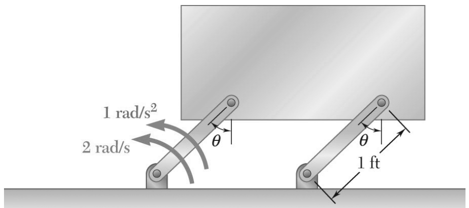
CHAPTER 15



PROBLEM 15.CQ1

A rectangular plate swings from arms of equal length as shown below. What is the magnitude of the angular velocity of the plate?

(*a*) 0 rad/s

(*b*) 1 rad/s

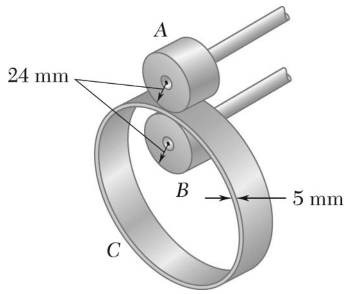
(*c*) 2 rad/s

(*d*) 3 rad/s

(*e*) Need to know the location of the center of gravity

SOLUTION

Answer: (*a*) ◀



PROBLEM 15.CQ2

Knowing that wheel *A* rotates with a constant angular velocity and that no slipping occurs between ring *C* and wheel *A* and wheel *B*, which of the following statements concerning the angular speeds are true?

(*a*) $\omega_a = \omega_b$

(*b*) $\omega_a > \omega_b$

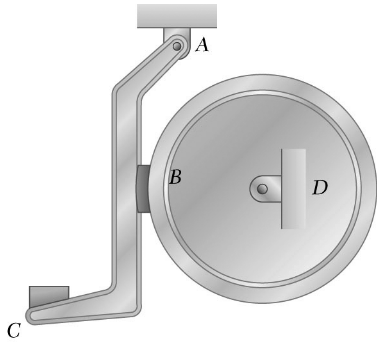
(*c*) $\omega_a < \omega_b$

(*d*) $\omega_a = \omega_c$

(*e*) the contact points between *A* and *C* have the same acceleration

SOLUTION

Answer: (*b*) ◀



PROBLEM 15.1

The brake drum is attached to a larger flywheel that is not shown. The motion of the brake drum is defined by the relation $\theta = 36t - 1.6t^2$, where θ is expressed in radians and t in seconds. Determine (*a*) the angular velocity at $t = 2$ s, (*b*) the number of revolutions executed by the brake drum before coming to rest.

SOLUTION

Given: $\theta = 36t - 1.6t^2$ radians

Differentiate to obtain the angular velocity.

$$\omega = \frac{d\theta}{dt} = 36 - 3.2t \quad \text{rad/s}$$

(*a*) At $t = 2$ s, $\omega = 36 - (3.2)(2)$ $\qquad \omega = 29.6$ rad/s ◀

(*b*) When the rotor stops, $\omega = 0.$

$$0 = 36 - 3.2t \qquad t = 11.25 \text{ s}$$

$$\theta = (36)(11.25) - (1.6)(11.25)^2 = 202.5 \text{ radians}$$

In revolutions, $\theta = \dfrac{202.5}{2\pi}$ $\qquad \theta = 32.2$ rev ◀

PROBLEM 15.2

The motion of an oscillating crank is defined by the relation $\theta = \theta_0 \sin(\pi t/T) - (0.5\theta_0)\sin(2\pi t/T)$, where θ is expressed in radians and t in seconds. Knowing that $\theta_0 = 6$ rad and $T = 4$ s, determine the angular coordinate, the angular velocity, and the angular acceleration of the crank when (*a*) $t = 0$, (*b*) $t = 2$ s.

SOLUTION

$$\omega = \frac{d\theta}{dt} = \theta_0 \frac{\pi}{T}\cos\left(\frac{\pi t}{T}\right) - 0.5\theta_0 \frac{2\pi}{T}\cos\left(\frac{2\pi t}{T}\right)$$

$$\alpha = \frac{d\omega}{dt} = -\theta_0\left(\frac{\pi}{T}\right)^2 \sin\left(\frac{\pi t}{T}\right) + 0.5\theta_0\left(\frac{2\pi}{T}\right)^2 \sin\left(\frac{2\pi t}{T}\right)$$

(*a*) $t = 0$: $\theta = 0$ ◀

$$\omega = 6\frac{\pi}{4} - 0.5(6)\frac{2\pi}{4}$$ $\omega = 0$ ◀

$\alpha = 0$ ◀

(*b*) $t = 2$ s:

$$\theta = 6\sin\left(\frac{2\pi}{4}\right) - 0.5(6)\sin\left(\frac{4\pi}{4}\right) = 6 - 0$$ $\theta = 6.00$ rad ◀

$$\omega = 6\left(\frac{\pi}{4}\right)\cos\left(\frac{2\pi}{4}\right) - 0.5(6)\frac{2\pi}{4}\cos\left(\frac{4\pi}{4}\right)$$

$$= 6\frac{\pi}{4}(0) - 0.5(6)\frac{2\pi}{4}(-1)$$

$$= \frac{6\pi}{4}$$ $\omega = 4.71$ rad/s ◀

$$\alpha = -6\left(\frac{\pi}{4}\right)^2 \sin\left(\frac{2\pi}{4}\right) + 0.5(6)\left(\frac{2\pi}{4}\right)^2 \sin\left(\frac{4\pi}{4}\right)$$

$$= -6\left(\frac{\pi}{4}\right)^2 (1) + 3\left(\frac{2\pi}{4}\right)^2 (0)$$

$$= -\frac{3}{8}\pi^2$$ $\alpha = -3.70$ rad/s^2 ◀

PROBLEM 15.3

The motion of a disk rotating in an oil bath is defined by the relation $\theta = \theta_0(1 - e^{-t/4})$, where θ is expressed in radians and t in seconds. Knowing that $\theta_0 = 0.40$ rad, determine the angular coordinate, velocity, and acceleration of the disk when (*a*) $t = 0$, (*b*) $t = 3$ s, (*c*) $t = \infty$.

SOLUTION

$$\theta = 0.40(1 - e^{-t/4})$$

$$\omega = \frac{d\theta}{dt} = \frac{1}{4}(0.40)e^{-t/4} = 0.10e^{-t/4}$$

$$\alpha = \frac{d\omega}{dt} = -\frac{1}{4}(0.10)e^{-t/4} = -0.025e^{-t/4}$$

(*a*) $t = 0$:

$\theta = 0.40(1 - e^{0})$ $\quad\theta = 0$ ◀

$\omega = 0.10e^{0}$ $\quad\omega = 0.1000$ rad/s ◀

$\alpha = -0.025e^{0}$ $\quad\alpha = -0.0250$ rad/s^2 ◀

(*b*) $t = 3$ s:

$\theta = 0.40(1 - e^{-3/4})$

$= 0.40(1 - 0.4724)$ $\quad\theta = 0.211$ rad ◀

$\omega = 0.10e^{-3/4}$

$= 0.10(0.4724)$ $\quad\omega = 0.0472$ rad/s ◀

$\alpha = -0.025e^{-3/4}$

$= -0.025(0.4724)$ $\quad\alpha = -0.01181$ rad/s^2 ◀

(*c*) $t = \infty$:

$\theta = 0.40(1 - e^{-\infty})$

$= 0.40(1 - 0)$ $\quad\theta = 0.400$ rad ◀

$\omega = 0.10e^{-\infty}$ $\quad\omega = 0$ ◀

$\alpha = -0.025e^{-\infty}$ $\quad\alpha = 0$ ◀

PROBLEM 15.4

The rotor of a gas turbine is rotating at a speed of 6900 rpm when the turbine is shut down. It is observed that 4 min is required for the rotor to coast to rest. Assuming uniformly accelerated motion, determine (*a*) the angular acceleration, (*b*) the number of revolutions that the rotor executes before coming to rest.

SOLUTION

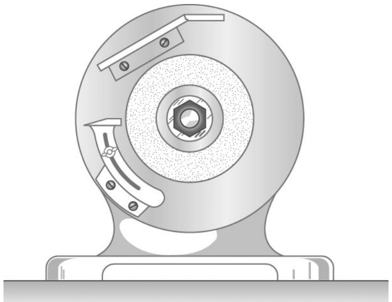
$$\omega_0 = 6900 \text{ rpm}$$
$$= 722.57 \text{ rad/s}$$
$$t = 4 \text{ min} = 240 \text{ s}$$

(*a*) $$\omega = \omega_0 + \alpha t; \quad 0 = 722.57 + \alpha(240)$$

$$\alpha = -3.0107 \text{ rad/s}$$ $\alpha = -3.01 \text{ rad/s}^2$ ◀

(*b*) $$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (722.57)(240) + \frac{1}{2}(-3.0107)(240)^2$$
$$\theta = 173,416 - 86,708 = 86,708 \text{ rad}$$

$$\theta = 86,708 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)$$ $\theta = 13,80 \text{ rev}$ ◀



PROBLEM 15.5

A small grinding wheel is attached to the shaft of an electric motor which has a rated speed of 3600 rpm. When the power is turned on, the unit reaches its rated speed in 5 s, and when the power is turned off, the unit coasts to rest in 70 s. Assuming uniformly accelerated motion, determine the number of revolutions that the motor executes (*a*) in reaching its rated speed, (*b*) in coasting to rest.

SOLUTION

For uniformly accelerated motion,

$$\omega = \omega_0 + \alpha t \qquad (1)$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad (2)$$

(*a*) Data for start up: $\theta_0 = 0, \ \omega_0 = 0,$

At $t = 5$ s, $\omega = 3600 \text{ rpm} = \dfrac{2\pi(3600)}{60} = 120\pi \text{ rad/s}$

From Eq. (1), $120\pi = \alpha(5) \qquad \alpha = 24\pi \text{ rad/s}^2$

From Eq. (2), $\theta = 0 + 0 + \dfrac{1}{2}(24\pi)(5)^2 = 300\pi$ radians

In revolutions, $\theta = \dfrac{300\pi}{2\pi}$ $\qquad \theta = 150 \text{ rev}$ ◀

(*b*) Data for coasting to rest:

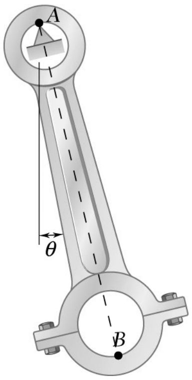
$\theta_0 = 0, \quad \omega_0 = 120\pi$ rad/s

At $t = 70$ s, $\omega = 0$

From Eq. (1), $0 = 120\pi - \alpha(70) \qquad \alpha = \dfrac{120\pi}{70}$ rad/s

From Eq. (2), $\theta = 0 + (120\pi)(70) - \dfrac{(120\pi)(70)^2}{2(70)} = 4200\pi$ radians

In revolutions, $\theta = \dfrac{4200\pi}{2\pi}$ $\qquad \theta = 2100 \text{ rev}$ ◀



PROBLEM 15.6

A connecting rod is supported by a knife-edge at Point A. For small oscillations the angular acceleration of the connecting rod is governed by the relation $\alpha = -6\theta$ where α is expressed in rad/s^2 and θ in radians. Knowing that the connecting rod is released from rest when $\theta = 20°$, determine (*a*) the maximum angular velocity, (*b*) the angular position when $t = 2$ s.

SOLUTION

Angular motion relations:

$$\alpha = \frac{d\omega}{dt} = \frac{\omega d\omega}{d\theta} = -6\theta \qquad (1)$$

Separation of variables ω and θ gives

$$\omega d\omega = -6\theta d\theta$$

Integrating, using $\omega = 0$ when $\theta = \theta_0$,

$$\int_0^{\omega} \omega d\omega = -6\int_{\theta_0}^{\theta} \theta d\theta$$

$$\frac{1}{2}\omega^2 = -3(\theta^2 - \theta_0^2) = 3(\theta_0^2 - \theta^2)$$

$$\omega^2 = 6(\theta_0^2 - \theta^2) \qquad \omega = \sqrt{6(\theta_0^2 - \theta^2)}$$

(*a*) ω is maximum when $\theta = 0$.

Data: $\theta_0 = 20° = 0.34907$ radians

$$\omega_{max}^2 = 6(0.34907^2 - 0) = 0.73108 \text{ rad}^2/\text{s}$$

$\omega_{max} = 0.855$ rad/s ◀

(*b*) From $\omega = \dfrac{d\theta}{dt}$ we get $dt = \dfrac{d\theta}{\omega} = \dfrac{1}{\sqrt{6}} \dfrac{d\theta}{\sqrt{\theta_0^2 - \theta^2}}$

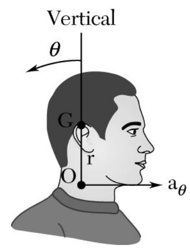
Integrating, using $t = 0$ when $\theta = \theta_0$,

$$\int_0^t dt = \frac{1}{\sqrt{6}} \int_{\theta_0}^{\theta} \frac{d\theta}{\sqrt{\theta_0^2 - \theta^2}}$$

$$t = -\frac{1}{\sqrt{6}} \cos^{2-1} \frac{\theta}{\theta_0}\Bigg|_{\theta_0}^{\theta} = -\frac{1}{\sqrt{6}}\left[0 - \cos^{-1}\frac{\theta}{\theta_0}\right] = -\frac{1}{\sqrt{6}}\cos^{-1}\frac{\theta}{\theta_0}$$

$$\theta = \theta_0 \cos(\sqrt{6}t) = 0.34907 \cos[(\sqrt{6})2] = (0.34907)(0.18551) = 0.064756 \text{ radians}$$

$\theta = 3.71°$ ◀



PROBLEM 15.7

When studying whiplash resulting from rear end collisions, the rotation of the head is of primary interest. An impact test was performed, and it was found that the angular acceleration of the head is defined by the relation $\alpha = 700\cos\theta + 70\sin\theta$ where α is expressed in rad/s^2 and θ in radians. Knowing that the head is initially at rest, determine the angular velocity of the head when $\theta = 30°$.

SOLUTION

Angular motion relations:

$$\alpha = \frac{d\omega}{dt} = \frac{\omega d\omega}{d\theta} = 700\cos\theta + 70\sin\theta$$

Separating variables ω and θ gives

$$\omega d\omega = (700\cos\theta + 70\sin\theta)d\theta$$

Integrating, using $\omega = 0$ when $\theta = 0$,

$$\int_0^\omega \omega d\omega = \int_0^\theta (700\cos\theta + 70\sin\theta)d\theta$$

$$\frac{1}{2}\omega^2 = (700\sin\theta - 70\cos\theta)\Big|_0^\theta$$

$$= 700\sin\theta + 70(1-\cos\theta)$$

$$\omega = \sqrt{1400\sin\theta + 140(1-\cos\theta)}$$

Data: $\theta = 30° = \dfrac{\pi}{6}\,\text{rad}$

With calculator set to "degrees" for trigonometric functions,

$$\omega = \sqrt{1400\sin 30° + 140(1-\cos 30°)} = 26.8 \text{ rad/s}$$

$\omega = 26.8$ rad/s ◀

With calculator set to "radians" for trigonometric functions,

$$\omega = \sqrt{1400\sin(\pi/6) + 140(1-\cos(\pi/6))} = 26.8 \text{ rad/s}$$

PROBLEM 15.8

The angular acceleration of an oscillating disk is defined by the relation $\alpha = -k\theta$. Determine (*a*) the value of k for which $\omega = 8$ rad/s when $\theta = 0$ and $\theta = 4$ rad when $\omega = 0$, (*b*) the angular velocity of the disk when $\theta = 3$ rad.

SOLUTION

$$\alpha = -k\theta$$

$$\omega = \frac{d\omega}{d\theta} = -k\theta$$

$$\omega d\omega = -k\theta d\theta$$

(*a*) $$\int_{8 \text{ rad/s}}^{0} \omega d\omega = -\int_0^{4 \text{ rad}} k\theta d\theta; \qquad \left|\frac{1}{2}\omega^2\right|_8^0 = -\left|\frac{1}{2}k\theta^2\right|_0^4$$

$$\frac{1}{2}(0 - 8^2) = -\frac{1}{2}k(4^2 - 0)$$

$k = 4.00 \text{ s}^{-2}$ ◀

(*b*) $$\int_{8 \text{ rad/s}}^{\omega} \omega d\omega = -\int_0^{3 \text{ rad}} k\theta d\theta; \qquad \left|\frac{1}{2}\omega^2\right|_0^{\omega} = -\left|\frac{1}{2}(4 \text{ s}^{-1})\theta^2\right|_0^3$$

$$\frac{1}{2}(\omega^2 - 8^2) = -\frac{1}{2}(4)(3^2 - 0)$$

$$\omega^2 - 64 = -36; \quad \omega^2 = 64 - 36 = 28$$

$\omega = 5.29$ rad/s ◀

PROBLEM 15.9

The angular acceleration of a shaft is defined by the relation $\alpha = -0.25\omega$, where α is expressed in rad/s^2 and ω in rad/s. Knowing that at $t = 0$ the angular velocity of the shaft is 20 rad/s, determine (*a*) the number of revolutions the shaft will execute before coming to rest, (*b*) the time required for the shaft to come to rest, (*c*) the time required for the angular velocity of the shaft to be reduced to 1 percent of its initial value.

SOLUTION

$$\alpha = -0.25\omega$$

$$\omega \frac{d\omega}{d\theta} = -0.25\omega$$

$$d\omega = -0.25 d\theta$$

(*a*) $$\int_{20\text{ rad/s}}^{0} d\omega = -0.25\int_0^\theta d\theta; \quad (0-20) = -0.25\theta; \quad \theta = 80 \text{ rad}$$

$$\theta = (80 \text{ rad})\frac{\text{rev}}{2\pi \text{ rad}}$$ $\theta = 12.73$ rev ◀

(*b*) $$\alpha = -0.25\omega; \quad \frac{d\omega}{dt} = -0.25\omega; \quad \frac{d\omega}{\omega} = -0.25dt$$

$$\int_{20\text{ rad/s}}^{\omega} \frac{d\omega}{\omega} = -0.25\int_0^t dt \qquad |\ln\omega|_{20}^{\omega} = -0.25t$$

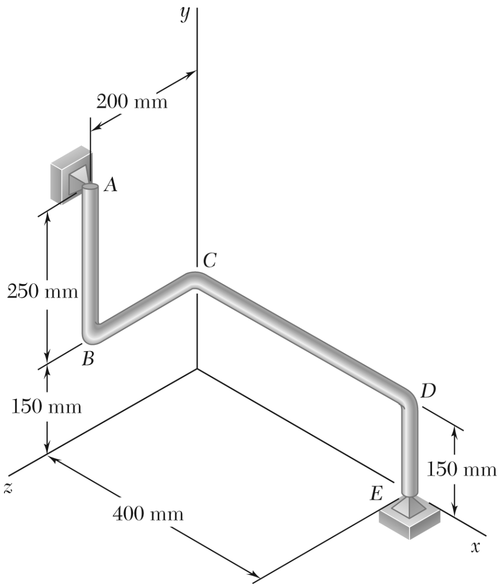
$$t = -\frac{1}{0.25}(\ln\omega - \ln 20) = 4(\ln 20 - \ln\omega)$$

$$t = 4\ln\frac{20}{\omega} \tag{1}$$

For $\omega = 0$ $$t = 4\ln\frac{20}{0} = 4\ln\infty$$ $t = \infty$ ◀

(*c*) For $$\omega = 0.01\omega_0 = 0.01(20) = 0.2 \text{ rad}$$

Use Eq. (1): $$t = 4\ln\left(\frac{20}{0.2}\right) = 4\ln 100 = 4(4.605)$$ $t = 18.42$ s ◀



PROBLEM 15.10

The bent rod *ABCDE* rotates about a line joining Points *A* and *E* with a constant angular velocity of 9 rad/s. Knowing that the rotation is clockwise as viewed from *E*, determine the velocity and acceleration of corner *C*.

SOLUTION

$$EA^2 = 0.4^2 + 0.4^2 + 0.2^2$$

$$EA = 0.6 \text{ m}$$

$$\mathbf{r}_{C/E} = -(0.4 \text{ m})\mathbf{i} + (0.15 \text{ m})\mathbf{j}$$

$$\overrightarrow{EA} = -(0.4 \text{ m})\mathbf{i} + (0.4 \text{ m})\mathbf{j} + (0.2 \text{ m})\mathbf{k}$$

$$\boldsymbol{\lambda}_{EA} = \frac{\overrightarrow{EA}}{EA} = \frac{1}{0.6}(-0.4\mathbf{i} + 0.4\mathbf{j} + 0.2\mathbf{k}) = \frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$\boldsymbol{\omega} = \omega_{AE}\boldsymbol{\lambda}_{EA} = (9 \text{ rad/s})\frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$\boldsymbol{\omega} = -(6 \text{ rad/s})\mathbf{i} + (6 \text{ rad/s})\mathbf{j} + (3 \text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_{C/E} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 6 & 3 \\ -0.4 & 0.15 & 0 \end{vmatrix} = -0.45\mathbf{i} - 1.2\mathbf{j} + (-0.9 + 2.4)\mathbf{k}$$

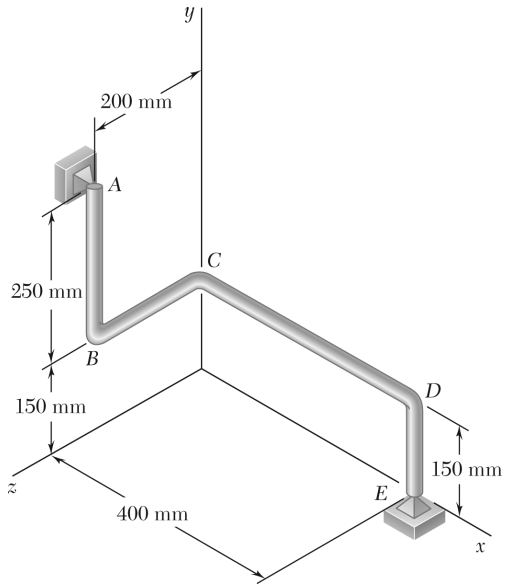
$$\mathbf{v}_C = -(0.45 \text{ m/s})\mathbf{i} - (1.2 \text{ m/s})\mathbf{j} + (1.5 \text{ m/s})\mathbf{k} \blacktriangleleft$$

$$\mathbf{a}_C = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/E} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{C/E}) = \boldsymbol{\alpha}_{AC} \times \mathbf{r}_{C/E} + \boldsymbol{\omega} \times \mathbf{v}_C$$

$$\mathbf{a}_C = 0 + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 6 & 3 \\ -0.45 & -1.2 & 1.5 \end{vmatrix}$$

$$= (9 + 3.6)\mathbf{i} + (-1.35 + 9)\mathbf{j} + (7.2 + 2.7)\mathbf{k}$$

$$\mathbf{a}_C = (12.60 \text{ m/s}^2)\mathbf{i} + (7.65 \text{ m/s}^2)\mathbf{j} + (9.90 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.11

In Problem 15.10, determine the velocity and acceleration of corner *B*, assuming that the angular velocity is 9 rad/s and increases at the rate of 45 rad/s^2.

PROBLEM 15.10 The bent rod *ABCDE* rotates about a line joining Points *A* and *E* with a constant angular velocity of 9 rad/s. Knowing that the rotation is clockwise as viewed from *E*, determine the velocity and acceleration of corner *C*.

SOLUTION

$$EA^2 = 0.4^2 + 0.4^2 + 0.2^2$$

$$EA = 0.6 \text{ m}$$

$$\mathbf{r}_{B/A} = -(0.25 \text{ m})\mathbf{j}$$

$$\overrightarrow{EA} = -(0.4 \text{ m})\mathbf{i} + (0.4 \text{ m})\mathbf{j} + (0.2 \text{ m})\mathbf{k}$$

$$\boldsymbol{\lambda}_{EA} = \frac{\overrightarrow{EA}}{EA} = \left(\frac{-0.4\mathbf{i} + 0.4\mathbf{j} + 0.2\mathbf{k}}{0.6}\right) = \frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$\boldsymbol{\omega} = \omega_{AE}\boldsymbol{\lambda}_{EA} = (9 \text{ rad/s})\frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

$$\boldsymbol{\omega} = -(6 \text{ rad/s})\mathbf{i} + (6 \text{ rad/s})\mathbf{j} + (3 \text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_{B/A} = (-6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) \times (-0.25)\mathbf{j} = 1.5\mathbf{k} + 0.75\mathbf{i}$$

$$\mathbf{v}_B = (0.75 \text{ m/s})\mathbf{i} + (1.5 \text{ m/s})\mathbf{k} \blacktriangleleft$$

$$\boldsymbol{\alpha} = \alpha_{AE}\boldsymbol{\lambda}_{EA} = (45 \text{ rad/s}^2)\frac{1}{3}(-2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

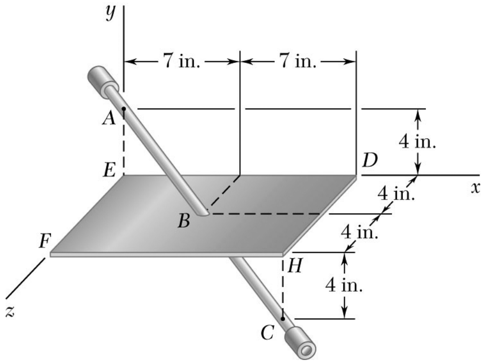
$$\boldsymbol{\alpha} = -(30 \text{ rad/s}^2)\mathbf{i} + (30 \text{ rad/s}^2)\mathbf{j} + (15 \text{ rad/s}^2)\mathbf{k}$$

$$\mathbf{a}_B = \boldsymbol{\alpha} \times \mathbf{r}_{B/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{B/A}) = \boldsymbol{\alpha} \times \mathbf{r}_{B/A} + \boldsymbol{\omega} \times \mathbf{v}_B$$

$$\mathbf{a}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -30 & 30 & 15 \\ 0 & -0.25 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 6 & 3 \\ 0.75 & 0 & 1.5 \end{vmatrix}$$

$$= 3.75\mathbf{i} + 7.5\mathbf{k} + 9\mathbf{i} + (2.25 + 9)\mathbf{j} - 4.5\mathbf{k}$$

$$\mathbf{a}_B = (12.75 \text{ m/s}^2)\mathbf{i} + (11.25 \text{ m/s}^2)\mathbf{j} + (3 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.12

The assembly shown consists of the straight rod ABC which passes through and is welded to the rectangular plate $DEFH$. The assembly rotates about the axis AC with a constant angular velocity of 9 rad/s. Knowing that the motion when viewed from C is counterclockwise, determine the velocity and acceleration of corner F.

SOLUTION

$$\overrightarrow{AC} = (14 \text{ in.})\mathbf{i} - (8 \text{ in.})\mathbf{j} + (8 \text{ in.})\mathbf{k} \qquad AC = 18 \text{ in.}$$

$$\lambda_{AC} = \frac{\overrightarrow{AC}}{AC} = \frac{14\mathbf{i} - 8\mathbf{j} + 8\mathbf{k}}{18} = (0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\omega} = \omega\lambda_{AC} = (9 \text{ rad/s})(0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\omega} = (7 \text{ rad/s})\mathbf{i} - (4 \text{ rad/s})\mathbf{j} + (4 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\alpha} = 0$$

Corner F:

$$\mathbf{r}_{F/B} = (-7 \text{ in.})\mathbf{i} + (4 \text{ in.})\mathbf{k}$$

$$= -(0.58333 \text{ ft})\mathbf{i} + (0.33333 \text{ ft})\mathbf{k}$$

$$\mathbf{v}_F = \boldsymbol{\omega} \times \mathbf{r}_{F/B}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & 4 \\ -0.58333 & 0 & 0.33333 \end{vmatrix}$$

$$= -1.3333\mathbf{i} - 4.6667\mathbf{j} - 2.3333\mathbf{k}$$

$$\mathbf{v}_F = -(1.333 \text{ ft/s})\mathbf{i} - (4.67 \text{ ft/s})\mathbf{j} - (2.33 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

$$\alpha = 0$$

$$\mathbf{a}_F = \boldsymbol{\alpha} \times \mathbf{r}_{F/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{F/B})$$

$$= 0 + \boldsymbol{\omega} \times \mathbf{v}_F$$

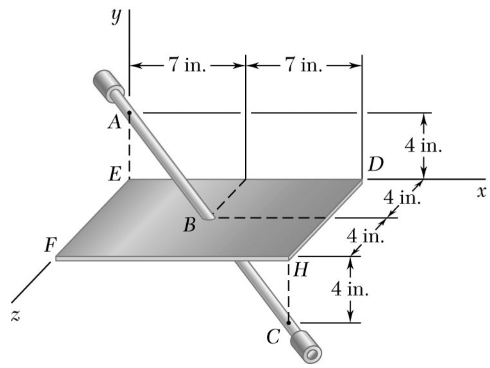
$$\mathbf{a}_F = \boldsymbol{\omega} \times \mathbf{v}_F$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & 4 \\ -1.3333 & -4.6667 & -2.3333 \end{vmatrix}$$

$$= (28.0)\mathbf{i} + (11.0)\mathbf{j} + (-38.0)\mathbf{k}$$

$$\mathbf{a}_F = (28.0 \text{ ft/s}^2)\mathbf{i} + (11.00 \text{ ft/s}^2)\mathbf{j} - (38.0 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.13



In Problem 15.12, determine the acceleration of corner H, assuming that the angular velocity is 9 rad/s and decreases at a rate of 18 rad/s^2.

PROBLEM 15.12 The assembly shown consists of the straight rod ABC which passes through and is welded to the rectangular plate $DEFH$. The assembly rotates about the axis AC with a constant angular velocity of 9 rad/s. Knowing that the motion when viewed from C is counterclockwise, determine the velocity and acceleration of corner F.

SOLUTION

$$\overrightarrow{AC} = (14 \text{ in.})\mathbf{i} - (8 \text{ in.})\mathbf{j} + (8 \text{ in.})\mathbf{k} \qquad AC = 18 \text{ in.}$$

$$\lambda_{AC} = \frac{\overrightarrow{AC}}{AC} = \frac{14\mathbf{i} - 8\mathbf{j} + 8\mathbf{k}}{18} = (0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\omega} = \omega\lambda_{AC} = (9 \text{ rad/s})(0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\omega} = (7 \text{ rad/s})\mathbf{i} - (4 \text{ rad/s})\mathbf{j} + (4 \text{ rad/s})\mathbf{k}$$

$$\alpha = -18 \text{ rad/s}^2; \qquad \boldsymbol{\alpha} = \alpha\lambda_{AC} = (-18 \text{ rad/s}^2)(0.77778\mathbf{i} - 0.44444\mathbf{j} + 0.44444\mathbf{k})$$

$$\boldsymbol{\alpha} = -(14 \text{ rad/s}^2)\mathbf{i} + (8 \text{ rad/s}^2)\mathbf{j} - (8 \text{ rad/s}^2)\mathbf{k}$$

Corner H:

$$\mathbf{r}_{H/B} = (7 \text{ in.})\mathbf{i} + (4 \text{ in.})\mathbf{k}$$

$$= (0.58333 \text{ ft})\mathbf{i} + (0.33333 \text{ ft})\mathbf{k}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & 4 \\ 0.58333 & 0 & 0.33333 \end{vmatrix}$$

$$= -1.3333\mathbf{i} + 2.3333\mathbf{k}$$

$$\mathbf{v}_H = -(1.333 \text{ ft/s})\mathbf{i} + (2.33 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

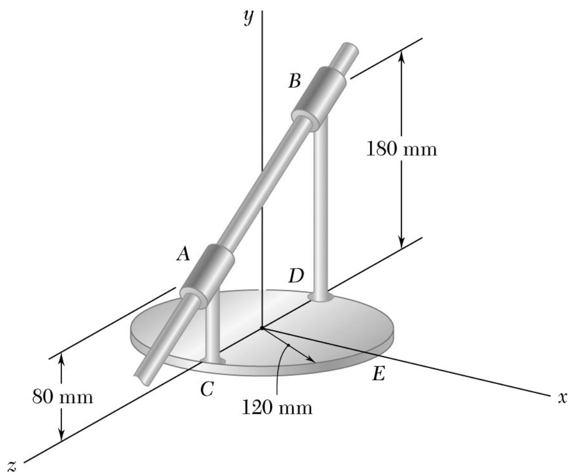
$$\mathbf{a}_H = \boldsymbol{\alpha} \times \mathbf{r}_{H/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{H/B})$$

$$= \boldsymbol{\alpha} \times \mathbf{r}_{H/B} + \boldsymbol{\omega} \times \mathbf{v}_H$$

$$\mathbf{a}_H = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -14 & 8 & -8 \\ 0.58333 & 0 & 0.33333 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & -4 & +4 \\ -1.3333 & 0 & 2.3333 \end{vmatrix}$$

$$= 2.6667\mathbf{i} + 0\mathbf{j} - 4.46667\mathbf{k} - 9.3333\mathbf{i} - 21.667\mathbf{j} - 5.3333\mathbf{k}$$

$$\mathbf{a}_H = -(6.67 \text{ ft/s}^2)\mathbf{i} - (21.7 \text{ ft/s}^2)\mathbf{j} - (10.00 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.14

A circular plate of 120 mm radius is supported by two bearings A and B as shown. The plate rotates about the rod joining A and B with a constant angular velocity of 26 rad/s. Knowing that, at the instant considered, the velocity of Point C is directed to the right, determine the velocity and acceleration of Point E.

SOLUTION

$$\overrightarrow{BA} = -(100\text{ mm})\mathbf{j} + (240\text{ mm})\mathbf{k} \qquad BA = 260\text{ mm}$$

$$\lambda_{BA} = \frac{\overrightarrow{BA}}{BA} = \frac{-(100)\mathbf{j} + (240)\mathbf{k}}{260}, \qquad \alpha = 0$$

$$\boldsymbol{\omega} = \omega\lambda_{BA} = 26\left(\frac{1}{260}\right)(-(100)\mathbf{j} + (240)\mathbf{k})$$

Point E:

$$\mathbf{r}_{E/A} = (120\text{ mm})\mathbf{i} - (80\text{ mm})\mathbf{j} - (120\text{ mm})\mathbf{k}$$

$$\mathbf{v}_E = \boldsymbol{\omega} \times \mathbf{r}_{E/A}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 24 \\ 120 & -80 & -120 \end{vmatrix}$$

$$= (3120\text{ mm/s})\mathbf{i} + (2880\text{ mm/s})\mathbf{j} + (1200\text{ mm/s})\mathbf{k}$$

$$\mathbf{v}_E = (3.12\text{ m/s})\mathbf{i} + (2.88\text{ m/s})\mathbf{j} + (1.200\text{ m/s})\mathbf{k} \blacktriangleleft$$

$$\mathbf{a}_E = \boldsymbol{\alpha} \times \mathbf{r}_{E/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{E/A}) = 0 + \boldsymbol{\omega} \times \mathbf{v}_B$$

$$\mathbf{a}_E = \boldsymbol{\omega} \times \mathbf{v}_B$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 24 \\ 3120 & 2880 & 1200 \end{vmatrix}$$

$$= -(81120\text{ mm/s}^2)\mathbf{i} + (74880\text{ mm/s}^2)\mathbf{j} + (31200\text{ mm/s}^2)$$

$$\mathbf{a}_E = -(81.1\text{ m/s}^2)\mathbf{i} + (74.9\text{ m/s}^2)\mathbf{j} + (31.2\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.15

In Problem 15.14, determine the velocity and acceleration of Point E, assuming that the angular velocity is 26 rad/s and increases at the rate of 65 rad/s^2.

SOLUTION

See Problem 15.14 for $\boldsymbol{\lambda}_{BA}$ and $\boldsymbol{\omega}$

$$\boldsymbol{\lambda}_{BA} = \frac{-(100)\mathbf{j} + (240)\mathbf{k}}{260}$$

$$\omega = (10\ \text{rad/s})\mathbf{j} = (24\ \text{rad/s})\mathbf{k}$$

$$\boldsymbol{\alpha} = +65\ \text{rad/s}^2; \qquad \boldsymbol{\alpha} = \boldsymbol{\alpha}\lambda_{BA} = (65\ \text{rad/s}^2)\left(\frac{1}{260}\right) - (100)\mathbf{j} + (240)\mathbf{k}$$

$$\boldsymbol{\alpha} = -(25\ \text{rad/s}^2)\mathbf{j} + (60\ \text{rad/s}^2)\mathbf{k}$$

Point E:

$$\mathbf{r}_{E/A} = (120\ \text{mm})\mathbf{i} - (80)\mathbf{j} - (120)\mathbf{k}$$

$$\mathbf{v}_E = \boldsymbol{\omega} \times \mathbf{r}_{E/A}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 24 \\ 120 & -80 & -120 \end{vmatrix}$$

$$= (3120\ \text{mm/s})\mathbf{i} + (2880\ \text{mm/s})\mathbf{j} + (1200\ \text{mm/s})\mathbf{k}$$

$$\mathbf{v}_E = (3.12\ \text{m/s})\mathbf{i} + (2.88\ \text{m/s})\mathbf{j} + (1.200\ \text{m/s})\mathbf{k} \blacktriangleleft$$

$$\mathbf{a}_D = \boldsymbol{\alpha} \times \mathbf{r}_{E/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{E/A}) = \boldsymbol{\alpha} \times \mathbf{r}_{E/A} + \boldsymbol{\omega} \times \mathbf{v}_E$$

$$\mathbf{a}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -25 & 60 \\ 120 & -80 & -120 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 24 \\ 3120 & 2880 & 1200 \end{vmatrix}$$

$$= -(7800\ \text{mm/s}^2)\mathbf{i} + (7200\ \text{mm/s}^2)\mathbf{j} + (3000\ \text{mm/s}^2)\mathbf{k}$$

$$-(81120\ \text{mm/s}^2)\mathbf{i} + (74880\ \text{mm/s}^2)\mathbf{j} + (31200\ \text{mm/s}^2)\mathbf{k}$$

$$\mathbf{a}_B = -(73320\ \text{mm/s}^2)\mathbf{i} + (82080\ \text{mm/s}^2)\mathbf{j} + (34200\ \text{mm/s}^2)\mathbf{k}$$

$$= -(73.3\ \text{m/s}^2)\mathbf{i} + (82.1\ \text{m/s}^2)\mathbf{j} + (34.2\ \text{m/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.16

The earth makes one complete revolution around the sun in 365.24 days. Assuming that the orbit of the earth is circular and has a radius of 93,000,000 mi, determine the velocity and acceleration of the earth.

SOLUTION

$$\omega = \frac{2\pi \text{ rad}}{(365.24 \text{ days})\left(\frac{24 \text{ h}}{\text{day}}\right)\left(\frac{3600 \text{ s}}{\text{h}}\right)}$$

$$= 199.11 \times 10^{-9} \text{ rad/s}$$

$$v = r\omega$$

$$= (93 \times 10^6 \text{ mi})\left(\frac{5280 \text{ ft}}{\text{mi}}\right)(199.11 \times 10^{-9} \text{ rad/s})$$

$$v = 97,770 \text{ ft/s} \qquad v = 66,700 \text{ mi/h} \blacktriangleleft$$

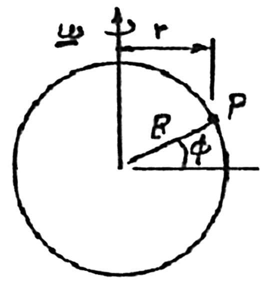
$$a = r\omega^2$$

$$= (93 \times 10^6)(5280)(199.11 \times 10^{-9} \text{ rad/s})^2 \qquad a = 19.47 \times 10^{-3} \text{ ft/s}^2 \blacktriangleleft$$

PROBLEM 15.17

The earth makes one complete revolution on its axis in 23 h 56 min. Knowing that the mean radius of the earth is 3960 mi, determine the linear velocity and acceleration of a point on the surface of the earth (*a*) at the equator, (*b*) at Philadelphia, latitude 40° north, (*c*) at the North Pole.

SOLUTION



$$23\text{ h }56\text{ m} = 23.933\text{ h}$$

$$\omega = \frac{2\pi\text{ rad}}{(23.933\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right)}$$

$$= 72.925\times 10^{-6}\text{ rad/s}$$

$$R = (3960\text{ mi})\left(\frac{5280\text{ ft}}{\text{mi}}\right)$$

$$= 20.91\times 10^{6}\text{ ft}$$

$$r = \text{radius of path}$$

$$= R\cos\phi$$

(*a*) Equator: Latitude $= \phi = 0$

$$v = r\omega$$

$$= R(\cos 0)\omega$$

$$= (20.91\times 10^{6}\text{ ft})(1)(72.925\times 10^{-6}\text{ rad/s}) \qquad v = 1525\text{ ft/s} \blacktriangleleft$$

$$a = r\omega^2$$

$$= R(\cos 0)\omega^2$$

$$= (20.91\times 10^{6}\text{ ft})(1)(72.925\times 10^{-6}\text{ rad/s})^2 \qquad a = 0.1112\text{ ft/s}^2 \blacktriangleleft$$

(*b*) Philadelphia: Latitude $= \phi = 40°$

$$v = r\omega$$

$$= R(\cos 40°)\omega$$

$$= (20.91\times 10^{6}\text{ ft})(\cos 40°)(72.925\times 10^{-6}\text{ rad/s}) \qquad v = 1168\text{ ft/s} \blacktriangleleft$$

$$a = r\omega^2$$

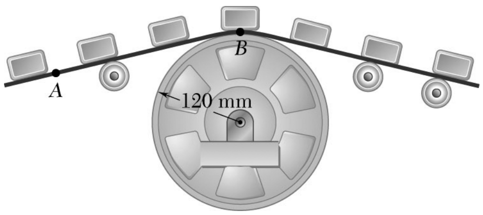
$$= R(\cos 40°)\omega^2$$

$$= (20.91\times 10^{6}\text{ ft})(\cos 40°)(72.925\times 10^{-6}\text{ rad/s})^2 \qquad a = 0.0852\text{ ft/s}^2 \blacktriangleleft$$

(*c*) North Pole: Latitude $= \phi = 0$

$$r = R\cos 0 = 0 \qquad v = a = 0 \blacktriangleleft$$

PROBLEM 15.18



A series of small machine components being moved by a conveyor belt pass over a 120 mm radius idler pulley. At the instant shown, the velocity of Point A is 300 mm/s to the left and its acceleration is 180 mm/s^2 to the right. Determine (*a*) the angular velocity and angular acceleration of the idler pulley, (*b*) the total acceleration of the machine component at B.

SOLUTION

$$v_B = v_A = 300 \text{ mm/s} \longleftarrow \qquad r_B = 120 \text{ mm}$$

$$(a_B)_t = a_A = 180 \text{ mm/s} \longrightarrow$$

(*a*) $v_B = \omega r_B,$

$$\omega = \frac{v_B}{r_B} = \frac{300}{120} = 2.5 \text{ rad/s}$$

$\boldsymbol{\omega} = 2.50$ rad/s ◀

$(a_B)_t = \alpha r_B,$

$$\alpha = \frac{(a_B)_t}{r_B} = \frac{180}{120} = 1.5 \text{ rad/s}$$

$\boldsymbol{\alpha} = 1.500$ rad/s^2 ◀

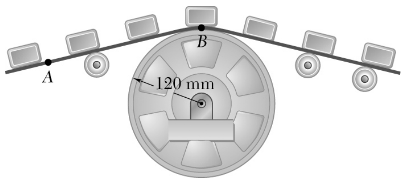
(*b*)

$$(a_B)_n = r_B\omega^2 = (120)(2.5)^2 = 750 \text{ mm/s}^2 \downarrow$$

$$a_B = \sqrt{(a_B)_t^2 + (a_B)_n^2} = \sqrt{(180)^2 + (750)^2} = 771 \text{ mm/s}^2$$

$$\tan\beta = \frac{750}{180}, \qquad \beta = 76.5°$$

$\mathbf{a}_B = 771$ mm/s^2 ⦪ 76.5° ◀



PROBLEM 15.19

A series of small machine components being moved by a conveyor belt pass over a 120-mm-radius idler pulley. At the instant shown, the angular velocity of the idler pulley is 4 rad/s clockwise. Determine the angular acceleration of the pulley for which the magnitude of the total acceleration of the machine component at B is 2400 mm/s^2.

SOLUTION

$$\omega_B = 4 \text{ rad/s} \circlearrowright, \quad r_B = 120 \text{ mm}$$

$$(a_B)_n = r_B\omega_B^2 = (120)(4)^2 = 1920 \text{ mm/s}^2$$

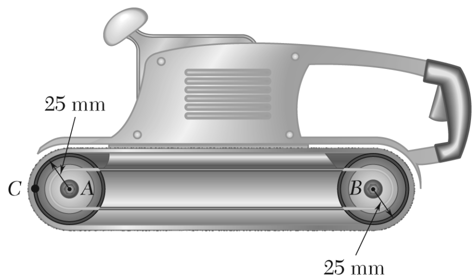
$$a_B = 2400 \text{ mm/s}^2$$

$$(a_B)_t = \sqrt{a_B^2 - (a_B)_n^2} = \sqrt{2400^2 - 1920^2} = \pm 1440 \text{ mm/s}^2$$

$$(a_B)_t = r_B\alpha, \qquad \alpha = \frac{(a_B)_t}{r_B} = \frac{\pm 1440}{120} = \pm 12 \text{ rad/s}^2$$

$$12.00 \text{ rad/s}^2 \circlearrowleft \text{ or } \circlearrowright \blacktriangleleft$$

PROBLEM 15.20

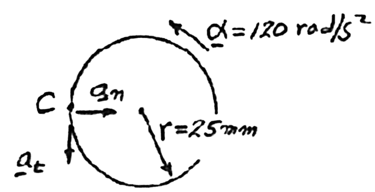


The belt sander shown is initially at rest. If the driving drum B has a constant angular acceleration of 120 rad/s^2 counter-clockwise, determine the magnitude of the acceleration of the belt at Point C when (*a*) $t = 0.5$ s, (*b*) $t = 2$ s.

SOLUTION

$$a_t = r\alpha = (0.025 \text{ m})(120 \text{ rad/s}^2)$$

$$\mathbf{a}_t = 3 \text{ m/s}^2 \downarrow$$



(*a*) $t = 0.5$ s: $\omega = \alpha t = (120 \text{ rad/s}^2)(0.5 \text{ s}) = 60 \text{ rad/s}$

$$a_n = r\omega^2 = (0.025 \text{ m})(60 \text{ rad/s})^2$$

$$\mathbf{a}_n = 90 \text{ m/s}^2 \longrightarrow$$

$$a_B^2 = a_t^2 + a_n^2 = 3^2 + 90^2$$

$a_B = 90.05 \text{ m/s}^2$ ◀

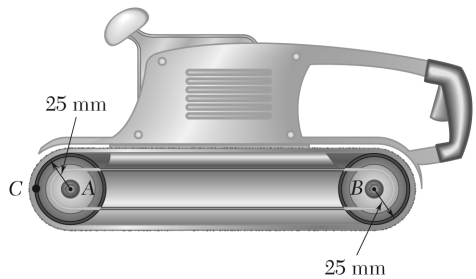
(*b*) $t = 2$ s: $\omega = \alpha t = (120 \text{ rad/s}^2)(2 \text{ s}) = 240 \text{ rad/s}$

$$a_n = r\omega^2 = (0.025 \text{ m})(240 \text{ rad/s})^2$$

$$a_n = 1440 \text{ m/s}^2$$

$$a_B^2 = a_t^2 + a_n^2 = 3^2 + 1440^2$$

$a_B = 1440 \text{ m/s}^2$ ◀



PROBLEM 15.21

The rated speed of drum B of the belt sander shown is 2400 rpm. When the power is turned off, it is observed that the sander coasts from its rated speed to rest in 10 s. Assuming uniformly decelerated motion, determine the velocity and acceleration of Point C of the belt, (*a*) immediately before the power is turned off, (*b*) 9 s later.

SOLUTION

$$\omega_0 = 2400 \text{ rpm}$$
$$= 251.3 \text{ rad/s}$$
$$r = 0.025 \text{ m}$$

(*a*) $v_C = r\omega = (0.025 \text{ m})(251.3 \text{ rad/s})$ $\qquad v_C = 6.28 \text{ m/s}$ ◀

$a_C = r\omega^2 = (0.025 \text{ m})(251.3 \text{ rad/s})^2$ $\qquad a_C = 1579 \text{ m/s}^2$ ◀

(*b*) When $t = 10$ s: $\omega = 0.$

$$\omega = \omega_0 + \alpha t$$
$$0 = 251.3 \text{ rad/s} + \alpha(10 \text{ s})$$
$$\alpha = -25.13 \text{ rad/s}^2$$

When $t = 9$ s: $\omega = \omega_0 + \alpha t$

$$\omega_9 = 251.3 \text{ rad/s} - (25.13 \text{ rad/s}^2)(9 \text{ s})$$
$$= 25.13 \text{ rad/s}$$

$$v_C = r\omega_9$$
$$= (0.025 \text{ m})(25.13 \text{ rad/s})$$

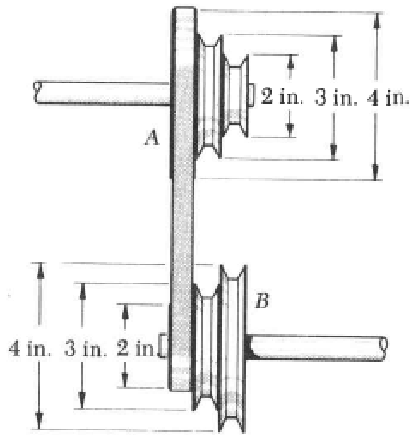
$v_9 = 0.628 \text{ m/s}$ ◀

$$(a_C)_t = r\alpha_n$$
$$= (0.025 \text{ m})(-25.13 \text{ rad/s}^2)$$
$$(a_C)_t = 0.628 \text{ m/s}^2$$
$$(a_C)_n = r\omega_9^2$$
$$= (0.025 \text{ m})(25.13 \text{ rad/s})^2$$
$$(a_C)_n = 15.79 \text{ m/s}^2$$

$$a_C^2 = (a_C)_t^2 + (a_C)_n^2$$
$$= (0.628 \text{ m/s}^2)^2 + (15.79 \text{ m/s}^2)^2$$

$a_C = 15.80 \text{ m/s}^2$ ◀

PROBLEM 15.22



The two pulleys shown may be operated with the V belt in any of three positions. If the angular acceleration of shaft A is 6 rad/s^2 and if the system is initially at rest, determine the time required for shaft B to reach a speed of 400 rpm with the belt in each of the three positions.

SOLUTION

Angular velocity of shaft A: $\omega_A = \alpha_A t$

Belt speed: $v = r_A\omega_A = r_B\omega_B$

Angular speed of shaft B: $\omega_B = \dfrac{v}{r_B} = \dfrac{r_A\alpha_A t}{r_B}$

Solving for t, $t = \dfrac{r_B\omega_B}{r_A\alpha_A}$

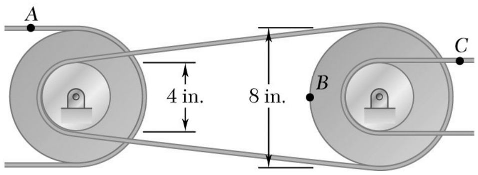
Data: $\alpha_A = 6$ rad/s, $\omega_B = 400 \text{ rpm} = 41.889 \text{ rad/s}$

$$t = \frac{r_B}{r_A}\cdot\frac{41.889}{6} = 6.9813\frac{r_B}{r_A} = 6.9813\frac{d_B}{d_A}$$

Belt at left: $\dfrac{d_B}{d_A} = \dfrac{2 \text{ in.}}{4 \text{ in.}}$ $t = 3.49$ s ◀

Belt in middle: $\dfrac{d_B}{d_A} = \dfrac{3 \text{ in.}}{3 \text{ in.}}$ $t = 6.98$ s ◀

Belt at right: $\dfrac{d_B}{d_A} = \dfrac{4 \text{ in.}}{2 \text{ in.}}$ $t = 13.96$ s ◀

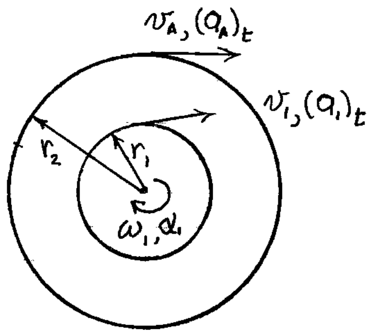


PROBLEM 15.23

Three belts move over two pulleys without slipping in the speed reduction system shown. At the instant shown the velocity of Point A on the input belt is 2 ft/s to the right, decreasing at the rate of 6 ft/s^2. Determine, at this instant, (*a*) the velocity and acceleration of Point C on the output belt, (*b*) the acceleration of Point B on the output pulley.

SOLUTION

Left pulley.



Inner radius $r_1 = 2$ in.

Outer radius $r_2 = 4$ in.

$$v_A = 2 \text{ ft/s} \longrightarrow$$

$$(a_A)_t = -6 \text{ ft/s}^2 = 6 \text{ ft/s}^2 \longleftarrow$$

$$\omega_1 = \frac{v_A}{r_2} = \frac{2}{\frac{4}{12}} = 6 \text{ rad/s}$$

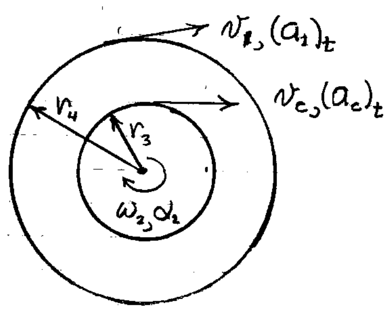
$$\alpha_1 = \frac{(a_A)_t}{r_2} = \frac{6}{\frac{4}{12}} = 18 \text{ rad/s}^2$$

Intermediate belt.

$$v_1 = r_1\omega_1 = \left(\frac{2}{12}\right)(6) = 1 \text{ ft/s}$$

$$(a_1)_t = r_1\alpha_1 = \left(\frac{2}{12}\right)(18) = 3 \text{ ft/s}^2$$

Right pulley.



Inner radius $r_3 = 2$ in.

Outer radius $r_4 = 4$ in.

$$\omega_2 = \frac{v_1}{r_4} = \frac{1}{\left(\frac{4}{12}\right)} = 3 \text{ rad/s}$$

$$\alpha_2 = \frac{(a_1)_t}{r_4} = \frac{3}{\left(\frac{4}{12}\right)} = 9 \text{ rad/s}^2$$

PROBLEM 15.23 (Continued)

(*a*) *Velocity and acceleration of Point C.*

$$v_C = r_3\omega_2 = \left(\frac{2}{12}\right)(3) = 0.5\text{ ft/s}$$

$\mathbf{v}_C = 0.5\text{ ft/s} \longrightarrow$ ◀

$$(a_C)_t = r_3\alpha_2 = \left(\frac{2}{12}\right)(9) = 1.5\text{ ft/s}^2$$

$\mathbf{a}_C = 1.5\text{ ft/s}^2 \longleftarrow$ ◀

(*b*) *Acceleration of Point B.*

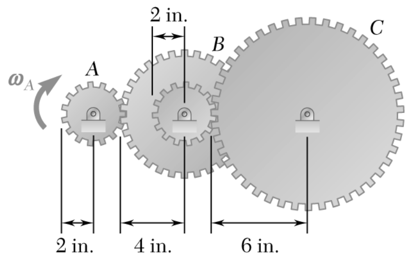
$$(a_B)_n = r_4\omega_2^2 = \left(\frac{4}{12}\right)(3)^2 = 3\text{ ft/s}^2$$

$(\mathbf{a}_B)_n = 3\text{ ft/s}^2 \longrightarrow$

$$(a_B)_t = r_4\alpha_2 = \left(\frac{4}{12}\right)(9) = 3\text{ ft/s}^2$$

$(\mathbf{a}_B)_t = 3\text{ ft/s}^2 \downarrow$

$\mathbf{a}_B = 4.24\text{ ft/s}^2 \searrow 45°$ ◀



PROBLEM 15.24

A gear reduction system consists of three gears A, B, and C. Knowing that gear A rotates clockwise with a constant angular velocity ω_A = 600 rpm, determine (*a*) the angular velocities of gears B and C, (*b*) the accelerations of the points on gears B and C which are in contact.

SOLUTION

(*a*)

$$\omega_A = 600 \text{ rpm} = \frac{(600)(2\pi)}{60} = 20\pi \text{ rad/s.}$$

Let Points A, B, and C lie at the axles of gears A, B, and C, respectively.

Let D be the contact point between gears A and B.

$$v_D = r_{D/A}\omega_A = (2)(20\pi) = 40\pi \text{ in./s} \downarrow$$

$$\omega_B = \frac{v_D}{r_{D/B}} = \frac{40\pi}{4} = 10\pi \text{ rad/s} = 10\pi \cdot \frac{60}{2\pi} = 300 \text{ rpm} \circlearrowleft$$

$$\boldsymbol{\omega}_B = 300 \text{ rpm} \circlearrowleft \blacktriangleleft$$

Let E be the contact point between gears B and C.

$$v_E = r_{E/B}\omega_B = (2)(10\pi) = 20\pi \text{ in./s} \uparrow$$

$$\omega_C = \frac{v_E}{r_{E/C}} = \frac{20\pi}{6} = 3.333\pi \text{ rad/s} = (3.333\pi)\frac{60}{2\pi} = 100 \text{ rpm} \circlearrowright$$

$$\boldsymbol{\omega}_C = 100 \text{ rpm} \circlearrowright \blacktriangleleft$$

(*b*) *Accelerations at Point E.*

On gear B:

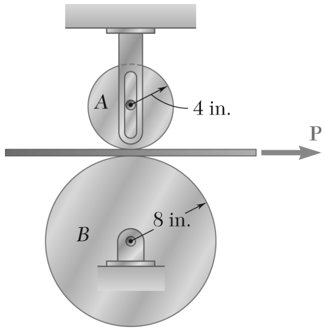
$$a_B = \frac{v_E^2}{r_{E/B}} = \frac{(20\pi)^2}{2} = 1973.9 \text{ in./s}^2 \longleftarrow$$

$$\mathbf{a}_B = 1974 \text{ in./s}^2 \longleftarrow \blacktriangleleft$$

On gear C:

$$a_C = \frac{v_E^2}{r_{E/C}} = \frac{(20\pi)^2}{6} = 658 \text{ in./s}^2 \longrightarrow$$

$$\mathbf{a}_C = 658 \text{ in./s}^2 \longrightarrow \blacktriangleleft$$



PROBLEM 15.25

A belt is pulled to the right between cylinders A and B. Knowing that the speed of the belt is a constant 5 ft/s and no slippage occurs, determine (a) the angular velocities of A and B, (b) the accelerations of the points which are in contact with the belt.

SOLUTION

(a) Angular velocities.

Disk A: $\omega_A = \dfrac{v_P}{r_A} = \dfrac{5 \text{ ft/s}}{(4/12) \text{ ft}}$ $\qquad \boldsymbol{\omega}_A = 15.00 \text{ rad/s} \circlearrowright$ ◀

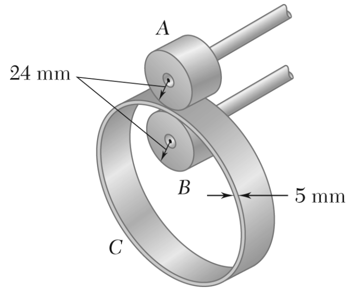
Disk B: $\omega_B = \dfrac{v_P}{r_B} = \dfrac{5 \text{ ft/s}}{(8/12) \text{ ft}}$ $\qquad \boldsymbol{\omega}_B = 7.50 \text{ rad/s} \circlearrowright$ ◀

(b) Accelerations of contact points.

Disk A: $a_A = \omega_A^2 r_A = (15.00 \text{ rad/s})^2((4/12) \text{ ft})$ $\qquad \mathbf{a}_A = 75.00 \text{ rad/s}^2 \uparrow$ ◀

Disk B: $a_B = \omega_B^2 r_B = (7.50 \text{ rad/s})^2((8/12) \text{ ft})$ $\qquad \mathbf{a}_B = 37.5 \text{ rad/s}^2 \downarrow$ ◀

PROBLEM 15.26



Ring C has an inside radius of 55 mm and an outside radius of 60 mm and is positioned between two wheels A and B, each of 24-mm outside radius. Knowing that wheel A rotates with a constant angular velocity of 300 rpm and that no slipping occurs, determine (a) the angular velocity of the ring C and of wheel B, (b) the acceleration of the Points of A and B which are in contact with C.

SOLUTION

$$\omega_A = 300 \text{ rpm}\left(\frac{2\pi}{60}\right)$$

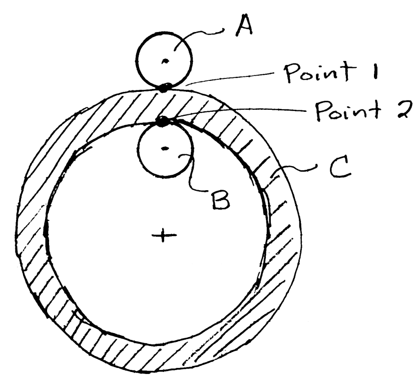
$$= 31.416 \text{ rad/s}$$

$$r_A = 24 \text{ mm}$$

$$r_B = 24 \text{ mm}$$

$$r_1 = 60 \text{ mm}$$

$$r_2 = 55 \text{ mm}$$



[We assume senses of rotation shown for our computations.]

(a) Velocities:

Point 1 (Point of contact of A and C)

$$v_1 = r_A\omega_A = r_1\omega_C$$

$$\omega_C = \frac{r_A}{r_1}\omega_A$$

$$= \frac{24 \text{ mm}}{60 \text{ mm}}(300 \text{ rpm})$$

$$= 120 \text{ rpm}$$

$\omega_C = 120 \text{ rpm}$ ◀

Point 2 (Point of contact of B and C)

$$v_2 = r_B\omega_B = r_2\omega_C$$

$$\omega_B = \frac{r_2}{r_B}\omega_C$$

$$= \frac{r_2}{r_B}\left(\frac{r_A}{r_1}\right)\omega_A$$

$$= \frac{55 \text{ mm}}{24 \text{ mm}}\left(\frac{24 \text{ mm}}{60 \text{ mm}}\right)300 \text{ rpm}$$

$$\omega_B = 275 \text{ mm}$$

$\omega_B = 275 \text{ rpm}$ ◀

PROBLEM 15.26 (Continued)

(*b*) Accelerations:

Point on rim of *A*: $r_A = 24 \text{ mm} = 0.024 \text{ m}$

$$a_A = r_A \omega_A^2$$

$$= (0.024 \text{ m})(31.416 \text{ rad/s})^2$$

$$= 23.687 \text{ m/s}^2 \qquad \mathbf{a}_A = 23.7 \text{ m/s}^2 \uparrow \blacktriangleleft$$

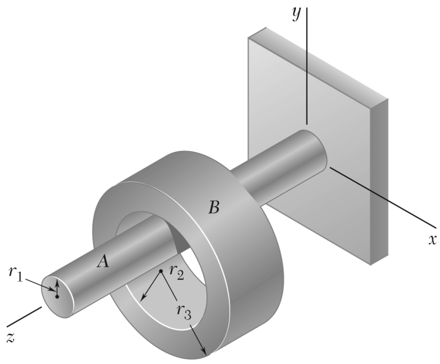
Point on rim of *B*: $\omega_B = 275 \text{ rpm}\left(\dfrac{2\pi}{60}\right)$

$$= 28.798 \text{ rad/s}$$

$$a_B = r_B \omega_B^2$$

$$= (0.024 \text{ m})(28.798 \text{ rad/s})^2$$

$$= 19.904 \text{ m/s}^2 \qquad \mathbf{a}_B = 19.90 \text{ m/s}^2 \downarrow \blacktriangleleft$$



PROBLEM 15.27

Ring B has an inside radius r_2 and hangs from the horizontal shaft A as shown. Shaft A rotates with a constant angular velocity of 25 rad/s and no slipping occurs. Knowing that $r_1 = 12$ mm, $r_2 = 30$ mm, and $r_3 = 40$ mm, determine (*a*) the angular velocity of ring B, (*b*) the accelerations of the points of shaft A and ring B which are in contact, (*c*) the magnitude of the acceleration of a point on the outside surface of ring B.

SOLUTION

Let Point C be the point of contact between the shaft and the ring.

$$v_C = r_1\omega_A$$

$$\omega_B = \frac{v_C}{r_2}$$

$$= \frac{r_1\omega_A}{r_2} \qquad \omega_B = \frac{r_1\omega_A}{r_2} \downarrow$$

On shaft A: $$a_A = r_1\omega_A^2 \qquad \mathbf{a}_A = r_1\omega_A^2 \downarrow$$

On ring B: $$a_B = r_2\omega_B^2 = r_2\left(\frac{r_1\omega_A}{r_2}\right)^2 \qquad \mathbf{a}_B = \frac{r_1^2\omega_A^2}{r_2} \downarrow$$

<u>Acceleration of Point D on outside of ring</u>.

$$a_D = r_3\omega_B^2 = r_3\left(\frac{r_1}{r_2}\omega_A\right)^2$$

$$a_D = r_3\left(\frac{r_1}{r_2}\right)^2 \omega_A^2 \downarrow$$

<u>Data</u>:

$$\omega_A = 25 \text{ rad/s}$$
$$r_1 = 12 \text{ mm}$$
$$r_2 = 30 \text{ mm}$$
$$r_3 = 40 \text{ mm}$$

(*a*)

$$\omega_B = \frac{r_1}{r_2}\omega_A$$

$$= \frac{12 \text{ mm}}{30 \text{ mm}}(25 \text{ rad/s}) \qquad \boldsymbol{\omega}_B = 10 \text{ rad/s} \circlearrowright \blacktriangleleft$$

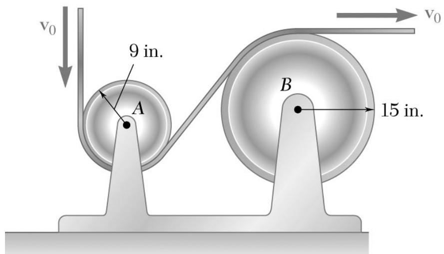
(*b*)

$$a_A = r_1 \omega_A^2$$
$$= (12 \text{ mm})(25 \text{ rad/s})^2$$
$$= 7.5 \times 10^3 \text{ mm/s}^2 \qquad \mathbf{a}_A = 7.50 \text{ m/s}^2 \downarrow \blacktriangleleft$$

$$a_B = \frac{r_1^2}{r_2} \omega_A^2$$
$$= \frac{(12 \text{ mm})^2}{(30 \text{ mm})} (25 \text{ rad/s})^2$$
$$= 3 \times 10^3 \text{ mm/s}^2 \qquad \mathbf{a}_B = 3.00 \text{ m/s}^2 \downarrow \blacktriangleleft$$

(*c*)

$$a_D = r_3 \left(\frac{r_1}{r_2} \right)^2 \omega_A^2$$
$$= (40 \text{ mm}) \left(\frac{12 \text{ mm}}{30 \text{ mm}} \right)^2 (25 \text{ rad/s})^2$$
$$a_D = 4 \times 10^3 \text{ mm/s}^2 \qquad \mathbf{a}_D = 4.00 \text{ m/s}^2 \downarrow \blacktriangleleft$$



PROBLEM 15.28

A plastic film moves over two drums. During a 4-s interval the speed of the tape is increased uniformly from v_0 = 2 ft/s to v_1 = 4 ft/s. Knowing that the tape does not slip on the drums, determine (*a*) the angular acceleration of drum *B*, (*b*) the number of revolutions executed by drum *B* during the 4-s interval.

SOLUTION

Belt motion:

$$v = v_0 + at$$

$$4 \text{ ft/s} = 2 \text{ ft/s} + a(4 \text{ s})$$

$$a = \frac{4 \text{ ft/s} - 2 \text{ ft/s}}{4 \text{ s}} = 0.5 \text{ ft/s}^2 = 6 \text{ in./s}^2$$

Since the belt does not slip relative to the periphery of the drum, the tangential accelation at the periphery of the drum is

$$a_t = 6 \text{ in./s}^2$$

(*a*) *Angular acceleration of drum B.*

$$\alpha_B = \frac{a_t}{r_B} = \frac{6 \text{ in./s}^2}{15 \text{ in.}}$$

$$\boldsymbol{\alpha}_B = 0.400 \text{ rad/s}^2 ◀$$

(*b*) *Angular displacement of drum B.*

At $t = 0$,

$$\omega_0 = \frac{v_0}{r_B} = \frac{24 \text{ in./s}}{15 \text{ in.}} = 1.6 \text{ rad/s}$$

At $t = 4$ s,

$$\omega_1 = \frac{v_1}{r_B} = \frac{48 \text{ in./s}}{15 \text{ in.}} = 3.2 \text{ rad/s}$$

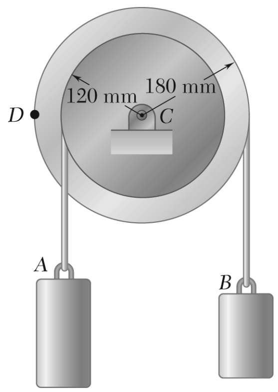
$$\omega_1^2 = \omega_0^2 + 2\alpha_B\theta_B$$

$$\theta_B = \frac{\omega_1^2 - \omega_0^2}{2\alpha_B} = \frac{(3.2)^2 - (1.6)^2}{(2)(0.400)} = 9.6 \text{ radians}$$

In revolutions,

$$\theta_B = \frac{9.6}{2\pi}$$

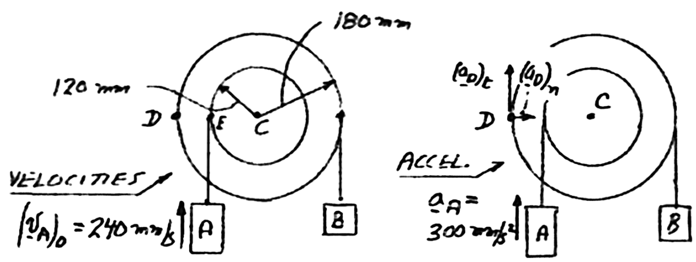
$$\theta_B = 1.528 \text{ rev} ◀$$



PROBLEM 15.29

A pulley and two loads are connected by inextensible cords as shown. Load A has a constant acceleration of 300 mm/s^2 and an initial velocity of 240 mm/s, both directed upward. Determine (a) the number of revolutions executed by the pulley in 3 s, (b) the velocity and position of load B after 3 s, (c) the acceleration of Point D on the rim of the pulley at $t = 0$.

SOLUTION



(a) Motion of pulley:

$$(\mathbf{v}_E)_0 = (v_A)_0 = 240 \text{ mm/s} \uparrow$$

$$(\mathbf{a}_E)_t = \mathbf{a}_A = 300 \text{ mm/s}^2 \uparrow$$

$$(v_E)_0 = r\omega_0: \quad 240 \text{ mm/s} = (120 \text{ mm})\omega_0 \qquad \boldsymbol{\omega}_0 = 2 \text{ rad/s} \circlearrowright$$

$$(a_E)_t = r\alpha: \quad 300 \text{ mm}^2\text{/s} = (120 \text{ mm})\alpha \qquad \boldsymbol{\alpha} = 2.5 \text{ rad/s}^2 \circlearrowright$$

For $t = 3$ s:

$$\omega = \omega_0 + \alpha t$$
$$= 2 \text{ rad/s} + (2.5 \text{ rad/s}^2)(3 \text{ s})$$
$$= 9.5 \text{ rad/s}$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$= (2 \text{ rad/s})(3 \text{ s}) + \frac{1}{2}(2.5 \text{ rad/s}^2)(3 \text{ s})^2$$

$$\theta = 17.25 \text{ rad} \qquad \theta = 17.25\left(\frac{1}{2\pi}\right) \qquad \theta = 2.75 \text{ rev} \blacktriangleleft$$

(b) Load B:

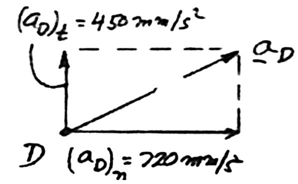
$$r = 180 \text{ mm}$$
$$t = 3 \text{ s}$$

$$v_B = r\omega = (0.180 \text{ m})(9.5 \text{ rad/s}) = 1.710 \text{ m/s} \qquad \mathbf{v}_B = 1.710 \text{ m/s} \downarrow \blacktriangleleft$$

$$\Delta y_B = r\theta = (0.180 \text{ m})(17.25 \text{ rad}) = 3.105 \text{ m} \qquad \Delta y_B = 3.11 \text{ m} \downarrow \blacktriangleleft$$

PROBLEM 15.29 (Continued)

(*c*) Point *D*: $r = 180\text{ mm} \qquad t = 0$

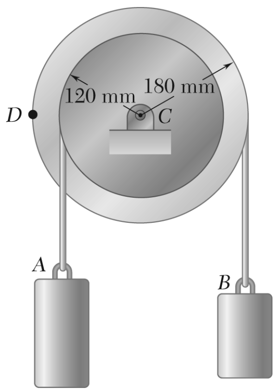


$$(\mathbf{a}_D)_t = r\alpha = (180\text{ mm})(2.5\text{ rad/s}^2) = 450\text{ mm/s}^2$$

$$(a_D)_n = r\omega_0^2 = (180\text{ mm})(2\text{ rad/s})^2 = 720\text{ mm/s}^2$$

$$(\mathbf{a}_D)_n = 720\text{ mm/s}^2 \longrightarrow$$

$\mathbf{a}_D = 849\text{ mm/s}^2 \measuredangle\ 32.0°$ ◀



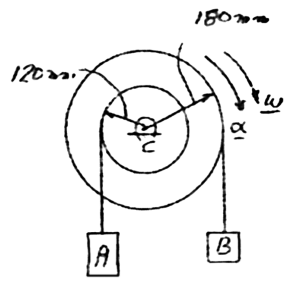
PROBLEM 15.30

A pulley and two loads are connected by inextensible cords as shown. The pulley starts from rest at $t = 0$ and is accelerated at the uniform rate of 2.4 rad/s^2 clockwise. At $t = 4$ s, determine the velocity and position (*a*) of load *A*, (*b*) of load *B*.

SOLUTION

Uniformly accelerated motion. $\omega_0 = 0$

$$\alpha = 2.4 \text{ rad/s}^2$$



At $t = 4$ s:

$$\omega = \omega_0 + \alpha t = 0 + (2.4 \text{ rad/s}^2)(4 \text{ s})$$

$$\boldsymbol{\omega} = 9.6 \text{ rad/s}^2$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= 0 + 0 + \frac{1}{2}(2.4 \text{ rad/s}^2)(4 \text{ s})^2$$

$$\theta = 19.20 \text{ rad}$$

(*a*) Load *A*. At $t = 4$ s: $r_A = 120$ mm

$$v_A = r_A \omega$$
$$= (120 \text{ mm})(9.6 \text{ rad/s})$$
$$= 1152 \text{ mm/s}$$

$$\mathbf{v}_A = 1.152 \text{ m/s} \uparrow \blacktriangleleft$$

$$y_A = r_A \theta$$
$$= (120 \text{ mm})(19.2 \text{ rad})$$
$$= 2304 \text{ mm}$$

$$\mathbf{y}_A = 2.30 \text{ m} \uparrow \blacktriangleleft$$

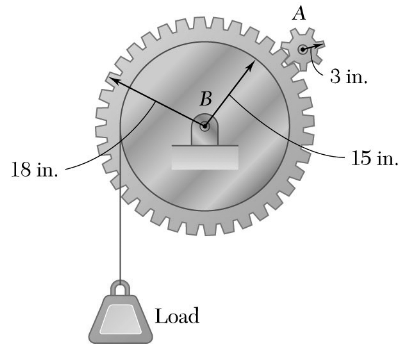
(*b*) Load *B*. At $t = 4$ s: $r_B = 180$ mm

$$v_B = r_B \omega$$
$$= (180 \text{ mm})(9.6 \text{ rad/s})$$
$$= 1728 \text{ mm/s}$$

$$\mathbf{v}_B = 1.728 \text{ m/s} \downarrow \blacktriangleleft$$

$$y_B = r_B \theta$$
$$= (180 \text{ mm})(19.2 \text{ rad})$$
$$= 3456 \text{ mm}$$

$$\mathbf{y}_B = 3.46 \text{ m} \downarrow \blacktriangleleft$$



PROBLEM 15.31

A load is to be raised 20 ft by the hoisting system shown. Assuming gear A is initially at rest, accelerates uniformly to a speed of 120 rpm in 5 s, and then maintains a constant speed of 120 rpm, determine (*a*) the number of revolutions executed by gear A in raising the load, (*b*) the time required to raise the load.

