Chapter 9:- Hypothesis Test

Section 9.1

* We will use sample to test a hypothesis about μ

* The hypothesized (current) value of μ is called μ_0

* In any hypothesis testing, there are two hypotheses :—

① The null hypothesis : H_0

② The Alternative hypothesis : H_a "The researcher claim"

[H_a :— is the researcher claim
(μ greater than / OR less than // OR different from μ_0)
H_0 :— is the opposite of H_a.

** Three form of tests :—

① Upper tail Test —
$H_0: \mu \leq \mu_0$
$H_a: \mu > \mu_0$

→ One-tailed Test

② Lower tail Test —
$H_0: \mu \geq \mu_0$
$H_a: \mu < \mu_0$

③ Two tailed Test
$H_0: \mu = \mu_0$
$H_a: \mu \neq \mu_0$

* Conclusion (Desicion) of Test

Based on the sample, we either

→ Reject H_0 :- [The sample support the researcher claim]

→ Accept H_0 :- [Don't Reject H_0]

[The sample does not support the researcher claim]

9.2 Type I and Type II error :-

	H_0 True	H_0 False
Reject H_0	Type I error	Correct Conclusion
Accept H_0	Correct Conclusion	Type II error

Type I error :- Rejecting H_0 when its true

Type II error : Accepting H_0 when its False

** Significant level α :- the probability of making type I error

α usually is 0.1 (10%) or 0.05 (5%) or 0.01 (1%)

In General :- The steps for testing are

1. Write the hypotheses H_0, H_a.
2. Find Test statistics :- A value of "Z" OR "t"
3. Test using $\boxed{\text{Critical value Approache}}$, OR $\boxed{\text{P\_value Approaches}}$
4. Conclusion.

→ Test Statistics :- a value found based on the sample "Z" or "t"

Case 1 : σ known ⟹ Test statistics is

$$\boxed{Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}}$$

n: sample size

$\bar{x}$: sample mean

μ_0: hypothesized mean

σ: pop. standard deviation

S: sample standard deviation

Case 2: σ unknown ⟹ Test statistics is

$$\boxed{t = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}}$$

Critical Value Approach.

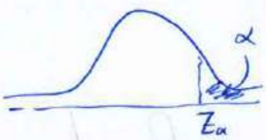
- depends on α, type of test, σ Known or unknown

- We compare the test statistics with the Critical value(s) to conclude

σ Known (Test statistic Z)

① upper tailed Test :-

$\longrightarrow$ critical value = Z_α

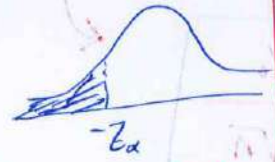


Z_α found by t-table with $df = \infty$

$$\left[\begin{array}{ll} \text{Reject } H_0 & Z \geq Z_\alpha \\ \text{Accept } H_0 & Z < Z_\alpha \end{array}\right]$$

② Lower tailed test :-

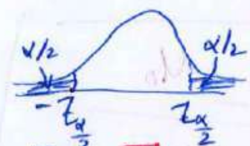
$\longrightarrow$ Critical value = $-Z_\alpha$



$$\left[\begin{array}{ll} \text{Reject } H_0 & Z \leq -Z_\alpha \\ \text{Accept } H_0 & Z > -Z_\alpha \end{array}\right]$$

③ two tailed Test

$\rightarrow$ critical value = $\pm Z_{\frac{\alpha}{2}}$

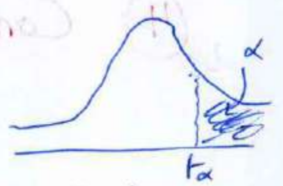


$$\left[\begin{array}{l} \text{Reject } H_0 \quad Z \leq -Z_{\frac{\alpha}{2}} \text{ OR } Z \geq Z_{\frac{\alpha}{2}} \\ \text{Accept } H_0 \quad -Z_{\frac{\alpha}{2}} < Z < Z_{\frac{\alpha}{2}} \end{array}\right]$$

σ unknown (Test statistic t)

① upper tailed Test

$\rightarrow$ Critical value = t_α



t_α found by t-table $df = n-1$

$$\left[\begin{array}{ll} \text{Reject } H_0 & t \geq t_\alpha \\ \text{Accept } H_0 & t < t_\alpha \end{array}\right]$$

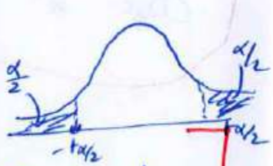
② Lower tailed test

$\rightarrow$ Critical value = $-t_\alpha$

$$\left[\begin{array}{ll} \text{Reject } H_0 & t \leq -t_\alpha \\ \text{Accept } H_0 & t > -t_\alpha \end{array}\right]$$

③ Two tailed Test

$\rightarrow$ critical value = $\pm t_{\frac{\alpha}{2}}$



$$\left[\begin{array}{l} \text{Reject } H_0 \quad t \leq -t_{\frac{\alpha}{2}} \text{ OR } t \geq t_{\frac{\alpha}{2}} \\ \text{Accept } H_0 \quad -t_{\frac{\alpha}{2}} < t < t_{\frac{\alpha}{2}} \end{array}\right]$$

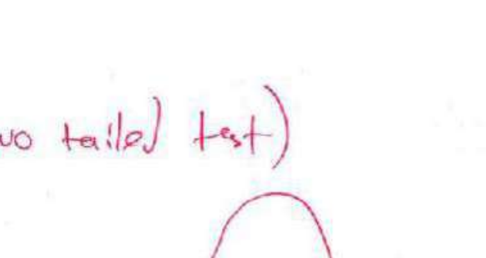
9.3 مطلوب σ Known 9.4 σ Un Know

In general, the steps for testing are:-

1. Write the hypotheses H_0, H_a.
2. Find the test Statistics: a value of Z or t
3. Test Using Critical Value or P-value Approach
4. Conclusion

