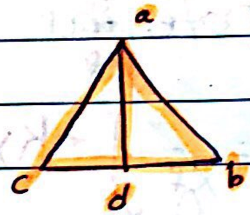
Potential Energy and Conservation Of Energy

→ Forces : [1] non conservation forces : Ex: friction.

[2] Conservation forces "F_{cons}" :

[i] work done by F_{cons} between 2 Points don't depand on the path.

$$W_{a\to c\to d} = W_{a\to b\to d}$$



[2] work done by F_{cons} around a Closed path equal zero.

$$W_{a\to b\to d\to c\to a} = zero$$

→ $W_{cons} = -\Delta U$, U: potential energy

$$\int F_{(x)} dx = -\Delta U$$

$$\Delta U = -\int_{x_i}^{x_f} F_{(x)} dx$$

$$U_f - U_i = -\int_{x_i}^{x_f} F_{(x)} dx$$

Gravitational potential energy:

$$\Rightarrow U_f - U_i = -\int_{y_i}^{y_f} mg\,dy\,\cancel{(\cos(180))}$$

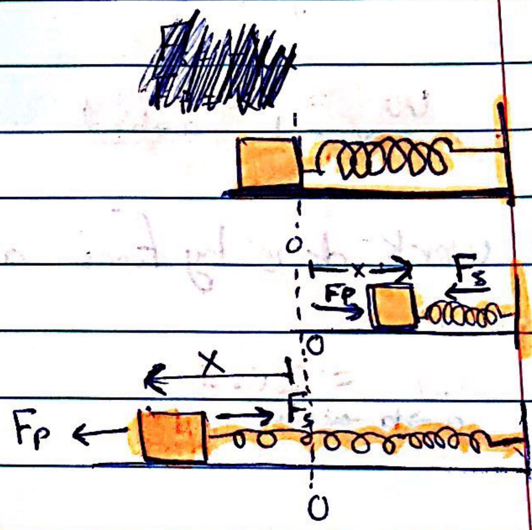
$$= -\int_{y_i}^{y_f} mg\,dy$$

Spring potential energy:

$$\Rightarrow U_f - U_i = -\int_{x_i}^{x_f} -kx\,dx$$

$$= \int_{x_i}^{x_f} kx\,dx$$

$$= k\frac{x^2}{2}\Big|_{x_i}^{x_f}$$



$$\Rightarrow U_f - U_i = \frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

$(\Delta x)^2 \neq (x_f^2 - x_i^2)$

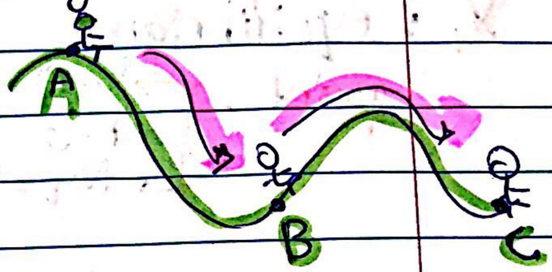
Mechanical energy:

$\Rightarrow$ Mechanical energy $= K + U$

⇒ Conservation of mechanical energy.

the only for acting on the system is

$\boxed{F_{cons}:}$



$W_{done\ by\ F_{cons}} = \Delta K$

$W_{done\ by\ F_{cons}} = -\Delta U$

$\therefore \boxed{\Delta K = -\Delta U} \Rightarrow \Delta K + \Delta U = \text{zero}$ "constant"

$(K+U)_i = (K+U)_f$ [F_{cons} is acting only]

$M_E(A) = M_E(B) = M_E(C)$.