SOLUTION

The load is raised a distance $h = 20\text{ ft} = 240\text{ in.}$

For gear-pulley B, radius to rope groove is $r_1 = 15\text{ in.}$

Required angle change for B: $\theta_B = \dfrac{h}{r_1} = \dfrac{240}{15} = 16\text{ radians}$

Circumferential travel of gears A and B:

$$s = r_2\theta_B = r_A\theta_A \qquad \text{where } r_2 = 18\text{ in. and } r_A = 3\text{ in.}$$

$$s = (18\text{ in.})(16\text{ radians}) = 288\text{ in.}$$

(*a*) *Angle change of gear A*: $\theta_A = \dfrac{s}{r_A} = \dfrac{288}{3} = 96\text{ radians}$

In revolutions, $\theta_A = \dfrac{96}{2\pi}$ $\qquad \theta_A = 15.28\text{ rev}$ ◀

(*b*) *Motion of gear A*. $\omega_0 = 0, \quad \omega_f = 120\text{ rpm} = 4\pi\text{ rad/s}$

Gear A is uniformly accelerated over the first 5 seconds.

$$\alpha = \frac{\omega_f - \omega_0}{t} = \frac{4\pi\text{ rad/s}}{5\text{ s}} = 2.5133\text{ rad/s}^2$$

$$\theta_A = \frac{1}{2}\alpha t^2 = \frac{1}{2}(2.5133)(5)^2 = 31.416\text{ radians}$$

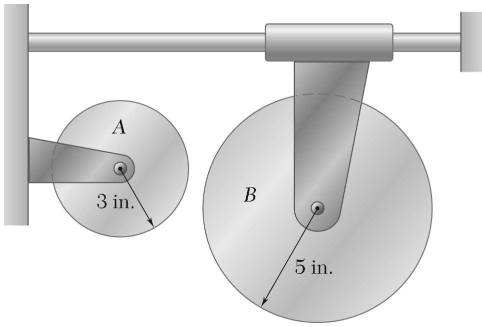
The angle change over the constant speed phase is

$$\Delta\theta = \theta_A - \theta = 96 - 31.416 = 64.584\text{ radians}$$

For uniform motion, $\Delta\theta = \omega_f(\Delta t)$

$$\Delta t = \frac{\Delta\theta}{\omega_f} = \frac{64.584}{4\pi} = 5.139\text{ s}$$

Total time elapsed: $t_f = 5\text{ s} + \Delta t$ $\qquad t_f = 10.14\text{ s}$ ◀



PROBLEM 15.32

Disk B is at rest when it is brought into contact with disk A which is rotating freely at 450 rpm clockwise. After 6 s of slippage, during which each disk has a constant angular acceleration, disk A reaches a final angular velocity of 140 rpm clockwise. Determine the angular acceleration of each disk during the period of slippage.

SOLUTION

Disk A: $(\omega_A)_0 = 450 \text{ rpm} = 47.124 \text{ rad/s}$

When $t = 6$ s: $\boldsymbol{\omega}_A = 140 \text{ rpm} = 14.661 \text{ rad/s}$

$$\omega_A = (\omega_A)_0 + \alpha_A t$$

$$14.661 \text{ rad/s} = 47.124 \text{ rad/s} + \alpha_A (6 \text{ s})$$

$$\alpha_A = -5.41 \text{ rad/s}$$

$\boldsymbol{\alpha}_A = 5.41 \text{ rad/s}^2$ ◀

Disk B: $\omega_0 = 0$

When $t = 6$ s: (end of slippage)

$+\downarrow\ r_A\omega_A = r_B\omega_B$: $(3 \text{ in.})(14.661 \text{ rad/s}) = (5 \text{ in.})(\omega_B)$

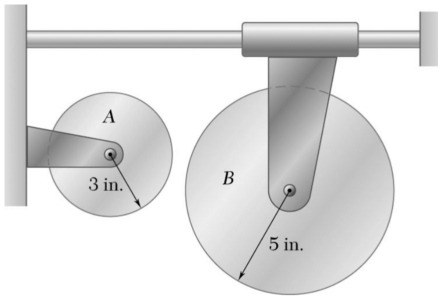
$$\omega_B = 8.796 \text{ rad/s}$$

$$\omega_B = (\omega_B)_0 + \alpha_B t$$

$$8.796 \text{ rad/s} = 0 + \alpha_B (6 \text{ s})$$

$$\alpha_B = 1.466 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_B = 1.466 \text{ rad/s}^2$ ◀



PROBLEM 15.33

A simple friction drive consists of two disks A and B. Initially, disk A has a clockwise angular velocity of 500 rpm and disk B is at rest. It is known that disk A will coast to rest in 60 s. However, rather than waiting until both disks are at rest to bring them together, disk B is given a constant angular acceleration of 2.5 rad/s^2 counterclockwise. Determine (a) at what time the disks can be brought together if they are not to slip, (b) the angular velocity of each disk as contact is made.

SOLUTION

<u>Disk A</u>: $(\omega_A)_0 = 500 \text{ rpm} = 52.36 \text{ rad/s}$ ↻

Disk A will coast to rest in 60 s.

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad 0 = 52.36 \text{ rad/s} + \alpha_A (60 \text{ s})$$

$$\alpha_A = -0.87266 \text{ rad/s}^2$$

At time t:

$$\omega_A = (\omega_A)_0 + \alpha_A t$$

$$\omega_A = 52.36 - 0.87266t \qquad (1)$$

<u>Disk B</u>: $\alpha_B = 2.5 \text{ rad/s}^2 \quad (\omega_B)_0 = 0$

At time t: $\omega_B = (\omega_B)_0 + \alpha_B t; \quad \omega_B = 2.5t \qquad (2)$

(a) Bring disks together when: $r_A\omega_A = r_B\omega_B$

$$(3 \text{ in.})(52.36 - 0.87266t) = (5 \text{ in.})(2.5t)$$

$$157.08 - 2.618t = 12.5t$$

$$157.08 = 15.118t$$

$t = 10.39$ s ◀

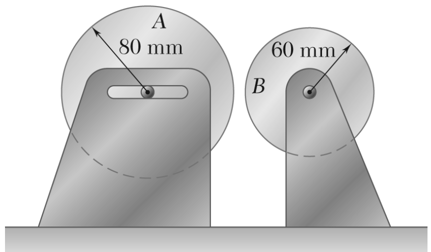
(b) When contact is made ($t = 10.39$ s)

Eq. (1): $\omega_A = 52.36 - 0.87266(10.39)$

$\omega_A = 43.29$ rad/s $\qquad \boldsymbol{\omega}_A = 413$ rpm ↻ ◀

Eq. (2): $\omega_B = 2.5(10.39)$

$\omega_B = 25.975$ rad/s $\qquad \boldsymbol{\omega}_B = 248$ rpm ↺ ◀



PROBLEM 15.34

A simple friction drive consists of two disks A and B. Initially, disk A has a clockwise angular velocity of 500 rpm and disk B is at rest. It is known that disk A will coast to rest in 60 s. However, rather than waiting until both disks are at rest to bring them together, disk B is given a constant angular acceleration of 2.5 rad/s^2 counterclockwise. Determine (a) at what time the disks can be brought together if they are not to slip, (b) the angular velocity of each disk as contact is made.

SOLUTION

Disk A: $(\boldsymbol{\omega}_A)_0 = 500 \text{ rpm} = 52.36 \text{ rad/s} \circlearrowright$

Disk a will coast to rest in 60 s.

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad 0 = 52.36 + \alpha_A (60 \text{ s})$$

$$\alpha_A = -0.87266 \text{ rad/s}^2$$

At time t:

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad \omega_A = 52.36 - 0.87266t \tag{1}$$

Disk B: $\alpha_B = 2.5 \text{ rad/s}^2 \quad (\omega_B)_0 = 0$

At time t: $\omega_B = (\omega_B)_0 + \alpha_B t; \quad \omega_B = 2.5t \quad (2)$

(a) Bring disks together when: $r_A \omega_A = r_B \omega_B$

$$(80 \text{ mm})(52.36 - 0.87266t) = (60 \text{ mm})(2.5t)$$

$$4188.8 - 69.813t = 150t$$

$$4188.8 = 219.813t$$

$$t = 19.056 \text{ s} \qquad t = 19.06 \text{ s} \blacktriangleleft$$

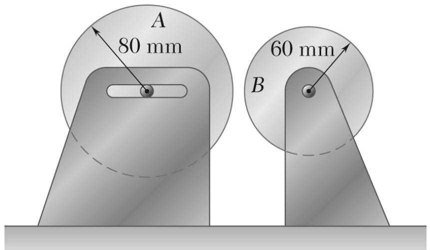
(b) Contact is made:

Eq. (1): $\omega_A = 52.36 - 0.87266(19.056)$

$$\omega_A = 35.73 \text{ rad/s} \qquad \boldsymbol{\omega}_A = 341 \text{ rpm} \circlearrowright \blacktriangleleft$$

Eq. (2): $\omega_B = 2.5(19.056)$

$$\omega_B = 47.64 \text{ rad/s} \qquad \boldsymbol{\omega}_B = 455 \text{ rpm} \circlearrowleft \blacktriangleleft$$



PROBLEM 15.35

Two friction disks A and B are both rotating freely at 240 rpm counterclockwise when they are brought into contact. After 8 s of slippage, during which each disk has a constant angular acceleration, disk A reaches a final angular velocity of 60 rpm counterclockwise. Determine (*a*) the angular acceleration of each disk during the period of slippage, (*b*) the time at which the angular velocity of disk B is equal to zero.

SOLUTION

(*a*) Disk A: $(\boldsymbol{\omega}_A)_0 = 240 \text{ rpm} = 25.133 \text{ rad/s}$

When $t = 8$ s, $\boldsymbol{\omega}_A = 60 \text{ rpm} = 6.283 \text{ rad/s}$

$$\omega_A = (\omega_A)_0 + \alpha_A t; \quad 6.283 \text{ rad/s} = 25.133 \text{ rad/s} + \alpha_A (8 \text{ s})$$

$$\alpha_A = -2.356 \text{ rad/s}^2$$

$$\boldsymbol{\alpha}_A = 2.36 \text{ rad/s}^2 \blacktriangleleft$$

Disk B: $(\omega_B)_0 = 240 \text{ rpm} = 25.123 \text{ rad/s}$

When $t = 8$ s: (slippage stops)

$$r_A \omega_A = r_B \omega_B$$

$$(80 \text{ mm})(6.283 \text{ rad/s}) = (60 \text{ mm})\omega_B$$

$$\omega_B = 8.378 \text{ rad/s} \qquad \boldsymbol{\omega}_B = 8.38 \text{ rad/s}$$

For $+$:

$$\omega_B = (\omega_B)_0 + \alpha_B t$$

$$8.375 \text{ rad/s} = -25.133 \text{ rad/s} + \alpha_B (8 \text{ s})$$

$$\alpha_B = 4.188 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_B = 4.19 \text{ rad/s}^2 \blacktriangleleft$$

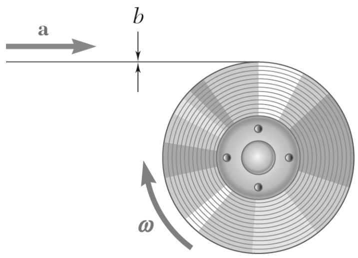
(*b*) Time when $\omega_B = 0$

For $+$:

$$\omega_B = (\omega_B)_0 + \alpha_B t$$

$$0 = -25.133 \text{ rad/s} + (4.188 \text{ rad/s}^2)t$$

$$t = 6.00 \text{ s} \qquad t = 6.00 \text{ s} \blacktriangleleft$$



PROBLEM 15.36*

Steel tape is being wound onto a spool which rotates with a constant angular velocity ω_0. Denoting by r the radius of the spool and tape at any given time and by b the thickness of the tape, derive an expression for the acceleration of the tape as it approaches the spool.

SOLUTION

Let one layer of tape be wound and let v be the tape speed.

$$v\Delta t = 2\pi r \quad \text{and} \quad \Delta r = b$$

$$\frac{\Delta r}{\Delta t} = \frac{bv}{2\pi r} = \frac{b\omega}{2\pi}$$

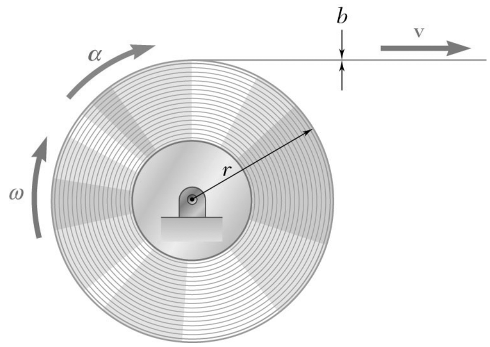
For the spool:

$$\frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{v}{r}\right) = \frac{1}{r}\frac{dv}{dt} + v\frac{d}{dt}\left(\frac{1}{r}\right)$$

$$= \frac{a}{r} - \frac{v}{r^2}\frac{dr}{dt} = \frac{a}{r} - \frac{v}{r^2}\frac{b\omega}{2\pi}$$

$$= \frac{1}{r}\left[a - \frac{b\omega^2}{2\pi}\right] = 0$$

$$\mathbf{a} = \frac{b\omega_0^2}{2\pi} \longrightarrow \blacktriangleleft$$



PROBLEM 15.37*

In a continuous printing process, paper is drawn into the presses at a constant speed v. Denoting by r the radius of the paper roll at any given time and by b the thickness of the paper, derive an expression for the angular acceleration of the paper roll.

SOLUTION

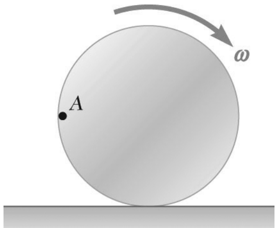
Let one layer of paper be unrolled.

$$v\Delta t = 2\pi r \quad \text{and} \quad \Delta r = -b$$

$$\frac{\Delta r}{\Delta t} = \frac{-bv}{2\pi r} = \frac{dr}{dt}$$

$$\begin{aligned}\alpha &= \frac{d\omega}{dt}\\ &= \frac{d}{dt}\left(\frac{v}{r}\right)\\ &= \frac{1}{r}\frac{dv}{dt} + v\frac{d}{dt}\left(\frac{1}{r}\right)\\ &= 0 - \frac{v}{r^2}\frac{dr}{dt}\\ &= \left(-\frac{v}{r^2}\right)\left(\frac{-bv}{2\pi r}\right)\\ &= \frac{bv^2}{2\pi r^3}\end{aligned}$$

$$\alpha = \frac{bv^2}{2\pi r^3} \circlearrowright \blacktriangleleft$$



PROBLEM 15.CQ3

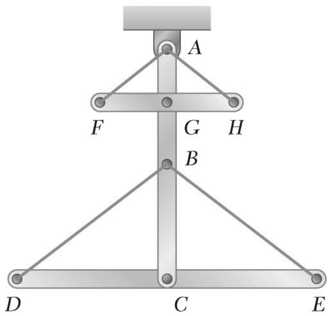
The ball rolls without slipping on the fixed surface as shown. What is the direction of the velocity of Point *A*?

(*a*) $\longrightarrow$ (*b*) $\nearrow$ (*c*) $\uparrow$ (*d*) $\downarrow$ (*e*) $\searrow$

SOLUTION

Answer: (*b*) ◀

PROBLEM 15.CQ4



Three uniform rods, *ABC*, *DCE* and *FGH* are connected as shown. Which of the following statements are true?

(*a*) $\omega_{ABC} = \omega_{DCE} = \omega_{FGH}$

(*b*) $\omega_{DCE} > \omega_{ABC} > \omega_{FGH}$

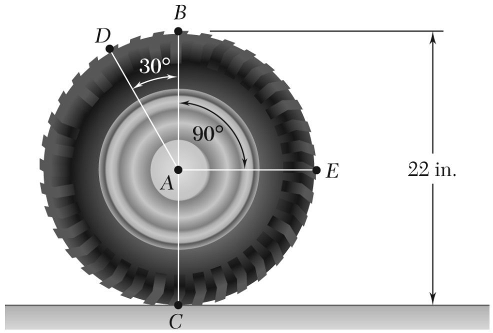
(*c*) $\omega_{DCE} < \omega_{ABC} < \omega_{FGH}$

(*d*) $\omega_{ABC} > \omega_{DCE} > \omega_{FGH}$

(*e*) $\omega_{FGH} = \omega_{DCE} < \omega_{ABC}$

SOLUTION

Answer: (*a*) ◀



PROBLEM 15.38

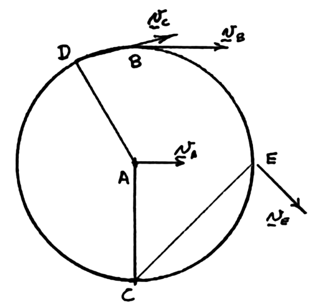
An automobile travels to the right at a constant speed of 48 mi/h. If the diameter of a wheel is 22 in., determine the velocities of Points B, C, D, and E on the rim of the wheel.

SOLUTION

$$\mathbf{v}_A = 48 \text{ mi/h} = 70.4 \text{ ft/s}$$

$$\mathbf{v}_C = 0 \blacktriangleleft$$

$$d = 22 \text{ in.} \quad r = \frac{d}{2} = 11 \text{ in.} = 0.91667 \text{ ft}$$



$$\omega = \frac{v_A}{r} = \frac{70.4}{0.91667} = 76.8 \text{ rad/s} \circlearrowright$$

$$v_{B/A} = v_{D/A} = v_{E/A} = r\omega$$

$$= (0.91667)(76.8) = 70.4 \text{ ft/s}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = [70.4 \text{ ft/s} \longrightarrow] + [70.4 \text{ ft/s} \longrightarrow]$$

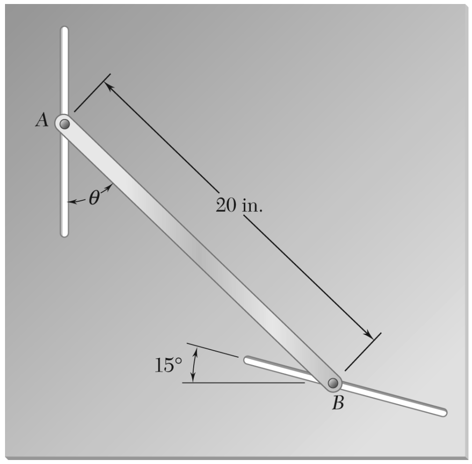
$$\mathbf{v}_B = 140.8 \text{ ft/s} \longrightarrow \blacktriangleleft$$

$$\mathbf{v}_D = \mathbf{v}_A + \mathbf{v}_{D/A} = [70.4 \text{ ft/s} \longrightarrow] + [70.4 \text{ ft/s} \measuredangle 30°]$$

$$\mathbf{v}_D = 136.0 \text{ ft/s} \measuredangle 15.0° \blacktriangleleft$$

$$\mathbf{v}_E = \mathbf{v}_A + \mathbf{v}_{E/A} = [70.4 \text{ ft/s} \longrightarrow] + [70.4 \text{ ft/s} \downarrow]$$

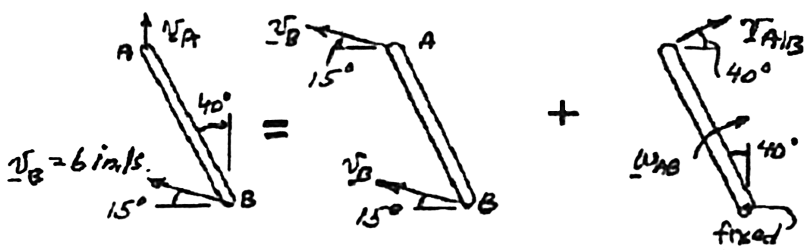
$$\mathbf{v}_E = 99.6 \text{ ft/s} \searrow 45.0° \blacktriangleleft$$



PROBLEM 15.39

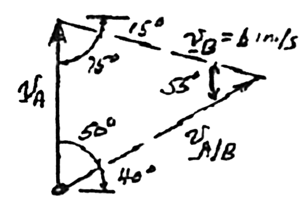
The motion of rod AB is guided by pins attached at A and B which slide in the slots shown. At the instant shown, $\theta = 40°$ and the pin at B moves upward to the left with a constant velocity of 6 in./s. Determine (*a*) the angular velocity of the rod, (*b*) the velocity of the pin at end A.

SOLUTION



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$[v_A \uparrow] = [6 \text{ in./s} \nwarrow 15°] + [v_{A/B} \measuredangle 40°]$$



Law of sines.

$$\frac{v_A}{\sin 55°} = \frac{v_{A/B}}{\sin 75°} = \frac{6 \text{ in./s}}{\sin 50°}$$

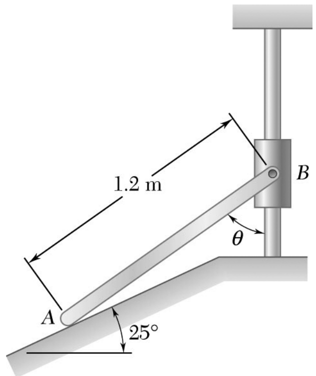
(*b*) $\mathbf{v}_A = 6.42 \text{ in./s} \uparrow$ ◀

$$\mathbf{v}_{A/B} = 7.566 \text{ in./s} \measuredangle 40°$$

$$v_{A/B} = (AB)\omega_{AB} \quad AB = 20 \text{ in.}$$

$$7.566 \text{ in./s} = (20 \text{ in.})\omega_{AB}$$

(*a*) $\omega_{AB} = 0.3783 \text{ rad/s}$ $\qquad$ $\boldsymbol{\omega}_{AB} = 0.378 \text{ rad/s} \circlearrowright$ ◀

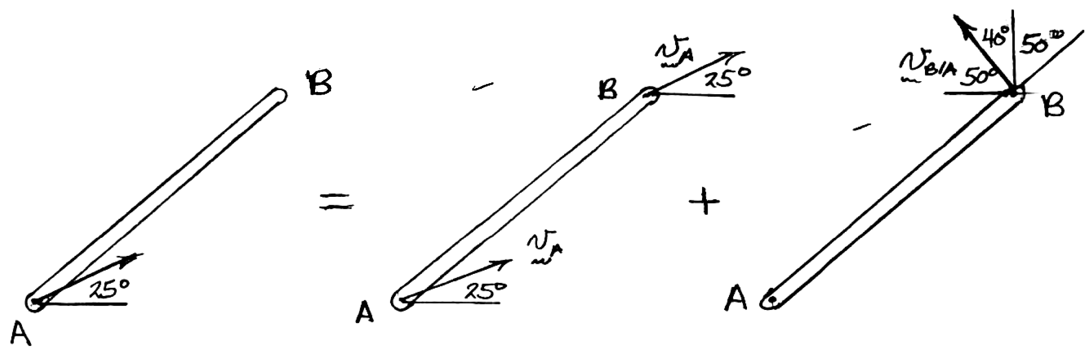


PROBLEM 15.40

Collar B moves upward with a constant velocity of 1.5 m/s. At the instant when $\theta = 50°$, determine (*a*) the angular velocity of rod AB, (*b*) the velocity of end A of the rod.

SOLUTION

Draw a diagram showing the motion of rod AB.



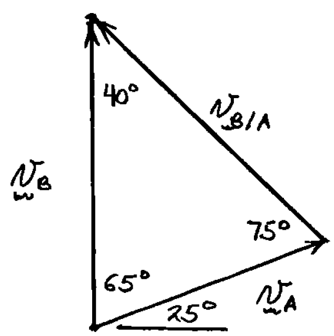
Plane motion = Translation + Rotation

$$\mathbf{v}_A = v_A \measuredangle 25° \qquad \mathbf{v}_B = 1.5 \text{ m/s} \uparrow \qquad \mathbf{v}_{B/A} = v_{B/A} \measuredangle 50°$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[1.5 \text{ m/s}\uparrow] = [v_A \measuredangle 25°] + [v_{B/A} \measuredangle 50°]$$

Draw the velocity vector diagram.



Interior angles of the triangle.

$$90° - 25° = 65°$$
$$90° - 50° = 40°$$
$$25° + 50° = 75°$$

Law of sines.

$v_B = 1.5$ m/s

$$\frac{v_B}{\sin 75°} = \frac{v_A}{\sin 40°} = \frac{v_{B/A}}{\sin 65°}$$

PROBLEM 15.40 (Continued)

(*a*) *Angular velocity of AB.*

$$v_{B/A} = \frac{\sin 65^\circ}{\sin 75^\circ}(1.5 \text{ m/s}) = 1.4074 \text{ m/s}$$

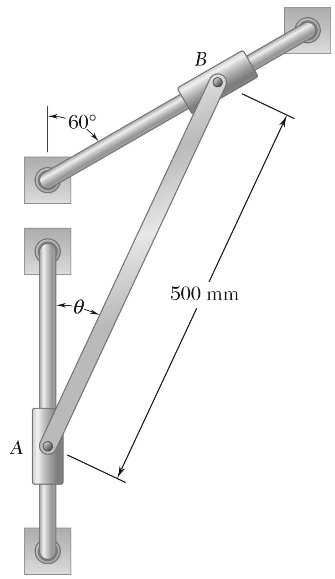
$$\omega_{AB} = \frac{v_{B/A}}{l_{AB}} = \frac{1.4074 \text{ m/s}}{1.2 \text{ m}}$$

$\boldsymbol{\omega}_{AB} = 1.173 \text{ rad/s}$ ◀

(*b*) *Velocity of end A.*

$$v_A = \frac{\sin 40^\circ}{\sin 75^\circ}(1.5 \text{ m/s})$$

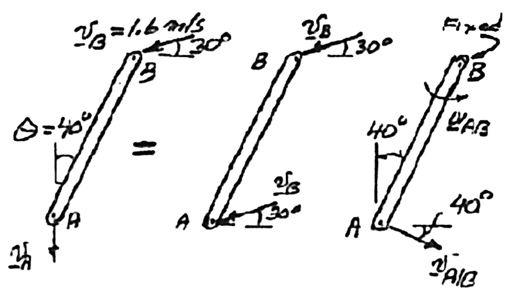
$\mathbf{v}_A = 0.998 \text{ m/s} \measuredangle\ 25^\circ$ ◀



PROBLEM 15.41

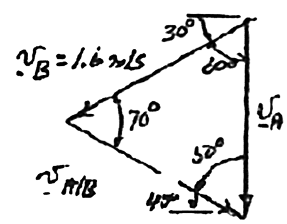
Collar B moves downward to the left with a constant velocity of 1.6 m/s. At the instant shown when $\theta = 40°$, determine (*a*) the angular velocity of rod AB, (*b*) the velocity of collar A.

SOLUTION



$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$[v_A \downarrow] = [1.6 \text{ m/s} \measuredangle 30°] + [v_{A/B} \measuredangle 40°]$$



Law of sines.

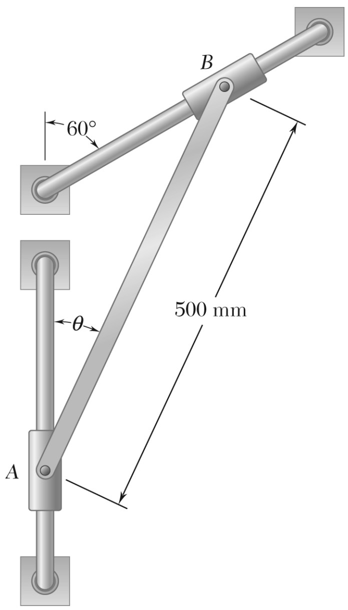
$$\frac{v_A}{\sin 70°} = \frac{v_{A/B}}{\sin 60°} = \frac{1.6 \text{ m/s}}{\sin 50°}$$

(*b*) $\mathbf{v}_A = 1.963 \text{ m/s} \downarrow$ ◀

$$\mathbf{v}_{A/B} = 1.809 \text{ m/s}^2 \measuredangle 40°$$
$$AB = 0.5 \text{ m}$$
$$v_{A/B} = (AB)\omega_{AB}$$
$$1.809 \text{ m/s} = (0.5 \text{ m})\omega_{AB}$$

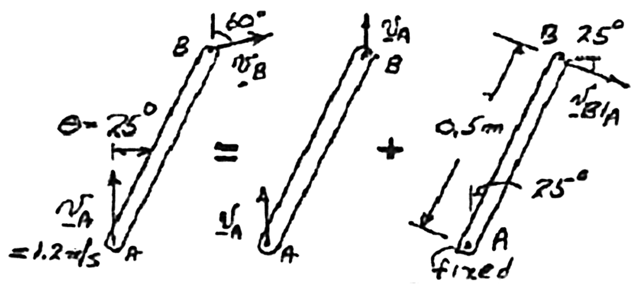
(*a*) $\omega_{AB} = 3.618 \text{ rad/s}$ $\qquad \boldsymbol{\omega}_{AB} = 3.62 \text{ rad/s} \circlearrowright$ ◀

PROBLEM 15.42



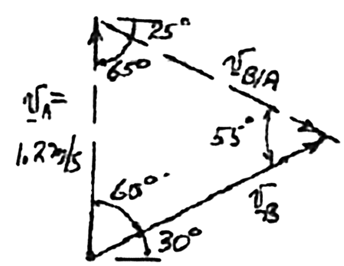
Collar A moves upward with a constant velocity of 1.2 m/s. At the instant shown when $\theta = 25°$, determine (*a*) the angular velocity of rod AB, (*b*) the velocity of collar B.

SOLUTION



$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$[v_B \measuredangle 30°] = [1.2 \text{ m/s} \uparrow] + [v_{B/A} \searrow 25°]$$



Law of sines.

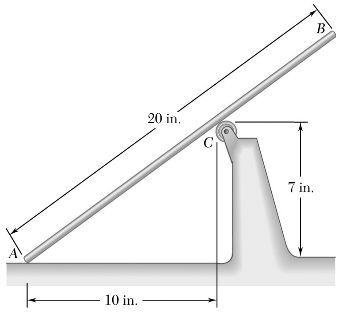
$$\frac{v_B}{\sin 65°} = \frac{v_{B/A}}{\sin 60°} = \frac{1.2 \text{ m/s}}{\sin 55°}$$

(*b*) $\mathbf{v}_B = 1.328 \text{ m/s} \measuredangle 30°$ ◀

$$\mathbf{v}_{B/A} = 1.269 \text{ m/s} \searrow 65°$$

$$v_{B/A} = (AB)\omega_{AB}$$
$$1.269 \text{ m/s} = (0.5 \text{ m})\omega_{AB}$$
$$\omega_{AB} = 2.538 \text{ rad/s}$$

(*a*) $\boldsymbol{\omega}_{AB} = 2.54 \text{ rad/s} \circlearrowright$ ◀



PROBLEM 15.43

Rod AB moves over a small wheel at C while end A moves to the right with a constant velocity of 25 in./s. At the instant shown, determine (a) the angular velocity of the rod, (b) the velocity of end B of the rod.

SOLUTION

Slope angle of rod. $\tan\theta = \dfrac{7}{10} = 0.7, \qquad \theta = 35°$

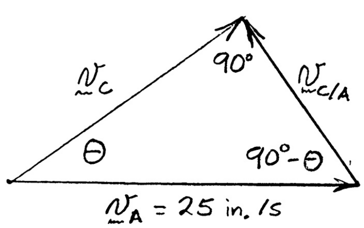
$$\overline{AC} = \frac{10}{\cos\theta} = 12.2066 \text{ in.} \qquad \overline{CB} = 20 - \overline{AC} = 7.7934 \text{ in.}$$

Velocity analysis.

$$\mathbf{v}_A = 25 \text{ in./s} \longrightarrow, \qquad \mathbf{v}_C = v_C \measuredangle\, \theta$$

$$\mathbf{v}_{C/A} = \overline{AC}\omega_{AB} \searrow \theta$$

$$\mathbf{v}_C = \mathbf{v}_A + v_{C/A}$$

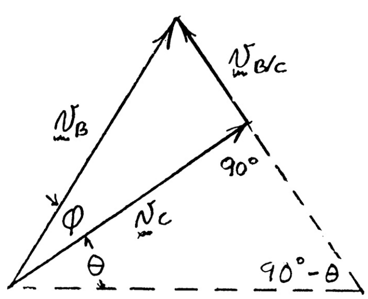


Draw corresponding vector diagram.

$$v_{C/A} = v_A \sin\theta = 25\sin 35° = 14.34 \text{ in./s}$$

(a) $$\omega_{AB} = \frac{v_{C/A}}{AC} = \frac{14.34}{12.2066} = 1.175 \text{ rad/s}$$

$$\omega_{AB} = 1.175 \text{ rad/s} \circlearrowleft \blacktriangleleft$$



$$v_C = v_A \cos\theta = 25\cos\theta = 20.479 \text{ in./s}$$

$$v_{B/C} = \overline{CB}\omega_{AB} = (7.7934)(1.175) = 9.1551 \text{ in./s}$$

$v_{B/C}$ has same direction as $v_{C/A}$.

$$\mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C}$$

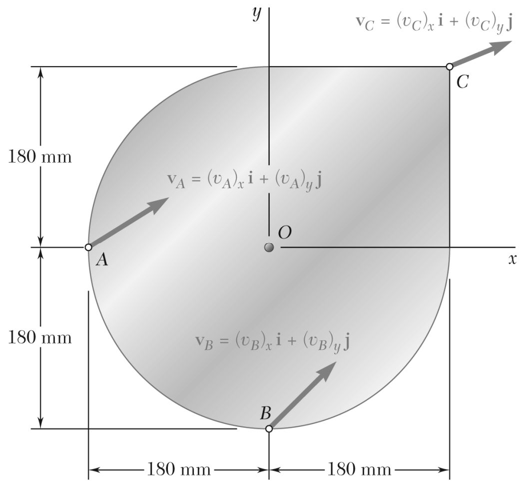
Draw corresponding vector diagram.

$$\tan\phi = \frac{v_{B/C}}{v_C} = \frac{9.1551}{20.479}, \quad \phi = 24.09°$$

(b) $$v_B = \frac{v_C}{\cos\phi} = \frac{20.479}{\cos 24.09°} = 22.4 \text{ in./s} = 1.869 \text{ ft/s}$$

$$\phi + \theta = 59.1°$$

$$\mathbf{v}_B = 1.869 \text{ ft/s} \measuredangle\, 59.1° \blacktriangleleft$$



PROBLEM 15.44

The plate shown moves in the xy plane. Knowing that $(v_A)_x = 120$ mm/s, $(v_B)_y = 300$ mm/s, and $(v_C)_y = -60$ mm/s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of Point A.

SOLUTION

$$\mathbf{r}_{C/B} = (180 \text{ mm})\mathbf{i} + (360 \text{ mm})\mathbf{j}$$

$$\boldsymbol{\omega} = \omega\mathbf{k}$$

$$\mathbf{v}_B = (v_B)_x\mathbf{i} + (300 \text{ mm/s})\mathbf{j}$$

$$\mathbf{v}_C = (v_C)_x\mathbf{i} - (60 \text{ mm/s})\mathbf{j}$$

(*a*) $$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$(v_C)_x\mathbf{i} - (60 \text{ mm/s})\mathbf{j} = (v_B)_x\mathbf{i} + (300 \text{ mm/s})\mathbf{j} + \boldsymbol{\omega}\times\mathbf{r}_{C/B}$$

$$(v_C)_x\mathbf{i} - 60\mathbf{j} = (v_B)_x\mathbf{i} + 300\mathbf{j} + \omega\mathbf{k}\times(180\mathbf{i} + 360\mathbf{j})$$

$$(v_C)_x\mathbf{i} - 60\mathbf{j} = (v_B)_x\mathbf{i} + 300\mathbf{j} + 180\omega\mathbf{j} - 360\omega\mathbf{i}$$

Coefficients of $\mathbf{j}$: $$-60 = 300 + 180\omega$$

$$\omega = -2 \text{ rad/s}$$

$\boldsymbol{\omega} = 2$ rad/s ↻ ◀

(*b*) Velocity of A: $$\mathbf{r}_{A/B} = -(180 \text{ mm})\mathbf{i} + (180 \text{ mm})\mathbf{j}$$

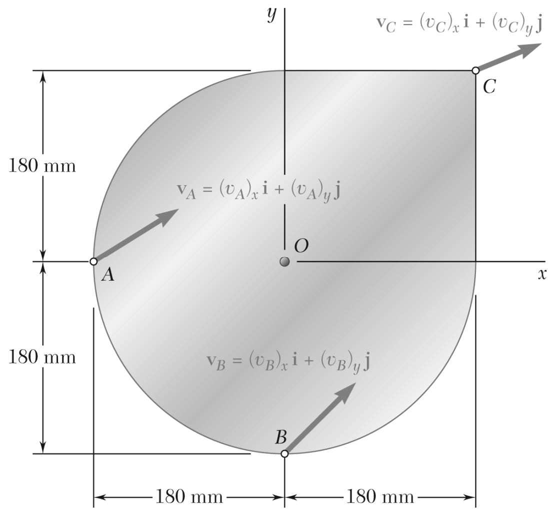
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = \mathbf{v}_B + \boldsymbol{\omega}\times\mathbf{r}_{A/B}$$

$$120\mathbf{i} + (v_A)_y\mathbf{j} = (v_B)_x\mathbf{i} + 300\mathbf{j} + (-2\mathbf{k})\times(-180\mathbf{i} + 180\mathbf{j})$$

$$120\mathbf{i} + (v_A)_y\mathbf{j} = (v_B)_x\mathbf{i} + 300\mathbf{j} + 360\mathbf{j} + 360\mathbf{i}$$

Coefficients of $\mathbf{j}$: $$(v_A)_y = 300 + 360 = 660 \text{ mm/s}$$

$\mathbf{v}_A = (120 \text{ mm/s})\mathbf{i} + (660 \text{ mm/s})\mathbf{j}$ ◀



PROBLEM 15.45

In Problem 15.44, determine (*a*) the velocity of Point *B*, (*b*) the point of the plate with zero velocity.

PROBLEM 15.44 The plate shown moves in the *xy* plane. Knowing that $(v_A)_x = 120$ mm/s, $(v_B)_y = 300$ mm/s, and $(v_C)_y = -60$ mm/s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of Point *A*.

SOLUTION

$$\mathbf{r}_{B/A} = (180 \text{ mm})\mathbf{i} - (180 \text{ mm})\mathbf{j}$$

From the answer of Problem 15.44, we have

$$\boldsymbol{\omega} = -(2 \text{ rad/s})\mathbf{k}$$
$$\mathbf{v}_A = (120 \text{ mm/s})\mathbf{i} + (660 \text{ mm/s})\mathbf{j}$$

(*a*) *Velocity of B*:

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \boldsymbol{\omega}\times\mathbf{r}_{B/A}$$
$$= 120\mathbf{i} + 660\mathbf{j} - 2\mathbf{k}\times(180\mathbf{i} - 180\mathbf{j})$$
$$= 120\mathbf{i} + 660\mathbf{j} - 360\mathbf{j} - 360\mathbf{i}$$

$$\mathbf{v}_B = -(240 \text{ mm/s})\mathbf{i} + (300 \text{ mm/s})\mathbf{j} \blacktriangleleft$$

(*b*) *Point with* $v = 0$:

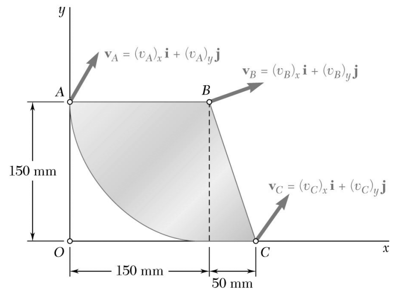
Let $\mathbf{P} = x\mathbf{i} + y\mathbf{j}$ be an arbitrary point.

Thus

$$\mathbf{r}_{P/A} = (180 + x)\mathbf{i} + y\mathbf{j}$$
$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{v}_{P/A} = \mathbf{v}_A + \boldsymbol{\omega}\times\mathbf{r}_{P/A}$$
$$\mathbf{v}_P = 120\mathbf{i} + 660\mathbf{j} + (-2\mathbf{k})\times[(180 + x)\mathbf{i} + y\mathbf{j}]$$
$$\mathbf{v}_P = 120\mathbf{i} + 660\mathbf{j} - (360 + 2x)\mathbf{j} + 2y\mathbf{i}$$
$$\mathbf{v}_P = (120 + 2y)\mathbf{i} + (300 - 2x)\mathbf{j}$$

For $\mathbf{v}_P = 0$: $\quad 120 + 2y = 0 \quad$ and $\quad 300 - 2x = 0$

$\mathbf{v} = 0$ at: $\quad y = -60$ mm, $\quad x = 150$ mm $\blacktriangleleft$



PROBLEM 15.46

The plate shown moves in the xy plane. Knowing that $(v_A)_x = 250$ mm/s, $(v_B)_x = -450$ mm/s, and $(v_C)_x = -500$ mm/s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of Point A.

SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega\mathbf{k}$

Relative position vectors:

$$\mathbf{r}_{B/A} = (150\text{ mm})\mathbf{i}$$

$$\mathbf{r}_{C/A} = (200\text{ mm})\mathbf{i} + (150\text{ mm})\mathbf{j}$$

Velocity vectors:

$$\mathbf{v}_A = (250\text{ mm/s})\mathbf{i} + (v_A)_y\mathbf{j}$$
$$\mathbf{v}_B = (v_B)_x\mathbf{i} - (450\text{ mm/s})\mathbf{j}$$
$$\mathbf{v}_C = -(500\text{ mm/s})\mathbf{i} + (v_C)_y\mathbf{j}$$

Unknowns are ω, $(v_A)_y$, $(v_B)_x$, and $(v_C)_y$.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \omega\mathbf{k}\times\mathbf{r}_{B/A}$$

$$(v_B)_x\mathbf{i} - 450\mathbf{j} = 250\mathbf{i} + (v_A)_y\mathbf{j} + \omega\mathbf{k}\times 150\mathbf{i}$$
$$= 250\mathbf{i} + (v_A)_y\mathbf{j} + 150\omega\mathbf{j}$$

i: $\quad (v_B)_x = 250 \qquad (1)$

j: $\quad -450 = (v_A)_y + 150\omega \qquad (2)$

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_A + \omega\mathbf{k}\times\mathbf{r}_{C/A}$$

$$-500\mathbf{i} + (v_C)_y\mathbf{j} = 250\mathbf{i} + (v_A)_y\mathbf{j} + \omega\mathbf{k}\times(200\mathbf{i} - 150\mathbf{j})$$
$$= 250\mathbf{i} + (v_A)_y\mathbf{j} + 200\omega\mathbf{j} + 150\omega\mathbf{i}$$

i: $\quad -500 = 250 + 150\omega \qquad (3)$

j: $\quad (v_C)_y = (v_A)_y + 150\omega \qquad (4)$

(*a*) *Angular velocity of the plate*.

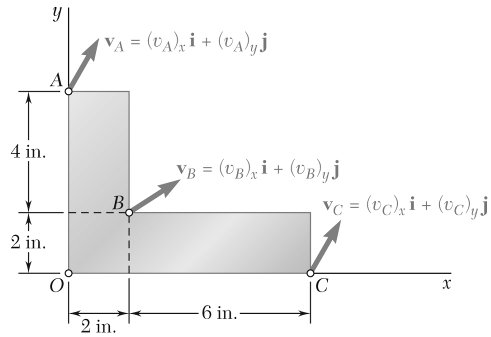
From Eq. (3), $\quad \omega = -\dfrac{750}{150} = -5$

$$\boldsymbol{\omega} = -(5.00\text{ rad/s})\mathbf{k} = 5.00\text{ rad/s} \circlearrowright \blacktriangleleft$$

(*b*) *Velocity of Point A*.

From Eq. (2), $(v_A)_y = -450 - 150\omega = -450 - (150)(-5) = 300$ mm/s

$$\mathbf{v}_A = (250\text{ mm/s})\mathbf{i} + (300\text{ mm/s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.47

The plate shown moves in the xy plane. Knowing that $(v_A)_x$ = 12 in./s, $(v_B)_x$ = −4 in./s, and $(v_C)_y$ = −24 in./s, determine (*a*) the angular velocity of the plate, (*b*) the velocity of Point *B*.

SOLUTION

Angular velocity: $\boldsymbol{\omega} = \omega\mathbf{k}$

Relative position vectors:

$$\mathbf{r}_{A/B} = -(2\text{ in.})\mathbf{i} + (4\text{ in.})\mathbf{j}$$

$$\mathbf{r}_{C/B} = (6\text{ in.})\mathbf{i} - (2\text{ in.})\mathbf{j}$$

Velocity vectors:

$$\mathbf{v}_A = (12\text{ in./s})\mathbf{i} + (v_A)_y\mathbf{j}$$

$$\mathbf{v}_B = -(4\text{ in./s})\mathbf{i} + (v_B)_y\mathbf{j}$$

$$\mathbf{v}_C = (v_C)_x\mathbf{i} - (24\text{ in./s})\mathbf{j}$$

Unknowns are ω, $(v_A)_y$, $(v_B)_y$, and $(v_C)_x$.

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = \mathbf{v}_B + \omega\mathbf{k}\times\mathbf{r}_{A/B}$$

$$12\mathbf{i} + (v_A)_y = -4\mathbf{i} + (v_B)_y\mathbf{j} + \omega\mathbf{k}\times(-2\mathbf{i} + 4\mathbf{j})$$

$$= -4\mathbf{i} + (v_B)_y\mathbf{j} - 2\omega\mathbf{j} - 4\omega\mathbf{i}$$

$\mathbf{i}$: $$12 = -4 - 4\omega \qquad (1)$$

$\mathbf{j}$: $$(v_A)_y = (v_B)_y - 2\omega \qquad (2)$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B} = \mathbf{v}_B + \omega\mathbf{k}\times\mathbf{r}_{C/B}$$

$$(v_C)_x\mathbf{i} - 24\mathbf{j} = -4\mathbf{i} + (v_B)_y\mathbf{j} + \omega\mathbf{k}\times(6\mathbf{i} - 2\mathbf{j})$$

$$= -4\mathbf{i} + (v_B)_y\mathbf{j} + 6\omega\mathbf{j} + 2\omega\mathbf{i}$$

$\mathbf{i}$: $$(v_C)_x = -4 + 2\omega \qquad (3)$$

$\mathbf{j}$: $$-24 = (v_B)_y + 6\omega \qquad (4)$$

PROBLEM 15.47 (Continued)

(*a*) *Angular velocity of the plate.*

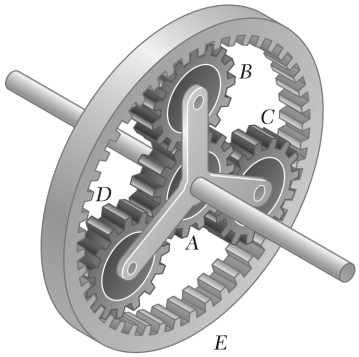
From Eq. (1), $\omega = -\frac{16}{4} = -4 \text{ rad/s}$

$\boldsymbol{\omega} = -(4.00 \text{ rad/s})\mathbf{k} = 4.00 \text{ rad/s}$ ◀

(*b*) *Velocity of Point B.*

From Eq. (4), $(v_B)_y = -24 - (6)(-4) = 0$

$\mathbf{v}_B = -(4.00 \text{ in./s})\mathbf{i}$ ◀



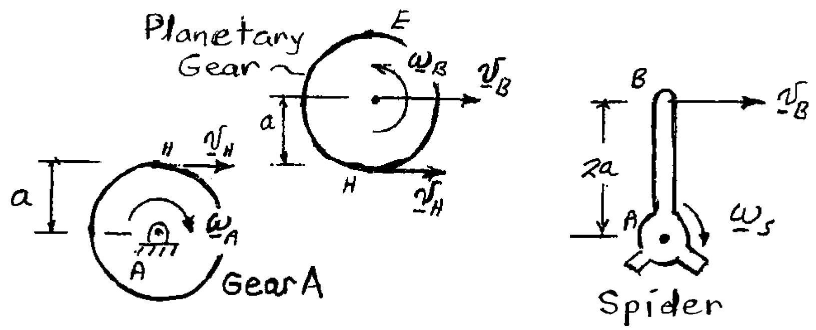
PROBLEM 15.48

In the planetary gear system shown, the radius of gears A, B, C, and D is a and the radius of the outer gear E is $3a$. Knowing that the angular velocity of gear A is ω_A clockwise and that the outer gear E is stationary, determine (*a*) the angular velocity of each planetary gear, (*b*) the angular velocity of the spider connecting the planetary gears.

SOLUTION

Gear E is stationary. $\qquad v_E = 0$

Let A be the center of gear A and the spider. Since the motions of gears B, C, and D are similar, only gear B is considered. Let H be the effective contact point between gears A and B.



Gear A: $\qquad \mathbf{v}_H = a\omega_A \longrightarrow$

(*a*) *Planetary gears B, C, and D*: $\qquad \mathbf{v}_H = \mathbf{v}_E + \mathbf{v}_{H/E}$

$$+\longrightarrow: a\omega_A = 0 + (2a)\omega_B \qquad \omega_B = \frac{1}{2}\omega_A$$

$$\boldsymbol{\omega}_B = \boldsymbol{\omega}_C = \boldsymbol{\omega}_D = \frac{1}{2}\omega_A \circlearrowright \blacktriangleleft$$

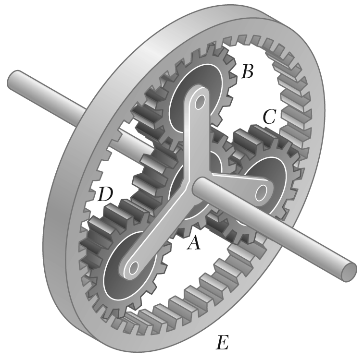
$$\mathbf{v}_B = \mathbf{v}_E + \mathbf{v}_{B/E}$$

$$+\longrightarrow: v_B = 0 + a\left(\frac{1}{2}\omega_A\right) \qquad \mathbf{v}_B = \frac{1}{2}a\omega_A \longrightarrow \qquad (1)$$

(*b*) *Spider*. $\qquad \mathbf{v}_B = (2a)\omega_s \longrightarrow \qquad (2)$

Equating expressions (1) and (2) for $\mathbf{v}_B$,

$$\frac{1}{2}a\omega_A = (2a)\omega_s \qquad \omega_s = \frac{1}{4}\omega_A \qquad \boldsymbol{\omega}_s = \frac{1}{4}\omega_A \circlearrowright \blacktriangleleft$$



PROBLEM 15.49

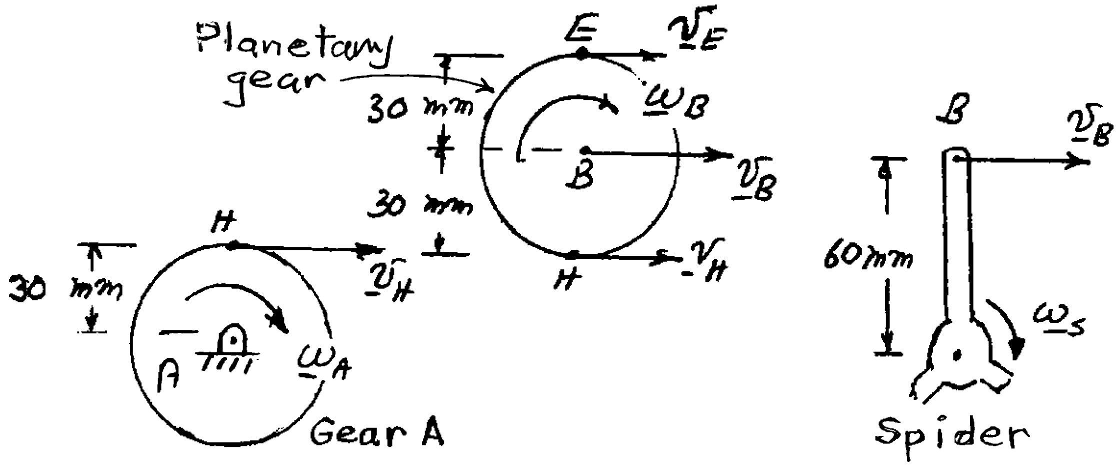
In the planetary gear system shown, the radius of gears *A*, *B*, *C*, and *D* is 30 mm and the radius of the outer gear *E* is 90 mm. Knowing that gear *E* has an angular velocity of 180 rpm clockwise and that the central gear *A* has an angular velocity of 240 rpm clockwise, determine (*a*) the angular velocity of each planetary gear, (*b*) the angular velocity of the spider connecting the planetary gears.

SOLUTION

Since the motions of the planetary gears *B*, *C*, and *D* are similar, only gear *B* is considered. Let Point *H* be the effect contact point between gears *A* and *B* and let Point *E* be the effective contact point between gears *B* and *E*.

Given angular velocities: $\boldsymbol{\omega}_E = 180 \text{ rpm} \circlearrowright = 6\pi \text{ rad/s} \circlearrowright$

$\boldsymbol{\omega}_A = 240 \text{ rpm} \circlearrowright = 8\pi \text{ rad/s} \circlearrowright$



Outer gear *E*: radius $= r_E = 90$ mm

$$v_E = r_E \omega_E = (90 \text{ mm})(6\pi \text{ rad/s}) = 540\pi \text{ mm/s}$$

$$\mathbf{v}_E = 540\pi \text{ mm/s} \longrightarrow$$

Gear *A*: radius $= r_A = 30$ mm

$$v_H = r_A \omega_A = (30 \text{ mm})(8\pi \text{ rad/s}) = 240\pi \text{ mm/s}$$

$$\mathbf{v}_H = 240\pi \text{ mm/s} \longrightarrow$$

Planetary gear B: radius $= r_B = 30$ mm, $\boldsymbol{\omega}_B = \omega_B \circlearrowright$

$$\mathbf{v}_H = \mathbf{v}_E + \mathbf{v}_{H/E}$$

$$[(30 \text{ mm})\omega_A \longrightarrow] = [540\pi \text{ mm/s} \longrightarrow] + [(60 \text{ mm})\omega_B \longleftarrow]$$

$$30\omega_A = 540\pi - 60\omega_B$$

$$\omega_B = \frac{540\pi + 30\omega_A}{60} = 9\pi + \frac{1}{2}\omega_A = 9\pi - \frac{1}{2}(8\pi) = 5\pi \text{ rad/s} \circlearrowright$$

(*a*) *Angular velocity of planetary grears*:

$$\boldsymbol{\omega}_B = \boldsymbol{\omega}_C = \boldsymbol{\omega}_D = 5\pi \text{ rad/s} \circlearrowright = 150 \text{ rpm} \circlearrowright \blacktriangleleft$$

$$\mathbf{v}_B = \mathbf{v}_H + v_{B/H} = [(30 \text{ mm})\omega_A \longrightarrow] + [30 \text{ mm } \omega_B \longrightarrow]$$

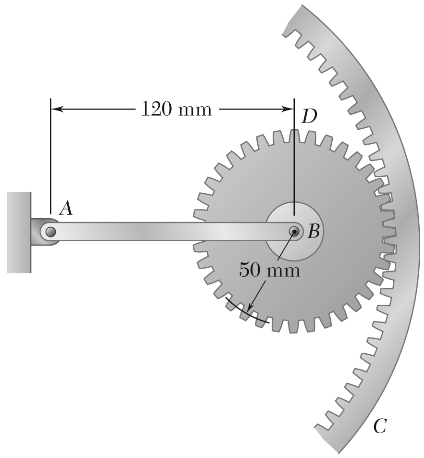
$$v_B = (30 \text{ mm})(8\pi \text{ rad/s}) + (30 \text{ mm})(5\pi \text{ rad/s}) = 390\pi \text{ mm/s}$$

(*b*) *Spider*: arm $= r_s = 60$ mm, $\boldsymbol{\omega}_s = \omega_s \circlearrowright$

$$v_B = r_s\omega_s$$

$$\omega_s = \frac{v_B}{r_s} = \frac{390\pi \text{ mm/s}}{60 \text{ mm}} = 6.5\pi \text{ rad/s}$$

$$\boldsymbol{\omega}_s = 6.5\pi \text{ rad/s} \circlearrowright = 195 \text{ rpm} \circlearrowright \blacktriangleleft$$

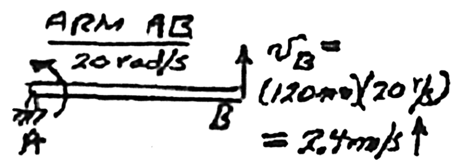


PROBLEM 15.50

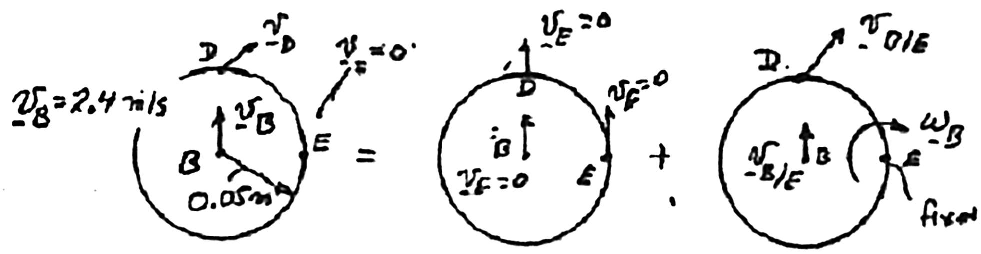
Arm AB rotates with an angular velocity of 20 rad/s counterclockwise. Knowing that the outer gear C is stationary, determine (a) the angular velocity of gear B, (b) the velocity of the gear tooth located at Point D.

SOLUTION

<u>Arm AB</u>:



<u>Gear B</u>:



(a) $BE = 0.05$ m:

$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/E} = 0 + (BE)\omega_B$$

$$2.4 \text{ m/s}\uparrow = 0 + (0.05 \text{ m})\omega_B \uparrow$$

$$\omega_B = 48 \text{ rad/s}$$

$\boldsymbol{\omega}_B = 48$ rad/s ↻ ◀

(b) $DE = (0.05\sqrt{2})$:

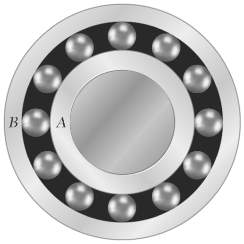
$$\mathbf{v}_D = \mathbf{v}_E + \mathbf{v}_{D/E} = 0 + (DE)\omega_B$$

$$v_D = 0 + (0.05\sqrt{2})(48)$$

$$v_D = 3.39 \text{ m/s}$$

$\mathbf{v}_D = 3.39$ m/s ∠ 45° ◀

PROBLEM 15.51



In the simplified sketch of a ball bearing shown, the diameter of the inner race A is 60 mm and the diameter of each ball is 12 mm. The outer race B is stationary while the inner race has an angular velocity of 3600 rpm. Determine (*a*) the speed of the center of each ball, (*b*) the angular velocity of each ball, (*c*) the number of times per minute each ball describes a complete circle.

SOLUTION

Data: $\omega_A = 3600 \text{ rpm} = 376.99 \text{ rad/s}, \quad \omega_B = 0$

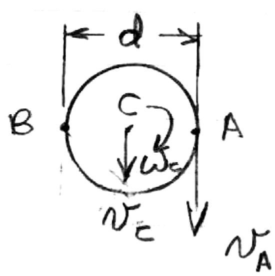
$$r_A = \frac{1}{2}d_A = 30 \text{ mm}$$

$$d = \text{diameter of ball} = 12 \text{ mm}$$

Velocity of point on inner race in contact with a ball.

$$v_A = r_A\omega_A = (30)(376.99) = 11310 \text{ mm/s}$$

Consider a ball with its center at Point C.



$$v_A = v_B + v_{A/B}$$

$$v_A = 0 + \omega_C d$$

$$\omega_C = \frac{v_A}{d} = \frac{11310}{12}$$

$$= 942.48 \text{ rad/s}$$

$$v_C = v_B + v_{C/B}$$

$$= 0 + \frac{1}{2}d\omega = (6)(942.48) = 5654.9 \text{ mm/s}$$

(*a*) $v_C = 5.65 \text{ m/s}$ ◀

(*b*) Angular velocity of ball.

$$\omega_C = 942.48 \text{ rad/s}$$

$\omega_C = 9000 \text{ rpm}$ ◀

(*c*) Distance traveled by center of ball in 1 minute.

$$l_C = v_C t = 5654.9(60) = 339290 \text{ mm}$$

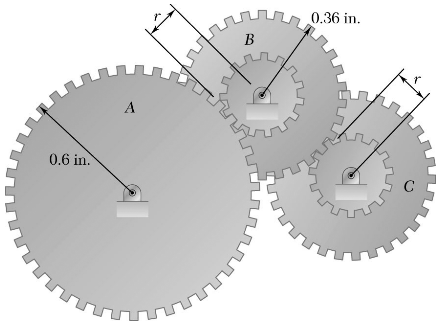
Circumference of circle: $2\pi r = 2\pi(30+6)$

$$= 226.19 \text{ mm}$$

Number of circles completed in 1 minute:

$$n = \frac{l}{2\pi r} = \frac{339290}{226.19}$$

$n = 1500$ ◀



PROBLEM 15.52

A simplified gear system for a mechanical watch is shown. Knowing that gear A has a constant angular velocity of 1 rev/h and gear C has a constant angular velocity of 1 rpm, determine (a) the radius r, (b) the magnitudes of the accelerations of the points on gear B that are in contact with gears A and C.

SOLUTION

Point where A contacts B:

$$v_1 = r_A\omega_A = r\omega_B \qquad \omega_B = \frac{r_A\omega_A}{r} \quad (1)$$

Point where B contacts C:

$$v_2 = r_B\omega_B = r\omega_C \qquad \omega_C = \frac{r_B}{r}\omega_B \quad (2)$$

From Eqs. (1) and (2),

$$\omega_C = \frac{r_A r_B}{r^2}\omega_A$$

$$r^2 = r_A r_B \frac{\omega_A}{\omega_C}$$

Data:

$$r_A = 0.6 \text{ in.}, \quad r_B = 0.36 \text{ in.} \quad \frac{\omega_A}{\omega_C} = \frac{1 \text{ rev/h}}{1 \text{ rev/m}} = \frac{1}{60}$$

$$r^2 = \frac{(0.6 \text{ in.})(0.36 \text{ in.})}{60} = 0.0036 \text{ in}^2$$

(a) *Radius r*: $r = 0.0600$ in. ◀

Angular velocity of B.

$$\omega_C = 1 \text{ rpm} = \frac{2\pi}{60} \text{ rad/s}$$

$$\omega_B = \frac{r}{r_B}\omega_C = \frac{0.060}{0.36}\frac{2\pi}{60} = 0.017453 \text{ rad/s}$$

(b) *Point where B contacts A.*

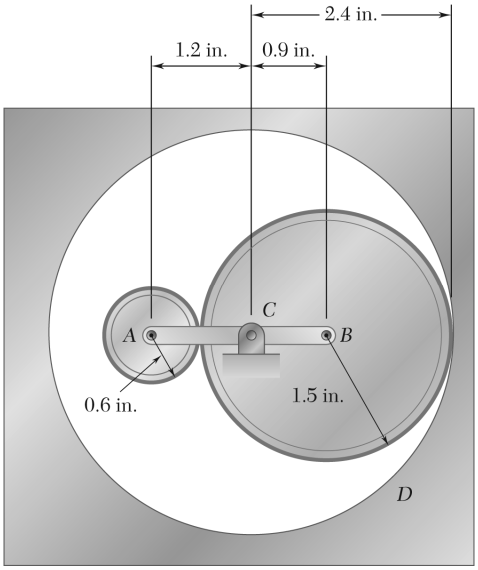
$$a_n = r\omega_B^2 = (0.0600 \text{ in.})(0.017453 \text{ rad/s})^2$$

$$a_n = 18.28\times10^{-6} \text{ in./s}^2 \quad ◀$$

Point where B contacts C.

$$a_n = r_B\omega_B^2 = (0.36 \text{ in.})(0.017453 \text{ rad/s})^2$$

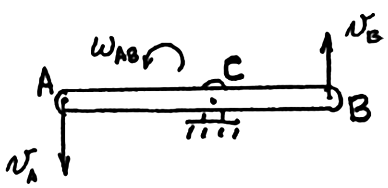
$$a_n = 109.7\times10^{-6} \text{ in./s}^2 \quad ◀$$

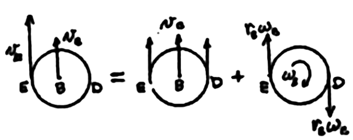


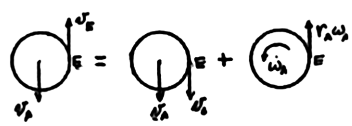
PROBLEM 15.53

Arm ACB rotates about Point C with an angular velocity of 40 rad/s counterclockwise. Two friction disks A and B are pinned at their centers to arm ACB as shown. Knowing that the disks roll without slipping at surfaces of contact, determine the angular velocity of (*a*) disk A, (*b*) disk B.

SOLUTION







Arm ACB: Fixed axis rotation.

$$r_{A/C} = 24 \text{ mm}, \quad \mathbf{v}_A = r_{A/C}\omega_{AB} = (24)(40) = 960 \text{ mm/s} \downarrow$$

$$r_{B/C} = 18 \text{ mm}, \quad \mathbf{v}_B = r_{B/C}\omega_{AB} = (18)(40) = 720 \text{ mm/s} \uparrow$$

Disk B: Plane motion = Translation with B + Rotation about B.

$$r_B = 30 \text{ mm}, \quad \mathbf{v}_D = \mathbf{v}_B - \mathbf{v}_{D/B}$$

$$0 = 720 \uparrow + 30\ \omega_B \downarrow \text{ mm/s}$$

$$\omega_B = \frac{720}{30} = 24 \text{ rad/s} \circlearrowright$$

$$\mathbf{v}_E = \mathbf{v}_B + \mathbf{v}_{E/B}$$

$$= 720 \uparrow + (30)(24) \uparrow = 1440 \text{ mm/s} \uparrow$$

Disk A: Plane motion = Translation with A + Rotation about A.

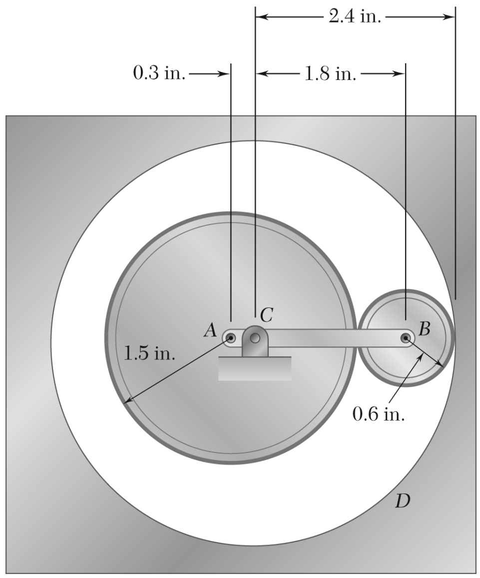
$$r_A = 12 \text{ mm}, \quad \mathbf{v}_E = \mathbf{v}_A - \mathbf{v}_{E/A}$$

$$1440 \uparrow = 960 \downarrow + 12\omega_A \uparrow$$

$$\omega_A = \frac{1440 + 960}{12} = 200 \text{ rad/s} \circlearrowleft$$

(*a*) $\boldsymbol{\omega}_A = 200 \text{ rad/s} \circlearrowleft$ ◀

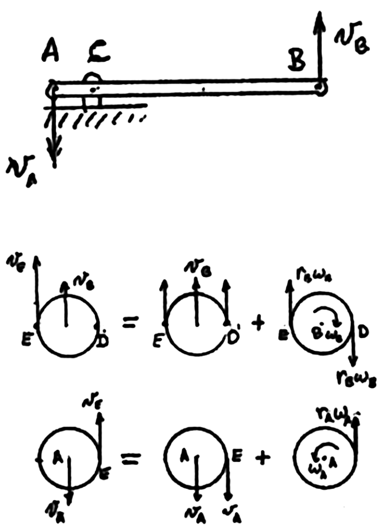
(*b*) $\boldsymbol{\omega}_B = 24.0 \text{ rad/s} \circlearrowright$ ◀



PROBLEM 15.54

Arm ACB rotates about Point C with an angular velocity of 40 rad/s counterclockwise. Two friction disks A and B are pinned at their centers to arm ACB as shown. Knowing that the disks roll without slipping at surfaces of contact, determine the angular velocity of (*a*) disk A, (*b*) disk B.

SOLUTION



Arm ACB: Fixed axis rotation.

$$r_{A/C} = 0.3 \text{ in.}, \quad \mathbf{v}_A = r_{A/C}\omega_{AB} \downarrow = (0.3)(40) \downarrow = 12 \text{ in./s} \downarrow$$

$$r_{B/C} = 1.8 \text{ in.}, \quad \mathbf{v}_B = r_{B/C}\omega_{AB} \uparrow = (1.8)(40) \uparrow = 72 \text{ in./s} \uparrow$$

Disk B: Plane motion = Translation with B + Rotation about B.

$$r_B = 0.6 \text{ in.}, \quad \mathbf{v}_D = \mathbf{v}_B - \mathbf{v}_{B/A}$$

$$0 = 72 \uparrow + 0.6\omega_B \downarrow$$

$$\omega_B = \frac{72}{0.6} = 120 \text{ rad/s} \circlearrowright$$

$$\mathbf{v}_E = \mathbf{v}_B + \mathbf{v}_{E/B}$$

$$= 72 \uparrow + (0.6)(120) \uparrow = 144 \text{ in./s} \uparrow$$

Disk A: Plane motion = Translation with A + Rotation about A.

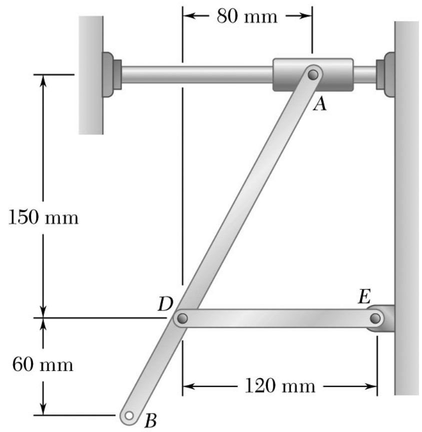
$$r_A = 1.5 \text{ in.}, \quad \mathbf{v}_E = \mathbf{v}_A - \mathbf{v}_{E/A}$$

$$144 \uparrow = 12 \downarrow \; 1.5\omega_A$$

$$\omega_A = \frac{144 + 12}{1.5} = 104 \text{ rad/s} \circlearrowright$$

(*a*) $\boldsymbol{\omega}_A = 104.0 \text{ rad/s} \circlearrowright$ ◀

(*b*) $\boldsymbol{\omega}_B = 120.0 \text{ rad/s} \circlearrowright$ ◀



PROBLEM 15.55

Knowing that at the instant shown the velocity of collar A is 900 mm/s to the left, determine (a) the angular velocity of rod ADB, (b) the velocity of Point B.

SOLUTION

Consider rod ADB.

$$\mathbf{v}_D = v_D\mathbf{j}, \quad \mathbf{v}_A = -(900 \text{ mm/s})\mathbf{i}$$

$$\mathbf{r}_{D/A} = -(80 \text{ mm})\mathbf{i} - (150 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_{D/A} = \boldsymbol{\omega}_{AD} \times \mathbf{r}_{D/A} = \omega_{AD}\mathbf{k} \times (-80\mathbf{i} - 150\mathbf{j})$$

$$= 150\omega_{AD}\mathbf{i} - 80\omega_{AD}\mathbf{j}$$

$$\mathbf{v}_0 = \mathbf{v}_A + \mathbf{v}_{D/A}$$

$$v_D\mathbf{j} = -900\mathbf{i} + 150\omega_{AD}\mathbf{i} - 80\omega_{AD}\mathbf{j}$$

Equate components.

$$\mathbf{i}: \quad 0 = -900 + 150\omega_{AD} \qquad \omega_{AD} = 6 \text{ rad/s}$$

(a) *Angular velocity of ADB.*

$$\boldsymbol{\omega}_{AD} = (6.00 \text{ rad/s})\mathbf{k} = 6.00 \text{ rad/s} \circlearrowleft \blacktriangleleft$$

By proportions,

$$\mathbf{r}_{B/A} = \frac{150+60}{150}\mathbf{r}_{D/A} = 1.4\,\mathbf{r}_{D/A}$$

$$= -(112 \text{ mm})\mathbf{i} - (210 \text{ mm})\mathbf{j}$$

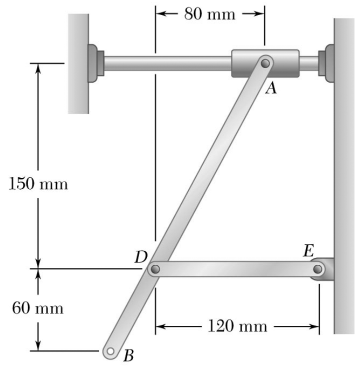
$$\mathbf{v}_B = \mathbf{v}_A + \omega_{AD}\mathbf{k} \times \mathbf{r}_{B/A}$$

$$= -900\mathbf{i} + 6\mathbf{k} \times (-112\mathbf{i} - 210\mathbf{j})$$

$$= -900\mathbf{i} - 672\mathbf{j} + 1260\mathbf{i}$$

(b) *Velocity of B.*

$$\mathbf{v}_B = (360 \text{ mm/s})\mathbf{i} - (672 \text{ mm/s})\mathbf{j} = 762 \text{ mm/s} \;\searrow 61.8° \blacktriangleleft$$



PROBLEM 15.56

Knowing that at the instant shown the angular velocity of rod DE is 2.4 rad/s clockwise, determine (*a*) the velocity A, (*b*) the velocity of Point B.

SOLUTION

Rod DE: Point E is fixed. $\boldsymbol{\omega}_{DE} = 2.4 \text{ rad}$ ↻

$$v_D = \omega_{DB} r_{DE} = (2.4 \text{ rad/s})(120 \text{ mm}) = 288 \text{ mm/s}$$

$$\mathbf{v}_D = 288 \text{ mm/s} \uparrow = (288 \text{ mm/s})\mathbf{j}$$

Rod ADB: $\mathbf{r}_{A/D} = (80 \text{ mm})\mathbf{i} + (150 \text{ mm})\mathbf{j}, \;\; \boldsymbol{\omega}_{AD} = \boldsymbol{\omega}_{AD}\mathbf{k}, \quad \mathbf{v}_A = v_A\mathbf{i}$

$$\mathbf{v}_A = \mathbf{v}_D + \mathbf{v}_{D/A} = \mathbf{v}_D + \omega_{AD}\mathbf{k} \times \mathbf{r}_{A/D}$$

$$v_A\mathbf{i} = (288 \text{ mm/s})\mathbf{j} + \omega_{AD}\mathbf{k} \times [(80 \text{ mm})\mathbf{i} + (150 \text{ mm})\mathbf{j}]$$

$$v_A\mathbf{i} = 288\mathbf{j} + 80\omega_{AD}\mathbf{j} - 150\omega_{AD}\mathbf{i}$$

Equate components.

$$\mathbf{i}: \quad v_A = -150\omega_{AD} \tag{1}$$

$$\mathbf{j}: \quad 0 = 288 + 80\omega_{AD} \tag{2}$$

From Eq. (2), $\omega_{AD} = -\dfrac{288}{80}$ $\qquad \boldsymbol{\omega}_{AD} = (-3.6 \text{ rad/s})\mathbf{k}$

From Eq. (1), $v_A = -(150)(-3.6) = 540 \text{ mm/s}$

(*a*) *Velocity of collar A.* $\qquad \mathbf{v}_A = 540 \text{ mm/s} \longrightarrow$ ◀

(*b*) *Velocity of Point B.*

By proportions $\quad \mathbf{r}_{B/D} = -\dfrac{60}{150}\mathbf{r}_{A/D} = -(32 \text{ mm})\mathbf{i} - 60 \text{ mm } \mathbf{j}$

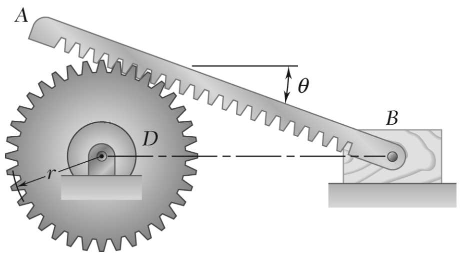
$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D} = \mathbf{v}_D + \boldsymbol{\omega}_{AD} \times \mathbf{r}_{B/D}$$

$$= (288 \text{ mm/s})\mathbf{j} + [-(3.6 \text{ rad/s})\mathbf{k}] \times [-(32 \text{ mm})\mathbf{i} - (60 \text{ mm})\mathbf{j}]$$

$$= (288 \text{ mm/s})\mathbf{j} + (115.2 \text{ mm/s})\mathbf{j} - (216 \text{ mm/s})\mathbf{i}$$

$$\mathbf{v}_B = -(216 \text{ mm/s})\mathbf{i} + (403.2 \text{ mm/s})\mathbf{j}$$

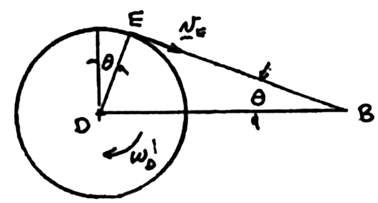
$\mathbf{v}_B = 457 \text{ mm/s} \;\measuredangle\; 61.8°$ ◀



PROBLEM 15.57

A straight rack rests on a gear of radius r and is attached to a block B as shown. Denoting by ω_D the clockwise angular velocity of gear D and by θ the angle formed by the rack and the horizontal, derive expressions for the velocity of block B and the angular velocity of the rack in terms of r, θ, and ω_D.