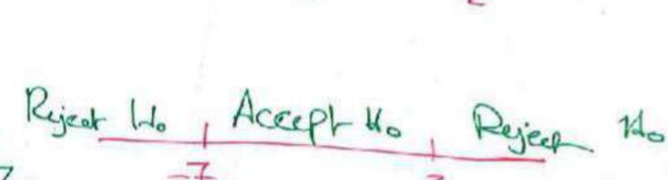
σ: known
test statistics is $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

*** Hypothesis:- $H_0: \mu = \mu_0$
$H_a: \mu \neq \mu_0$ (two tailed test)



Test statistics :- $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

Critical value : $\pm Z_{\frac{\alpha}{2}}$

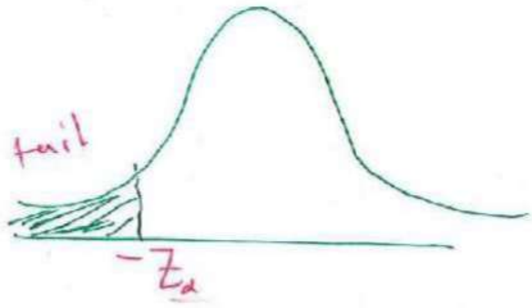


Conclusion :- Reject $H_0 \rightarrow |Z| \geq Z_{\frac{\alpha}{2}}$

Accept $H_0 \rightarrow |Z| < Z_{\frac{\alpha}{2}}$

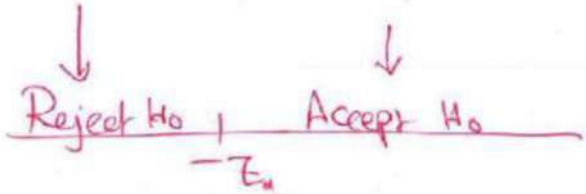
** Hypothesis:- $H_0 : \mu \geq \mu_0$ Lower tailed test

$H_a : \mu < \mu_0$



Test Statistics $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$

Critical Value: $-Z_\alpha$

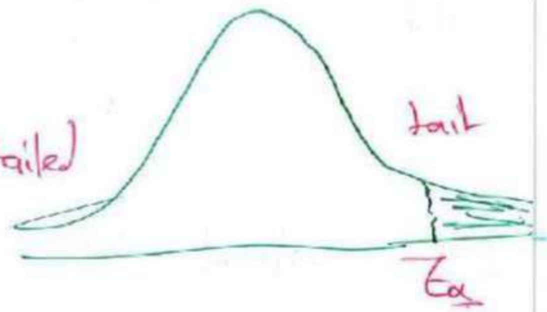


Conclusion $Z \leq -Z_\alpha$ Reject Ho

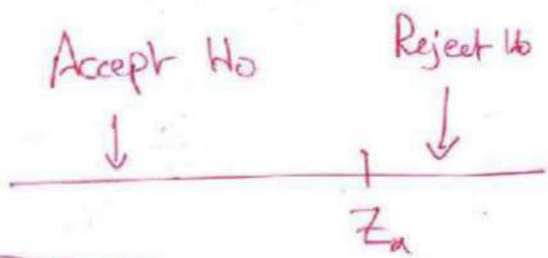
$Z > -Z_\alpha$ Accept Ho

** Hypothesis : $H_0 : \mu \leq \mu_0$ upper tailed test

$H_a : \mu > \mu_0$



test statistics :- $Z = \dfrac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$



Critical value

C.V. = $\boxed{Z_\alpha}$

Conclusion

$Z \geq Z_\alpha$ Reject Ho

$Z < Z_\alpha$ Accept Ho

Exp: $H_0 : \mu \leq 20$ given $n = 35$

$H_a : \mu > 20$ $\bar{x} = 23$

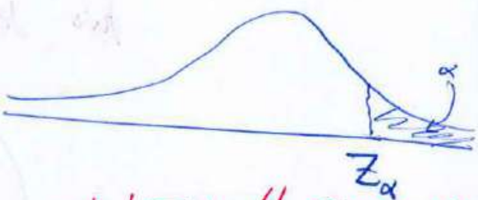
upper tail Test $\alpha = 10\%$

$\sigma = 4.5$

① Find the test Statistics :- (σ known)

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = 3.94$$

② Find the critical value



critical value depend on $\alpha = 0.1$ // Type: upper tailed Test // σ known

$$\text{Critical value} = Z_\alpha = Z_{0.1}$$

using t-table with $df = \infty \longrightarrow Z_\alpha = 1.282$

③ What is your conclusion

Since $Z \geq Z_\alpha \longrightarrow$ Reject H_0.

Exp: $H_0: \mu = 80.42$ Given that

$H_a: \mu \neq 80.42$

Sample size = 100

$\overline{X} = 81$

$\sigma = 15.2$

$\alpha = 1\%$

① Hypothesis $H_0: \mu = 80.42$

$H_a: \mu \neq 80.42$

two tailed Test

② Test Statistics (σ Known) $Z = \dfrac{\overline{X} - \mu_0}{\sigma / \sqrt{n}} = \dfrac{81 - 80.42}{15.2/\sqrt{100}}$

$= 0.38$

③ Critical Value (Two-tailed Test / σ Known)

$\alpha = 0.01$

Critical value $= \pm Z_{\frac{\alpha}{2}} = \pm 2.576$

$\frac{\alpha}{2} = 0.005$

$df = \infty$

③ What is your conclusion ??

Since $-Z_{\frac{\alpha}{2}} < Z < Z_{\frac{\alpha}{2}}$

Dont Reject H_0 (Accept H_0).

