Ch.8 : Interval Estimation

Recall :- Statistical Inference :- "Inferential Statistic"

means using Sample to predict (estimate) a characteristic of population.

* Parameter :- a numerical measure of population [Ex: μ, σ]

* Statistic :- a numerical measure of sample [Ex: $\bar{x}, s$]

* Two type of Inferential Statistic :-

  * Estimation (Ch.8) $\rightarrow$ Point estimator

    $\rightarrow$ Interval Estimation "Confidence Interval"

  * Hypothesis (Ch.9)

Point Estimator :-

- $\overline{X}$ is a point estimator of μ

$$(\mu \approx \overline{X})$$

True exact Average of pop.   estimate

- S is a point estimator of σ

$$(\sigma \approx S)$$

8.1 / 8.2 Interval Estimation

* we will use the sample to construct an interval estimation for the population (True) mean μ.

* General from of an interval Estimate of μ is

$$\boxed{\overline{X} \pm E = [\overline{X} - E, \overline{X} + E]}$$

where $\overline{X}$: Sample mean

E : Margin of error

<u>Note:</u> The interval we compute is called "confidence interval" because it has a confidence level

Exp: If $\boxed{\bar{X} = 10}$ & $\boxed{E = 2}$

and the confidence level is 95%, then

a 95% confidence interval for μ is :-

Sol. $10 \pm 2 = \underbrace{[8, 12]}_{}$

CI = 95% confidence Interval for μ.

which means that we are 95% confident that the true Mean μ is between 8 and 12

that is there is 5% Chance that we are Wrong

(5% is called ~~Confe~~ Significance level")

* We denote confidence level :- $1-\alpha$

we denote Significance level :- α

The most common confidene level are 90%, 95%, 99%.

" " " significance level are 10%, 5%, 1%.

99% confidence level $\longrightarrow$ $1-\alpha = 0.99$, $\alpha = 0.01$

95% confidence level $\longrightarrow$ $1-\alpha = 0.95$, $\alpha = 0.05$

97% confidence level $\longrightarrow$ $1-\alpha = 0.97$, $\alpha = 0.03$

How to find E ??

case 1: σ: unknown (we use s instead)

Case 2: σ known (from previous or historical data)

* case 1:- σ Unknown:-

$$E = \frac{t_{\frac{\alpha}{2}}\, s}{\sqrt{n}},$$

s: Sample standard deviation

n: Sample size.

$t_{\frac{\alpha}{2}}$: t value from t distribution

t-distribution:— a probability distribution look like the normal Dist.

To find $t_{\frac{\alpha}{2}}$ we need $\frac{\alpha}{2}$, df, t-table

df = degree of freedom, $\frac{\alpha}{2}$ = Area in upper tail

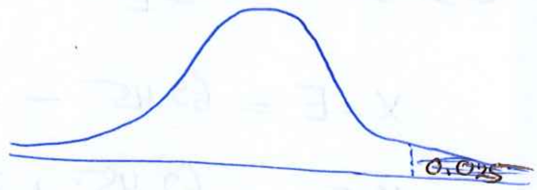
Exp: If confidence level 90% and $n = 16$

$\longrightarrow 1-\alpha = 0.90 \longrightarrow \alpha = 0.10 \longrightarrow \frac{\alpha}{2} = 0.05$

$\longrightarrow df = n-1 = 15$

df \ $\alpha/2$	0.2	0.1	0.05	0.025	----
1					
2					
⋮					
14					
15			1.753		
16			$t_{\frac{\alpha}{2}}$		

Exp: 95% $n=30$



$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$df = 29$

$t_{\frac{\alpha}{2}} = t_{0.025} = 2.045$

Exp: To estimate the weight of B.Z.U Students we took a sample of 50 student

The sample mean 62.45 and the sample standard deviation was 14.80

(1) Construct a 90% CI for the true Mean weight

(2) = = 95% CI = = = = = Interpret your answer

(3) = = 99% CI = = = = = Interpret your answer

Sol. $n=50 \longrightarrow df=49$

(1) $\bar{X} = 62.45$, $S = 14.80$ (σ unknown)

$\frac{\alpha}{2} = 0.05$

$df = 49$

$t_{\frac{\alpha}{2}} = 1.677$

$1-\alpha = 0.90 \longrightarrow \alpha = 0.10 \longrightarrow \frac{\alpha}{2} = 0.05$