SOLUTION



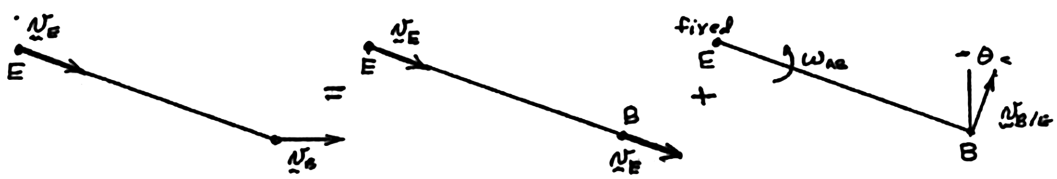
Gear D: Rotation about D. Tooth E is in contact with rack AB.

$$\mathbf{v}_E = r\omega_D \searrow \theta$$

Rack AB:

$$l_{EB} = \frac{r}{\tan\theta}$$

Plane motion = Translation with E + Rotation about E.

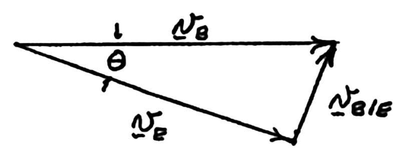


$$\mathbf{v}_B = \mathbf{v}_E + \mathbf{v}_{B/E} \qquad [v_B \longrightarrow] = [v_E \searrow \theta] + [v_{B/E} \nearrow \theta]$$

Draw velocity vector diagram.

$$v_B = \frac{v_E}{\cos\theta} = \frac{r\omega_D}{\cos\theta}$$

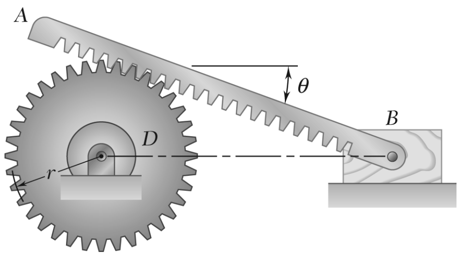
$$\mathbf{v}_B = \frac{r\omega_D}{\cos\theta} \longrightarrow \blacktriangleleft$$



$$v_{B/E} = v_E \tan\theta$$
$$= r\omega_D \tan\theta$$

$$\omega_{AB} = \frac{v_{B/E}}{l_{EB}}$$
$$= \frac{r\omega_D \tan\theta}{\frac{r}{\tan\theta}}$$
$$= \omega_D \tan^2\theta$$

$$\boldsymbol{\omega}_{AB} = \omega_D \tan^2\theta \circlearrowright \blacktriangleleft$$

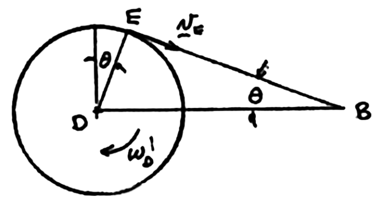


PROBLEM 15.58

A straight rack rests on a gear of radius $r = 2.5$ in. and is attached to a block B as shown. Knowing that at the instant shown the velocity of block B is 8 in./s to the right and $\theta = 25°$, determine (*a*) the angular velocity of gear D, (*b*) the angular velocity of the rack.

SOLUTION

Gear D: Rotation about D. Tooth E is in contact with rack AB.

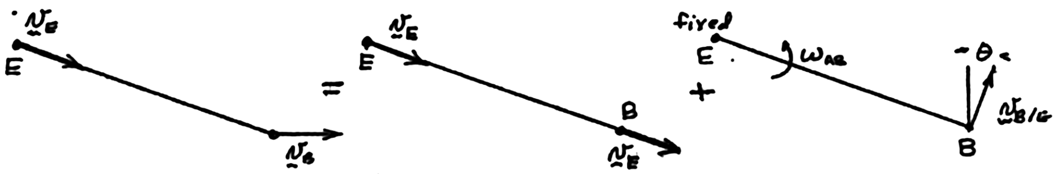


$$\mathbf{v}_E = r\omega_D \searrow \theta$$

Rack AB:

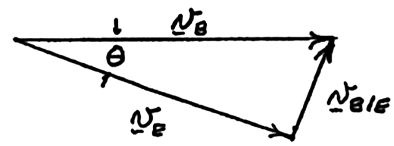
$$l_{EB} = \frac{r}{\tan\theta}$$

Plane motion = Translation with E + Rotation about E.



$$\mathbf{v}_B = \mathbf{v}_E + \mathbf{v}_{B/E} \qquad [v_B \longrightarrow] = [v_E \searrow \theta] + [v_{B/E} \nearrow \theta]$$

Draw velocity vector diagram.



$$v_B = \frac{v_E}{\cos\theta} = \frac{r\omega_D}{\cos\theta}$$

$$\mathbf{v}_B = \frac{r\omega_D}{\cos\theta} \longrightarrow$$

$$v_{B/E} = v_E \tan\theta = r\omega_D \tan\theta$$

$$\omega_{AB} = \frac{v_{B/E}}{l_{EB}} = \frac{r\omega_D \tan\theta}{\frac{r}{\tan\theta}} = \omega_D \tan^2\theta$$

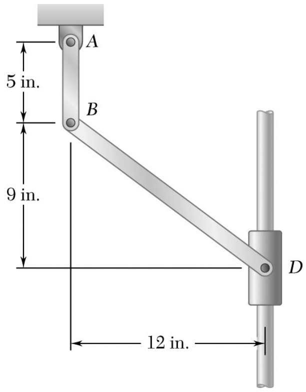
$$\omega_{AB} = \omega_D \tan^2\theta \circlearrowright$$

Data: $\quad r = 2.5 \text{ in.} \quad \theta = 25° \quad \mathbf{v}_B = 8 \text{ in./s} \longrightarrow$

(*a*) $$\omega_D = \frac{v_B \cos\theta}{r} = \frac{8\cos 25°}{2.5} \qquad \boldsymbol{\omega}_D = 2.90 \text{ rad/s} \circlearrowright \blacktriangleleft$$

(*b*) $$\omega_{AB} = 2.90\tan^2 25° \qquad \boldsymbol{\omega}_{AB} = 0.631 \text{ rad/s} \circlearrowright \blacktriangleleft$$

PROBLEM 15.59



Knowing that at the instant shown the angular velocity of crank AB is 2.7 rad/s clockwise, determine (*a*) the angular velocity of link BD, (*b*) velocity of collar D, (*c*) the velocity of the midpoint of link BD.

SOLUTION

Crank AB: Point A is fixed. $\quad \boldsymbol{\omega}_{AB} = 2.7 \text{ rad/s} \circlearrowright$

$$v_B = \omega_{AB} r_{AB} = (2.7 \text{ rad/s})(5 \text{ in.}) = 13.5 \text{ in./s}$$

$$\mathbf{v}_B = 13.5 \text{ in./s} \longleftarrow = -(13.5 \text{ in./s})\mathbf{i}$$

Link BD: $\quad \mathbf{r}_{D/B} = (12 \text{ in.})\mathbf{i} - (9 \text{ in.})\mathbf{j}, \quad \boldsymbol{\omega}_{BD} = \omega_{BD} \circlearrowleft = \omega_{AB}\mathbf{k},$

$$\mathbf{v}_D = v_D \uparrow = v_D \mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{B/D} = \mathbf{v}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{B/D}$$

$$v_D \mathbf{j} = -(13.5 \text{ in./s})\mathbf{i} + \omega_{BD}\mathbf{k} \times [(12 \text{ in.})\mathbf{i} - (9 \text{ in.})\mathbf{j}]$$

$$= -13.5\mathbf{i} + 12\omega_{BD}\mathbf{j} + 9\omega_{BD}\mathbf{i}$$

Equate components.

$$\mathbf{i}: \quad 0 = -13.5 + 9\omega_{BD} \qquad (1)$$

$$\mathbf{j}: \quad v_D = 12\omega_{BD} \qquad (2)$$

(*a*) *Angular velocity of link BD.*

From Eq. (1), $\quad \omega_{BD} = \dfrac{13.5}{9} \qquad \omega_{BD} = 1.500 \text{ rad/s} \circlearrowleft$ ◀

(*b*) *Velocity of collar D.*

From Eq. (2), $\quad v_D = (12)(1.5) \qquad \mathbf{v}_D = 18.00 \text{ in./s} \uparrow$ ◀

(*c*) *Velocity of midpoint M of link BD.*

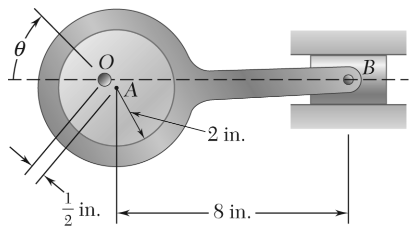
$$\mathbf{r}_{M/B} = \frac{1}{2}\mathbf{r}_{D/B} = (6 \text{ in.})\mathbf{i} - (4.5 \text{ in.})\mathbf{j}$$

$$\mathbf{v}_M = \mathbf{v}_B + \mathbf{v}_{M/B} = \mathbf{v}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{M/D}$$

$$= -13.5\mathbf{i} + (1.500\mathbf{k}) \times (6\mathbf{i} - 4.5\mathbf{j})$$

$$= -13.5\mathbf{i} + 9\mathbf{j} + 6.75\mathbf{i}$$

$$\mathbf{v}_M = -(6.75 \text{ in./s})\mathbf{i} + (9.00 \text{ in./s})\mathbf{j} = 11.25 \text{ in./s} \measuredangle 53.1^\circ$$ ◀



PROBLEM 15.60

In the eccentric shown, a disk of 2-in.-radius revolves about shaft O that is located 0.5 in. from the center A of the disk. The distance between the center A of the disk and the pin at B is 8 in. Knowing that the angular velocity of the disk is 900 rpm clockwise, determine the velocity of the block when $\theta = 30°$.

SOLUTION

Geometry.

$$(OA)\sin\theta = (AB)\sin\beta$$

$$\sin\beta = \frac{(OA)\sin\theta}{AB} = \frac{0.5\sin 30°}{8}, \quad \beta = 1.79°$$

Shaft and eccentric disk. (Rotation about O)

$$\omega_{OA} = 900 \text{ rpm} = 30\pi \text{ rad/s}$$

$$\mathbf{v}_A = (OA)\,\omega_{OA} = (0.5)(30\pi) = 15\pi \text{ in/s}$$

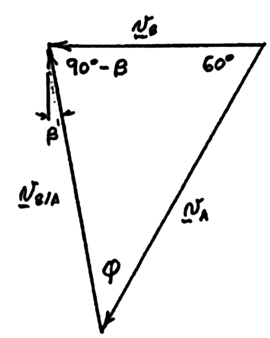
Rod AB. (Plane motion = Translation with A + Rotation about A.)

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \quad [v_B \longleftarrow] = [v_A \swarrow 60°] + [v_{A/B} \ \beta]$$

Draw velocity vector diagram.

$$90° - \beta = 88.21°$$

$$\varphi = 180° - 60° - 88.21° = 31.79°$$

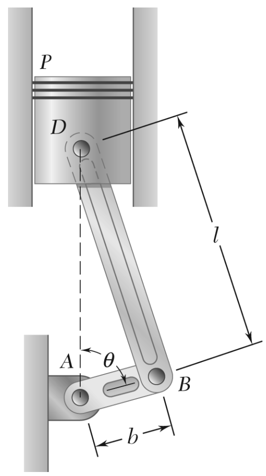


Law of sines.

$$\frac{v_B}{\sin\phi} = \frac{v_A}{\sin(90° - \beta)}$$

$$v_B = \frac{v_A\sin\varphi}{\sin(90° - \beta)} = \frac{(15\pi)\sin 31.79°}{\sin 88.21°} = 24.837 \text{ in./s}$$

$$\mathbf{v}_B = 24.8 \text{ in./s} \longleftarrow$$



PROBLEM 15.61

In the engine system shown, $l = 160$ mm and $b = 60$ mm. Knowing that the crank AB rotates with a constant angular velocity of 1000 rpm clockwise, determine the velocity of the piston P and the angular velocity of the connecting rod when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

SOLUTION

$$\omega_{AB} = 1000 \text{ rpm} \circlearrowright = \frac{(1000)(2\pi)}{60} = 104.72 \text{ rad/s} \circlearrowright$$

(*a*) $\theta = 0°$. *Crank AB.* (Rotation about A) $\quad \mathbf{r}_{B/A} = 0.06 \text{ m} \uparrow$

$$\mathbf{v}_B = v_{B/A}\omega_{AB} = (0.06)(104.72) = 6.2832 \text{ m/s} \longrightarrow$$

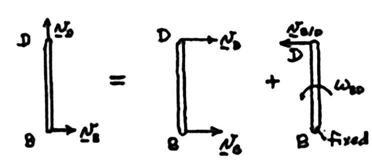
Rod BD. (Plane motion = Translation with B + Rotation about B)

$$\mathbf{v}_D = \mathbf{v}_B + v_{D/B}$$

$$v_D \uparrow = [6.2832 \longrightarrow] + [v_{D/B} \longleftarrow]$$

$$v_D = 0$$

$$v_{D/B} = 6.2832 \text{ m/s}$$



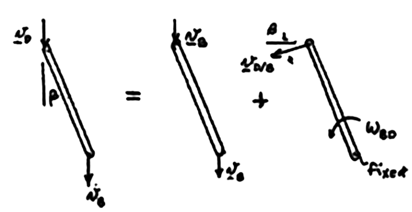
$$\mathrm{v}_P = \mathrm{v}_D \qquad\qquad \mathbf{v}_P = 0 \blacktriangleleft$$

$$\omega_{BD} = \frac{v_B}{l} = \frac{6.2832}{0.16} \qquad\qquad \boldsymbol{\omega}_{BD} = 39.3 \text{ rad/s} \circlearrowleft \blacktriangleleft$$

(*b*) $\theta = 90°$. *Crank AB.* (Rotation about A) $\quad \mathbf{r}_{B/A} = 0.06 \text{ m} \longrightarrow$

$$\mathbf{v}_B = r_{B/A}\omega_{AB} = (0.06)(104.72) = 6.2832 \text{ m/s} \downarrow$$

Rod BD. (Plane motion = Translation with B + Rotation about B.)



$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

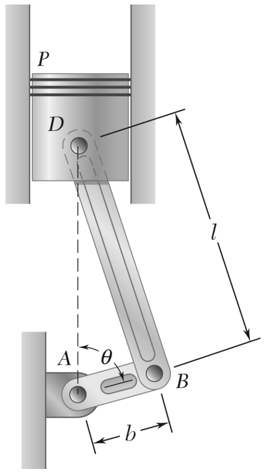
$$[v_D] \downarrow] = [6.2832] \downarrow + [v_{D/B} \nearrow \beta]$$

PROBLEM 15.61 (Continued)

$$v_{D/B} = 0, \quad v_D = 6.2832 \text{ m/s}$$

$$\omega_{BD} = \frac{v_{D/B}}{l} \qquad \omega_{BD} = 0 \blacktriangleleft$$

$$\mathbf{v}_P = \mathbf{v}_D = 6.2832 \text{ m/s} \downarrow \qquad \mathbf{v}_P = 6.28 \text{ m/s} \downarrow \blacktriangleleft$$



PROBLEM 15.62

In the engine system shown $l = 160$ mm and $b = 60$ mm. Knowing that crank AB rotates with a constant angular velocity of 1000 rpm clockwise, determine the velocity of the piston P and the angular velocity of the connecting rod when $\theta = 60°$.

SOLUTION

$$\omega_{AB} = 1000 \text{ rpm} = \frac{(1000)(2\pi)}{60} = 104.72 \text{ rad/s} \circlearrowright$$

$\underline{\theta = 60°}$. *Crank AB*. (Rotation about A) $\quad \mathbf{r}_{B/A} = 3$ in. $\measuredangle$ 30°

$$\mathbf{v}_B = r_{B/A}\omega_{AB} = (0.06)(104.72) = 6.2832 \text{ m/s} \searrow 60°$$

Rod BD. (Plane motion = Translation with B + Rotation about B.)

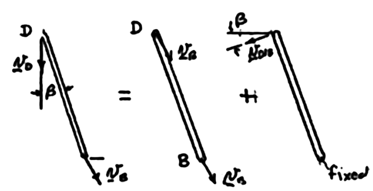
Geometry.

$$l \sin\beta = r\sin\theta$$

$$\sin\beta = \frac{r}{l}\sin\theta = \frac{0.06}{0.16}\sin 60°$$

$$\beta = 18.95°$$

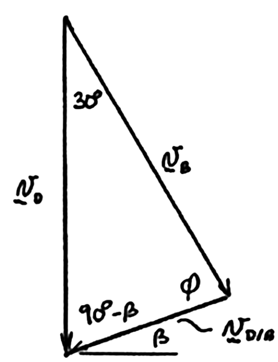
$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$



$$[v_D \downarrow] = [314.16 \searrow 60°] + [v_{D/B} \swarrow \beta]$$

Draw velocity vector diagram.

$$\varphi = 180° - 30° - (90° - \beta) = 78.95°$$



<u>Law of sines.</u>

$$\frac{v_D}{\sin\varphi} = \frac{v_{D/B}}{\sin 30°} = \frac{v_B}{\sin(90° - \beta)}$$

$$v_D = \frac{v_B \sin\varphi}{\cos\beta}$$

$$= \frac{6.2832 \ \sin 78.95°}{\cos 18.95°}$$

$$= 6.52 \text{ m/s}$$

$$v_P = v_D$$

$\mathbf{v}_P = 6.52$ m/s $\downarrow$ ◀

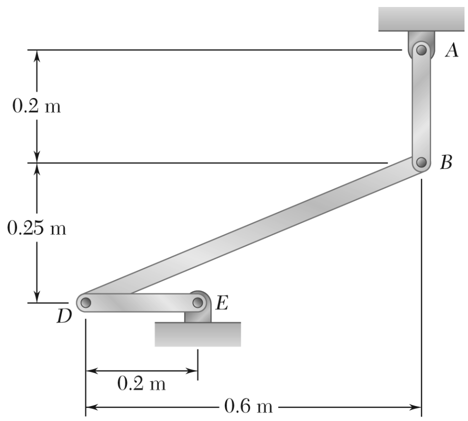
PROBLEM 15.62 (Continued)

$$v_{D/B} = \frac{v_B \sin 30°}{\cos \beta}$$

$$= \frac{6.2832 \sin 30°}{\cos 18.95°}$$

$$= 3.3216 \text{ m/s}$$

$$\omega_{BD} = \frac{v_{D/B}}{l} = \frac{3.3216}{0.16} \qquad \omega_{BD} = 20.8 \text{ rad/s} \circlearrowright \blacktriangleleft$$



PROBLEM 15.63

Knowing that at the instant shown the angular velocity of rod *AB* is 15 rad/s clockwise, determine (*a*) the angular velocity of rod *BD*, (*b*) the velocity of the midpoint of rod *BD*.

SOLUTION

Rod *AB*: $\boldsymbol{\omega}_{AB} = 15 \text{ rad/s} \circlearrowright$

$$v_B = (AB)\omega_{AB} = (0.200)(15) = 3 \text{ m/s} \qquad \mathbf{v}_B = 3 \text{ m/s} \longleftarrow$$

Rod *BD*: $\mathbf{v}_B = -(3 \text{ m/s})\mathbf{i}, \quad \mathbf{v}_D = v_D\mathbf{j}, \quad \boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k}$

$$\mathbf{r}_{B/D} = (0.6 \text{ m})\mathbf{i} + (0.25 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_D + \mathbf{v}_{B/D} = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$$

$$-3\mathbf{i} = v_D\mathbf{j} + \omega_{BD}\mathbf{k} \times (0.6\mathbf{i} + 0.25\mathbf{j})$$

$$= v_D\mathbf{j} + 0.6\omega_{BD}\mathbf{j} - 0.25\omega_{BD}\mathbf{i}$$

Equate components.

$$\mathbf{i}: \quad -3 = -0.25\omega_{BD} \qquad (1)$$

$$\mathbf{j}: \quad 0 = v_D + 0.6\omega_{BD} \qquad (2)$$

(*a*) *Angular velocity of rod BD.*

From Eq. (1), $\omega_{BD} = \dfrac{3}{0.25}$ $\qquad \boldsymbol{\omega}_{BD} = 12.00 \text{ rad/s} \circlearrowleft$ ◀

From Eq. (2), $v_D = -0.6\omega_{BD}$ $\qquad v_D = -7.2 \text{ m/s}$

(*b*) *Velocity of midpoint M of rod BD.*

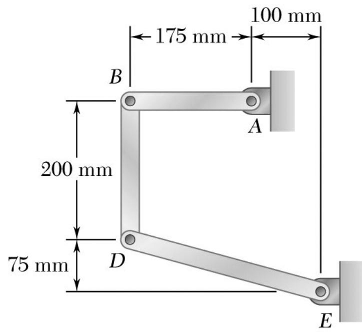
$$\mathbf{r}_{M/D} = \frac{1}{2}\mathbf{r}_{B/D} = (0.3 \text{ m})\mathbf{i} + (0.125 \text{ m})\mathbf{j}$$

$$\mathbf{v}_M = \mathbf{v}_D + \mathbf{v}_{M/D} = v_D\mathbf{j} + \omega_{BD}\mathbf{k} \times \mathbf{r}_{M/D}$$

$$= -7.2\mathbf{j} + 12.00\mathbf{k} \times (0.3\mathbf{i} + 0.125\mathbf{j})$$

$$= -(1.500 \text{ m/s})\mathbf{i} - (3.60 \text{ m/s})\mathbf{j}$$

$$\mathbf{v}_M = 3.90 \text{ m/s} \measuredangle 67.4° \text{ ◀}$$



PROBLEM 15.64

In the position shown, bar AB has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars BD and DE.

SOLUTION

Bar AB: (Rotation about A) $\quad \boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright = -(4 \text{ rad/s})\mathbf{k}$

$$\mathbf{r}_{B/A} = -(175 \text{ mm})\mathbf{i} \quad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-175\mathbf{i})$$

$$\mathbf{v}_B = (700 \text{ mm/s})\mathbf{j}$$

Bar BD: (Plane motion $=$ Translation with B + Rotation about B.)

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \quad \mathbf{r}_{D/B} = -(200 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = 700\mathbf{j} + (\omega_{BD}\mathbf{k}) \times (-200\mathbf{j})$$

$$\mathbf{v}_D = 700\mathbf{j} + 200\omega_{BD}\mathbf{i}$$

Bar DE: (Rotation about E) $\quad \boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$

$$\mathbf{r}_{D/E} = -(275 \text{ mm})\mathbf{i} + (75 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (-275\mathbf{i} + 75\mathbf{j})$$

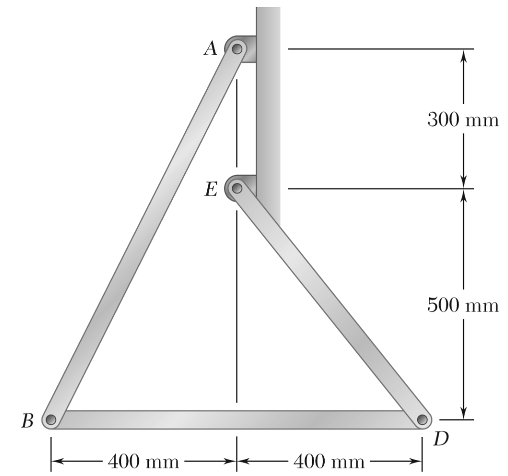
$$\mathbf{v}_D = -275\omega_{DE}\mathbf{j} - 75\omega_{DE}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_D$,

$\mathbf{j}$: $\quad 700 = -275\omega_{DE} \qquad \omega_{DE} = -2.5455 \text{ rad/s} \qquad\qquad \boldsymbol{\omega}_{DE} = 2.55 \text{ rad/s} \circlearrowright$ ◀

$\mathbf{i}$: $\quad 200\omega_{BD} = -75\omega_{DE} \qquad \omega_{BD} = -\dfrac{3}{8}\omega_{BD}$

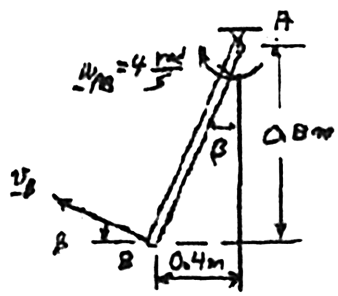
$$\omega_{BD} = -\left(\frac{3}{8}\right)(-2.5455) = 0.95455 \text{ rad/s} \qquad\qquad \boldsymbol{\omega}_{BD} = 0.955 \text{ rad/s} \circlearrowleft \blacktriangleleft$$



PROBLEM 15.65

In the position shown, bar AB has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars BD and DE.

SOLUTION



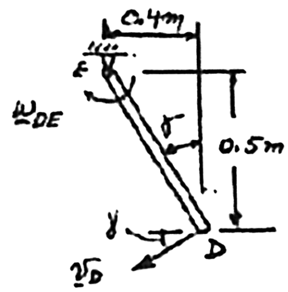
Bar AB:

$$\beta = \tan^{-1}\frac{0.4}{0.8} = 26.56°$$

$$AB = \frac{0.8}{\cos\beta} = 0.8944 \text{ m}$$

$$\mathbf{v}_B = (AB)\omega_{AB} = (0.8944 \text{ m})(4 \text{ m/s})$$

$$\mathbf{v}_B = 3.578 \text{ m/s} \measuredangle 26.56°$$



Bar DE:

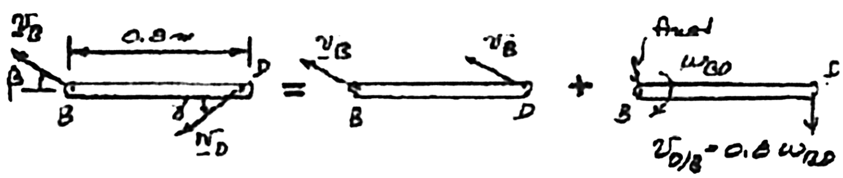
$$\gamma = \tan^{-1}\frac{0.4}{0.5} = 38.66°$$

$$DE = \frac{0.5}{\cos\gamma} = 0.6403 \text{ m}$$

$$v_D = (DE)\omega_{DE}$$

$$\mathbf{v}_D = (0.6403 \text{ m})\omega_{DE} \measuredangle 38.66°$$

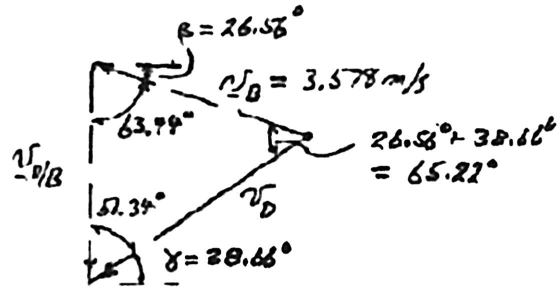
Bar BD:



$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[\mathbf{v}_D \measuredangle \gamma] = [\mathbf{v}_B \measuredangle \beta] + [\mathbf{v}_{D/B} \downarrow]$$

PROBLEM 15.65 (Continued)



Law of sines.

$$\frac{v_D}{\sin 63.44°} = \frac{v_{D/B}}{\sin 65.22°} = \frac{3.578 \text{ m/s}}{\sin 51.34°}$$

$$v_D = 4.099 \text{ m/s}$$

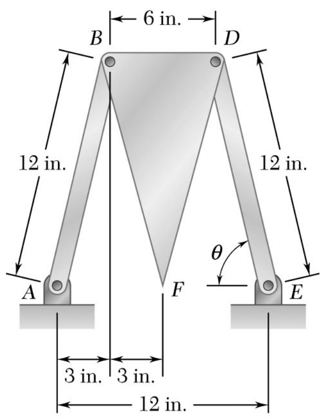
$$(0.6403 \text{ m})\omega_{DE} = 4.099 \text{ m/s}$$

$\boldsymbol{\omega}_{DE} = 6.4$ rad/s ◀

$$v_{D/B} = 4.160 \text{ m/s}$$

$$(0.8 \text{ m})v_{BD} = 4.16 \text{ m/s}$$

$\boldsymbol{\omega}_{BD} = 5.2$ rad/s ◀



PROBLEM 15.66

Robert's linkage is named after Richard Robert (1789–1864) and can be used to draw a close approximation to a straight line by locating a pen at Point F. The distance AB is the same as BF, DF and DE. Knowing that the angular velocity of bar AB is 5 rad/s clockwise in the position shown, determine (a) the angular velocity of bar DE, (b) the velocity of Point F.

SOLUTION

Bar AB: $\boldsymbol{\omega}_{AB} = 5 \text{ rad/s} \circlearrowright = -(5 \text{ rad/s})\mathbf{k}$

In inches,

$$\mathbf{r}_{B/A} = 3\mathbf{i} + \sqrt{12^2 - 3^2}\,\mathbf{j} = 3\mathbf{i} + \sqrt{135}\mathbf{j}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = -5\mathbf{k} \times (3\mathbf{i} + \sqrt{135}\mathbf{j})$$

$$= 5\sqrt{135}\mathbf{i} - 15\mathbf{j}$$

Object BDF: $\mathbf{r}_{D/B} = (6 \text{ in.})\mathbf{i}, \quad \mathbf{r}_{F/B} = 3\mathbf{i} - \sqrt{135}\mathbf{j} \text{ (in.)}, \quad \boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k}$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{B/D} = \mathbf{v}_B + \omega_{BD}\mathbf{k} \times \mathbf{r}_{D/B}$$

$$= 5\sqrt{135}\mathbf{i} - 15\mathbf{j} + \omega_{BD}\mathbf{k} \times 6\mathbf{i}$$

$$= 5\sqrt{135}\mathbf{i} - 15\mathbf{j} + 6\omega_{BD}\mathbf{j} \qquad (1)$$

Bar DE: $\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}, \quad \mathbf{r}_{D/E} = -(3 \text{ in.})\mathbf{i} + (\sqrt{135} \text{ in.})\mathbf{j},$

Point E is fixed so $\mathbf{v}_E = 0$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = \omega_{DE}\mathbf{k} \times (-3\mathbf{i} + \sqrt{135}\mathbf{j})$$

$$= -\sqrt{135}\omega_{DE}\mathbf{i} - 3\omega_{DE}\mathbf{j} \qquad (2)$$

Equating like components of $\mathbf{v}_D$ from Eqs. (1) and (2),

i: $5\sqrt{135} = -\sqrt{135}\omega_{DE}$ (3)

j: $-15 + 6\omega_{BD} = -3\omega_{DE}$ (4)

(a) *Angular velocity of bar DE.*

From Eq. (3), $\omega_{DE} = -5 \text{ rad/s}$ $\qquad \boldsymbol{\omega}_{DE} = 5.00 \text{ rad/s} \circlearrowright$ ◀

From Eq. (4), $\omega_{BD} = \dfrac{1}{6}(15 - 3\omega_{DE}) = \dfrac{1}{6}(15 + 15)$ $\qquad \boldsymbol{\omega}_{BD} = 5.00 \text{ rad/s} \circlearrowleft$

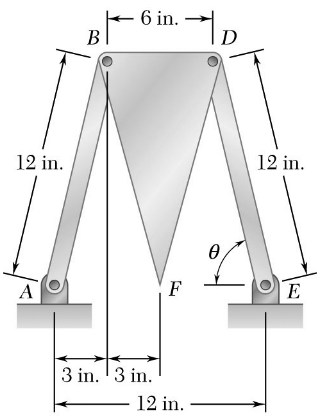
PROBLEM 15.66 (Continued)

(*b*) *Velocity of Point F.*

$$\mathbf{v}_{F/B} = 3i - \sqrt{135}\,\mathbf{j}$$

$$\begin{aligned}\mathbf{v}_F &= \mathbf{v}_B = \mathbf{v}_{F/B} = \mathbf{v}_B + \omega_{BD}\mathbf{k}\times\mathbf{r}_{F/B}\\ &= 5\sqrt{135}\mathbf{i} - 15\mathbf{j} + 5\mathbf{k}\times(3\mathbf{i} - \sqrt{135}\,\mathbf{j})\\ &= 5\sqrt{135}\mathbf{i} - 15\mathbf{j} + 15\mathbf{j} + 5\sqrt{135}\mathbf{i} = 10\sqrt{135}\mathbf{i}\end{aligned}$$

$\mathbf{v}_F = 116.2$ in./s $\longrightarrow$ ◀

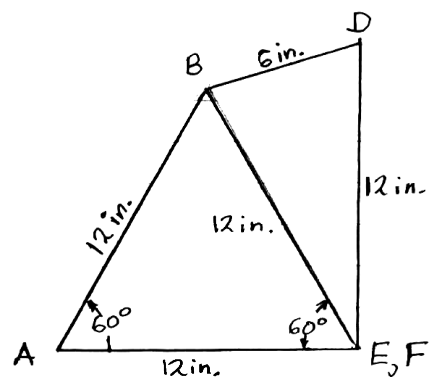


PROBLEM 15.67

Robert's linkage is named after Richard Robert (1789–1864) and can be used to draw a close approximation to a straight line by locating a pen at Point *F*. The distance *AB* is the same as *BF*, *DF* and *DE*. Knowing that the angular velocity of plate *BDF* is 2 rad/s counterclockwise when $\theta = 90°$, determine (*a*) the angular velocities of bars *AB* and *DE*, (*b*) the velocity of Point *F*. When $\theta = 90°$, determine (*a*) the angular velocity of bar **DE** (*b*) the velocity of Point **F**.

SOLUTION

When $\theta = 90°$, the configuration of the linkage is close to that shown at the right.



Bar *AB*: $\boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k}$

In inches, $\mathbf{r}_{B/A} = 6\mathbf{i} + 6\sqrt{3}\mathbf{j}$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = \omega_{AB}\mathbf{k} \times (6\mathbf{i} + 6\sqrt{3}\mathbf{j}) = -6\sqrt{3}\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j}$$

Object *BDF*: $\mathbf{r}_{D/B} = 6\mathbf{i} + 6(2-\sqrt{3})\mathbf{j}$

$$\mathbf{r}_{F/B} = 6\mathbf{i} - 6\sqrt{3}\mathbf{j} \qquad \boldsymbol{\omega}_{BD} = (2 \text{ rad/s})\mathbf{k}$$

$$\begin{aligned}\mathbf{v}_D &= \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_B = \omega_{BD}\mathbf{k} \times \mathbf{r}_{D/B}\\ &= -6\sqrt{3}\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j} + 2\mathbf{k} \times [6\mathbf{i} + 6(2-\sqrt{3})\mathbf{j}]\\ &= -6\sqrt{3}\omega_{AB}\mathbf{i} + 6\omega_{AB}\mathbf{j} + 24\mathbf{i} - 12\sqrt{3}\mathbf{i} + 12\mathbf{j} \qquad (1)\end{aligned}$$

Bar *DE*: $\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}, \qquad \mathbf{r}_{D/E} = 12\mathbf{j}$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = \omega_{DE}\mathbf{k} \times 12\mathbf{j} = -12\omega_{DE}\mathbf{i} \qquad (2)$$

Equating like components of $\mathbf{v}_D$ from Eqs. (1) and (2),

i: $$-6\sqrt{3}\omega_{AB} + 12 - 6\sqrt{3} = -12\omega_{DE} \qquad (3)$$

j: $$6\omega_{AB} + 12 = 0$$

$$\omega_{AB} = -2 \text{ rad/s} \qquad \omega_{AB} = 2.00 \text{ rad/s} \circlearrowright \blacktriangleleft$$

From Eq. (3), $$-12\omega_{DE} = -(6\sqrt{3})(-2) + 24 - 12\sqrt{3}$$

$$\omega_{DE} = 2 - 2\sqrt{3} = -1.4641 \qquad \boldsymbol{\omega}_{DE} = 1.464 \text{ rad/s} \circlearrowright \blacktriangleleft$$

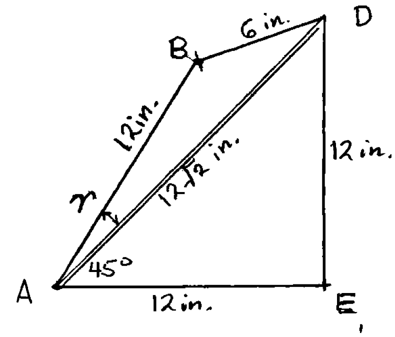
$$\mathbf{v}_D = -(12)(-1.4641)\mathbf{i} = 17.569\mathbf{i}$$

$$\mathbf{v}_F = \mathbf{v}_D + \omega_{BD}\mathbf{k} \times \mathbf{r}_{F/D}$$

$$= 17.569\mathbf{i} + 2\mathbf{k} \times (-12\mathbf{j}) = (41.569 \text{ in./s})\mathbf{i}$$

$$\mathbf{v}_F = 41.6 \text{ in./s} \longrightarrow \blacktriangleleft$$

Note: The exact configuration of the linkage when $\theta = 90°$ may be calculated from trigonometry using the figure given below.



Applying the law of cosines to triangle *ADB* gives

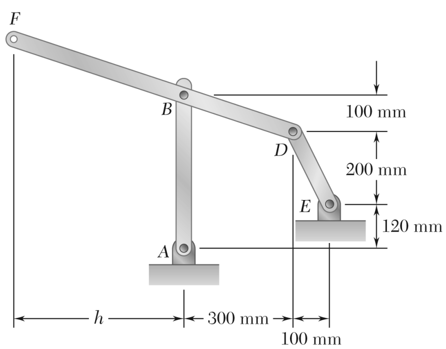
$$\gamma = 13.5°$$

so that angle *EAB* is

$$45° + 13.5° = 58.5°.$$

We used 60° in the approximate analysis.

Point *F* then lies about 0.53 in. to the right of Point *E*.



PROBLEM 15.68

In the position shown, bar DE has a constant angular velocity of 10 rad/s clockwise. Knowing that $h = 500$ mm, determine (*a*) the angular velocity of bar FBD, (*b*) the velocity of Point F.

SOLUTION

<u>Bar *DE*</u>: (Rotation about E)

$$\boldsymbol{\omega}_{DE} = 10 \text{ rad/s} = -(10 \text{ rad/s})\mathbf{k}$$

$$\mathbf{r}_{D/E} = -(0.1 \text{ m})\mathbf{i} + (0.2 \text{ m})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (-10\mathbf{k}) \times (-0.1\mathbf{i} + 0.2\mathbf{j})$$

$$= (1 \text{ m/s})\mathbf{j} + (2 \text{ m/s})\mathbf{i}$$

<u>Bar *FBD*</u>: (Plane motion = Translation with D + Rotation about D.)

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \quad \mathbf{r}_{B/D} = -(0.3 \text{ m})\mathbf{i} + (0.1 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$$

$$= \mathbf{j} + 2\mathbf{i} + (\omega_{BD}\mathbf{k}) \times (-0.3\mathbf{i} + 0.1\mathbf{j})$$

$$= \mathbf{j} + 2\mathbf{i} - 0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i}$$

<u>Bar *AB*</u>: (Rotation about A)

$$\boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k} \quad \mathbf{r}_{B/A} = (0.42 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (\omega_{AB}\mathbf{k}) \times (0.42\mathbf{j}) = -0.42\omega_{AB}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_B$,

(*a*) $\mathbf{j}$: $1 - 0.3\omega_{BD} = 0 \quad \omega_{BD} = 3.3333$ rad/s $\quad \boldsymbol{\omega}_{BD} = 3.33$ rad/s ◀

$\mathbf{i}$: $2 - 0.1\omega_{BD} = -0.42\omega_{AB} \quad 2 - (0.1)(3.3333) = -0.42\omega_{AB}$

$$\omega_{AB} = -3.9683 \text{ rad/s} \quad \omega_{AB} = 3.97 \text{ rad/s}$$

<u>Bar *FBD*</u>:

$$\mathbf{r}_{F/D} = C\mathbf{r}_{B/D} \quad \text{where} \quad C = \frac{h + 0.3}{0.3}$$

$$\mathbf{v}_F = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{F/D}$$

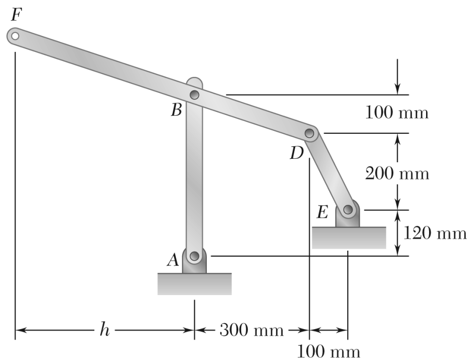
$$= \mathbf{j} + 2\mathbf{i} + C(-0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i})$$

$$= \mathbf{j} + 2\mathbf{i} + C(-\mathbf{j} - 0.33333\mathbf{i})$$

With $\quad h = 500 \text{ mm} = 0.5 \text{ m}, \quad C = \dfrac{0.8}{0.3} = 2.6667$

$$\mathbf{v}_F = \mathbf{j} + 2\mathbf{i} - 2.6667\mathbf{j} - 0.88889\mathbf{i}$$

(*b*) $\mathbf{v}_F = (1.11111 \text{ m/s})\mathbf{i} - (1.66667 \text{ m/s})\mathbf{j} \quad \mathbf{v}_F = 2.00 \text{ m/s} \; 56.3°$ ◀



PROBLEM 15.69

In the position shown, bar DE has a constant angular velocity of 10 rad/s clockwise. Determine (a) the distance h for which the velocity of Point F is vertical, (b) the corresponding velocity of Point F.

SOLUTION

Bar DE: (Rotation about E)

$$\boldsymbol{\omega}_{DE} = 10 \text{ rad/s} \circlearrowright = -(10 \text{ rad/s})\mathbf{k}$$

$$\mathbf{r}_{D/E} = -(0.1 \text{ m})\mathbf{i} + (0.2 \text{ m})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (-10\mathbf{k}) \times (-0.1\mathbf{i} + 0.2\mathbf{j})$$

$$= (1 \text{ m/s})\mathbf{j} + (2 \text{ m/s})\mathbf{i}$$

Bar FBD: (Plane motion = Translation with D + Rotation about D.)

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \quad \mathbf{r}_{B/D} = -(0.3 \text{ m})\mathbf{i} + (0.1 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_D + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{B/D}$$

$$= \mathbf{j} + 2\mathbf{i} + (\omega_{BD}\mathbf{k}) \times (-0.3\mathbf{i} + 0.1\mathbf{j})$$

$$= \mathbf{j} + 2\mathbf{i} - 0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i}$$

Bar AB: (Rotation about A)

$$\boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k} \quad \mathbf{r}_{B/A} = (0.42 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (\omega_{AB}\mathbf{k}) \times (0.42\mathbf{j}) = -0.42\omega_{AB}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_B$,

(a) $\mathbf{j}$: $1 - 0.3\omega_{BD} = 0 \qquad \omega_{BD} = 3.3333 \text{ rad/s}$

$\mathbf{i}$: $2 - 0.1\omega_{BD} = -0.42\omega_{AB} \qquad 2 - (0.1)(3.3333) = -0.42\omega_{AB}$

$$\omega_{AB} = -3.9683 \text{ rad/s} \qquad \omega_{AB} = 3.97 \text{ rad/s} \circlearrowright$$

Bar FBD: $\mathbf{r}_{F/D} = C\mathbf{r}_{B/D}$ where $C = \dfrac{h + 0.3}{0.3}$

$$\mathbf{v}_F = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{F/D}$$

$$= \mathbf{j} + 2\mathbf{i} + C(-0.3\omega_{BD}\mathbf{j} - 0.1\omega_{BD}\mathbf{i})$$

$$= \mathbf{j} + 2\mathbf{i} + C(-\mathbf{j} - 0.33333\mathbf{i})$$

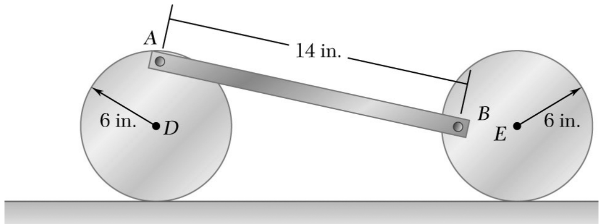
PROBLEM 15.69 (Continued)

But $\mathbf{v}_F = v_F\mathbf{j}$. Equating components of the two expressions for $\mathbf{v}_F$,

$$\mathbf{i}: \quad 0 = 2 - 0.33333C \qquad C = 6$$

(*a*) $$h = 0.3C - 0.3 = (0.3)(6) - 0.3$$ $h = 1.500 \text{ m}$ ◀

(*b*) $$\mathbf{j}: \quad v_F = \mathbf{j} - C\mathbf{j} = (1-6)\mathbf{j}$$ $\mathbf{v}_F = 5.00 \text{ m/s} \downarrow$ ◀



PROBLEM 15.70

Both 6-in.-radius wheels roll without slipping on the horizontal surface. Knowing that the distance *AD* is 5 in., the distance *BE* is 4 in. and *D* has a velocity of 6 in./s to the right, determine the velocity of Point *E*.

SOLUTION

Disk *D*: Velocity at the contact Point *P* with the ground is zero.

$$\mathbf{v}_0 = 6 \text{ in./s} \longrightarrow$$

$$\omega_D = \frac{v_D}{r_{D/P}} = \frac{6 \text{ in./s}}{6 \text{ in.}} = 1 \text{ rad/s} \qquad \boldsymbol{\omega}_D = 1 \text{ rad/s} \circlearrowright$$

At Point *A*,
$$v_A = r_{A/P}\omega_D = (6 \text{ in.} + 5 \text{ in.})(1 \text{ rad/s}) = 11 \text{ in./s}$$

$$\mathbf{v}_A = 11 \text{ in./s} \longrightarrow$$

Disk *E*: Velocity at the contact Point *Q* with the ground is zero. $\boldsymbol{\omega}_E = \omega_E \circlearrowleft = \omega_E\mathbf{k}$.

$$\mathbf{r}_{B/Q} = -(4 \text{ in.})\mathbf{i} + (6 \text{ in.})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_{B/Q} = \boldsymbol{\omega}_E \times \mathbf{r}_{B/Q} = \omega_E\mathbf{k} \times (-4\mathbf{i} + 6\mathbf{j})$$

$$\mathbf{v}_B = -6\omega_E\mathbf{i} - 4\omega_E\mathbf{j} \qquad (1)$$

Connecting rod *AB*:
$$\mathbf{r}_{B/A} = (\sqrt{14^2 - 5^2})\mathbf{i} - 5\mathbf{j} \text{ in inches.}$$

$$\mathbf{v}_{B/A} = \sqrt{171}\mathbf{i} - 5\mathbf{j} \qquad \boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \omega_{AB}\mathbf{k} \times (\sqrt{171}\mathbf{i} - 5\mathbf{j})$$

$$= 11\mathbf{i} + 5\omega_{AB}\mathbf{i} + \sqrt{171}\omega_{AB}\mathbf{j} \qquad (2)$$

Equating expressions (1) and (2) for $\mathbf{v}_B$ gives

$$-6\omega_E\mathbf{i} - 4\omega_E\mathbf{j} = 11\mathbf{i} + 5\omega_{AB}\mathbf{i} + \sqrt{171}\omega_{AB}\mathbf{j}$$

Equating like components and transposing terms,

$$\mathbf{i}: \qquad 5\omega_{AB} + 6\omega_E = -11 \qquad (3)$$

$$\mathbf{j}: \qquad \sqrt{171}\omega_{AB} + 4\omega_E = 0 \qquad (4)$$

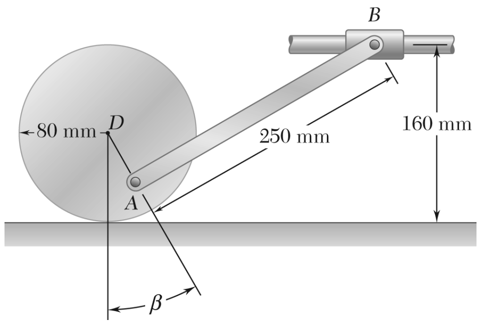
PROBLEM 15.70 (Continued)

Solving the simultaneous equaitons (3) and (4),

$$\omega_{AB} = 0.75265 \text{ rad/s}, \quad \omega_E = -2.4605 \text{ rad/s}$$

Velocity of Point E. $\mathbf{v}_E = \omega_E \mathbf{k} \times \mathbf{r}_{E/Q} = -2.4605\mathbf{k} \times 6\mathbf{j}$

$$\mathbf{v}_E = 14.76 \text{ in./s } \mathbf{i} = 14.76 \text{ in./s} \longrightarrow \blacktriangleleft$$



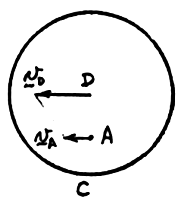
PROBLEM 15.71

The 80-mm-radius wheel shown rolls to the left with a velocity of 900 mm/s. Knowing that the distance AD is 50 mm, determine the velocity of the collar and the angular velocity of rod AB when (a) $\beta = 0$, (b) $\beta = 90°$.

SOLUTION

(a) $\beta = 0$.

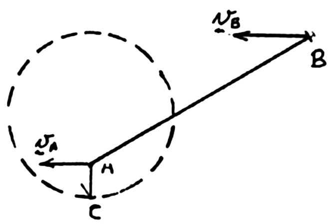
Wheel AD. $\mathbf{v}_C = 0, \quad \mathbf{v}_D = 45 \text{ in./s} \longleftarrow$



$$\omega_{AD} = \frac{v_D}{CD} = \frac{900}{80} = 11.25 \text{ rad/s} \circlearrowleft$$

$$CA = (CD) - (DA) = 80 - 50 = 30 \text{ mm}$$

$$v_A = (CA)\omega_{AD} = (30)(11.25) = 337.5 \text{ mm/s} \longleftarrow$$



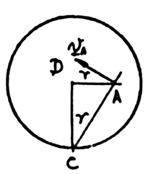
Rod AB. $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$$[v_B \longleftarrow] = [337.5 \longleftarrow] + [v_{B/A} \measuredangle \varphi] \qquad \mathbf{v}_B = 338 \text{ mm/s} \longleftarrow \blacktriangleleft$$

$$v_{B/A} = 0 \qquad \omega_{AB} = 0 \blacktriangleleft$$

(b) $\beta = 90°$.

Wheel AD. $\mathbf{v}_C = 0, \quad \omega_{AD} = 11.25 \text{ rad/s} \circlearrowleft$

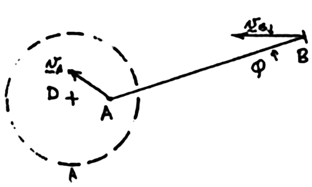


$$\tan\gamma = \frac{DA}{DC} = \frac{50}{80}, \qquad \gamma = 32.005°$$

$$CA = \frac{DC}{\cos\gamma} = 94.34 \text{ mm}$$

$$v_A = (CA)\omega_{AD} = (94.34)(11.25) = 1061.3 \text{ mm/s}$$

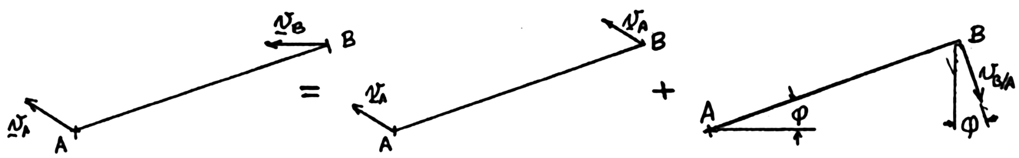
$$\mathbf{v}_A = [1061.3 \text{ mm/s} \searrow 32.005°]$$



Rod AB. $\mathbf{v}_B = v_B \longleftarrow$

$$\sin\varphi = \frac{80}{250}, \quad \varphi = 18.663°$$

Plane motion = Translation with A + Rotation about A.

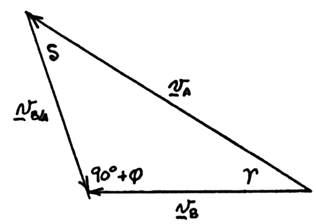


$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ $\qquad [v_B \leftarrow] = [v_A \searrow \gamma] + [v_{B/A} \searrow \varphi]$

Draw velocity vector diagram.

$$\delta = 180° - \gamma - (90° + \varphi)$$

$$= 90° - 32.005° - 18.663° = 39.332°$$



Law of sines.

$$\frac{v_B}{\sin\delta} = \frac{v_{B/A}}{\sin\gamma} = \frac{v_A}{\sin(90° + \varphi)}$$

$$v_B = \frac{v_A \sin\delta}{\sin(90° + \varphi)} = \frac{(1061.3)\sin 39.332°}{\sin 108.663°}$$

$$= 710 \text{ mm/s}$$

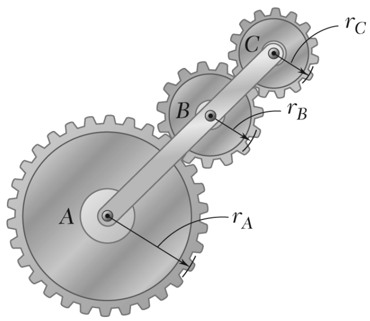
$\mathbf{v}_B = 710 \text{ mm/s} \leftarrow$ ◀

$$v_{B/A} = \frac{v_A \sin\gamma}{\sin(90° + \varphi)} = \frac{(1061.3)\sin 32.005°}{\sin 108.663°}$$

$$= 593.8 \text{ mm/s}$$

$$\omega_{AB} = \frac{v_{B/A}}{\overline{AB}} = \frac{593.8}{250} = 2.37 \text{ rad/s}$$

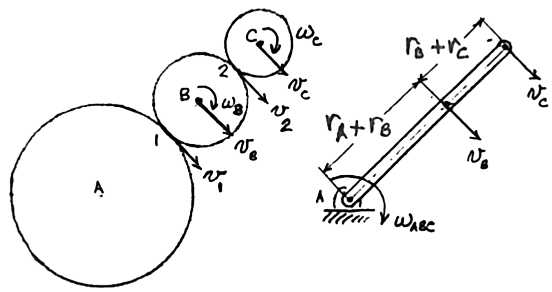
$\boldsymbol{\omega}_{AB} = 2.37 \text{ rad/s} \circlearrowright$ ◀



PROBLEM 15.72*

For the gearing shown, derive an expression for the angular velocity ω_C of gear C and show that ω_C is independent of the radius of gear B. Assume that Point A is fixed and denote the angular velocities of rod ABC and gear A by ω_{ABC} and ω_A respectively.

SOLUTION



Label the contact point between gears A and B as 1 and that between gears B and C as 2.

Rod ABC: $\qquad \omega_{ABC} = \omega_{ABC}$ $\qquad$ Assume $\curvearrowright$ for sketch.

$$v_A = 0$$

$$v_B = (r_A + r_B)\omega_{ABC} \searrow$$

$$v_C = (r_A + 2r_B + r_C)\omega_{ABC} \searrow$$

Gear A: $\qquad \omega_A = 0, \quad v_A = 0, \quad v_1 = 0$

Gear B: $\qquad v_1 = v_B - r_B\,\omega_B = 0$

$$(r_A + r_B)\omega_{ABC} - r_B\,\omega_B = 0$$

$$\omega_B = \left(\frac{r_A + r_B}{r_B}\right)\omega_{ABC} \curvearrowright$$

$$v_2 = v_B + r_B\,\omega_B$$

$$= 2(r_A + r_B)\omega_{ABC} \searrow$$

PROBLEM 15.72* (Continued)

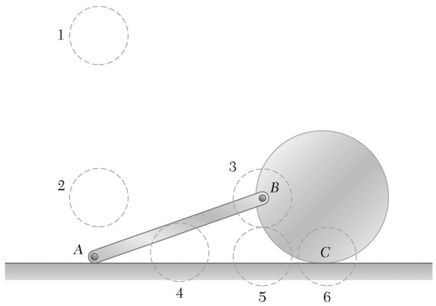
Gear C: $$v_2 = v_C - r_C\,\omega_C$$

$$2(r_A + r_B)\omega_{ABC} = (r_A + 2r_B + r_C)\omega_{ABC} - r_C\omega_C$$

$$\omega_C = (r_A - r_C)\omega_{ABC} = -r_C\,\omega_C \qquad \omega_C = \left(1 - \frac{r_A}{r_C}\right)\omega_{ABC} \blacktriangleleft$$

Note that the result is independent of r_B.

PROBLEM 15.CQ5



The disk rolls without sliding on the fixed horizontal surface. At the instant shown, the instantaneous center of zero velocity for rod *AB* would be located in which region?

(*a*) region 1

(*b*) region 2

(*c*) region 3

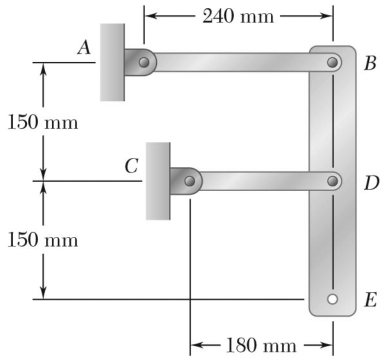
(*d*) region 4

(*e*) region 5

(*f*) region 6

SOLUTION

Answer: (*a*) ◀



PROBLEM 15.CQ6

Bar *BDE* is pinned to two links, *AB* and *CD*. At the instant shown the angular velocities of link *AB*, link *CD* and bar *BDE* are ω_{AB}, ω_{CD}, and ω_{BDE}, respectively. Which of the following statements concerning the angular speeds of the three objects is true at this instant?

(*a*) $\omega_{AB} = \omega_{CD} = \omega_{BDE}$

(*b*) $\omega_{BDE} > \omega_{AB} > \omega_{CD}$

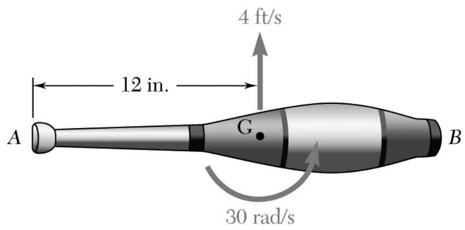
(*c*) $\omega_{AB} = \omega_{CD} > \omega_{BDE}$

(*d*) $\omega_{AB} > \omega_{CD} > \omega_{BDE}$

(*e*) $\omega_{CD} > \omega_{AB} > \omega_{BDE}$

SOLUTION

Answer: (*e*) ◀



PROBLEM 15.73

A juggling club is thrown vertically into the air. The center of gravity G of the 20 in. club is located 12 in. from the knob. Knowing that at the instant shown G has a velocity of 4 ft/s upwards and the club has an angular velocity of 30 rad/s counterclockwise, determine (*a*) the speeds of Point A and B, (*b*) the location of the instantaneous center of rotation.

SOLUTION

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

Relative positions: $\mathbf{r}_{A/G} = -(1\text{ ft})\mathbf{i}, \quad \mathbf{r}_{B/A} = \left(\frac{8}{12}\text{ ft}\right)\mathbf{i}$

Angular velocity: $\boldsymbol{\omega} = 30\text{ rad/s} \circlearrowleft = (30\text{ rad/s})\mathbf{k}$

Velocity at A:

$$\begin{aligned}\mathbf{v}_A &= \mathbf{v}_G + \mathbf{v}_{A/G} = \mathbf{v}_G + \boldsymbol{\omega} \times \mathbf{r}_{A/G}\\ &= (4\text{ ft/s})\mathbf{j} + (30\text{ rad/s})\mathbf{k} \times (-1\text{ ft})\mathbf{i}\\ &= (4\text{ ft/s})\mathbf{j} - (30\text{ ft/s})\mathbf{j} = -(26\text{ ft/s})\mathbf{j}\\ &= 26\text{ ft/s}\downarrow\end{aligned}$$

$v_A = 26.0\text{ ft/s}$ ◀

Velocity at B:

$$\begin{aligned}\mathbf{v}_B &= \mathbf{v}_G + \mathbf{v}_{B/G} = \mathbf{v}_G + \boldsymbol{\omega} \times \mathbf{r}_{B/G}\\ &= (4\text{ ft/s})\mathbf{j} + (30\text{ rad/s})\mathbf{k} \times \left(\frac{8}{12}\text{ ft}\right)\mathbf{i}\\ &= (4\text{ ft/s})\mathbf{j} + (20\text{ ft/s})\mathbf{j} = (24\text{ ft/s})\mathbf{j}\\ &= 24\text{ ft/s}\uparrow\end{aligned}$$

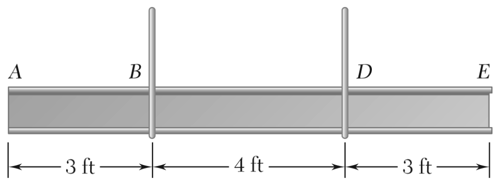
$v_B = 24.0\text{ ft/s}$ ◀

Let $\mathbf{r}_{C/G} = x\mathbf{i}$ bet the position of the instantaneous center C relative to G.

$$\begin{aligned}\mathbf{v}_C &= \mathbf{v}_G + \mathbf{v}_{C/G} = \mathbf{v}_G + \boldsymbol{\omega} \times (x\,\mathbf{i})\\ &= (4\text{ ft/s})\mathbf{j} + (30\text{ rad/s})\mathbf{k} \times (x\,\mathbf{i})\\ &= (4\text{ ft/s})\mathbf{j} + (30\text{ ft/s})\,x\,\mathbf{j} = 0\end{aligned}$$

$$x = -\frac{4\text{ ft/s}}{30\text{ rad/s}} = -\frac{4}{30}\text{ ft} = -1.6\text{ in.}$$

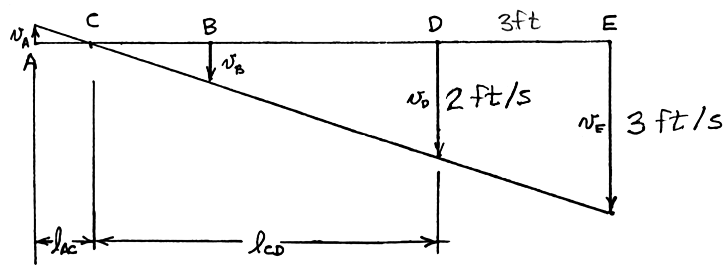
Point C lies 1.6 in. to the left of G. ◀



PROBLEM 15.74

A 10-ft beam AE is being lowered by means of two overhead cranes. At the instant shown, it is known that the velocity of Point D is 24 in./s downward and the velocity of Point E is 36 in./s downward. Determine (*a*) the instantaneous center of rotation of the beam, (*b*) the velocity of Point A.

SOLUTION

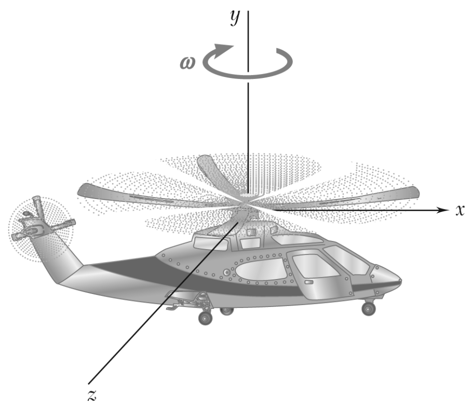


$$\omega = \frac{v_E - v_D}{l_{ED}} = \frac{3-2}{3} = \frac{1}{3} \text{ rad/s} \circlearrowright$$

$$l_{CE} = \frac{v_D}{\omega} = \frac{2}{\frac{1}{3}} = 6 \text{ ft}$$

(*a*) $l_{AC} = 3 + 4 - 6 = 1 \text{ ft}$ — C lies 1 ft to the right of A. ◀

(*b*) $v_A = l_{AC}\omega = (1)\left(\frac{1}{3}\right) = 0.3333 \text{ ft/s}$ — $\mathbf{v}_A = 4.00 \text{ in./s} \uparrow$ ◀



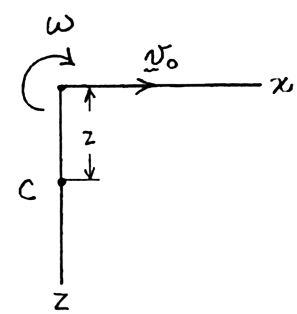
PROBLEM 15.75

A helicopter moves horizontally in the x direction at a speed of 120 mi/h. Knowing that the main blades rotate clockwise with an angular velocity of 180 rpm, determine the instantaneous axis of rotation of the main blades.

SOLUTION

$$\mathbf{v}_0 = 120 \text{ mi/h} = 176 \text{ ft/s} \longrightarrow$$

$$\omega = 180 \text{ rpm} = \frac{(180)(2\pi)}{60} = 18.85 \text{ rad/s}$$



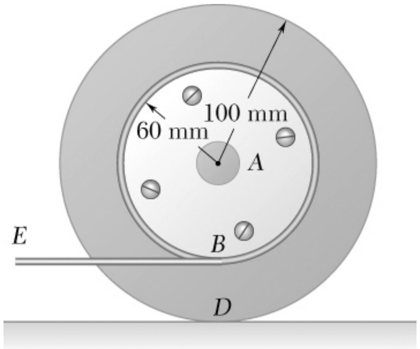
$$v_0 = z\omega$$

$$z = \frac{v_0}{\omega} = \frac{176}{18.85} = 9.34 \text{ ft}$$

Instantaneous axis is parallel to the y axis and passes through the point $x = 0$ ◀

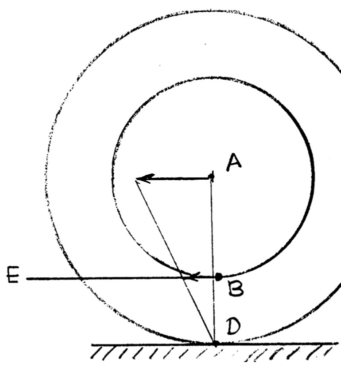
$z = 9.34$ ft ◀

PROBLEM 15.76



A 60-mm-radius drum is rigidly attached to a 100-mm-radius drum as shown. One of the drums rolls without sliding on the surface shown, and a cord is wound around the other drum. Knowing that end E of the cord is pulled to the left with a velocity of 120 mm/s, determine (*a*) the angular velocity of the drums, (*b*) the velocity of the center of the drums, (*c*) the length of cord wound or unwound per second.

SOLUTION



Since the drum rolls without sliding, its instantaneous center lies at D.

$$\mathbf{v}_E = \mathbf{v}_B = 120 \text{ mm/s} \longleftarrow$$

$$v_A = v_{A/D}\omega, \quad v_B = r_{B/D}\omega$$

(*a*) $$\omega = \frac{v_B}{v_{B/D}} = \frac{120}{100-60} = 3 \text{ rad/s}$$

$$\boldsymbol{\omega} = 3.00 \text{ rad/s} \circlearrowleft \blacktriangleleft$$

(*b*) $$v_A = (100)(3) = 300 \text{ mm/s}$$

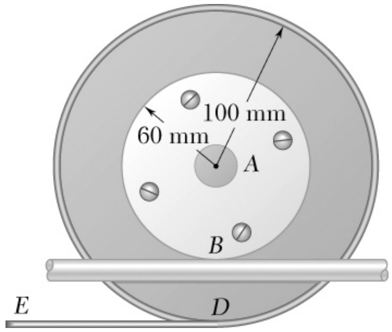
$$\mathbf{v}_A = 300 \text{ mm/s} \longleftarrow \blacktriangleleft$$

Since v_A is greater than v_B, cord is being wound.

$$v_A - v_B = 300 - 120 = 180 \text{ mm/s}$$

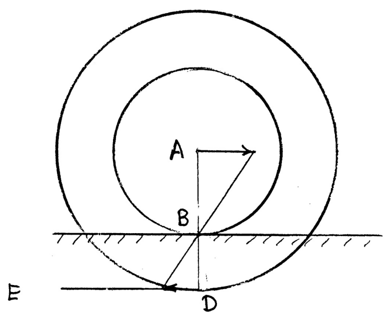
(*c*) Cord wound per second $= 180.0$ mm $\blacktriangleleft$

PROBLEM 15.77



A 60-mm-radius drum is rigidly attached to a 100-mm-radius drum as shown. One of the drums rolls without sliding on the surface shown, and a cord is wound around the other drum. Knowing that end E of the cord is pulled to the left with a velocity of 120 mm/s, determine (a) the angular velocity of the drums, (b) the velocity of the center of the drums, (c) the length of cord wound or unwound per second.

SOLUTION



Since the drum rolls without sliding, its instantaneous center lies at B.

$$\mathbf{v}_E = \mathbf{v}_D = 120 \text{ mm/s} \longleftarrow$$

$$v_A = r_{A/B}\omega, \; v_D = r_{D/B}\omega$$

(a)
$$\omega = \frac{v_D}{r_{D/B}} = \frac{120}{100-60} = 3 \text{ rad/s}$$

$$\boldsymbol{\omega} = 3.00 \text{ rad/s} \circlearrowright \blacktriangleleft$$

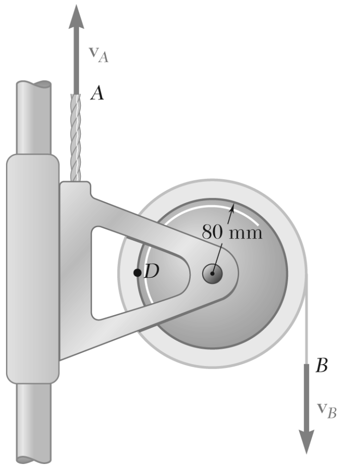
(b)
$$v_A = (60)(3.00) = 180 \text{ mm/s}$$

$$\mathbf{v}_A = 180 \text{ mm/s} \longrightarrow \blacktriangleleft$$

Since $\mathbf{v}_A$ is to the right and $\mathbf{v}_D$ is to the left, cord is being unwound.

$$v_A - v_E = 180 + 120 = 300 \text{ mm/s}$$

(c) Cord unwound per second $= 300$ mm ◀



PROBLEM 15.78

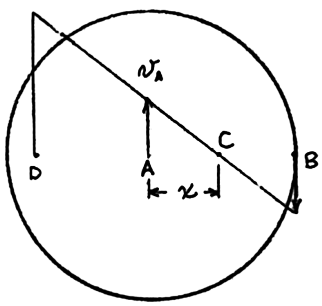
The spool of tape shown and its frame assembly are pulled upward at a speed $v_A = 750$ mm/s. Knowing that the 80-mm-radius spool has an angular velocity of 15 rad/s clockwise and that at the instant shown the total thickness of the tape on the spool is 20 mm, determine (*a*) the instantaneous center of rotation of the spool, (*b*) the velocities of Points *B* and *D*.

SOLUTION

$$\mathbf{v}_A = 750 \text{ mm/s} \uparrow$$

$$\omega = 15 \text{ rad/s} \circlearrowright$$

$$x = \frac{v_A}{\omega} = \frac{750}{15} = 50 \text{ mm}$$



(*a*) The instantaneous center lies 50 mm to the right of the axle. ◀

$$CB = 80 + 20 - 50 = 50 \text{ mm}$$

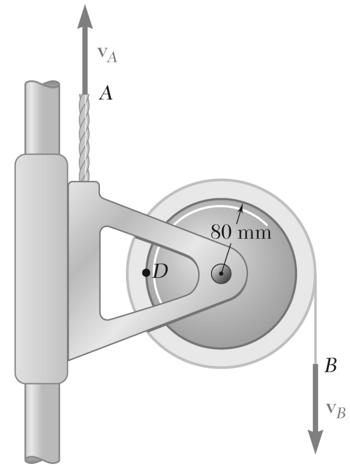
(*b*) $v_B = (CB)\omega = (50)(15) = 750$ mm/s

$$\mathbf{v}_B = 750 \text{ mm/s} \downarrow \; ◀$$

$$CD = 80 + 50 = 130 \text{ mm}$$

$$v_D = (CD)\omega = (130)(15) = 1950 \text{ mm/s}$$

$$\mathbf{v}_D = 1.950 \text{ m/s} \uparrow \; ◀$$



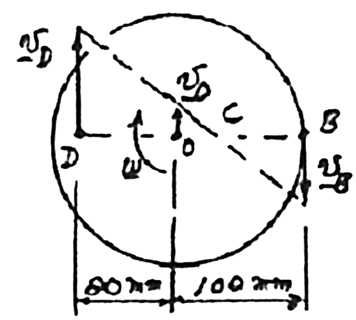
PROBLEM 15.79

The spool of tape shown and its frame assembly are pulled upward at a speed $v_A = 100$ mm/s. Knowing that end B of the tape is pulled downward with a velocity of 300 mm/s and that at the instant shown the total thickness of the tape on the spool is 20 mm, determine (*a*) the instantaneous center of rotation of the spool, (*b*) the velocity of Point D of the spool.

SOLUTION

$$v_D = v_A = 100 \text{ mm/s}$$

(*a*) Since $\mathbf{v}_0$ and $\mathbf{v}_B$ are parallel, instantaneous center C is located at intersection of BC and line joining end points of $\mathbf{v}_D$ and $\mathbf{v}_B$.



Similar triangles.

$$\frac{OC}{v_0} = \frac{BC}{v_B} = \frac{OC + BC}{v_0 + v_B}$$

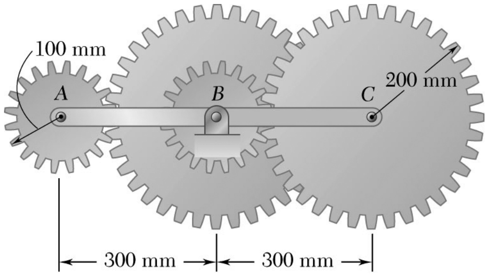
$$OC = \frac{v_0}{v_0 + v_B}(OC + BC)$$

$$OC = \frac{100 \text{ mm/s}}{(100 + 300) \text{ mm/s}}(100 \text{ mm})$$

$$= 25 \text{ mm} \quad ◀$$

(*b*) $$\frac{v_D}{(DO) + (OC)} = \frac{v_0}{(OC)}; \qquad \frac{v_D}{(80 + 25) \text{ mm}} = \frac{100 \text{ mm/s}}{25 \text{ mm}}$$

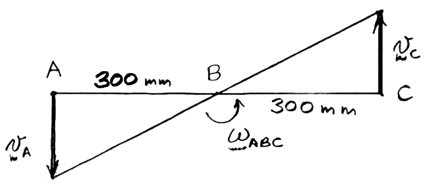
$$\mathbf{v}_D = 420 \text{ mm/s} \uparrow \quad ◀$$



PROBLEM 15.80

The arm ABC rotates with an angular velocity of 4 rad/s counterclockwise. Knowing that the angular velocity of the intermediate gear B is 8 rad/s counterclockwise, determine (a) the instantaneous centers of rotation of gears A and C, (b) the angular velocities of gears A and C.

SOLUTION



Contact points:

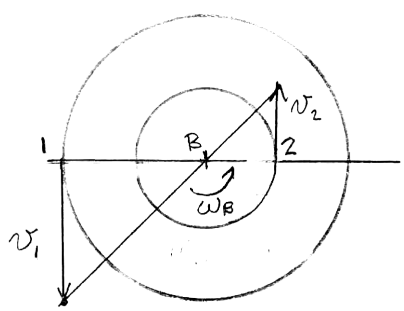
1 between gears A and B.

2 between gears B and C.