Ex. $\mu = 154$ $\quad$ $n = 40$

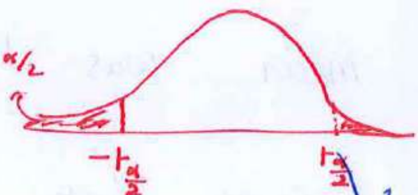
$\mu \neq 154$ $\quad$ $\bar{X} = 157$

$S = 10$

two tailed Test $\quad$ $\alpha = 5\%$

① Find the Test statistics: (σ unknown)

$$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} = -0.63$$



② Find the critical value (σ unknown) / (two tailed test)

$\alpha = 0.05$

$$\text{Critical value} = \pm t_{\frac{\alpha}{2}} = \pm 2.023$$

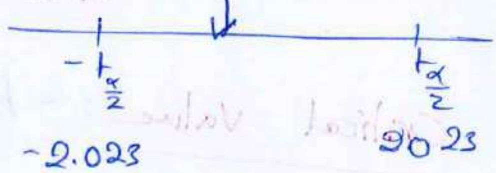
t-table

$df = 39$

$\frac{\alpha}{2} = 0.025$

③ What is your Conclusion ??

Since $-t_{\frac{\alpha}{2}} < t < t_{\frac{\alpha}{2}}$



Accept H_0 (Dont Reject H_0)

Student say that they study an average 5 hour per day. I claim that the real average is much less than that

A sample of 30 students was taken. The sample mean was 4.6 hours

Assume that the sample standard deviation is 2.1.

At 10% significance, Test my claim

Sol.

$n = 30$

$\bar{X} = 4.6$

$\mu_0 = 5$

$S = 2.1$ (σ unknown)

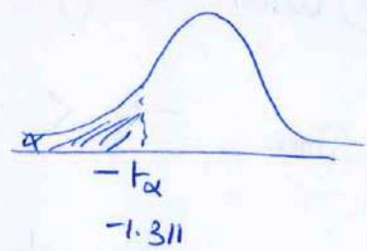
① $H_0 : \mu \geq 5$

$H_a : \mu < 5$ Lower tailed Test

② Test Statistics $t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} = -1.04$

③ Critical value : $-t_\alpha = -1.311$

t-value by using t-table, $df = 29$, $\alpha = 0.1$



④ Conclusion : Accept H_0 (Dont Reject H_0)

Since $t > -t_\alpha$

P-value Approach (σ Known)

In general, The steps for testing are :-

(1) Write the hypothesis H_0 & H_a

(2) Test Statistics :- Value of $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

(3) Test : P-value Approach

(4) Conclusion.

P-value : a probability (area) found using Z table
based on test statistic and type of test

In any test

If P-value $\leq \alpha$: Reject H_0

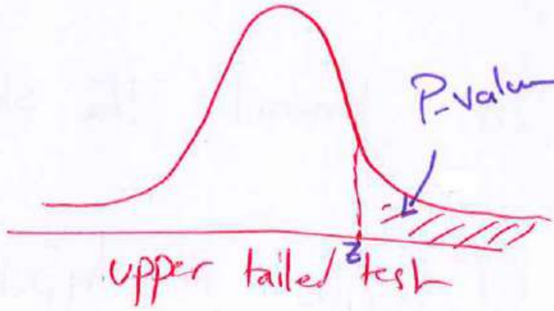
P-value $> \alpha$: Accept H_0 (Don't Reject H_0)

How to Find p-value (Use Z table)

[1] Upper tail Test

*** ① Hypothesis $H_0: \mu \leq \mu_0$

$H_a: \mu > \mu_0$



② Test statistics : $Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$

③ P-value : Area above Z

④ conclusion :- P-value $\leq \alpha$ Reject H_0

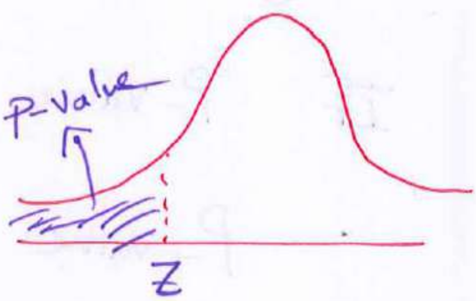
P-value $> \alpha$ Accept H_0

[2] Lower tail Test :-

*** ① Hypothesis $H_0: \mu \geq \mu_0$

$H_a: \mu < \mu_0$

② Test Statistics $Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$



③ P-value : Area below Z

④ conclusion: P-value $\leq \alpha$ Reject H_0

P-value $> \alpha$ Accept H_0

** Two tailed Test

① Hypothesis : $H_0 : \mu = \mu_0$

$H_a : \mu \neq \mu_0$

② Test statistics $Z = \dfrac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$

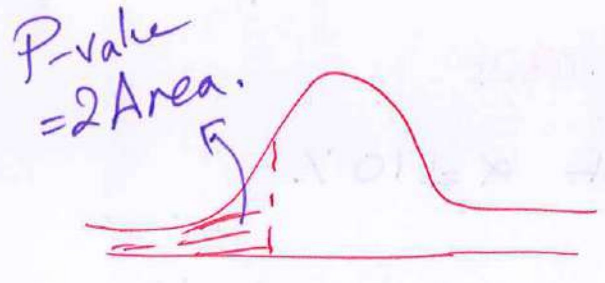
③ P-value = 2 (area above Z) if $Z > 0$

OR

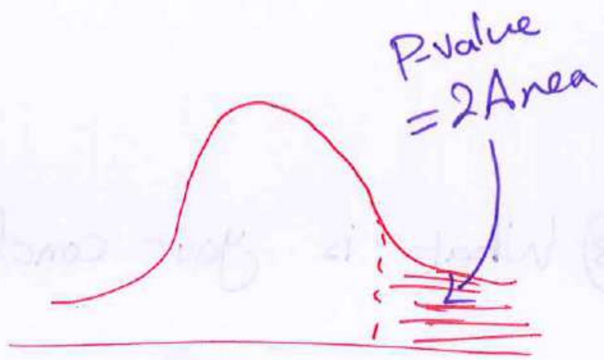
= 2 (area below Z) if $Z < 0$

④ Conclusion P-value $\leq \alpha$ Reject H_0

P-value $> \alpha$ Accept H_0.