Finding Conservative Force from U :-

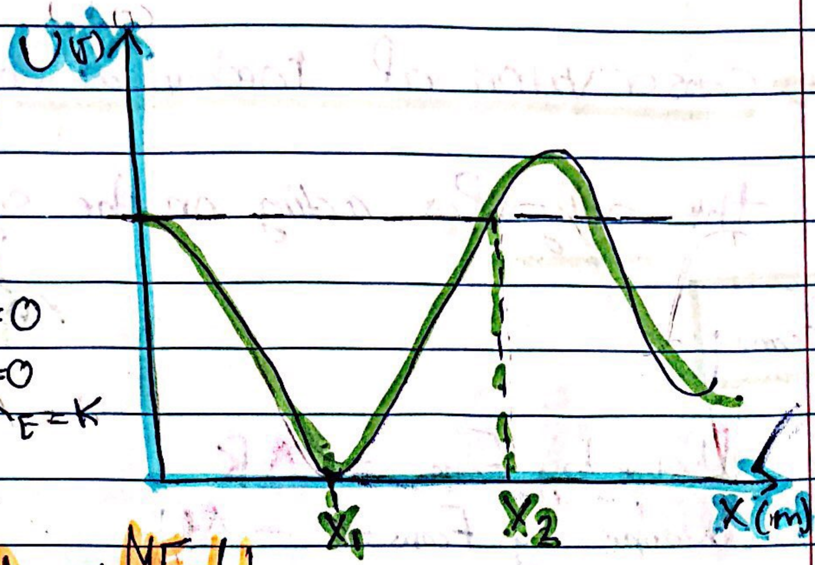
$\rightarrow F_{cons}\,dx = -du$

$\rightarrow F_{cons} = -\frac{du}{dx}$

$\rightarrow W = -\Delta U$

Reading potiental energy curve:

$\boxed{m}$ moves along the x-axis



→ $F_{cons} = -\frac{dU}{dx}$ "slop"

x_1 : equilibriam point
- → $U=0$
- → $F=0$
- → $M_E = K$

x_2 : Turning point
- → $ME = U$
- → $K = 0$
- → $F \neq 0$

* When the system consists of conservative and non-conservative force:

$$W_{cons} + W_{non\text{-}cons} = \Delta K$$
$$-\Delta U + W_{non\text{-}cons} = \Delta K$$

$$W_{non\text{-}cons} = \Delta E = \Delta K + \Delta U$$

$$\Delta E_{th} = f_k d$$
$$W = \Delta E_{mec} + \Delta E_{th}$$

Systems

1 without applied force

② closed with friction

$\Delta E_{mec} + \Delta E_{th} = 0$

① closed and No friction

$\Delta E_{mec} = 0$

2 with applied force

① with friction

$\Delta E_{mec} + \Delta E_{th} = W_{app}$

② without friction

$\Delta E_{mec} = W_{app}$

* Note:

$\Rightarrow \Delta E_{th} = f_k d$

$\Rightarrow \Delta U_g = -W_g$

$\Rightarrow \Delta U_s = -W_s$

Chapter 9

Center of Mass and linear Momentum

9.1 Center of mass:

mass of a system on particles

x_{com} → $\frac{m_1x_1+m_2x_2\dots}{m_1+m_2\dots}$

y_{com} → $\frac{m_1y_1+m_2y_2\dots}{m_1+m_2\dots}$

z_{com} → $\frac{m_1z_1+m_2z_2\dots}{m_1+m_2\dots}$

Solid Bodies

$x_{com}=\frac{1}{M}\int x\,dm$

$y_{com}=\frac{1}{M}\int y\,dm$

$z_{com}=\frac{1}{M}\int z\,dm$

Note: $\rho=\frac{dm}{dV}=\frac{m}{V}$

density

9.2 Newtons second law for a system of particles

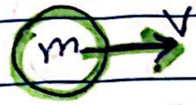
$$\frac{d}{dt}\left(M\vec{r}_{com}=m_1\vec{r}_1+m_2\vec{r}_2+\dots+m_n\vec{r}_n\right)$$

$$\frac{d}{dt}\left(M\vec{v}_{com}=m_1\vec{v}_1+m_2\vec{v}_2+\dots+m_n\vec{v}_n\right)$$

$$\left(M\vec{a}_{com}=m_1\vec{a}_1+m_2\vec{a}_2+\dots+m_n\vec{a}_n\right)$$

9.3 linear momentum

For one particle $\Rightarrow \frac{d}{dt}\left(\vec{p} = m\vec{v}\right)$

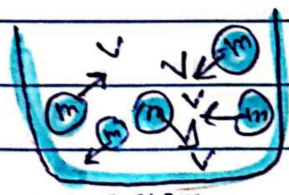


$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt}$$

$$\frac{d\vec{p}}{dt} = m\vec{a}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

of a system of particles $\Rightarrow$ $\vec{p} = \vec{p}_1 + \vec{p}_2 + \dots + \vec{p}_n$



system of particles

$$M = m_1 + m_2 + \dots + m_n$$

$$\frac{d\vec{p}}{dt} = M\frac{d\vec{v}_{com}}{dt}$$

$$\frac{d\vec{p}}{dt} = M\vec{a}_{com}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_{net}$$