Arm ABC: $\omega_{ABC} = 4$ rad/s ⟲

$$v_A = (0.300)(4) = 1.2 \text{ m/s} \downarrow$$

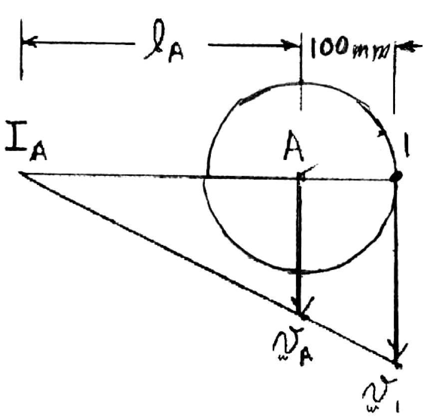
$$v_C = (0.300)(4) = 1.2 \text{ m/s} \uparrow$$



Gear B: $\omega_B = 8$ rad/s ⟲

$$v_1 = (0.200)(8) = 1.6 \text{ m/s} \downarrow$$

$$v_2 = (0.100)(8) = 0.8 \text{ m/s} \uparrow$$



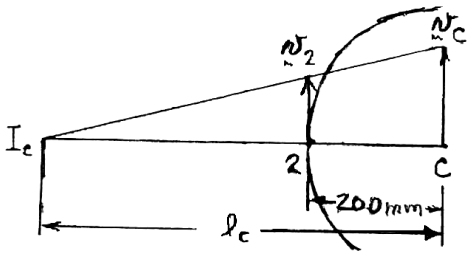
Gear A:

$$\omega_A = \frac{v_1 - v_A}{0.100} = \frac{1.6 - 1.2}{0.100}$$

$$\omega_A = 4 \text{ rad/s} ⟳$$

$$\ell_A = \frac{v_A}{\omega_A} = \frac{1.2}{4} = 0.3 \text{ m} = 300 \text{ mm}$$

PROBLEM 15.80 (Continued)



Gear C:

$$\omega_C = \frac{v_C - v_2}{0.200} = \frac{1.2 - 0.8}{0.2}$$

$$\omega_C = 2 \text{ rad/s}$$

$$\ell_C = \frac{v_C}{\omega_C} = \frac{1.2}{2} = 0.6 \text{ m}$$

(*a*) *Instantaneous centers.*

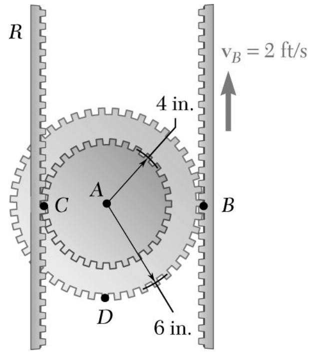
Gear *A*: 300 mm left of *A* ◀

Gear *C*: 600 mm left of *C* ◀

(*b*) *Angular velocities.*

$\boldsymbol{\omega}_A = 4.00$ rad/s ◀

$\boldsymbol{\omega}_C = 2.00$ rad/s ◀



PROBLEM 15.81

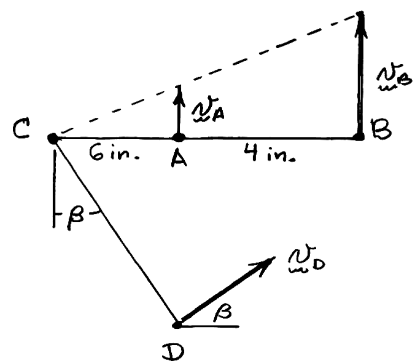
The double gear rolls on the stationary left rack R. Knowing that the rack on the right has a constant velocity of 2 ft/s, determine (a) the angular velocity of the gear, (b) the velocities of Points A and D.

SOLUTION

Since the rack R is stationary, Point C is the instantaneous center of the double gear.

Given: $\mathbf{v}_B = 2 \text{ ft/s} \uparrow = 24 \text{ in./s} \uparrow$

Make a diagram showing the locations of Points A, B, C, and D on the double gear.



$$v_B = \omega l_{CB}$$

$$\omega = \frac{v_B}{l_{CB}} = \frac{24 \text{ in./s}}{10 \text{ in.}} = 2.40 \text{ rad/s}$$

(a) *Angular velocity of the gear.* $\boldsymbol{\omega} = 2.40 \text{ rad/s} \circlearrowleft$ ◀

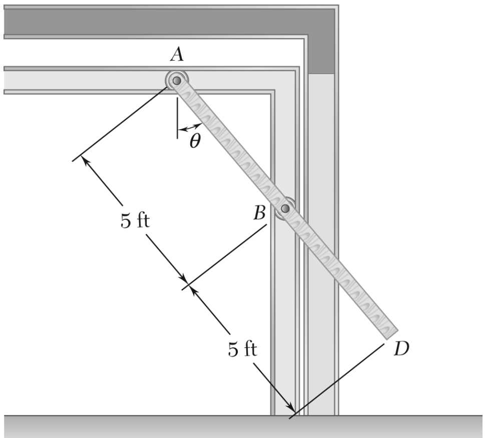
(b) *Velocity of Point A.* $v_A = l_{AC}\omega = (4 \text{ in.})(2.40 \text{ rad/s})$ $\mathbf{v}_A = 9.60 \text{ in./s} = 0.800 \text{ ft/s} \uparrow$ ◀

Geometry: $l_{CD} = \sqrt{(4 \text{ in.})^2 + (6 \text{ in.})^2} = \sqrt{52} \text{ in.}$

$$\tan\beta = \frac{4 \text{ in.}}{6 \text{ in.}} \qquad \beta = 33.7°$$

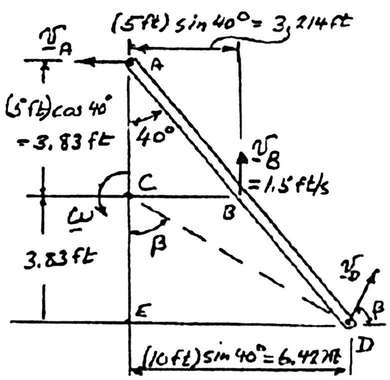
Velocity of Point D. $\mathbf{v}_D = l_{CD}\omega = \sqrt{52}(2.40) = 17.31 \text{ in./s}$ $\mathbf{v}_D = 1.442 \text{ ft/s} \measuredangle 33.7°$ ◀

PROBLEM 15.82



An overhead door is guided by wheels at A and B that roll in horizontal and vertical tracks. Knowing that when $\theta = 40°$ the velocity of wheel B is 1.5 ft/s upward, determine (*a*) the angular velocity of the door, (*b*) the velocity of end D of the door.

SOLUTION



Locate instantaneous center at intersection of lines drawn perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

(*a*) Angular velocity.

$$v_B = (BC)\omega$$

$$1.5 \text{ ft/s} = (3.214 \text{ ft})\omega$$

$$\omega = 0.4667 \text{ rad/s} \qquad \boldsymbol{\omega} = 0.467 \text{ rad/s} \circlearrowright \blacktriangleleft$$

(*b*) Velocity of D:

In ΔCDE:

$$\beta = \tan^{-1}\frac{6.427}{3.83} = 59.2°$$

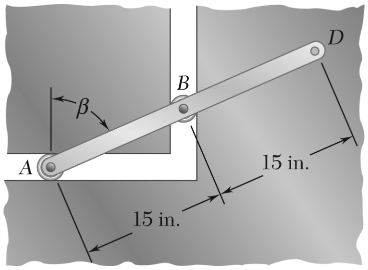
$$CD = \frac{6.427}{\sin\beta} = 7.482 \text{ ft}$$

$$v_D = (CD)\omega$$

$$= (7.482 \text{ ft})(0.4667 \text{ rad/s})$$

$$= 3.49 \text{ ft/s}$$

$$\mathbf{v}_D = 3.49 \text{ ft/s} \measuredangle\, 59.2° \blacktriangleleft$$

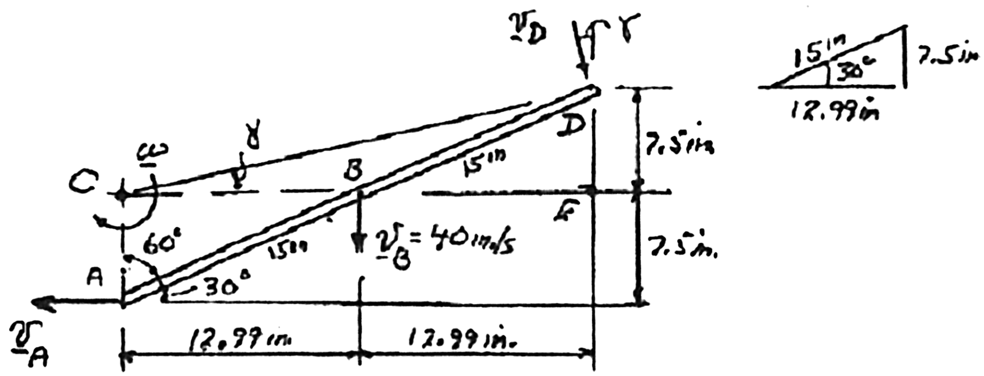


PROBLEM 15.83

Rod *ABD* is guided by wheels at *A* and *B* that roll in horizontal and vertical tracks. Knowing that at the instant shown $\beta = 60°$ and the velocity of wheel *B* is 40 in./s downward, determine (*a*) the angular velocity of the rod, (*b*) the velocity of Point *D*.

SOLUTION

Rod *ABD*:



We locate the instantaneous center by drawing lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_D$.

(*a*) Angular velocity.

$$v_B = (BC)\omega$$

$$40 \text{ in./s} = (12.99 \text{ in.})\omega$$

$$\omega = 3.079 \text{ rad/s}$$

$\boldsymbol{\omega} = 3.08$ rad/s ◀

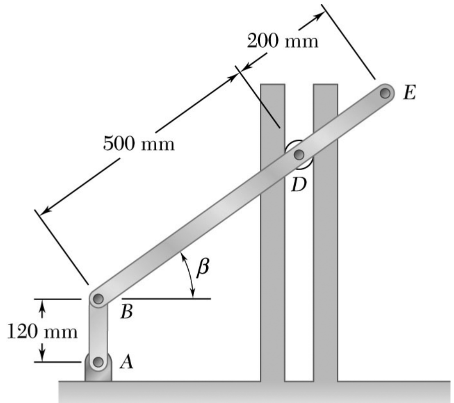
(*b*) Velocity of *D*:

In ΔCDE: $\gamma = \tan^{-1}\dfrac{7.5}{25.98} = 16.1°$; $CD = \dfrac{25.98}{\cos\gamma} = 27.04$ in.

$$v_D = (CD)\omega = (27.04 \text{ in.})(3.079 \text{ rad/s}) = 83.3 \text{ in./s}$$

$$\mathbf{v}_D = 83.3 \text{ in./s} \; 16.1°$$

$\mathbf{v}_D = 83.3$ in./s 73.9° ◀



PROBLEM 15.84

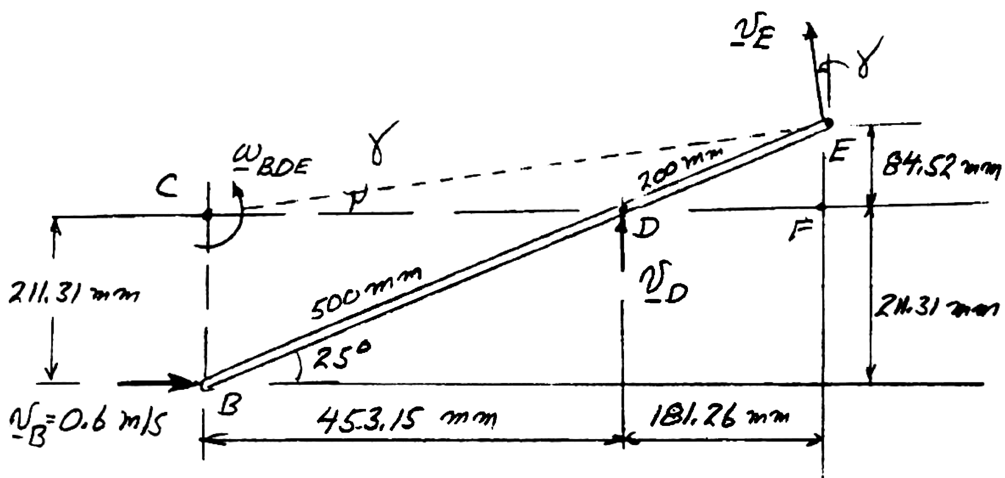
Rod *BDE* is partially guided by a roller at *D* which moves in a vertical track. Knowing that at the instant shown the angular velocity of crank *AB* is 5 rad/s clockwise and that $\beta = 25°$, determine (*a*) the angular velocity of the rod, (*b*) the velocity of Point *E*.

SOLUTION

Crank *AB*: $\boldsymbol{\omega}_{AB} = 5 \text{ rad/s} \curvearrowright \qquad \mathbf{r}_{B/A} = 120 \text{ mm} \uparrow$

$$v_B = \omega_{AB} r_{B/A} = (5)(0.120) \qquad \mathbf{v}_B = 0.6 \text{ m/s} \longrightarrow$$

Rod *BDE*: Draw a diagram of the geometry of the rod and note that $\mathbf{v}_B = 0.6 \text{ m/s} \longrightarrow$ and $\mathbf{v}_D = \mathbf{v}_D \uparrow$.



Locate Point *C*, the instantaneous center, by noting that *BC* is perpendicular to $\mathbf{v}_B$ and *DC* is perpendicular to $\mathbf{v}_D$. Calculate lengths of *BC* and *CD*.

$$l_{BC} = 500 \sin 25° = 211.31 \text{ mm}$$
$$l_{CD} = 500 \cos 25° = 453.15.$$

(*a*) *Angular velocity of the rod.*

$$\omega_{BCD} = \frac{v_B}{l_{BC}} = \frac{0.6 \text{ m/s}}{0.21131 \text{ m}} = 2.8394 \text{ rad/s}$$

$\boldsymbol{\omega}_{BCD} = 2.84 \text{ rad/s} \curvearrowleft$ ◀

(*b*) *Velocity of Point E.*

Locate Point *F* on the diagram.

$$CF = 700\cos 25°\text{ mm} \qquad FE = 200\sin 25°$$

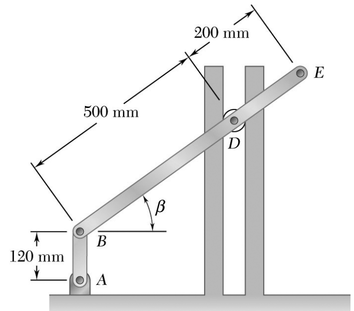
$$\tan\gamma = \frac{FE}{CF} = \frac{200\sin 25°}{700\cos 25°} = \frac{2}{7}\tan 25° = 0.13323$$

$$\gamma = 7.6° \qquad \beta = 90° - \gamma = 82.4°$$

$$l_{CE} = \sqrt{(CF)^2 + (FE)^2} = 640.02\text{ mm} = 0.64002\text{ m}$$

$$v_E = l_{CE}\omega = (0.64002)(2.8394)$$

$\mathbf{v}_E = 1.817\text{ m/s} \measuredangle 82.4°$ ◀



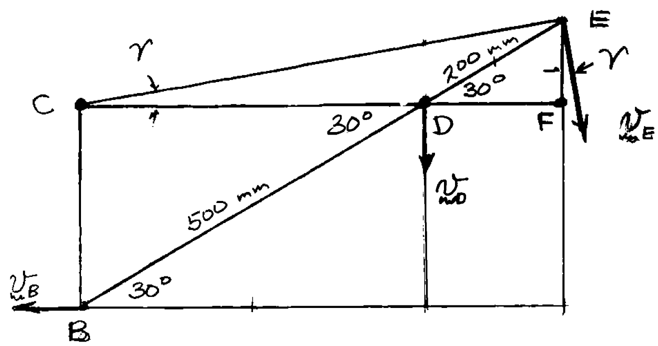
PROBLEM 15.85

Rod *BDE* is partially guided by a roller at *D* which moves in a vertical track. Knowing that at the instant shown $\beta = 30°$, Point *E* has a velocity of 2 m/s down and to the right, determine the angular velocities of rod *BDE* and crank *AB*.

SOLUTION

Crank *AB*: When *AB* is vertical, the velocity $\mathbf{v}_B$ at Point *B* is horizontal.

Rod *BDE*: Draw a diagram of the geometry of the rod and note that $\mathbf{v}_B$ is horizontal and $\mathbf{v}_D$ is vertical.



Locate Point *C*, the instantaneous center *C*, by noting that *CB* is vertical and *CD* is horizontal. From the diagram, with Point *F* added,

$$CF = 700\cos 30° \text{ mm} \qquad FE = 200\sin 30° \text{ mm}$$

$$CE = \sqrt{(CF)^2 + (FE)^2} = 614.41 \text{ mm} = 0.61441 \text{ m}$$

Angular velocity of rod BDE

$$\omega_{BDE} = \frac{v_E}{(CE)} = \frac{2 \text{ m/s}}{0.61441 \text{ m}} = 3.2552 \text{ rad/s}$$

$$\boldsymbol{\omega}_{BDE} = 3.26 \text{ rad/s} \circlearrowright \blacktriangleleft$$

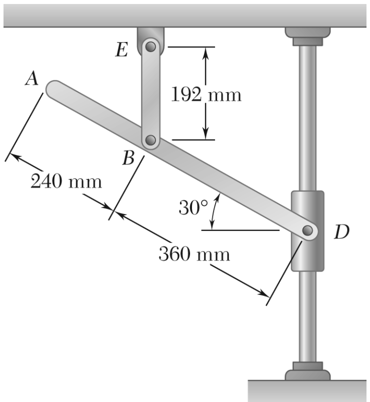
Velocity of B. $\quad CB = 500\sin 30° \text{ mm} = 250 \text{ mm} = 0.250 \text{ m}$

$$v_B = (CB)\omega_{BDE} = (0.250)(3.2552)$$

$$\mathbf{v}_B = 0.81379 \text{ m/s} \longleftarrow$$

Angular velocity of crank AB: $\quad AB = 120 \text{ mm} = 0.120 \text{ m}$

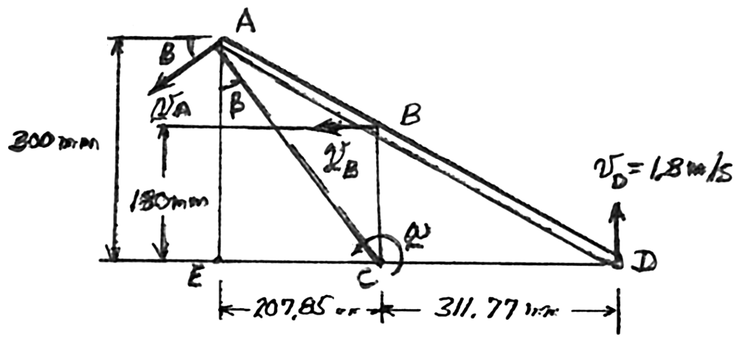
$$\omega_{AB} = \frac{v_B}{(AB)} = \frac{0.81379 \text{ m/s}}{0.120 \text{ m}} \qquad \boldsymbol{\omega}_{AB} = 6.78 \text{ rad/s} \circlearrowright \blacktriangleleft$$



PROBLEM 15.86

Knowing that at the instant shown the velocity of collar D is 1.6 m/s upward, determine (a) the angular velocity of rod AD, (b) the velocity of Point B, (c) the velocity of Point A.

SOLUTION



We draw perpendiculars to $\mathbf{v}_B$ and $\mathbf{v}_D$ to locate instantaneous center C.

(a) Angular velocity:

$$v_D = (CD)\omega \qquad 1.6 \text{ m/s} = (0.31177 \text{ m})\omega$$

$$\omega = 5.132 \text{ rad/s} \qquad \boldsymbol{\omega} = 5.13 \text{ rad/s} ◀$$

(b)

$$v_B = (BC)\omega = (180 \text{ mm})(5.132 \text{ rad/s})$$

$$v_D = 923.76 \text{ mm/s} \qquad \mathbf{v}_D = 0.924 \text{ m/s} \leftarrow ◀$$

(c)

$$v_A = (AC)\omega$$

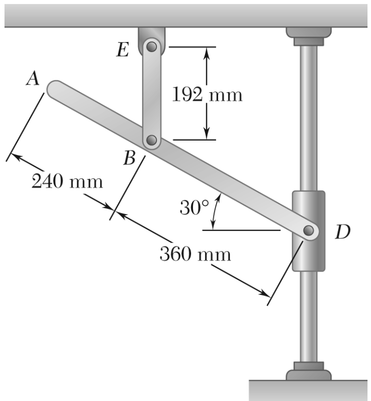
In triangle ACE:

$$\tan\beta = \frac{207.85 \text{ mm}}{300 \text{ mm}} \qquad \beta = 34.72°$$

$$AC = \sqrt{(207.85)^2 + (300)^2} \qquad AC = 364.97 \text{ mm}$$

$$v_A = (364.97 \text{ mm})(5.132 \text{ rad/s}) = 1873.0 \text{ mm/s}$$

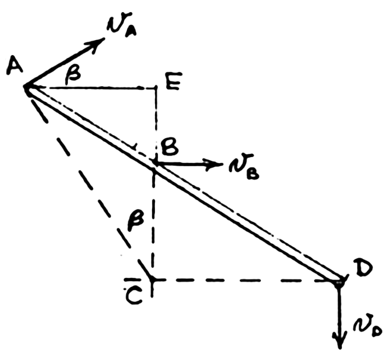
$$\mathbf{v}_A = 1.870 \text{ m/s} \measuredangle 34.7° ◀$$



PROBLEM 15.87

Knowing that at the instant shown the angular velocity of rod BE is 4 rad/s counterclockwise, determine (a) the angular velocity of rod AD, (b) the velocity of collar D, (c) the velocity of Point A.

SOLUTION



Rod AD.

$$\mathbf{v}_B = r_{B/E}\omega_{BE} = (0.192)(4) = 0.768 \text{ m/s} \longrightarrow$$

(a) Instantaneous center C is located by noting that CD is perpendicular to $\mathbf{v}_D$ and CB is perpendicular to $\mathbf{v}_B$.

$$r_{B/C} = 0.360 \ \sin 30° = 0.180 \text{ m}$$

$$\omega_{AD} = \frac{v_B}{r_{B/C}} = \frac{0.768}{0.180} = 4.2667$$

$$\boldsymbol{\omega}_{AD} = 4.27 \text{ rad/s} \circlearrowright \blacktriangleleft$$

(b) Velocity of D.

$$r_{D/C} = 0.360 \ \cos 30° = 0.31177 \text{ m}$$

$$v_D = r_{D/C}\omega = (0.31177)(4.2667)$$

$$\mathbf{v}_D = 1.330 \text{ m/s} \downarrow \blacktriangleleft$$

(c) Velocity of A.

$$l_{AE} = 0.240 \cos 30° = 0.20785 \text{ m}$$

$$l_{CE} = 0.600 \sin 30° = 0.300 \text{ m}$$

$$\tan\beta = \frac{0.20785}{0.300} \qquad \beta = 34.7°$$

$$l_{CA} = \sqrt{(0.20785)^2 + (0.300)^2}$$

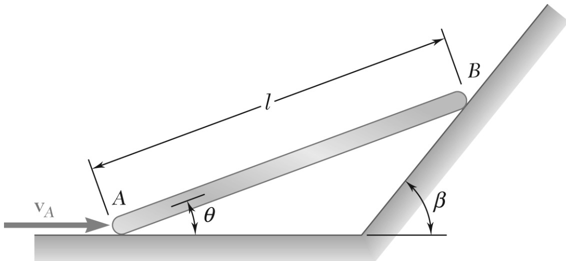
$$= 0.36497 \text{ m}$$

$$v_A = l_{CA}\omega_{AD}$$

$$= (0.36497)(4.2667)$$

$$= 1.557 \text{ m/s}$$

$$\mathbf{v}_A = 1.557 \text{ m/s} \ \angle\!\!\nearrow 34.7° \blacktriangleleft$$



PROBLEM 15.88

Rod AB can slide freely along the floor and the inclined plane. Denoting by $\mathbf{v}_A$ the velocity of Point A, derive an expression for (*a*) the angular velocity of the rod, (*b*) the velocity of end B.

SOLUTION

Locate the instantaneous center at intersection of lines drawn perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

Law of sines.

$$\frac{AC}{\sin[90° - (\beta - \theta)]} = \frac{BC}{\sin(90° - \theta)}$$

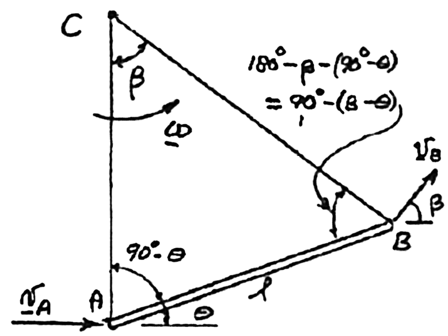
$$= \frac{l}{\sin\beta}$$

$$\frac{AC}{\cos(\beta - \theta)} = \frac{BC}{\cos\theta}$$

$$= \frac{l}{\sin\beta}$$

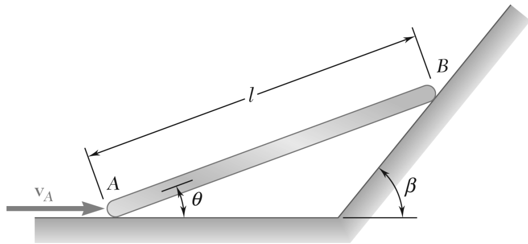
$$AC = l\frac{\cos(\beta - \theta)}{\sin\beta}$$

$$BC = l\frac{\cos\theta}{\sin\beta}$$



(*a*) Angular velocity: $v_A = (AC)\omega = l\dfrac{\cos(\beta - \theta)}{\sin\beta}\omega$ $\qquad \omega = \dfrac{v_A}{l} \cdot \dfrac{\sin\beta}{\cos(\beta - \theta)}$ ◀

(*b*) Velocity of B: $v_B = (BC)\omega = l\dfrac{\cos\theta}{\sin\beta} \cdot \left[\dfrac{v_\theta}{l} \cdot \dfrac{\sin\beta}{\cos(\beta - \theta)}\right]$ $\qquad v_B = v_A \dfrac{\cos\theta}{\cos(\beta - \theta)}$ ◀



PROBLEM 15.89

Rod AB can slide freely along the floor and the inclined plane. Knowing that $\theta = 20°$, $\beta = 50°$, $l = 2$ ft, and $v_A = 8$ ft/s, determine (*a*) the angular velocity of the rod, (*b*) the velocity of end B.

SOLUTION

Locate the instantaneous center at intersection of lines draw perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

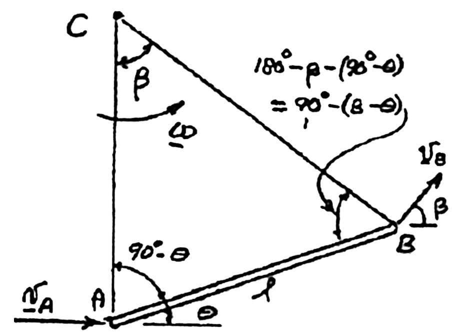
Law of sines.

$$\frac{AC}{\sin[90° - (\beta - \theta)]} = \frac{BC}{\sin(90° - \theta)} = \frac{l}{\sin\beta}$$

$$\frac{AC}{\cos(\beta - \theta)} = \frac{BC}{\cos\theta} = \frac{l}{\sin\beta}$$

$$AC = l\frac{\cos(\beta - \theta)}{\sin\beta}$$

$$BC = l\frac{\cos\theta}{\sin\beta}$$



Angular velocity:
$$v_A = (AC)\omega = l\frac{\cos(\beta - \theta)}{\sin\beta}\omega$$

$$\omega = \frac{v_A}{l}\cdot\frac{\sin\beta}{\cos(\beta - \theta)}$$

Velocity of B:
$$v_B = (BC)\omega = l\frac{\cos\theta}{\sin\beta}\cdot\left[\frac{v_A}{l}\cdot\frac{\sin\beta}{\cos(\beta - \theta)}\right]$$

$$v_B = v_A\frac{\cos\theta}{\cos(\beta - \theta)}$$

Data: $\theta = 20°$, $\beta = 50°$, $l = 2$ ft, $v_A = 8$ ft/s

(*a*)
$$\omega = \frac{v_A}{l}\frac{\sin\beta}{\cos(\beta - \theta)} = \frac{8 \text{ ft/s}}{2 \text{ ft}}\cdot\frac{\sin 50°}{\cos(50° - 20°)}$$

$$\omega = 3.5382 \text{ rad/s}$$

$\boldsymbol{\omega} = 3.54$ rad/s ↻ ◀

PROBLEM 15.89 (Continued)

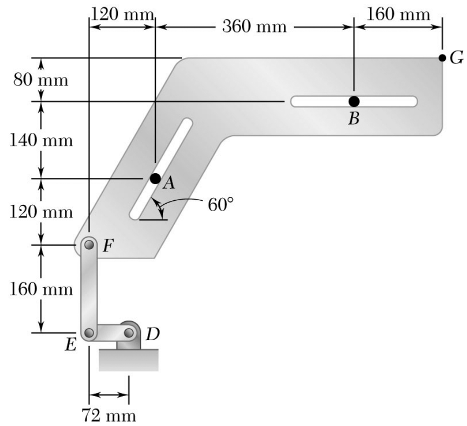
(*b*)

$$v_B = v_A \frac{\cos\theta}{\cos(\beta-\theta)}$$

$$= (8 \text{ ft/s}) \frac{\cos 20^\circ}{\cos(50^\circ - 20^\circ)}$$

$$v_B = 8.6805 \text{ ft/s}$$

$\mathbf{v}_B = 8.68 \text{ ft/s} \measuredangle 50^\circ$ ◀



PROBLEM 15.90

Two slots have been cut in plate *FG* and the plate has been placed so that the slots fit two fixed pins *A* and *B*. Knowing that at the instant shown the angular velocity of crank *DE* is 6 rad/s clockwise, determine (*a*) the velocity of Point *F*, (*b*) the velocity of Point *G*.

SOLUTION

Crank *DE*: $v_E = (DE)\omega_{DE} = (72 \text{ mm})(6 \text{ rad/s})$

$\mathbf{v}_E = 432 \text{ mm/s} \uparrow$

Rod *EF*: $(v_F)_y = v_E = 432 \text{ mm/s} \uparrow$

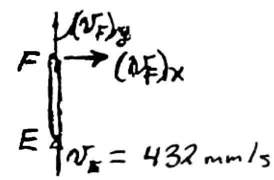
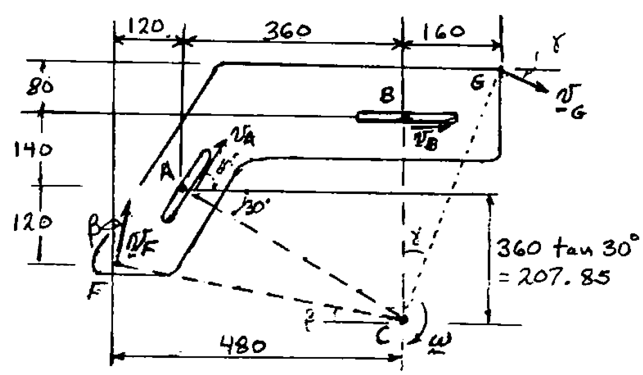


Plate *FG*:



$\mathbf{v}_A$ and $\mathbf{v}_B$ are velocities of points on the plate next to the pins *A* and *B*. We draw lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$ to locate the instantaneous center *C*.

(*a*) Velocity of Point *F*:

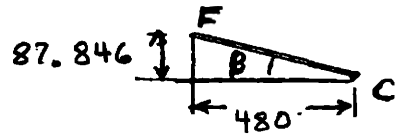
$$v_F = (CF)\omega$$
$$(v_F)_y = [(CF)\omega]\cos\beta = [(CF)\cos\beta]\omega$$

But $(v_F)_y = 432 \text{ mm/s}$ and $(CF)\cos\beta = 480 \text{ mm}$:

$$432 \text{ mm/s} = (480 \text{ mm})\omega$$
$$\omega = 0.9 \text{ rad/s}$$

$\boldsymbol{\omega} = 0.9 \text{ rad/s}$ ↻

PROBLEM 15.90 (Continued)



$$CF = 487.97 \text{ mm}$$

$$\beta = 10.37°$$

$$v_F = (CF)\omega$$

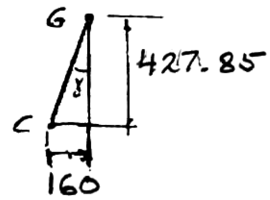
$$= (487.97 \text{ mm})(0.9 \text{ rad/s})$$

$$= 439.18 \text{ mm/s}$$

$$\mathbf{v}_F = 439 \text{ mm/s} \measuredangle 10.4°$$

$\mathbf{v}_F = 439$ mm/s $\measuredangle$ 79.6° ◀

(*b*) Velocity of Point *G*:



$$CG = 456.78 \text{ mm}$$

$$\gamma = 20.50°$$

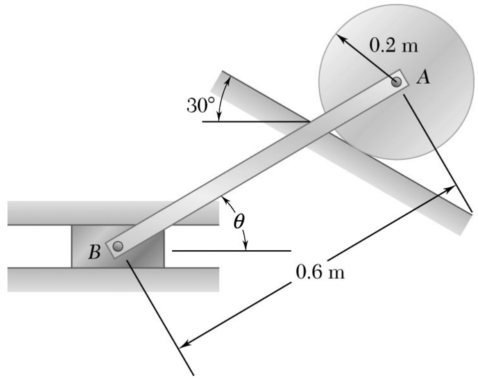
$$v_G = (CG)\omega$$

$$= (456.78 \text{ in.})(0.9 \text{ rad/s})$$

$$v_G = 411.11 \text{ mm/s}$$

$\mathbf{v}_G = 411$ mm/s $\measuredangle$ 20.5° ◀

PROBLEM 15.91

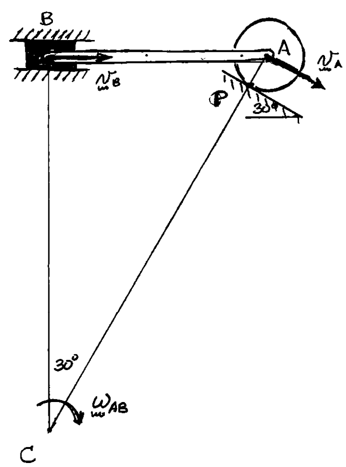


The disk is released from rest and rolls down the incline. Knowing that the speed of A is 1.2 m/s when $\theta = 0°$, determine at that instant (*a*) the angular velocity of the rod, (*b*) the velocity of B. Only portions of the two tracks are shown.

SOLUTION

Draw the slider, rod, and disk at $\theta = 0°$.

Let Point P be the contact point between the disk and the incline. It is the instantaneous center of the disk. $\mathbf{v}_A$ is parallel to the incline. So that



$$\mathbf{v}_A = v_A \searrow 30°$$

Constraint of slider:

$$\mathbf{v}_B = v_B \longrightarrow$$

To locate the instantaneous center C of the rod AB, extend the line AP to meet the vertical line through P at Point C.

$$l_{AC} = l_{AB} / \sin 30°$$
$$l_{BC} = l_{AB} / \tan 30°$$

Angular velocity of rod AB.

$$\omega_{AB} = \frac{v_A}{l_{AC}} = \frac{v_A \sin 30°}{l_{AB}} = \frac{(1.2 \text{ m/s}) \sin 30°}{0.6 \text{ m}}$$

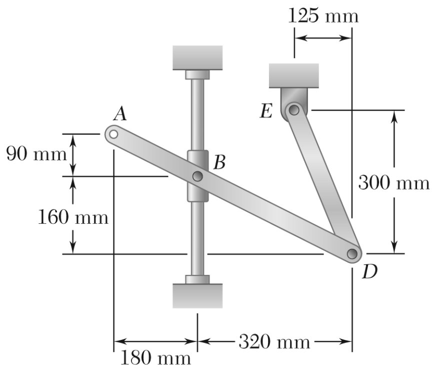
$$\boldsymbol{\omega}_{AB} = 1.000 \text{ rad/s} \circlearrowright \blacktriangleleft$$

(*b*) *Velocity of Point B.*

$$v_B = l_{BC}\omega_{AB}$$

$$v_B = \frac{l_{AB}}{\tan 30°} \frac{v_A \sin 30°}{l_{AB}} = v_A \cos 30° = 1.2 \cos 30°$$

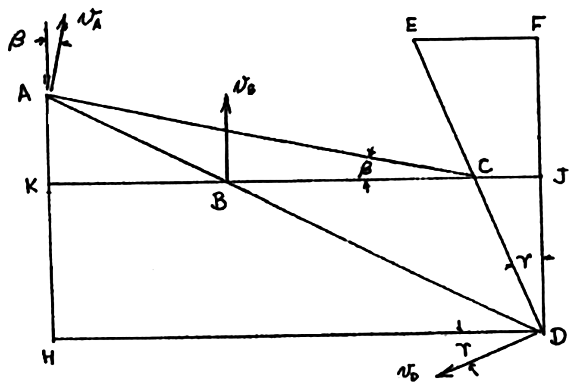
$$\mathbf{v}_B = 1.039 \text{ m/s} \longrightarrow \blacktriangleleft$$



PROBLEM 15.92

Arm ABD is connected by pins to a collar at B and to crank DE. Knowing that the velocity of collar B is 400 mm/s upward, determine (a) the angular velocity of arm ABD, (b) the velocity of Point A.

SOLUTION



$\mathbf{v}_B = 16$ in./s$\uparrow$ $\qquad \tan\gamma = \dfrac{EF}{DF} = \dfrac{125}{300} \qquad \mathbf{v}_D = v_D \measuredangle \gamma$

Locate the instantaneous center (Point C) of bar ABD by noting that velocity directions at Points B and D are known. Draw BC perpendicular to $\mathbf{v}_B$ and DC perpendicular to $\mathbf{v}_D$.

$$CJ = (DJ)\tan\gamma = (160)\left(\frac{125}{300}\right) = 66.667 \text{ mm}$$

$$CB = JB - CJ = 320 - 66.667 = 253.33 \text{ mm}$$

(a)

$$\omega_{ABD} = \frac{v_B}{CB} = \frac{400}{253.33} = 1.57895 \text{ rad/s} \qquad \boldsymbol{\omega}_{ABD} = 1.579 \text{ rad/s} \circlearrowright \blacktriangleleft$$

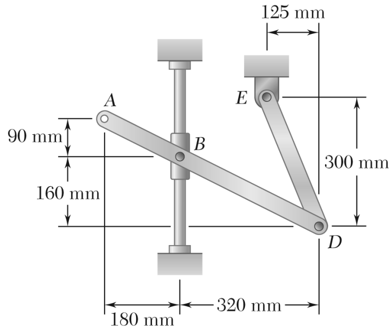
$$CK = CB + BK = 253.33 + 180 = 433.33 \text{ mm}$$

$$\tan\beta = \frac{KA}{CK} = \frac{90}{433.33}, \quad \beta = 11.733°, \quad 90° - \beta = 78.3°$$

$$AC = \frac{CK}{\cos\beta} = \frac{433.33}{\cos 11.733°} = 442.58 \text{ mm}$$

(b)

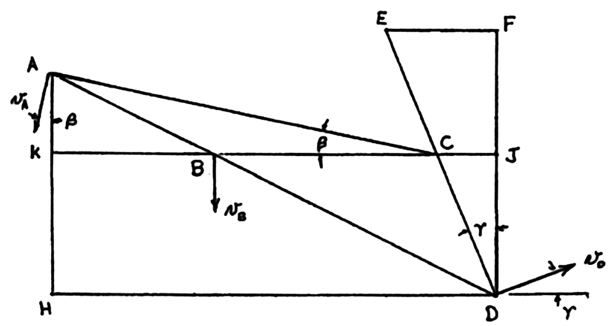
$$v_A = (AC)\omega_{ABD} = (442.58)(1.57895) = 699 \text{ mm/s} \qquad \mathbf{v}_A = 699 \text{ mm/s} \measuredangle 78.3° \blacktriangleleft$$



PROBLEM 15.93

Arm *ABD* is connected by pins to a collar at *B* and to crank *DE*. Knowing that the angular velocity of crank *DE* is 1.2 rad/s counterclockwise, determine (*a*) the angular velocity of arm *ABD*, (*b*) the velocity of Point *A*.

SOLUTION



$$\tan\gamma = \frac{EF}{DF} = \frac{125}{300}, \quad \gamma = 22.620°, \quad ED = \frac{FD}{\cos\gamma} = \frac{300}{\cos\gamma} = 325 \text{ mm}$$

$$v_D = (ED)\omega_{DE} = (325)(1.2) = 390 \text{ mm/s} \qquad \mathbf{v}_D = 390 \text{ mm/s} \measuredangle \gamma$$

$$\mathbf{v}_B = v_B \downarrow$$

Locate the instantaneous center (Point C) of bar *ABD* by noting that velocity directions at Points *B* and *D* are known. Draw *BC* perpendicular to $\mathbf{v}_B$ and *DC* perpendicular to $\mathbf{v}_D$.

$$CJ = (DJ)\tan\gamma = (160)\left(\frac{125}{300}\right) = 66.667 \text{ mm}, \quad CD = \frac{DJ}{\cos\gamma} = \frac{160}{\cos\gamma} = 173.33 \text{ mm}$$

(*a*)

$$\omega_{ABD} = \frac{v_D}{CD} = \frac{390}{173.33} = 2.25 \text{ rad/s} \circlearrowright \qquad \omega_{ABD} = 2.25 \text{ rad/s} \circlearrowright \blacktriangleleft$$

$$CK = KJ - CJ = 500 - 66.667 = 433.33 \text{ mm}$$

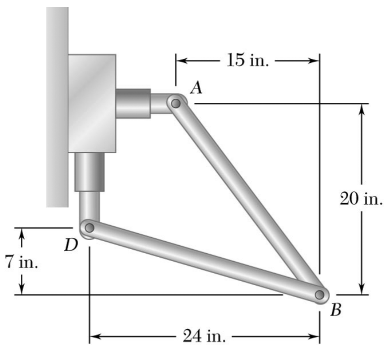
$$\tan\beta = \frac{AK}{CK} = \frac{90}{433.33} \qquad \beta = 11.733° \qquad 90° - \beta = 78.3°$$

$$AC = \frac{CK}{\cos\beta} = \frac{433.33}{\cos\beta} = 442.58 \text{ mm}$$

(*b*)

$$v_A = (AC)\omega_{ABD} = (442.58)(2.25) = 996 \text{ mm/s} \qquad \mathbf{v}_A = 996 \text{ mm/s} \measuredangle 78.3° \blacktriangleleft$$

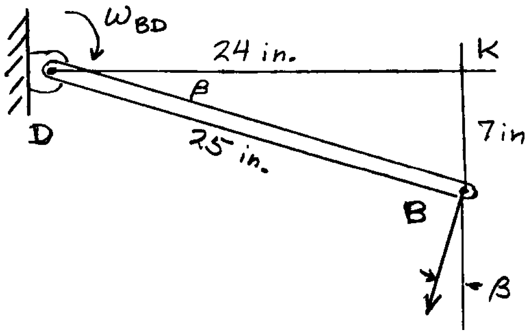
PROBLEM 15.94



Two links AB and BD, each 25 in. long, are connected at B and guided by hydraulic cylinders attached at A and D. Knowing that D is stationary and that the velocity of A is 30 in./s to the right, determine at the instant shown (*a*) the angular velocity of each link, (*b*) the velocity of B.

SOLUTION

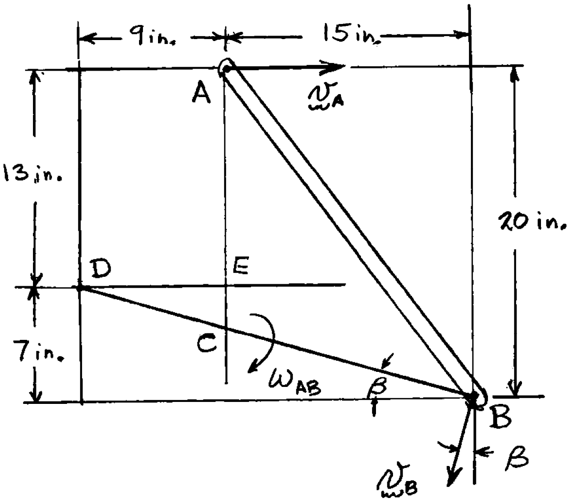
Link DB: Point D is stationary. Assume $\boldsymbol{\omega}_{BD} = \omega_{BD}$ ↻.



$v_B = (DB)\omega_{BD}$ $\mathbf{v}_B$ is perpendicular to DB.

$\tan\beta = \dfrac{7 \text{ in.}}{24 \text{ in.}}$ $\beta = 16.3°$ $90° - \beta = 73.7°$

Link AB: Draw the configuration. Locate the instantaneous center C of link AB by noting that the line BC is perpendicular to $\mathbf{v}_B$, i.e., along DB, and that AC is perependucular to $\mathbf{v}_A = v_A \longrightarrow$. ($v_A = 30$ in./s).



PROBLEM 15.94 (Continued)

$$\overline{AC} = \overline{AE} + \overline{EC} = 13\text{ in.} + (7\text{ in.})\frac{9\text{ in.}}{24\text{ in.}} = 15.625\text{ in.}$$

$$\overline{BC} = \frac{15\text{ in.}}{24\text{ in.}}(25\text{ in.}) = 15.625\text{ in.}$$

$$\omega_{AB} = \frac{v_A}{(AC)} = \frac{30\text{ in./s}}{15.625\text{ in.}} = 1.92\text{ rad/s}$$

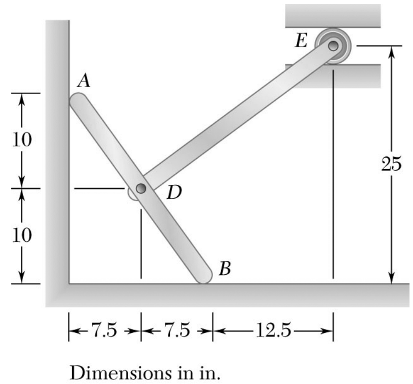
$$v_B = (\overline{BC})\omega_{AB} = (15.625)(1.92) = 30\text{ in./s}$$

Returning to link *DB*, $$\omega_{BD} = \frac{v_B}{(DB)} = \frac{30\text{ in./s}}{25\text{ in.}} = 1.20\text{ rad/s}$$

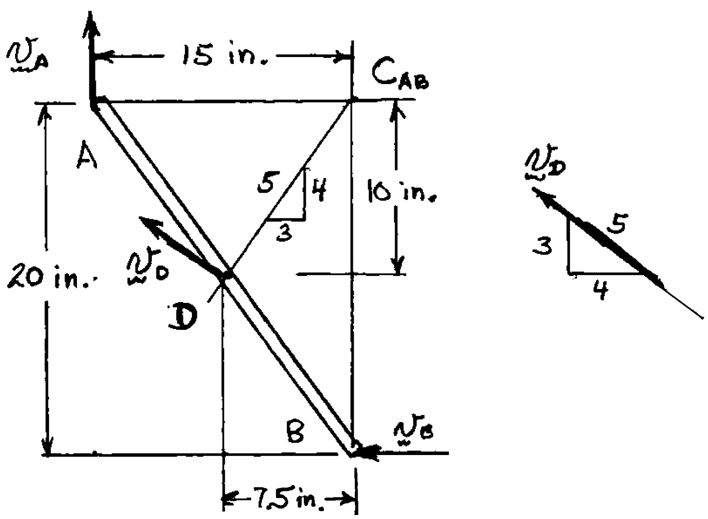
(*a*) *Angular velocities*: $\boldsymbol{\omega}_{AB} = 1.920\text{ rad/s} \circlearrowright$ ◀

$\boldsymbol{\omega}_{BD} = 1.200\text{ rad/s} \circlearrowright$ ◀

(*b*) *Velocity of Point B*: $\mathbf{v}_B = 30.0\text{ in./s} \measuredangle\, 73.7°$ ◀

	**PROBLEM 15.95** Two 25-in. rods are pin-connected at D as shown. Knowing that B moves to the left with a constant velocity of 24 in./s, determine at the instant shown (a) the angular velocity of each rod, (b) the velocity of E.

SOLUTION



Rod AB: Draw lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$ to locate instantaneous center C_{AB}.

$$v_B = (BC_{AB})\omega_{AB}$$

$$24 \text{ in./s} = (20 \text{ in.})\omega_{AB}$$

$$\boldsymbol{\omega}_{AB} = 1.200 \text{ rad/s} \curvearrowright \blacktriangleleft$$

Velocity of D:

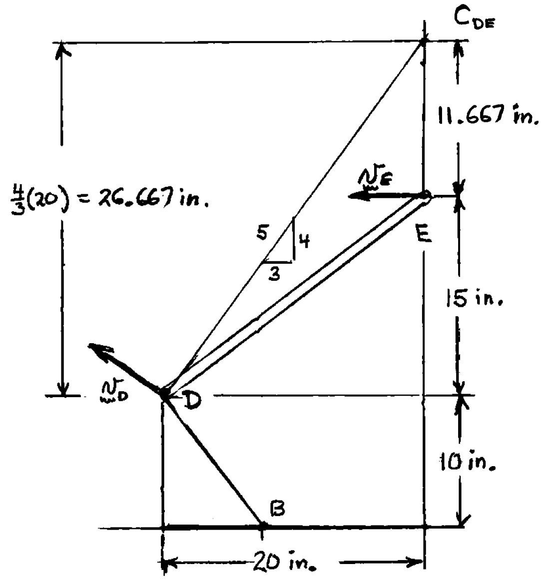
$$DC_{AB} = 12.5 \text{ in.}$$

$$v_D = (DC_{AB})\omega_{AB}$$

$$= (12.5 \text{ in.})(1.2 \text{ rad/s})$$

$$\mathbf{v}_D = 15 \text{ in./s} \measuredangle^{4}_{3}$$

Rod *DE*:



Draw lines perpendicular to $\mathbf{v}_D$ and $\mathbf{v}_E$ to locate instantaneous center C_{DE}.

$$DC_{DE} = \sqrt{(20)^2 + (26.667)^2} = 33.333 \text{ in.}$$

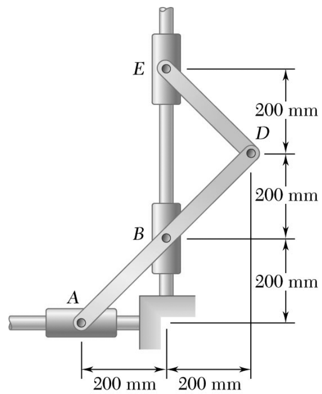
$$v_D = (DC_{DE})\omega_{DE}$$

(*a*) $15 \text{ in./s} = (33.333 \text{ in.})\omega_{DE}; \quad \omega_{DE} = 0.45 \text{ rad/s}$ $\qquad \boldsymbol{\omega}_{DE} = 0.450 \text{ rad/s}$ ◀

$$v_E = (EC_{DE})\omega_{DE} = (11.667 \text{ in.})(0.45 \text{ rad/s})$$

(*b*) $v_E = 5.25 \text{ in./s}$ $\qquad \mathbf{v}_E = 5.25 \text{ in./s} \longleftarrow$ ◀

PROBLEM 15.96



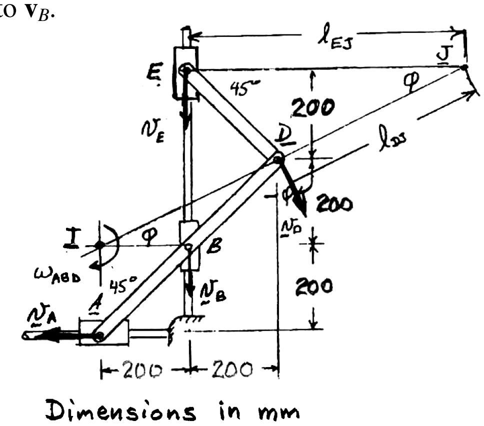
Two rods ABD and DE are connected to three collars as shown. Knowing that the angular velocity of ABD is 5 rad/s clockwise, determine at the instant shown (*a*) the angular velocity of DE, (*b*) the velocity of collar E.

SOLUTION

$$\omega_{ABC} = 5 \text{ rad/s} \circlearrowright \qquad \mathbf{v}_A = v_A \leftarrow$$

$$\mathbf{v}_B = v_B \downarrow \qquad \mathbf{v}_E = v_E \downarrow$$

Locate Point I, the instantaneous center of rod ABD by drawing IA perependicular to $\mathbf{v}_A$ and IB perpendicular to $\mathbf{v}_B$.



$$\tan\phi = \frac{200}{400} \qquad \phi = 26.565°$$

$$I_{ID} = \frac{400}{\cos\phi} = 447.21 \text{ mm}$$

$$v_D = \omega_{ABD} l_{ID} = (5)(447.21 \text{ mm})$$

$$v_D = 2236.1 \text{ mm/s} \searrow \phi$$

Locate Point J, the instantaneous center of rod DE by drawing JD perpendicular to $\mathbf{v}_D$ and JE perpendicular to $\mathbf{v}_E$.

$$l_{JD} = \frac{400}{\cos\phi} = 447.21 \text{ mm}$$

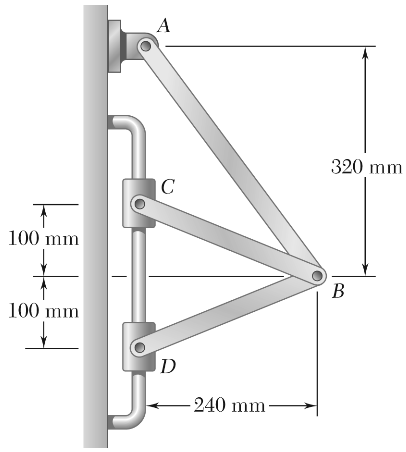
$$\omega_{DE} = \frac{v_D}{l_{JD}} = \frac{2236.1 \text{ mm/s}}{447.21} = 5 \text{ rad/s}$$

(*a*) $\boldsymbol{\omega}_{DE} = 5.00 \text{ rad/s} \circlearrowleft$ ◀

$$l_{JE} = 200 + l_{JD}\cos\phi = 600 \text{ mm}$$

$$v_E = l_{JE}\,\omega_{DE} = (600)(5) = 3000 \text{ mm/s}$$

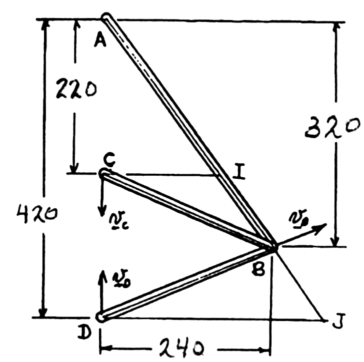
(*b*) $\mathbf{v}_E = 3.00 \text{ m/s} \downarrow$ ◀



PROBLEM 15.97

Two collars C and D move along the vertical rod shown. Knowing that the velocity of collar C is 660 mm/s downward, determine (a) the velocity of collar D, (b) the angular velocity of member AB.

SOLUTION



$AB = 400$ mm

Instantaneous centers: at I for BC.

at J for BD.

Geometry.

$$IC = \left(\frac{240}{320}\right)(220) = 165 \text{ mm}$$

$$JD = \left(\frac{240}{320}\right)(420) = 315 \text{ mm}$$

$$AI = \left(\frac{220}{320}\right)(400) = 275 \text{ mm}$$

$$BI = AB - AI = 400 - 275 = 125 \text{ mm}$$

$$BJ = BI = 125 \text{ mm}$$

Member BC. $\mathbf{v}_C = 660 \text{ mm/s}\downarrow$

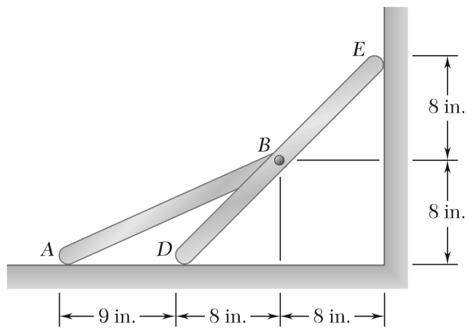
$$\omega_{BC} = \frac{v_C}{IC} = \frac{660}{165} = 4 \text{ rad/s}$$

$$v_B = (BI)\omega_{BC} = (125 \text{ mm})(4 \text{ rad/s}) = 500 \text{ mm/s}$$

Member BD. $$\omega_{BD} = \frac{v_B}{BJ} = \frac{500 \text{ mm/s}}{125 \text{ mm}} = 4 \text{ rad/s}$$

(a) $\mathbf{v}_D = (JD)\omega_{BD} = (315 \text{ mm})(4 \text{ rad/s})$ $\qquad \mathbf{v}_D = 1260 \text{ mm/s}\uparrow$ ◀

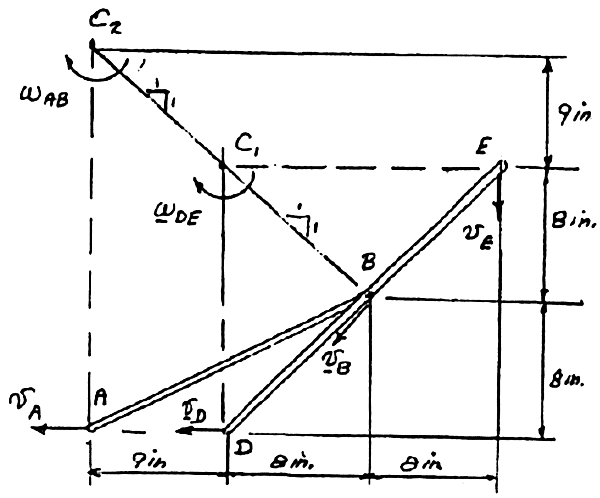
(b) $\omega_{AB} = \dfrac{v_B}{AB} = \dfrac{500 \text{ mm/s}}{400 \text{ mm}}$ $\qquad \boldsymbol{\omega}_{AB} = 1.250 \text{ rad/s}$ ↻ ◀



PROBLEM 15.98

Two rods AB and DE are connected as shown. Knowing that Point D moves to the left with a velocity of 40 in./s, determine (a) the angular velocity of each rod, (b) the velocity of Point A.

SOLUTION



We locate two instantaneous centers at intersections of lines drawn as follows:

C_1: For rod DE, draw lines perpendicular to $\mathbf{v}_D$ and $\mathbf{v}_E$.

C_2: For rod AB, draw lines perpendicular to $\mathbf{v}_A$ and $\mathbf{v}_B$.

Geometry:

$$BC_1 = (8\text{ in.})\sqrt{2} = 8\sqrt{2}\text{ in.}$$
$$DC_1 = 16\text{ in.}$$
$$BC_2 = (9\text{ in.} + 8\text{ in.})\sqrt{2} = 17\sqrt{2}\text{ in.}$$
$$AC_2 = 25\text{ in.}$$

(a) Rod DE:

$$v_D = (DC_1)\omega_{DE}$$
$$40\text{ in./s} = (16\text{ in.})\omega_{DE}$$
$$\omega_{DE} = 2.5\text{ rad/s}$$

$\boldsymbol{\omega}_{DE} = 2.5\text{ rad/s}$ ◀

$$v_B = (BC)\omega_{DE}$$
$$= (8\sqrt{2}\text{ in.})(2.5\text{ rad/s})$$
$$\mathbf{v}_B = 20\sqrt{2}\text{ in./s} \measuredangle 45^\circ$$

PROBLEM 15.98 (Continued)

Rod *AB*:

$$v_B = (BC_2)\omega_{AB}$$

$$20\sqrt{2} \text{ in./s} = (17\sqrt{2} \text{ in.})\omega_{AB}$$

$$\omega_{AB} = \frac{20}{17} \text{ rad/s} = 1.1765 \text{ rad/s}$$

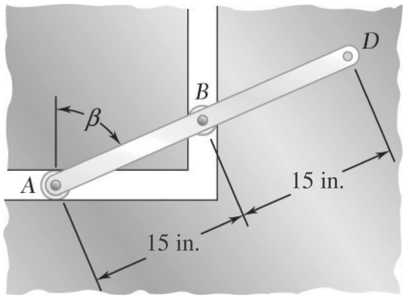
$$\boldsymbol{\omega}_{AB} = 1.177 \text{ rad/s} \circlearrowright \blacktriangleleft$$

(*b*)

$$v_A = (AC_2)\omega_{AB}$$

$$= (25 \text{ in.})(1.1765 \text{ rad/s})$$

$$v_A = 29.41 \text{ in./s} \qquad \mathbf{v}_A = 29.4 \text{ in./s} \longleftarrow \blacktriangleleft$$

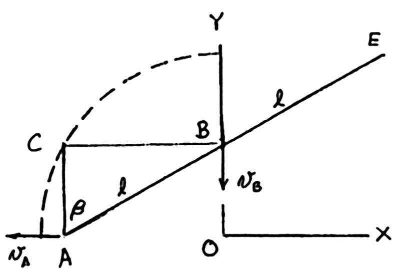


PROBLEM 15.99

Describe the space centrode and the body centrode of rod *ABD* of Problem 15.83. (*Hint:* The body centrode need not lie on a physical portion of the rod.)

PROBLEM 15.83 Rod *ABD* is guided by wheels at *A* and *B* that roll in horizontal and vertical tracks. Knowing that at the instant shown $\beta = 60°$ and the velocity of wheel *B* is 40 in./s downward, determine (*a*) the angular velocity of the rod, (*b*) the velocity of Point *D*.

SOLUTION



Draw *x* and *y* axes as shown with origin at the intersection of the two slots. These axes are fixed in space.

$$\mathbf{v}_A = v_A \longleftarrow, \qquad \mathbf{v}_B = v_B \downarrow$$

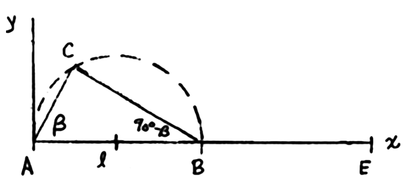
Locate the space centrode (Point *C*) by noting that velocity directions at Points *A* and *B* are known. Draw *AC* perpendicular to $\mathbf{v}_A$ and *BC* perpendicular to $\mathbf{v}_B$.

The coordinates of Point *C* are $x_C = -l \sin\beta$ and $y_C = l\cos\beta$

$$x_C^2 + y_C^2 = l^2 = (15 \text{ in.})^2$$

The *space centrode* is a quarter circle of 15 in. radius centered at *O*. ◀

Redraw the figure, but use axes *x* and *y* that move with the body. Place origin at *A*.



$$x_C = (AC)\cos\beta$$

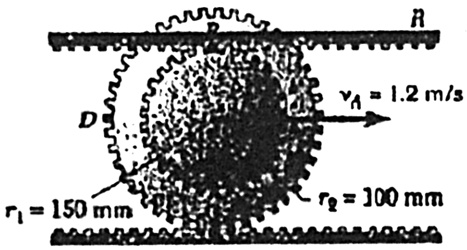
$$= l\cos^2\beta = \frac{l}{2}(1+\cos 2\beta)$$

$$y_C = (AC)\sin\beta$$

$$= l\cos\beta\sin\beta = \frac{l}{2}\sin 2\beta$$

$$\left(x_C - \frac{l}{2}\right)^2 + y_C^2 = \left(\frac{l}{2}\right)^2 = (x_C - 7.5)^2 + y_C^2 = 7.5^2$$

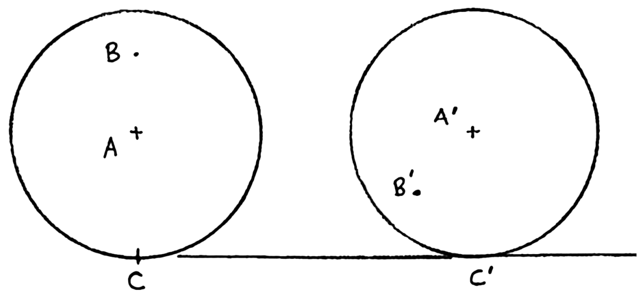
The *body centrode* is a semicircle of 7.5 in. radius centered midway between *A* and *B*. ◀



PROBLEM 15.100

Describe the space centrode and the body centrode of the gear of Sample Problem 15.2 as the gear rolls on the stationary horizontal rack.

SOLUTION



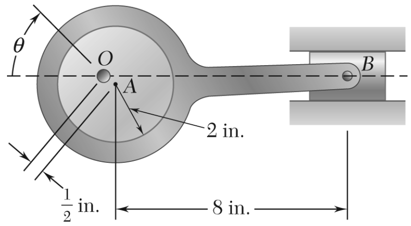
Let Points, *A, B,* and *C* move to *A*′, *B*′, and *C*′ as shown.

Since the instantaneous center always lies on the fixed lower rack, the space centrode is the lower rack.

space centrode: lower rack ◀

Since the point of contact of the gear with the lower rack is always a point on the circumference of the gear, the body centrode is the circumference of the gear.

body centrode: circumference of gear ◀

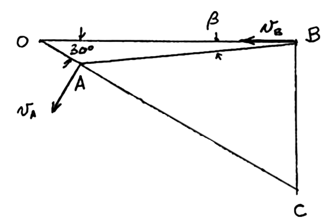


PROBLEM 15.101

Using the method of Section 15.7, solve Problem 15.60.

PROBLEM 15.60 In the eccentric shown, a disk of 2-in.-radius revolves about shaft O that is located 0.5 in. from the center A of the disk. The distance between the center A of the disk and the pin at B is 8 in. Knowing that the angular velocity of the disk is 900 rpm clockwise, determine the velocity of the block when $\theta = 30°$.

SOLUTION



$$(OA) = 0.5 \text{ in.} \quad \omega_{OA} = 900 \text{ rpm} = 30\pi \text{ rad/s} \circlearrowright$$

$$\mathbf{v}_A = (OA)\omega_{OA} = (0.5)(30\pi) = 15\pi \text{ in./s}$$

$$\mathbf{v}_A = v_A \measuredangle 60°, \qquad \mathbf{v}_B = v_A \longleftarrow$$

Locate the instantaneous center (Point C) of bar BD by noting that velocity directions at Point B and A are known. Draw BC perpendicular to $\mathbf{v}_B$ and AC perpendicular to $\mathbf{v}_A$.

$$\sin\beta = \frac{(OA)\sin 30°}{AB} = \frac{0.5\sin 30°}{8}, \quad \beta = 1.79°$$

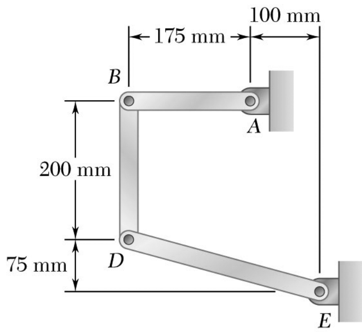
$$OB = (OA)\cos 30° + (AB)\cos\beta = 0.5\cos 30° + 8\cos\beta$$
$$= 8.4291 \text{ in.}$$

$$AC = \frac{OB}{\cos 30°} - OA = \frac{8.4291}{\cos 30°} - 0.5 = 9.2331 \text{ in.}$$

$$BC = (OB)\tan 30° = 4.8665 \text{ in.}$$

$$\omega_{AB} = \frac{v_A}{AC} = \frac{v_B}{BC}$$

$$v_B = \left(\frac{BC}{AC}\right)v_A = \frac{(4.8665)(15\pi)}{9.2331} = 24.84 \text{ in./s} \qquad \mathbf{v}_B = 24.8 \text{ in./s} \longleftarrow \blacktriangleleft$$



PROBLEM 15.102

Using the method of Section 15.7, solve Problem 15.64.

PROBLEM 15.64 In the position shown, bar *AB* has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars *BD* and *DE*.

SOLUTION

Bar *AB*: (Rotation about *A*) $\qquad \boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright$

$$\overline{AB} = 175 \text{ mm} \qquad v_B = \omega_{AB}(\overline{AB}) = (4)(175)$$

$$\mathbf{v}_B = 700 \text{ mm/s} \uparrow$$

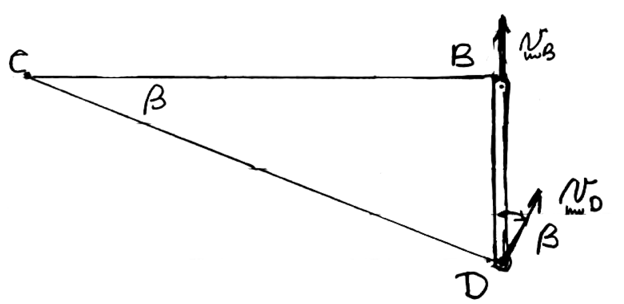
Bar *DE*: (Rotation about *E*) $\qquad \boldsymbol{\omega}_{DE} = \omega_{DE} \circlearrowright$

$$\overline{DE} = \sqrt{(275)^2 + (75)^2} = 285.04 \text{ mm}$$

$$\mathbf{v}_D = 285.04\omega_{DE} \measuredangle \beta$$

$$\tan\beta = \frac{75 \text{ mm}}{275 \text{ mm}} = 0.27273$$

Bar *BD*: $\qquad \mathbf{v}_B = 700 \text{ mm/s} \uparrow, \qquad \mathbf{v}_D = 285.04\omega_{DE} \measuredangle \beta$



Locate the instantaneous center of bar *BD* by drawing line *BC* perpendicular to $\mathbf{v}_B$ and line *DC* perpendicular to $\mathbf{v}_D$.

$$\overline{BD} = 200 \text{ mm}$$

$$\overline{CB} = \frac{\overline{BD}}{\tan\beta} = \frac{(200)(275)}{75} = 733.3 \text{ mm}$$

$$\overline{CD} = \frac{\overline{BD}}{\sin\beta} = \frac{(200)(285.04)}{75} = 760.11 \text{ mm}$$

PROBLEM 15.102 (Continued)

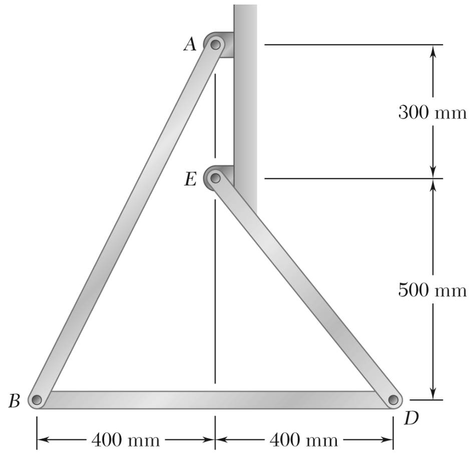
$$\omega_{BD} = \frac{v_B}{CB} = \frac{700 \text{ mm/s}}{733.33 \text{ mm}} = 0.95455 \text{ rad/s}$$

$$v_D = \omega_{BD}(CD) = (0.95455 \text{ rad/s})(760.11 \text{ mm}) = 725.56 \text{ mm/s}$$

$$\omega_{DE} = \frac{v_D}{285.04} = \frac{725.56}{285.04} = 2.5455 \text{ rad/s}$$

Angular velocities: $\boldsymbol{\omega}_{BD} = 0.955 \text{ rad/s} \circlearrowright$ ◀

$\boldsymbol{\omega}_{DE} = 2.55 \text{ rad/s} \circlearrowleft$ ◀

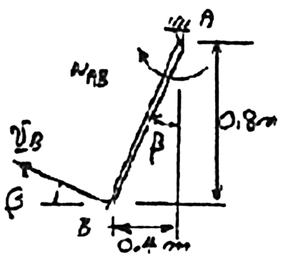


PROBLEM 15.103

Using the method of Section 15.7, solve Problem 15.65.

PROBLEM 15.65 In the position shown, bar *AB* has an angular velocity of 4 rad/s clockwise. Determine the angular velocity of bars *BD* and *DE*.

SOLUTION

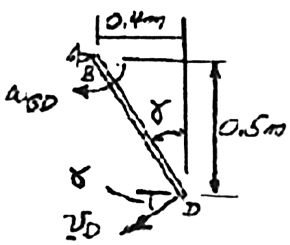


Bar *AB*: $\beta = \tan^{-1}\dfrac{0.4 \text{ m}}{0.8 \text{ m}} = 26.56°$

$$AB = \frac{0.8 \text{ m}}{\cos\beta} = 0.8944 \text{ m}$$

$$v_B = (AB)\omega_{AB} = (0.8944 \text{ m})(4 \text{ rad/s})$$

$$\mathbf{v}_B = 3.578 \text{ m/s} \searrow 26.56°$$

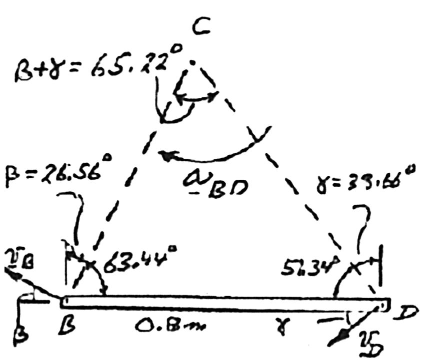


Bar *DE*: $\gamma = \tan^{-1}\dfrac{0.4 \text{ m}}{0.5 \text{ m}} = 38.66°$

$$DE = \frac{0.5 \text{ m}}{\cos\gamma} = 0.6403 \text{ m}$$

$$v_D = (DE)\omega_{DE}$$

$$\mathbf{v}_D = (0.6403 \text{ m})\omega_{DE} \swarrow 38.66° \qquad (1)$$



Bar *BD*: Locate instantaneous center at intersection of lines drawn perpendicular to $\mathbf{v}_B$ and $\mathbf{v}_D$.

<u>Law of sines</u>. $\dfrac{BC}{\sin 51.34°} = \dfrac{CD}{\sin 63.44°} = \dfrac{0.8 \text{ m}}{\sin 65.22°}$

$$BC = 0.688 \text{ m}$$

$$CD = 0.7881 \text{ m}$$

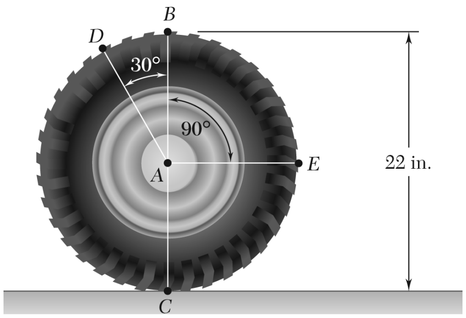
$$v_B = (BC)\omega_{BD}$$

$$3.578 \text{ m/s} = (0.688 \text{ m})\omega_{BD}; \qquad \boldsymbol{\omega}_{BD} = 5.2 \text{ rad/s} \circlearrowright \blacktriangleleft$$

$$v_D = (CD)\omega_{BD} = (0.7881 \text{ m})(5.2 \text{ m/s})$$

$$= 4.098 \text{ m/s}$$

Eq. (1): $4.098 \text{ m/s} = (0.6403 \text{ m})\omega_{DE}; \qquad \boldsymbol{\omega}_{DE} = 6.4 \text{ rad/s} \circlearrowright \blacktriangleleft$



PROBLEM 15.104

Using the method of section 15.7, solve Problem 15.38.

PROBLEM 15.38 An automobile travels to the right at a constant speed of 48 mi/h. If the diameter of a wheel is 22 in., determine the velocities of Points *B*, *C*, *D*, and *E* on the rim of the wheel.

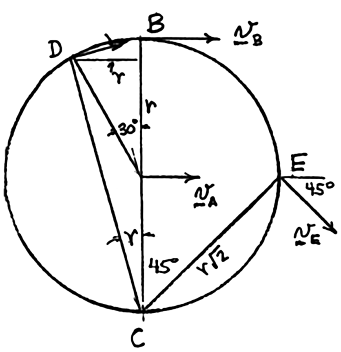
SOLUTION

$$v_A = 48 \text{ mi/h} = 70.4 \text{ ft/s}$$

$$\mathbf{v}_C = 0 \blacktriangleleft$$

$$d = 22 \text{ in.,} \quad r = \frac{1}{2}d = 11 \text{ in.} = 0.91667 \text{ ft}$$

Point *C* is the instantaneous center.



$$\omega = \frac{v_A}{r} = \frac{70.4}{9.1667} = 76.8 \text{ rad/s}$$

$$CB = 2r = 1.8333 \text{ ft}$$

$$v_B = (CB)\omega = (1.8333)(76.8) = 140.8 \text{ ft/s}$$

$$\mathbf{v}_B = 140.8 \text{ ft/s} \longrightarrow \blacktriangleleft$$

$$\gamma = \frac{1}{2}(30°) = 15°$$

$$CD = 2r\cos 15° = (2)(0.91667)\cos 15° = 1.7709 \text{ ft}$$

$$v_D = (CD)\omega = (1.7709)(76.8) = 136.0 \text{ ft/s}$$

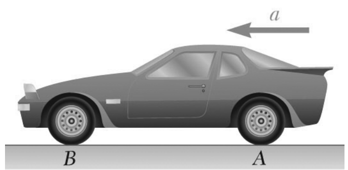
$$\mathbf{v}_D = 136.0 \text{ ft/s} \measuredangle 15.0° \blacktriangleleft$$

$$CE = r\sqrt{2} = 0.91667\sqrt{2} = 1.2964 \text{ ft}$$

$$v_E = (CE)\omega = (1.2964)(76.8) = 99.56 \text{ ft/s}$$

$$\mathbf{v}_E = 99.6 \text{ ft/s} \searrow 45.0° \blacktriangleleft$$

PROBLEM 15.CQ7



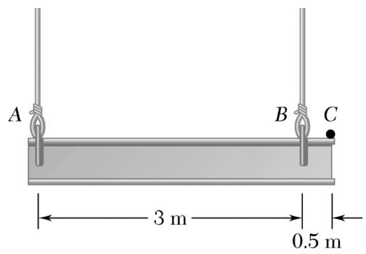
A rear wheel drive car starts from rest and accelerates to the left so that the tires do not slip on the road. What is the direction of the acceleration of the point on the tire in contact with the road, that is, Point *A*?

(*a*) ← (*b*) ↖ (*c*) ↑ (*d*) ↓ (*e*) ↙

SOLUTION

The tangential acceleration will be zero since the tires do not slip, but there will be an acceleration component perpendicular to the ground.