If $Z<0$



If $Z>0$

Ex: $H_0: \mu \leq 30.5$

$H_a: \mu > 30.5$

$n = 70$

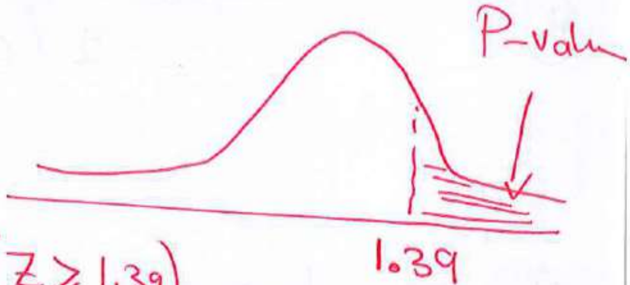
$\bar{X} = 31.5$

$\sigma = 6.02$ (Known)

① Find the test Statistics

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{31.5 - 30.5}{6.02 / \sqrt{70}} = 1.39$$

② Find the P-Value



Sol: upper tail test $\longrightarrow$ P-value $= P(Z \geq 1.39)$

$$= 1 - P(Z \leq 1.39)$$

$$= 1 - 0.9177$$

$$= 0.0823$$

③ What is your conclusion at $\alpha = 10\%$.

P-value $\leq \alpha \Longrightarrow$ Reject H_0

$0.0823 \leq 0.1$

④ What is your conclusion at $\alpha = 5\%$.

P-Value $> \alpha \Longrightarrow$ Acept H_0

$0.0823 > 0.05$

Notice: Changing α may change your decision.

Ex: $H_0 : \mu \geq 1220$ $\quad n = 95$

$H_a : \mu < 1220$ $\quad \bar{X} = 1210$

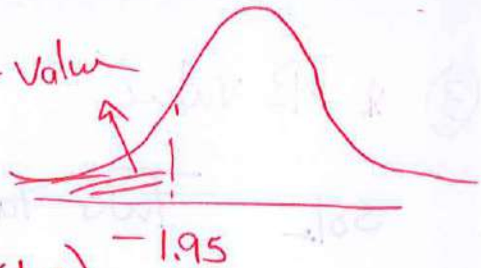
$\sigma = 50$

$\alpha = 1\%$

(1) Find the test statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \frac{1210 - 1220}{50 / \sqrt{95}} = -1.95$$

(2) Find P-value:



Sol:- Lower tail test $\rightarrow$ P-value $= P(Z < -1.95)$

$$= P(Z \geq 1.95)$$

$$= 1 - P(Z \leq 1.95)$$

$$= 1 - 0.9744$$

$$= 0.0256$$

(3) What is your conclusion.-

P-value $> \alpha \longrightarrow$ Accept H_0

$0.0256 > 0.01$

Ex: $H_0 : \mu = 250$ $\quad n = 140$

$H_1 : \mu \neq 250$ $\quad \sigma = 20.25$

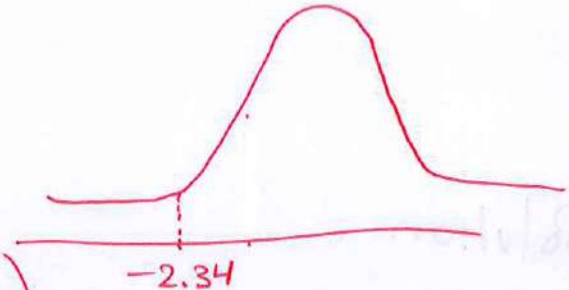
$\alpha = 5\%$

$\bar{x} = 246$

(1) Find Test Statistic :-

$$Z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} = \frac{246 - 250}{20.25 / \sqrt{140}} = -2.34$$

(2) Find P- Value.



Sol.

Two tailed Test $\longrightarrow$ P-value $= 2P(Z \leq -2.34)$

$$= 2\,P(Z \geq 2.34)$$

$$= 2\left[1 - P(Z \leq 2.34)\right]$$

$$= 2\left[1 - 0.9904\right] = 0.0192$$

(3) Conclusion: P-value $\leq \alpha \longrightarrow$ Reject H_0

$0.0192 \leq 0.05$

Ex: The mean weight of high school student is 70 kg. A sample of 50 Students is taken

This sample produced a mean of 73.