$E = \frac{t_{\frac{\alpha}{2}} S}{\sqrt{n}} = 3.51$

CI :- $\bar{X} \pm E$

$X - E = 62.45 - 3.51 = 58.94$

$X + E = 62.45 + 3.51 = 65.96$

90% CI is = $[X - E, X + E] = [58.94, 65.96]$

Interpret :— We are 90% confident that the mean weight (μ) is between 58.94 and 65.96 kg

(2) 95% CI is $[X - E, X + E]$

$t_{\frac{\alpha}{2}} = 2.010$
because
$\frac{\alpha}{2} = 0.025$
$df = 49$

$E = \frac{t_{\frac{\alpha}{2}} S}{\sqrt{n}} = 4.21$

95% CI = $[58.24, 66.66]$

We are 95% confident that mean weight for pop. is between 58.24 and 66.66 kg

(3) 99% CI is $[56.84, 68.06]$

we are 99% confident that True mean is between 56.84 and 68.06 kg

Note

(1) When confidence level $\uparrow \rightarrow E \uparrow \rightarrow$ CI become wider

(2) When $n \uparrow \rightarrow E \downarrow \rightarrow$ CI become shorter

* Case 2:- σ known :-

$$E = \frac{Z_{\frac{\alpha}{2}}\, \sigma}{\sqrt{n}}$$

σ: population std. dev

n: sample size

$Z_{\frac{\alpha}{2}} = t_{\frac{\alpha}{2}}$ with $df = \infty$

Exp. For 99% confidence level, Find $Z_{\frac{\alpha}{2}}$

$\alpha = 0.01$, $\frac{\alpha}{2} = 0.005$

$df = \infty$

$Z_{\frac{\alpha}{2}} = 2.576$

Exp: To estimate the weekly time spent on facebook by students, we took a sample of 65 student

Assume $\bar{X} = 34$ and $\sigma = 8.5$

Find 95% confidence Interval for the true mean.

$n = 65 \longrightarrow df = 64$

$\bar{X} = 34$

$\sigma = 8.5$ (σ known)

$$E = \frac{Z_{\frac{\alpha}{2}}\, \sigma}{\sqrt{n}}$$

$$E = \frac{Z_{\frac{\alpha}{2}} \sigma}{\sqrt{n}} = \frac{(1.960)(8.5)}{\sqrt{65}}$$

$$= 2.07$$

to find $Z_{\frac{\alpha}{2}}$

$df = \infty$

$\frac{\alpha}{2} = 0.025$

$Z_{\frac{\alpha}{2}} = 1.96$.

95% C.I = $[31.93, 36.07]$

We are 95% confident that true mean of weekly time spent on facebook is between 31.93 and 36.07.

8.3 Determining the sample size.

* Given a describe margin of error E, we want to find the (Minimum) Sample size needed to achieve this error E for some confidence level

$$n = \left(\frac{Z_{\frac{\alpha}{2}} \sigma}{E} \right)^2$$

E: margin of Error

$Z_{\frac{\alpha}{2}} = t_{\frac{\alpha}{2}}$ with $df = \infty$

σ: planning value for σ

How??

σ could be

$\longrightarrow \sigma$ (Historical Data)

$\longrightarrow \sigma$ (guessed / estimated)

$\longrightarrow$ Range/4 (Range is given)

$\longrightarrow S$ (pilot study)

Exp:- Find the minimum sample size needed to achieve 95% confidence level with $E = 10, \sigma = 40$

Sol.

$E = 10$

$\sigma = 40$

$1 - \alpha = 0.95 \longrightarrow \alpha = 0.05 \longrightarrow \frac{\alpha}{2} = 0.025$

$Z_{\frac{\alpha}{2}} = 1.96$

$$n = \left(\frac{Z_{\frac{\alpha}{2}} \sigma}{E} \right)^2 = 61.47$$

round up

minimum number of Sample size $\boxed{n = 62}$

Exp: At 99% confidence, How large a sample size would provided a margin of error of 2

Assume range is 36

Sol $E = 2$

$$\sigma = \frac{Range}{4} = 9$$

$$Z_{\frac{\alpha}{2}} = 2.576$$

$$n = \left(\frac{Z_{\frac{\alpha}{2}}\sigma}{E}\right)^2 = 134.37$$

Sample size $= n = 135$

$1-\alpha = 0.99$

$\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

$Z_{\frac{\alpha}{2}} = 2.576$