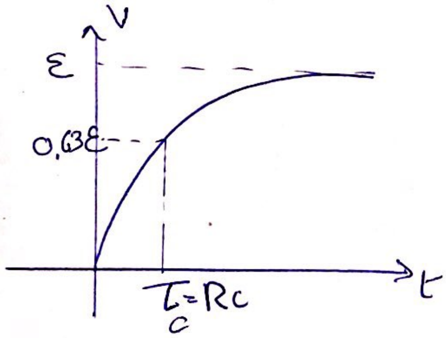


$V = \frac{Q}{C}$ الشحنة / الوسع

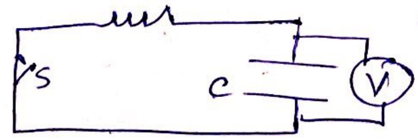
$[C] = \text{Farad} = \frac{\text{colum}}{\text{Volts}}$

$\tau = RC$

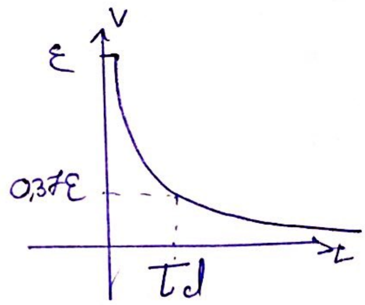
$V_c = 0.63\mathcal{E}$



discharging



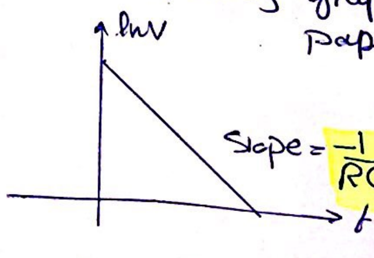
$V_c = 0.37\mathcal{E}$



$\tau_c = \tau_d$ نظرياً

$\bar{\tau} = \frac{\tau_c + \tau_d + \tau_s}{3}$ عملياً

• on a semi-log graph paper



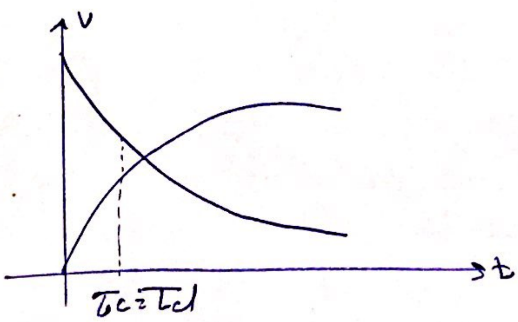
$\text{slope} = \frac{-1}{RC} = \frac{-1}{\tau_s}$

$\Rightarrow C = \frac{\bar{\tau}}{R}$

$\frac{\Delta C}{C} = \frac{\Delta \bar{\tau}}{\bar{\tau}} + \frac{\Delta R}{R}$

where $\Delta\bar{\tau}$ is calculated $= \sigma_m$

Theoritically

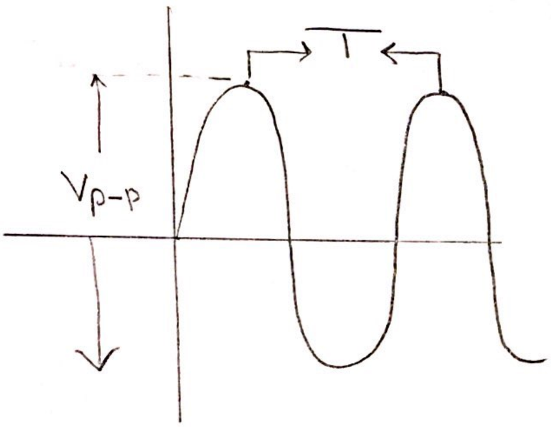


The Cathode-Ray Oscilloscope (CRO)