Assume the population Standard deviation is 20

① Use significance level of 1% to test the claim that the mean weight is greater than 70

Solution

① Hypothesis: $H_0: \mu \leq 70$

$H_a: \mu > 70$ upper tail. test

② Test statistic: $Z = \dfrac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$

$= \dfrac{73 - 70}{20/\sqrt{50}} = 1.06$

$\begin{cases} \mu_0 = 70 \\ n = 50 \\ \bar{X} = 73 \\ \sigma = 20 \text{ (Known)} \\ \alpha = 1\% \end{cases}$

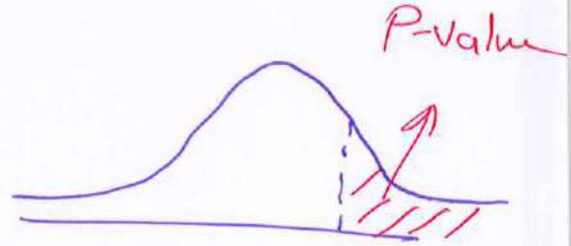
③ Test: critical value $= Z_\alpha = 2.326$

④ Conclusion:- $Z < Z_\alpha$: Accept H_0
(Don't Reject H_0)

① Hypothesis $H_0 : \mu \leq 70$

$H_a \; \mu > 70$ Upper tail Test

② Test statistic $Z = 1.06$



③ Test : P- value = Area above Z

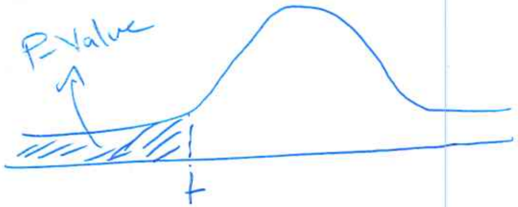
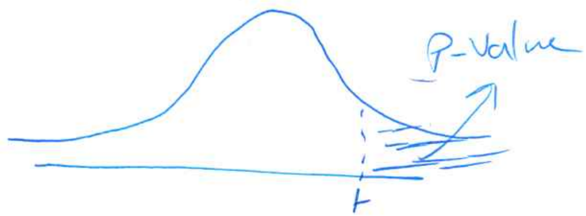
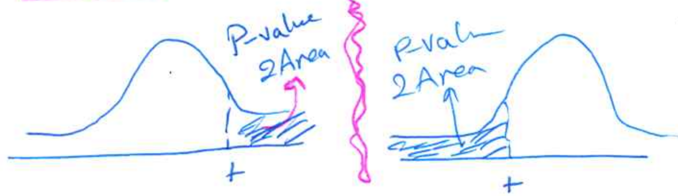
$= P(Z \geq 1.06)$

$= 1 - P(Z \leq 1.06)$

$= 1 - 0.8554 = 0.1446$

④ Conclusion: P-value $> \alpha$ Accept H_0

$0.1446 > 0.01$ (Don't Reject H_0)

	Lower Tail Test	Upper tail Test	Two tailed Test
Hypothesis	$H_0: \mu \geq \mu_0$ $H_a: \mu < \mu_0$	$H_0: \mu \leq \mu_0$ $H_a: \mu > \mu_0$	$H_0: \mu = \mu_0$ $H_a: \mu \neq \mu_0$
Test statistic	$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$	$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$	$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$
Test	P_value : Area below t  	P-value : Area above t  	P-value : 2 Area $\boxed{t > 0}$ OR $\boxed{t < 0}$ 
Conclusion	Pvalue ≤ α Reject H_0 P-value > α Accept H_0	P-value ≤ α Reject H_0 P-value > α Accept H_0	P-value ≤ α Reject H_0 P-value > α Accept H_0

Ex: Consider the ~~#~~ Following Hypothesis

$H_0: \mu \leq 12$

$H_a: \mu > 12$

A Sample of 25 produced Sample mean 14 and Sample standard deviation $S = 4.32$

[a] Compute the value of Test statistics

[b] Compute the range of the P-value

[c] At $\alpha = 0.05$, what is your conclusion

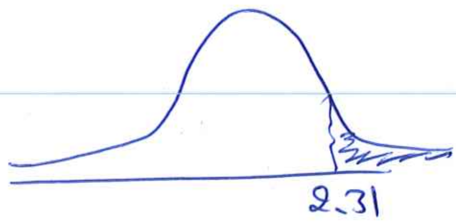
Solution: $n = 25$ $\quad \bar{x} = 14$ $\quad S = 4.32$ (σ unknown) $\quad \mu_0 = 12$

[1] $H_0: \mu \leq 12$

$H_a: \mu > 12$

upper tail test

[2] Test Statistic :- $t = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}} = \dfrac{14 - 12}{4.32/\sqrt{25}} = 2.31$



[3] P-value = Area above $t = 2.31$

$P(t \geq 2.31)$

$P(t \geq 2.31)$ From t-table $df = 24$

∴ P-value is between 0.01 & 0.025

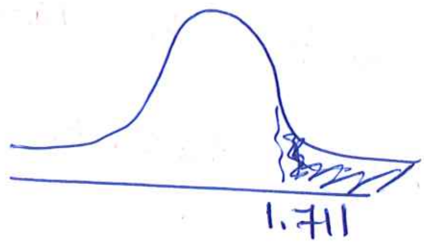
Conclusion P-Value $\leq$ α

$0.01 \rightarrow 0.025 \leq 0.05$

Reject Ho.

By using critical value

* Test statistic $t = 2.31$



* Critical value $= t_\alpha$

$= t_{0.05}$

$= 1.711$

* Conclusion Reject Ho $t \geq t_\alpha$

Consider the following Hypothesis $H_0: \mu \geq 45$
$H_a: \mu < 45$

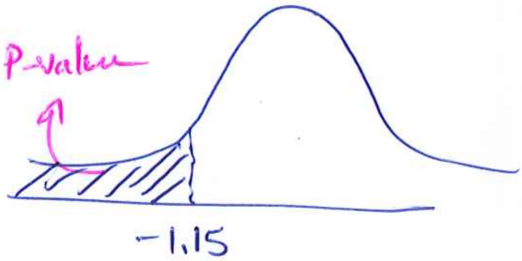
A Sample of 36 is used. Identify the P-value and State your conclusion for the following Sample result

By using $\alpha = 0.01$, $n = 36$, $\bar{X} = 44$, $S = 5.2$

$\boxed{1}$ Hypothesis :- $H_0: \mu \geq 45$
$H_a: \mu < 45$ Lower tail test

$\boxed{2}$ Test statistic :- $t = \dfrac{\bar{X} - \mu_0}{S/\sqrt{n}} = \dfrac{44 - 45}{5.2/\sqrt{36}} = -1.15$

