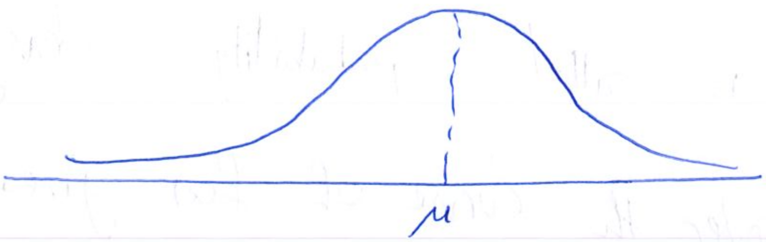
6.2 Normal Distribution



Proparties :-

① Most comman and widely used in application such as weight, Age, price, grades

② X: Contincous Random variable ⟶ $f(x)$: continuous density function

③ It has a bell shaped Dist

④ Any normal Dist is determined by μ and σ

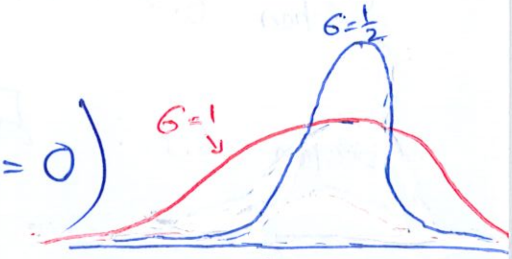
$N(\mu, \sigma)$

⑤ Mean = Median = Mode (at the center

⑥ σ determine the flatness of the curve (variation)

$\sigma \uparrow \longrightarrow$ wider

⑦ Symmetric about μ (skewness = 0)



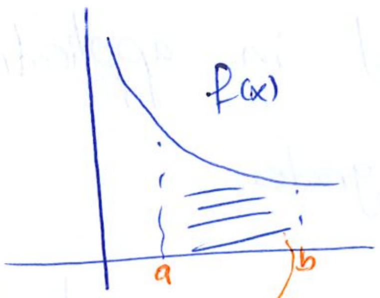
⑧ Probability = $f(x)$ = area under the Normal Dist

Ch 6 Continuous Probability Distribution

* We will deal with continuous prob. Dist

* f(x) = is called probability density function P.D.F

* Area under the curve of f(x) gives the Probability.



Area = probability between a and b $= \int_a^b f(x)\,dx$

* Total Area under curve = 1

* $P(x=a)=0$

* $P(x \geq a) = P(x > a)$

* $P(x \leq a) = P(x < a)$

Notes Probability (Area) ~~under~~ at a particular value of x is Zero

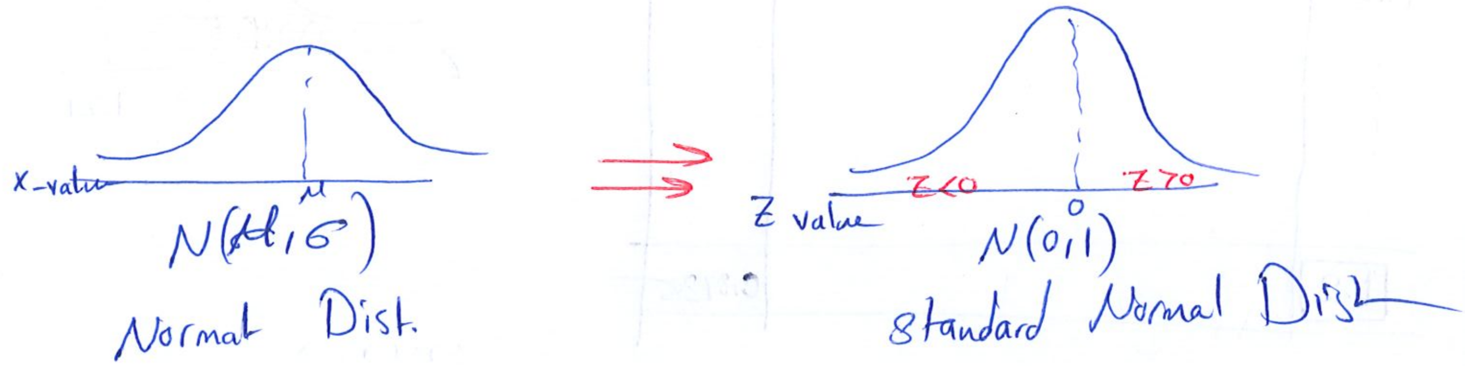
Section 6.1 : Uniform Continuous probability Dist : Skip

