4.2 Events and their probabilities

Event: a collection of outcomes (sample point)

* Event is a subset of the sample space
* Usually we denote events by capital letter: A, B, C ...

Probability of an event $P(E)$ = The sum of the probability of the out comes in their event

Ex:- In Rolling a die $S = \{1, 2, 3, 4, 5, 6\}$

(1) let A be the event of even number

Find A and P(A)

(2) let B be the event of number less than or equal 5

Find B and P(B)

→ (1) **Solution:** $A = \{2, 4, 6\}$

$$P(A) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$$

→ (2) Solution $B = \{1, 2, 3, 4, 5\}$

$$P(B) = P(1) + P(2) + P(3) + P(4) + P(5) = \frac{5}{6}.$$

Note → $0 \leq P(A) \leq 1$ for any event A

Note → $\sum P(A) = 1$

Corollary $P(S) = 1$, since S = Sample space

$P(\emptyset) = 0$, since $\emptyset$ = empty set

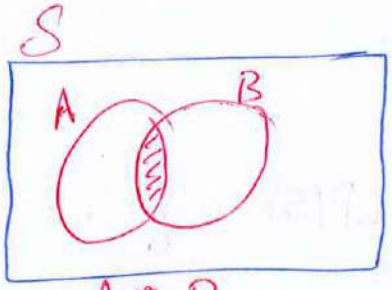
Recall: From Math 1351

Given a large set S and A, B subset of S

$A \subseteq S$

$B \subseteq S$

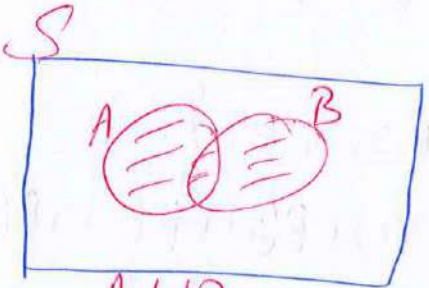
* Intersection: $A \cap B$: element in Both A and B
* Union: $A \cup B$: element in A or B (either in A or B or Both)
* Complement: A^c: element not in A



$A \cap B$

Both

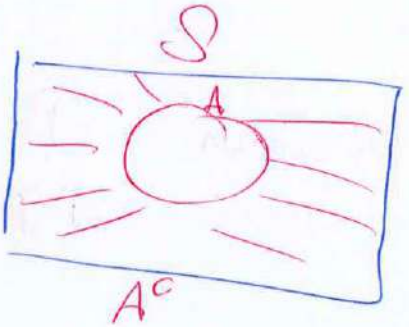
A and B



$A \cup B$

A OR B

either A or B



A^c

element not in A

4.3 & 4.4 Relationships in Probability

[1] Complement

[2] Joint Probability (Intersection)

[3] Union

[4] Conditional Probability

* Complement

If A is an event $\longrightarrow$ A^c : the sample point not in A

[A^c Means A does not occur]

Exp: A : event of sale $\longrightarrow$ A^c event of no sale

B : event of smoker $\longrightarrow$ B^c event of nonsmoker

$$P(A) + P(A^c) = 1 \Longrightarrow P(A^c) = 1 - P(A)$$

* Intersection :—

A, B events $\longrightarrow$ $A \cap B$: Sample points in A and B

$A \cap B$: Means A and B occurs at the same time

Exp: A : Male , B : smoker.

$A \cap B$ $\longrightarrow$ Males and smoker

* Union:

A, B event ⟶ $A \cup B$: Sample points in A or B or Both

$(A \cup B)$: Means the occurence of either A or B

Exp: A: male B: Smoker

$A \cup B$ ⟶ Male or Smoker

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Additional Law

Exp: In a Sample of 40 Student

30 are smoker 25 are males

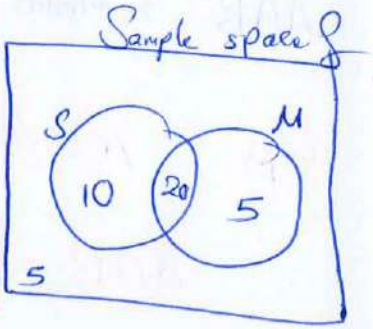
20 are both males and smoker

If we select a random student, What is the probability that he/she is either male or Smokers??

Sol: $P(M \cup S) = P(M) + P(S) - P(M \cap S)$

$= \frac{25}{40} + \frac{30}{40} - \frac{20}{40} = \frac{35}{40}$

$= 0.875$



Note: Approximation for four decimal (Probability)

التقريب لأربع منازل (للاحتمالات)

Mutually exclusive event

متبادلة الحدث

Def Two event A, B are called mutually exclusive if

$$A \cap B = \phi$$

$$P(A \cap B) = 0 \longrightarrow P(A \cup B) = P(A) + P(B)$$

that is A and B can't occur at the same time

Exp: P: Pass the exam
F: Fail the exam
$\longrightarrow$ P, F are mutually exclusive
$P(P \cap F) = 0$

Exp: $S: \{E_1, E_2, E_3, E_4\}$

Event: $A = \{E_1, E_2\}$ $B = \{E_2, E_3, E_4\}$ $C = \{E_1\}$

Suppose: $P(E_1) = 0.1$ $P(E_2) = 0.2$ $P(E_3) = 0.2$ $P(E_4) = 0.5$

Find ① $P(A) = 0.3$

$P(B) = 0.9$

$P(C) = 0.1$

② $P(A \cap B)$?? $\longrightarrow A \cap B = \{E_2\} \longrightarrow P(A \cap B) = 0.2 \neq 0$