used for :- 1- measuring the peak to peak voltage V_{p-p}

2- measuring the frequency of a sinusoidal signal supplied by a signal generator

3- To display lissajous figures

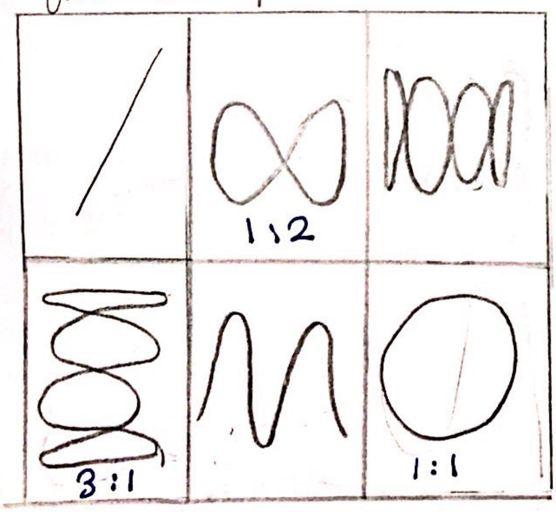


$$f = \frac{1}{T} \quad \text{(frequency)}$$

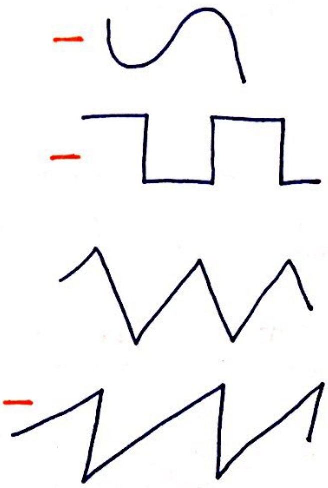
$$V_p = \frac{V_{p-p}}{2}$$

lissajous figures

for example



• formes of waves



- Sine wave
- Square wave
- Triangle wave
- Saw tooth

* CRO can display graphs of potential differences Vs. time
* it can be used to measure AC and DC voltages
* Can misure amplitude & freequency of a given AC signal As well as the phase (ϕ) betwee two AC Signals

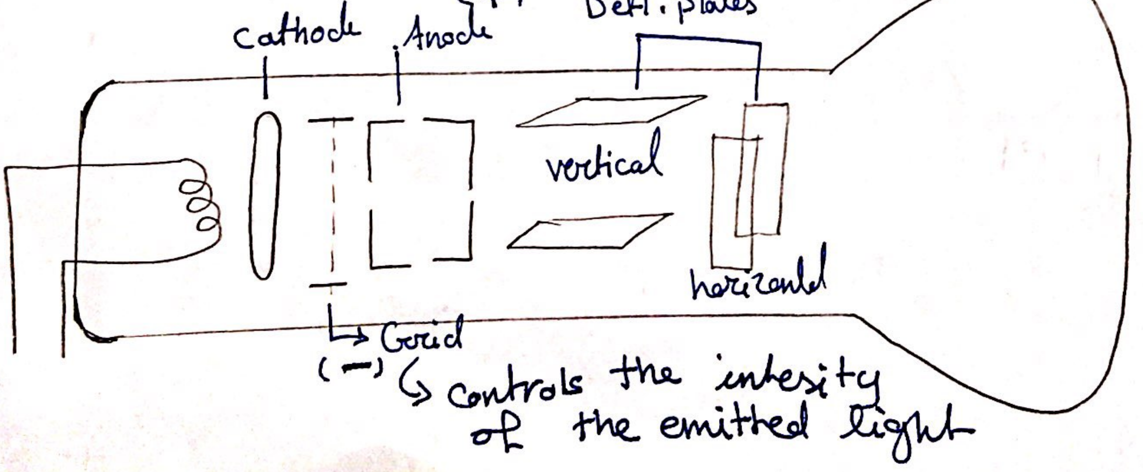
Structure

1- evacuated glass tube (CRT) — C: cathode, R: Ray, T: Tube
2- Cathode and Anode
3- Deflection plates
4- Grid (charged -)
5- vertical and horizontal deflection plates

The process :-

① filaments heates the cathode ⇒ ② electrons are emitted by the Cathode ⇒ ③ electrons are accelerated due the high Positive potential at the accelerating anode

⇒ ④ electrons move toward the florescent screen ⇒ ⑤ electrons hit the screen ⇒ ⑥ The material covering the screen emits light