$\boxed{3}$ P Value = Area below $t = -1.15$



From the t-table $df = 35$

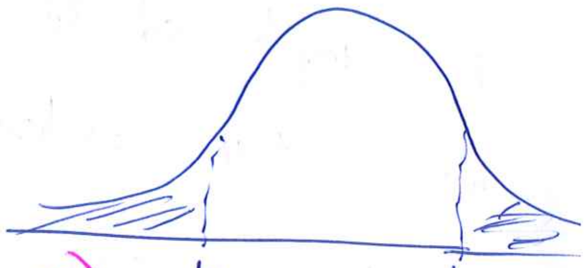
P-value between 0.1 and 0.2

$\boxed{4}$ Conclusion: P-value $>$ α Accept Ho [Don't Reject Ho]
$0.1 \to 0.2$ $>$ 0.01

By using Critical Value :-

$\boxed{*}$ Test statistic $t = -1.54$

$\boxed{*}$ Critical value $t_{\frac{\alpha}{2}}$, $-t_{\frac{\alpha}{2}}$



$t_{\frac{\alpha}{2}} = t_{0.025}$ (from t-table df = 47)

$= 2.012$

$t_{\frac{\alpha}{2}} = 2.012$

$-t_{\frac{\alpha}{2}} = -2.012$

* Conclusion :- Accept Ho $\left[-t_{\frac{\alpha}{2}} < t < t_{\frac{\alpha}{2}} \right]$

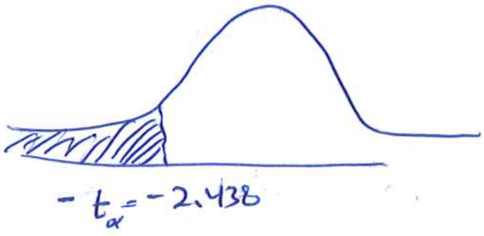
By using critical value.

(*) Test statistic $t = -1.15$

(*) Critical value $= -t_\alpha$

$= -t_{0.01}$

$= -2.438$



* Conclusion:- Accept H_0 [Don't Reject H_0]

since $t > -t_\alpha$

Ex:- Consider the following Hypothesis $H_0: \mu = 18$

$H_a: \mu \neq 18$

A sample of 48 ~~paid~~ provided a sample mean 17 and sample standard deviation 4.5

(a) Compute the value of test statistic

(b) Compute a range for P-Value

(c) At $\alpha = 0.05$, what is your conclusion ??

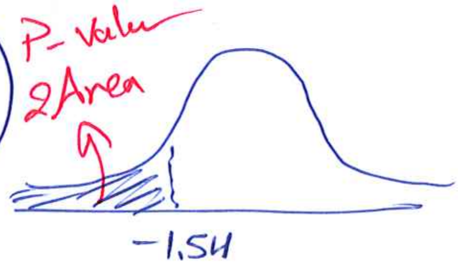
$\bar{x} = 17$ / $S = 4.5$ / $n = 48$ / $\mu_0 = 18$

$\boxed{1}$ Hypothesis $H_0: \mu = 18$

$H_a: \mu \neq 18$ Two tailed Test

$\boxed{2}$ Test statistic $t = \dfrac{\bar{x} - \mu_0}{S/\sqrt{n}} = \dfrac{17 - 18}{4.5/\sqrt{48}} = -1.54$

$\boxed{3}$ P-value $= 2\left(\text{Area below } t\ -1.54\right)$



$= 2\left(P(t \leq -1.54)\right)$

from t-table with df=47

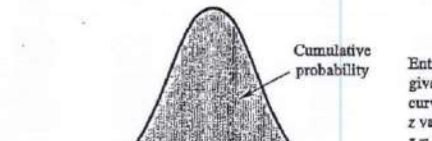
$P(t \leq -1.54)$ between 0.1 and 0.05

P-Value between 0.2 & 0.1

$\boxed{4}$ Conclusion :- P-Value > α (Accept H_0)

0.1 → 0.2 0.05

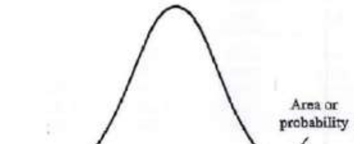
TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)



Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

TABLE 2 t DISTRIBUTION



Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921
17	.863	1.333	1.740	2.110	2.567	2.898
18	.862	1.330	1.734	2.101	2.552	2.878
19	.861	1.328	1.729	2.093	2.539	2.861
20	.860	1.325	1.725	2.086	2.528	2.845
21	.859	1.323	1.721	2.080	2.518	2.831
22	.858	1.321	1.717	2.074	2.508	2.819
23	.858	1.319	1.714	2.069	2.500	2.807
24	.857	1.318	1.711	2.064	2.492	2.797
25	.856	1.316	1.708	2.060	2.485	2.787
26	.856	1.315	1.706	2.056	2.479	2.779
27	.855	1.314	1.703	2.052	2.473	2.771
28	.855	1.313	1.701	2.048	2.467	2.763
29	.854	1.311	1.699	2.045	2.462	2.756
30	.854	1.310	1.697	2.042	2.457	2.750
31	.853	1.309	1.696	2.040	2.453	2.744
32	.853	1.309	1.694	2.037	2.449	2.738
33	.853	1.308	1.692	2.035	2.445	2.733

TABLE 2 *t* DISTRIBUTION (*Continued*)