$\longrightarrow$ A, B are not mutually exclusive

$P(A \cap C)$?? $\longrightarrow A \cap C = \{E_1\} \longrightarrow P(A \cap C) = 0.1 \neq 0$

$\longrightarrow$ A, C are not mutually exclusive

$P(B \cap C)$?? $\longrightarrow B \cap C = \phi \longrightarrow P(B \cap C) = 0$

$\longrightarrow$ B, C are mutually exclusive

③ $P(A \cup B)$?? $\longrightarrow$ $A \cup B = S \longrightarrow P(A \cup B) = 1$

OR

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.3 + 0.9 - 0.2 = 1$$

④ $P(A \cap B)^c = ??$

$$P(A \cap B)^c = 1 - P(A \cap B)$$

$$= 1 - 0.2 = 0.8$$

⑤ $P(A^c \cup B^c) = 0.8$

Since De Morgan's Law :-

$$A^c \cup B^c = (A \cap B)^c$$

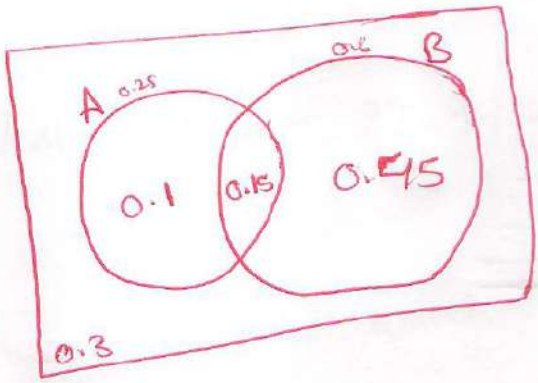
$$P(A^c \cup B^c) = P(A \cap B)^c.$$

⑥ $P(S) = 1$

4.4 Conditional probability :-

Exp:- let A, B two events such that $P(A) = 0.25$, $P(B) = 0.6$, $P(A \cap B) = 0.15$

1. Find $P(A \cup B)$
2. Find $P(A|B)$
3. Find $P(B|A)$
4. Are the event A, B independent, Why?
5. Are the event A, B mutually exclusive, why?
6. $P(A|B')$
7. $P(B'|A)$



Solution :-

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.25 + 0.6 - 0.15$
$P(A \cup B) = 0.7$

2. $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.15}{0.6} = \boxed{0.25}$

3. $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.15}{0.25} = \boxed{0.6}$

6. $P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A - B)}{0.4} = \frac{P(A) - P(A \cap B)}{0.4} = \frac{0.1}{0.4} = \boxed{0.25}$

7. $P(B'|A) = \frac{P(B' \cap A)}{P(A)} = \frac{P(A - B)}{P(A)} = \frac{0.1}{0.25} = \boxed{0.4}$

4. Yes, A, B : Independent event since $P(A|B) = P(A)$

5. No, A, B : not mutually exclusive since $P(A \cap B) \neq 0$

Exp:- The follow Crosstabulation summarizes the gender and Religion of a sample of 400 people

Religion / Gender	Muslim	Christon	Jewish	Total
Male	20	80	40	140
Female	10	90	160	260
Total	30	170	200	400

If we select a person randomly.

[1] What is the probability that she/he is a Muslim?

$$P(\text{Muslim}) = \frac{30}{400} = 0.075$$

$$P(\text{Christon}) = \frac{170}{400} = 0.425$$

$$P(\text{Jewish}) = \frac{200}{400} = 0.5$$

notice Sum = 1

$$P(\text{Male}) = \frac{140}{400} = 0.35$$

$$P(\text{Female}) = \frac{260}{400} = 0.65$$

notice Sum = 1

[2] What is the probability that a person is Muslim and Male

$$P(\text{Muslim} \cap \text{Male}) = \frac{20}{400} = 0.05$$

$$P(\text{Jewish} \cap \text{Female}) = \frac{160}{400} = 0.4$$

[3] What is the probability that a person is Muslim or Male

$$P(\text{Muslim} \cup \text{Male}) = \frac{150}{400} = 0.375$$

[4] What is the probability that a person is male given that he is christion??

$$P(\text{Male} \mid \text{Christion}) = \frac{80}{170} = 0.4706.$$

[5] If a person is female, What is the probability that she is Muslium

$$P(\text{Muslim} \mid \text{female}) = \frac{10}{260} = 0.0385.$$

[6] Are the varable: Religion, gender motually exclusive?? Why??

check any Joint probability

Take $P(\text{Muslim} \cap \text{Male}) = 0.05 \neq 0$

$\longrightarrow$ not motually exclusive

[7] Are the variable: Religion, gender independent? Why??

check any conditional probability (OR Joint)

$P(\text{Male}) \neq P(\text{Male} \mid \text{christion})$

$0.35 \neq 0.4706$ $\longrightarrow$ Variables are Not Independent

Extra question :-

A survey about a sample of B.ZU Students is summarized below :-

	Watch Horror Movies		
weight	Yes	No	Total
50 – 65	6	8	14
66 – 81	8	10	18
82 – 97	9	5	14
More than 97	12	2	14
Total	35	25	60

(1) What is the prob. that student weight 82-97 kg

(2) What is the prob. that a student does not watch horror movies ??

(3) What is the prob. that a student weight more that 65kg?

(4) What is the prob. that a student watchs horror movies and weights less than 82 kg ?

(5) If a student watchs horror movies, what is the Prob. that she/he weights more than 97

(6) Are the variables Independent, Explain your answer ?