Chapter 4 :-

4.1 Experiments, Counting Rules, and assigning Problem

Experiment : a process that generates outcomes (Sample point)

Sample space :- The set of all outcomes of an experiments denoted by S

$$S = \{E_1, E_2, E_3, \dots\ E_n\} = \begin{cases} E_1 : \text{First outcomes} \\ E_2 : \text{Second outcomes} \\ \vdots \end{cases}$$

Examples :-

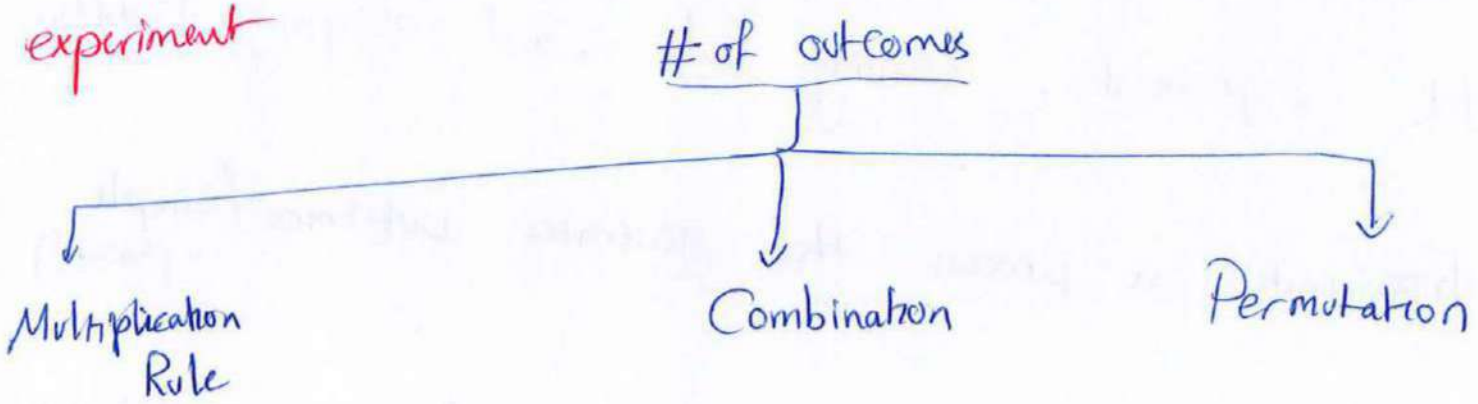
Experiment	outcomes	Sample space (S)	# of total outcomes
Toss a coin	Head, Tail	$S = \{H, T\}$	2
Roll a die	1, 2, 3, 4, 5, 6	$S = \{1, 2, 3, 4, 5, 6\}$	6
play a football	win, lose, tie	$S = \{win, lose, tie\}$	3

Multiple-step experiment :- experiment with more than one step.

Exp : * Tossing two coins (Two step)

* Rolling two dice and Five coins (seven step).

How to Find # of total outcomes of a multiple_step experiment



* Multiplication Rule

If an multiple step experiment ~~Consit~~ Consists k-steps

with step 1 has n_1 outcomes

step 2 has n_2 outcomes

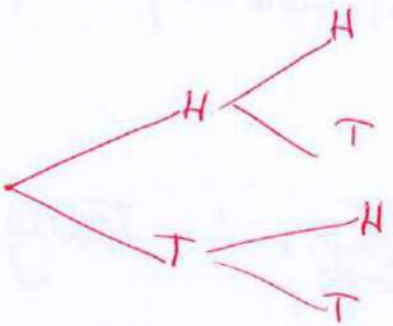
⋮

step k has n_k outcomes

Then # of outcomes = $n_1 \times n_2 \times \cdots \cdots n_k$

Exp:- In Tossing two coins, Find the total number of outcomes, Then Find the sample space

Sol. Step 1 / 2 Step 2 / 2



# of outcomes = $2 \times 2 = 4$

$S = \{HH, HT, TH, TT\}$.

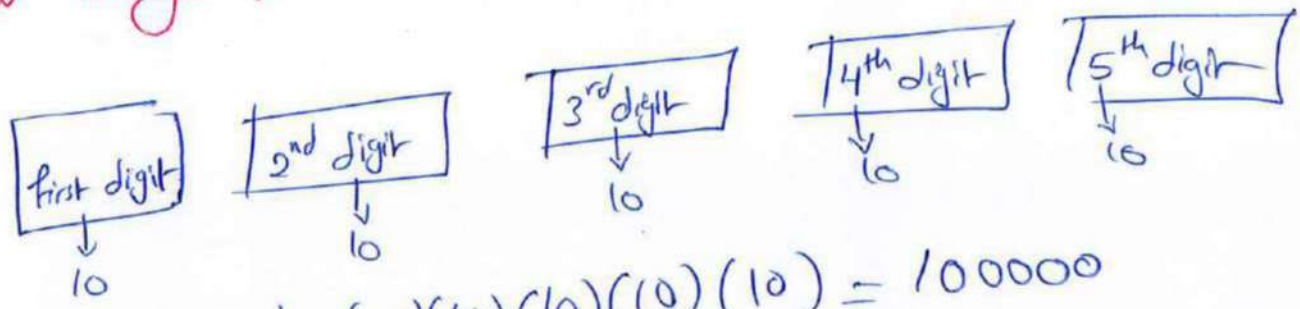
Exp: Tossing three dice and four coins
What is the total number of outcomes

Sol.