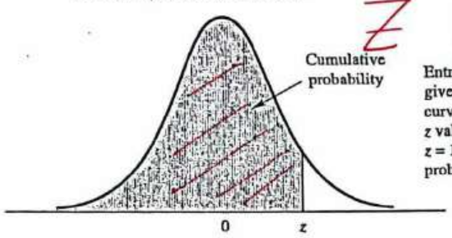
Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
35	.852	1.306	1.690	2.030	2.438	2.724
36	.852	1.306	1.688	2.028	2.434	2.719
37	.851	1.305	1.687	2.026	2.431	2.715
38	.851	1.304	1.686	2.024	2.429	2.712
39	.851	1.304	1.685	2.023	2.426	2.708
40	.851	1.303	1.684	2.021	2.423	2.704
41	.850	1.303	1.683	2.020	2.421	2.701
42	.850	1.302	1.682	2.018	2.418	2.698
43	.850	1.302	1.681	2.017	2.416	2.695
44	.850	1.301	1.680	2.015	2.414	2.692
45	.850	1.301	1.679	2.014	2.412	2.690
46	.850	1.300	1.679	2.013	2.410	2.687
47	.849	1.300	1.678	2.012	2.408	2.685
48	.849	1.299	1.677	2.011	2.407	2.682
49	.849	1.299	1.677	2.010	2.405	2.680
50	.849	1.299	1.676	2.009	2.403	2.678
51	.849	1.298	1.675	2.008	2.402	2.676
52	.849	1.298	1.675	2.007	2.400	2.674
53	.848	1.298	1.674	2.006	2.399	2.672
54	.848	1.297	1.674	2.005	2.397	2.670
55	.848	1.297	1.673	2.004	2.396	2.668
56	.848	1.297	1.673	2.003	2.395	2.667
57	.848	1.297	1.672	2.002	2.394	2.665
58	.848	1.296	1.672	2.002	2.392	2.663
59	.848	1.296	1.671	2.001	2.391	2.662
60	.848	1.296	1.671	2.000	2.390	2.660
61	.848	1.296	1.670	2.000	2.389	2.659
62	.847	1.295	1.670	1.999	2.388	2.657
63	.847	1.295	1.669	1.998	2.387	2.656
64	.847	1.295	1.669	1.998	2.386	2.655
65	.847	1.295	1.669	1.997	2.385	2.654
66	.847	1.295	1.668	1.997	2.384	2.652
67	.847	1.294	1.668	1.996	2.383	2.651
68	.847	1.294	1.668	1.995	2.382	2.650
69	.847	1.294	1.667	1.995	2.382	2.649
70	.847	1.294	1.667	1.994	2.381	2.648
71	.847	1.294	1.667	1.994	2.380	2.647
72	.847	1.293	1.666	1.993	2.379	2.646
73	.847	1.293	1.666	1.993	2.379	2.645
74	.847	1.293	1.666	1.993	2.378	2.644
75	.846	1.293	1.665	1.992	2.377	2.643
76	.846	1.293	1.665	1.992	2.376	2.642
77	.846	1.293	1.665	1.991	2.376	2.641
78	.846	1.292	1.665	1.991	2.375	2.640
79	.846	1.292	1.664	1.990	2.374	2.639

TABLE 2 *t* DISTRIBUTION (*Continued*)

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
80	.846	1.292	1.664	1.990	2.374	2.639
81	.846	1.292	1.664	1.990	2.373	2.638
82	.846	1.292	1.664	1.989	2.373	2.637
83	.846	1.292	1.663	1.989	2.372	2.636
84	.846	1.292	1.663	1.989	2.372	2.636
85	.846	1.292	1.663	1.988	2.371	2.635
86	.846	1.291	1.663	1.988	2.370	2.634
87	.846	1.291	1.663	1.988	2.370	2.634
88	.846	1.291	1.662	1.987	2.369	2.633
89	.846	1.291	1.662	1.987	2.369	2.632
90	.846	1.291	1.662	1.987	2.368	2.632
91	.846	1.291	1.662	1.986	2.368	2.631
92	.846	1.291	1.662	1.986	2.368	2.630
93	.846	1.291	1.661	1.986	2.367	2.630
94	.845	1.291	1.661	1.986	2.367	2.629
95	.845	1.291	1.661	1.985	2.366	2.629
96	.845	1.290	1.661	1.985	2.366	2.628
97	.845	1.290	1.661	1.985	2.365	2.627
98	.845	1.290	1.661	1.984	2.365	2.627
99	.845	1.290	1.660	1.984	2.364	2.626
100	.845	1.290	1.660	1.984	2.364	2.626
∞	.842	1.282	1.645	1.960	2.326	2.576

TABLE 1 CUMULATIVE PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION (*Continued*)

Z table



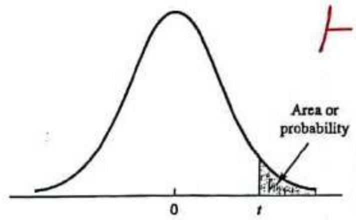
Entries in the table give the area under the curve to the left of the z value. For example, for $z = 1.25$, the cumulative probability is .8944.