Section 6.2 : Normal continuous Prob. Dis : very important

Section 6.3 : Exponential Prob. Dist : Skip

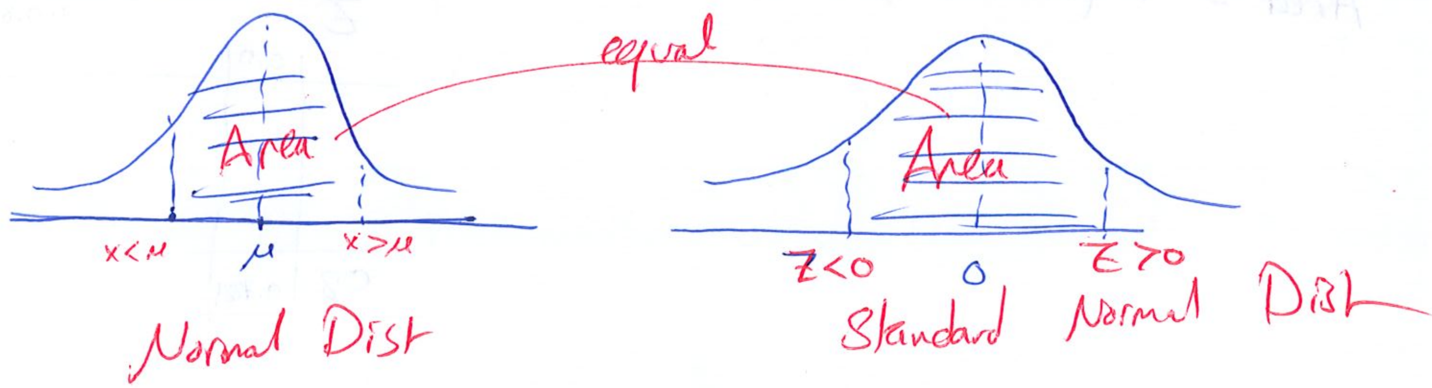
* Standard Normal Dist.

- It's a normal Distribution with $\mu=0$ and $\sigma=1$
- It's the Distribution of the Z score.



Recall $x_i \rightarrow Z_i$

$$Z_i = \frac{X_i - \mu}{\sigma}$$



* We will Find probability [Area under the curve of standard Normal Distribution By using Z table