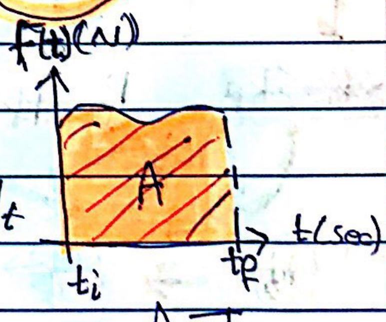
9.4 Collision and Impulse

Impulse $(\vec{J})$:-

$\rightarrow \vec{J} = \Delta\vec{p}$

$\vec{J} = \vec{p}_f - \vec{p}_i$

$\rightarrow \vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt$



$A = J$

$\rightarrow J = F_{avg}\,\Delta t$

9.5 Conservation of linear momentum

- In the (closed, isolated systems) like Collisions explosion and roket motion in free space, $\vec{F}_{net} = 0$ so $\frac{d\vec{p}}{dt} = 0$ which mean that:-

→ $\vec{P} = \text{Constant}$

→ $\vec{P}_i = \vec{P}_f$

→ (total linear momentum at some initial time ti) = (total linear momentum at some later time tf)

9.6 Collisions «1D»

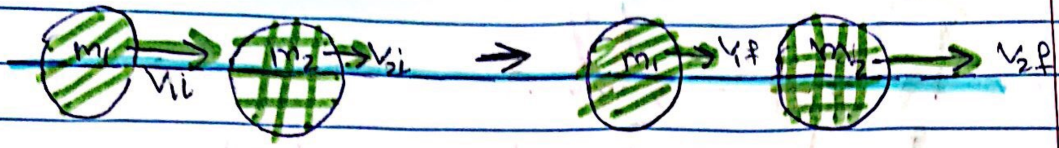
- **Elastic**
  - $\vec{P}_i = \vec{P}_f$
  - $\Sigma K_i = \Sigma K_f$
- **InElastic**
  - $\vec{P}_i = \vec{P}_f$
  - $\Sigma K_i \neq \Sigma K_f$
- **Completely InElastic**
  - $\vec{P}_i = \vec{P}_f$
  - $\Sigma K_i \neq \Sigma K_f$

$m_1 \rightarrow v_i \quad m_2 \;\rightarrow\; m_1 \rightarrow v_{1f} \quad m_2 \rightarrow v_{2f}$

$m_1 \rightarrow v_i \quad m_2 \;\rightarrow\; (m_1 + m_2) \rightarrow v_f$

$$\vec{V}_{com_i} = \vec{V}_{com_f}$$

9.7 Elastic collisions in One dimension



"before" "after"

• In Elastic collisions the total kinetic energy of the system does not change:

$$\sum K_i = \sum K_f \quad (1)$$

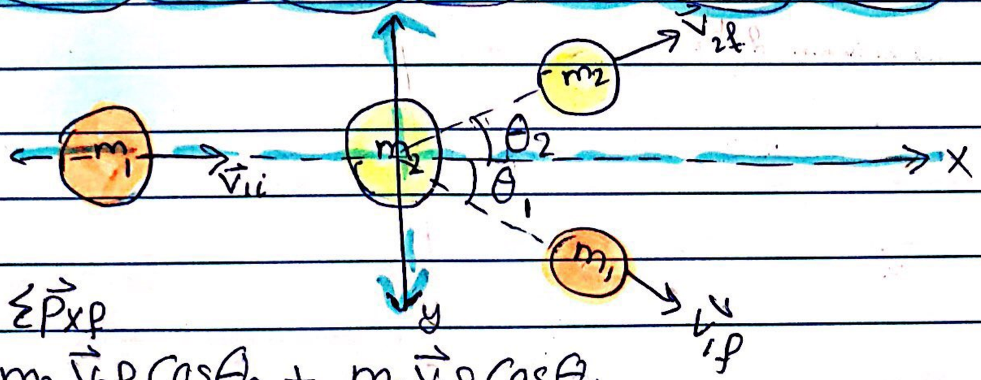
$$\sum \vec{P}_i = \sum \vec{P}_f \quad (2)$$