Lower Area

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

TABLE 2 t DISTRIBUTION

t table



Entries in the table give t values for an area or probability in the upper tail of the t distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

upper Area

df = n-1

t Value

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604
5	.920	1.476	2.015	2.571	3.365	4.032
6	.906	1.440	1.943	2.447	3.143	3.707
7	.896	1.415	1.895	2.365	2.998	3.499
8	.889	1.397	1.860	2.306	2.896	3.355
9	.883	1.383	1.833	2.262	2.821	3.250
10	.879	1.372	1.812	2.228	2.764	3.169
11	.876	1.363	1.796	2.201	2.718	3.106
12	.873	1.356	1.782	2.179	2.681	3.055
13	.870	1.350	1.771	2.160	2.650	3.012
14	.868	1.345	1.761	2.145	2.624	2.977
15	.866	1.341	1.753	2.131	2.602	2.947
16	.865	1.337	1.746	2.120	2.583	2.921
17	.863	1.333	1.740	2.110	2.567	2.898
18	.862	1.330	1.734	2.101	2.552	2.878
19	.861	1.328	1.729	2.093	2.539	2.861
20	.860	1.325	1.725	2.086	2.528	2.845
21	.859	1.323	1.721	2.080	2.518	2.831
22	.858	1.321	1.717	2.074	2.508	2.819
23	.858	1.319	1.714	2.069	2.500	2.807
24	.857	1.318	1.711	2.064	2.492	2.797
25	.856	1.316	1.708	2.060	2.485	2.787
26	.856	1.315	1.706	2.056	2.479	2.779
27	.855	1.314	1.703	2.052	2.473	2.771
28	.855	1.313	1.701	2.048	2.467	2.763
29	.854	1.311	1.699	2.045	2.462	2.756
30	.854	1.310	1.697	2.042	2.457	2.750
31	.853	1.309	1.696	2.040	2.453	2.744
32	.853	1.309	1.694	2.037	2.449	2.738
33	.853	1.308	1.692	2.035	2.445	2.733

TABLE 2 *t* DISTRIBUTION (*Continued*)

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
35	.852	1.306	1.690	2.030	2.438	2.724
36	.852	1.306	1.688	2.028	2.434	2.719
37	.851	1.305	1.687	2.026	2.431	2.715
38	.851	1.304	1.686	2.024	2.429	2.712
39	.851	1.304	1.685	2.023	2.426	2.708
40	.851	1.303	1.684	2.021	2.423	2.704
41	.850	1.303	1.683	2.020	2.421	2.701
42	.850	1.302	1.682	2.018	2.418	2.698
43	.850	1.302	1.681	2.017	2.416	2.695
44	.850	1.301	1.680	2.015	2.414	2.692
45	.850	1.301	1.679	2.014	2.412	2.690
46	.850	1.300	1.679	2.013	2.410	2.687
47	.849	1.300	1.678	2.012	2.408	2.685
48	.849	1.299	1.677	2.011	2.407	2.682
49	.849	1.299	1.677	2.010	2.405	2.680
50	.849	1.299	1.676	2.009	2.403	2.678
51	.849	1.298	1.675	2.008	2.402	2.676
52	.849	1.298	1.675	2.007	2.400	2.674
53	.848	1.298	1.674	2.006	2.399	2.672
54	.848	1.297	1.674	2.005	2.397	2.670
55	.848	1.297	1.673	2.004	2.396	2.668
56	.848	1.297	1.673	2.003	2.395	2.667
57	.848	1.297	1.672	2.002	2.394	2.665
58	.848	1.296	1.672	2.002	2.392	2.663
59	.848	1.296	1.671	2.001	2.391	2.662
60	.848	1.296	1.671	2.000	2.390	2.660
61	.848	1.296	1.670	2.000	2.389	2.659
62	.847	1.295	1.670	1.999	2.388	2.657
63	.847	1.295	1.669	1.998	2.387	2.656
64	.847	1.295	1.669	1.998	2.386	2.655
65	.847	1.295	1.669	1.997	2.385	2.654
66	.847	1.295	1.668	1.997	2.384	2.652
67	.847	1.294	1.668	1.996	2.383	2.651
68	.847	1.294	1.668	1.995	2.382	2.650
69	.847	1.294	1.667	1.995	2.382	2.649
70	.847	1.294	1.667	1.994	2.381	2.648
71	.847	1.294	1.667	1.994	2.380	2.647
72	.847	1.293	1.666	1.993	2.379	2.646
73	.847	1.293	1.666	1.993	2.379	2.645
74	.847	1.293	1.666	1.993	2.378	2.644
75	.846	1.293	1.665	1.992	2.377	2.643
76	.846	1.293	1.665	1.992	2.376	2.642
77	.846	1.293	1.665	1.991	2.376	2.641
78	.846	1.292	1.665	1.991	2.375	2.640
79	.846	1.292	1.664	1.990	2.374	2.639

TABLE 2 *t* DISTRIBUTION (*Continued*)

Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
80	.846	1.292	1.664	1.990	2.374	2.639
81	.846	1.292	1.664	1.990	2.373	2.638
82	.846	1.292	1.664	1.989	2.373	2.637
83	.846	1.292	1.663	1.989	2.372	2.636
84	.846	1.292	1.663	1.989	2.372	2.636
85	.846	1.292	1.663	1.988	2.371	2.635
86	.846	1.291	1.663	1.988	2.370	2.634
87	.846	1.291	1.663	1.988	2.370	2.634
88	.846	1.291	1.662	1.987	2.369	2.633
89	.846	1.291	1.662	1.987	2.369	2.632
90	.846	1.291	1.662	1.987	2.368	2.632
91	.846	1.291	1.662	1.986	2.368	2.631
92	.846	1.291	1.662	1.986	2.368	2.630
93	.846	1.291	1.661	1.986	2.367	2.630
94	.845	1.291	1.661	1.986	2.367	2.629
95	.845	1.291	1.661	1.985	2.366	2.629
96	.845	1.290	1.661	1.985	2.366	2.628
97	.845	1.290	1.661	1.985	2.365	2.627
98	.845	1.290	1.661	1.984	2.365	2.627
99	.845	1.290	1.660	1.984	2.364	2.626
100	.845	1.290	1.660	1.984	2.364	2.626
∞	.842	1.282	1.645	1.960	2.326	2.576

$df = \infty$ $\quad Z_{\frac{\alpha}{2}} = t_{\frac{\alpha}{2}}$ with $df = \infty$