Step 1	Step 2	Step 3	Step 4	Step 5	Step 6	Step 7
6	6	6	2	2	2	2

# of outcomes $= 6 * 6 * 6 * 2 * 2 * 2 * 2 = 3456$

Exp: If we want to create a password of five numbe
How many password can you form??



# of password $= (10)(10)(10)(10)(10) = 100000$

Exp: If we want to create a password of five numb
How many password can form if the first digit can't be zero??

# of password $= (9)(10)(10)(10)(10) = 90000$.

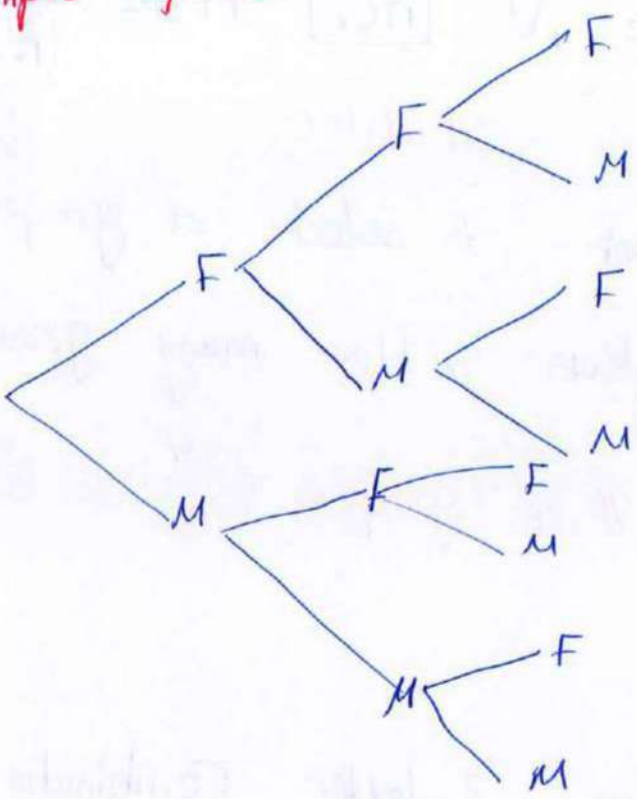
Tree diagram:- A graphical tool used to find the outcomes of a multiple step experiment

Exp:- Suppose we randomly select three worker from a company and observe whether the worker selected each time is a male M or Female F

∴ ① Find the # of outcomes

answer : # of outcomes = 8

② Find the sample space



$S = \{FFF, FFM, FMF, FMM, MFF, MFM, MMF, MMM\}$.

* Combination: Selecting n objects from a Larger set of N object, where

(i) repetition is not allowed

(ii) Order is not important

The # of outcomes C_n^N

where C_n^N can be found using calculator as follow

$$N \boxed{nCr} n = \frac{N!}{(N-n)!\, n!}$$

calculator — formula

Exp: If we want to select a group of two student from four student, How many group we can form??

Answer: # of group $= C_2^4 = 6$ group.

Exp: How many 3-letter Combination can be formed from the english alphabetical ??

# of Combination $= C_3^{26} = 2600$ Combination

** Permutation :- Selecting n objects from a Large set of N object where

(i) repetition is not allowed

(ii) Order is important

The # of permutation $P_n^N = \frac{N!}{(N-n)!}$

Calculator : N [shift] [nCr] n [=]

Exp: We want to form a group of two student [consists of a president and vice president] from four student, How many group we can form??

Sol # of groups = $P_2^4 = 12$

4 [shift] [nCr] 2 [=]

Exp:- In how many way can we arrange 3 students in a row of 5 chairs ??

Sol # of outcomes = $P_3^5 = \frac{5!}{2!} = (5)(4)(3) = 60$ Choices

Assigning Probability

Probability : A numerical measure of the likelihood that an event will occure.

$$0 \leq \text{Probability} \leq 1$$

Two requirment for assigning prob:-

① $0 \leq P(E_i) \leq 1$

② $\sum_{i=1}^{n} P(E_i) = 1$

** Three Method to assign probability

(1) classical Method

(2) Relative Frequency Method

(3) Subjective Method

→① classical Method :-

Condition:- All outcomes are equally likely

$$P(E_1) = P(E_2) = \text{---} \quad P(E_n) = \frac{1}{n}$$

Exp: Roll a die:

** Sample space = $S = \{1, 2, 3, 4, 5, 6\}$

# of outcome = 6

$P(1) = P(2) - - - - - - = P(6) = \frac{1}{6}$

Notice: $0 \leq P(E_i) \leq 1$ & $\sum_{i=1}^{6} P(E_i) = 1$

→ (2) Relative Frequency Method

Probability = R.F = $\frac{F}{n}$

Exp:- If we select a student randomely what is the probability that his blood type is A ??

Blood type	Freq.
A	10
B	15
O	20
AB	5
Total	50

-Answer $P(A) = \frac{10}{50} = 0.2$

→ (3) Subjective Method:-

Probability assigned based on experience or belief

Exp: I believe that the probability that Corona crisis will end soon is 0.2.