

5.4 Binomial Probability distribution

* Discrete Probability distribution

* It deal with experiment called binomial experiment

Ex:- Tossing a coin 10 times $n=10$

Define X : # of Heads

$X = 0, 1, 2, \dots, 10$

Notice that :-

[1] This experiment consist of 10 identical trials $n=10$

[2] The trials are independent

[3] Each trial has two outcomes

Head : (success)

Tail : (Failure)

[4] In each trial :-

The probability of success $= P(\text{success})$ is Fixed
 $= P$

& the probability of failure $= P(\text{Failure})$ is fixed
 $= 1 - P$

This experiment is called a binomial experiment

In general : A binomial experiment satisfies

[1] The experiment consists of n identical trials
 n : Sample size $n \geq 2$

[2] The trials are independent

[3] Each trial has two outcomes
success : the one we study (depends on X)

failure : the complement (no success)

[4] In each trial :-

The probability of success = $P(\text{success})$ is fixed, denoted by P .

& The probability of failure = $P(\text{failure})$ is fixed $(1-P)$

Ex: 65% of BZU employees own private cars
We took a random sample 25 employees

Define X :- number of employees who own private cars.

Find $n = ??$ $P =$ $1-P =$

$X =$

How to find $P(x)$?

x : number of success

We use the formula:-

$$f(x) = P(x) = C_x^n P^x (1-P)^{n-x}$$

n : # of trials (sample size)

P : probability of success (if any trial)

Note: ① $P(0) + P(1) + \dots + P(n) = 1$

② $E(x) = \mu = nP$

③ $\text{Var}(x) = \sigma^2 = nP(1-P)$

④ Standard deviation of $x = \sigma = \sqrt{nP(1-P)}$

Ex:- The percentage of students who like early classes is 40%.

In a random sample of 8 students.

① What is the probability that 3 students like early classes?

② What is the probability that ALL of them like early classes?

③ What is the expected number of students who like early classes?

④ What is the variance = = = = = dislike early classes

⑤ What is the variance = = = = = like early classes

⑥ What is the prob. that at most 2 students like early classes

⑦ What is the prob. that at least 2 students like early classes

Exp:- let $X = B(6, \underline{0.4})$ $P = 0.4$
 $1-P = 0.6$

[1] Write the prob. function

[2] Find $E(x)$ $Var(x)$, $\sigma(x)$

[4] Find the prob. of ~~exactly~~ exactly 4 successes

[5] Find the prob of no more than (at most) 2 failures.

Solution:-

[1] $f(x) = C_x^6 (0.4)^x (0.6)^{6-x}$ $x = 0, 1, \dots, 6$

[2] $E(x) = n \cdot p = 6 \cdot 0.4 = 2.4$

[3] $Var(x) = n p \cdot (1-p) = 6 \cdot 0.4 \cdot 0.6 = 1.44$

[4] $f(4) = C_4^6 (0.4)^4 (0.6)^2 = 0.1382$

Calculator $[6 \text{ nCr } 4 \text{ } \times \text{ } 0.4 \text{ } \wedge \text{ } 4 \text{ } \times \text{ } 0.6 \text{ } \wedge \text{ } 2 =]$

[5] ~~At~~ At most 2 failures $\xrightarrow{\text{Mean}}$ At least 4 Success.

* Define a new Binomial variable with 6 trial

Prob. of success (0.6)

$$X = B(6, 0.6)$$

$$P(\text{At most 2 failures}) = f(2) + f(1) + f(0)$$

$$= C_2^6 (0.6)^2 (0.4)^4 + C_1^6 (0.6)^1 (0.4)^5 + C_0^6 (0.6)^0 (0.4)^6$$

A survey founded that one out of five Palestinians
say he or she visited a doctor last month.
If 10 people are ~~selected~~ selected at random, what is
the probability that none of them visited a doctor last ~~month~~
month

Solution:- X :- # of Palestinian visited a doctor last month

$$n = 10$$

$$p = \frac{1}{5}$$

$$1 - p = \frac{4}{5}$$

$$X = 0, 1, 2, 3, 4, \dots, 10$$

$$P(0) = C_0^{10} \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^{10}$$

$$= 0.1074$$