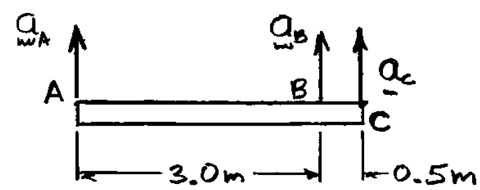
Answer: (*c*) ◀



PROBLEM 15.105

A 3.5-m steel beam is lowered by means of two cables unwinding at the same speed from overhead cranes. As the beam approaches the ground, the crane operators apply brakes to slow down the unwinding motion. At the instant considered, the deceleration of the cable attached at A is 4 m/s^2, while that of the cable at B is 1.5 m/s^2. Determine (a) the angular acceleration of the beam, (b) the acceleration of Point C.

SOLUTION

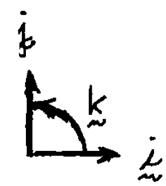


$$\mathbf{a}_A = 4 \text{ m/s} \uparrow$$

$$\mathbf{a}_B = 1.5 \text{ m/s} \uparrow$$

Assume $\omega = 0$.

(a) Angular acceleration. $\boldsymbol{\alpha} = \alpha \mathbf{k}$



$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$1.5\mathbf{j} = 4\mathbf{j} + \alpha\mathbf{k} \times (3\mathbf{i}) = 4\mathbf{j} + 3\alpha\mathbf{j}$$

$$\alpha = \frac{1.5 - 4}{3} = -0.83333$$

$$\boldsymbol{\alpha} = -0.833\mathbf{k}$$

$$\boldsymbol{\alpha} = 0.833 \text{ rad/s}^2 \circlearrowright$$

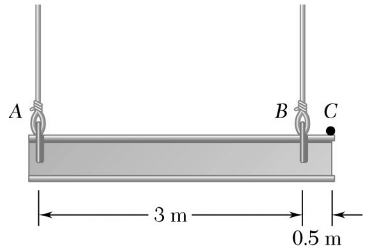
(b) Acceleration of Point C. Because the cables are unwinding at the same speed, $\omega = 0$

$$\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A} = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{C/A}$$

$$= 4\mathbf{j} + (-0.83333\mathbf{k} \times 3.5\mathbf{i})$$

$$= 4\mathbf{j} - 2.9167\mathbf{j} = 1.0833\mathbf{j}$$

$$\mathbf{a}_C = 1.083 \text{ m/s}^2 \uparrow \blacktriangleleft$$

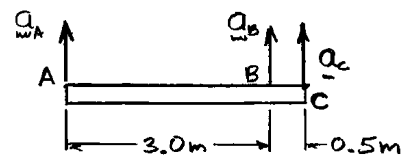


PROBLEM 15.106

The acceleration of Point C is $0.3\ \text{m/s}^2$ downward and the angular acceleration of the beam is $0.8\ \text{rad/s}^2$ clockwise. Knowing that the angular velocity of the beam is zero at the instant considered, determine the acceleration of each cable.

SOLUTION

$$\omega = 0$$
$$\boldsymbol{\alpha} = (-0.8\ \text{rad/s})\mathbf{k}$$
$$\mathbf{a}_C = 0.3\ \text{m/s}\downarrow$$

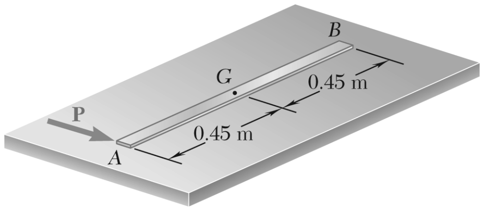


Acceleration of cable A.

$$\mathbf{a}_A = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{A/C}$$
$$= -0.3\mathbf{j} + [-0.8\mathbf{k} \times (-3.5\mathbf{i})]$$
$$= -0.3\mathbf{j} + 2.8\mathbf{j} = (2.5\ \text{m/s}^2)\mathbf{j}\uparrow \qquad \mathbf{a}_A = 2.50\ \text{m/s}^2\uparrow \blacktriangleleft$$

Acceleration of cable B.

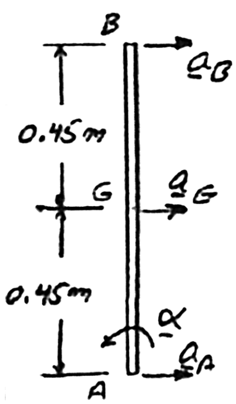
$$\mathbf{a}_B = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{B/C}$$
$$= -0.3\mathbf{j} + [-0.8\mathbf{k} \times (-0.5\mathbf{i})]$$
$$= -0.3\mathbf{j} + 0.4\mathbf{j} = 0.1\mathbf{j} = (0.1\ \text{m/s}^2)\mathbf{j} \qquad \mathbf{a}_B = 0.100\ \text{m/s}^2\uparrow$$



PROBLEM 15.107

A 900-mm rod rests on a horizontal table. A force **P** applied as shown produces the following accelerations: $\mathbf{a}_A = 3.6 \text{ m/s}^2$ to the right, $\alpha = 6 \text{ rad/s}^2$ counterclockwise as viewed from above. Determine the acceleration (*a*) of Point *G*, (*b*) of Point *B*.

SOLUTION



(*a*)

$$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A} = [a_A \longrightarrow] + [(AG)\alpha \longleftarrow]$$

$$\mathbf{a}_G = [3.6 \text{ m/s}^2 \longrightarrow] + [(0.45 \text{ m})(6 \text{ rad/s}^2) \longleftarrow]$$

$$\mathbf{a}_G = [3.6 \text{ m/s}^2 \longrightarrow] + [2.7 \text{ m/s}^2 \longleftarrow]$$

$$\mathbf{a}_G = 0.9 \text{ m/s}^2 \longrightarrow \blacktriangleleft$$

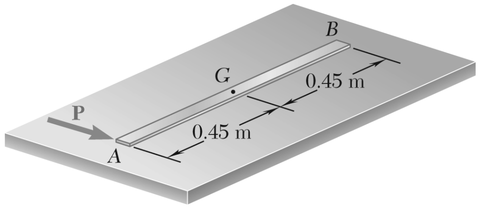
(*b*)

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = [a_A \longrightarrow] + [(AB)\alpha \longleftarrow]$$

$$\mathbf{a}_B = [3.6 \text{ m/s}^2 \longrightarrow] + [(0.9 \text{ m})(6 \text{ rad/s}^2) \longleftarrow]$$

$$\mathbf{a}_B = [3.6 \text{ m/s}^2 \longrightarrow] + [5.4 \text{ m/s}^2 \longleftarrow]$$

$$\mathbf{a}_B = 1.8 \text{ m/s}^2 \longleftarrow \blacktriangleleft$$

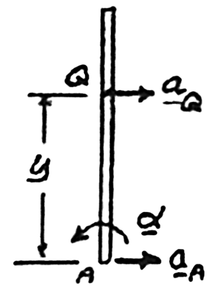


PROBLEM 15.108

In Problem 15.107, determine the point of the rod that (*a*) has no acceleration, (*b*) has an acceleration of $2.4\ \text{m/s}^2$ to the right.

SOLUTION

(*a*) For $a_Q = 0$:



$$\mathbf{a}_Q = \mathbf{a}_A + \mathbf{a}_{Q/A} = \mathbf{a}_A \longrightarrow + (AQ)\alpha \longleftarrow$$

$$0 = 3.6\ \text{m/s}^2 \longrightarrow + (y)(6\ \text{rad/s}^2) \longleftarrow$$

$$y = \frac{3.6\ \text{m/s}^2}{6\ \text{rad/s}} = 0.6\ \text{m}$$

$\mathbf{a} = 0$ at $0.6\ \text{m}$ from A ◀

(*b*) For $\mathbf{a}_Q = 2.4\ \text{m/s}^2 \longrightarrow$:

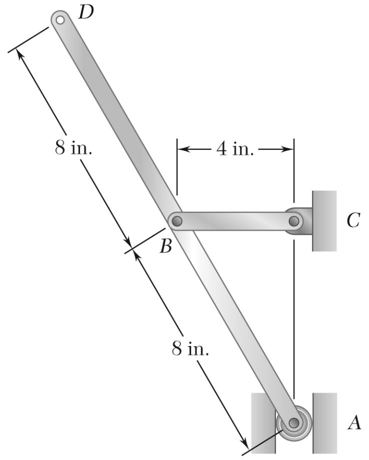
$$\mathbf{a}_Q = \mathbf{a}_A + \mathbf{a}_{Q/A} = [a_A \longrightarrow] + [(AQ)\alpha \longleftarrow]$$

$$2.4\ \text{m/s}^2 \longrightarrow = [3.6\ \text{m/s}^2 \longrightarrow] + [(y)(6\ \text{rad/s}^2) \longleftarrow]$$

$$1.2\ \text{m/s}^2 \longleftarrow = (y)(6\ \text{rad/s}^2) \longleftarrow$$

$$y = 0.2\ \text{m}$$

$\mathbf{a} = 2.4\ \text{m/s}^2 \longrightarrow$ at $0.2\ \text{m}$ from A ◀



PROBLEM 15.109

Knowing that at the instant shown crank BC has a constant angular velocity of 45 rpm clockwise, determine the acceleration (*a*) of Point A, (*b*) of Point D.

SOLUTION

<u>Geometry.</u> Let β be angle BAC.

$$\sin\beta = \frac{4 \text{ in.}}{8 \text{ in.}} \qquad \beta = 30°$$

<u>Velocity analysis</u>. $\quad \boldsymbol{\omega}_{BC} = 45 \text{ rpm} \circlearrowright = 4.7124 \text{ rad/s} \circlearrowright \qquad \mathbf{v}_A = v_A \uparrow$

$$v_B = (BC)\omega_{BC} = (4)(4.7124) = 18.8496 \text{ in./s} \qquad \mathbf{v}_B = 18.8496 \text{ in./s} \uparrow$$

$\mathbf{v}_A$ and $\mathbf{v}_B$ are parallel; hence, the instantaneous center of rotation of rod AD lies at infinity.

$$\boldsymbol{\omega}_{AD} = 0 \qquad \mathbf{v}_A = \mathbf{v}_B = 18.8496 \text{ in./s} \uparrow$$

<u>Acceleration analysis</u>. $\quad \alpha_{BC} = 0$

Crank BC: $\quad (a_B)_t = (BC)\alpha = 0$

$$(a_B)_n = (BC)\omega_{BC}^2 = (4)(4.7124)^2$$

$$\mathbf{a}_B = 88.827 \text{ in./s}^2 \longrightarrow$$

Rod ABD: $\quad \boldsymbol{\alpha}_{AD} = \alpha_{AD} \circlearrowleft \qquad \mathbf{a}_A = a_A \uparrow$

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t + (\mathbf{a}_{A/B})_A$$

$$[a_A \uparrow] = [88.827 \longrightarrow] + [8\alpha_{AD} \measuredangle 30°] + [8\omega_{AD}^2 \searrow 60°]$$

Resolve into components.

$\xrightarrow{+}$: $\quad 0 = 88.827 + 8\alpha_{AD}\cos 30° + 0 \qquad \alpha_{AD} = -12.821 \text{ rad/s}^2$

(*a*) $\quad +\uparrow$: $\quad a_A = 8\alpha_{AD}\sin 30° = (8)(-12.821)\sin 30° = -51.284 \text{ in./s}^2$

$$\mathbf{a}_A = 51.3 \text{ in./s}^2 \downarrow \blacktriangleleft$$

PROBLEM 15.109 (Continued)

(*b*)

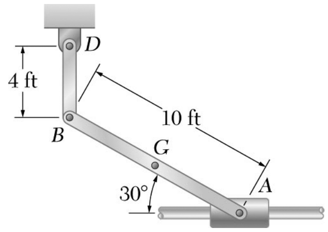
$$\mathbf{a}_D = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_t$$

$$= [88.827 \longrightarrow] + [8\alpha_{BD} \swarrow 30°] + [8\omega_{BD}^2 [\searrow 60°]$$

$$= [88.827 \longrightarrow] + [(8)(-12.821) \swarrow 30°] + 0$$

$$= [88.827 \longrightarrow] + [102.568 \nearrow 30°] = [177.653 \longrightarrow \; + 51.284 \uparrow]$$

$$\mathbf{a}_D = 184.9 \text{ in./s}^2 \nearrow 16.1° \blacktriangleleft$$



PROBLEM 15.110

End A of rod AB moves to the right with a constant velocity of 6 ft/s. For the position shown, determine (a) the angular acceleration of rod AB, (b) the acceleration of the midpoint G of rod AB.

SOLUTION

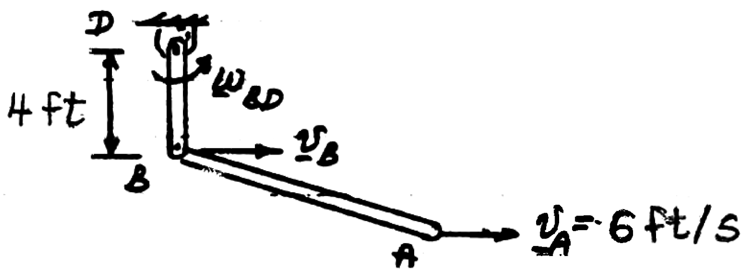
Use units of ft and seconds.

Geometry and unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

$$\mathbf{r}_{B/A} = -(10\cos 30°)\mathbf{i} + (10\sin 30°)\mathbf{j} \qquad \mathbf{r}_{B/D} = -4\mathbf{j}$$

Velocity analysis.

Rod AB: $\mathbf{v}_A = 6 \text{ ft/s} \longrightarrow, \quad \mathbf{v}_B = v_B \longrightarrow$



Since $\mathbf{v}_A$ and $\mathbf{v}_B$ are parallel, the instantaneous center lies at infinity, so $\omega_{AB} = 0$ and $\mathbf{v}_B = \mathbf{v}_A$.

Acceleration analysis.

Rod AB: $\mathbf{a}_A = 0$ since $\mathbf{v}_A$ is constant. $\qquad \boldsymbol{\alpha}_{AB} = \alpha_{AB} \circlearrowleft = \alpha_{AB}\mathbf{k}$

$$\begin{aligned}\mathbf{a}_B &= \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_A + \mathbf{a}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\ &= 0 + \alpha_{AB}\mathbf{k} \times (-10\cos 30°\mathbf{i} + 5\mathbf{j}) - 0 \\ \mathbf{a}_B &= -(10\cos 30°)\alpha_{AB}\mathbf{j} - 5\alpha_{AB}\mathbf{j} \end{aligned} \qquad (1)$$

Rod BD: $\mathbf{a}_D = 0, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}$

$$\begin{aligned}\mathbf{a}_B &= \mathbf{a}_D + \mathbf{a}_{B/D} = 0 + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{B/D} - \omega_{BD}^2 \mathbf{r}_{B/D} \\ &= \alpha_{BD}\mathbf{k} \times (-4\mathbf{j}) - (1.5)^2(-4\mathbf{j}) \\ &= 4\alpha_{BD}\mathbf{i} + 9\mathbf{j} \end{aligned} \qquad (2)$$

Equating the coefficients of $\mathbf{j}$ in the expressions (1) and (2) for $\mathbf{a}_B$.

$$-(10\cos 30°)\alpha_{AB} = 9 \qquad \alpha_{AB} = -1.0392$$

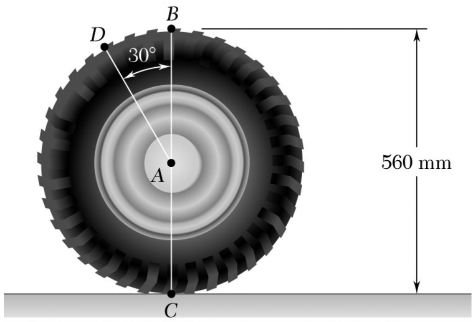
PROBLEM 15.110 (Continued)

(*a*) *Angular acceleration of rod AB*: $\boldsymbol{\alpha}_{AB} = 1.039 \text{ rad/s}^2$ ◀

(*b*) *Acceleration of midpoint G of rod AB*.

$$\mathbf{a}_G = \mathbf{a}_A + \mathbf{a}_{G/A} = \mathbf{a}_A + \boldsymbol{\alpha}_{AB}\mathbf{k} \times \mathbf{r}_{G/A} - \omega_{AB}^2 \mathbf{r}_{G/A}$$
$$= 0 - 1.0392\mathbf{k} \times (-5\cos 30°\mathbf{i} + 5\sin 30°\mathbf{j})$$

$\mathbf{a}_G = (2.60 \text{ ft/s}^2)\mathbf{i} + (4.50 \text{ ft/s}^2)\mathbf{j} = 5.20 \text{ ft/s}^2 \measuredangle 60°$ ◀



PROBLEM 15.111

An automobile travels to the left at a constant speed of 72 km/h. Knowing that the diameter of the wheel is 560 mm, determine the acceleration (*a*) of Point *B*, (*b*) of Point *C*, (*c*) of Point *D*.

SOLUTION

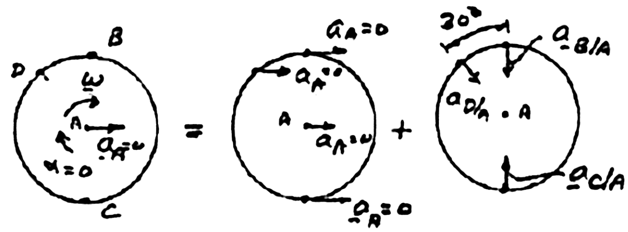
$$\mathbf{v}_A = 72 \text{ km/h} \cdot \frac{\text{h}}{3600 \text{ s}} \cdot \frac{1000 \text{ m}}{\text{km}} = 20 \text{ m/s} \longrightarrow$$

Rolling with no sliding, instantaneous center is at *C*.

$$v_A = (AC)\omega; \quad 20 \text{ m/s} = (0.28 \text{ m})\omega$$

$$\boldsymbol{\omega} = 71.429 \text{ rad/s} \circlearrowright$$

Acceleration.



Plane motion = Trans. with *A* + Rotation about *A*

$$a_{B/A} = a_{C/A} = a_{D/A} = r\omega^2 = (0.280 \text{ m})(71.429 \text{ rad/s})^2 = 1428.6 \text{ m/s}^2$$

(*a*) $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A} = 0 + 1428.6 \text{ m/s}^2 \downarrow$

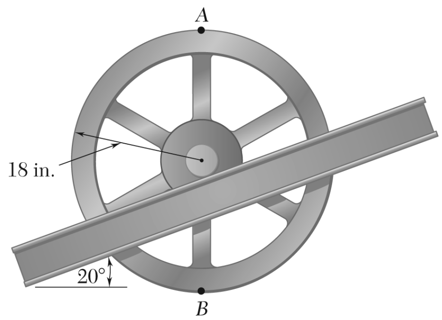
$\mathbf{a}_B = 1430 \text{ m/s}^2 \downarrow$ ◀

(*b*) $\mathbf{a}_C = \mathbf{a}_A + \mathbf{a}_{C/A} = 0 + 1428.6 \text{ m/s}^2 \uparrow$

$\mathbf{a}_C = 1430 \text{ m/s}^2 \uparrow$ ◀

(*c*) $\mathbf{a}_D = \mathbf{a}_A + \mathbf{a}_{D/A} = 0 + 1428.6 \text{ m/s}^2 \searrow 60°$

$\mathbf{a}_D = 1430 \text{ m/s}^2 \searrow 60°$ ◀



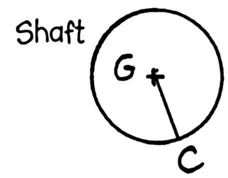
PROBLEM 15.112

The 18-in.-radius flywheel is rigidly attached to a 1.5-in.-radius shaft that can roll along parallel rails. Knowing that at the instant shown the center of the shaft has a velocity of 1.2 in./s and an acceleration of 0.5 in./s^2, both directed down to the left, determine the acceleration (*a*) of Point *A*, (*b*) of Point *B*.

SOLUTION

Velocity analysis.

Let Point *G* be the center of the shaft and Point *C* be the point of contact with the rails. Point *C* is the instantaneous center of the wheel and shaft since that point does not slip on the rails.



$$\mathbf{v}_G = r\omega, \quad \omega = \frac{v_G}{r} = \frac{1.2}{1.5} = 0.8 \text{ rad/s} \circlearrowleft$$

Acceleration analysis.

Since the shaft does not slip on the rails,

$$\mathbf{a}_C = a_C \nwarrow 20^\circ$$

Also,

$$\mathbf{a}_G = [0.5 \text{ in./s}^2 \swarrow 20^\circ]$$

$$\mathbf{a}_C = \mathbf{a}_G + (\mathbf{a}_{C/G})_t + (\mathbf{a}_{C/G})_n$$

$$[a_C \nwarrow 20^\circ] = [0.5 \text{ in./s}^2 \swarrow 20^\circ] + [1.5\alpha \swarrow 20^\circ] + [1.5\omega^2 \nwarrow 20^\circ]$$

Components $\swarrow$ 20°: $\quad 0.5 = -1.5\alpha \qquad \alpha = 0.33333 \text{ rad/s}^2 \circlearrowleft$

(*a*) *Acceleration of Point A.*

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$= [0.5 \swarrow 20^\circ] + [18\alpha \longleftarrow] + [18\omega^2 \downarrow]$$

$$= [0.4698 \longleftarrow] + [0.1710 \downarrow] + [6 \longleftarrow] + [11.52 \downarrow]$$

$$= [6.4698 \longleftarrow] + [11.670 \downarrow] \qquad \mathbf{a}_A = 13.35 \text{ in./s}^2 \swarrow 61.0^\circ \blacktriangleleft$$

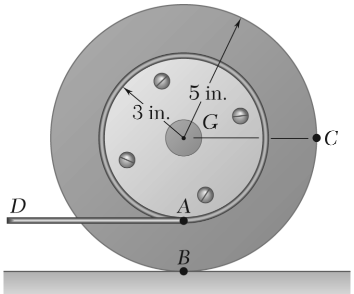
(*b*) *Acceleration of Point B.*

$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

$$= [0.5 \swarrow 20^\circ] + [18\alpha \longrightarrow] + [18\omega^2 \uparrow]$$

$$= [0.4698 \longleftarrow] + [0.1710 \downarrow] + [6 \longrightarrow] + [11.52 \uparrow]$$

$$= [5.5302 \longrightarrow] + [11.349 \uparrow] \qquad \mathbf{a}_B = 12.62 \text{ in./s}^2 \nearrow 64.0^\circ \blacktriangleleft$$



PROBLEM 15.113

A 3-in.-radius drum is rigidly attached to a 5-in.-radius drum as shown. One of the drums rolls without sliding on the surface shown, and a cord is wound around the other drum. Knowing that at the instant shown end D of the cord has a velocity of 8 in./s and an acceleration of 30 in./s^2, both directed to the left, determine the accelerations of Points A, B, and C of the drums.

SOLUTION

Velocity analysis. $\quad v_D = v_A = 8\text{ in./s}$

Instantaneous center is at Point B. $\quad v_A = (AB)\omega, \quad 8 = (5-3)\omega$

$$\omega = 4\text{ rad/s} \circlearrowright$$

Acceleration analysis. $\quad \mathbf{a}_B = [a_B \uparrow]$ for no slipping.

$$\alpha = \alpha \circlearrowright$$

$$\mathbf{a}_A = [30\text{ in./s}^2 \longleftarrow] + [(a_A)_n \uparrow]$$

$$\mathbf{a}_G = [a_G \longleftarrow]$$

$$\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

$$[a_B \uparrow] = [30 \longleftarrow] + [(a_A)_n \uparrow] + [(5-3)\alpha \longrightarrow] + [5-3)\omega^2 \uparrow]$$

Components $\xrightarrow{+}$: $\quad 0 = -30 + 2\alpha \qquad \alpha = 15\text{ rad/s}^2 \circlearrowright$

$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

$$[a_B \uparrow] = [a_G \longleftarrow] + [5\alpha \longrightarrow] + [5\omega^2 \uparrow]$$

Components $\xrightarrow{+}$: $\quad 0 = -a_G + 5\alpha \qquad a_G = 5\alpha = 75\text{ in./s}^2$

$+\uparrow$: $\quad a_B = (5)(4)^2 = 80\text{ in./s}^2 \qquad \mathbf{a}_B = 80.0\text{ in./s}^2 \uparrow$ ◀

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$= [75 \longleftarrow] + [3\alpha \longrightarrow] + [3\omega^2 \uparrow]$$

$$= [75 \longleftarrow] + [45 \longrightarrow] + [48 \uparrow]$$

$$= [30\text{ in./s}^2 \longleftarrow] + [48\text{ in./s}^2 \uparrow]$$

$$\mathbf{a}_A = 56.6\text{ in./s}^2 \measuredangle 58.0° \text{ ◀}$$

PROBLEM 15.113 (Continued)

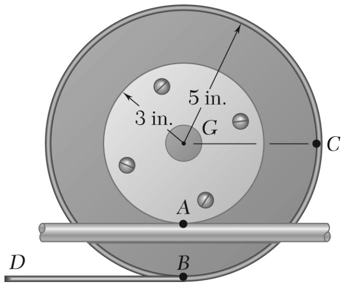
$$\mathbf{a}_C = \mathbf{a}_G + (\mathbf{a}_{C/G})_t + (\mathbf{a}_{C/G})_n$$

$$= [75 \longleftarrow] + [5\alpha \uparrow] + [5\omega^2 \longleftarrow]$$

$$= [75 \longleftarrow] + [75 \uparrow] + [80 \longleftarrow]$$

$$= [155 \text{ in./s}^2 \longleftarrow] + [75 \text{ in./s}^2 \uparrow]$$

$\mathbf{a}_C = 172.2 \text{ in./s}^2 \measuredangle 25.8°$ ◀



PROBLEM 15.114

A 3-in.-radius drum is rigidly attached to a 5-in.-radius drum as shown. One of the drums rolls without sliding on the surface shown, and a cord is wound around the other drum. Knowing that at the instant shown end D of the cord has a velocity of 8 in./s and an acceleration of 30 in./s^2, both directed to the left, determine the accelerations of Points A, B, and C of the drums.

SOLUTION

Velocity analysis. $\mathbf{v}_D = \mathbf{v}_B = 8 \text{ in./s}$

Instantaneous center is at Point A. $v_B = (AB)\omega, \quad 8 = (5-3)\omega$

$$\omega = 4 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\mathbf{a}_A = [a_A \uparrow]$ for no slipping. $\alpha = \alpha \circlearrowright$

$$\mathbf{a}_B = [30 \text{ in./s}^2 \longleftarrow] + [(a_B)_n \uparrow]$$

$$\mathbf{a}_G = [a_G \longrightarrow]$$

$$\mathbf{a}_A = \mathbf{a}_B + (\mathbf{a}_{A/B})_t + (\mathbf{a}_{A/B})_n$$

$$[a_A \uparrow] = [30 \longleftarrow] + [(a_B)_n \uparrow] + [(5-3)\alpha \longrightarrow] + [(5-3)]\omega^2 \downarrow]$$

Components $\xrightarrow{+}$: $0 = -30 + 2\alpha \qquad \alpha = 15 \text{ rad/s}^2 \circlearrowright$

$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$

$$a_A \uparrow = [a_G \longrightarrow] + [3\alpha \longleftarrow] + [3\omega^2 \uparrow]$$

Components $\xrightarrow{+}$: $0 = a_G - 3\alpha \qquad a_G = 3\alpha = 45 \text{ in./s}^2$

$+\uparrow$: $a_A = 3\omega^2 = (3)(4)^2 = 48 \text{ in./s}^2$ $\qquad \mathbf{a}_A = 48.0 \text{ in./s}^2 \uparrow$ ◀

$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$

$$= [45 \longrightarrow] + [5\alpha \longleftarrow] + [5\omega^2 \uparrow]$$

$$= [45 \longrightarrow] + [75 \longleftarrow] + [80 \uparrow]$$

$$= [30 \text{ in./s}^2 \longleftarrow] + [80 \text{ in./s}^2 \uparrow]$$

$\mathbf{a}_B = 85.4 \text{ in./s}^2 \measuredangle 69.4°$ ◀

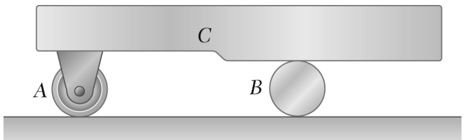
PROBLEM 15.114 (Continued)

$$\mathbf{a}_C = \mathbf{a}_G + (\mathbf{a}_{C/G})_t + (\mathbf{a}_{C/G})_n$$

$$= [45 \longrightarrow] + [5\alpha \downarrow] + [5\omega^2 \longleftarrow]$$

$$= [45 \longrightarrow] + [75 \downarrow] + [80 \longleftarrow]$$

$$= [35 \text{ in./s}^2 \longleftarrow] + [75 \text{ in./s}^2 \downarrow] \qquad \mathbf{a}_C = 82.8 \text{ in./s}^2 \swarrow 65.0° \blacktriangleleft$$



PROBLEM 15.115

A carriage C is supported by a caster A and a cylinder B, each of 50-mm diameter. Knowing that at the instant shown the carriage has an acceleration of 2.4 m/s^2 and a velocity of 1.5 m/s, both directed to the left, determine (*a*) the angular accelerations of the caster and of the cylinder, (*b*) the accelerations of the centers of the caster and of the cylinder.

SOLUTION

Rolling occurs at all surfaces of contact. Instantaneous centers are at points of contact with floor.

Caster: $r = 0.025 \text{ m}$

$$\mathbf{a}_A = \mathbf{a}_C = 2.4 \text{ m/s}^2 \longleftarrow$$

$$(a_D)_x = 0 \text{ (rolling with no sliding)}$$

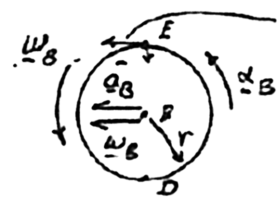
$$\mathbf{a}_A = \mathbf{a}_D + \mathbf{a}_{A/D}$$

$$[a_A \longleftarrow] = [(a_D)_x \longleftarrow] + [(a_D)_y \uparrow] + [r d_A \longleftarrow] + [r\omega_A^2 \downarrow]$$

$$\overset{+}{\longleftarrow} \quad a_A = 0 + r\alpha_A$$

$$2.4 \text{ m/s}^2 \longleftarrow = (0.025 \text{ m})\alpha_A \qquad \boldsymbol{\alpha}_A = 96 \text{ rad/s}^2 \circlearrowright$$

Cylinder: $r = 0.025 \text{ m}$



$$(\mathbf{a}_E)_x = \mathbf{a}_C = 2.4 \text{ m/s}^2 \longleftarrow$$

$$(a_E)_x = 0$$

$$\mathbf{a}_E = \mathbf{a}_D + \mathbf{a}_{E/D}$$

$$[(a_E)_x \longleftarrow] + [(a_E)_y \downarrow] = [(a_D)_x \longleftarrow] + [(a_D)_y \downarrow] + [2r\alpha_B \longleftarrow] + 2r\omega_B^2 \downarrow]$$

$$\overset{+}{\longleftarrow}: \quad (a_E)_x = (a_D)_y + 2r\alpha_B$$

$$[2.4 \text{ m/s}^2 \longleftarrow] = 0 + 2(0.025 \text{ m})\alpha_B$$

$$\boldsymbol{\alpha}_B = 48 \text{ rad/s}^2 \circlearrowright$$

$$[a_B \longleftarrow] = [(a_D)_x \longleftarrow] + [(a_D)_y \uparrow] + [r\alpha \longleftarrow] + [r\omega^2 \downarrow]$$

$$\overset{+}{\longleftarrow}: \quad a_B = 0 + r\alpha_B$$

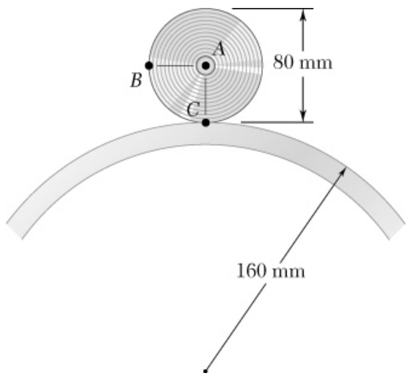
$$a_B = (0.025 \text{ m})(48 \text{ rad/s}^2); \qquad \mathbf{a}_B = 1.2 \text{ m/s}^2 \longleftarrow$$

Answers:

(*a*) $\boldsymbol{\alpha}_A = 96.0 \text{ rad/s}^2 \circlearrowright, \quad \mathbf{a}_A = 2.40 \text{ m/s}^2 \longleftarrow$ ◀

(*b*) $\boldsymbol{\alpha}_B = 48.0 \text{ rad/s}^2 \circlearrowright, \quad \mathbf{a}_B = 1.200 \text{ m/s}^2 \longleftarrow$ ◀

PROBLEM 15.116



A wheel rolls without slipping on a fixed cylinder. Knowing that at the instant shown the angular velocity of the wheel is 10 rad/s clockwise and its angular acceleration is 30 rad/s^2 counterclockwise, determine the acceleration of (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*.

SOLUTION

Velocity analysis. $r = 0.04$ m $\quad \boldsymbol{\omega} = 10$ rad/s $\circlearrowright$

Point *C* is the instantaneous center of the wheel.

$$\mathbf{v}_A = [(r\omega) \longrightarrow] = [(0.04)(10) \longrightarrow] = 0.4 \text{ m/s} \longrightarrow]$$

Acceleration analysis. $\alpha = 30$ rad/s^2 $\circlearrowleft$

Point *A* moves on a circle of radius $\rho = R + r = 0.16 + 0.04 = 0.2$ m.

Since the wheel does not slip, $\mathbf{a}_C = a_C \uparrow$

$$\mathbf{a}_C = \mathbf{a}_A + (\mathbf{a}_{C/A})_t + (\mathbf{a}_{C/A})_n$$

$$[a_C \uparrow] = [(a_A)_t \longleftarrow] + \left[\frac{v_A^2}{\rho} \downarrow\right] + [r\alpha \longrightarrow] + [r\omega^2 \uparrow]$$

$$= [(a_A)_t \longleftarrow] + \left[\frac{(0.4)^2}{0.2} \downarrow\right] + [(0.04)(30) \longrightarrow] + [(0.04)(10)^2 \uparrow]$$

$$= [(a_A)_t \longleftarrow] + [0.8 \downarrow] + [1.2 \longrightarrow] + [4 \uparrow]$$

Components. $\xrightarrow{+}$: $-(a_A)_t + 1.2 = 0 \qquad (a_A)_t = 1.2 \text{ m/s}^2$

$+\uparrow$: $a_C = -0.8 + 4.0 \qquad a_C = 3.2 \text{ m/s}^2$

(*a*) *Acceleration of Point A.*

$$\mathbf{a}_A = [1.2 \text{ m/s}^2 \longleftarrow] + [0.8 \text{ m/s}^2 \downarrow]$$

$\mathbf{a}_A = 1.442 \text{ m/s}^2 \swarrow 33.7°$ ◀

PROBLEM 15.116 (Continued)

(*b*) *Acceleration of Point B.* $\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$

$$\mathbf{a}_B = [1.2 \longleftarrow] + [0.8 \downarrow] + [r\alpha \downarrow] + [r\omega^2 \longrightarrow]$$

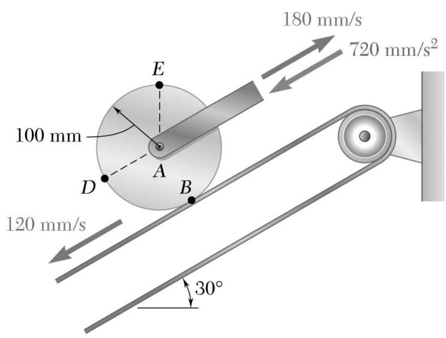
$$= [1.2 \longleftarrow] + [0.8 \downarrow] + [(0.04)(30) \downarrow] + [(0.04)(10)^2 \longrightarrow]$$

$$= [2.8 \text{ m/s}^2 \longrightarrow] + [2 \text{ m/s}^2 \downarrow]$$

$\mathbf{a}_B = 3.44 \text{ m/s}^2 \searrow 35.5°$ ◀

(*c*) *Acceleration of Point C.* $\mathbf{a}_C = a_C \uparrow$

$\mathbf{a}_C = 3.20 \text{ m/s}^2 \uparrow$ ◀



PROBLEM 15.117

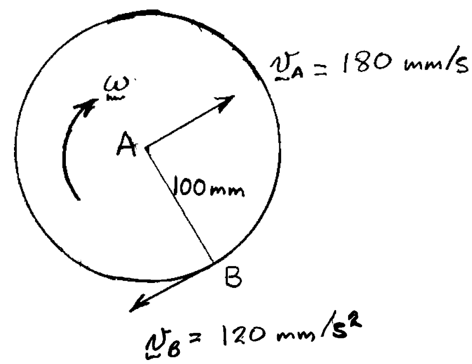
The 100 mm radius drum rolls without slipping on a portion of a belt which moves downward to the left with a constant velocity of 120 mm/s. Knowing that at a given instant the velocity and acceleration of the center A of the drum are as shown, determine the acceleration of Point D.

SOLUTION

Velocity analysis.

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$[180 \text{ mm/s} \nearrow] = [120 \text{ mm/s} \swarrow] + [(100 \text{ mm})\omega \nearrow]$$



Components $\nearrow$:

$$180 = -120 = 100\omega$$

$$\omega = 3 \text{ rad/s} \circlearrowright$$

Acceleration analysis.

Point A moves on a path parallel to the belt. The path is assumed to be straight.

$$\mathbf{a}_A = 720 \text{ mm/s}^2 \swarrow 30°$$

Since the drum rolls without slipping on the belt, the component of acceleration of Point B on the drum parallel to the belt is the same as the belt acceleration. Since the belt moves at constant velocity, this component of acceleration is zero. Thus

$$\mathbf{a}_B = a_B \nwarrow 60°$$

Let the angular acceleration of the drum be $\alpha \circlearrowleft$.

$$\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

$$[a_B \nwarrow] = [720 \swarrow] + [r\alpha \nearrow] + [r\omega^2 \nwarrow]$$

Components $\swarrow$:

$$0 = 720 - 100\alpha$$

$$\alpha = 7.2 \text{ rad/s} \circlearrowleft$$

Acceleration of Point D.

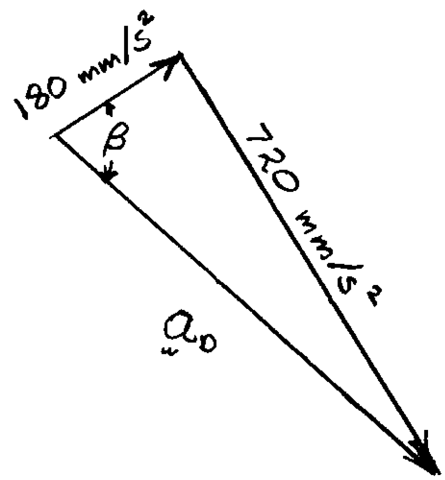
$$\mathbf{a}_D = \mathbf{a}_A + (\mathbf{a}_{D/A})_t + (\mathbf{a}_{D/A})_n$$

$$= [a_A \swarrow 30°] + [r\alpha \searrow 60°] + [r\omega^2 \nearrow 30°]$$

$$= [720 \swarrow 30°] + [(100)(7.2) \searrow 60°] + [(100)(3)^2 \nearrow 30°]$$

Components: $\nearrow 30°$: $-720 + 900 = 180 \text{ mm/s}^2$

Components: $\searrow 60°$: 720 mm/s^2

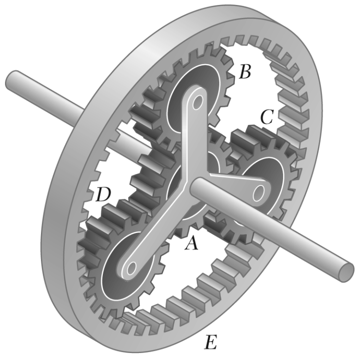


$$a_D = \sqrt{180^2 + 720^2} = 742.16 \text{ mm/s}^2$$

$$\tan\beta = \frac{720}{180} \qquad \beta = 76.0°$$

$$\beta - 30° = 46.0°$$

$\mathbf{a}_D = 742 \text{ mm/s}^2 \searrow 46.0°$ ◀



PROBLEM 15.118

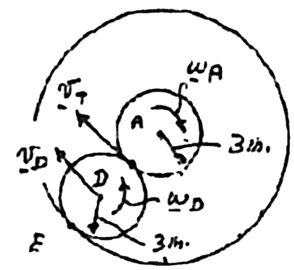
In the planetary gear system shown the radius of gears A, B, C, and D is 3 in. and the radius of the outer gear E is 9 in. Knowing that gear A has a constant angular velocity of 150 rpm clockwise and that the outer gear E is stationary, determine the magnitude of the acceleration of the tooth of gear D that is in contact with (*a*) gear A, (*b*) gear E.

SOLUTION

Velocity. $\quad T =$ Tooth of gear D in contact with gear A

Gears: $\quad v_T = r\omega_A = (3 \text{ in.})\omega_A$

Since $v_E = 0$, E is instantaneous center of gear D.



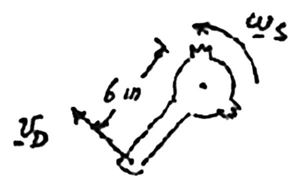
$$v_T = 2r\omega_D$$

$$(3 \text{ in.})\omega_A = 2(3 \text{ in.})\omega_D$$

$$\omega_D = \frac{1}{2}\omega_A$$

$$v_D = r\omega_D = (3 \text{ in.})\frac{1}{2}\omega_A = (1.5 \text{ in.})\omega_A$$

Spider:



$$v_D = (6 \text{ in.})\omega_S$$

$$(1.5 \text{ in.})\omega_A = (6 \text{ in.})\omega_S$$

$$\omega_S = \frac{1}{4}\omega_A$$

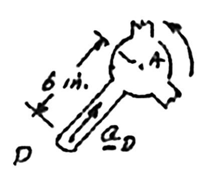
$$\boldsymbol{\omega}_A = 150 \text{ rpm} = 15.708 \text{ rad/s}$$

$$\boldsymbol{\omega}_D = \frac{1}{2}\omega_A = 7.854 \text{ rad/s}$$

$$\boldsymbol{\omega}_S = \frac{1}{4}\omega_A = 3.927 \text{ rad/s}$$

Acceleration.

Spider: $\quad \omega_S = 3.927 \text{ rad/s}$

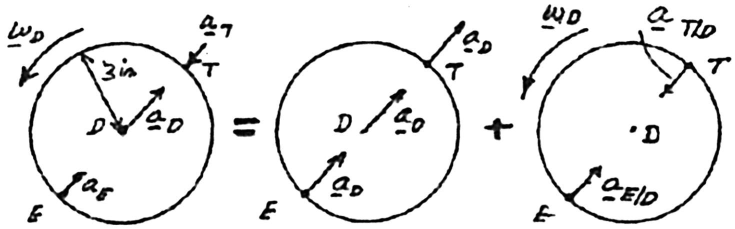


$$a_D = (AD)\omega_S^2 = (6 \text{ in.})(3.927 \text{ rad/s})^2$$

$$\mathbf{a}_D = 92.53 \text{ in./s}^2$$

PROBLEM 15.118 (Continued)

Gear D:



Plane motion = Trans. with D + Rotation about D

(*a*) Tooth T in contact with gear A.

$$\mathbf{a}_T = \mathbf{a}_D + \mathbf{a}_{T/D} = a_D + (DT)\omega_D^2$$

$$= 92.53 \text{ in./s}^2 \nearrow + (3 \text{ in.})(7.854 \text{ rad/s})^2 \swarrow$$

$$= 92.53 \text{ in./s}^2 \nearrow + 185.06 \text{ in./s}^2 \swarrow$$

$$\mathbf{a}_T = 92.53 \text{ in./s}^2 \swarrow$$

$a_T = 92.5 \text{ in./s}^2$ ◀

(*b*) Tooth E in contact with gear E.

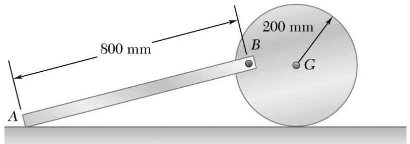
$$\mathbf{a}_E = \mathbf{a}_D + \mathbf{a}_{E/D} = \mathbf{a}_D + (ED)\omega_D^2$$

$$= 92.53 \text{ in./s}^2 \nearrow + (3 \text{ in.})(7.854 \text{ rad/s})^2 \nearrow$$

$$= 92.53 \text{ in./s}^2 \nearrow + 185.06 \text{ in./s}^2 \nearrow$$

$$\mathbf{a}_E = 277.6 \text{ in./s}^2 \nearrow$$

$a_E = 278 \text{ in./s}^2$ ◀



PROBLEM 15.119

The 200-mm-radius disk rolls without sliding on the surface shown. Knowing that the distance BG is 160 mm and that at the instant shown the disk has an angular velocity of 8 rad/s counterclockwise and an angular acceleration of 2 rad/s^2 clockwise, determine the acceleration of A.

SOLUTION

Units: m, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow$, $\mathbf{j} = 1 \uparrow$, $\mathbf{k} = 1 \circlearrowleft$

Geometric analysis. Let P be the point where the disk contacts the flat surface.

$$\mathbf{r}_{G/A} = 0.200\mathbf{j} \qquad \mathbf{r}_{B/G} = -0.16\mathbf{i}$$
$$\mathbf{r}_{A/B} = -\sqrt{0.600}\mathbf{i} - 0.200\mathbf{j}$$

Velocity analysis. $\boldsymbol{\omega}_G = (8 \text{ rad/s})\mathbf{k}, \quad \mathbf{v}_P = 0, \ \mathbf{v}_A = v_A\mathbf{i}$

$$\mathbf{v}_B = \mathbf{v}_P + \mathbf{v}_{G/P} = \mathbf{v}_P + \boldsymbol{\omega}_G \times \mathbf{r}_{G/P}$$
$$= 0 + 8\mathbf{k} \times (-0.160\mathbf{i} + 0.200\mathbf{j}) = -1.6\mathbf{i} - 1.28\mathbf{j}$$
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$
$$v_A\mathbf{i} = -1.6\mathbf{i} - 1.28\mathbf{j} + \omega_{AB}\mathbf{k} \times (-\sqrt{0.600}\mathbf{i} - 0.200\mathbf{j})$$
$$= -1.6\mathbf{i} - 1.28\mathbf{j} - 0.77460\omega_{AB}\mathbf{j} + 0.2\omega_{AB}\mathbf{i}$$

Resolve into components and transpose terms.

$\mathbf{j}$: $\quad 0.77460\omega_{AB} = -1.28 \qquad \omega_{AB} = -1.6525 \text{ rad/s}$ ◀

Acceleration analysis: $\mathbf{a}_A = \mathbf{a}_A\mathbf{j}, \quad \mathbf{a}_G = -2 \text{ rad/s}^2\mathbf{k}$

$$\mathbf{a}_P = (\omega_G^2 r)\mathbf{j} = (8)^2(0.2)\mathbf{j} = (12.8 \text{ m/s}^2)\mathbf{j}$$
$$\mathbf{a}_B = \mathbf{a}_P + \mathbf{a}_{B/P} = \mathbf{a}_P + \mathbf{a}_G \times \mathbf{r}_{B/P} - \omega_G^2\mathbf{r}_{B/P}$$
$$= 12.8\mathbf{j} + (-2\mathbf{k}) \times (-0.160\mathbf{i} + 0.200\mathbf{j}) - (8)^2(-0.160\mathbf{i} + 0.200\mathbf{j})$$
$$= 12.8\mathbf{j} + 0.32\mathbf{j} + 0.4\mathbf{i} + 10.24\mathbf{i} - 12.8\mathbf{j}$$
$$= 10.64\mathbf{i} + 0.32\mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B} = \mathbf{a}_B + \alpha_{AB}\mathbf{k} \times \mathbf{r}_{A/B} - \omega_{AB}^2\mathbf{r}_{A/B}$$
$$= 10.64\mathbf{i} + 0.32\mathbf{j} + \alpha_{AB}\mathbf{k} \times (-\sqrt{0.600}\mathbf{i} - 0.200\mathbf{j}) - (1.6525)^2(-\sqrt{0.600}\mathbf{i} - 0.200\mathbf{j})$$
$$= 10.64\mathbf{i} + 0.32\mathbf{j} - 0.77460\alpha_{AB}\mathbf{j} + 0.2\alpha_{AB}\mathbf{i} + 2.115\mathbf{i} + 0.54615\mathbf{j}$$
$$a_A\mathbf{i} = 12.755\mathbf{i} + 0.86615\mathbf{j} + 0.2\alpha_{AB}\mathbf{i} - 0.77460\alpha_{AB}\mathbf{j}$$

PROBLEM 15.119 (Continued)

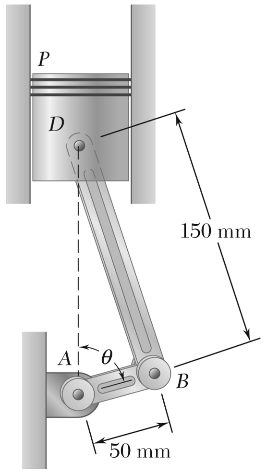
Resolve into components and transpose terms.

j: $0 = 0.86615 - 0.77460\alpha_{AB} \qquad \alpha_{AB} = 1.1182$

i: $a_A = 12.755 + 0.2\alpha_{AB} = 12.755 + (0.2)(1.1182) = 12.98$

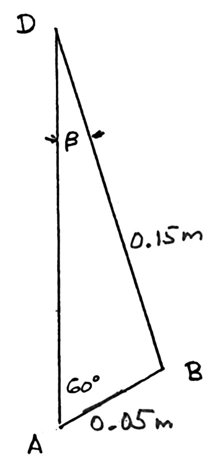
$\mathbf{a}_A = (12.98 \text{ m/s}^2)\mathbf{i} = 12.98 \text{ m/s}^2 \longrightarrow$ ◀

PROBLEM 15.120



Knowing that crank AB rotates about Point A with a constant angular velocity of 900 rpm clockwise, determine the acceleration of the piston P when $\theta = 60°$.

SOLUTION



Law of sines.

$$\frac{\sin\beta}{0.05} = \frac{\sin 60°}{0.15} \qquad \beta = 16.779°$$

Velocity analysis.

$$\omega_{AB} = 900 \text{ rpm} = 30\pi \text{ rad/s} \circlearrowright$$

$$\mathbf{v}_B = 0.05\omega_{AB} = 1.5\pi \text{ m/s} \searrow 60°$$

$$\mathbf{v}_D = v_D \downarrow \qquad \omega_{BD} = \omega_{BD} \circlearrowleft$$

$$\mathbf{v}_{D/B} = 0.15\,\omega_{BD} \swarrow \beta$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[v_D \downarrow] = [1.5\pi \searrow 60°] + [0.15\omega_{BD} \swarrow \beta]$$

Components $\xrightarrow{+}$:

$$0 = 1.5\pi\cos 60° - 0.15\omega_{BD}\cos\beta$$

$$\omega_{BD} = \frac{1.5\pi\cos 60°}{0.15\cos\beta} = 16.4065 \text{ rad/s} \circlearrowleft$$

Acceleration analysis.

$$\alpha_{AB} = 0$$

$$\mathbf{a}_B = 0.05\omega_{AB}^2 = (0.05)(30\pi)^2 = 444.13 \text{ m/s}^2 \swarrow 30°$$

$$\mathbf{a}_D = a_D \downarrow \qquad \alpha_{BD} = \alpha_{BD} \circlearrowright$$

$$\mathbf{a}_{D/B} = [0.15\alpha_{AB} \nearrow \beta] + [0.15\omega_{BD}^2 \searrow \beta]$$

$$= [0.15\alpha_{BD} \nearrow \beta] + [40.376 \searrow \beta]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \qquad \text{Resolve into components.}$$

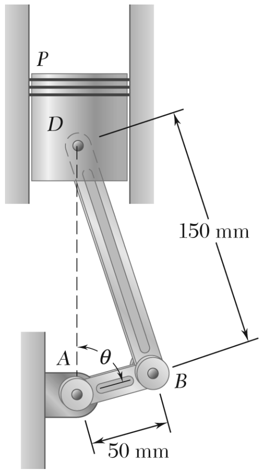
PROBLEM 15.120 (Continued)

$\xrightarrow{+}$: $0 = -444.13\cos 30° + 0.15\alpha_{BD}\cos\beta + 40.376\sin\beta$

$\alpha_{BD} = 2597.0 \text{ rad/s}^2$

$+\downarrow$: $a_D = 444.13\sin 30° - (0.15)(2597.0)\sin\beta + 40.376\cos\beta$

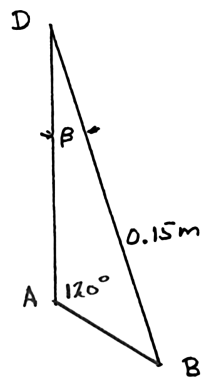
$= 148.27 \text{ m/s}^2 \qquad \mathbf{a}_P = \mathbf{a}_D \qquad \mathbf{a}_P = 148.3 \text{ m/s}^2 \downarrow$ ◀



PROBLEM 15.121

Knowing that crank AB rotates about Point A with a constant angular velocity of 900 rpm clockwise, determine the acceleration of the piston P when $\theta = 120°$.

SOLUTION



Law of sines. $\dfrac{\sin\beta}{0.05} = \dfrac{\sin 120°}{0.15}, \qquad \beta = 16.779°$

Velocity analysis. $\omega_{AB} = 900 \text{ rpm} = 30\pi \text{ rad/s}$

$$\mathbf{v}_B = 0.05\omega_{AB} = 1.5\pi \text{ m/s} \swarrow 60°$$

$$\mathbf{v}_D = v_D \downarrow \qquad \omega_{BD} = \omega_{BD}$$

$$\mathbf{v}_{D/B} = 0.15\omega_{BD} \swarrow \beta$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[v_D \downarrow] = [1.5\pi \swarrow 60°] + [0.15\omega_{BD} \swarrow \beta]$$

Components $\xrightarrow{+}$: $\quad 0 = -1.5\pi \cos 60° - 0.15\omega_{BD} \cos\beta$

$$\omega_{BD} = -\frac{1.5\pi \cos 60°}{0.15 \cos\beta} = 16.4065 \text{ rad/s}$$

Acceleration analysis. $\alpha_{AB} = 0$

$$\mathbf{a}_B = 0.05\omega_{AB}^2 = (0.05)(30\pi)^2 = 444.13 \text{ m/s}^2 \searrow 30°$$

$$\mathbf{a}_D = a_D \downarrow \qquad \alpha_{BD} = \alpha_{BD}$$

$$\mathbf{a}_{D/B} = [0.15\alpha_{AB} \nearrow \beta] + [0.15\omega_{BD}^2 \searrow \beta]$$

$$= [6\alpha_{BD} \nearrow \beta] + [40.376 \searrow \beta]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \qquad \text{Resolve into components.}$$

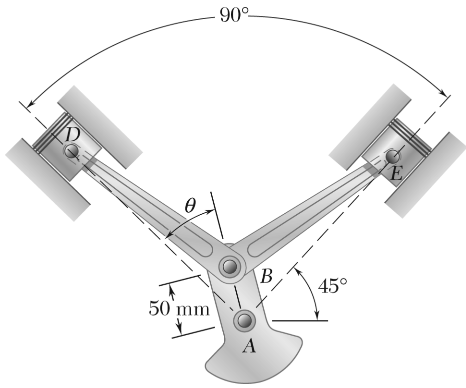
PROBLEM 15.121 (Continued)

$$\xrightarrow{+}: \quad 0 = -444.13\cos 30° + 0.15\alpha_{BD}\cos\beta + 40.376\sin\beta$$

$$\alpha_{BD} = 2597.0 \text{ rad/s}^2$$

$$+\downarrow: \quad a_D = -444.13\sin 30° - (0.15)(2597.0)\sin\beta + 40.376\cos\beta$$

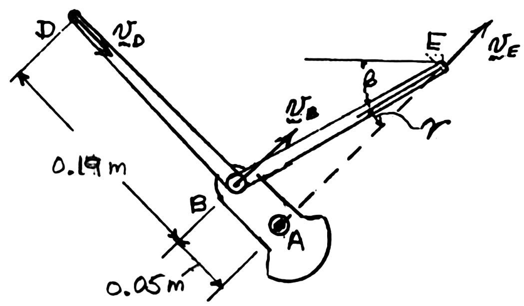
$$= -296 \text{ m/s}^2 \qquad \mathbf{a}_P = \mathbf{a}_D \qquad \mathbf{a}_P = 296 \text{ m/s}^2 \uparrow \blacktriangleleft$$



PROBLEM 15.122

In the two-cylinder air compressor shown the connecting rods BD and BE are each 190 mm long and crank AB rotates about the fixed Point A with a constant angular velocity of 1500 rpm clockwise. Determine the acceleration of each piston when $\theta = 0$.

SOLUTION



Crank AB.

$$\mathbf{v}_A = 0, \quad \mathbf{a}_A = 0, \quad \omega_{AB} = 1500 \text{ rpm} = 157.08 \text{ rad/s} \circlearrowright, \quad \alpha_{AB} = 0$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = 0 + [0.05\,\omega_{AB} \nearrow 45^\circ] = [7.854 \text{ m/s} \nearrow 45^\circ]$$

$$\mathbf{a}_B = \mathbf{a}_A + (\mathbf{a}_{B/A})_t + (\mathbf{a}_{B/A})_n$$

$$= 0 + [0.05\alpha_{AB} \swarrow 45^\circ] + [0.05\omega_{AB}^2 \searrow 45^\circ]$$

$$= [(0.05)(157.08)^2 \searrow 45^\circ] = 1233.7 \text{ m/s}^2 \searrow 45^\circ$$

Rod BD. $\quad \mathbf{v}_D = v_D \searrow 45^\circ \qquad \boldsymbol{\omega}_{BD} = \omega_{BD} \circlearrowright$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{B/D}$$

$$v_D \searrow 45^\circ = [7.854 \nearrow 45^\circ] + [0.19\omega_{BD} \swarrow 45^\circ]$$

Components $\nearrow 45^\circ$: $\quad 0 = 7.854 - 0.19\omega_{BD} \qquad \omega_{BD} = 41.337 \text{ rad/s}$

$$\mathbf{a}_D = a_D \searrow 45^\circ$$

$$\mathbf{a}_D = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_n$$

$$[a_D \searrow 45^\circ] = [1233.7 \searrow 45^\circ] + [0.19\,\alpha_{BD} \swarrow 45^\circ] + [0.19\,\omega_{BD}^2 \searrow 45^\circ]$$

Components ⦨ 45°: $a_D = 1233.7 + (0.19)(41.337)^2 = 1558.4 \text{ m/s}^2$

$\mathbf{a}_D = 1558 \text{ m/s}^2$ ⦨ 45° ◀

Rod BE. $\sin\gamma = \dfrac{0.05}{0.19}, \quad \gamma = 15.258°, \quad \beta = 45° - \gamma = 29.742°$

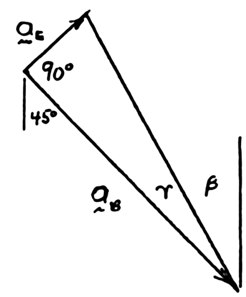
$\mathbf{v}_E = v_E$ ⦨ 45°

Since $\mathbf{v}_E$ is parallel to $\mathbf{v}_B$, $\omega_{BE} = 0.$

$\mathbf{a}_E = a_E$ ⦨ 45° $\quad (a_{B/E})_n = 0.19\,\omega_{BE}^2 = 0$

$(\mathbf{a}_{E/B})_t = (a_{E/B})_t$ ⦨ $\beta \qquad \mathbf{a}_E = \mathbf{a}_B + (\mathbf{a}_{B/E})_t$

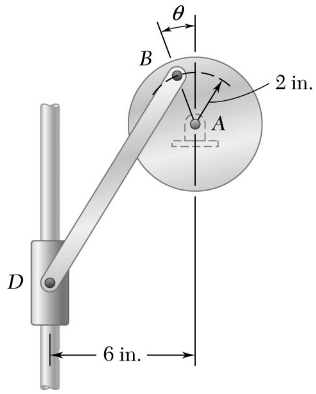
Draw vector addition diagram.



$$\begin{aligned} \gamma &= 45° - \beta \\ &= 15.258° \\ a_E &= a_B \tan\gamma \\ &= 1233.7 \tan\gamma \\ &= 336.52 \text{ m/s}^2 \end{aligned}$$

$\mathbf{a}_E = 337 \text{ m/s}^2$ ⦨ 45° ◀

PROBLEM 15.123



The disk shown has a constant angular velocity of 500 rpm counter-clockwise. Knowing that rod BD is 10 in. long, determine the acceleration of collar D when (a) $\theta = 90°$, (b) $\theta = 180°$.

SOLUTION

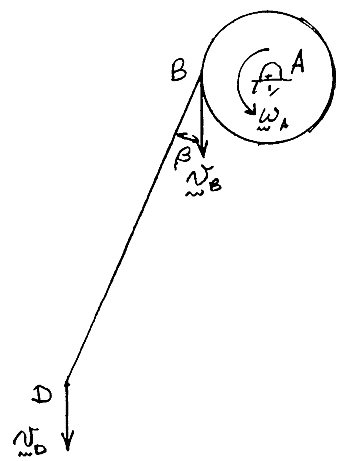
Disk A.

$$\boldsymbol{\omega}_A = 500 \text{ rpm} \circlearrowleft = 52.36 \text{ rad/s} \circlearrowleft$$

$$\boldsymbol{\alpha}_A = 0, \quad (AB) = 2 \text{ in.}$$

$$v_B = (AB)\omega_A = (2)(52.36) = 104.72 \text{ in./s}$$

$$a_B = (AB)\omega_A^2 = (2)(52.36)^2 = 5483.1 \text{ in./s}^2$$



(a) $\theta = 90°$.

$$\mathbf{v}_B = 104.72 \text{ m/s} \downarrow, \quad \mathbf{v}_D = v_D \downarrow$$

$$\sin\beta = \frac{2 \text{ in.}}{5 \text{ in.}} = 0.4 \qquad \beta = 23.58°$$

$\mathbf{v}_D$ and $\mathbf{v}_B$ are parallel.

$$\omega_{BD} = 0$$

$$\mathbf{a}_B = 5483.1 \text{ in./s}^2 \longrightarrow, \quad \mathbf{a}_D = a_D \uparrow, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD} \circlearrowleft$$

$$\mathbf{a}_{D/B} = [(BD)\alpha_{BD} \searrow \beta] + [(BD)\omega_{BD}^2 \nearrow \beta]$$

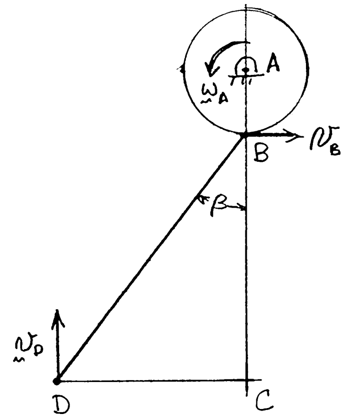
$$= [10\,\alpha_{BD} \searrow \beta] + 0$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \quad \text{Resolve into components.}$$

$\xrightarrow{+}$: $\quad 0 = 5483.1 + (10\cos\beta)\alpha_{BD} \qquad \alpha_{BD} = -598.26 \text{ rad/s}^2$

$+\uparrow$: $\quad a_D = 0 - (10\sin\beta)(-598.26) + 0 = 2393.0 \text{ in./s}^2 \qquad \mathbf{a}_D = 199.4 \text{ ft/s}^2 \uparrow$ ◀

PROBLEM 15.123 (Continued)



(*b*) $\theta = 180°$. $\mathbf{v}_B = 104.72$ in./s $\longrightarrow$, $\mathbf{v}_D = v_D \uparrow$

$$\sin\beta = \frac{6 \text{ in.}}{10 \text{ in.}} = 0.6 \qquad \beta = 36.87$$

$$\mathbf{v}_B = 104.72 \text{ in./s} \longrightarrow, \quad \mathbf{v}_D = v_D \uparrow$$

Instantaneous center of bar *BD* lies at Point *C*.

$$\omega_{BD} = \frac{v_B}{(BD)} = \frac{104.72}{10 \cos\beta} = 13.09 \text{ rad/s}$$

$$\mathbf{a}_B = 5483.1 \text{ in./s}^2 \uparrow, \quad \mathbf{a}_D = a_D \uparrow, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD} \circlearrowright$$

$$\mathbf{a}_{D/B} = [(BD)\alpha_{BD} \searrow \beta] + [(BD)\omega_{BD}^2 \nearrow \beta]$$

$$= [10\alpha_{BD} \searrow \beta] + [1713.5 \nearrow \beta]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \quad \text{Resolve in components.}$$

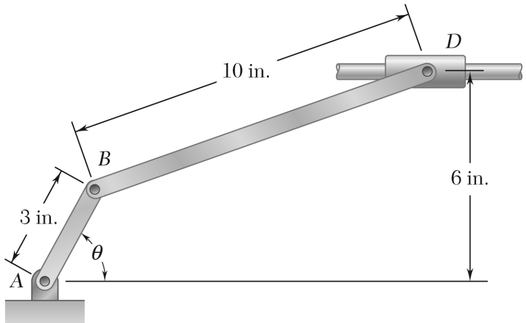
$\xrightarrow{+}$: $\quad 0 = 0 + (10 \cos\beta)\alpha_{BD} + 1713.5 \sin\beta$

$$\alpha_{BD} = -128.51 \text{ rad/s}^2$$

$+\uparrow$: $\quad a_D = 5483.1 - (10 \sin\beta)(-128.51) + 1713.5 \cos\beta$

$$= 7625.0 \text{ in./s}^2$$

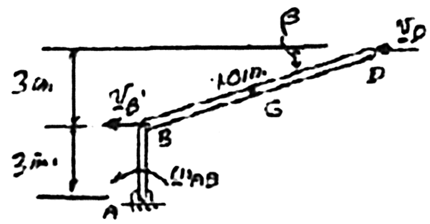
$\mathbf{a}_D = 635 \text{ ft/s}^2 \uparrow$ ◀



PROBLEM 15.124

Arm AB has a constant angular velocity of 16 rad/s counterclockwise. At the instant when $\theta = 90°$, determine the acceleration (*a*) of collar D, (*b*) of the midpoint G of bar BD.

SOLUTION



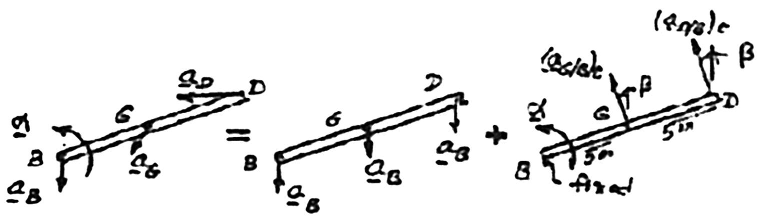
Rod AB:

$$a_B = (AB)\omega_{AB}^2$$
$$= (3\text{ in.})(16\text{ rad/s})^2$$
$$\mathbf{a}_B = 768\text{ in./s}^2 \downarrow$$

Rod BD: instantaneous center is at OD; $\boldsymbol{\omega}_{BD} = 0$

$$\sin\beta = (3\text{ in.})/(10\text{ in.}) = 0.3; \quad \beta = 17.46°$$

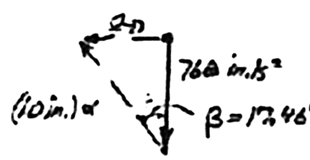
<u>Acceleration</u>.



<u>Plane motion</u> = <u>Trans. with B</u> + <u>Rotation about B</u>

(*a*)
$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} = a_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_n$$
$$= [a_B \downarrow] + [(BD)\alpha \nwarrow \beta] + [(BD)\omega_{BD}^2 \measuredangle \beta]$$
$$= [768\text{ in./s}^2 \downarrow] + [(10\text{ in.})\,\alpha \nwarrow \beta] + [(10\text{ in.})(0)^2 \measuredangle \beta]$$
$$\mathbf{a}_D \leftrightarrow = [768\text{ in./s} \downarrow] + [(10\text{ in.})\alpha \nwarrow \beta]$$

Vector diagram:



$$\mathbf{a}_D = (768\text{ in./s}^2)\tan 17.46°$$
$$= 241.62\text{ in./s}^2$$

$$\mathbf{a}_D = 242\text{ in./s}^2 \longleftarrow \blacktriangleleft$$

$$(10\text{ in.})\alpha = (768\text{ in./s}^2)/\cos 17.46°$$
$$(10\text{ in.})\alpha = 805.08\text{ in./s}^2$$
$$\boldsymbol{\alpha} = 80.5\text{ rad/s}^2 \circlearrowleft$$

PROBLEM 15.124 (Continued)

(*b*)

$$\mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B} = \mathbf{a}_B + (\mathbf{a}_{G/B})_t + (\mathbf{a}_{G/B})_n$$

$$= [\mathbf{a}_B \downarrow] + [(BG)\alpha \nwarrow \beta] + [(BG)\omega_{BD}^2 \measuredangle \beta]$$

$$= [768 \text{ in./s}^2 \downarrow] + [(5 \text{ in.})(80.5 \text{ rad/s}^2) \nwarrow \beta] + [(BG)(0)^2]$$

$$\mathbf{a}_G = [768 \text{ in./s}^2 \downarrow] + [402.5 \text{ in./s}^2 \nwarrow 17.46°]$$

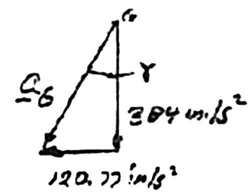
$\xleftarrow{+}$ components: $(a_G)_x = (402.5 \text{ in./s}^2)\sin 17.46°$

$$(\mathbf{a}_G)_x = 120.77 \text{ in./s}^2 \longleftarrow$$

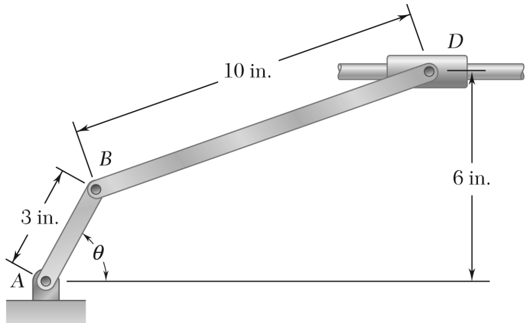
$+\!\downarrow$ components: $(a_G)_y = 768 \text{ in./s}^2 - (402.5 \text{ in./s}^2)\cos 17.46°$

$$= 768 \text{ in./s}^2 - 384 \text{ in./s}^2$$

$$(\mathbf{a}_G)_y = 384 \text{ in./s}^2 \downarrow$$



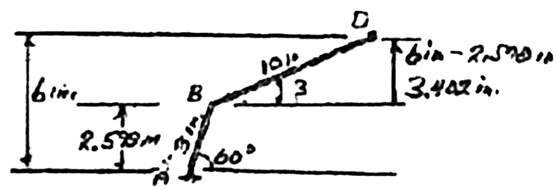
$$\mathbf{a}_G = 403 \text{ in./s}^2 \swarrow 72.5° \blacktriangleleft$$



PROBLEM 15.125

Arm AB has a constant angular velocity of 16 rad/s counterclockwise. At the instant when $\theta = 60°$, determine the acceleration of collar D.

SOLUTION

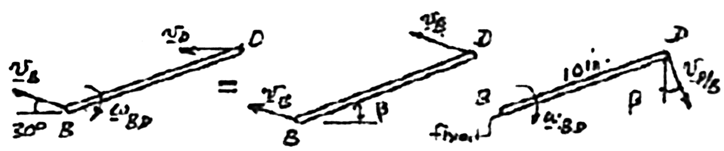


$$\beta = \sin^{-1}\frac{3.403 \text{ in.}}{10 \text{ in.}}$$

$$\beta = 19.89°$$

Velocity.
$$\mathbf{v}_B = (AB)\omega_{AB} = (3 \text{ in.})(16 \text{ rad/s}) = 48 \text{ in./s} \searrow 30°$$

Rod BD:



Plane motion = Trans. with B + Rotation about B

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_B + [(BD)\omega_{BD} \searrow \beta]$$

$$\mathbf{v}_D \leftrightarrow = [48 \text{ in./s} \searrow 30°] + [(10 \text{ in.})\omega_{BD} \searrow 19.89°]$$

$+\uparrow$ components:
$$(48 \text{ in./s})\sin 30° - (10 \text{ in.})\omega_{BD}\cos 19.89°$$

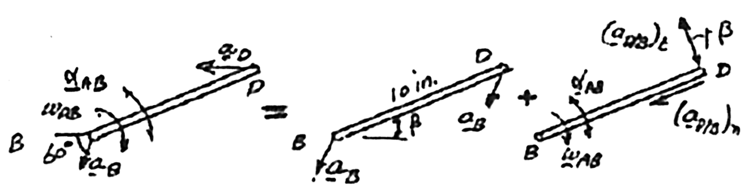
$$\boldsymbol{\omega}_{BD} = \frac{(48 \text{ in./s})\sin 30°}{(10 \text{ in.})\cos 19.890°} = 2.552 \text{ rad/s} \circlearrowright$$

Acceleration.

Rod AB:
$$\mathbf{a}_B = [(AB)\omega_{AB}^2 \swarrow 60°] = [(3 \text{ in.})(16 \text{ rad/s})^2 \swarrow 60°]$$

$$\mathbf{a}_B = 768 \text{ in./s}^2 \swarrow 60°$$

Rod BD:



Plane motion = Trans. with B + Rotation about B

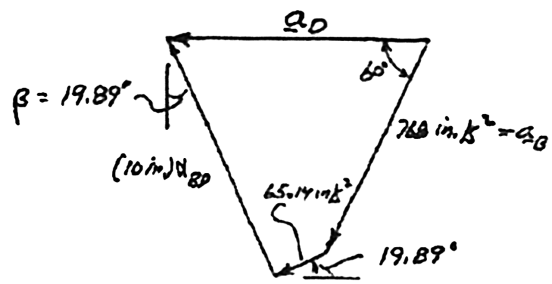
$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{B/D} = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_n$$

$$a_D \leftrightarrow = [a_B \swarrow 60°] + [(BD)\alpha_{BD} \searrow \beta] + [(BD)\omega_{DB}^2 \swarrow \beta]$$

$$= [768 \text{ in./s}^2 \swarrow 60°] + [(10 \text{ in.})\alpha_{BD} \searrow \beta] + [(10 \text{ in.})(2.552 \text{ rad/s}^2) \swarrow \beta]$$

$$a_D \leftrightarrow = [768 \text{ in./s}^2 \swarrow] + [(10 \text{ in.})\alpha_{BD} \searrow 19.89°] + [65.14 \text{ in./s}^2 \swarrow 19.89°]$$

Vector diagram.



y components:

$$+\downarrow: \quad 768\sin 60° + 65.14\sin 19.89° - 10\alpha_{BD}\cos 19.89° = 0$$

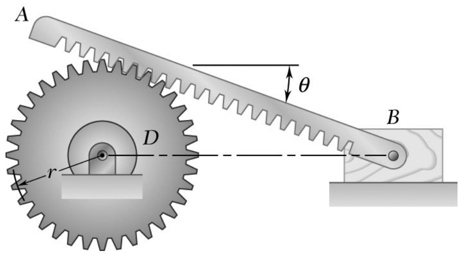
$$\alpha_{BD} = 73.09 \text{ rad/s}^2$$

x components:

$$\overset{+}{\leftarrow}: \quad a_D = 768\cos 60° + 65.14\cos 19.89° + (10)(73.09)\sin 19.89°$$

$$a_D = 693.9 \text{ in./s}^2$$

$\mathbf{a}_D = 694 \text{ in./s}^2 \longleftarrow$ ◀



PROBLEM 15.126

A straight rack rests on a gear of radius $r = 3$ in. and is attached to a block B as shown. Knowing that at the instant shown $\theta = 20°$, the angular velocity of gear D is 3 rad/s clockwise, and it is speeding up at a rate of 2 rad/s^2, determine (*a*) the angular acceleration of AB, (*b*) the acceleration of block B.

SOLUTION

Let Point P on the gear and Point Q on the rack be located at the contact point between them.

Units: inches, in./s, in./s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

Geometry:

$$\mathbf{r}_{P/D} = 3(\sin\theta\mathbf{i} + \cos\theta\mathbf{j})$$

$$\mathbf{r}_{B/Q} = \frac{3}{\tan\theta}(\cos\theta\mathbf{i} - \sin\theta\mathbf{j})$$

$$\theta = 20°$$

Gear D:

$$\boldsymbol{\omega}_{\text{gear}} = 3 \text{ rad/s} \circlearrowright \qquad \boldsymbol{\alpha}_{\text{gear}} = 2 \text{ rad/s}^2 \circlearrowright$$

$$v_P = \omega_{\text{gear}} r = (3)(3) = 9 \text{ in./s} \qquad \mathbf{v}_P = 9 \text{ in./s} \searrow \theta$$

$$(a_P)_n = \omega_{\text{gear}}^2 r = (3)^2(3) = 27 \text{ in./s}^2 \qquad (\mathbf{a}_P)_n = 27 \text{ in./s}^2 \swarrow \theta$$

$$(a_P)_t = \alpha_{\text{gear}} r = (2)(3) = 6 \text{ in./s}^2 \qquad (\mathbf{a}_P)_t = 6 \text{ in./s}^2 \searrow \theta$$

Velocity analysis.

