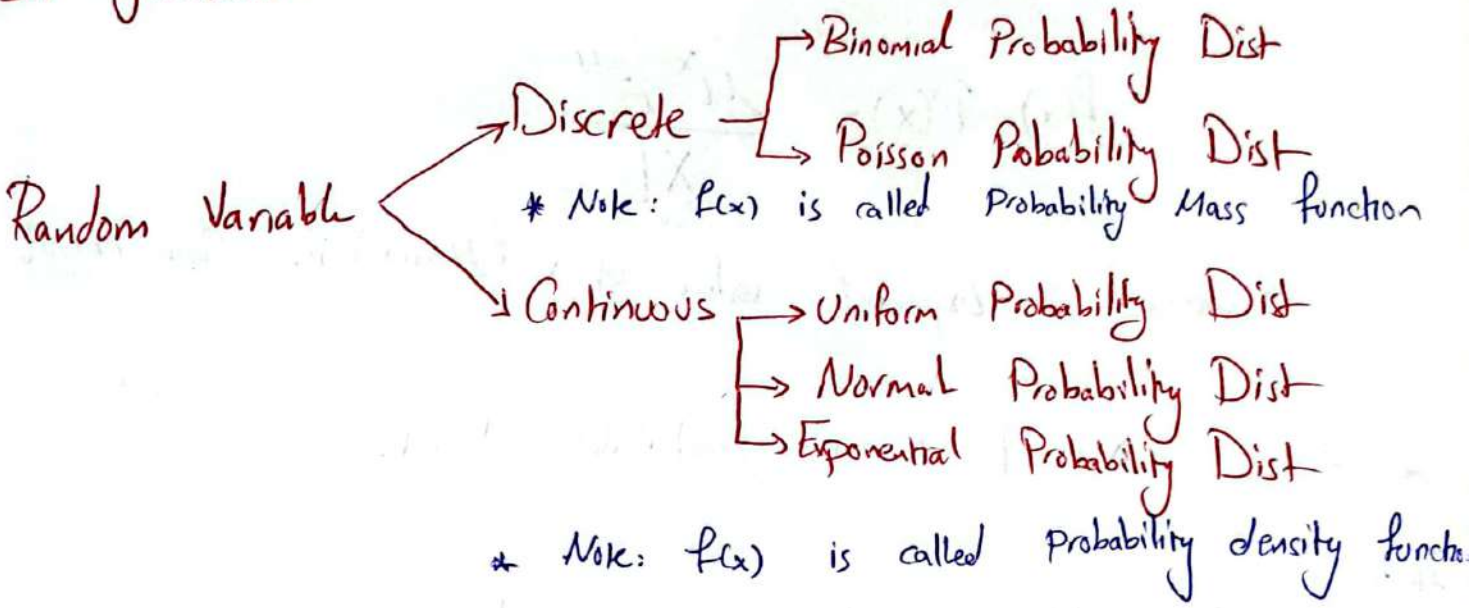
Ch. 5 Random Variable

5.5 Poisson Probability Distribution

In general-




5.5 Poisson Probability Distribution

* Discrete prob. Dist.

* X :- number of something in an interval (time, space).

Exp: X: # of cars arrive a car wash in a day

X: # of calls received by Jawwal Center per hour

X: # of Rita's visitors every 15 minute

* If X has a Poisson Distribution then

.. $E(X) = \mu$

.. $Var(X) = \sigma^2 = \mu$ .. Std. dev. of $X = \sigma = \sqrt{\mu}$

Note: $X = 0, 1, 2, \ldots$ "Discrete" finite number

How to find $f(x)$ in Poisson Dist ??

$$f(x) = P(x) = \frac{\mu^x e^{-\mu}}{x!}$$

where $\mu$ : Expected value of x (Mean) in Some interval

* Since X is Poisson Probability Dist.

then

(1) $P(0) + P(1) + P(2) + \cdots = 1$

(2) $\mu$ changes according to the interval

Exp If the Mean of calls per hour is 100

$\Longrightarrow \mu = 100$ (Per hour)

$\Longrightarrow \mu = 2400$ (Per day)

$\Longrightarrow \mu = 1.67$ (Per minute)

** If X has a poisson dist. ~~Then~~ Then

.. $E(x) = \mu$

.. $\text{Var}(x) = \sigma^2 = \mu$

.. Std. dev. of $x = \sigma = \sqrt{\mu}$

**Exp:** The number of Ritaj's visitor has poisson Dist. with an average of 10 visitors per minute

**Sol.** Define X : # of Ritaj's visitor

$X = 0, 1, 2, 3, \dots$

$\mu = 10$ (Per min)

① What is the Probability that 6 will ~~be~~ visit ritaj in a minute ?

$$P(X) = \frac{\mu^x e^{-\mu}}{X!}$$

$\mu = 10$ (Per min)

$X = 6$

$$P(6) = \frac{10^6 e^{-10}}{6!}$$

10 [^] 6 [x] [Shift] [ln] -10 [÷]

6 [Shift] [$x^{-1}$] [=]

$= 0.0631$

② What is the probability of no visitor in 30 second

$\mu = 5$ (Per 30 second)

$X = 0$

$$P(0) = \frac{5^0 e^{-5}}{0!}$$

$= 0.0067$

(3) what is the expected number of visitor in an hour

Expected number $= \mu = 600$ (per hour)

(4) What is the standard deviation of visitor in a d

Std. deviation $= \sigma = \sqrt{\mu} = \sqrt{14400} = 120$

(5) what is the probability at most one visitor in a minute

$\mu = 10$ (per minute)

$x = 0, 1$

$P(X \leq 1) = P(0) + P(1)$

$= \frac{10^0 e^{-10}}{0!} + \frac{10^1 e^{-10}}{1!}$

$= 0.0005$

(6) What is the probability at least 2 visitor in a minute

$\mu = 10$ (per Min)

$X = 2, 3, 4, \ldots\ldots$

$P(X \geq 2) = P(2) + P(3) + \ldots$

$= 1 - [P(0) + P(1)]$

$= 1 - 0.0005$

$= 0.9995$