Z table

It only evaluate area below a positive Z value

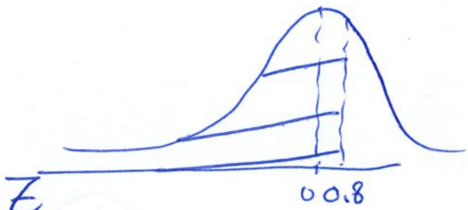
Exp① $P(Z < 1.24) = P(Z \leq 1.24)$

by using Z table $\longrightarrow$ Area below $Z = 1.24 \longrightarrow 0.8925$

	0.00	0.01	0.02	0.03	$\boxed{0.04}$
0.0					
⋮					
$\boxed{1.2}$					0.8925

Exp②: Find the area below (to the left) of $Z = 0.8$

$\text{Area} = P(Z < 0.8) = 0.7881$

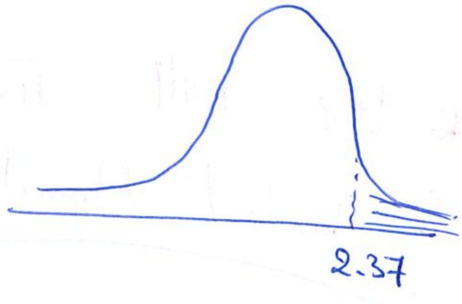


	0.0
0.8	0.7881

Exp③ $P(Z > 2.37) = 1 - P(Z < 2.37)$

$= 1 - 0.9911$

$= 0.0089$



$P(Z > a) = 1 - P(Z < a)$

Exp④ $P(Z \geq 0) = 0.5$

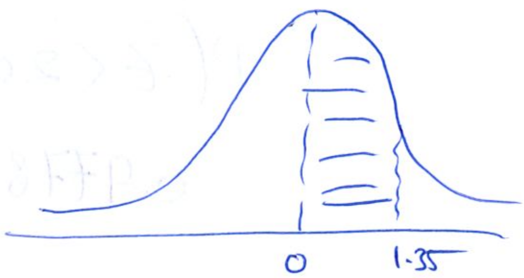
$P(Z \leq 0) = 0.5$

Exp. 5 $P(0 < Z < 1.35)$

$P(Z < 1.35) - P(Z < 0)$

$= 0.9115 - 0.5$

$= 0.4115$

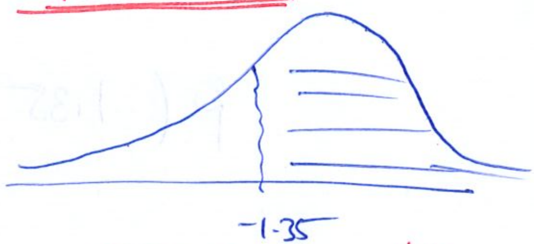


* To solve the following question, we use Symmetry and the fact Total Area = 1

Exp 6 $P(Z > -1.35)$

$= P(Z < 1.35)$

$= 0.9115$



Note

$P(Z > -a) = P(Z < a)$

$P(Z < -a) = P(Z > a)$

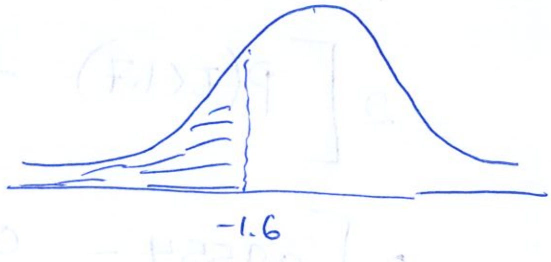
Exp 7 $P(Z < -1.6)$

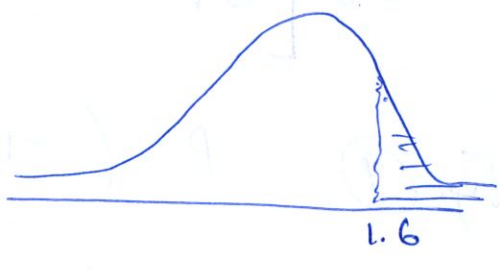
$= P(Z > 1.6)$

$= 1 - P(Z < 1.6)$

$= 1 - 0.9452$

$= 0.0548$





Exp 6 $P(1.35 < Z < 2.01)$

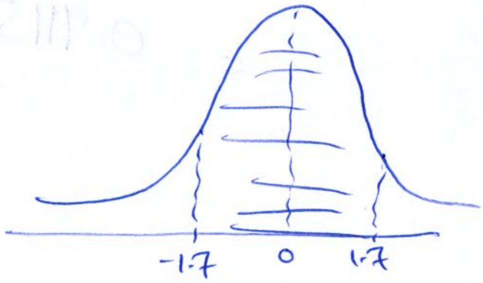
$P(Z<2.01) - P(Z<1.35)$

$0.9778 - 0.9115 = 0.0663$

Exp 7 $P(-2.01 < Z < -1.35)$

$P(1.35 < Z < 2.01) = 0.0663.$

Exp. 8 $P(-1.7 < Z < 1.7)$

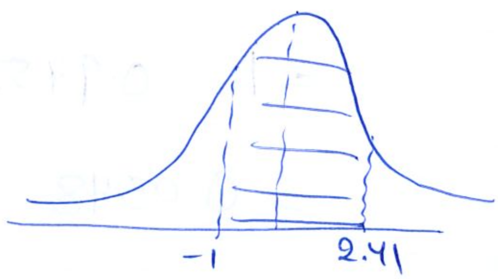


$2P(0 < Z < 1.7)$

$= 2\left[P(Z<1.7) - P(Z<0)\right]$

$= 2\left[0.9554 - 0.5\right] = 0.9108$

Exp. 9 $P(-1 < Z < 2.41)$



$= \underbrace{P(Z<2.41)}_{\text{from table}} - \underbrace{P(Z<-1)}_{\text{from } P(Z>1)\ [1-P(Z<1)]}$

$= 0.9920 - [1 - 0.8413] = 0.8333.$

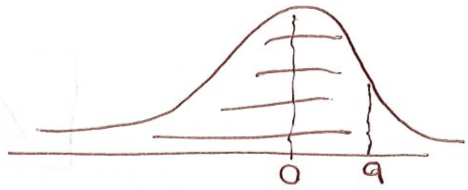
Now :- We will find Z values if area is given

Exp: Find the Z value(s) in the following cases

(1) Area to left of Z is 0.8980

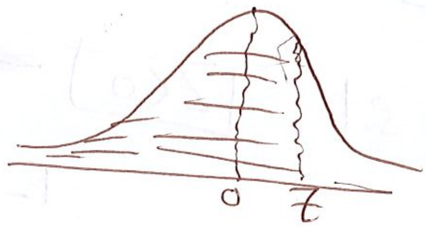
Solution : In this case $Z > 0$ since Area > 0.5