Gear to rack contact: $\mathbf{v}_Q = \mathbf{v}_P = 9 \text{ in./s} \searrow \theta$

Rack AQB:

$$\boldsymbol{\omega}_{AB} = \omega_{AB} \circlearrowleft, \qquad \boldsymbol{\alpha}_{AB} = \alpha_{AB} \circlearrowleft$$

$$\mathbf{v}_B = v_B \longrightarrow, \qquad \boldsymbol{\alpha}_B = \alpha_B \longrightarrow$$

$$\mathbf{v}_B = \mathbf{v}_Q + \mathbf{v}_{B/Q} = \mathbf{v}_Q + \omega_{AB}\mathbf{k} \times \mathbf{r}_{B/Q}$$

$$v_B\mathbf{i} = 9(\cos\theta\mathbf{i} - \sin\theta\mathbf{j}) + \omega_{AB}\mathbf{k} \times (7.74535\mathbf{i} - 2.81908\mathbf{j})$$

$$= (9\cos\theta + 2.81907\omega_{AB})\mathbf{i} + (-9\sin\theta + 2.81907\omega_{AB})\mathbf{j}$$

Equating like components,

$\mathbf{j}$: $$0 = -9\sin\theta + 7.74535\omega_{AB}$$

$$\omega_{AB} = 0.39742 \qquad \omega_{AB} = 0.39742 \text{ rad/s} \circlearrowleft$$

Acceleration analysis.

Gear to rack contact: $(\mathbf{a}_Q)_t = (\mathbf{a}_P)_t = 6 \text{ in./s}^2 \searrow \theta$

$$(\mathbf{a}_Q)_n = (\mathbf{a}_P)_n + r\omega_{rd}^2 \nearrow \theta$$

where $\boldsymbol{\omega}_{rd} = \boldsymbol{\omega}_{AB} - \boldsymbol{\omega}_D = 3.39742 \text{ rad/s} \circlearrowright$

$$(\mathbf{a}_Q)_n = 27 \text{ in./s}^2 \swarrow 20° + (3)(3.39742)^2 \nearrow 20°$$

$$= 7.6274 \text{ in./s}^2 \nearrow 20°$$

Then, $\mathbf{a}_Q = 6(\cos\theta\,\mathbf{i} - \sin\theta\,\mathbf{j}) + 7.6274(\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})$

$$= (8.2469 \text{ in./s}^2)\mathbf{i} + (5.1153 \text{ in./s}^2)\mathbf{j}$$

$$\mathbf{a}_B = \mathbf{a}_Q + \mathbf{a}_{B/Q} = \mathbf{a}_Q + \alpha_{AB}\mathbf{k} \times \mathbf{r}_{B/Q} - \omega_{AB}^2 \mathbf{r}_{B/Q}$$

$$a_B\mathbf{i} = 8.2469\mathbf{i} + 5.1153\mathbf{j} + \alpha_{AB}\mathbf{k} \times (7.74535\mathbf{i} - 2.81908\mathbf{j})$$

$$- (0.39742)^2(7.74535\mathbf{i} - 2.81908\mathbf{j})$$

$$= (8.2469 + 2.81908\alpha_{AB} - 1.22332)\mathbf{i}$$

$$+ (5.1153 + 7.74535\alpha_{AB} + 0.44526)\mathbf{j}$$

Equating like components of $\mathbf{a}_B$,

j: $0 = 5.1153 + 7.74535\alpha_{AB} + 0.44526$

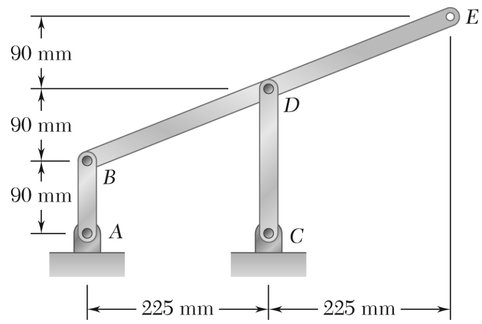
$$\alpha_{AB} = -0.71792 \text{ rad/s}^2$$

i: $a_B = 8.2469 + (2.81908)(-0.71792) - 1.22332$

$$a_B = 5.00 \text{ in./s}^2$$

(*a*) *Angular acceleration of AB*: $\boldsymbol{\alpha}_{AB} = 0.718 \text{ rad/s}^2 \circlearrowright$ ◀

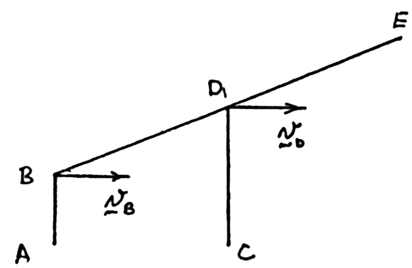
(*b*) *Acceleration of block B*: $\mathbf{a}_B = 5.00 \text{ in./s}^2 \longrightarrow$ ◀



PROBLEM 15.127

Knowing that at the instant shown rod AB has a constant angular velocity of 6 rad/s clockwise, determine the acceleration of Point D.

SOLUTION



Velocity analysis. $\omega_{AB} = 6 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_B = (AB)\omega_{AB} = (90)(6) = 540 \text{ mm/s}$$

$$\mathbf{v}_B = v_B \longrightarrow, \quad \mathbf{v}_D = v_D \longrightarrow$$

The instantaneous center of bar BDE lies at ∞.

Then $\omega_{BD} = 0$ and $v_D = v_B = 540$ mm/s

$$\omega_{CD} = \frac{v_D}{CD} = \frac{540}{180} = 3 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\alpha_{AB} = 0$

$$\mathbf{a}_B = (AB)\omega_{AB}^2 = [(90)(6)^2 \downarrow] = 3240 \text{ mm/s}^2 \downarrow$$

$$\mathbf{a}_D = [(CD)\alpha_{CD} \longleftarrow] + [(CD)\omega_{CD}^2 \downarrow] = [180\alpha_{CD} \longleftarrow] + [(180)(3)^2 \downarrow]$$

$$= [180\alpha_{CD} \longleftarrow] + [1620 \text{ mm/s}^2 \downarrow]$$

$$\mathbf{a}_{D/B} = [90\alpha_{BD} \longleftarrow] + [225\alpha_{BD} \uparrow] + [225\omega_{BD}^2 \longleftarrow] + [90\omega_{BD}^2 \downarrow]$$

$$= [90\alpha_{BD} \longleftarrow] + [225\alpha_{BD} \uparrow]$$

$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$ Resolve into components.

$+\uparrow$: $-1620 = -3240 + 225\alpha_{BD}$, $\alpha_{BD} = 7.2 \text{ rad/s}^2 \circlearrowleft$

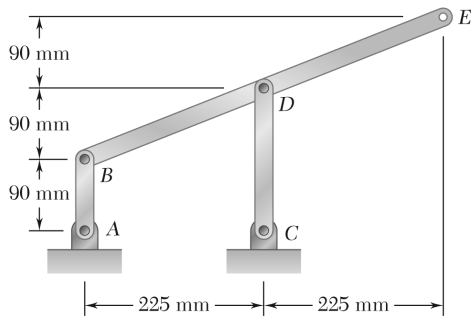
$\overset{+}{\longleftarrow}$: $180\alpha_{CD} = 0 + (90)(7.2)$, $\alpha_{CD} = 3.6 \text{ rad/s}^2 \circlearrowleft$

$$\mathbf{a}_D = [3240 \downarrow] + [(90)(7.2) \longleftarrow] + [(225)(7.2) \uparrow]$$

$$= [648 \longleftarrow] + [1620 \text{ mm/s}^2 \downarrow]$$

$$= 1745 \text{ mm/s}^2 \measuredangle 68.2^\circ$$

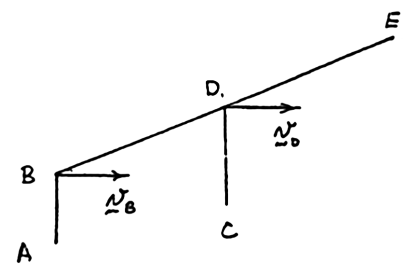
$\mathbf{a}_D = 1.745 \text{ m/s}^2 \measuredangle 68.2^\circ$ ◀



PROBLEM 15.128

Knowing that at the instant shown rod AB has a constant angular velocity of 6 rad/s clockwise, determine (*a*) the angular acceleration of member BDE, (*b*) the acceleration of Point E.

SOLUTION



Velocity analysis. $\omega_{AB} = 6 \text{ rad/s}$ ↻

$$\mathbf{v}_B = (AB)\omega_{AB} = (90)(6) = 540 \text{ mm/s}$$

$$\mathbf{v}_B = v_B \longrightarrow, \quad \mathbf{v}_D = v_D \longrightarrow$$

The instantaneous center of bar BDE lies at ∞.

Then $\omega_{BD} = 0$ and $v_D = v_B = 540 \text{ mm/s}$

$$\omega_{CD} = \frac{v_D}{CD} = \frac{27}{9} = 3 \text{ rad/s} ↻$$

Acceleration analysis. $\alpha_{AB} = 0$

$$\mathbf{a}_B = (AB)\omega_{AB}^2 = [(90)(6)^2 \downarrow] = 3240 \text{ mm/s}^2 \downarrow$$

$$\mathbf{a}_D = [(CD)\alpha_{CD} \longleftarrow] + [(CD)\omega_{CD}^2 \downarrow] = [180\alpha_{CD} \longleftarrow] + [(180)(3)^2 \downarrow]$$

$$= [180\alpha_{CD} \longleftarrow] + [1620 \text{ mm/s}^2 \downarrow]$$

$$\mathbf{a}_{D/B} = [90\alpha_{BD} \longleftarrow] + [225\alpha_{BD} \uparrow] + [225\omega_{BD}^2 \longleftarrow] + [90\omega_{BD}^2 \downarrow]$$

$$= [90\alpha_{BD} \longleftarrow] + [225\alpha_{BD} \uparrow]$$

$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$ Resolve into components.

(*a*) $+\uparrow$: $-1620 = -3240 + 225\alpha_{BD}$, $\alpha_{BD} = 7.20 \text{ rad/s}^2$ ↺

$$\mathbf{a}_{E/B} = [180\alpha_{BD} \longleftarrow] + [450\alpha_{BD} \uparrow] + [450\omega_{BD}^2 \longleftarrow] + [180\omega_{BD}^2 \downarrow]$$

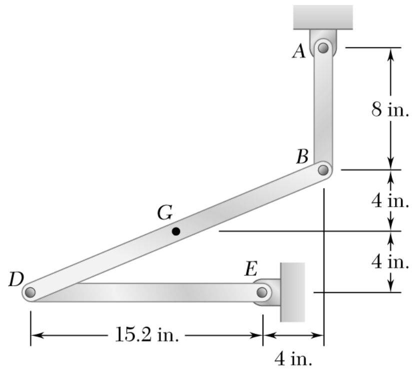
$$= [(180)(7.2) \longleftarrow] + [(450)(7.2) \uparrow] + [0 \longleftarrow] + [0 \downarrow]$$

$$= [1296 \text{ mm/s}^2 \longleftarrow] + [3240 \text{ mm/s}^2 \uparrow]$$

(*b*) $\mathbf{a}_E = \mathbf{a}_B + \mathbf{a}_{B/E} = [3240 \text{ mm/s}^2 \downarrow] + [1296 \text{ mm/s}^2 \longleftarrow] + [3240 \text{ mm/s}^2 \uparrow]$

$$= 1296 \text{ mm/s}^2 \longleftarrow$$

$\mathbf{a}_E = 1.296 \text{ m/s}^2 \longleftarrow$ ◀



PROBLEM 15.129

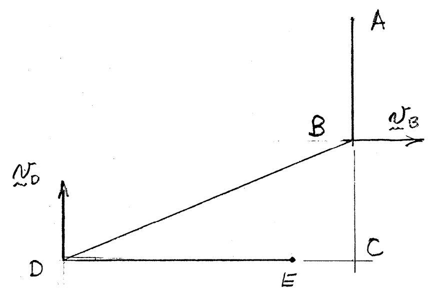
Knowing that at the instant shown bar AB has a constant angular velocity of 19 rad/s clockwise, determine (*a*) the angular acceleration of bar BGD, (*b*) the angular acceleration of bar DE.

SOLUTION

Velocity analysis. $\omega_{AB} = 19 \text{ rad/s}$

$$v_B = (AB)\omega_{AB} = (8)(19) = 152 \text{ in./s}$$

$$\mathbf{v}_B = v_B \longrightarrow, \quad \mathbf{v}_D = v_D \uparrow$$



Instantaneous center of bar BD lies at C.

$$\omega_{BD} = \frac{v_B}{BC} = \frac{152}{8} = 19 \text{ rad/s}$$

$$v_D = (CD)\omega_{BD} = (19.2)(19) = 364.8 \text{ in./s}$$

$$\omega_{DE} = \frac{v_D}{DE} = \frac{364.8}{15.2} = 24 \text{ rad/s}^2$$

Acceleration analysis. $\alpha_{AB} = 0.$

$$\mathbf{a}_B = [(AB)\omega_{AB}^2 \uparrow] = [(8)(19)^2 \uparrow] = 2888 \text{ in./s} \uparrow$$

$$\mathbf{a}_D = [(DE)\alpha_{DE} \downarrow] + [(DE)\omega_{DE}^2 \longrightarrow]$$

$$= [15.2\alpha_{DE} \downarrow] + [8755.2 \text{ in./s}^2 \longrightarrow]$$

$$(\mathbf{a}_{D/B})_t = [19.2\alpha_{BD} \downarrow] + [8\alpha_{DB} \longrightarrow]$$

$$(\mathbf{a}_{D/B})_n = [19.2\omega_{BD}^2 \longrightarrow] + [8\omega_{BD}^2 \uparrow]$$

$$= [6931.2 \text{ in./s}^2 \longrightarrow] + [2888 \text{ in./s}^2 \uparrow]$$

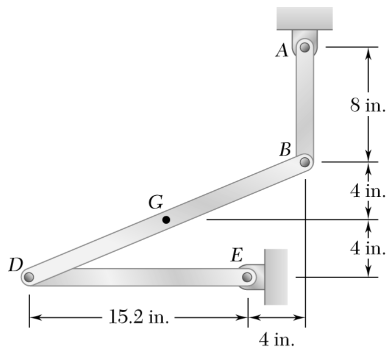
$\mathbf{a}_D = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (a_{D/B})_n$ Resolve into components.

$\xrightarrow{+}$: $8755.2 = 0 + 8\alpha_{BD} + 6931.2$

(*a*) $\boldsymbol{\alpha}_{BD} = 228 \text{ rad/s}^2$ ◀

$+\downarrow$: $15.2\alpha_{DE} = -2888 + (19.2)(228) - 2888$

(*b*) $\alpha_{DE} = -92 \text{ rad/s}$ $\boldsymbol{\alpha}_{DE} = 92.0 \text{ rad/s}^2$ ◀



PROBLEM 15.130

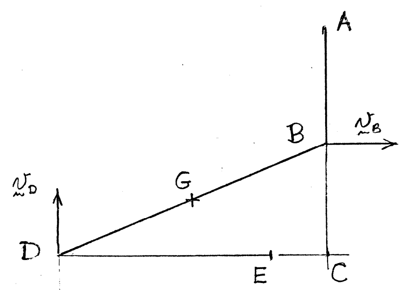
Knowing that at the instant shown bar DE has a constant angular velocity of 18 rad/s clockwise, determine (*a*) the acceleration of Point B, (*b*) the acceleration of Point G.

SOLUTION

Velocity analysis. $\omega_{DE} = 18 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_D = (DE)\omega_{DE} = (15.2)(18) = 273.6 \text{ in./s}$$

$$\mathbf{v}_D = v_D \uparrow, \quad \mathbf{v}_B = v_B \longrightarrow$$



Point C is the instantaneous center of bar BD.

$$\omega_{BD} = \frac{v_D}{CD} = \frac{273.6}{19.2} = 14.25 \text{ rad/s} \circlearrowright$$

$$v_B = (CB)\omega_{BD} = (8)(14.25) = 114 \text{ in./s}$$

$$\omega_{AB} = \frac{v_B}{AB} = \frac{114}{8} = 14.25 \text{ rad/s} \circlearrowleft$$

Acceleration analysis. $\alpha_{DE} = 0$

$$\mathbf{a}_D = [(DE)\omega_{DE}^2 \longrightarrow] = [(15.2)(18)^2 \longrightarrow] = [4924.8 \text{ in./s}^2 \longrightarrow]$$

$$\mathbf{a}_B = [(AB)\alpha_{AB} \longrightarrow] + [(AB)\omega_{AB}^2 \uparrow]$$

$$= [8\alpha_{AB} \longrightarrow] + [1624.5 \text{ in./s}^2 \uparrow]$$

$$(\mathbf{a}_{D/B})_t = [19.2\alpha_{BD} \downarrow] + [8\alpha_{BD} \longrightarrow]$$

$$(\mathbf{a}_{D/B})_n = [19.2\omega_{BD}^2 \longrightarrow] + [8\omega_{BD}^2 \uparrow]$$

$$\mathbf{a}_D = \mathbf{a}_B + (\mathbf{a}_{D/B})_t + (\mathbf{a}_{D/B})_n \qquad \text{Resolve into components.}$$

$+\downarrow$: $\quad 0 = 1624.5 - 19.2\alpha_{BD} + 1624.5,$

$$\alpha_{BD} = 169.21875 \text{ rad/s}^2$$

$\xrightarrow{+}$: $\quad 4924.8 = 8\alpha_{AB} + (8)(169.21875) + 3898.8$

$$\alpha_{AB} = -40.96875 \text{ rad/s}^2$$

(*a*) $\quad \mathbf{a}_B = [(8)(-40.96875) \longrightarrow] + [1624.5 \text{ in./s}^2 \uparrow]$

$$= [327.75 \text{ in./s}^2 \longleftarrow] + [1624.5 \text{ in./s}^2 \uparrow],$$

$$\mathbf{a}_B = 138.1 \text{ ft/s}^2 \measuredangle\ 78.6° \blacktriangleleft$$

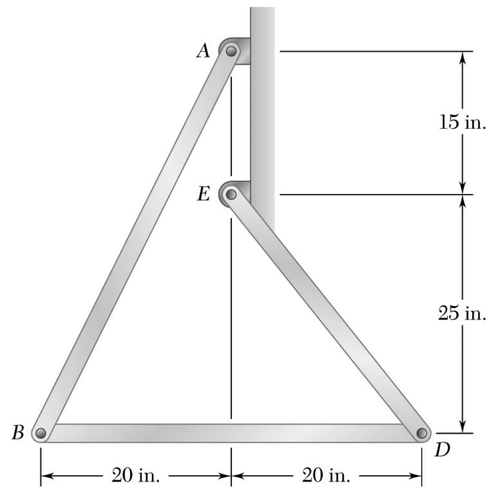
(*b*) $\quad \mathbf{a}_G = \mathbf{a}_B + \mathbf{a}_{G/B} = \mathbf{a}_B + \frac{1}{2}\mathbf{a}_{D/B}$

$$= \mathbf{a}_B + \frac{1}{2}(\mathbf{a}_D - \mathbf{a}_B) = \frac{1}{2}(\mathbf{a}_B + \mathbf{a}_D)$$

$$= \left[\frac{-327.75 + 4924.8}{2} \longrightarrow\right] + \left[\frac{1624.5}{2} \uparrow\right]$$

$$= [2298.5 \text{ in./s}^2] \longrightarrow + [812.25 \text{ in./s}^2 \uparrow]$$

$$\mathbf{a}_G = 203 \text{ ft/s}^2 \measuredangle\ 19.5° \blacktriangleleft$$



PROBLEM 15.131

Knowing that at the instant shown bar *AB* has a constant angular velocity of 4 rad/s clockwise, determine the angular acceleration (*a*) of bar *BD*, (*b*) of bar *DE*.

SOLUTION

Relative position vectors.

$$\mathbf{r}_{B/A} = -(20\text{ in.})\mathbf{i} - (40\text{ in.})\mathbf{j}$$
$$\mathbf{r}_{D/B} = (40\text{ in.})\mathbf{i}$$
$$\mathbf{r}_{D/E} = (20\text{ in.})\mathbf{i} - (25\text{ in.})\mathbf{j}$$

Velocity analysis.

Bar *AB* (Rotation about *A*):

$$\boldsymbol{\omega}_{AB} = 4\text{ rad/s} \circlearrowright = -(4\text{ rad/s})\mathbf{k}$$

$$\mathbf{r}_{B/A} = -(20\text{ in.})\mathbf{i} - (40\text{ in.})\mathbf{j} \qquad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-20\mathbf{i} - 40\mathbf{j})$$
$$\mathbf{v}_B = -(160\text{ in./s})\mathbf{i} + (80\text{ in./s})\mathbf{j}$$

Bar *BD* (Plane motion = Translation with *B* + Rotation about *B*):

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \mathbf{r}_{D/B} = (40\text{ in.})\mathbf{i}$$
$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = \mathbf{v}_B + (\omega_{BD}\mathbf{k}) \times (40\mathbf{i})$$
$$\mathbf{v}_D = -(160\text{ in/s})\mathbf{i} + (40\omega_{BD} + 80\text{ in./s})\mathbf{j}$$

Bar *DE* (Rotation about *E*):

$$\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$$
$$\mathbf{r}_{D/E} = (20\text{ in.})\mathbf{i} - (25\text{ in.})\mathbf{j}$$
$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (20\mathbf{i} - 25\mathbf{j})$$
$$\mathbf{v}_D = 20\omega_{DE}\mathbf{j} + 25\omega_{DE}\mathbf{i}$$

Equating components of the two expression for $\mathbf{v}_D$,

$\mathbf{i}$: $\quad -160 = 25\omega_{DE} \qquad \omega_{DE} = -6.4\text{ rad/s}$

$\mathbf{j}$: $\quad 40\omega_{BE} + 80 = 20\omega_{DE} \qquad 40\omega_{BD} + 80 = 20(-6.4) \qquad \omega_{BD} = -5.2\text{ rad/s}$

PROBLEM 15.131 (Continued)

Summary of angular velocities: $\boldsymbol{\omega}_{AB} = 4$ rad/s ↻ $\boldsymbol{\omega}_{DE} = 6.4$ rad/s ↻ $\boldsymbol{\omega}_{BD} = 5.2$ rad/s ↻

Acceleration analysis. $\boldsymbol{\alpha}_{AB} = 0, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}, \quad \boldsymbol{\alpha}_{DE} = \alpha_{DE}\mathbf{k}$

<u>Bar *AB*</u> (Rotation about *A*):

$$\begin{aligned}\mathbf{a}_B &= \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\ &= 0 - (4)^2(-20\mathbf{i} - 40\mathbf{j}) \\ &= (320 \text{ in./s}^2)\mathbf{i} + (640 \text{ in./s}^2)\mathbf{j}\end{aligned}$$

<u>Bar *BD*</u> (Translation with B + Rotation about B):

$$\begin{aligned}\mathbf{a}_D &= \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B} \\ &= 320\mathbf{i} + 640\mathbf{j} + \alpha_{BD}\mathbf{k} \times (40\mathbf{i}) - (5.2)^2(40)\mathbf{i} \\ &= 320\mathbf{i} + 640\mathbf{j} + 40\alpha_{BD}\mathbf{j} - 1081.6\mathbf{i} \\ &= -761.60\mathbf{i} + (640 + 40\alpha_{BD})\mathbf{j} \qquad (1)\end{aligned}$$

<u>Bar *DE*</u> (Rotation about E):

$$\begin{aligned}\mathbf{a}_D &= \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 \mathbf{r}_{D/E} \\ &= \alpha_{DE}\mathbf{k} \times (20\mathbf{i} - 25\mathbf{j}) - (6.4)^2(20\mathbf{i} - 25\mathbf{j}) \\ &= -20\alpha_{DE}\mathbf{j} + 25\alpha_{DE}\mathbf{i} - 819.20\mathbf{i} + 1024\mathbf{j} \\ &= (25\alpha_{DE} - 819.20)\mathbf{i} + (20\alpha_{DE} + 1024)\mathbf{j} \qquad (2)\end{aligned}$$

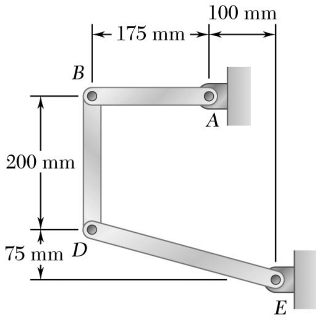
Equate like components of $\mathbf{a}_D$ expressed by Eqs. (1) and (2).

i: $-761.60 = 25\alpha_{DE} - 819.20 \qquad \alpha_{DE} = 2.3040 \text{ rad/s}^2$

j: $640 + 40\alpha_{BD} = (20)(2.304) + 1024 \qquad \alpha_{BD} = 10.752 \text{ rad/s}^2$

(*a*) <u>Angular acceleration of bar *BD*.</u> $\boldsymbol{\alpha}_{BD} = 10.75 \text{ rad/s}^2$ ↺ ◀

(*b*) <u>Angular acceleration of bar *DE*.</u> $\boldsymbol{\alpha}_{DE} = 2.30 \text{ rad/s}^2$ ↺ ◀



PROBLEM 15.132

Knowing that at the instant shown bar AB has a constant angular velocity of 4 rad/s clockwise, determine the angular acceleration (a) of bar BD, (b) of bar DE.

SOLUTION

Velocity analysis.

Bar AB (Rotation about A): $\quad \boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright = -(4 \text{ rad/s})\mathbf{k}$

$$\mathbf{r}_{B/A} = -(175 \text{ mm})\mathbf{i} \qquad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-175\mathbf{i})$$

$$\mathbf{v}_B = (700 \text{ mm/s})\mathbf{j}$$

Bar BD (Plane motion = Translation with B + Rotation about B):

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \mathbf{r}_{D/B} = -(200 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = 700\mathbf{j} + (\omega_{BD}\mathbf{k}) \times (-200\mathbf{j})$$

$$\mathbf{v}_D = 700\mathbf{j} + 200\omega_{BD}\mathbf{i}$$

Bar DE (Rotation about E): $\quad \boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$

$$\mathbf{r}_{D/E} = -(275 \text{ mm})\mathbf{i} + (75 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (-275\mathbf{i} + 75\mathbf{j})$$

$$\mathbf{v}_D = -275\omega_{DE}\mathbf{j} - 75\omega_{DE}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_D$,

$\mathbf{j}$: $\quad 700 = -275\omega_{DE} \qquad \omega_{DE} = -2.5455 \text{ rad/s} \qquad\qquad \boldsymbol{\omega}_{DE} = 2.55 \text{ rad/s} \circlearrowright$

$\mathbf{i}$: $\quad 200\omega_{BD} = -75\omega_{BD} \qquad \omega_{DE} = -\dfrac{3}{8}\omega_{BD}$

$$\omega_{BD} = -\left(\frac{3}{8}\right)(-2.5455) = 0.95455 \text{ rad/s} \qquad\qquad \boldsymbol{\omega}_{BD} = 0.955 \text{ rad/s} \circlearrowleft$$

Acceleration analysis. $\quad \boldsymbol{\alpha}_{AB} = 0$

Bar AB: $\quad \mathbf{a}_B = -\omega_{AB}^2\mathbf{r}_{B/A} = -(4)^2(-175\mathbf{i}) = (2800 \text{ mm/s}^2)\mathbf{i}$

Bar BD: $\quad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}$

$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2\mathbf{r}_{D/B}$$

$$= 2800\mathbf{i} + \alpha_{BD}\mathbf{k} \times (-200\mathbf{j}) - (0.95455)^2(-200\mathbf{j})$$

$$= (2800 + 200\,\alpha_{BD})\mathbf{i} + 182.23\mathbf{j} \qquad (1)$$

PROBLEM 15.132 (Continued)

Bar DE:

$$\alpha_{DE} = \alpha_{DE}\mathbf{k}$$

$$\mathbf{a}_D = \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 r_{D/E}$$

$$= \alpha_{DE}\mathbf{k} \times (-275\mathbf{i} + 75\mathbf{j}) - (2.5455)^2(-275\mathbf{i} + 75\mathbf{j})$$

$$= -275\alpha_{DE}\mathbf{j} - 75\alpha_{DE}\mathbf{i} + 1781.8\mathbf{i} - 485.95\mathbf{j}$$

$$= (-75\alpha_{DE} + 1781.8)\mathbf{i} - (275\alpha_{DE} + 485.95)\mathbf{j} \qquad (2)$$

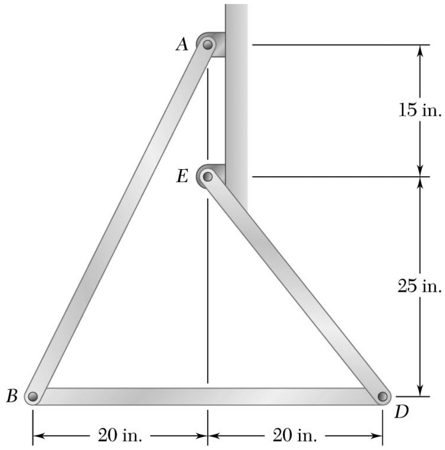
Equate like components of $\mathbf{a}_D$ expressed by Eqs. (1) and (2).

$\mathbf{j}$: $\quad 182.23 = -(275\alpha_{DE} + 485.95) \qquad \alpha_{DE} = -2.4298 \text{ rad/s}^2$

$\mathbf{i}$: $\quad (2800 + 200\alpha_{BD}) = [-(75)(-2.4298) + 1781.8] \qquad \alpha_{BD} = -4.1795 \text{ rad/s}^2$

(*a*) Angular acceleration of bar *BD*. $\qquad \boldsymbol{\alpha}_{BD} = 4.18 \text{ rad/s}^2$ ◀

(*b*) Angular acceleration of bar *DE*. $\qquad \boldsymbol{\alpha}_{DE} = 2.43 \text{ rad/s}^2$ ◀



PROBLEM 15.133

Knowing that at the instant shown bar AB has an angular velocity of 4 rad/s and an angular acceleration of 2 rad/s^2, both clockwise, determine the angular acceleration (*a*) of bar BD, (*b*) of bar DE by using the vector approach as is done in Sample Problem 15.8.

SOLUTION

Relative position vectors.

$$\mathbf{r}_{B/A} = -(20 \text{ in.})\mathbf{i} - (40 \text{ in.})\mathbf{j}$$
$$\mathbf{r}_{D/B} = (40 \text{ in.})\mathbf{i}$$
$$\mathbf{r}_{D/E} = (20 \text{ in.})\mathbf{i} - (25 \text{ in.})\mathbf{j}$$

Velocity analysis.

Bar *AB* (Rotation about *A*):

$$\boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright = -(4 \text{ rad/s})\mathbf{k}$$

$$\mathbf{r}_{B/A} = -(20 \text{ in.})\mathbf{i} - (40 \text{ in.})\mathbf{j} \qquad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-20\mathbf{i} - 40\mathbf{j})$$
$$\mathbf{v}_B = -(160 \text{ in./s})\mathbf{i} + (80 \text{ in./s})\mathbf{j}$$

Bar *BD* (Plane motion = Translation with B + Rotation about B):

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \mathbf{r}_{D/B} = (40 \text{ in.})\mathbf{i}$$
$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = \mathbf{v}_B + (\omega_{BD}\mathbf{k}) \times (40\mathbf{i})$$
$$\mathbf{v}_D = -(160 \text{ in/s})\mathbf{i} + (40\omega_{BD} + 80 \text{ in./s})\mathbf{j}$$

Bar *DE* (Rotation about *E*):

$$\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$$
$$\mathbf{r}_{D/E} = (20 \text{ in.})\mathbf{i} - (25\text{in.})\mathbf{j}$$
$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (20\mathbf{i} - 25\mathbf{j})$$
$$\mathbf{v}_D = 20\omega_{DE}\mathbf{j} + 25\omega_{DE}\mathbf{i}$$

Equating components of the two expression for $\mathbf{v}_D$,

$\mathbf{i}$: $\qquad -160 = 25\omega_{DE} \qquad \omega_{DE} = -6.4 \text{ rad/s}$

$\mathbf{j}$: $\qquad 40\omega_{BE} + 80 = 20\omega_{DE} \qquad 40\omega_{BD} + 80 = 20(-6.4) \qquad \omega_{BD} = -5.2 \text{ rad/s}$

PROBLEM 15.133 (Continued)

Summary of angular velocities: $\omega_{AB} = 4 \text{ rad/s} \circlearrowright \quad \omega_{DE} = 6.4 \text{ rad/s} \circlearrowright \quad \omega_{BD} = 5.2 \text{ rad/s} \circlearrowright$

Acceleration analysis. $\boldsymbol{\alpha}_{AB} = -(2 \text{ rad/s}^2)\mathbf{k}, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}, \quad \boldsymbol{\alpha}_{DE} = \alpha_{DE}\mathbf{k}$

<u>Bar *AB*</u> (Rotation about *A*)

$$\begin{aligned}
\mathbf{a}_B &= \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\
&= (-2\mathbf{k}) \times (-20\mathbf{i} - 40\mathbf{j}) - (4)^2(-20\mathbf{i} - 40\mathbf{j}) \\
&= -(80 \text{ in./s}^2)\mathbf{i} + (40 \text{ in./s}^2)\mathbf{j} + (320 \text{ in./s}^2)\mathbf{i} + (640 \text{ in./s}^2)\mathbf{j} \\
&= (240 \text{ in./s}^2)\mathbf{i} + (680 \text{ in./s}^2)\mathbf{j}
\end{aligned}$$

<u>Bar *BD*</u> (Translation with *B* + Rotation about *B*):

$$\begin{aligned}
\mathbf{a}_D &= \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B} \\
&= 240\mathbf{i} + 680\mathbf{j} + \alpha_{BD}\mathbf{k} \times (40\mathbf{i}) - (5.2)^2(40)\mathbf{i} \\
&= 240\mathbf{i} + 680\mathbf{j} + 40\alpha_{BD}\mathbf{j} - 1081.6\mathbf{i} \\
&= -841.60\mathbf{i} + (680 + 40\alpha_{BD})\mathbf{j} \qquad (1)
\end{aligned}$$

<u>Bar *DE*</u> (Rotation about *E*):

$$\begin{aligned}
\mathbf{a}_D &= \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 \mathbf{r}_{D/E} \\
&= \alpha_{DE}\mathbf{k} \times (20\mathbf{i} - 25\mathbf{j}) - (6.4)^2(20\mathbf{i} - 25\mathbf{j}) \\
&= 20\alpha_{DE}\mathbf{j} + 25\alpha_{DE}\mathbf{i} - 819.20\mathbf{i} + 1024\mathbf{j} \\
&= (25\alpha_{DE} - 819.20)\mathbf{i} + (20\alpha_{DE} + 1024)\mathbf{j} \qquad (2)
\end{aligned}$$

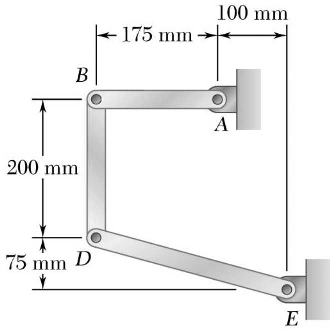
Equate like components of $\mathbf{a}_D$ expressed by Eqs. (1) and (2).

i: $\quad -841.60 = 25\alpha_{DE} - 819.20 \qquad \alpha_{DE} = -0.896 \text{ rad/s}^2$

j: $\quad 680 + 40\alpha_{BD} = (20)(-0.896) + 1024 \qquad \alpha_{BD} = 8.152 \text{ rad/s}^2$

(*a*) <u>Angular acceleration of bar *BD*.</u> $\boldsymbol{\alpha}_{BD} = 8.15 \text{ rad/s}^2 \circlearrowleft$ ◀

(*b*) <u>Angular acceleration of bar *DE*.</u> $\boldsymbol{\alpha}_{DE} = 0.896 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 15.134

Knowing that at the instant shown bar *AB* has an angular velocity of 4 rad/s and an angular acceleration of 2 rad/s^2, both clockwise, determine the angular acceleration (*a*) of bar BD, (*b*) of bar *DE* by using the vector approach as is done in Sample Problem 15.8.

SOLUTION

Velocity analysis.

<u>Bar *AB*</u> (Rotation about *A*): $\boldsymbol{\omega}_{AB} = 4 \text{ rad/s} = -(4 \text{ rad/s})\mathbf{k}$

$$\mathbf{r}_{B/A} = -(175 \text{ mm})\mathbf{i} \qquad \mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = (-4\mathbf{k}) \times (-175\mathbf{i})$$

$$\mathbf{v}_B = (700 \text{ mm/s})\mathbf{j}$$

<u>Bar *BD*</u> (Plane motion = Translation with *B* + Rotation about *B*):

$$\boldsymbol{\omega}_{BD} = \omega_{BD}\mathbf{k} \qquad \mathbf{r}_{D/B} = -(200 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B} = 700\mathbf{j} + (\omega_{BD}\mathbf{k}) \times (-200\mathbf{j})$$

$$\mathbf{v}_D = 700\mathbf{j} + 200\omega_{BD}\mathbf{i}$$

<u>Bar *DE*</u> (Rotation about *E*): $\boldsymbol{\omega}_{DE} = \omega_{DE}\mathbf{k}$

$$\mathbf{r}_{D/E} = -(275 \text{ mm})\mathbf{i} + (75 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_{DE} \times \mathbf{r}_{D/E} = (\omega_{DE}\mathbf{k}) \times (-275\mathbf{i} + 75\mathbf{j})$$

$$\mathbf{v}_D = -275\omega_{DE}\mathbf{j} - 75\omega_{DE}\mathbf{i}$$

Equating components of the two expressions for $\mathbf{v}_D$,

$$\mathbf{j}: \quad 700 = -275\omega_{DE} \qquad \omega_{DE} = -2.5455 \text{ rad/s} \qquad \boldsymbol{\omega}_{DE} = 2.55 \text{ rad/s}$$

$$\mathbf{i}: \quad 200\omega_{BD} = -75\omega_{DE} \qquad \omega_{BD} = -\frac{3}{8}\omega_{DE}$$

$$\omega_{BD} = -\left(\frac{3}{8}\right)(-2.5455) = 0.95455 \text{ rad/s} \qquad \boldsymbol{\omega}_{BD} = 0.955 \text{ rad/s}$$

Acceleration analysis. $\boldsymbol{\alpha}_{AB} = 2 \text{ rad/s} = -(2 \text{ rad/s}^2)\mathbf{k}$

<u>Bar *AB*</u>:

$$\mathbf{a}_B = \alpha_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 r_{B/A}$$

$$= (-2\mathbf{k}) \times (-175\mathbf{i}) - (4)^2(-175\mathbf{i}) = 2800 \text{ mm/s}^2\mathbf{i} + 350 \text{ mm/s}^2\mathbf{j}$$

<u>Bar *BD*</u>: $\boldsymbol{\alpha}_{BD} = \alpha_{BD}\mathbf{k}$

$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2\mathbf{r}_{D/B}$$

$$= 2800\mathbf{i} + 350\mathbf{j} + \alpha_{BD}\mathbf{k} \times (-200\mathbf{j}) - (0.95455)^2(-200\mathbf{j})$$

$$= (2800 + 200\,\alpha_{BD})\mathbf{i} + 532.23\mathbf{j} \qquad (1)$$

PROBLEM 15.134 (Continued)

Bar *DE*:

$$\alpha_{DE} = \alpha_{DE}\mathbf{k}$$

$$\mathbf{a}_D = \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 r_{D/E}$$

$$= \alpha_{DE}\mathbf{k} \times (-275\mathbf{i} + 75\mathbf{j}) - (2.5455)^2(-275\mathbf{i} + 75\mathbf{j})$$

$$= -275\alpha_{DE}\mathbf{j} - 75\alpha_{DE}\mathbf{i} + 1781.8\mathbf{i} - 485.95\mathbf{j}$$

$$= (-75\alpha_{DE} + 1781.8)\mathbf{i} - (275\alpha_{DE} + 485.95)\mathbf{j} \qquad (2)$$

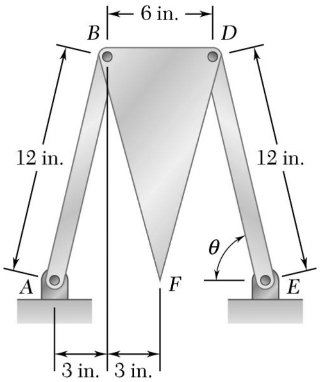
Equate like components of $\mathbf{a}_D$ expressed by Eqs. (1) and (2).

$\mathbf{j}$: $\quad 532.23 = -(275\alpha_{DE} + 485.95) \qquad \alpha_{DE} = -3.7025 \text{ rad/s}^2$

$\mathbf{i}$: $\quad (2800 + 200\alpha_{BD}) = [-(75)(-3.7025) + 1781.8] \qquad \alpha_{BD} = -3.7025 \text{ rad/s}^2$

(*a*) Angular acceleration of bar *BD*. $\qquad \boldsymbol{\alpha}_{BD} = 3.70 \text{ rad/s}^2$ ◀

(*b*) Angular acceleration of bar *DE*. $\qquad \boldsymbol{\alpha}_{DE} = 3.70 \text{ rad/s}^2$ ◀



PROBLEM 15.135

Robert's linkage is named after Richard Robert (1789–1864) and can be used to draw a close approximation to a straight line by locating a pen at Point *F*. The distance *AB* is the same as *BF*, *DF* and *DE*. Knowing that at the instant shown bar *AB* has a constant angular velocity of 4 rad/s clockwise, determine (*a*) the angular acceleration of bar *DE*, (*b*) the acceleration of Point *F*.

SOLUTION

Units: inches, in./s, in./s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \ \mathbf{j} = 1 \uparrow, \ \mathbf{k} = 1 \circlearrowleft.$

Geometry:

$$\mathbf{r}_{B/A} = 3\mathbf{i} + \sqrt{12^2 - 3^2}\,\mathbf{j} = 3\mathbf{i} + \sqrt{135}\,\mathbf{j}$$

$$\mathbf{r}_{D/B} = 6\mathbf{i} \qquad \mathbf{r}_{F/B} = 3\mathbf{i} - \sqrt{135}\,\mathbf{j}$$

$$\mathbf{r}_{D/E} = -3\mathbf{i} + \sqrt{135}\,\mathbf{j}$$

Velocity analysis: $\boldsymbol{\omega}_{AB} = 4 \text{ rad/s}^2 \circlearrowright = -4\,\mathbf{k}$

Bar *AB*: $\mathbf{v}_B = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{B/A} = -4\mathbf{k} \times (3\mathbf{i} + \sqrt{135}\,\mathbf{j}) = 4\sqrt{135}\,\mathbf{i} - 12\mathbf{j}$

Object *BDF*:

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_B + \boldsymbol{\omega}_{BD} \times \mathbf{r}_{D/B}$$

$$= 4\sqrt{135}\,\mathbf{i} - 12\mathbf{j} + \omega_{BD}\mathbf{k} \times 6\mathbf{i}$$

$$= 4\sqrt{135}\,\mathbf{i} - 12\mathbf{j} + 6\omega_{BD}\mathbf{j} \qquad (1)$$

Bar *DE*:

$$\mathbf{v}_D = \omega_{DE} \times \mathbf{r}_{D/E} = \omega_{DE}\mathbf{k} \times (-3\mathbf{i} + \sqrt{135}\,\mathbf{j})$$

$$= -\sqrt{135}\,\omega_{DE}\mathbf{i} - 3\omega_{DE}\mathbf{j} \qquad (2)$$

Equating like components of $\mathbf{v}_D$ from Eqs. (1) and (2),

$$\mathbf{i}: \quad 4\sqrt{135} = -\sqrt{135}\,\omega_{DE} \qquad (3)$$

$$\mathbf{j}: \quad -12 + 6\omega_{BD} = -3\omega_{DE} \qquad (4)$$

From Eq. (3), $\omega_{DE} = -4 \qquad \omega_{DE} = 5 \text{ rad/s} \circlearrowright$

From Eq. (4), $\omega_{BD} = \dfrac{1}{6}(12 - 3\omega_{DE}) = 5 \qquad \omega_{BD} = 4 \text{ rad/s} \circlearrowright$

PROBLEM 15.135 (Continued)

Acceleration Analysis: $\boldsymbol{\alpha}_{AB} = 0$

Bar AB:

$$\mathbf{a}_B = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A}$$
$$= 0 - (4)^2 (3\mathbf{i} + \sqrt{135}\mathbf{j})$$
$$= -48\mathbf{i} - 16\sqrt{135}\mathbf{j}$$

Object BDF:

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}$$

$$\mathbf{a}_D = -48\mathbf{i} - 16\sqrt{135}\mathbf{j} + \alpha_{BD}\mathbf{k} \times (6\mathbf{i}) - (4)^2 (6\mathbf{i})$$
$$= -144\mathbf{i} - 16\sqrt{135}\mathbf{j} + 6\alpha_{BD}\mathbf{j} \qquad (5)$$

Bar DE:

$$\mathbf{a}_D = \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 \mathbf{r}_{D/E}$$

$$\mathbf{a}_D = \alpha_{DE}\mathbf{k} \times (-3\mathbf{i} + \sqrt{135}\mathbf{j}) - (4)^2 (-3\mathbf{i} + \sqrt{135}\mathbf{j})$$
$$= -\sqrt{135}\alpha_{DE}\mathbf{i} - 3\alpha_{DE}\mathbf{j} + 48\mathbf{i} - 16\sqrt{135}\mathbf{j} \qquad (6)$$

Equating like components of $\mathbf{a}_D$ from Eqs. (5) and (6),

$$\mathbf{i}: \quad -144 = -\sqrt{135}\alpha_{DE} + 48 \qquad (7)$$

$$\mathbf{j}: \quad -16\sqrt{135} + 6\alpha_{BD} = -3\alpha_{DE} - 16\sqrt{135} \qquad (8)$$

From Eq. (7), $\alpha_{DE} = \dfrac{192}{\sqrt{135}}$

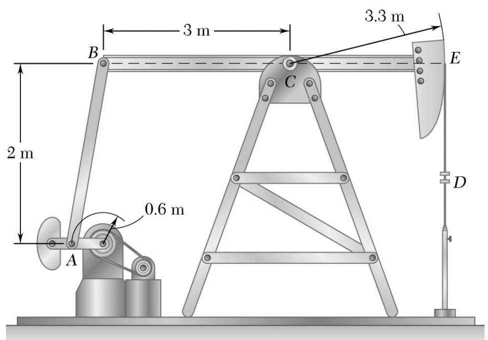
From Eq. (8), $\alpha_{BD} = -\dfrac{1}{2}\alpha_{BD} = -\dfrac{96}{\sqrt{135}}$

(*a*) Angular acceleration of bar DE: $\boldsymbol{\alpha}_{DE} = 16.53 \text{ rad/s}^2$ ↻ ◀

(*b*) Acceleration of Point F:

$$\mathbf{a}_F = \mathbf{a}_B + \mathbf{a}_{F/B} = \mathbf{a}_B + \alpha_{BD} \times \mathbf{r}_{F/B} - \omega_{BD}^2 \mathbf{r}_{F/B}$$
$$= -48\mathbf{i} - 16\sqrt{135}\mathbf{j} + \left(-\frac{96}{\sqrt{135}}\mathbf{k}\right) \times (3\mathbf{i} - \sqrt{135}\mathbf{j}) - (4)^2 (3\mathbf{i} - \sqrt{135}\mathbf{j})$$
$$= -48\mathbf{i} - 16\sqrt{135}\mathbf{j} - \frac{288}{\sqrt{135}}\mathbf{j} - 96\mathbf{i} - 48\mathbf{i} + 16\sqrt{135}\mathbf{j}$$
$$= -192\mathbf{i} - \frac{288}{\sqrt{135}}\mathbf{j}$$

$\mathbf{a}_F = -(192.0 \text{ in./s}^2)\mathbf{i} - (24.8 \text{ in./s}^2)\mathbf{j} = 193.6 \text{ in./s}^2 \measuredangle\, 7.36°$ ◀



PROBLEM 15.136

For the oil pump rig shown, link AB causes the beam BCE to oscillate as the crank OA revolves. Knowing that OA has a radius of 0.6 m and a constant clockwise angular velocity of 20 rpm, determine the velocity and acceleration of Point D at the instant shown.

SOLUTION

Units: meters, m/s, m/s^2

Unit vectors: $\quad \mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft.$

Crank OA: $\quad \mathbf{r}_{OA} = 0.6 \text{ m}, \quad \boldsymbol{\omega}_{OA} = 20 \text{ rpm} \circlearrowright = 2.0944 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_A = \omega_{OA} r_{OA} = (2.0944)(0.6) \qquad \mathbf{v}_A = 1.25664 \text{ m/s} \uparrow$$

$$\boldsymbol{\alpha}_{OA} = 0 \qquad (a_A)_t = 0$$

$$(\mathbf{a}_A)_n = \omega_{OA}^2 r_{OA} = (2.0944)^2(0.6) = 2.6319 \text{ m/s}^2$$

$$\mathbf{a}_A = 2.6319 \text{ m/s}^2 \longrightarrow$$

Rod AB: $\quad \mathbf{v}_B = v_A \uparrow,$

Since $\mathbf{v}_B$ and $\mathbf{v}_A$ are parallel, $\mathbf{v}_A = \mathbf{v}_B$ and $\omega_{AB} = 0.$

$$\mathbf{v}_B = 1.25664 \text{ m/s} \uparrow$$

$$\begin{aligned} \mathbf{a}_B &= \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_A + \alpha_{AB}\mathbf{k} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\ &= 2.6319\mathbf{i} + \alpha_{AB}\mathbf{k} \times (0.6\mathbf{i} + 2\mathbf{j}) - 0 \\ &= (2.6319 - 2\alpha_{AB})\mathbf{i} + 0.6\alpha_{AB}\mathbf{j} \end{aligned} \qquad (1)$$

Beam BCE: Point C is a pivot.

$$v_B = \omega_{BCE} r_{BC} \qquad \omega_{BCE} = \frac{v_B}{r_{BC}} = \frac{1.25664}{3} = 0.41888$$

$$v_E = \omega_{BCE} r_{CE} = (0.41888)(3.3) = 1.38230$$

$$\boldsymbol{\omega}_{BCE} = 0.41888 \text{ rad/s} \circlearrowright \qquad \mathbf{v}_E = 1.38230 \text{ m/s} \downarrow$$

$$\begin{aligned} \mathbf{a}_B &= \boldsymbol{\alpha}_{BCE} \times \mathbf{r}_{B/C} - \omega_{BCE}^2 r_{B/C} \\ &= \alpha_{BCE}\mathbf{k} \times (-3\mathbf{i}) - (0.41888)^2(-3\mathbf{i}) \\ &= 0.52638\mathbf{i} - 3\alpha_{BCE}\mathbf{j} \end{aligned} \qquad (2)$$

PROBLEM 15.136 (Continued)

Equating like components of $\boldsymbol{\alpha}_B$ expressed by Eqs. (1) and (2),

i: $2.6319 - 2\alpha_{AB} = 0.52638$ $\qquad \alpha_{AB} = 1.05276 \text{ rad/s}$

j: $0.6\alpha_{AB} = -3\alpha_{BCE}$ $\qquad \alpha_{BCE} = -0.21055 \text{ rad/s}^2$

$$\boldsymbol{\alpha}_{AB} = 1.055276 \text{ rad/s} \qquad \boldsymbol{\alpha}_{BCE} = 0.21055 \text{ rad/s}^2$$

$$\mathbf{a}_E = (\mathbf{a}_{E/C})_t + (\mathbf{a}_{E/C})_n$$

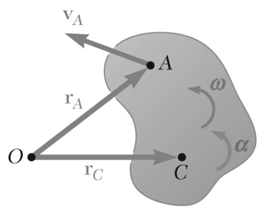
$$(\mathbf{a}_{E/C})_t = \boldsymbol{\alpha}_{BCE} \times \mathbf{r}_{E/C} = (-0.21055\mathbf{k}) \times (3.3\mathbf{i})$$

$$= -(0.69482 \text{ m/s}^2)\mathbf{j}$$

String *ED*: $\mathbf{v}_D = \mathbf{v}_E$ $\qquad \mathbf{v}_D = 1.382 \text{ m/s} \downarrow$ ◀

$\mathbf{a}_D = (\mathbf{a}_{E/C})_t = -(0.69482 \text{ m/s}^2)\mathbf{j}$ $\qquad \mathbf{a}_D = 0.695 \text{ m/s}^2 \downarrow$ ◀

PROBLEM 15.137



Denoting by $\mathbf{r}_A$ the position vector of Point A of a rigid slab that is in plane motion, show that (a) the position vector $\mathbf{r}_C$ of the instantaneous center of rotation is

$$\mathbf{r}_C = \mathbf{r}_A + \frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2}$$

where $\boldsymbol{\omega}$ is the angular velocity of the slab and $\mathbf{v}_A$ is the velocity of Point A, (b) the acceleration of the instantaneous center of rotation is zero if, and only if,

$$\mathbf{a}_A = \frac{\alpha}{\omega}\mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{v}_A$$

where $\boldsymbol{\alpha} = \alpha\mathbf{k}$ is the angular acceleration of the slab.

SOLUTION

(a) At the instantaneous center C, $\mathbf{v}_C = 0$

$$\mathbf{v}_A = \mathbf{v}_C + \boldsymbol{\omega} \times \mathbf{r}_{A/C} = \boldsymbol{\omega} \times \mathbf{r}_{A/C}$$

$$\boldsymbol{\omega} \times \mathbf{v}_A = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/C}) = -\omega^2 \mathbf{r}_{A/C}$$

$$\mathbf{r}_{A/C} = -\frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2} = -\mathbf{r}_{C/A} \quad \text{or} \quad \mathbf{r}_{C/A} = \frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2}$$

$$\mathbf{r}_C - \mathbf{r}_A = -\frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2}$$

$$\mathbf{r}_C = \mathbf{r}_A + \frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2} \blacktriangleleft$$

(b)

$$\mathbf{a}_A = \mathbf{a}_C + \boldsymbol{\alpha} \times \mathbf{r}_{A/C} + \boldsymbol{\omega} \times \mathbf{v}_{A/C}$$

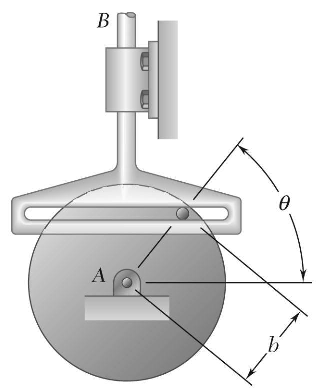
$$= \mathbf{a}_C - \alpha\mathbf{k} \times \frac{\boldsymbol{\omega} \times \mathbf{v}_A}{\omega^2} + \boldsymbol{\omega} \times (\mathbf{v}_A - \mathbf{v}_C)$$

$$= \mathbf{a}_C - \frac{\alpha\omega}{\omega^2}\mathbf{k} \times (\mathbf{k} \times \mathbf{v}_A) + \boldsymbol{\omega} \times \mathbf{v}_A$$

$$= \mathbf{a}_C + \frac{\alpha}{\omega}\mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{v}_A$$

Set $\mathbf{a}_C = 0$.

$$\mathbf{a}_A = \frac{\alpha}{\omega}\mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{v}_A \blacktriangleleft$$

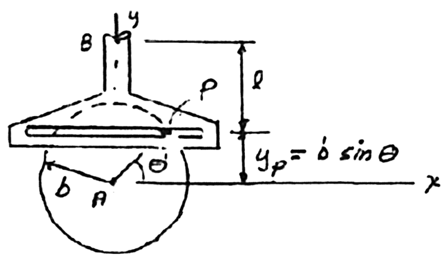


PROBLEM 15.138*

The drive disk of the scotch crosshead mechanism shown has an angular velocity ω and an angular acceleration α, both directed counterclockwise. Using the method of Section 15.9, derive expressions for the velocity and acceleration of Point B.

SOLUTION

Origin at A.



$$y_B = l + y_P = l + b\sin\theta$$

$$v_B = \dot{y}_B = b\cos\theta\dot{\theta} = b\cos\theta\,\omega \qquad v_B = b\omega\cos\theta \blacktriangleleft$$

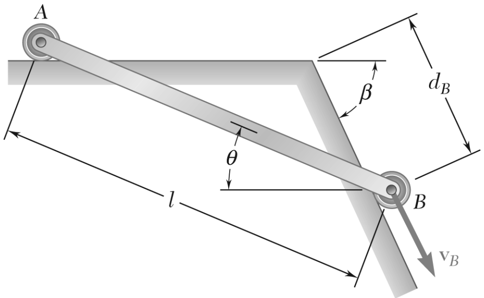
$$a_B = \ddot{y}_B$$

$$= \frac{d}{dt}v_B$$

$$= \frac{d}{dt}(b\cos\theta\dot{\theta})$$

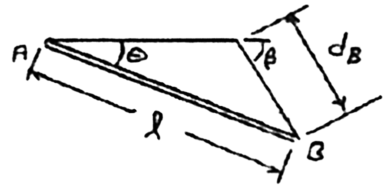
$$a_B = -b\sin\theta\dot{\theta}^2 + b\cos\theta\ddot{\theta} \qquad a_B = b\alpha\cos\theta - b\omega^2\sin\theta \blacktriangleleft$$

PROBLEM 15.139*



The wheels attached to the ends of rod AB roll along the surfaces shown. Using the method of Section 15.9, derive an expression for the angular velocity of the rod in terms of v_B, θ, l, and β.

SOLUTION



Law of sines.

$$\frac{d_B}{\sin\theta} = \frac{l}{\sin\beta}$$

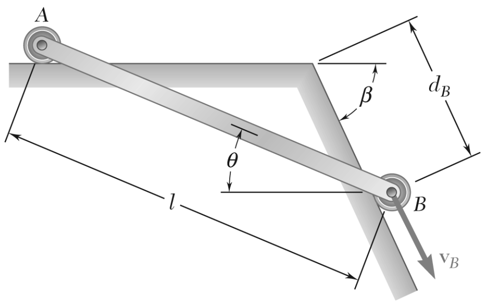
$$d_B = \frac{l}{\sin\beta}\sin\theta$$

$$v_B = \frac{d}{dt}(d_B)$$

$$= \frac{l}{\sin\beta}\cos\theta\frac{d\theta}{dt}$$

$$= \frac{l}{\sin\beta}\cos\theta\,\omega$$

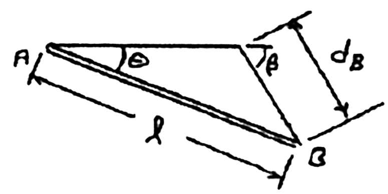
$$\omega = \frac{v_B\sin\beta}{l\cos\theta} \blacktriangleleft$$



PROBLEM 15.140*

The wheels attached to the ends of rod AB roll along the surfaces shown. Using the method of Section 15.9 and knowing that the acceleration of wheel B is zero, derive an expression for the angular acceleration of the rod in terms of v_B, θ, l, and β.

SOLUTION



Law of sines.

$$\frac{d_B}{\sin\theta} = \frac{l}{\sin\beta}$$

$$d_B = \frac{l}{\sin\beta}\sin\theta$$

$$v_B = \frac{d}{dt}(d_B)$$

$$= \frac{l}{\sin\beta}\cos\theta\frac{d\theta}{dt}$$

$$= \frac{l}{\sin\beta}\cos\theta\,\omega$$

$$\omega = \frac{v_B\sin\beta}{l\cos\theta}$$

Note that

$$a_B = \frac{dv_B}{dt} = 0.$$

$$\alpha = \frac{d\omega}{dt} = \frac{v_B\sin\beta}{l}\cdot\frac{\sin\theta}{\cos^2\theta}\cdot\frac{d\theta}{dt}$$

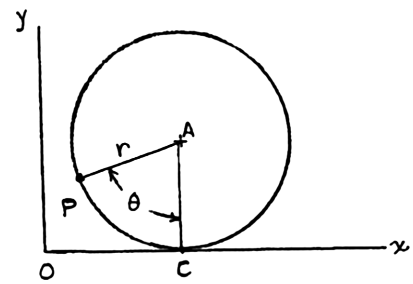
$$\alpha = \frac{v_B\sin\beta\sin\theta}{l\cos^2\theta}\cdot\frac{v_B\sin\beta}{l\cos\theta}$$

$$\alpha = \left[\frac{v_B\sin\beta}{l}\right]^2\frac{\sin\theta}{\cos^3\theta} \blacktriangleleft$$

PROBLEM 15.141*

A disk of radius r rolls to the right with a constant velocity $\mathbf{v}$. Denoting by P the point of the rim in contact with the ground at $t = 0$, derive expressions for the horizontal and vertical components of the velocity of P at any time t.

SOLUTION



$$x_A = r\theta, \quad y_A = r$$

$$x_P = x_A - r\sin\theta$$

$$= r\theta - r\sin\theta$$

$$y_P = y_A - r\cos\theta$$

$$= r - r\cos\theta$$

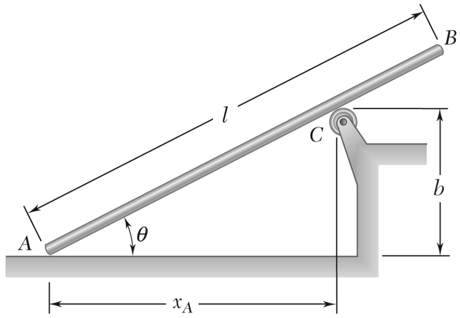
$$\theta = \frac{x_A}{r}$$

$$\dot{x}_A = v, \quad \dot{y}_A = 0, \quad \dot{\theta} = \frac{v}{r}$$

$$x_A = vt, \quad \theta = \frac{vt}{r}$$

$$\dot{x}_P = v_x = r\dot{\theta} - r\cos\theta\dot{\theta} = r\left(1 - \cos\frac{vt}{r}\right)\frac{v}{r} \qquad v_x = v\left(1 - \cos\frac{vt}{r}\right) \blacktriangleleft$$

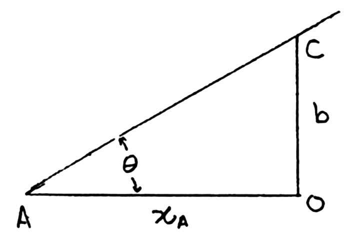
$$\dot{y}_P = v_y = r\sin\theta\dot{\theta} = r\left(\sin\frac{vt}{r}\right)\frac{v}{r} \qquad v_y = v\sin\frac{vt}{r} \blacktriangleleft$$



PROBLEM 15.142*

Rod AB moves over a small wheel at C while end A moves to the right with a constant velocity $\mathbf{v}_A$. Using the method of Section 15.9, derive expressions for the angular velocity and angular acceleration of the rod.

SOLUTION



$$\tan\theta = \frac{b}{x_A} \qquad \cot\theta = \frac{x_A}{b} = u$$

$$\theta = \cot^{-1} u$$

$$\dot{\theta} = -\frac{\dot{u}}{1+u^2}$$

$$\ddot{\theta} = \frac{(2u\dot{u})\dot{u}}{(1+u^2)^2} - \frac{\ddot{u}}{1+u^2}$$

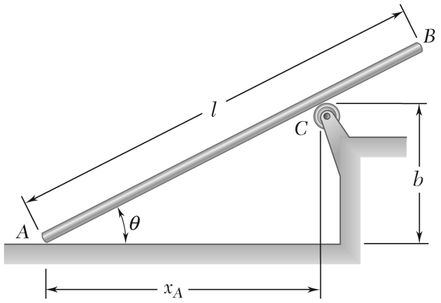
But $\omega = \dot{\theta}$ and $\alpha = \ddot{\theta}$

$$u = \frac{x_A}{b}, \quad \dot{u} = \frac{\dot{x}_A}{b} = -\frac{v_A}{b}, \quad \ddot{u} = -\frac{\dot{v}_A}{b} = 0$$

Then

$$\omega = \frac{\frac{v_A}{b}}{1+\left(\frac{x_A}{b}\right)^2} = \frac{bv_A}{b^2+x_A^2}, \qquad \omega = \frac{bv_A}{b^2+x_A^2} \circlearrowright \blacktriangleleft$$

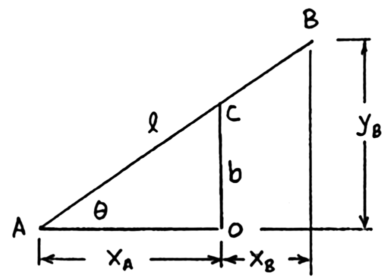
$$\alpha = \frac{2\left(\frac{x_A}{b}\right)\left(\frac{v_A}{b}\right)^2}{\left[1+\left(\frac{x_A}{b}\right)^2\right]^2} - 0 = \frac{2bx_Av_A^2}{\left(b^2+x_A^2\right)^2}, \qquad \alpha = \frac{2bx_Av_A^2}{\left(b^2+x_A^2\right)^2} \circlearrowright \blacktriangleleft$$



PROBLEM 15.143*

Rod AB moves over a small wheel at C while end A moves to the right with a constant velocity $\mathbf{v}_A$. Using the method of Section 15.9, derive expressions for the horizontal and vertical components of the velocity of Point B.

SOLUTION



$$\sin\theta = \frac{b}{(b^2 + x_A^2)^{1/2}}, \qquad \cos\theta = \frac{x_A}{(b^2 + x_A^2)^{1/2}}$$

$$x_B = l\cos\theta - x_A$$

$$= \frac{l x_A}{(b^2 + x_A^2)^{1/2}} - x_A$$

$$y_B = l\sin\theta = \frac{lb}{(b^2 + x_A^2)^{1/2}}$$

$$\dot{x}_B = \frac{l\dot{x}_A}{(b^2 + x_A^2)^{1/2}} - \frac{l x_A x_A \dot{x}_A}{(b^2 + x_A^2)^{3/2}} - \dot{x}_A$$

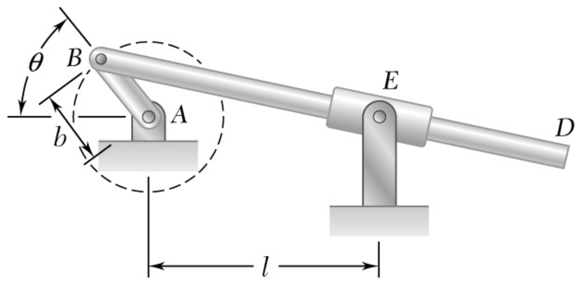
$$= \frac{lb^2 \dot{x}_A}{(b^2 + x_A^2)^{3/2}} - \dot{x}_A$$

$$\dot{y}_B = -\frac{lb x_A \dot{x}_A}{(b^2 + x_A^2)^{3/2}}$$

But $\qquad \dot{x}_A = -v_A, \qquad \dot{x}_B = (v_B)_x, \qquad \dot{y}_B = (v_B)_y$

$$(v_B)_x = v_A - \frac{lb^2 v_A}{(b^2 + x_A^2)^{3/2}} \longrightarrow \blacktriangleleft$$

$$(v_B)_y = \frac{lb x_A v_A}{(b^2 + x_A^2)^{3/2}} \uparrow \blacktriangleleft$$

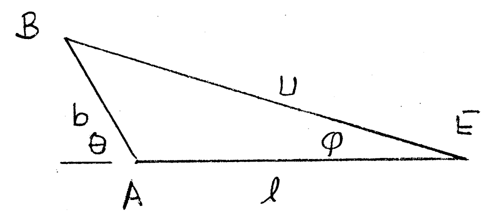


PROBLEM 15.144

Crank AB rotates with a constant clockwise angular velocity $\boldsymbol{\omega}$. Using the method of Section 15.9, derive expressions for the angular velocity of rod BD and the velocity of the point on the rod coinciding with Point E in terms of θ, ω, b, and l.

SOLUTION

Law of cosines for triangle ABE.



$$u^2 = l^2 + b^2 - 2bl\cos(180° - \theta)$$

$$= l^2 + b^2 + 2bl\cos\theta$$

$$\cos\varphi = \frac{l + b\cos\theta}{u}$$

$$\tan\varphi = \frac{b\sin\theta}{l + b\cos\theta}$$

$$\frac{d}{dt}(\tan\varphi) = \sec^2\varphi\dot{\varphi} = \frac{(l + b\cos\theta)(b\cos\theta)\dot{\theta} + (b\sin\theta)(b\cos\theta)\dot{\theta}}{(l + b\cos\theta)^2}$$

$$\dot{\varphi} = \frac{(\cos^2\varphi)[bl\cos\theta + b^2(\cos^2\theta + \sin^2\theta)]\dot{\theta}}{(l + b\cos\theta)^2}$$

$$= \frac{bl\cos\theta + b^2}{u^2}\dot{\theta} = \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\dot{\theta}$$

But, $\dot{\theta} = \omega,\quad \dot{\varphi} = \omega_{BD},\quad$ and $\quad v_E = -\dot{u}$

Hence,

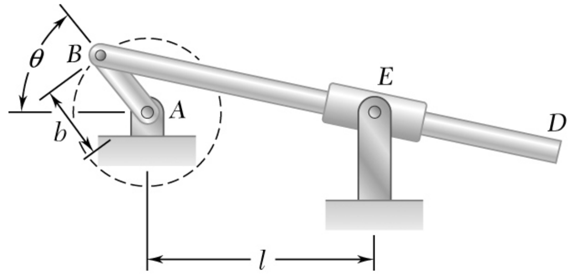
$$\omega_{BD} = \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\omega \circlearrowright \blacktriangleleft$$

Differentiate the expression for u^2.

$$2u\dot{u} = -2bl\sin\theta\dot{\theta}$$

$$v_E = -\dot{u} = \frac{bl\sin\theta}{l^2 + b^2 + 2bl\cos\theta}\omega$$

$$\mathbf{v}_E = \frac{bl\sin\theta}{l^2 + b^2 + 2bl\cos\theta}\omega \searrow \tan^{-1}\left(\frac{b\sin\theta}{l + b\cos\theta}\right) \blacktriangleleft$$

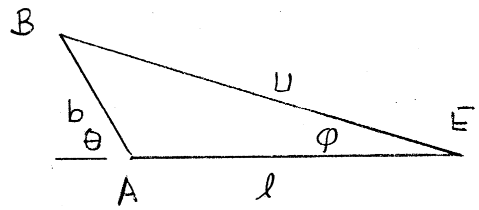


PROBLEM 15.145

Crank AB rotates with a constant clockwise angular velocity $\boldsymbol{\omega}$. Using the method of Section 15.9, derive an expression for the angular acceleration of rod BD in terms of θ, ω, b, and l.

SOLUTION

Law of cosines for triangle ABE.



$$u^2 = l^2 + b^2 - 2bl\cos(180° - \theta)$$
$$= l^2 + b^2 + 2bl\cos\theta$$
$$\cos\varphi = \frac{l + b\cos\theta}{u}$$
$$\tan\varphi = \frac{b\sin\theta}{l + b\cos\theta}$$

$$\frac{d}{dt}(\tan\varphi) = \sec^2\varphi\dot{\varphi} = \frac{(l + b\cos\theta)(b\cos\theta)\dot{\theta} + (b\sin\theta)(b\cos\theta)\dot{\theta}}{(l + b\cos\theta)^2}$$

$$\dot{\varphi} = \frac{(\cos^2\varphi)[bl\cos\theta + b^2(\cos^2\theta + \sin^2\theta)]\dot{\theta}}{(l + b\cos\theta)^2}$$

$$= \frac{bl\cos\theta + b^2}{u^2}\dot{\theta} = \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\dot{\theta}$$

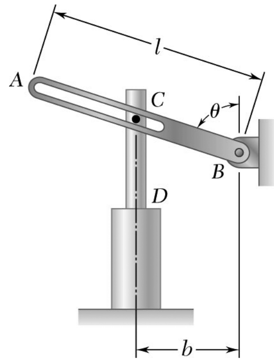
$$\ddot{\varphi} = \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\ddot{\theta} + \frac{(l^2 + b^2 + 2bl\cos\theta)(-bl\sin\theta) - b(b + l\cos\theta)(-2bl\sin\theta)}{(l^2 + b^2 + 2bl\cos\theta)^2}\dot{\theta}^2$$

$$= \frac{b(b + l\cos\theta)}{l^2 + b^2 + 2bl\cos\theta}\ddot{\theta} - \frac{bl(l^2 - b^2)\sin\theta}{(l^2 + b^2 + 2bl\cos\theta)^2}\dot{\theta}^2$$

But, $\dot{\theta} = \omega,\quad \ddot{\theta} = \dot{\omega} = 0,\quad \ddot{\varphi} = \alpha_{BD}$

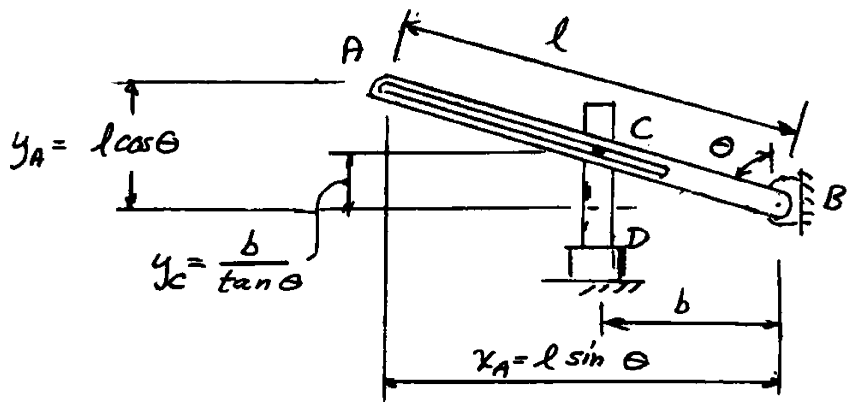
$$\alpha_{BD} = \frac{bl(l^2 - b^2)\sin\theta}{l^2 + b^2 + 2bl\cos\theta}\omega^2 \circlearrowleft \blacktriangleleft$$

PROBLEM 15.146*



Pin C is attached to rod CD and slides in a slot cut in arm AB. Knowing that rod CD moves vertically upward with a constant velocity $\mathbf{v}_0$, derive an expression for (a) the angular velocity of arm AB, (b) the components of the velocity of Point A; and (c) an expression for the angular acceleration of arm AB.

SOLUTION



(a)

$$y_C = \frac{b\cos\theta}{\sin\theta}$$

$$v_C = v_0 = \frac{dy_C}{dt} = b\frac{d}{d\theta}\left(\frac{\cos\theta}{\sin\theta}\right)\frac{d\theta}{dt}$$

$$= b\frac{-\sin^2\theta - \cos^2\theta}{\sin^2\theta}\dot{\theta} = -\frac{b}{\sin^2\theta}\dot{\theta}$$

$$\dot{\theta} = -\frac{v_0\sin^2\theta}{b} \qquad (1)$$

But $\omega = \dot{\theta}$

$$\boldsymbol{\omega} = \frac{v_0}{b}\sin^2\theta \circlearrowright \blacktriangleleft$$

(b)

$$x_A = l\sin\theta$$

$$\dot{x}_A = l\cos\theta\dot{\theta} = -\frac{v_0 l}{b}(\sin^2\theta\cos\theta)$$

$$y_A = l\cos\theta$$

$$\dot{y}_A = -l\sin\theta\dot{\theta} = \frac{v_0 l}{b}\sin^3\theta \uparrow$$

PROBLEM 15.146* (Continued)

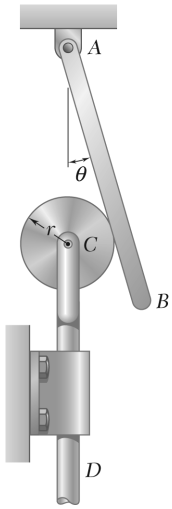
Components: $$\mathbf{v}_A = \frac{v_0 l}{b}\sin^2\theta\cos\theta \longrightarrow + \frac{v_0 l}{b}\sin^3\theta \uparrow \blacktriangleleft$$

(*c*) Differentiating Eq. (1),

$$\ddot{\theta} = \frac{d}{dt}\left(-\frac{v_0\sin^2\theta}{b}\right) = -\frac{2v_0\sin\theta\cos\theta}{b}\frac{d\theta}{dt}$$

$$= -\left(\frac{2v_0\sin\theta\cos\theta}{b}\right)\left(-\frac{v_0\sin^2\theta}{b}\right) = \frac{2v_0^2}{b^2}\sin^3\theta\cos\theta$$

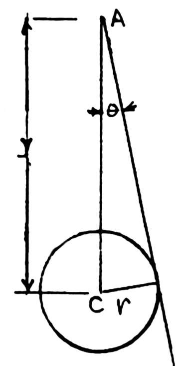
$$\boldsymbol{\alpha} = \ddot{\theta} \circlearrowleft \qquad \boldsymbol{\alpha} = \frac{2v_0^2}{b^2}\sin^3\theta\cos\theta \circlearrowleft \blacktriangleleft$$



PROBLEM 15.147*

The position of rod AB is controlled by a disk of radius r which is attached to yoke CD. Knowing that the yoke moves vertically upward with a constant velocity $\mathbf{v}_0$, derive expression for the angular velocity and angular acceleration of rod AB.