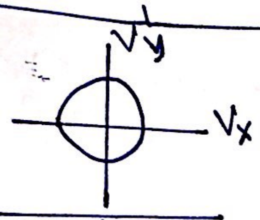


To Summerize the process :- 6 steps

Cathode is heated → electrons are emitted → electrons are accelerated → electrons move toward the screen → electrons hit the screen → screen emits light

Modes of the CRO

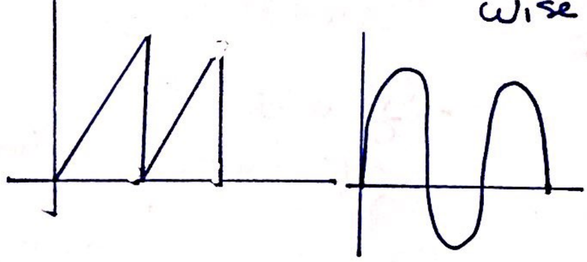
external Mode



- selected by the time base button to the x-y ext. mode
- screen acts as an x-y plotter
- The Voltage .Vs Time plot appears on the screen
- lissajous figures
- Time base button Clockwise

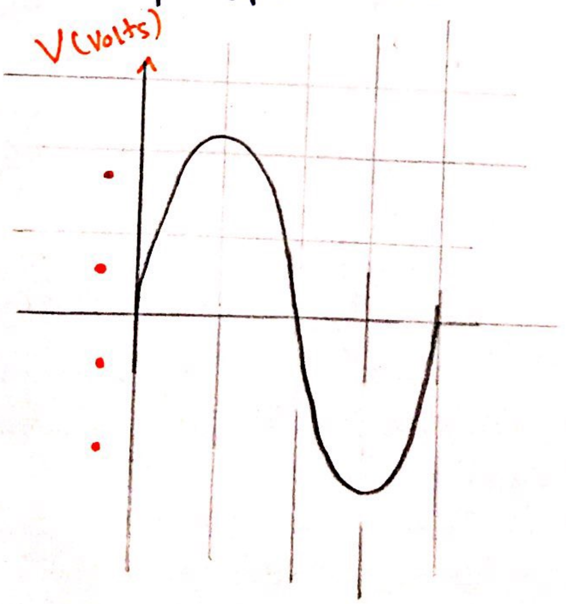
internal Mode

- x-axis becomes a time axis
- a sawtooth ~~wave~~ potential difference
- Time base button Counterclock wise



- x- input :- recieves external signals
- y- 〃 :- 〃 〃 〃

T (period) = No of boxes of one wave × time base Reading



$V_{p-p} = 4 \times 1$ Volts

$= 4$ Volts

$T = 4 \times 1$ se

$= 4$ sec

In case That Time base Reading is

1 volts / square

1 sec / square

Exp 11) RC Circuit using Oscilloscope

- measuring τ and $t_{\frac{1}{2}}$ using a signal generator and a CRO

- connecting a signal generator and R and C on series
- (provides a square wave voltage to the circuit)

$$t_{\frac{1}{2}} = RC \ln 2$$

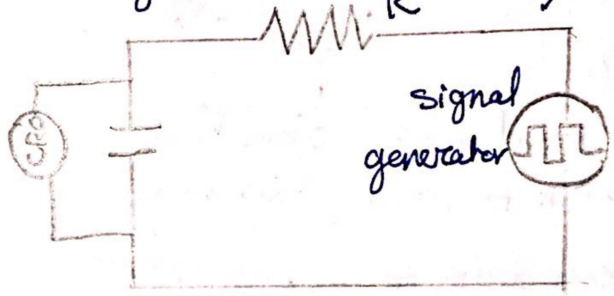
$$t_{\frac{1}{2}} = \tau \ln 2$$

$$V(t) = \frac{Q_0}{C}(1 - e^{-\frac{t}{RC}}) \quad \text{(charging)}$$

$$V(t) = \frac{Q_0}{C} e^{-\frac{t}{RC}} \quad \text{(discharging)}$$

$\tau_c = 0.63 V_{max}$

$\tau_d = 0.37 V_{max}$



ما نظهر على شاشة CRO :-