$P(Z < a) = 0.8980$



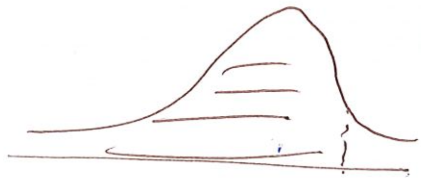
$\longrightarrow \boxed{Z = a = 1.27}$

(2) Area to left of Z is 0.7385



Sol. $Z > 0$ closest area is 0.7389

$\boxed{\longrightarrow Z = 0.64}$



(3) Area to left of Z is 0.9

Sol. $Z > 0 \longrightarrow$ closest area is 0.8997

$\boxed{Z = 1.28}$

		0.08	0.09
1.2		0.8997	0.9015

(4) Area to left of Z is 0.9540

Sol: $Z > 0$: closest area : 0.9535 , 0.9545

1.68   1.69

$Z = \frac{1.68 + 1.69}{2}$

$= 1.685$

④ Area to right of Z is 0.1

Sol. $\boxed{Z > 0}$

Area to left of Z is 0.9

$\longrightarrow$ closest area : 0.8997

$\boxed{Z = 1.28}$

⑤ Area to left of Z is 0.2

Sol. $\boxed{Z < 0}$ $\longrightarrow$

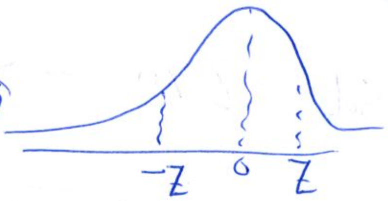
$1 - 0.2 = 0.8$

closest area $= 0.7995$

$Z = -0.89$.

⑥ Area between Z and $-Z$ is 0.85

$\frac{0.85}{2} = 0.42 \longrightarrow 0.42 + 0.5 = 0.9250$



~~P(Z~~

closest area = 0.9251

$\boxed{Z = \pm 1.44}$

Exp: If the weight of B.Z.U student is normally Dist with mean of 60 kg and std dev. of 8.5 kg

X: weight (Normal) $\longrightarrow$ $X: N(60, 8.5)$

① What is the probability that a student weight less than 80 kg

② " " " " " " " " between 45 & 55 kg

③ A student is considered 'fat' if he is in the top "upper 1%, what is the minimum weight of a 'fat' student

sol.

① $X = 80 \longrightarrow Z = \frac{80-60}{8.5} = 2.35$

$P(Z < 2.35) = 0.9906$

② $P(45 < X < 55) = P(-1.76 < Z < -0.59)$

$= P(0.59 < Z < 1.76)$

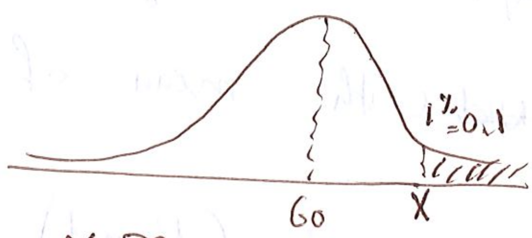
$= P(Z < 1.76) - \cancel{P(0.59)}$ $P(Z<0.59)$

$= 0.9608 - 0.7224$

$= 0.2384$

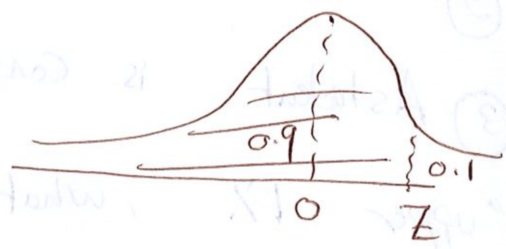
③

In this part



we need to ~~tot~~ Find weight = X ??

First : we find Z , then $X = Z\sigma + \mu$ to find X



Area below Z = 0.99

closest area = 0.9901

⟶ Z = 2.33

$X = (2.33)(8.5) + 60 = 79.81$ Kg Min weight of fat student

[A student who weight 79.81 kg or More is "fat"]

Exp: The final exam is ~~to~~ Stat 2311 is N(65, 10)

(a) Find the prob. that a student scored less than 78

(b) = = percentage of the scores that are between 50 and 70 ??

(c) calculate the minimum score of the top 15% student on the exam