SOLUTION



From geometry,
$$y = \frac{r}{\sin\theta}$$
$$\frac{dy}{dt} = -\frac{r\cos\theta}{\sin^2\theta}\frac{d\theta}{dt}$$

But,
$$\frac{dy}{dt} = -v_0 \quad \text{and} \quad \frac{d\theta}{dt} = \omega$$
$$v_0 = \frac{r\cos\theta}{\sin^2\theta}\omega$$
$$\omega = \frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta}$$

$$\boldsymbol{\omega} = \frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta} \circlearrowleft \blacktriangleleft$$

From geometry,
$$y = \frac{r}{\sin\theta}$$
$$\frac{dy}{dt} = -\frac{r\cos\theta}{\sin^2\theta}\frac{d\theta}{dt}$$

But,
$$\frac{dy}{dt} = -v_0 \quad \text{and} \quad \frac{d\theta}{dt} = \omega$$
$$v_0 = \frac{r\cos\theta}{\sin^2\theta}\omega$$
$$\omega = \frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta}$$

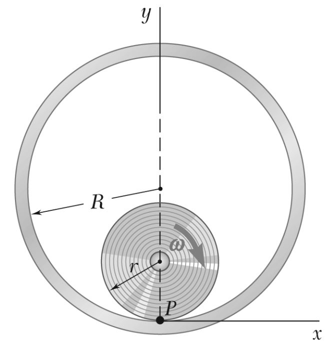
Angular acceleration.

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{d\omega}{d\theta}\omega$$

$$= \frac{v_0}{r}\frac{(2\cos^2\theta\sin\theta + \sin^3\theta)}{\cos^2\theta}\left(\frac{v_0}{r}\frac{\sin^2\theta}{\cos\theta}\right)$$

$$= \left(\frac{v_0}{r}\right)^2 \frac{(1+\cos^2\theta)\sin^3\theta}{\cos^3\theta}$$

$$\boldsymbol{\alpha} = \left(\frac{v_0}{r}\right)^2 (1+\cos^2\theta)\tan^3\theta \circlearrowright \blacktriangleleft$$



PROBLEM 15.148*

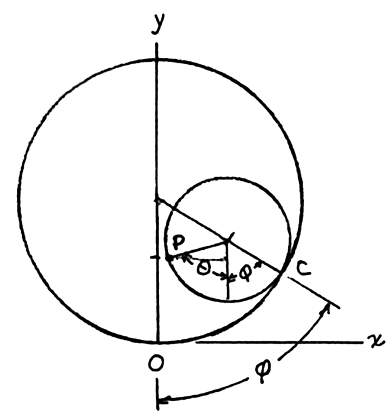
A wheel of radius r rolls without slipping along the inside of a fixed cylinder of radius R with a constant angular velocity $\boldsymbol{\omega}$. Denoting by P the point of the wheel in contact with the cylinder at $t = 0$, derive expressions for the horizontal and vertical components of the velocity of P at any time t. (The curve described by Point P is a *hypocycloid*.)

SOLUTION

Define angles θ and φ as shown.

$$\dot{\theta} = \omega, \qquad \theta = \omega t$$

Since the wheel rolls without slipping, the arc OC is equal to arc PC.



$$r(\varphi + \theta) = R\varphi$$

$$\varphi = \frac{r\theta}{R-r}$$

$$\dot{\varphi} = \frac{r\dot{\theta}}{R-r} = \frac{r\omega}{R-r}$$

$$\varphi = \frac{r\omega t}{R-r}$$

$$x_P = (R-r)\sin\varphi - r\sin\theta$$

$$(v_P)_x = \dot{x}_P$$

$$= (R-r)\cos\varphi\dot{\varphi} - r\cos\theta\dot{\theta}$$

$$= (R-r)\left(\cos\frac{r\omega t}{R-r}\right)\left(\frac{r\omega}{R-r}\right) - r(\cos\omega t)(\omega)$$

$$(v_P)_x = r\omega\left(\cos\frac{r\omega t}{R-r} - \cos\omega t\right) \blacktriangleleft$$

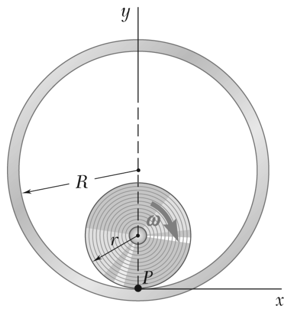
$$y_P = R - (R-r)\cos\varphi - r\cos\theta$$

$$(v_P)_y = \dot{y}_P$$

$$= (R-r)\sin\varphi\dot{\varphi} + r\sin\theta\dot{\theta}$$

$$= (R-r)\left(\sin\frac{r\omega t}{R-r}\right)\left(\frac{r\omega}{R-r}\right) + r(\sin\omega t)(\omega)$$

$$(v_P)_y = r\omega\left(\sin\frac{r\omega t}{R-r} + \sin\omega t\right) \blacktriangleleft$$

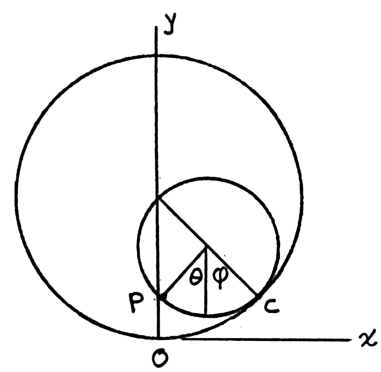


PROBLEM 15.149*

In Problem 15.148, show that the path of P is a vertical straight line when $r = R/2$. Derive expressions for the corresponding velocity and acceleration of P at any time t.

SOLUTION

Define angles θ and φ as shown.



$$\dot{\theta} = \omega, \quad \theta = \omega t, \quad \ddot{\theta} = 0$$

Since the wheel rolls without slipping, the arc OC is equal to arc PC.

$$r(\varphi + \theta) = R\theta$$
$$= 2r\theta$$
$$\varphi = 0$$
$$\dot{\varphi} = \dot{\theta} = \omega$$
$$\ddot{\varphi} = \ddot{\theta} = 0$$

$$x_P = (R - r)\sin\varphi - r\sin\theta$$
$$= r\sin\theta - r\sin\theta$$
$$= 0$$

The path is the y axis. ◀

$$y_P = R - (R - r)\cos\varphi - r\cos\theta$$
$$= R - r\cos\theta - r\cos\theta$$
$$= R(1 - \cos\theta)$$
$$v = \dot{y}_P = R\sin\theta\dot{\theta}$$

$\mathbf{v} = (R\omega\sin\omega t)\mathbf{j}$ ◀

$$a = \dot{v}$$
$$= (R\cos\theta\dot{\theta}^2 - \sin\theta\ddot{\theta})$$
$$= R\omega^2\cos\theta$$

$\mathbf{a} = (R\omega^2\cos\omega t)\mathbf{j}$ ◀

PROBLEM 15.CQ8

A person walks radially inward on a platform that is rotating counterclockwise about its center. Knowing that the platform has a constant angular velocity **ω** and the person walks with a constant speed **u** relative to the platform, what is the direction of the acceleration of the person at the instant shown?

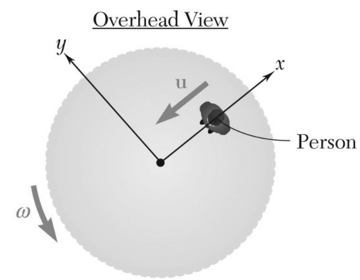
(*a*) Negative *x*

(*b*) Negative *y*

(*c*) Negative *x* and positive *y*

(*d*) Positive *x* and positive *y*

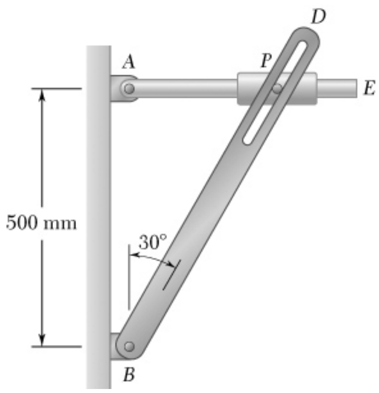
(*e*) Negative *x* and negative *y*



SOLUTION

The $\omega^2 r$ term will be in the negative *x*-direction and the Coriolis acceleration will be in the negative *y*-direction.

Answer: (*e*) ◀



PROBLEM 15.150

Pin P is attached to the collar shown; the motion of the pin is guided by a slot cut in rod BD and by the collar that slides on rod AE. Knowing that at the instant considered the rods rotate clockwise with constant angular velocities, determine for the given data the velocity of pin P.

$\omega_{AE} = 8$ rad/s, $\omega_{BD} = 3$ rad/s

SOLUTION

$$AB = 500 \text{ mm} = 0.5 \text{ m}, \quad AP = 0.5\tan 30°, \quad BP = \frac{0.5}{\cos 30°}$$

$$\boldsymbol{\omega}_{AE} = 8 \text{ rad/s} \circlearrowright, \qquad \boldsymbol{\omega}_{BD} = 3 \text{ rad/s} \circlearrowright$$

Let P' be the coinciding point on AE and u_1 be the outward velocity of the collar along the rod AE.

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AE} = [(AP)\omega_{AE} \downarrow] + [u_1 \longrightarrow]$$

Let P'' be the coinciding point on BD and u_2 be the outward speed along the slot in rod BD.

$$\mathbf{v}_P = \mathbf{v}_{P''} + \mathbf{v}_{P/BD} = [(BP)\omega_{BD} \searrow 30°] + [u_2 \nearrow 60°]$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\xrightarrow{+}: \quad u_1 = \left(\frac{0.5}{\cos 30°}\right)(3)(\cos 30°) + u_2 \cos 60°$$

or

$$u_1 = 1.5 + 0.5u_2 \qquad (1)$$

$$+\uparrow: \quad -(0.5\tan 30°)(8) = -\left(\frac{0.5}{\cos 30°}\right)(3)\sin 30° + u_2 \sin 60°$$

$$u_2 = \frac{1}{\sin 60°}[1.5\tan 30° - 4\tan 30°] = -1.66667 \text{ m/s}$$

From (1),

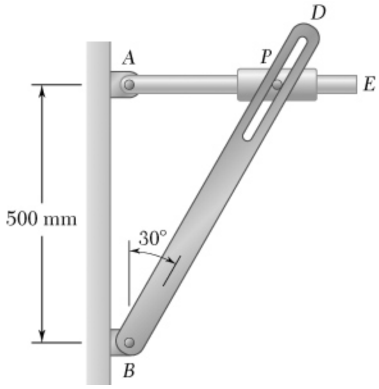
$$u_1 = 1.5 + (0.5)(-1.66667) = 0.66667 \text{ m/s}$$

$$\mathbf{v}_P = [(0.5\tan 30°)(8) \downarrow] + [0.66667 \longrightarrow] = [2.3094 \text{ m/s} \downarrow] + [0.66667 \text{ m/s} \longrightarrow]$$

$$v_P = -\sqrt{2.3094^2 + 0.66667^2} = 2.4037 \text{ m/s}$$

$$\tan\beta = \frac{2.3094}{0.66667} \qquad \beta = 73.9°$$

$$\mathbf{v}_P = 2.40 \text{ m/s} \searrow 73.9° \blacktriangleleft$$



PROBLEM 15.151

Pin P is attached to the collar shown; the motion of the pin is guided by a slot cut in rod BD and by the collar that slides on rod AE. Knowing that at the instant considered the rods rotate clockwise with constant angular velocities, determine for the given data the velocity of pin P.

$\omega_{AE} = 7$ rad/s, $\omega_{BD} = 4.8$ rad/s

SOLUTION

$$AB = 500 \text{ mm} = 0.5 \text{ m}, \quad AP = 0.5\tan 30°, \quad BP = \frac{0.5}{\cos 30°}$$

$$\boldsymbol{\omega}_{AE} = 7 \text{ rad/s} \circlearrowright, \qquad \boldsymbol{\omega}_{BD} = 4.8 \text{ rad/s} \circlearrowright$$

Let P' be the coinciding point on AE and u_1 be the outward velocity of the collar along the rod AE.

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AE} = [(AP)\omega_{AE} \downarrow] + [u_1 \longrightarrow]$$

Let P'' be the coinciding point on BD and u_2 be the outward speed along the slot in rod BD.

$$\mathbf{v}_P = \mathbf{v}_{P''} + \mathbf{v}_{P/BD} = [(BP)\omega_{BD} \nwarrow 30°] + [u_2 \nearrow 60°]$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\xrightarrow{+}: \quad u_1 = \left(\frac{0.5}{\cos 30°}\right)(4.8)(\cos 30°) + u_2 \cos 60°$$

or

$$u_1 = 2.4 + 0.5u_2 \tag{1}$$

$$+\uparrow: \quad -(0.5\tan 30°)(7) = -\left(\frac{0.5}{\cos 30°}\right)(4.8)\sin 30° + u_2 \sin 60°$$

$$u_2 = \frac{1}{\sin 60°}[2.4\tan 30° - 3.5\tan 30°] = -0.73333 \text{ m/s}$$

From (1), $\quad u_1 = 2.4 + (0.5)(-0.73333) = 2.0333$ m/s

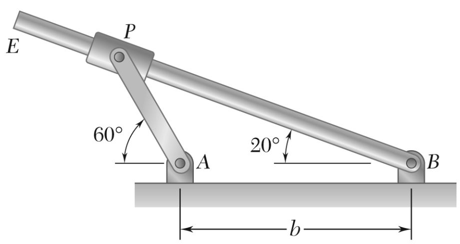
$$\mathbf{v}_P = [(0.5\tan 30°)(7) \downarrow] + [2.0333 \longrightarrow] = [2.0207 \text{ m/s} \downarrow] + [2.0333 \text{ m/s} \longrightarrow]$$

$$v_P = \sqrt{(2.0333)^2 + (2.0207)^2} = 2.87 \text{ m/s}$$

$$\tan\beta = -\frac{2.0207}{2.0333}, \qquad \beta = -44.8°$$

$$\mathbf{v}_P = 2.87 \text{ m/s} \searrow 44.8° \blacktriangleleft$$

PROBLEM 15.152



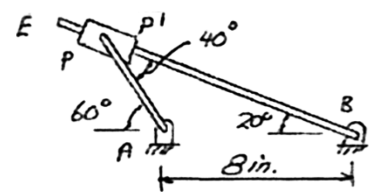
Two rotating rods are connected by slider block P. The rod attached at A rotates with a constant angular velocity ω_A. For the given data, determine for the position shown (a) the angular velocity of the rod attached at B, (b) the relative velocity of slider block P with respect to the rod on which it slides.

$b = 8$ in., $\quad \omega_A = 6$ rad/s.

SOLUTION

Dimensions:

Law of sines.



$$\frac{AP}{\sin 20°} = \frac{BP}{\sin 120°} = \frac{8 \text{ in.}}{\sin 40°}$$

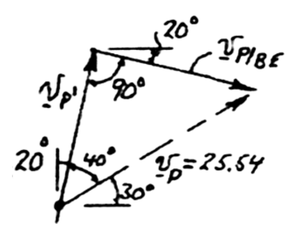
$$AP = 4.2567 \text{ in.}$$

$$BP = 10.7784 \text{ in.}$$

$$\boldsymbol{\omega}_{AP} = 6 \text{ rad/s} \circlearrowright$$

Velocities.

Note: $P' =$ Point of BE coinciding with P.



$$v_P = (AP)\omega_{AP}$$

$$= (4.2567 \text{ in.})(6 \text{ rad/s})$$

$$= 25.540 \text{ in./s} \measuredangle 30°$$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/BE}$$

$$[25.540 \measuredangle 30°] = [v_{P'} \measuredangle 70°] + [v_{P/BE} \searrow 30°]$$

(a)

$$v_{P'} = (25.54)\cos 40°$$

$$= 19.565 \text{ in./s}$$

$$\omega_{BE} = \frac{v_{P'}}{BP}$$

$$= \frac{19.565 \text{ in./s}}{10.7784 \text{ in.}}$$

$$= 1.8152 \text{ rad/s}$$

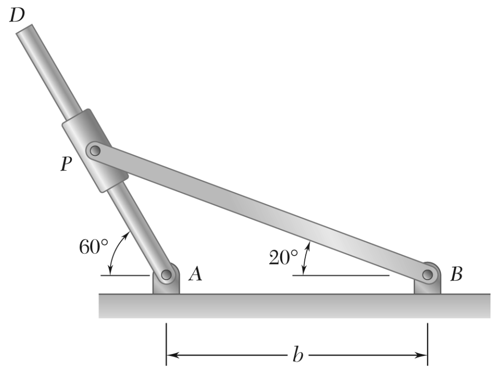
$\boldsymbol{\omega}_{BE} = 1.815 \text{ rad/s} \circlearrowright$ ◀

(b)

$$v_{P/BE} = (25.54)\sin 40°$$

$$= 16.417 \text{ in./s}$$

$\mathbf{v}_{P/BE} = 16.42 \text{ in./s} \searrow 20°$ ◀



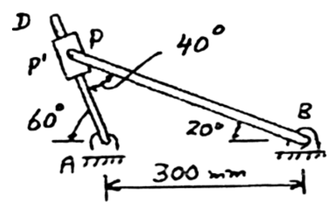
PROBLEM 15.153

Two rotating rods are connected by slider block P. The rod attached at A rotates with a constant angular velocity ω_A. For the given data, determine for the position shown (*a*) the angular velocity of the rod attached at B, (*b*) the relative velocity of slider block P with respect to the rod on which it slides.

$b = 300$ mm, $\omega_A = 10$ rad/s.

SOLUTION

Dimensions:



Law of sines.

$$\frac{AP}{\sin 20°} = \frac{BP}{\sin 120°} = \frac{300 \text{ mm}}{\sin 40°}$$

$$AP = 159.63 \text{ mm}$$

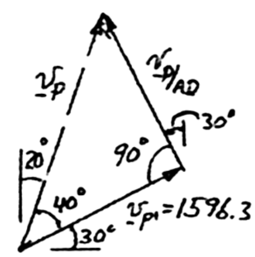
$$BP = 404.19 \text{ mm}$$

$$\boldsymbol{\omega}_{AD} = 10 \text{ rad/s}$$

Velocities.

Note: $P' =$ Point of AD coinciding with P.

$$\mathbf{v}_{P'} = (AP')\omega_{AD}$$
$$= (159.63 \text{ mm})(10 \text{ rad/s})$$
$$= 1596.3 \text{ mm/s} \measuredangle 30°$$



$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AD}$$

$$[v_P \measuredangle 70°] = [1596.3 \measuredangle 30°] + [v_{P/AD} \searrow 60°]$$

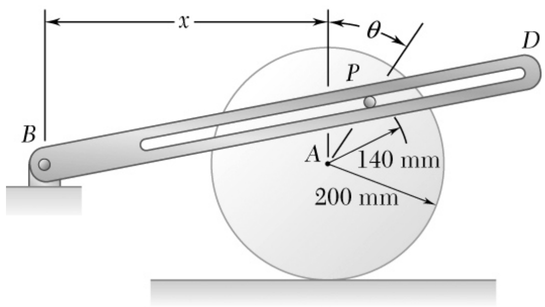
(*a*)
$$v_P = (1596.3)/\cos 40°$$
$$= 2083.8 \text{ mm/s}$$

$$\omega_{BP} = \frac{v_P}{BP}$$
$$= \frac{2083.8 \text{ mm/s}}{404.19 \text{ mm}}$$
$$= 5.155 \text{ rad/s}$$

$\boldsymbol{\omega}_{BD} = 5.16$ rad/s ◀

(*b*)
$$v_{P/AD} = (1596.3)\tan 40° = 1339.5 \text{ mm/s}$$

$\mathbf{v}_{P/AD} = 1.339$ m/s $\searrow 60°$ ◀

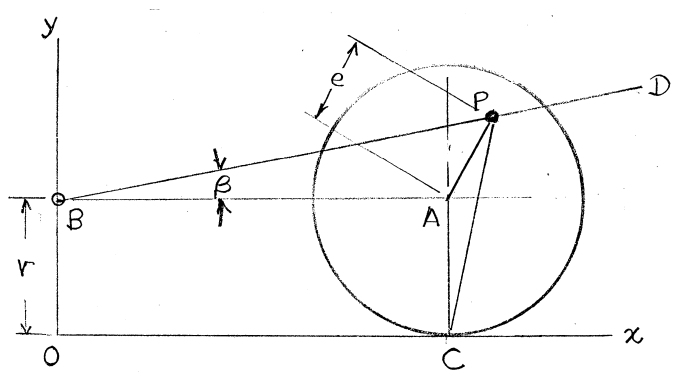


PROBLEM 15.154

Pin P is attached to the wheel shown and slides in a slot cut in bar BD. The wheel rolls to the right without slipping with a constant angular velocity of 20 rad/s. Knowing that $x = 480$ mm when $\theta = 0$, determine the angular velocity of the bar and the relative velocity of pin P with respect to the rod for the given data.

(*a*) $\theta = 0$, (*b*) $\theta = 90°$.

SOLUTION



Coordinates.

$$x_A = (x_A)_0 + r\theta, \qquad y_A = r$$

$$x_B = 0, \qquad y_B = r$$

$$x_C = x_A, \qquad y_C = 0$$

$$x_P = x_A + e\sin\theta$$

$$y_P = r + e\cos\theta$$

Data:

$$(x_A)_0 = 480 \text{ mm} = 0.48 \text{ m}$$

$$r = 200 \text{ mm} = 0.20 \text{ m}$$

$$e = 140 \text{ mm} = 0.14 \text{ m}$$

Velocity analysis.

$$\boldsymbol{\omega}_{AC} = \omega_{AC} \circlearrowright, \qquad \boldsymbol{\omega}_{BD} = \omega_{BD} \circlearrowright,$$

$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{v}_{P/A} = [r\omega_{AC} \longrightarrow] + [e\omega_{AC} \measuredangle \theta]$$

$$\mathbf{v}_{P'} = [x_P\omega_{BD} \downarrow] + [(e\cos\theta)]\omega_{BD} \longrightarrow]$$

$$\mathbf{v}_{P/F} = [u\cos\beta \longrightarrow] + [u\sin\beta \uparrow]$$

Use $\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/F}$ and resolve into components.

$$\xrightarrow{+}: \quad (r + e\cos\theta)\omega_{AC} = (e\cos\theta)\omega_{BD} + (\cos\beta)u \qquad (1)$$

$$+\downarrow: \quad (e\sin\theta)\omega_{AC} = x_P\omega_{BD} - (\sin\beta)u \qquad (2)$$

(*a*) $\theta = 0.$ $\quad x_A = 0.48 \text{ m}, \quad x_P = 0.48 \text{ m}, \quad \omega_{AC} = 20 \text{ rad/s}$

$$\tan\beta = \frac{e\cos\theta}{x_P} = \frac{0.14}{0.48}, \quad \beta = 16.26°$$

Substituting into Eqs. (1) and (2),

$$(0.20 + 0.14)(20) = 0.14\omega_{BD} + (\cos 16.26°)u \qquad (1)$$

$$0 = 0.48\omega_{BD} - (\sin 16.26°)u \qquad (2)$$

Solving simultaneously, $\omega_{BD} = 3.81$ rad/s, $\qquad \boldsymbol{\omega}_{BD} = 3.81 \text{ rad/s} \circlearrowright$ ◀

$u = 6.53$ m/s, $\qquad \mathbf{v}_{P/F} = 6.53 \text{ m/s} \measuredangle 16.26°$ ◀

(*b*) $\theta = 90°.$ $\quad x_P = 0.48 + (0.20)\left(\dfrac{\pi}{2}\right) + 0.14 = 0.93416 \text{ m}$

$$\beta = 0$$

Substituting into Eqs. (1) and (2),

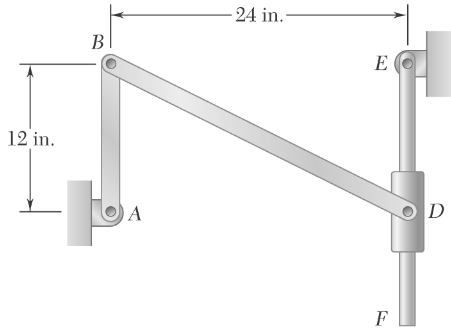
$$(0.20)(20) = u \qquad (1)$$

$$u = 4 \text{ m/s}$$

$$(0.14)(20) = 0.93416\omega_{BD} \qquad (2)$$

$\omega_{BD} = 2.9973$ rad/s, $\qquad \boldsymbol{\omega}_{BD} = 3.00 \text{ rad/s} \circlearrowright$ ◀

$\mathbf{v}_{P/F} = 4.00 \text{ m/s} \longrightarrow$ ◀



PROBLEM 15.155

Bar AB rotates clockwise with a constant angular velocity of 8 rad/s and rod EF rotates clockwise with a constant angular velocity of 6 rad/s. Determine at the instant shown (*a*) the angular velocity of bar BD, (*b*) the relative velocity of collar D with respect to rod EF.

SOLUTION

Bar AB. (Rotation about A) $\omega_{AB} = 8 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_B = (12)(8) = 96 \text{ in./s} \longrightarrow$$

Rod EF. (Rotation about E) $\omega_{EF} = 6 \text{ rad/s.} \circlearrowright$

$$\mathbf{v}_{D'} = (12)(6) = 72 \text{ in./s} \longleftarrow$$

Bar BD. Assume angular velocity is $\omega_{BD} \circlearrowleft$.

Plane motion = Translation with B + Rotation about B.

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = [96 \longrightarrow] + [24\omega_{BD} \uparrow] + [12\omega_{BD} \longrightarrow] \tag{1}$$

Collar D. Sliding on rotating rod EF with relative velocity $u \uparrow$.

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/EF} = [72 \longleftarrow] + [u \uparrow] \tag{2}$$

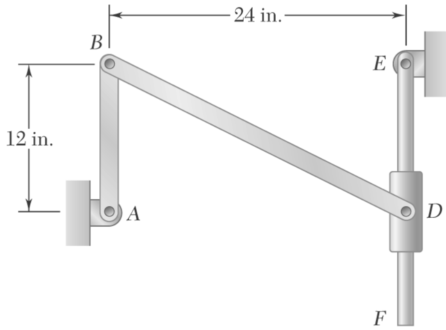
Matching the expressions (1) and (2) for v_D,

Components $\longrightarrow$: $96 + 12\omega_{BD} = -72$ $\omega_{BD} = -14$

(*a*) $\boldsymbol{\omega}_{BD} = 14.00 \text{ rad/s} \circlearrowright$ ◀

Components $\uparrow$: $24\omega_{BD} = u$ $u = (24)(-14) = -336 \text{ in./s}$

(*b*) $\mathbf{v}_{D/EF} = 28.0 \text{ ft/s} \downarrow$ ◀



PROBLEM 15.156

Bar AB rotates clockwise with a constant angular velocity of 4 rad/s. Knowing that the magnitude of the velocity of collar D is 20 ft/s and that the angular velocity of bar BD is counterclockwise at the instant shown, determine (*a*) the angular velocity of bar EF, (*b*) the relative velocity of collar D with respect to rod EF.

SOLUTION

Bar AB. (Rotation about A) $\quad \boldsymbol{\omega}_{AB} = 4 \text{ rad/s} \circlearrowright$

$$\mathbf{v}_B = (1 \text{ ft})(4 \text{ rad/s}) = 4 \text{ ft/s} \longrightarrow$$

Bar BD. Angular velocity is $\omega_{BD} \circlearrowleft$.

Plane motion = Translation with B + Rotation about B.

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = [4 \longrightarrow] + [2\omega_{BD} \uparrow] + [1\omega_{BD} \longrightarrow]$$

Magnitude of $\mathbf{v}_D$: $v_D = 20$ ft/s

$$v_D^2 = (4 + \omega_{BD})^2 + (2\omega_{BD})^2 = (20)^2$$

$$5\omega_{BD}^2 + 8\omega_{BD} - 384 = 0$$

$$\omega_{BD} = \frac{-8 \pm 88}{10} \qquad \text{Positive root} \quad \omega_{BD} = 8 \text{ rad/s} \circlearrowleft$$

$$\mathbf{v}_D = [4 \longrightarrow] + [(2)(8) \uparrow] + [(1)(8) \longrightarrow] = [12 \longrightarrow] + [16 \uparrow] \qquad (1)$$

Rod EF. (Rotation about E) $\quad$ Angular velocity = $\omega_{EF} \circlearrowleft$

$$\mathbf{v}_{D'} = [(1)\omega_{EF} \longrightarrow]$$

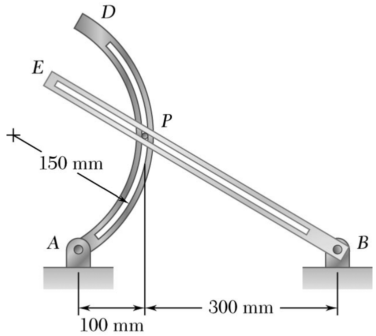
Collar D. Slides on rotating rod EF with relative velocity $u \uparrow$.

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/EF} = [1\omega_{EF} \longrightarrow] + [u \uparrow] \qquad (2)$$

Matching the expressions (1) and (2) for $\mathbf{v}_D$,

(*a*) Component $\longrightarrow$: $\quad 12 = 1\omega_{EF}$ $\qquad \boldsymbol{\omega}_{EF} = 12.00 \text{ rad/s} \circlearrowleft$ ◀

(*b*) Component $\uparrow$: $\quad 16 = u$ $\qquad \mathbf{v}_{D/EF} = 16 \text{ ft/s} \uparrow$ ◀



PROBLEM 15.157

The motion of pin P is guided by slots cut in rods AD and BE. Knowing that bar AD has a constant angular velocity of 4 rad/s clockwise and bar BE has an angular velocity of 5 rad/s counterclockwise and is slowing down at a rate of 2 rad/s^2, determine the velocity of P for the position shown.

SOLUTION

Units: meters, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft.$

Geometry: Slope angle θ of rod BE.

$$\tan\theta = \frac{0.15}{0.3} = 0.5 \qquad \theta = 26.565°$$

$$\mathbf{r}_{P/A} = 0.1\mathbf{i} + 0.15\mathbf{j} \qquad \mathbf{r}_{P/B} = -0.3\mathbf{i} + 0.15\mathbf{j}$$

Angular velocities: $\boldsymbol{\omega}_{AD} = -(4 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\omega}_{BE} = (5 \text{ rad/s})\mathbf{k}$

Angular accelerations: $\boldsymbol{\alpha}_{AD} = 0 \qquad \boldsymbol{\alpha}_{BE} = -(2 \text{ rad/s}^2)\mathbf{k}$

Velocity of Point P' on rod AD coinciding with the pin:

$$\mathbf{v}_{P'} = \boldsymbol{\omega}_{AD} \times \mathbf{r}_{P/A} = (-4\mathbf{k}) \times (0.1\mathbf{i} + 0.15\mathbf{j}) = 0.6\mathbf{i} - 0.4\mathbf{j}$$

Velocity of the pin relative to rod AD:

$$\mathbf{v}_{P/AD} = u_1 \uparrow = u_1\mathbf{j}$$

Velocity of P: $\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/AD} = 0.6\mathbf{i} - 0.4\mathbf{j} + u_1\mathbf{j}$

Velocity of Point P'' on rod BE coinciding with the pin:

$$\mathbf{v}_{P''} = \boldsymbol{\omega}_{BE} \times \mathbf{r}_{P/B} = 5\mathbf{k} \times (-0.3\mathbf{i} + 0.15\mathbf{j}) = -0.75\mathbf{i} - 1.5\mathbf{j}$$

Velocity of the pin relative to rod BE:

$$\mathbf{v}_{P/BE} = u_2 \searrow \theta = -u_2\cos\theta\,\mathbf{i} + u_2\sin\theta\,\mathbf{j}$$

Velocity of P:

$$\mathbf{v}_P = \mathbf{v}_{P''} + \mathbf{v}_{P/BE}$$
$$= -0.75\mathbf{i} - 1.5\mathbf{j} - u_2\cos\theta\,\mathbf{i} + u_2\sin\theta\,\mathbf{j}$$

PROBLEM 15.157 (Continued)

Equating the two expressions for $\mathbf{v}_P$ and resolving into components,

i: $$0.6 = -0.75 - u_2 \cos\theta$$

$$u_2 = -\frac{1.35}{\cos 26.565°} = -1.50965$$

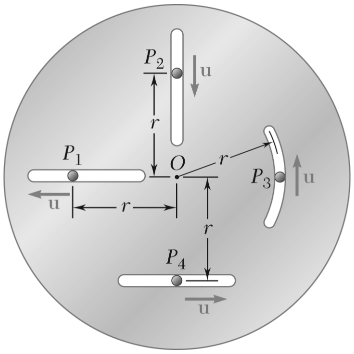
j: $$-0.4 + u_1 = -1.5 + u_2 \sin\theta$$

$$u_1 = -1.1 + (-1.50935)\sin 26.535° = -1.77500$$

Velocity of P:

$$\mathbf{v}_P = 0.6\mathbf{i} - 0.4\mathbf{j} - 1.775\mathbf{j} = 0.6\mathbf{i} - 2.175\mathbf{j}$$

$\mathbf{v}_P = 2.26$ m/s $\measuredangle$ 74.6° ◀



PROBLEM 15.158

Four pins slide in four separate slots cut in a circular plate as shown. When the plate is at rest, each pin has a velocity directed as shown and of the same constant magnitude u. If each pin maintains the same velocity relative to the plate when the plate rotates about O with a constant counterclockwise angular velocity ω, determine the acceleration of each pin.

SOLUTION

For each pin: $$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_C$$

Acceleration of the coinciding Point P′ of the plate.

For each pin $\mathbf{a}_{P'} = r\omega^2$ towards the center O.

Acceleration of the pin relative to the plate.

For pins P_1, P_2 and P_4, $$\mathbf{a}_{P/F} = 0$$

For pin P_3, $$\mathbf{a}_{P/F} = \frac{u^2}{r} \leftarrow$$

Coriolis acceleration $\mathbf{a}_C$.

For each pin $a_C = 2\omega u$ with $\mathbf{a}_C$ in a direction obtained by rotating $\mathbf{u}$ through $90°$ in the sense of $\boldsymbol{\omega}$, i.e., $\circlearrowleft$.

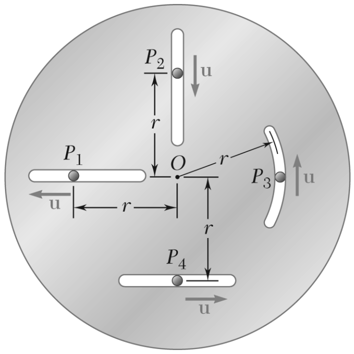
Then

$$\mathbf{a}_1 = [r\omega^2 \rightarrow] + [2\omega u \downarrow] \qquad \mathbf{a}_1 = r\omega^2\mathbf{i} - 2\omega u\mathbf{j} \blacktriangleleft$$

$$\mathbf{a}_2 = [r\omega^2 \downarrow] + [2\omega u \rightarrow] \qquad \mathbf{a}_2 = 2\omega u\mathbf{i} - r\omega^2\mathbf{j} \blacktriangleleft$$

$$\mathbf{a}_3 = [r\omega^2 \leftarrow] + \left[\frac{u^2}{r} \leftarrow\right] + [2\omega u \leftarrow] \qquad \mathbf{a}_3 = -\left(r\omega^2 + \frac{u^2}{r} + 2\omega u\right)\mathbf{i} \blacktriangleleft$$

$$\mathbf{a}_4 = [r\omega^2 \uparrow] + [2\omega u \uparrow] \qquad \mathbf{a}_4 = (r\omega^2 + 2\omega u)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.159

Solve Problem 15.158, assuming that the plate rotates about O with a constant clockwise angular velocity $\boldsymbol{\omega}$.

PROBLEM 15.158 Four pins slide in four separate slots cut in a circular plate as shown. When the plate is at rest, each pin has a velocity directed as shown and of the same constant magnitude u. If each pin maintains the same velocity relative to the plate when the plate rotates about O with a constant counterclockwise angular velocity ω, determine the acceleration of each pin.

SOLUTION

For each pin: $\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_C$

Acceleration of the coinciding Point P′ of the plate.

For each pin $\mathbf{a}_{P'} = r\omega^2$ towards the center O.

Acceleration of the pin relative to the plate.

For pins P_1, P_2 and P_4, $\mathbf{a}_{P/F} = 0$

For pin P_3, $\mathbf{a}_{P/F} = \dfrac{u^2}{r} \leftarrow$

Coriolis acceleration $\mathbf{a}_C$.

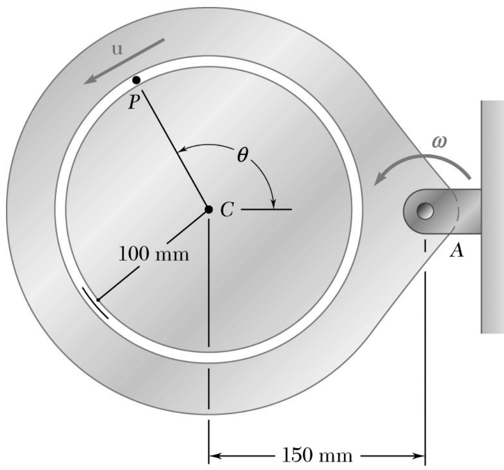
For each pin $a_C = 2\omega u$ with $\mathbf{a}_C$ in a direction obtained by rotating $\mathbf{u}$ through $90°$ in the sense of $\boldsymbol{\omega}$.

Then $\mathbf{a}_1 = [r\omega^2 \rightarrow] + [2\omega u \uparrow]$ $\quad \mathbf{a}_1 = r\omega^2\mathbf{i} + 2\omega u\mathbf{j}$ ◀

$\mathbf{a}_2 = [r\omega^2 \downarrow] + [2\omega u \leftarrow]$ $\quad \mathbf{a}_2 = -2\omega u\mathbf{i} - r\omega^2\mathbf{j}$ ◀

$\mathbf{a}_3 = [r\omega^2 \leftarrow] + \left[\dfrac{u^2}{r} \leftarrow\right] + [2\omega u \rightarrow]$ $\quad \mathbf{a}_3 = \left(2\omega u - r\omega^2 - \dfrac{u^2}{r}\right)\mathbf{i}$ ◀

$\mathbf{a}_4 = [r\omega^2 \uparrow] + [2\omega u \downarrow]$ $\quad \mathbf{a}_4 = (r\omega^2 - 2\omega u)\mathbf{j}$ ◀



PROBLEM 15.160

Pin P slides in the circular slot cut in the plate shown at a constant relative speed u = 500 mm/s. Assuming that at the instant shown the angular velocity of the plate is 6 rad/s and is increasing at the rate of 20 rad/s^2, determine the acceleration of pin P when $\theta = 90°$.

SOLUTION

$\theta = 90°$ Units: meters, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

$$\mathbf{r}_{P/A} = (0.15\mathbf{i} + 0.1\mathbf{j}) \qquad \mathbf{r}_{P/C} = 0.1\mathbf{j}$$

Motion of Point P' on the plate coinciding with P.

$$\boldsymbol{\omega} = (6 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\alpha} = (20 \text{ rad/s}^2)\mathbf{k}$$

$$\mathbf{v}_{P'} = \boldsymbol{\omega} \times \mathbf{r}_{P/A} = 6\mathbf{k} \times (-0.15\mathbf{i} + 0.1\mathbf{j}) = -0.6\mathbf{i} - 0.9\mathbf{j}$$

$$\begin{aligned}\mathbf{a}_{P'} &= \boldsymbol{\alpha} \times r_{P/A} - \omega^2 r_{B/A} \\ &= 20\mathbf{k} \times (-0.15\mathbf{i} + 0.1\mathbf{j}) - (6)^2(-0.15\mathbf{i} + 0.1\mathbf{j}) \\ &= -2\mathbf{i} - 3\mathbf{j} + 5.4\mathbf{i} - 3.6\mathbf{j} = 3.4\mathbf{i} - 6.6\mathbf{j}\end{aligned}$$

Motion of P relative to the plate AC.

$$u = 500 \text{ mm/s} = 0.5 \text{ m/s} \qquad \dot{u} = 0$$

$$\mathbf{v}_{P/AC} = -u\mathbf{i} = -0.5\mathbf{i}$$

$$\mathbf{a}_{P/AC} = -\dot{u}\mathbf{i} - \frac{u}{R}\mathbf{j} = 0 - \frac{(0.5)^2}{0.1}\mathbf{j} = -2.5\mathbf{j}$$

Coriolis acceleration: $2\boldsymbol{\omega} \times \mathbf{v}_{P/AC} = (2)(6\mathbf{k}) \times (-0.5\mathbf{i}) = -6\mathbf{j}$

Acceleration of P.

$$\begin{aligned}\mathbf{a}_P &= \mathbf{a}_{P'} + \mathbf{a}_{P/AC} + 2\boldsymbol{\omega} \times \mathbf{v}_{P/AC} \\ &= 3.4\mathbf{i} - 6.6\mathbf{j} - 2.5\mathbf{j} - 6\mathbf{j} \\ &= (3.4 \text{ m/s}^2)\mathbf{i} - (15.1 \text{ m/s})\mathbf{j}\end{aligned}$$

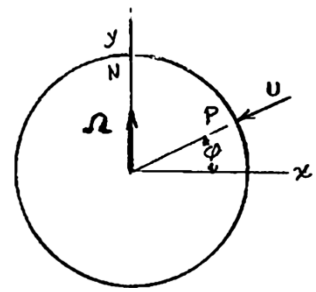
$\mathbf{a}_P = 15.47 \text{ m/s}^2 \measuredangle 77.3°$ ◀

PROBLEM 15.161

The cage of a mine elevator moves downward at a constant speed of 40 ft/s. Determine the magnitude and direction of the Coriolis acceleration of the cage if the elevator is located (*a*) at the equator, (*b*) at latitude 40° north, (*c*) at latitude 40° south.

SOLUTION

Earth makes one revolution $(2\pi \text{ radians})$ in 23.933 h (86,160 s).



$$\boldsymbol{\Omega} = \frac{2\pi}{86{,}160}\mathbf{j}$$
$$= (72.926\times 10^{-6} \text{ rad/s})\mathbf{j}$$

Velocity relative to the Earth at latitude angle φ.

$$\mathbf{v}_{P/\text{earth}} = 40(-\cos\varphi\mathbf{i} - \sin\varphi\mathbf{j})$$

Coriolis acceleration $\mathbf{a}_C$.

$$\mathbf{a}_C = 2\boldsymbol{\Omega}\times\mathbf{v}_{P/\text{earth}}$$
$$= (2)(72.926\times 10^{-6}\,\mathbf{j})\times[40(-\cos\varphi\mathbf{i} - \sin\varphi\mathbf{j})]$$
$$= (5.8341\times 10^{-3}\cos\varphi)\mathbf{k}$$

(*a*) $\varphi = 0°$, $\cos\varphi = 1.000$ $\mathbf{a}_C = 5.83\times 10^{-3}\,\text{ft/s}^2$ west ◀

(*b*) $\varphi = 40°$, $\cos\varphi = 0.76604$ $\mathbf{a}_C = 4.47\times 10^{-3}\,\text{ft/s}^2$ west ◀

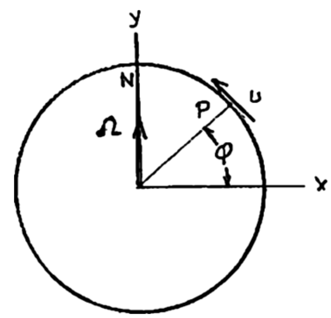
(*c*) $\varphi = -40°$, $\cos\varphi = 0.76604$ $\mathbf{a}_C = 4.47\times 10^{-3}\,\text{ft/s}^2$ west ◀

PROBLEM 15.162

A rocket sled is tested on a straight track that is built along a meridian. Knowing that the track is located at latitude $40°$ north, determine the Coriolis acceleration of the sled when it is moving north at a speed of 900 km/h.

SOLUTION

Earth makes one revolution $(2\pi$ radians) in $23.933 \text{ h} = 86{,}160 \text{ s}$.



$$\Omega = \frac{2\pi}{86{,}160}$$

$$= (72.926 \times 10^{-6} \text{ rad/s})\mathbf{j}$$

Speed of sled. $u = 900 \text{ km/h}$

$$= 250 \text{ m/s}$$

Velocity of sled relative to the Earth.

$$\mathbf{v}_{P/\text{earth}} = 250(-\sin\varphi\,\mathbf{i} + \cos\varphi\,\mathbf{j})$$

Coriolis acceleration. $\mathbf{a}_C = 2\mathbf{\Omega} \times \mathbf{v}_{P/\text{earth}}$

$$\mathbf{a}_C = (2)(72.926 \times 10^{-6}\,\mathbf{j}) \times [250(-\sin\varphi\,\mathbf{i} + \cos\varphi\,\mathbf{j})]$$

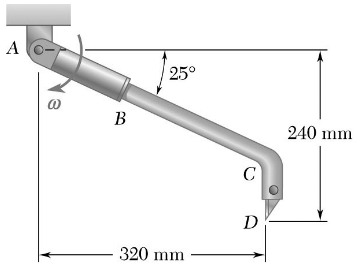
$$= 0.036463 \sin\varphi\,\mathbf{k}$$

At latitude $\varphi = 40°$,

$$\mathbf{a}_C = 0.036463 \sin 40°\mathbf{k}$$

$$= (0.0234 \text{ m/s}^2)\mathbf{k}$$

$\mathbf{a}_C = 0.0234 \text{ m/s}^2$ west ◀



PROBLEM 15.163

The motion of blade D is controlled by the robot arm ABC. At the instant shown, the arm is rotating clockwise at the constant rate $\omega = 1.8$ rad/s and the length of portion BC of the arm is being decreased at the constant rate of 250 mm/s. Determine (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = \circlearrowleft$

Units: meters, m/s, m/s^2

$$\mathbf{r}_{D/A} = (0.32\text{ m})\mathbf{i} - (0.24\text{ m})\mathbf{j}$$

Motion of Point D' of extended frame AB.

$$\boldsymbol{\omega} = -(1.8\text{ rad/s})\mathbf{k} \qquad \boldsymbol{\alpha} = 0$$

$$\begin{aligned}\mathbf{v}_{D'} &= \boldsymbol{\omega} \times \mathbf{r}_{D/A} = (-1.8\mathbf{k}) \times (0.32\mathbf{i} - 0.24\mathbf{j}) \\ &= -0.432\mathbf{i} - 0.576\mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{D'} &= \boldsymbol{\alpha} \times \mathbf{r}_{D/A} - \omega^2(\mathbf{r}_{D/A}) \\ &= 0 - (1.8)^2(0.32\mathbf{i} - 0.24\mathbf{j}) \\ &= -1.0368\mathbf{i} + 0.7776\mathbf{j}\end{aligned}$$

Motion of Point D relative to frame AB.

$$\begin{aligned}\mathbf{v}_{D/AB} &= 250\text{ mm/s} \nwarrow 25° \\ &= -(0.25\cos 25°)\mathbf{i} + (0.25\sin 25°)\mathbf{j} \\ &= -0.22658\mathbf{i} + 0.10565\mathbf{j}\end{aligned}$$

$$\mathbf{a}_{D/AB} = 0$$

Coriolis acceleration

$$\begin{aligned}2\boldsymbol{\omega} \times \mathbf{v}_{D/AB} &= (2)(-1.8\mathbf{k}) \times (-0.22658\mathbf{i} + 0.10565\mathbf{j}) \\ &= 0.38034\mathbf{i} + 0.81569\mathbf{j}\end{aligned}$$

(*a*) *Velocity of Point D.*

$$\begin{aligned}\mathbf{v}_D &= \mathbf{v}_{D'} + \mathbf{v}_{D/AB} \\ &= -0.432\mathbf{i} - 0.576\mathbf{j} - 0.22658\mathbf{i} + 0.10565\mathbf{j} \\ &= -0.65858\mathbf{i} - 0.47035\mathbf{j}\end{aligned}$$

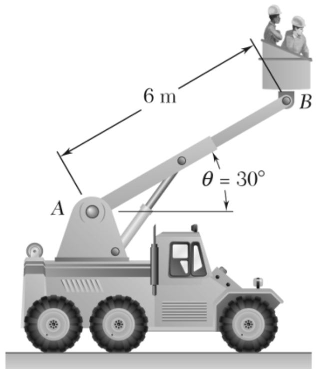
$$\mathbf{v}_D = (0.659\text{ m/s})\mathbf{i} - (0.470\text{ m/s})\mathbf{j} = 0.809\text{ m/s} \swarrow 35.5° \blacktriangleleft$$

PROBLEM 15.163 (Continued)

(*b*) *Acceleration of Point D.*

$$\begin{aligned}\mathbf{a}_D &= \mathbf{a}_{D'} + \mathbf{a}_{D/AB} + 2\boldsymbol{\omega} \times \mathbf{v}_{D/AB} \\ &= -1.0368\mathbf{i} + 0.7776\mathbf{j} + 0 + 0.38034\mathbf{i} + 0.81569\mathbf{j} \\ &= -0.6565\mathbf{i} + 1.5933\mathbf{j}\end{aligned}$$

$$\mathbf{a}_D = -(0.657 \text{ m/s}^2)\mathbf{i} + (1.593 \text{ m/s}^2)\mathbf{j} = 1.723 \text{ m/s}^2 \measuredangle 67.6° \blacktriangleleft$$



PROBLEM 15.164

At the instant shown the length of the boom AB is being *decreased* at the constant rate of 0.2 m/s and the boom is being lowered at the constant rate of 0.08 rad/s. Determine (*a*) the velocity of Point B, (*b*) the acceleration of Point B.

SOLUTION

Velocity of coinciding Point B′ on boom.

$$\mathbf{v}_{B'} = r\omega = (6)(0.08) = 0.48 \text{ m/s} \searrow 60°$$

Velocity of Point B relative to the boom.

$$\mathbf{v}_{B/\text{boom}} = 0.2 \text{ m/s} \swarrow 30°$$

(*a*) *Velocity of Point B.*

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/\text{boom}}$$

$$\xrightarrow{+}: \quad (v_B)_x = 0.48\cos 60° - 0.2\cos 30° = 0.06680 \text{ m/s}$$

$$+\uparrow: \quad (v_B)_y = -0.48\sin 60° - 0.2\sin 30° = -0.51569 \text{ m/s}$$

$$v_B = \sqrt{0.06680^2 + 0.51569^2}$$
$$= 0.520 \text{ m/s}$$

$$\tan\beta = \frac{0.51569}{0.06680}, \quad \beta = 82.6° \qquad \mathbf{v}_B = 0.520 \text{ m/s} \searrow 82.6° \blacktriangleleft$$

Acceleration of coinciding Point B′ on boom.

$$\mathbf{a}_{B'} = r\omega^2 = (6)(0.08)^2 = 0.0384 \text{ m/s}^2 \swarrow 30°$$

Acceleration of B relative to the boom.

$$\mathbf{a}_{B/\text{boom}} = 0$$

Coriolis acceleration.

$$2\omega u = (2)(0.08)(0.2) = 0.032 \text{ m/s}^2 \searrow 60°$$

PROBLEM 15.164 (Continued)

(*b*) *Acceleration of Point B.*

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\text{boom}} + 2\omega u$$

$$\xrightarrow{+} : \quad (a_B)_x = -0.0384\cos 30° + 0 - 0.032\cos 60° = -0.04926 \text{ m/s}^2$$

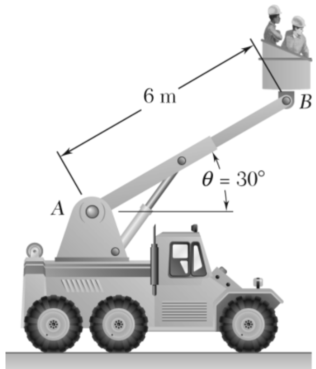
$$+\uparrow : \quad (a_B)_y = -0.0384\sin 30° + 0 + 0.032\sin 60° = 0.008513 \text{ m/s}^2$$

$$a_B = \sqrt{(0.04926)^2 + (0.008513)^2}$$

$$= 0.0500 \text{ m/s}^2$$

$$\tan\beta = \frac{0.008513}{0.04926}, \quad \beta = 9.8°$$

$\mathbf{a}_B = 50.0 \text{ mm/s}^2 \nwarrow 9.8°$ ◀



PROBLEM 15.165

At the instant shown the length of the boom AB is being *increased* at the constant rate of 0.2 m/s and the boom is being lowered at the constant rate of 0.08 rad/s. Determine (*a*) the velocity of Point B, (*b*) the acceleration of Point B.

SOLUTION

Velocity of coinciding Point B′ on boom.

$$\mathbf{v}_{B'} = r\omega = (6)(0.08) = 0.48 \text{ m/s} \measuredangle 60°$$

Velocity of Point B relative to the boom.

$$\mathbf{v}_{B/\text{boom}} = 0.2 \text{ m/s} \measuredangle 30°$$

(*a*) *Velocity of Point B.*

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/\text{boom}}$$

$$\xrightarrow{+}: \quad (v_B)_x = 0.48 \cos 60° + 0.2 \cos 30° = 0.4132 \text{ m/s}$$

$$+\uparrow: \quad (v_B)_y = -0.48 \sin 60° + 0.2 \sin 30° = -0.3157 \text{ m/s}$$

$$v_B = \sqrt{(0.4132)^2 + (0.3157)^2}$$
$$= 0.520 \text{ m/s}$$

$$\tan\beta = -\frac{0.3157}{0.4132}, \quad \beta = -37.4°$$

$$\mathbf{v}_B = 0.520 \text{ m/s} \measuredangle 37.4° \blacktriangleleft$$

Acceleration of coinciding Point B′ on boom.

$$\mathbf{a}_{B'} = r\omega^2 = (6)(0.08)^2 = 0.0384 \text{ m/s}^2 \measuredangle 30°$$

Acceleration of B relative to the boom.

$$\mathbf{a}_{B/\text{boom}} = 0$$

Coriolis acceleration.

$$2\omega u = (2)(0.08)(2) = 0.032 \text{ m/s}^2 \measuredangle 60°$$

PROBLEM 15.165 (Continued)

(*b*) *Acceleration of Point B.*

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/\text{boom}} + 2\omega u$$

$$\xrightarrow{+}: \quad (a_B)_x = -0.0384 \cos 30° + 0.032 \cos 60° = -0.017255 \text{ m/s}^2$$

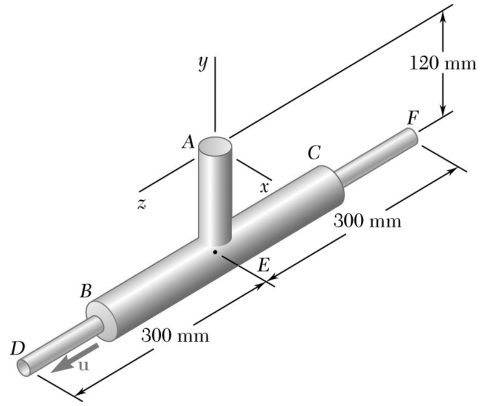
$$+\uparrow: \quad (a_B)_y = -0.0384 \sin 30° - 0.032 \sin 60° = -0.04691 \text{ m/s}^2$$

$$a_B = \sqrt{(0.017255)^2 + (0.04691)^2}$$

$$= 0.0500 \text{ m/s}^2$$

$$\tan\beta = \frac{0.04691}{0.017255}, \qquad \beta = 69.8°$$

$$\mathbf{a}_B = 50.0 \text{ mm/s}^2 \measuredangle\, 69.8° \blacktriangleleft$$



PROBLEM 15.166

The sleeve BC is welded to an arm that rotates about A with a constant angular velocity ω. In the position shown rod DF is being moved to the left at a constant speed $u = 400$ mm/s relative to the sleeve. For the given angular velocity ω, determine the acceleration (*a*) of Point D, (*b*) of the point of rod DF that coincides with Point E.

$\omega = (3 \text{ rad/s})\mathbf{i}$.

SOLUTION

(*a*) Point *D*.

$$\mathbf{v}_{D/F} = \mathbf{v}_{D/BC} = (0.4 \text{ m/s})\mathbf{k}; \quad a_{D/F} = 0$$

$$\overrightarrow{AD} = -(0.12 \text{ m})\mathbf{j} + (0.3 \text{ m})\mathbf{k}$$

$$\mathbf{a}_{D'} = \boldsymbol{\omega} \times (\omega \times \overrightarrow{AD})$$

$$= -\omega^2 (\overline{AD})$$

$$= -(3 \text{ rad/s})^2 \overline{AD}$$

$$= +(1.08 \text{ m/s}^2)\mathbf{j} - (2.70 \text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_c = 2\boldsymbol{\omega} \times \mathbf{v}_{D/F}$$

$$= 2[(3 \text{ rad/s})\mathbf{i}] \times (0.40 \text{ m/s})\mathbf{k}$$

$$= -(2.4 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$

$$= [+(1.08 \text{ m/s}^2)\mathbf{j} - (2.70 \text{ m/s}^2)\mathbf{k}] + 0 + [-(2.4 \text{ m/s})\mathbf{j}]$$

$$\mathbf{a}_D = -(1.32 \text{ m/s})\mathbf{j} - (2.70 \text{ m/s})\mathbf{k} \blacktriangleleft$$

(*b*) Point *P* of *DF* that coincides with *E*.

$$\mathbf{v}_{P/F} = \mathbf{v}_{P/BC} = (0.40 \text{ m/s})\mathbf{k}; \quad \mathbf{a}_{P/F} = 0$$

$$\overrightarrow{AE} = -(0.120 \text{ m})\mathbf{j}$$

$$\mathbf{a}_{P'} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \overrightarrow{AE}) = -\omega^2 AE = -(3 \text{ rad/s})^2 AE = (1.08 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_c = 2\boldsymbol{\omega} \times \mathbf{v}_{P/F}$$

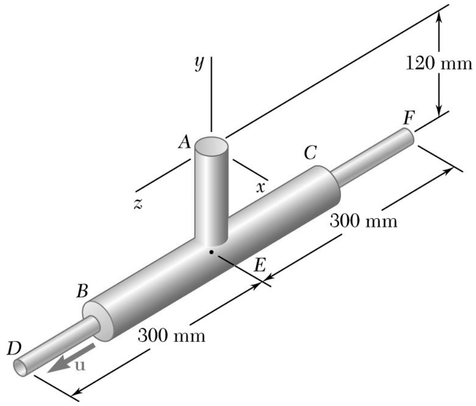
$$= 2[(3 \text{ rad/s})\mathbf{i}] \times (0.40 \text{ m/s})\mathbf{k}$$

$$= -(2.40 \text{ m/s}^2)\mathbf{j}$$

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_c$$

$$= [(1.08 \text{ m/s}^2)\mathbf{j}] + 0 + [-(2.4 \text{ m/s}^2)\mathbf{j}]$$

$$\mathbf{a}_P = -(1.32 \text{ m/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.167

The sleeve BC is welded to an arm that rotates about A with a constant angular velocity ω. In the position shown rod DF is being moved to the left at a constant speed $u = 400$ mm/s relative to the sleeve. For the given angular velocity ω, determine the acceleration (*a*) of Point D, (*b*) of the point of rod DF that coincides with Point E.

$\omega = (3 \text{ rad/s})\mathbf{j}$.

SOLUTION

(*a*) Point D.

$$\mathbf{v}_{D/F} = \mathbf{v}_{D/BC} = (0.4 \text{ m/s})\mathbf{k}; \quad a_{D/F} = 0$$

$$\overrightarrow{AD} = -(0.12 \text{ m})\mathbf{j} + (0.3 \text{ m})\mathbf{k}$$

$$\mathbf{a}_{D'} = \boldsymbol{\omega} \times (\omega \times \overrightarrow{AD})$$
$$= 3\mathbf{j} \times (3\mathbf{j} \times (-0.12\mathbf{j} + 0.3)\mathbf{k})$$
$$= -(2.70 \text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_c = 2\boldsymbol{\omega} \times \mathbf{v}_{D/F}$$
$$= 2[(3 \text{ rad/s})\mathbf{j}] \times (0.40 \text{ m/s})\mathbf{k}$$
$$= (2.4 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$
$$= [-(2.70 \text{ m/s}^2)\mathbf{k}] + 0 + [(2.4 \text{ m/s})\mathbf{i}]$$

$$\mathbf{a}_D = (2.4 \text{ m/s})\mathbf{i} - (2.70 \text{ m/s})\mathbf{k} \blacktriangleleft$$

(*b*) Point P of DF that coincides with E.

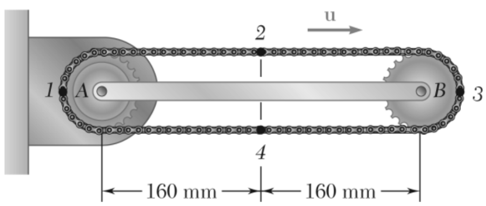
$$\mathbf{v}_{P/F} = \mathbf{v}_{P/BC} = (0.40 \text{ m/s})\mathbf{k}; \quad \mathbf{a}_{P/F} = 0$$

$$\overrightarrow{AE} = -(0.120 \text{ m})\mathbf{j}$$

$$\mathbf{a}_{P'} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \overrightarrow{AE})$$
$$= 3\mathbf{j} \times (3\mathbf{j} \times (-0.12\mathbf{j}) = 0$$

$$\mathbf{a}_c = 2\boldsymbol{\omega} \times \mathbf{v}_{P/F}$$
$$= 2[(3 \text{ rad/s})\mathbf{j}] \times (0.40 \text{ m/s})\mathbf{k}$$
$$= (2.40 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_c$$
$$= 0 + 0 + (2.4 \text{ m/s}^2)\mathbf{i} \qquad \mathbf{a}_P = (2.4 \text{ m/s}^2)\mathbf{i} \blacktriangleleft$$



PROBLEM 15.168

A chain is looped around two gears of radius 40 mm that can rotate freely with respect to the 320-mm arm AB. The chain moves about arm AB in a clockwise direction at the constant rate of 80 mm/s relative to the arm. Knowing that in the position shown arm AB rotates clockwise about A at the constant rate $\omega = 0.75$ rad/s, determine the acceleration of each of the chain links indicated.

Links 1 and 2.

SOLUTION

Let the arm AB be a rotating frame of reference. $\Omega = 0.75 \text{ rad/s} \circlearrowright = -(0.75 \text{ rad/s})\mathbf{k}$:

Link 1:

$$\mathbf{r}_1 = -(40 \text{ mm})\mathbf{i}, \quad \mathbf{v}_{1/AB} = u \uparrow = (80 \text{ mm/s})\mathbf{j}$$

$$\mathbf{a}_1' = -\Omega^2 \mathbf{r}_1 = -(0.75)^2(-40) = (22.5 \text{ mm/s})\mathbf{i}$$

$$\mathbf{a}_{1/AB} = \frac{u^2}{\rho} = \frac{80^2}{40} 160 \text{ mm/s} \rightarrow = (160 \text{ mm/s}^2)\mathbf{i}$$

$$2\Omega \times \mathbf{v}_{P/AB} = (2)(-0.75\mathbf{k}) \times (80\mathbf{j})$$

$$= (120 \text{ mm/s})\mathbf{i}$$

$$\mathbf{a}_1 = \mathbf{a}_1' + \mathbf{a}_{1/AB} + 2\Omega \times \mathbf{v}_{1/AB}$$

$$= (302.5 \text{ mm/s}^2)\mathbf{i}$$

$$\mathbf{a}_1 = 303 \text{ mm/s}^2 \longrightarrow \blacktriangleleft$$

Link 2:

$$\mathbf{r}_2 = (160 \text{ mm})\mathbf{i} + (40 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_{2/AB} = u \rightarrow = (80 \text{ mm/s})\mathbf{i}$$

$$\mathbf{a}_2' = -\Omega^2 \mathbf{r}_2$$

$$= -(0.75)^2(160\mathbf{i} + 40\mathbf{j})$$

$$= -(90 \text{ mm/s}^2)\mathbf{i} - (22.5 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_{2/AB} = 0$$

$$2\Omega \times \mathbf{v}_{2/AB} = (2)(-0.75\mathbf{k}) \times (80\mathbf{i})$$

$$= -(120 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}^2 = \mathbf{a}_2' + \mathbf{a}_{2/AB} + 2\Omega \times \mathbf{v}_{2/AB}$$

$$= -90\mathbf{i} - 22.5\mathbf{j} - 120\mathbf{j}$$

$$= -(90 \text{ mm/s}^2)\mathbf{i} - (142.5 \text{ mm/s}^2)\mathbf{j}$$

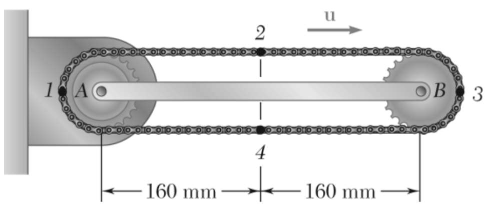
$$\mathbf{a}_2 = \sqrt{(90)^2 + (142.5)^2}$$

$$= 168.5 \text{ mm/s}^2$$

$$\tan \beta = \frac{142.5}{90}, \quad \beta = 57.7°$$

$$\mathbf{a}_2 = 168.5 \text{ mm/s}^2 \swarrow 57.7° \blacktriangleleft$$

PROBLEM 15.169



A chain is looped around two gears of radius 40 mm that can rotate freely with respect to the 320-mm arm *AB*. The chain moves about arm *AB* in a clockwise direction at the constant rate 80 mm/s relative to the arm. Knowing that in the position shown arm *AB* rotates clockwise about *A* at the constant rate $\omega = 0.75$ rad/s, determine the acceleration of each of the chain links indicated.

Links 3 and 4.

SOLUTION

Let arm *AB* be a rotating frame of reference. $\boldsymbol{\Omega} = 0.75 \text{ rad/s} \circlearrowright = -(0.75 \text{ rad/s})\mathbf{k}$

Link 3:

$$\mathbf{r}_3 = (360 \text{ mm})\mathbf{i} \quad \mathbf{v}_{3/AB} = u \downarrow = -(80 \text{ mm/s})\mathbf{j}$$

$$\mathbf{a}_{3'} = -\Omega^2 \mathbf{r}_3 = -(0.75)^2(360) = -(202.5 \text{ mm/s}^2)\mathbf{i}$$

$$\mathbf{a}_{3/AB} = \frac{u^2}{\rho} = \frac{(80)^2}{40} = 160 \text{ mm/s}^2\mathbf{i} \leftarrow = -(160 \text{ mm/s}^2)\mathbf{i}$$

$$2\Omega \times \mathbf{v}_{3/AB} = (2)(-0.75\mathbf{k}) \times (-80\mathbf{j}) = -(120 \text{ mm/s}^2)\mathbf{i}$$

$$\mathbf{a}_3 = \mathbf{a}_{3'} + \mathbf{a}_{3/AB} + 2\Omega \times \mathbf{v}_{3/AB}$$

$$= -(482.5 \text{ mm/s}^2)\mathbf{i}$$

$\mathbf{a}_3 = 483 \text{ mm/s}^2 \longleftarrow$ ◀

Link 4:

$$\mathbf{r}_4 = (160 \text{ mm})\mathbf{i} - (40 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_{4/AB} = u \leftarrow = -(80 \text{ mm/s}^2)\mathbf{i}$$

$$\mathbf{a}_{4'} = -\Omega^2 \mathbf{r}_4$$

$$= -(0.75)^2(160\mathbf{i} - 40\mathbf{j})$$

$$= -(90 \text{ mm/s}^2)\mathbf{i} + (22.5 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_{4/AB} = 0$$

$$2\Omega \times \mathbf{v}_{4/AB} = (2)(-0.75\mathbf{k}) \times (-80\mathbf{i})$$

$$= (120 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_4 = \mathbf{a}_{4'} + \mathbf{a}_{4/AB} + 2\Omega \times \mathbf{v}_{4/AB}$$

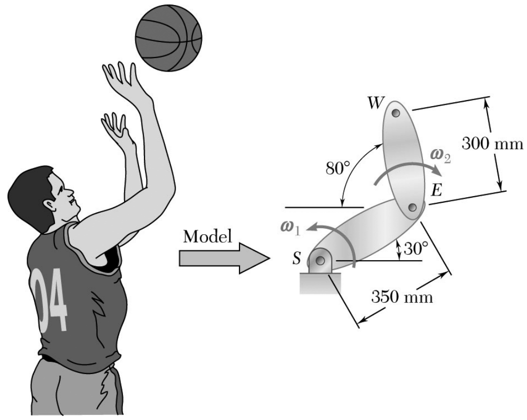
$$= -(90 \text{ mm/s}^2)\mathbf{i} + (142.5 \text{ mm/s}^2)\mathbf{j}$$

$$a_4 = \sqrt{(90)^2 + (142.5)^2}$$

$$= 168.5 \text{ mm/s}^2$$

$$\tan \beta = \frac{142.5}{90}, \quad \beta = 57.7°$$

$\mathbf{a}_4 = 168.5 \text{ mm/s}^2 \measuredangle 57.7°$ ◀



PROBLEM 15.170

A basketball player shoots a free throw in such a way that his shoulder can be considered a pin joint at the moment of release as shown. Knowing that at the instant shown the upper arm *SE* has a constant angular velocity of 2 rad/s counterclockwise and the forearm *EW* has a constant clockwise angular velocity of 4 rad/s with respect to *SE*, determine the velocity and acceleration of the wrist *W*.

SOLUTION

Units: meters, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

Relative positions:

$$\mathbf{r}_{E/S} = (0.35\cos 30°)\mathbf{i} + (0.35\sin 30°)\mathbf{j} = 0.30311\mathbf{i} + 0.175\mathbf{j}$$

$$\mathbf{r}_{W/E} = -(0.3\cos 80°)\mathbf{i} + (0.3\sin 80°)\mathbf{j} = -0.05209\mathbf{i} + 0.29544\mathbf{j}$$

$$\mathbf{r}_{W/S} = \mathbf{r}_{E/S} + \mathbf{r}_{W/E} = 0.25101\mathbf{i} + 0.47044\mathbf{j}$$

Use a frame of reference rotating with the upper arm *SE* with angular velocity

$$\mathbf{\Omega} = (2 \text{ rad/s})\mathbf{k} \quad (\dot{\mathbf{\Omega}} = 0)$$

The motion of the wrist *W* relative to this frame is a rotation about the elbow *E* with angular velocity

$$\boldsymbol{\omega} = -(4 \text{ rad/s})\mathbf{k} \quad (\dot{\boldsymbol{\omega}} = 0)$$

Motion of Point *W′* in the frame coinciding with *W*.

$$\mathbf{v}_{W'} = \mathbf{\Omega} \times \mathbf{r}_{W/S} = (2\mathbf{k}) \times (0.25101\mathbf{i} + 0.47044\mathbf{j})$$
$$= -0.94088\mathbf{i} + 0.50204\mathbf{j}$$

$$\mathbf{a}_{W'} = -\Omega^2 \mathbf{r}_{W/S} = -(2)^2(0.25101\mathbf{i} + 0.47044\mathbf{j})$$
$$= -1.00408\mathbf{i} - 1.88176\mathbf{j}$$

Motion of *W* relative to the frame.

$$\mathbf{v}_{W/SE} = \boldsymbol{\omega} \times \mathbf{r}_{W/E} = (-4\mathbf{k}) \times (-0.05210\mathbf{i} + 0.29544\mathbf{j})$$
$$= 1.18176\mathbf{i} + 0.2084\mathbf{j}$$

$$\mathbf{a}_{W/SE} = -\omega^2 \mathbf{r}_{W/E} = -(4)^2(-0.05210\mathbf{i} + 0.29544\mathbf{j})$$
$$= 0.8336\mathbf{i} - 4.72708\mathbf{j}$$

Velocity of *W*: $\mathbf{v}_W = \mathbf{v}_{W'} + \mathbf{v}_{W/SE} = 0.24088\mathbf{i} + 0.71044\mathbf{j}$

$$\mathbf{v}_W = 0.750 \text{ m/s} \measuredangle 71.3° \blacktriangleleft$$

PROBLEM 15.170 (Continued)

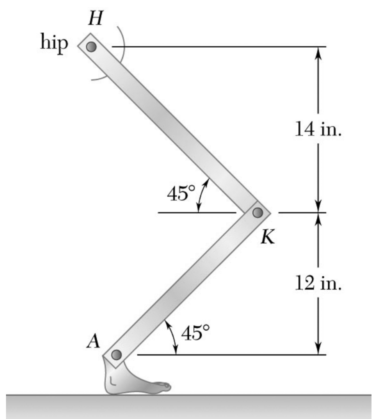
Coriolis acceleration:

$$2\boldsymbol{\Omega}\times\mathbf{v}_{W/SE} = (2)(2\mathbf{k})\times(1.18176\mathbf{i}+0.2084\mathbf{j})$$
$$= -0.8336\mathbf{i}+4.72704\mathbf{j}$$

Acceleration of W:

$$\mathbf{a}_W = \mathbf{a}_{W'}+\mathbf{a}_{W/SE}+2\boldsymbol{\Omega}\times\mathbf{v}_{W/SE}$$
$$= -1.00408\mathbf{i}-1.88176\mathbf{j}$$

$\mathbf{a}_W = 2.13\ \text{m/s}^2 \measuredangle\ 61.9° ◀$



PROBLEM 15.171

The human leg can be crudely approximated as two rigid bars (the femur and the tibia) connected with a pin joint. At the instant shown the veolcity and acceleration of the ankle is zero. During a jump, the velocity of the ankle A is zero, the tibia AK has an angular velocity of 1.5 rad/s counterclockwise and an angular acceleration of 1 rad/s^2 counterclockwise. Determine the relative angular velocity and angular acceleration of the femur KH with respect to AK so that the velocity and acceleration of H are both straight up at the instant shown.

SOLUTION

Units: inches, in./s, in./s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

Relative positions:

$$\mathbf{r}_{K/A} = (12 \text{ in.})\mathbf{i} + (12 \text{ in.})\mathbf{j}$$

$$\mathbf{r}_{H/K} = -(14 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{j}$$

$$\mathbf{r}_{H/A} = \mathbf{r}_{K/A} + \mathbf{r}_{H/K} = -(2 \text{ in.})\mathbf{i} + (26 \text{ in.})\mathbf{j}$$

Use a frame of reference moving with the lower leg AK with angular velocity

$$\boldsymbol{\Omega} = 1.5 \text{ rad/s} \circlearrowleft = (1.5 \text{ rad/s})\mathbf{k}$$

and angular acceleration $\dot{\boldsymbol{\Omega}} = 1.0 \text{ rad/s} \circlearrowleft = (1.0 \text{ rad/s}^2)\mathbf{k}$

The motion of the hip H relative to this frame is a rotation about the knee K with angular velocity

$$\boldsymbol{\omega} = \omega\mathbf{k}$$

and angular acceleration $\boldsymbol{\alpha} = \alpha\mathbf{k}$

Both $\boldsymbol{\omega}$ and $\boldsymbol{\alpha}$ are measured relative to the lower leg AK.

Motion of Point H' in the frame coinciding with H.

$$\mathbf{v}_{H'} = \boldsymbol{\Omega} \times \mathbf{r}_{H/A} = 1.5\mathbf{k} \times (-2\mathbf{i} + 26\mathbf{j})$$

$$= -(39 \text{ in./s})\mathbf{i} - (3 \text{ in./s})\mathbf{j}$$

$$\mathbf{a}_{H'} = \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{H/A} - \Omega^2 \mathbf{r}_{H/A}$$

$$= (1.0\mathbf{k}) \times (-2\mathbf{i} + 26\mathbf{j}) - (1.5)^2(-2\mathbf{i} + 26\mathbf{j})$$

$$= -26\mathbf{i} - 2\mathbf{j} + 4.5\mathbf{i} - 58.5\mathbf{j}$$

$$= -(21.5 \text{ in./s}^2)\mathbf{i} - (60.5 \text{ in./s}^2)\mathbf{j}$$

Motion of H relative to the frame.

$$\mathbf{v}_{H/AK} = \omega\mathbf{k}\times\mathbf{r}_{H/K} = \omega\mathbf{k}\times(-14\mathbf{i}+14\mathbf{j})$$
$$= -14\omega\mathbf{i} - 14\omega\mathbf{j}$$
$$\mathbf{a}_{H/AK} = \alpha\mathbf{k}\times\mathbf{r}_{H/K} - \omega^2\mathbf{r}_{H/K}$$
$$= \alpha\mathbf{k}\times(-14\mathbf{i}+14\mathbf{j}) - \omega^2(-14\mathbf{i}+14\mathbf{j})$$
$$= -14\alpha\mathbf{i} - 14\alpha\mathbf{j} + 14\omega^2\mathbf{i} - 14\omega^2\mathbf{j}$$

Velocity of H.
$$\mathbf{v}_H = v_H \uparrow = v_H\mathbf{j}$$
$$\mathbf{v}_H = \mathbf{v}_{H'} + \mathbf{v}_{H/AK}$$
$$v_H\mathbf{j} = -39\mathbf{i} - 3\mathbf{j} - 14\omega\mathbf{i} - 14\omega\mathbf{j}$$

Resolve into components.