By solve the two equations:

$$V_{1f} = \frac{m_1 - m_2}{m_1 + m_2} V_{1i}$$

$$V_{2f} = \frac{2m_1}{m_1 + m_2} V_{1i}$$

9.8 Collisions in Two dimension



$$\sum \vec{P}_{xi} = \sum \vec{P}_{xf}$$

$$m_1 \vec{V}_{1i} = m_2 \vec{V}_{2f} \cos\theta_2 + m_1 \vec{V}_{1f} \cos\theta_1$$

$$\sum \vec{P}_{yi} = \sum \vec{P}_{yf}$$

$$0 = m_2 \vec{V}_{2f} \sin\theta_2 - m_1 \vec{V}_{1f} \sin\theta_1$$

$$\sum K_i = \sum K_f$$

9.9 Systems with Varying mass

$$\Rightarrow R\,V_{rel} = M\,a$$

- M: rocket's instantaneous mass
- R: The fuel consumption rate
- V_{rel}: fuel's exhaust speed relative to the rocket

$$\Rightarrow V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$$

Rotation

angular position → θ "rad"

angular displacement → $\Delta\theta = \theta_2 - \theta_1$

average angular velocity → $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$

instantaneous angular velocity → $\omega_{ins} = \frac{d\theta}{dt}$

average angular acceleration → $\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$

instantaneous angular acceleration → $\alpha_{ins} = \frac{d\omega}{dt}$

1 revolutions = 2π radians

* Rotation with Constant angular acceleration:

→ $\omega = \omega_0 + \alpha t$

→ $\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$

→ $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

→ $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$

→ $\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$

easy come:

هاي المعادلات نفسه
معادلات الحركة بتسارع ثابت
ولكن بدل $\Delta x \rightarrow \Delta\theta$
$v \rightarrow \omega$
$a \rightarrow \alpha$

* relating the linear and angular Variables:

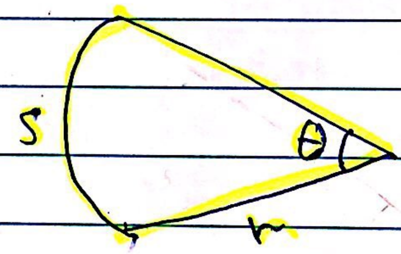
→ $S = \theta r$

→ $v = \omega r$

→ $a = \alpha r$

$a = \omega^2 r$

→ $T = \frac{2\pi}{\omega}$, T: predic time



* Kinetic energy of rotation

$$K = \frac{1}{2} I \omega^2$$

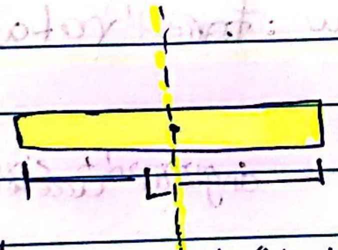
I: is the rotational inertia of the body defined as:

$I = \sum m_i r_i^2$, for a system of discrete particles and defined as:

$$I = \int r^2 \, dm = mr^2$$

r: the perpendicular distance from the axis of rotation to each mass element in the body

• The parallel-axis Theorem :-

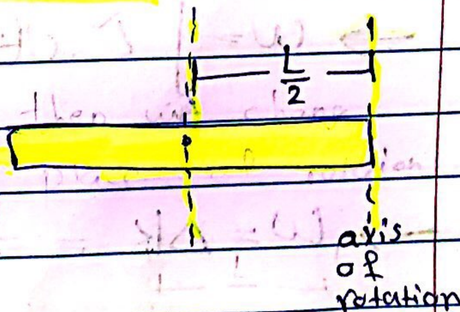


axis " in the center of mass ((com))"

$$I_{com} = \frac{1}{12} ML^2$$

⟹ Then when we change The place of rotation

$$I = I_{com} + M\left(\frac{L}{2}\right)^2$$



SO $$I = I_{com} + Mh^2$$

h: the distance the actual rotation axis has been shifted from the rotation axis through the center of mass.

Note: I_{cm} : will be given, you can look Page [238] Table (10-2)

* Torque (τ)

$(\vec{\tau}) = \vec{F} \times \vec{r}$ "cross product"

$|\tau| = |F||r| \sin\theta$ (N.m)

We can find the direction of (τ) By "right-hand rule"

Newton's Second law for rotation :-

$$\tau_{net} = I \alpha$$

τ_{net} → Net torque

I → inertia

α → angular acceleration

* Work and rotational Kinetic Energy =

$$\rightarrow W = \int_{\theta_i}^{\theta_f} \tau \, d\theta \rightarrow W = \tau(\theta_2 - \theta_1)$$

$$\rightarrow W = \Delta K = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$$

power $P = \frac{dW}{dt}$