Chapter 5 : Random variable

Random Variable :- is a numerical discription of the out comes of an experiment, usually denoted by X

Random Variable → Discrete
Random Variable → Cont.

Discrete random variable : assume finit number of values or infinite sequence of values (usually no decimal)

Experiment	Random variable X	Value of X
Tossing three coins	number of H	0, 1, 2, 3
20 questions exam	number of questions answered correctly	0, 1, 2, ... — 20
Inspect 20 radios	number of defective radios	0, 1, 2, ... — 20

Continous random variable : assumes numerical Values in an interval

Example : Age ($20 \leq x \leq 30$), time ($t \leq 3$), weight ($50 \leq x \leq 60$), distance, temperature ($x \geq 0$)

Area, volume.

Exp:- In the experiment of tossing two coins

Random Variable X - number of Heads

Total outcome = $2.2 = 4$ "Multiplication Rule"

$S = \{HH, HT, TH, TT\}$

Define X = number of Heads observed

$HH \longrightarrow 2$

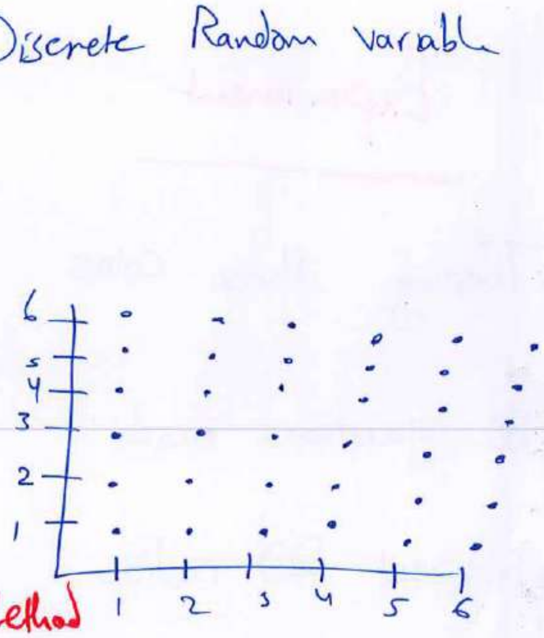
$TH \longrightarrow 1$

$HT \longrightarrow 1$

$TT \longrightarrow 0$

$X = \{0, 1, 2\}$ Discrete Random variable

Exp: Rolling 2 dice :-



Total outcomes : 36

Sample space = $\{(1,1), \ldots (6,6)\}$

$P(1,6) = \frac{1}{36}$ $\quad P(6,3) = \frac{1}{36}$ Classical Method (equally likely)

Define : Random Variable X : Sum of the two faces

$(1,1) \longrightarrow 2$

$(2,3) \longrightarrow 5$

$(6,6) \longrightarrow 12$

$X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

5.2 , 5.3 Discrete Probability Distribution

التوزيعات الاحتمالية المنفصلة .

Each random Variable X has a probability distribution

* Probability distribution :- describes how prob. are distributed over the value of X.

* Probability function $f(x) \equiv P(X=x)$ defines the probability distribution of X

→ الرسم

* $f(x)$ can be formula, table, graph.

Two condition for a probability function $f(x)$

* $0 \leq f(x) \leq 1$, $\forall x$

* $\sum_{i=1}^{n} f(x_i) = 1$ $\quad \underline{X} = \{x_1, x_2, x_3 \text{---} x_n\}$

Exp:- Tossing two Coins:- $\# S = 4$

$S = \{HH, HT, TH, TT\}$

let X = number of tails → $X = \{0, 1, 2\}$ Discrete

X=0 : when outcomes is (H, H) → $P(0) = \frac{1}{4} = f(0) = P(x=0)$

X=1 : when outcomes is (H,T) (TH) → $f(1) = P(X=1) = P(1) = \frac{2}{4}$

X=2 : when outcomes is (TT) → $f(2) = P(X=2) = P(2) = \frac{1}{4}$

$f(0) = P(X=0) = 0.25$

$f(1) = P(X=1) = 0.50$

$f(2) = P(X=2) = 0.25$

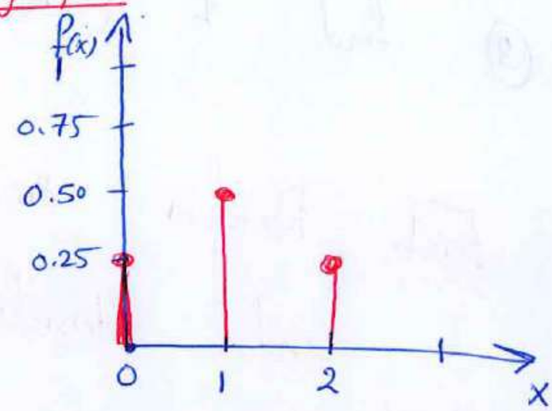
Notice:

(1) $0 \leq f(x) \leq 1$ for $x = 0, 1, 2$

(2) $\sum f(x) = f(0) + f(1) + f(2)$

$= 1$

* We can represent $f(x)$ as a graph:-



Exp: The following freq table represent # of test conducted on Sample of 50 people.

# of Test	Freq.
1	10
2	15
3	5
4	20
Total	50

(1) construct the probability dist.

(2) Find $f(X \leq 4)$

(3) Sketch $f(x)$.

Exp: Consider the following prob. dist. for the random variable x

X	-4	-2	-1	0	3
$f(x)$	0.05	0.15	0.2	0.25	0.35

① Find $f(0)$

② find $f(x<0)$

③ find $f(-4<X\leq 0)$

Each Random variable X has a mean, variance and standard deviation

① Expected value of X (Mean) $= E(X) = \mu = \sum x\, f(x)$

② Variance of $X = Var(x) = \sigma^2 = \sum (x-\mu)^2 f(x)$

③ Standard deviation of $X = \sigma$

** We will use the SD Mode to find them

"We deal with it as weighted mean".

Exp: The probability distribution below summarizes the # of cars sold in a week.

X	4	8	10	12	15
$P(X) = f(x)$	0.21	0.18	0.12		0.19

(1) Find $P(12)$ $\longrightarrow$ $P(12) = 1 - 0.70 = 0.30$

(2) Find $P(X < 10)$ $\longrightarrow$ $P(X<10) = P(4) + P(8) = 0.39$.

(3) Find $P(8 \leq X < 15)$ $\longrightarrow$ $P(8 \leq X < 15) = P(8) + P(10) + P(12) = 0.6$

(4) Find the expected value of X $\longrightarrow$ $\overset{E(x)}{\mu = 9.93}$

(5) Find the standard deviation of X $\longrightarrow$ $\sigma = 3.77$

(6) Find Variance of X $\longrightarrow$ $\overset{var(X)}{\sigma^2 = 14.23}$

$E(X) = \mu = 9.93$ cars : We expect to sell 9.93 cars in a week $\approx$ 10 cars.

Let the probability distribution below summarizes the # of cars sold in a week.

X	4	8	10	12	15
P(x)	0.21	0.18	0.12	.	0.19

① Find P(12) → P(12) = 1 − 0.70 = 0.30

② Find P(X < 10) → P(X < 10) = P(4) + P(8) = 0.39

③ Find P(8 ≤ X < 15) → P(8 ≤ X < 15) = P(8) + P(10) + P(12) = 0.60

④ Find the expected value of X → E(X) μ = 11.1

⑤ Find the standard deviation of X → σ = 1.7

⑥ Find Variance of X → σ² = 11.23

E(X) = μ = 9.73 cars; we expect to sell 9.73 cars in a week ≈ 10 cars.