$\mathbf{i}$: $\quad 0 = -39 - 14\omega \qquad \omega = -\dfrac{39}{14} = -2.7857 \text{ rad/s}$

$\mathbf{j}$: $\quad v_H = -3 - 14\omega \qquad v_H = -36 \text{ in./s}$

Relative angular velocity: $\boldsymbol{\omega} = -(2.79 \text{ rad/s})\mathbf{k} = 2.79 \text{ rad/s} \circlearrowright$ ◀

Coriolis acceleration:
$$2\boldsymbol{\Omega}\times\mathbf{v}_{H/AK} = (2)(1.5\mathbf{k})\times(-14\omega\mathbf{i} - 14\omega\mathbf{j})$$
$$= 42\omega\mathbf{i} - 42\omega\mathbf{j} = -(117 \text{ in./s}^2)\mathbf{i} + (117 \text{ in./s}^2)\mathbf{j}$$

Acceleration of H.
$$\mathbf{a}_H = a_H \uparrow = a_H\mathbf{j}$$
$$\mathbf{a}_H = \mathbf{a}_{H'} + \mathbf{a}_{H/AK} + 2\boldsymbol{\Omega}\times\mathbf{v}_{H/AK}$$
$$a_H\mathbf{j} = -21.5\mathbf{i} - 60.5\mathbf{j} - 14\alpha\mathbf{i} - 14\alpha\mathbf{j} + 14\omega^2\mathbf{i} - 14\omega^2\mathbf{j} - 117\mathbf{i} + 117\mathbf{j}$$

Resolve into components.

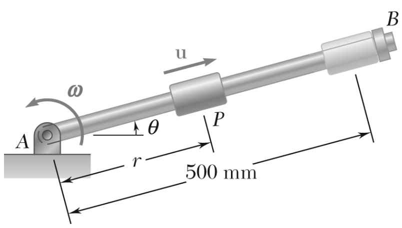
$\mathbf{i}$: $\quad 0 = -21.5 - 14\alpha + (14)(-2.7857)^2 - 117$

$$\alpha = -2.1327 \text{ rad/s}^2$$

$\mathbf{j}$: $\quad a_H = -60.5 - (14)(-2.1327) - (14)(-2.7857)^2 + 117$

$$= -22.284 \text{ in./s}^2$$

Relative angular acceleration: $\boldsymbol{\alpha} = -(2.13 \text{ rad/s}^2)\mathbf{k} = 2.13 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 15.172

The collar P slides outward at a constant relative speed u along rod AB, which rotates counterclockwise with a constant angular velocity of 20 rpm. Knowing that $r = 250$ mm when $\theta = 0$ and that the collar reaches B when $\theta = 90°$, determine the magnitude of the acceleration of the collar P just as it reaches B.

SOLUTION

$$\omega = 20 \text{ rpm} = \frac{(20)(2\pi)}{60} = \frac{2\pi}{3} \text{ rad/s}$$

$$\alpha = 0$$

$$\theta = 90° = \frac{\pi}{2} \text{ radians}$$

Uniform rotational motion. $\theta = \theta_0 + \omega t$

$$t = \frac{\theta - \theta_0}{\omega} = \frac{\frac{\pi}{2}}{\frac{2\pi}{3}} = 0.75 \text{ s}$$

Uniform motion along rod. $r = r_0 + ut$

$$u = \frac{r - r_0}{t} = \frac{0.5 - 0.25}{0.75} = \frac{1}{3} \text{ m/s},$$

$$\mathbf{v}_{P/AB} = \frac{1}{3} \text{ m/s} \uparrow$$

Acceleration of coinciding Point P′ on the rod. $(r = 0.5 \text{ m})$

$$\mathbf{a}_{P'} = r\omega^2$$

$$= (0.5)\left(\frac{2\pi}{3}\right)^2$$

$$= \frac{2\pi^2}{9} \text{ m/s}^2 \downarrow$$

$$= 2.1932 \text{ m/s}^2 \downarrow$$

Acceleration of collar P relative to the rod. $\mathbf{a}_{P/AB} = 0$

Coriolis acceleration. $2\boldsymbol{\omega} \times \mathbf{v}_{P/AB} = 2\omega u = (2)\left(\frac{2\pi}{3}\right)\left(\frac{1}{3}\right) = 1.3963 \text{ m/s}^2 \longleftarrow$

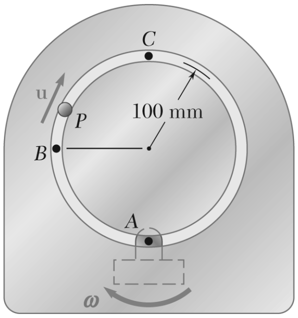
Acceleration of collar P.

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AB} + 2\boldsymbol{\omega} \times \mathbf{v}_{P/AB}$$

$$\mathbf{a}_P = [2.1932 \text{ m/s}^2 \downarrow] + [1.3963 \text{ m/s}^2 \longleftarrow]$$

$$\mathbf{a}_P = 2.60 \text{ m/s}^2 \swarrow 57.5°$$

$a_P = 2.60 \text{ m/s}^2$ ◀



PROBLEM 15.173

Pin P slides in a circular slot cut in the plate shown at a constant relative speed $u = 90$ mm/s. Knowing that at the instant shown the plate rotates clockwise about A at the constant rate $\omega = 3$ rad/s, determine the acceleration of the pin if it is located at (*a*) Point A, (*b*) Point B, (*c*) Point C.

SOLUTION

$$\omega = 3 \text{ rad/s} \curvearrowright, \quad \alpha = 0, \quad u = 90 \text{ mm/s} = 0.09 \text{ m/s}, \quad \dot{u} = 0$$

$$\rho = 100 \text{ mm}$$

$$\frac{u^2}{\rho} = \frac{(90)^2}{100} = 81 \text{ mm/s}^2 = 0.081 \text{ m/s}^2$$

$$\omega^2 = 36 \text{ rad}^2/\text{s}^2$$

$$2\omega u = (2)(3)(90) = 540 \text{ mm/s}^2 = 0.54 \text{ m/s}^2$$

(*a*) *Point A.*

$$\mathbf{r}_A = 0, \quad \mathbf{v}_{A/F} = 0.09 \text{ m/s} \leftarrow$$

$$\mathbf{a}_{A'} = 0, \quad \mathbf{a}_{A/F} = \frac{u^2}{\rho} \uparrow = 0.081 \text{ m/s}^2 \uparrow$$

Coriolis acceleration. $2\omega u \uparrow = 0.54 \text{ m/s}^2 \uparrow$

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + [2\omega u \uparrow] = 0.621 \text{ m/s}^2 \uparrow$$

$\mathbf{a}_A = 0.621 \text{ m/s}^2 \uparrow$ ◀

(*b*) *Point B.*

$$\mathbf{r}_B = 0.1\sqrt{2} \text{ m} \searrow 45°, \quad \mathbf{v}_{B/F} = 0.09 \text{ m/s} \uparrow$$

$$\mathbf{a}_{B'} = -\omega^2 \mathbf{r}_B = -(9)(0.1\sqrt{2}) \searrow 45° = 0.9\sqrt{2} \text{ m/s}^2 \nwarrow 45°$$

$$\mathbf{a}_{B/F} = \frac{u^2}{\rho}$$

$$= 0.081 \text{ m/s}^2 \rightarrow$$

Coriolis acceleration. $2\omega u = 0.54 \text{ m/s}^2 \rightarrow$

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + [2\omega u \rightarrow]$$

$$= [1.521 \text{ m/s}^2 \rightarrow] + [0.9 \text{ m/s}^2 \downarrow]$$

$\mathbf{a}_B = 1.767 \text{ m/s}^2 \nwarrow 30.6°$ ◀

(*c*) *Point C.*

$$\mathbf{r}_C = 0.2 \text{ m} \uparrow$$

$$\mathbf{v}_{C/F} = 0.09 \text{ m/s} \rightarrow$$

$$\mathbf{a}_{C'} = -\omega^2 \mathbf{r}_C = -(9)(0.2 \uparrow) = 1.8 \text{ m/s}^2 \downarrow$$

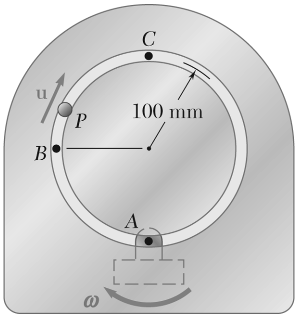
$$\mathbf{a}_{C/F} = \frac{u^2}{\rho} = 0.081 \text{ m/s}^2 \downarrow$$

Coriolis acceleration. $2\omega u = 0.54 \text{ m/s}^2 \downarrow$

$$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + [2\omega u \downarrow]$$

$$= 2.421 \text{ m/s}^2 \downarrow$$

$$\mathbf{a}_C = 2.42 \text{ m/s}^2 \downarrow \blacktriangleleft$$



PROBLEM 15.174

Pin P slides in a circular slot cut in the plate shown at a constant relative speed $u = 90$ mm/s. Knowing that at the instant shown the angular velocity ω of the plate is 3 rad/s clockwise and is decreasing at the rate of 5 rad/s^2, determine the acceleration of the pin if it is located at (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*.

SOLUTION

$$\omega = 3 \text{ rad/s} \circlearrowright, \quad \alpha = 5 \text{ rad/s} \circlearrowleft, \quad u = 90 \text{ mm/s} = 0.09 \text{ m/s}, \quad \dot{u} = 0$$

$$\rho = 100 \text{ mm}$$

$$\frac{u^2}{\rho} = \frac{(90)^2}{100} = 81 \text{ mm/s}^2 = 0.081 \text{ m/s}^2$$

$$\omega^2 = 36 \text{ rad}^2/\text{s}^2$$

$$2\omega u = (2)(3)(90) = 540 \text{ mm/s}^2 = 0.54 \text{ m/s}^2$$

(*a*) *Point A.*

$$\mathbf{r}_A = 0$$

$$\mathbf{v}_{A/F} = 0.09 \text{ m/s} \leftarrow$$

$$\mathbf{a}_{A'} = 0$$

$$\mathbf{a}_{A/F} = \frac{u^2}{\rho} \uparrow = 0.081 \text{ m/s}^2 \uparrow$$

Coriolis acceleration. $2\omega u \uparrow = 0.54 \text{ m/s}^2 \uparrow$

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + [2\omega u \uparrow]$$

$$= 0.621 \text{ m/s}^2 \uparrow$$

$$\mathbf{a}_A = 0.621 \text{ m/s}^2 \uparrow \blacktriangleleft$$

(*b*) *Point B.* $\mathbf{r}_B = 0.1\sqrt{2} \text{ m} \searrow 45°, \quad \mathbf{v}_{B/F} = 0.09 \text{ m/s} \uparrow$

$$\mathbf{a}_{B'} = \alpha \mathbf{k} \times \mathbf{r}_B - \omega^2 \mathbf{r}_B = [(0.1\sqrt{2})(5) \nearrow 45°] - [(9)(0.1\sqrt{2}) \searrow 45°]$$

$$= [0.5\sqrt{2} \text{ m/s}^2 \nearrow 45°] + [0.9\sqrt{2} \text{ m/s}^2 \nwarrow 45°]$$

$$\mathbf{a}_{B/F} = \frac{u^2}{\rho} = 0.081 \text{ m/s}^2 \rightarrow$$

Coriolis acceleration. $2\omega u = 0.54 \text{ m/s}^2 \rightarrow$

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + [2\omega u \rightarrow]$$

$$= [1.021 \text{ m/s}^2 \rightarrow] + [1.4 \text{ m/s}^2 \downarrow]$$

$$\mathbf{a}_B = 1.733 \text{ m/s}^2 \nwarrow 53.9° \blacktriangleleft$$

PROBLEM 15.174 (Continued)

(*c*) *Point C.*

$$\mathbf{r}_C = 0.2 \text{ m} \uparrow$$

$$\mathbf{v}_{C/F} = 0.09 \text{ m/s} \rightarrow$$

$$\mathbf{a}_{C'} = \alpha\mathbf{k} \times \mathbf{r}_C - \omega^2 \mathbf{r}_C$$

$$= [(0.2)(5) \leftarrow] - [(9)(0.2 \uparrow)]$$

$$= [1 \text{ m/s}^2 \leftarrow] + [1.8 \text{ m/s}^2 \downarrow]$$

$$\mathbf{a}_{C/F} = \frac{u^2}{\rho}$$

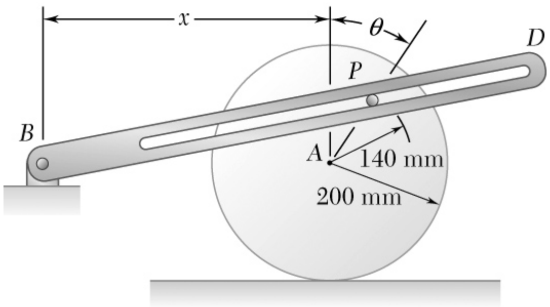
$$= 0.081 \text{ m/s}^2 \downarrow$$

Coriolis acceleration. $2\omega u = 0.54 \text{ m/s}^2 \downarrow$

$$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\omega u \downarrow$$

$$= [1 \text{ m/s}^2 \leftarrow] + [2.421 \text{ m/s}^2 \downarrow]$$

$\mathbf{a}_C = 2.62 \text{ m/s}^2 \swarrow 67.6°$ ◀

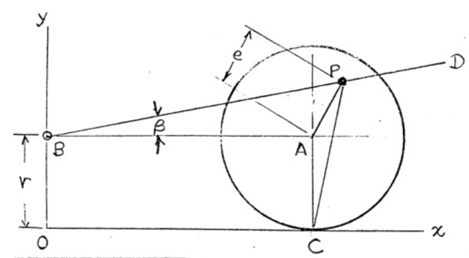


PROBLEM 15.175

Pin P is attached to the wheel shown and slides in a slot cut in bar BD. The wheel rolls to the right without slipping with a constant angular velocity of 20 rad/s. Knowing that x = 480 mm when $\theta = 0$, determine (*a*) the angular acceleration of the bar and (*b*) the relative acceleration of pin P with respect to the bar when $\theta = 0$.

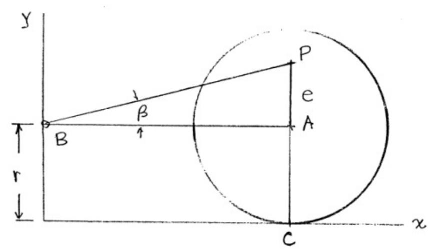
SOLUTION

Coordinates.



$$x_A = (x_A)_0 + r\theta, \qquad y_A = r$$
$$x_B = 0, \qquad y_B = r$$
$$x_C = x_A, \qquad y_C = 0$$
$$x_P = x_A + e\sin\theta, \qquad y_P = r + e\cos\theta$$

Data:
$$(x_A)_0 = 480 \text{ mm} = 0.48 \text{ m}$$
$$r = 200 \text{ mm} = 0.20 \text{ m}$$
$$e = 140 \text{ mm} = 0.14 \text{ m}$$
$$\theta = 0 \qquad x_P = 480 \text{ mm} = 0.48 \text{ m}$$



Velocity analysis.

$$\boldsymbol{\omega}_{AC} = 20 \text{ rad/s} \circlearrowright, \qquad \boldsymbol{\omega}_{BD} = \omega_{BD} \circlearrowright$$

$$\mathbf{v}_P = (r+e)\omega_{AC} \longrightarrow$$
$$= (0.20 + 0.14)(20)$$
$$= 6.8 \text{ m/s} \longrightarrow$$

$$\mathbf{v}_{P'} = [x_P\omega_{BD} \downarrow] + [e\omega_{BD} \longrightarrow]$$

$$\mathbf{v}_{P/F} = [u\cos\beta \longrightarrow] + [u\sin\beta \uparrow]$$

$$\tan\beta = \frac{e}{x_P} = \frac{0.14}{0.48}$$
$$\beta = 16.260°$$

Use $\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/F}$ and resolve into components.

$$\xrightarrow{+}: \quad 6.8 = 0.14\omega_{BD} + u\cos\beta \tag{1}$$

$$+\downarrow: \quad 0 = 0.48\omega_{BD} - u\sin\beta \tag{2}$$

Solving (1) and (2), $\omega_{BD} = 3.8080$ rad/s, $u = 6.528$ m/s

PROBLEM 15.175 (Continued)

Acceleration analysis. $\boldsymbol{\alpha}_{AC} = 0, \qquad \boldsymbol{\alpha}_{BD} = \alpha_{BD} \circlearrowright$

$$\mathbf{a}_A = 0 \qquad \mathbf{a}_{P/A} = r\omega_{AB}^2 = (0.14)(20)^2 = 56 \text{ m/s}^2 \downarrow$$

$$\mathbf{a}_P = \mathbf{a}_A + \mathbf{a}_{P/A} = 56 \text{ m/s}^2 \downarrow$$

$$\mathbf{a}_{P'} = [x_P\alpha_{BD} \downarrow] + [e\alpha_B \longrightarrow] + [x_P\omega_{BD}^2 \longleftarrow] + [e\omega_{BD}^2 \downarrow]$$

$$= [0.48\alpha_{BD} \downarrow] + [0.14\alpha_{BD} \longrightarrow] + [(0.48)(3.8080)^2 \longleftarrow]$$

$$+ [(0.14)(3.8080)^2 \downarrow]$$

$$= [0.48\alpha_{BD} \downarrow] + [0.14\alpha_{BD} \longrightarrow] + [6.9604 \text{ m/s}^2 \longleftarrow]$$

$$+ [2.0301 \text{ m/s}^2 \downarrow]$$

$$\mathbf{a}_{P/F} = [\dot{u}\cos\beta \longrightarrow] + [\dot{u}\sin\beta \uparrow]$$

Coriolis acceleration.

$$2\omega_{BD}u = (2)(3.8080)(6.528) = [49.717 \text{ m/s}^2 \searrow \beta]$$

Use $\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + [2\omega_{BD}u \searrow \beta]$ and resolve into components.

$$\xrightarrow{+}: \quad 0 = 0.14\alpha_{BD} - 6.9604 + \dot{u}\cos\beta + 49.717\sin\beta$$

or $\qquad 0.14\alpha_{BD} + \dot{u}\cos\beta = -6.9602 \qquad (3)$

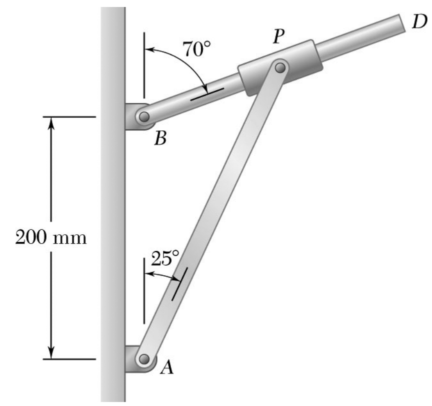
$$+\downarrow: \quad 56 = 0.48\alpha_{BD} + 2.0301 + \dot{u}\sin\beta + 49.717\cos\beta$$

or $\qquad 0.48\alpha_{BD} - \dot{u}\sin\beta = 6.2415 \qquad (4)$

Solving (3) and (4), $\qquad \alpha_{BD} = 8.09 \text{ rad/s}, \qquad \dot{u} = -8.43 \text{ m/s}^2$

(*a*) $\qquad \boldsymbol{\alpha}_{BD} = 8.09 \text{ rad/s}^2 \circlearrowright$ ◀

(*b*) $\qquad \mathbf{a}_{P/F} = 8.43 \text{ m/s}^2 \measuredangle\ 16.26°$ ◀



PROBLEM 15.176

Knowing that at the instant shown the rod attached at A has an angular velocity of 5 rad/s counterclockwise and an angular acceleration of 2 rad/s^2 clockwise, determine the angular velocity and the angular acceleration of the rod attached at B.

SOLUTION

Geometry: Apply the law of sines to the triangle ABP to determine the lengths $\overline{AP}$ and $\overline{BP}$.

$$\text{Angle } PBA = 180° - 70° = 110° \qquad \text{Angle } APB = 180° - 25° - 110° = 45°$$

$$\overline{AB} = 200 \text{ mm} = 0.2 \text{ m} \qquad \frac{0.2}{\sin 45°} = \frac{\overline{AP}}{\sin 110°} = \frac{\overline{BP}}{\sin 25°}$$

$$\overline{AP} = 0.265785 \text{ m} \qquad \overline{BP} = 0.119534$$

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

Relative position vectors:

$$\mathbf{r}_{P/A} = 0.265785(\sin 25°\mathbf{i} + \cos 25°\mathbf{j}) = 0.11233\mathbf{i} - 0.24088\mathbf{j}$$
$$\mathbf{r}_{P/B} = 0.119534(\sin 70°\mathbf{i} + \cos 70°\mathbf{j}) = 0.11233\mathbf{i} + 0.04088\mathbf{j}$$

Angular velocities: $\boldsymbol{\omega}_{AP} = 5 \text{ rad/s} \circlearrowleft = (5 \text{ rad/s})\mathbf{k}$

$$\boldsymbol{\omega}_{BP} = \omega_{BP}\mathbf{k}$$

Angular accelerations: $\boldsymbol{\alpha}_{AP} = 2 \text{ rad/s}^2 \circlearrowright = -(2 \text{ rad/s}^2)\mathbf{k}$

$$\boldsymbol{\alpha}_{BP} = \alpha_{BP}\mathbf{k}$$

Velocity of P:

$$\mathbf{v}_P = \boldsymbol{\omega}_{AP} \times \mathbf{r}_{P/A} = 5\mathbf{k} \times (0.11233\mathbf{i} + 0.24088\mathbf{j})$$
$$= -(1.2044 \text{ m/s})\mathbf{i} + (0.56165 \text{ m/s})\mathbf{j}$$

Acceleration of P:

$$\mathbf{a}_P = \boldsymbol{\alpha}_{AP} \times \mathbf{r}_{P/A} - \omega_{AP}^2\mathbf{r}_{P/A}$$
$$= (-2\mathbf{k}) \times (0.11233\mathbf{i} + 0.24088\mathbf{j}) - (5)^2(0.11233\mathbf{i} + 0.24088\mathbf{j})$$
$$= -(2.3265 \text{ m/s}^2)\mathbf{i} - (6.2467 \text{ m/s}^2)\mathbf{j}$$

Consider the slider P as a particle sliding along the rotating rod BP with a relative velocity

$$\mathbf{v}_{\text{rel}} = u \measuredangle 20° = u(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$$

and a relative acceleration

$$\mathbf{a}_{\text{rel}} = \dot{u} \measuredangle 20° = \dot{u}(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$$

Consider the rod *BP* as a rotating frame of reference.

Motion of Point *P′* on the rod currently at *P*.

$$\mathbf{v}_{P'} = \boldsymbol{\omega}_{BP} \times \mathbf{r}_{P/B} = \omega_{BP}\mathbf{k} \times (0.11233\mathbf{i} + 0.04088\mathbf{j})$$
$$= -0.04088\omega_{BP}\mathbf{i} + 0.11233\omega_{BP}\mathbf{j}$$

$$\mathbf{a}_{P'} = \boldsymbol{\alpha}_{BP} \times \mathbf{r}_{P/B} - \omega_{BP}^2\mathbf{r}_{P/B}$$
$$= \alpha_{BP}\mathbf{k} \times (0.11233\mathbf{i} + 0.04088\mathbf{j}) - \omega_{BP}^2(0.11233\mathbf{i} + 0.04088\mathbf{j})$$
$$= -0.04088\alpha_{BP}\mathbf{i} + 0.11233\alpha_{BP}\mathbf{j} - 0.11233\omega_{BP}^2\mathbf{i} - 0.04088\omega_{BP}^2\mathbf{j}$$

Velocity of *P*: $\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{\text{rel}}$

Resolve into components.

i: $-1.2044 = -0.04088\omega_{BP} + u\cos 20°$

j: $0.56165 = 0.11233\omega_{BP} + u\sin 20°$

Solving the simultaneous equations for ω_{BP} and u,

$$\omega_{BP} = 7.8612 \text{ rad/s} \qquad u = -0.93971 \text{ m/s}$$

Angular velocity of BP: $\boldsymbol{\omega}_{BP} = 7.86 \text{ rad/s}$ ↻ ◀

Relative velocity: $\mathbf{v}_{\text{rel}} = -0.93971(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$

Coriolis acceleration:

$$2\boldsymbol{\omega}_{BP} \times \mathbf{v}_{\text{rel}} = (2)(7.8612\mathbf{k}) \times (-0.93971\cos 20°\mathbf{i} - 0.93971\sin 20°\mathbf{j})$$
$$= (5.0532 \text{ m/s}^2)\mathbf{i} - (13.8835 \text{ m/s}^2)\mathbf{j}$$

Acceleration of *P*: $\mathbf{a}_{P'} = \mathbf{a}_{P'} + \mathbf{a}_{\text{rel}} + 2\boldsymbol{\omega}_{BP} \times \mathbf{v}_{\text{rel}}$

Resolve into components.

i: $-2.3265 = -0.04088\alpha_{BP} - 0.11233\omega_{BP}^2 + \dot{u}\cos 20° + 5.0532$

$$-0.04088\alpha_{BP} + \dot{u}\cos 20° = -0.43788 \qquad (1)$$

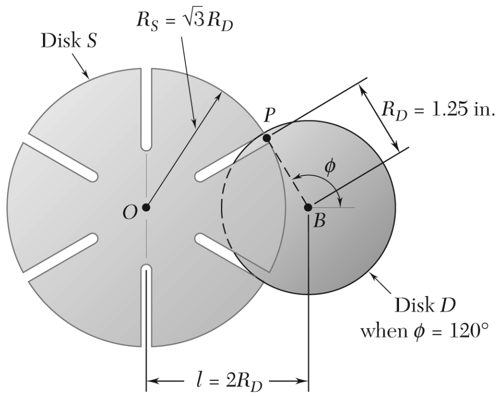
j: $-6.2467 = 0.11233\alpha_{BP} - 0.04088\omega_{BP}^2 + \dot{u}\sin 20° - 13.8835$

$$0.11233\alpha_{BP} + \dot{u}\sin 20° = 10.1631 \qquad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$\alpha_{BP} = 81.146 \text{ rad/s}^2 \qquad \dot{u} = 3.0641 \text{ m/s}^2$$

Angular acceleration of BP: $\boldsymbol{\alpha}_{BP} = 81.1 \text{ rad/s}^2$ ↻ ◀



PROBLEM 15.177

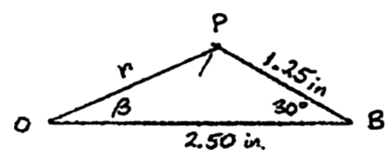
The Geneva mechanism shown is used to provide an intermittent rotary motion of disk S. Disk D rotates with a constant counterclockwise angular velocity ω_D of 8 rad/s. A pin P is attached to disk D and can slide in one of the six equally spaced slots cut in disk S. It is desirable that the angular velocity of disk S be zero as the pin enters and leaves each of the six slots; this will occur if the distance between the centers of the disks and the radii of the disks are related as shown. Determine the angular velocity and angular acceleration of disk S at the instant when $\phi = 150°$.

SOLUTION

Geometry:

Law of cosines. $\quad r^2 = 1.25^2 + 2.50^2 - (2)(1.25)(2.50)\cos 30°$

$$r = 1.54914 \text{ in.}$$



Law of sines. $\quad \dfrac{\sin\beta}{1.25} = \dfrac{\sin 30°}{r}$

$$\beta = 23.794°$$

Let disk S be a rotating frame of reference.

$$\mathbf{\Omega} = \omega_S \circlearrowright, \quad \dot{\mathbf{\Omega}} = \alpha_S \circlearrowright$$

Motion of coinciding Point P′ on the disk.

$$\mathbf{v}_{P'} = r\omega_S = 1.54914\omega_S \;\searrow\; \beta$$

$$\mathbf{a}_{P'} = -\alpha_S \mathbf{k}\times \mathbf{r}_{P/O} - \omega_S^2 \mathbf{r}_{P/O} = [1.54914\alpha_S \;\searrow\; \beta] + [1.54914\omega_S^2 \;\swarrow\; \beta]$$

Motion relative to the frame.

$$\mathbf{v}_{P/S} = u \;\swarrow\; \beta \quad \mathbf{a}_{P/S} = \dot{u} \;\swarrow\; \beta$$

Coriolis acceleration. $\quad 2\omega_S u \;\searrow\; \beta$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/S} = [1.54914\omega_S \;\searrow\; \beta] + [u \;\swarrow\; \beta]$$

$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/S} + 2\omega_S u \;\searrow$$

$$= [1.54914\alpha_S \;\searrow\; \beta] + [1.54914\omega_S^2 \;\swarrow\; \beta] + [\dot{u} \;\swarrow\; \beta] + [2\omega_S u \;\searrow\; \beta]$$

Motion of disk D. (Rotation about B)

$$\mathbf{v}_P = (BP)\omega_D = (1.25)(8) = 10 \text{ in./s} \;\swarrow\; 30°$$

$$\mathbf{a}_P = [(BP)\alpha_D \;\swarrow\; 60°] + [(BP)\omega_S^2 \;\searrow\; 30°] = 0 + [(1.25)(8)^2 \;\searrow\; 30°]$$

$$= 80 \text{ in./s}^2 \;\searrow\; 30°$$

PROBLEM 15.177 (Continued)

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\beta:\quad 1.54914\omega_S = 10\cos(30° + \beta)$$

$$\omega_S = \frac{10\cos 53.794°}{1.54914}$$

$$= 3.8130 \text{ rad/s}$$

$\boldsymbol{\omega}_S = 3.81$ rad/s ◀

$$\beta:\quad u = 10\sin(30° + \beta) = 10\sin 53.794° = 8.0690 \text{ in./s}$$

Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

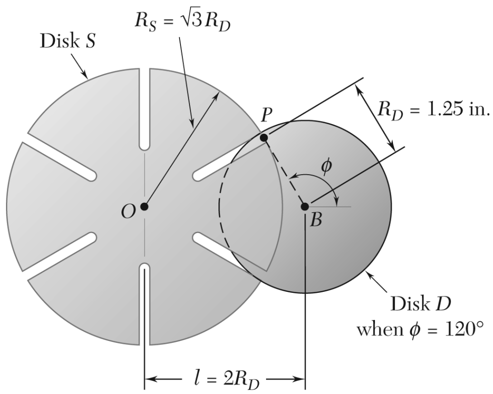
$$\beta:\quad 1.54914\alpha_S - 2\omega_S u = 80\sin(30° + \beta)$$

$$\alpha_S = \frac{80\sin 53.794° + (2)(3.8130)(8.0690)}{1.54914}$$

$$= 81.4 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_S = 81.4$ rad/s^2 ◀

PROBLEM 15.178

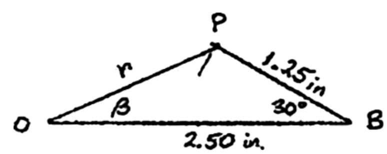


In Problem 15.177, determine the angular velocity and angular acceleration of disk S at the instant when $\phi = 135°$.

PROBLEM 15.177 The Geneva mechanism shown is used to provide an intermittent rotary motion of disk S. Disk D rotates with a constant counterclockwise angular velocity ω_D of 8 rad/s. A pin P is attached to disk D and can slide in one of the six equally spaced slots cut in disk S. It is desirable that the angular velocity of disk S be zero as the pin enters and leaves each of the six slots; this will occur if the distance between the centers of the disks and the radii of the disks are related as shown. Determine the angular velocity and angular acceleration of disk S at the instant when $\phi = 150°$.

SOLUTION

Geometry:



Law of cosines.

$$r^2 = 1.25^2 + 2.50^2 - (2)(1.25)(2.50)\cos 45°$$

$$r = 1.84203 \text{ in.}$$

Law of sines.

$$\frac{\sin\beta}{1.25} = \frac{\sin 45°}{r}$$

$$\beta = 28.675°$$

Let disk S be a rotating frame of reference.

$$\mathbf{\Omega} = \omega_S \circlearrowright, \quad \dot{\mathbf{\Omega}} = \alpha_S \circlearrowright$$

Motion of coinciding Point P′ on the disk.

$$\mathbf{v}_{P'} = r\omega_s = 1.84203\omega_s \searrow \beta$$

$$\mathbf{a}_{P'} = -\alpha_S \mathbf{k} \times \mathbf{r}_{P/O} - \omega_S^2 \mathbf{r}_{P/O} = [1.84203\alpha_S \searrow \beta] + [1.84203\omega_S^2 \swarrow \beta]$$

Motion relative to the frame.

$$\mathbf{v}_{P/S} = u \swarrow \beta \qquad \mathbf{a}_{P/S} = \dot{u} \swarrow \beta$$

Coriolis acceleration. $\qquad 2\omega_S u \nwarrow \beta$

$$\mathbf{v}_P = \mathbf{v}_{P'} + \mathbf{v}_{P/S} = [1.84203\omega_S \searrow \beta] + [u \swarrow \beta]$$

$$\mathbf{a}_P = \mathbf{a}_{P} + \mathbf{a}_{P/S} + 2\omega_S u \nwarrow$$

$$= [1.84203\alpha_S \searrow \beta] + [1.84203\omega_S^2 \swarrow \beta] + [\dot{u} \swarrow \beta] + [2\omega_S u \nwarrow \beta]$$

Motion of disk D. (Rotation about B)

$$\mathbf{v}_P = (BP)\omega_D = (1.25)(8) = 10 \text{ in./s} \swarrow 30^\circ$$

$$\mathbf{a}_P = [(BP)\alpha_D \searrow 45^\circ] + [(BP)\omega_S^2 \searrow 45^\circ] = 0 + [(1.25)(8)^2 \searrow 45^\circ]$$

$$= 80 \text{ in./s}^2 \searrow 45^\circ$$

Equate the two expressions for $\mathbf{v}_P$ and resolve into components.

$$\searrow \beta: \quad 1.84203\omega_S = 10\cos(45^\circ + \beta)$$

$$\omega_S = \frac{10\cos 73.675^\circ}{1.84203}$$

$$= 1.52595 \text{ rad/s} \qquad \boldsymbol{\omega}_S = 1.526 \text{ rad/s} \circlearrowright \blacktriangleleft$$

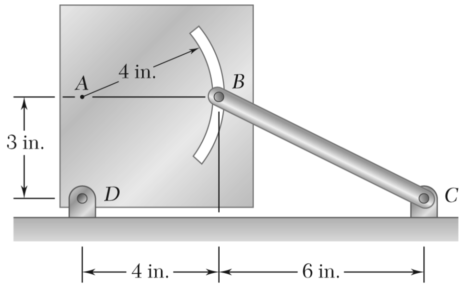
$$\swarrow \beta: \quad u = 10\sin(45^\circ + \beta) = 10\sin 73.675^\circ = 9.5968 \text{ in./s}$$

Equate the two expressions for $\mathbf{a}_P$ and resolve into components.

$$\searrow \beta: \quad 1.84203\alpha_S - 2\omega_S u = 80\sin(45^\circ + \beta)$$

$$\alpha_S = \frac{80\sin 73.675^\circ + (2)(1.52595)(9.5968)}{1.84203}$$

$$= 57.6 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_S = 57.6 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$



PROBLEM 15.179

At the instant shown, bar BC has an angular velocity of 3 rad/s and an angular acceleration of $2\ \text{rad/s}^2$, both counterclockwise. Determine the angular acceleration of the plate.

SOLUTION

Relative position vectors.
$$\mathbf{r}_{B/D} = (4\text{ in.})\mathbf{i} + (3\text{ in.})\mathbf{j}$$
$$\mathbf{r}_{B/C} = -(6\text{ in.})\mathbf{i} + (3\text{ in.})\mathbf{j}$$

Velocity analysis.
$$\boldsymbol{\omega}_{BC} = 3\text{ rad/s} \circlearrowleft$$

Bar BC (Rotation about C):
$$\boldsymbol{\omega}_{BC} = (3\text{ rad/s})\mathbf{k}$$
$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C} = 3\mathbf{k} \times (-6\mathbf{i} + 3\mathbf{j})$$
$$= -(9\text{ in./s})\mathbf{i} - (18\text{ in./s})\mathbf{j}$$

Plate (Rotation about D):
$$\omega_P = \omega_P\mathbf{k}$$

Let Point B' be the point in the plate coinciding with B.
$$\mathbf{v}_{B'} = \boldsymbol{\omega}_P \times \mathbf{r}_{B/D} = \omega_P\mathbf{k} \times (4\mathbf{i} + 3\mathbf{j})$$
$$= -3\omega_P\mathbf{i} + 4\omega_P\mathbf{j}$$

Let plate be a rotating frame.
$$\mathbf{v}_{B/F} = v_{\text{rel}}\mathbf{j}$$
$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$
$$= -3\omega_P\mathbf{i} + (4\omega_P + v_{\text{rel}})\mathbf{j}$$

Equate like components of $\mathbf{v}_B$.
$$\mathbf{i}:\quad -9 = -3\omega_P \qquad \boldsymbol{\omega}_P = (3\text{ rad/s})\mathbf{k}$$
$$\mathbf{j}:\quad -18 = (4)(3) + v_{\text{rel}} \qquad \mathbf{v}_{\text{rel}} = -(30\text{ in./s})\mathbf{j}$$

Acceleration analysis.
$$\boldsymbol{\alpha}_{BC} = 2\text{ rad/s}^2 \circlearrowleft$$

Bar BC:
$$\boldsymbol{\alpha}_{BC} = (2\text{ rad/s}^2)\mathbf{k}$$
$$\mathbf{a}_B = \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\,\mathbf{r}_{B/C}$$
$$= 2\mathbf{k} \times (-6\mathbf{i} + 3\mathbf{j}) - (3)^2(-6\mathbf{i} + 3\mathbf{j})$$
$$= -12\mathbf{j} - 6\mathbf{i} + 54\mathbf{i} - 27\mathbf{j} = (48\text{ in./s}^2)\mathbf{i} - (39\text{ in./s}^2)\mathbf{j}$$

Plate:
$$\boldsymbol{\alpha}_P = \alpha_P\mathbf{k}$$
$$\mathbf{a}_{B'} = \boldsymbol{\alpha}_P \times \mathbf{r}_{B/D} - \omega_P^2\,\mathbf{r}_{B/D}$$
$$= \alpha_P\mathbf{k} \times (4\mathbf{i} + 3\mathbf{j}) - (3)^2(4\mathbf{i} + 3\mathbf{j})$$
$$= -3\alpha_P\mathbf{i} + 4\alpha_P\mathbf{j} - 36\mathbf{i} - 27\mathbf{j}$$

PROBLEM 15.179 (Continued)

Relative to the frame (plate), the acceleration of pin B is

$$\mathbf{a}_{B/F} = (a_{\text{rel}})_t\mathbf{j} - \frac{v_{\text{rel}}^2}{\rho}\mathbf{i} = (a_{\text{rel}})_t\,\mathbf{j} - \frac{30^2}{4}\mathbf{i}$$

$$= -(225 \text{ in./s}^2)\mathbf{i} + (a_{\text{rel}})_t\,\mathbf{j}$$

Coriolis acceleration. $\quad 2\boldsymbol{\omega}_P \times \mathbf{v}_{P/F}$

$$\mathbf{a}_c = 2(3\mathbf{k})\times(-30\mathbf{j}) = (180 \text{ in./s}^2)\mathbf{i}$$

Then

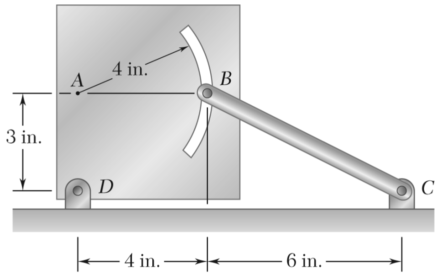
$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_c$$

$$\mathbf{a}_B = -(3\alpha_P + 36)\mathbf{i} + (4\alpha_P - 27)\mathbf{j} - 225\mathbf{i} + (a_{\text{rel}})_t\,\mathbf{j} + 180\mathbf{i}$$

$$\mathbf{a}_B = -(3\alpha_P + 81)\mathbf{i} + [4\alpha_P + (a_{\text{rel}})_t - 27]\mathbf{j}$$

Equate like components of $\mathbf{a}_B$.

$\mathbf{i}$: $\quad 48 = -(3\alpha_P + 81) \qquad \alpha_P = -43 \text{ rad/s}^2 \qquad \boldsymbol{\alpha}_P = 43.0 \text{ rad/s}^2$ ◀



PROBLEM 15.180

At the instant shown bar BC has an angular velocity of 3 rad/s and an angular acceleration of 2 rad/s^2, both clockwise. Determine the angular acceleration of the plate.

SOLUTION

Relative position vectors.

$$\mathbf{r}_{B/D} = (4\text{ in.})\mathbf{i} + (3\text{ in.})\mathbf{j}$$

$$\mathbf{r}_{B/C} = -(6\text{ in.})\mathbf{i} + (3\text{ in.})\mathbf{j}$$

Velocity analysis.

$$\boldsymbol{\omega}_{BC} = 3\text{ rad/s} \circlearrowright$$

Bar BC (Rotation about C):

$$\boldsymbol{\omega}_{BC} = -(3\text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

$$= (-3\mathbf{k}) \times (-6\mathbf{i} + 3\mathbf{j})$$

$$= (9\text{ in./s})\mathbf{i} + (18\text{ in./s})\mathbf{j}$$

Plate (Rotation about D):

$$\omega_P = \omega_P\mathbf{k}$$

Let Point B' be the point in the plate coinciding with B:

$$\mathbf{v}_{B'} = \boldsymbol{\omega}_P \times \mathbf{r}_{B/D}$$

$$= \omega_P\mathbf{k} \times (4\mathbf{i} + 3\mathbf{j})$$

$$= -3\omega_P\mathbf{i} + 4\omega_P\mathbf{j}$$

Let the plate be a rotating frame.

$$\mathbf{v}_{B/F} = v_{\text{rel}}\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$

$$= -3\omega_P\mathbf{i} + (4\omega_P + v_{\text{rel}})\mathbf{j}$$

Equate like components of $\mathbf{v}_B$.

$\mathbf{i}$: $\quad 9 = -3\omega_P \qquad \boldsymbol{\omega}_P = -(3\text{ rad/s})\mathbf{k}$

$\mathbf{j}$: $\quad 18 = (4)(3) + v_{\text{rel}} \qquad \mathbf{v}_{\text{rel}} = (30\text{ in./s})\mathbf{j}$

Acceleration analysis.

$$\boldsymbol{\alpha}_{BC} = 2\text{ rad/s}^2 \circlearrowright$$

Bar BC:

$$\boldsymbol{\alpha}_{BC} = -(2\text{ rad/s}^2)\mathbf{k}$$

$$\mathbf{a}_B = \boldsymbol{\alpha}_{BC} \times \mathbf{r}_{B/C} - \omega_{BC}^2\mathbf{r}_{B/C}$$

$$= (-2\mathbf{k}) \times (-6\mathbf{i} + 3\mathbf{j}) - (3)^2(-6\mathbf{i} + 3\mathbf{j})$$

$$= 12\mathbf{j} + 6\mathbf{i} + 54\mathbf{i} - 27\mathbf{j}$$

$$= (60\text{ in./s}^2)\mathbf{i} - (15\text{ in./s}^2)\mathbf{j}$$

Plate:
$$\boldsymbol{\alpha}_P = \alpha_P \mathbf{k}$$
$$\mathbf{a}_{B'} = \boldsymbol{\alpha}_P \times \mathbf{r}_{B/D} - \omega_P^2 \mathbf{r}_{B/D}$$
$$= \alpha_P \mathbf{k} \times (4\mathbf{i} + 3\mathbf{j}) - (3)^2 (4\mathbf{i} + 3\mathbf{j})$$
$$= -3\alpha_P \mathbf{i} + 4\alpha_P \mathbf{j} - 36\mathbf{i} - 27\mathbf{j}$$

Relative to the frame (plate), the acceleration of pin B is

$$\mathbf{a}_{B/F} = (a_{\text{rel}})_t \mathbf{j} - \frac{v_{\text{rel}}^2}{\rho} \mathbf{i}$$
$$= (a_{\text{rel}})_t \mathbf{j} - \frac{30^2}{4} \mathbf{i}$$
$$= -(225 \text{ in./s}^2)\mathbf{i} + (a_{\text{rel}})_t \mathbf{j}$$

Coriolis acceleration. $2\boldsymbol{\omega}_P \times \mathbf{v}_{P/F}$

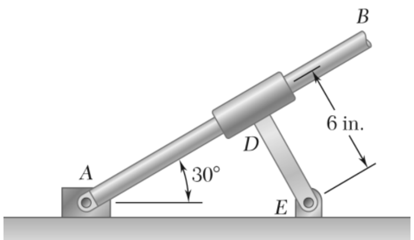
$$\mathbf{a}_c = 2(-3\mathbf{k}) \times (30\mathbf{j}) = (180 \text{ in./s}^2)\mathbf{i}$$

Then
$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_c$$
$$\mathbf{a}_B = -(3\alpha_P + 36)\mathbf{i} + (4\alpha_P - 27)\mathbf{j} - 225\mathbf{i} + (a_{\text{rel}})_t \mathbf{j} + 180\mathbf{i}$$
$$\mathbf{a}_B = -(3\alpha_P + 81)\mathbf{i} + [4\alpha_P + (a_{\text{rel}})_t - 27]\mathbf{j}$$

Equate like components of $\mathbf{a}_B$.

$\mathbf{i}$: $60 = -(3\alpha_P + 81)$ $\quad \alpha_P = -47 \text{ rad/s}^2$ $\quad \boldsymbol{\alpha}_P = 47.0 \text{ rad/s}^2$ ◀

PROBLEM 15.181*



Rod AB passes through a collar which is welded to link DE. Knowing that at the instant shown block A moves to the right at a constant speed of 75 in./s, determine (a) the angular velocity of rod AB, (b) the velocity relative to the collar of the point of the rod in contact with the collar, (c) the acceleration of the point of the rod in contact with the collar. (*Hint:* Rod AB and link DE have the same $\boldsymbol{\omega}$ and the same $\boldsymbol{\alpha}$.)

SOLUTION

Let $\boldsymbol{\omega} = \omega \circlearrowright$ and $\boldsymbol{\alpha} = \alpha \circlearrowright$ be the angular velocity and angular acceleration of the link DE and collar rigid body. Let F be a frame of reference moving with this body. The rod AB slides in the collar relative to the frame of reference with relative velocity $\mathbf{u} = u \measuredangle 30°$ and relative acceleration $\dot{\mathbf{u}} = \dot{u} \measuredangle 30°$. Note that this relative motion is a translation that applies to all points along the rod. Let Point A be moving with the end of the rod and A' be moving with the frame. Point E is a fixed point.

Geometry.
$$\mathbf{r}_{A'/E} = \frac{6 \text{ in.}}{\sin 30°} = 12 \text{ in.} \longleftarrow$$

Velocity analysis.
$$\mathbf{v}_A = 75 \text{ in./s} \longrightarrow$$
$$\mathbf{v}_{A'} = 12\omega \downarrow$$
$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{u} \qquad \text{Resolve into components.}$$

$$\xrightarrow{+}: \quad 75 = 0 + u\cos 30° \qquad u = \frac{75}{\cos 30°} = 86.603 \text{ in./s}$$

$$+\uparrow: \quad 0 = -12\omega + u\sin 30° \qquad \omega = \frac{u\sin 30°}{12} = 3.6085 \text{ rad/s}$$

(a) Angular velocity. $\boldsymbol{\omega} = 3.61 \text{ rad/s} \circlearrowright$ ◀

(b) Velocity of rod AB relative to the collar. $\mathbf{u} = 86.6 \text{ in./s} \measuredangle 30°$ ◀

(c) Acceleration analysis. $\mathbf{a}_A = 0$

$$\mathbf{a}_{A'} = [12\alpha \downarrow] + [12\omega^2 \longrightarrow] = [12\alpha \downarrow] + [156.25 \longrightarrow]$$

Coriolis acceleration.

$$\mathbf{a}_c = 2\omega u \searrow 60° = 625.01 \text{ in./s}^2 \searrow 60°$$
$$\mathbf{a}_A = \mathbf{a}_{A'} + \dot{\mathbf{u}} + \mathbf{a}_c \qquad \text{Resolve into components.}$$

$$\xrightarrow{+}: \quad 0 = 156.25 + \dot{u}\cos 30° - 625.01\cos 60°$$
$$\dot{u} = 180.43 \text{ in./s}^2$$
$$+\uparrow: \quad 0 = -12\alpha + \dot{u}\sin 30° + 625.01\sin 60°$$
$$\alpha = 52.624 \text{ rad/s}^2$$

For rod *AB*, $\qquad \boldsymbol{\omega}_{AB} = 3.6085 \text{ rad/s} \circlearrowright$

$$\boldsymbol{\alpha}_{AB} = 52.624 \text{ rad/s}^2 \circlearrowright$$

Let *P* be the point on *AB* coinciding with collar *D*.

$$\mathbf{r}_{P/A} = 12\cos 30^\circ \measuredangle 30^\circ = 10.392 \text{ in.} \measuredangle 30^\circ.$$

$$\mathbf{a}_P = \mathbf{a}_A + (\mathbf{a}_{P/A})_t + (\mathbf{a}_{P/A})_n$$

$$= 0 + [(10.392)(52.624) \searrow 60^\circ] + [(10.392)(3.6085)^2 \swarrow 30^\circ]$$

$$= [546.87 \searrow 60^\circ] + [135.32 \swarrow 30^\circ] = [390.63 \longleftarrow] + [405.94 \downarrow]$$

$$\mathbf{a}_P = 563 \text{ in./s}^2 \swarrow 46.1^\circ \blacktriangleleft$$

$\mathbf{a}_P$ may also be determined from $\mathbf{a}_P = \mathbf{a}_{P'} + \dot{\mathbf{u}} + \mathbf{a}_c$ using the rotating frame. The already calculated vectors $\dot{\mathbf{u}}$ and $\mathbf{a}_c$ also apply at Points P' and P.

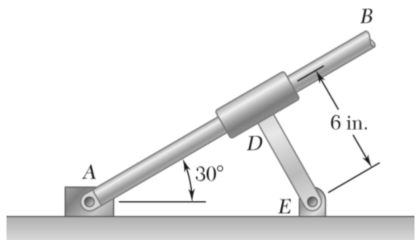
$$\mathbf{a}_{P'} = \mathbf{a}_D = 6\alpha \swarrow 30^\circ + 6\omega \nwarrow 60^\circ$$

$$= [315.74 \text{ in./s}^2 \swarrow 30^\circ] + [78.13 \text{ in./s}^2 \swarrow 60^\circ]$$

Then $\qquad \mathbf{a}_P = [315.74 \swarrow 30^\circ] + [78.13 \nwarrow 60^\circ] + (180.43 \measuredangle 30^\circ] + [625.01 \searrow 60^\circ]$

$$= (135.31 \swarrow 30^\circ] + [546.88 \searrow 60^\circ]$$

PROBLEM 15.182*



Solve Problem 15.181, assuming that block *A* moves to the left at a constant speed of 75 in./s.

PROBLEM 15.181 Rod *AB* passes through a collar which is welded to link *DE*. Knowing that at the instant shown block *A* moves to the right at a constant speed of 75 in./s, determine (*a*) the angular velocity of rod *AB*, (*b*) the velocity relative to the collar of the point of the rod in contact with the collar, (*c*) the acceleration of the point of the rod in contact with the collar. (*Hint:* Rod *AB* and link *DE* have the same $\boldsymbol{\omega}$ and the same $\boldsymbol{\alpha}$.)

SOLUTION

Let $\boldsymbol{\omega} = \omega$ ⤾ and $\boldsymbol{\alpha} = \alpha$ ⤾ be the angular velocity and angular acceleration of the link *DE* and collar rigid body. Let *F* be a frame of reference moving with this body. The rod *AB* slides in the collar relative to the frame of reference with relative velocity $\mathbf{u} = u \measuredangle 30°$ and relative acceleration $\dot{\mathbf{u}} = \dot{u} \measuredangle 30°$. Note that this relative motion is a translation that applies to all points along the rod. Let Point *A* be moving with the end of the rod and A' be moving with the frame. Point *E* is a fixed point.

Geometry.
$$\mathbf{r}_{A'/E} = \frac{6 \text{ in.}}{\sin 30°} = 12 \text{ in.} \longleftarrow$$

Velocity analysis.
$$\mathbf{v}_A = 75 \text{ in./s} \longleftarrow$$
$$\mathbf{v}_{A'} = 12\omega \downarrow$$
$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{u} \quad \text{Resolve into components.}$$

$$\xrightarrow{+}: \quad -75 = 0 + u\cos 30° \qquad u = \frac{75}{\cos 30°} = -86.603 \text{ in./s}$$

$$+\uparrow: \quad 0 = -12\omega + u\sin 30° \qquad \omega = \frac{u\sin 30°}{12} = -3.6085 \text{ rad/s}$$

(*a*) Angular velocity. $\boldsymbol{\omega} = 3.61 \text{ rad/s}$ ⤿ ◀

(*b*) Velocity of rod *AB* relative to the collar. $\mathbf{u} = 86.6 \text{ in./s} \measuredangle 30°$ ◀

Acceleration analysis.
$$\mathbf{a}_A = 0$$
$$\mathbf{a}_{A'} = [12\alpha \downarrow] + [12\omega^2 \longrightarrow] = [12\alpha \downarrow] + [156.25 \longrightarrow]$$

Coriolis acceleration.
$$\mathbf{a}_c = 2\omega u \searrow 60° = 625.01 \text{ in./s}^2 \searrow 60°$$
$$\mathbf{a}_A = \tilde{\mathbf{a}}_{A'} + \dot{\mathbf{u}} + \mathbf{a}_c \quad \text{Resolve into components.}$$

$$\xrightarrow{+}: \quad 0 = 156.25 + \dot{u}\cos 30° - 625.01\cos 60°$$
$$\dot{u} = 180.43 \text{ in./s}^2$$
$$+\uparrow: \quad 0 = -12\alpha + \dot{u}\sin 30° + 625.01\sin 60°$$
$$\alpha = 52.624 \text{ rad/s}^2$$

PROBLEM 15.182* (Continued)

For rod AB, $\quad \omega_{AB} = 3.6085 \text{ rad/s} \circlearrowright$

$$\alpha_{AB} = 52.624 \text{ rad/s}^2 \circlearrowright$$

Let P be the point on AB coinciding with collar D.

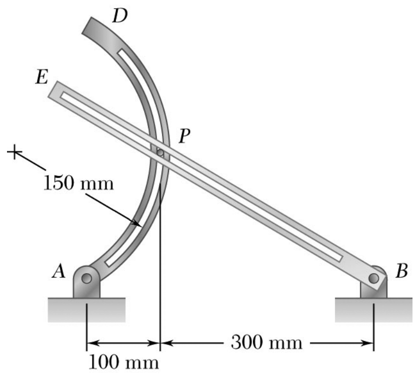
$$\mathbf{r}_{P/A} = 12\cos 30° \measuredangle 30° = 10.392 \text{ in.} \measuredangle 30°.$$

$$\mathbf{a}_P = \mathbf{a}_A + (\mathbf{a}_{P/A})_t + (\mathbf{a}_{P/A})_n$$

$$= 0 + [(10.392)(52.624) \searrow 60°] + [(10.392)(3.6085)^2 \swarrow 30°]$$

$$= [546.87 \searrow 60°] + [135.32 \swarrow 30°] = [390.63 \longleftarrow] + [405.94 \downarrow]$$

$$\mathbf{a}_P = 563 \text{ in./s}^2 \swarrow 46.1° \blacktriangleleft$$



PROBLEM 15.183*

In Problem 15.157, determine the acceleration of pin P.

PROBLEM 15.157 The motion of pin P is guided by slots cut in rods AD and BE. Knowing that bar AD has a constant angular velocity of 4 rad/s clockwise and bar BE has an angular velocity of 5 rad/s counterclockwise and is slowing down at a rate of 2 rad/s^2, determine the velocity of P for the position shown.

SOLUTION

Units: meters, m/s, m/s^2

Unit vectors: $\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

From the solution of Problem 15.157,

$$\theta = 26.565° \qquad R = 0.100 \text{ m}$$

$$\mathbf{r}_{P/A} = 0.1\mathbf{i} + 0.15\mathbf{j} \qquad \mathbf{r}_{P/B} = -0.3\mathbf{i} + 0.15\mathbf{j}$$

$$\boldsymbol{\omega}_{AD} = -(4 \text{ rad/s})\mathbf{k} \qquad \boldsymbol{\omega}_{BE} = (5 \text{ rad/s}^2)\mathbf{k}$$

$$\boldsymbol{\alpha}_{AD} = 0 \qquad \boldsymbol{\alpha}_{BE} = -(2 \text{ rad/s}^2)\mathbf{k}$$

$$\mathbf{v}_{P/AD} = u_1\mathbf{j} = -(1.775 \text{ m/s})\mathbf{j}$$

$$\begin{aligned}\mathbf{v}_{P/BE} &= -u_2 \cos\theta\mathbf{i} + u_2 \sin\theta\mathbf{j} \\ &= (-1.50935)(-\cos 26.565°\mathbf{i} + \sin 26.565°)\mathbf{j} \\ &= (1.35 \text{ m/s})\mathbf{i} + (0.675 \text{ m/s})\mathbf{j}\end{aligned}$$

Acceleration of Point P' on rod AD coinciding with the pin:

$$\begin{aligned}\mathbf{a}_{P'} &= \boldsymbol{\alpha}_{AD} \times \mathbf{r}_{P/A} - \omega_{AD}^2\mathbf{r}_{P/A} \\ &= 0 - (4)^2(0.1\mathbf{i} + 0.15\mathbf{j}) = -1.6\mathbf{i} - 2.4\mathbf{j}\end{aligned}$$

Acceleration of the pin relative to rod AD:

$$\begin{aligned}\mathbf{a}_{P/AD} &= \dot{u}_1\mathbf{j} - \frac{u_1^2}{R}\mathbf{i} = \dot{u}_1\mathbf{j} - \frac{(1.775)^2}{0.15}\mathbf{i} \\ &= \dot{u}_1\mathbf{j} - 21.004\mathbf{i}\end{aligned}$$

Coriolis acceleration: $\mathbf{a}_1 = 2\boldsymbol{\omega}_{AD} \times \mathbf{v}_{P/AD}$

$$\mathbf{a}_1 = 2(-4\mathbf{k}) \times (-1.775\mathbf{j}) = -14.2\mathbf{i}$$

Acceleration of P: $\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/AD} + \mathbf{a}_1$

$$\mathbf{a}_P = -36.804\mathbf{i} - 2.4\mathbf{j} + \dot{u}_1\mathbf{j}$$

PROBLEM 15.183* (Continued)

Acceleration of Point P'' on rod BE coinciding with the pin

$$\mathbf{a}_{P''} = \boldsymbol{\alpha}_{BE} \times \mathbf{r}_{P/B} - \omega_{BE}^2 \mathbf{r}_{P/E}$$
$$= (-2\mathbf{k}) \times (-0.3\mathbf{i} + 0.15\mathbf{j}) - (5)^2(-0.3\mathbf{i} + 0.15\mathbf{j})$$
$$= 0.6\mathbf{j} + 0.3\mathbf{i} + 7.5\mathbf{i} - 3.75\mathbf{j} = 7.8\mathbf{i} - 3.15\mathbf{j}$$

Acceleration of the pin relative to the rod BE:

$$\mathbf{a}_{P/BE} = \dot{u}_2(-\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$$

Coriolis acceleration:

$$\mathbf{a}_2 = 2\boldsymbol{\omega}_{BE} \times \mathbf{v}_{P/BE}$$
$$= (2)(5\mathbf{k}) \times (1.35\mathbf{i} - 0.675\mathbf{j}) = -13.5\mathbf{j} + 6.75\mathbf{i}$$

Acceleration of P:

$$\mathbf{a}_P = \mathbf{a}_{P''} + \mathbf{a}_{P/BE} + \mathbf{a}_2$$

$$\mathbf{a}_P = 7.8\mathbf{i} - 3.15\mathbf{j} + \dot{u}_2(-\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) + 13.5\mathbf{j} + 6.75\mathbf{i}$$
$$= 14.55\mathbf{i} + 10.35\mathbf{j} - \dot{u}_2\cos\theta\mathbf{i} + \dot{u}_2\sin\theta\mathbf{j}$$

Equating the two expressions for $\mathbf{a}_P$ and resolving into components,

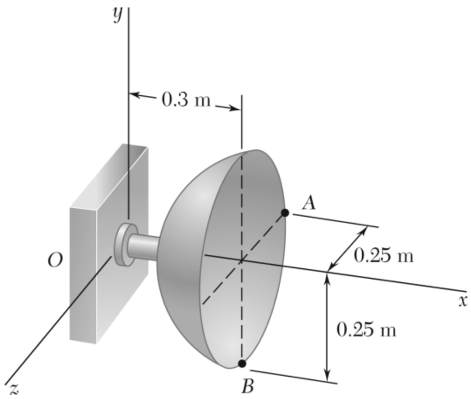
i:
$$-36.804 = 14.55 - \dot{u}_2\cos\theta$$
$$\dot{u}_2 = \frac{36.804 + 14.55}{\cos 26.565^\circ} = 57.415 \text{ m/s}^2$$

j:
$$-2.4 + \dot{u}_1 = 10.35 + \dot{u}_2\sin\theta$$
$$\dot{u}_1 = 12.75 + 57.415\sin 26.565^\circ = 38.426 \text{ m/s}^2$$

Acceleration of the pin.

$$\mathbf{a}_P = -36.804\mathbf{i} - 2.4\mathbf{j} + 38.427\mathbf{j}$$
$$= -36.804\mathbf{i} + 36.027\mathbf{j}$$

$$\mathbf{a}_P - (36.8 \text{ m/s}^2)\mathbf{i} + (36.0 \text{ m/s}^2)\mathbf{j} = 51.5 \text{ m/s}^2 \measuredangle 44.4^\circ \blacktriangleleft$$



PROBLEM 15.184

At the instant considered, the radar antenna shown rotates about the origin of coordinates with an angular velocity $\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$. Knowing that $(v_A)_y = 300$ mm/s, $(v_B)_y = 180$ mm/s, and $(v_B)_z = 360$ mm/s, determine (*a*) the angular velocity of the antenna, (*b*) the velocity of Point *A*.

SOLUTION

$$\mathbf{r}_A = (0.3 \text{ m})\mathbf{i} - (0.25 \text{ m})\mathbf{k}$$
$$\mathbf{v}_A = (v_A)_x\mathbf{i} + (0.3 \text{ m/s})\,\mathbf{j} + (v_A)_z\mathbf{k}$$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A: \quad (v_A)_x\mathbf{i} + 0.3\mathbf{j} + (v_A)_z\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & 0 & -0.25 \end{vmatrix}$$

$$(v_A)_x\mathbf{i} + 0.3\mathbf{j} + (v_A)_z\mathbf{k} = -0.25\omega_y\mathbf{i} + (0.3\omega_z + 0.25\omega_x)\mathbf{j} - 0.3\omega_y\mathbf{k}$$

$\mathbf{i}$: $\quad (v_A)_x = -0.25\omega_y$ (1)

$\mathbf{j}$: $\quad 0.3 = 0.3\omega_z + 0.25\omega_x$ (2)

$\mathbf{k}$: $\quad (v_A)_z = -0.3\omega_y$ (3)

$$\mathbf{r}_B = (0.3 \text{ m})\mathbf{i} - (0.25 \text{ m})\mathbf{j}$$
$$\mathbf{v}_B = (v_B)_x\mathbf{i} + (0.18 \text{ m/s})\mathbf{j} + (0.36 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B: \quad (v_B)_x\mathbf{i} + 0.18\mathbf{j} + 0.36\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & -0.25 & 0 \end{vmatrix}$$

$$(v_B)_x\mathbf{i} + 0.18\mathbf{j} + 0.36\mathbf{k} = 0.25\omega_z\mathbf{i} + 0.3\omega_z\mathbf{j} - (0.25\omega_x + 0.3\omega_y)\mathbf{k}$$

$\mathbf{i}$: $\quad (v_B)_x = 0.3\omega_z$ (4)

$\mathbf{j}$: $\quad 0.18 = 0.3\omega_z$ (5)

$\mathbf{k}$: $\quad 0.36 = -0.25\omega_x - 0.3\omega_y$ (6)

From Eq. (5), $\quad \omega_z = 0.6$ rad/s

PROBLEM 15.184 (Continued)

From Eq. (2),
$$\omega_x = \frac{1}{0.25}(0.3 - 0.3\omega_z)$$
$$= 0.48 \text{ rad/s}$$

From Eq. (6),
$$\omega_y = -\frac{1}{0.3}(0.36 + 0.25\omega_x)$$
$$= -1.6 \text{ rad/s}$$

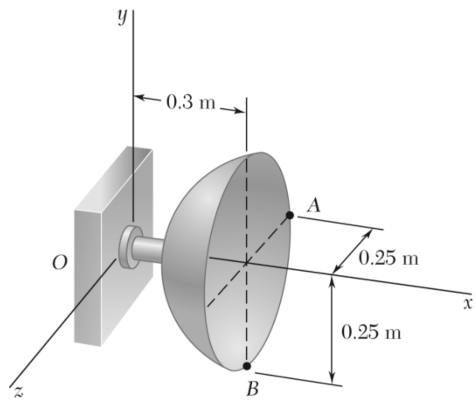
(*a*) *Angular velocity.* $\boldsymbol{\omega} = (0.480 \text{ rad/s})\mathbf{i} - (1.600 \text{ rad/s})\mathbf{j} + (0.600 \text{ rad/s})\mathbf{k}$ ◀

From Eq. (1),
$$(v_A)_x = -0.25\omega_y$$
$$= 0.400 \text{ m/s}$$

From Eq. (3),
$$(v_A)_z = -0.3\omega_y$$
$$= 0.480 \text{ m/s}$$

(*b*) *Velocity of Point A.* $\mathbf{v}_A = (0.400 \text{ m/s})\mathbf{i} + (0.300 \text{ m/s})\mathbf{j} + (0.480 \text{ m/s})\mathbf{k}$

or $\mathbf{v}_A = (400 \text{ mm/s})\mathbf{i} + (300 \text{ mm/s})\mathbf{j} + (480 \text{ mm/s})\mathbf{k}$ ◀



PROBLEM 15.185

At the instant considered the radar antenna shown rotates about the origin of coordinates with an angular velocity $\boldsymbol{\omega} = \omega_x \mathbf{i} + \omega_y \mathbf{j} + \omega_z \mathbf{k}$. Knowing that $(v_A)_x = 100$ mm/s, $(v_A)_y = -90$ mm/s, and $(v_B)_z = 120$ mm/s, determine (*a*) the angular velocity of the antenna, (*b*) the velocity of Point *A*.

SOLUTION

$$\mathbf{r}_A = (0.3 \text{ m})\mathbf{i} - 0.25 \text{ m})\mathbf{k}$$

$$\mathbf{v}_A = (0.1 \text{ m/s})\mathbf{i} - (0.09 \text{ m/s})\mathbf{j} + (v_A)_z \mathbf{k}$$

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A: \quad 0.1\mathbf{i} - 0.09\mathbf{j} + (v_A)_z \mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & 0 & -0.25 \end{vmatrix}$$

$$0.1\mathbf{i} - 0.09\mathbf{j} + (v_A)_z \mathbf{k} = -0.25\omega_y \mathbf{i} + (0.3\omega_z + 0.25\omega_x)\mathbf{j} - 0.3\omega_y \mathbf{k}$$

$$\mathbf{i}: \quad 0.1 = -0.25\omega_y \tag{1}$$

$$\mathbf{j}: \quad -0.09 = 0.3\omega_z + 0.25\omega_x \tag{2}$$

$$\mathbf{k}: \quad (v_A)_z = -0.3\omega_y \tag{3}$$

$$\mathbf{r}_B = (0.3 \text{ m})\mathbf{i} - (0.25 \text{ m})\mathbf{j}$$

$$\mathbf{v}_B = (v_B)_x \mathbf{i} + (v_B)_y \mathbf{j} + (0.12 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega} \times \mathbf{r}_B: \quad (v_B)_x \mathbf{i} + (v_B)_y \mathbf{j} + 0.12\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.3 & -0.25 & 0 \end{vmatrix}$$

$$(v_B)_x \mathbf{i} + (v_B)_y \mathbf{j} + 0.12\mathbf{k} = 0.25\omega_z \mathbf{i} + 0.3\omega_z \mathbf{j} - (0.25\omega_x + 0.3)\omega_y \mathbf{k}$$

$$\mathbf{i}: \quad (v_B)_x = 0.25\omega_z \tag{4}$$

$$\mathbf{j}: \quad (v_B)_y = 0.3\omega_z \tag{5}$$

$$\mathbf{k}: \quad 0.12 = -0.25\omega_x - 0.3\omega_y \tag{6}$$

From Eq. (1), $\qquad \omega_y = -\dfrac{0.1}{0.25} = -0.4 \text{ rad/s}$

PROBLEM 15.185 (Continued)

From Eq. (6),
$$\omega_x = -\frac{1}{0.25}(0.12 + 0.3\omega_y)$$
$$= 0$$

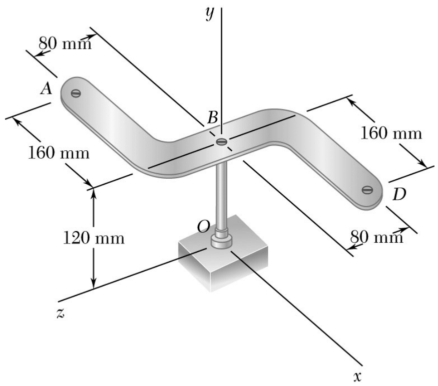
From Eq. (2),
$$\omega_z = -\frac{1}{0.25}(0.09 + 0.25\omega_x)$$
$$= -0.36 \text{ rad/s}$$

From Eq. (3),
$$(v_A)_z = -(0.3)(-0.4)$$
$$= 0.12 \text{ m/s}$$

(*a*) Angular velocity. $\boldsymbol{\omega} = -(0.400 \text{ rad/s})\mathbf{j} - (0.360 \text{ rad/s})\mathbf{k}$ ◀

(*b*) Velocity of Point *A*. $\mathbf{v}_A = (0.1 \text{ m/s})\mathbf{i} - (0.09 \text{ m/s})\mathbf{j} + (0.12 \text{ m/s})\mathbf{k}$

or $\mathbf{v}_A = (100 \text{ mm/s})\mathbf{i} - (90 \text{ mm/s})\mathbf{j} + (120 \text{ mm/s})\mathbf{k}$ ◀



PROBLEM 15.186

Plate ABD and rod OB are rigidly connected and rotate about the ball-and-socket joint O with an angular velocity $\boldsymbol{\omega} = \omega_x\,\mathbf{i} + \omega_x\,\mathbf{j} + \omega_z\,\mathbf{k}$. Knowing that $\mathbf{v}_A = (80 \text{ mm/s})\mathbf{i} + (360 \text{ mm/s})\mathbf{j} + (v_A)_z\,\mathbf{k}$ and $\omega_x = 1.5$ rad/s, determine (*a*) the angular velocity of the assembly, (*b*) the velocity of Point D.

SOLUTION

$$\omega_x = 1.5 \text{ rad/s} \qquad \boldsymbol{\omega} = (1.5 \text{ rad/s})\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{r}_A = -(160 \text{ mm})\mathbf{i} + (120 \text{ mm})\mathbf{j} + (80 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_D = +(160 \text{ mm})\mathbf{i} + (120 \text{ mm})\mathbf{j} - (80 \text{ mm})\mathbf{k}$$

(*a*)

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & \omega_y & \omega_z \\ -160 & +120 & +80 \end{vmatrix}$$

$$\mathbf{v}_A = (80\omega_y - 120\omega_z)\mathbf{i} + (-160\omega_z - 120)\mathbf{j} + (180 + 160\omega_y)\mathbf{k}$$

But we are given:

$$\mathbf{v}_A = (80 \text{ mm/s})\mathbf{i} + (360 \text{ mm/s})\mathbf{j} + (v_A)_z\mathbf{k}$$

$$(v_A)_x: \quad 80\omega_y - 120\omega_z = 80 \tag{1}$$

$$(v_A)_y: \quad -160\omega_x - 120 = 360 \qquad \omega_z = -3 \text{ rad/s} \tag{2}$$

$$(v_A)_z: \quad 180 + 160\omega_y = (v_A)_z \tag{3}$$

Substitute $\omega_z = -3.0$ rad/s into Eq. (1):

$$80\omega_y - 120(-3) = 80$$

$$\omega_y = -3.5 \text{ rad/s}$$

Substitute $\omega_y = -3.5$ rad/s into Eq. (3):

$$180 + 160(-3.5) = (v_A)_z$$

$$(v_A)_z = -380 \text{ in./s}$$

We have: $\boldsymbol{\omega} = (1.5 \text{ rad/s})\mathbf{i} - (3.5 \text{ rad/s})\mathbf{j} - (3.0 \text{ rad/s})\mathbf{k}$ ◀

PROBLEM 15.186 (Continued)

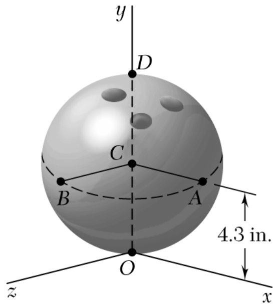
(*b*) Velocity of *D*.

$$\mathbf{v}_D = \boldsymbol{\omega} \times \mathbf{r}_D$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & -3.5 & -3.0 \\ +160 & +120 & -80 \end{vmatrix}$$

$$= (360 + 280)\mathbf{i} + (-480 + 120)\mathbf{j} + (180 + 560)\mathbf{k}$$

$$\mathbf{v}_D = (640 \text{ mm/s})\mathbf{i} - (360 \text{ mm/s})\mathbf{j} + (740 \text{ mm/s})\mathbf{k} \blacktriangleleft$$



PROBLEM 15.187

The bowling ball shown rolls without slipping on the horizontal xz plane with an angular velocity $\omega = \omega_x \mathbf{i} + \omega_y \mathbf{j} + \omega_z \mathbf{k}$. Knowing that $\mathbf{v}_A = (14.4 \text{ ft/s})\mathbf{i} - (14.4 \text{ ft/s})\mathbf{j} + (10.8 \text{ ft/s})\mathbf{k}$ and $\mathbf{v}_D = (28.8 \text{ ft/s})\mathbf{i} + (21.6 \text{ ft/s})\mathbf{k}$, determine (*a*) the angular velocity of the bowling ball, (*b*) the velocity of its center C.

SOLUTION

Radius of ball: 4.3 in. = 0.35833 ft

At the given instant, the origin is not moving.

$$\mathbf{v}_A = \boldsymbol{\omega} \times \mathbf{r}_A: \quad 14.4\mathbf{i} - 14.4\mathbf{j} + 10.8\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0.35833 & 0.35833 & 0 \end{vmatrix}$$

$$14.4\mathbf{i} - 14.4\mathbf{j} + 10.8\mathbf{k} = -0.35833\omega_z\mathbf{i} + 0.35833\omega_z\mathbf{j} + 0.35833(\omega_x - \omega_y)\mathbf{k}$$

$$\mathbf{i}: \quad -0.35833\omega_z = 14.4 \qquad \omega_z = -40.186 \text{ rad/s}$$

$$\mathbf{j}: \quad 0.35833\omega_z = -14.4 \qquad \omega_z = -40.186 \text{ rad/s}$$

$$\mathbf{k}: \quad 0.35833(\omega_x - \omega_y) = 10.8 \qquad \omega_x - \omega_y = 30.140 \text{ rad/s}$$

$$\mathbf{v}_D = \boldsymbol{\omega} \times \mathbf{r}_D: \quad 28.8\mathbf{i} + 21.6\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ 0 & 0.71667 & 0 \end{vmatrix}$$

$$28.8\mathbf{i} + 21.6\mathbf{k} = -0.71667\omega_z\mathbf{i} + 0.71667\omega_x\mathbf{k}$$

$$\mathbf{i}: \quad -0.71667\omega_z = 28.8 \qquad \omega_z = -40.186 \text{ rad/s}$$

$$\mathbf{k}: \quad 0.71667\omega_x = 21.6 \qquad \omega_x = 30.140 \text{ rad/s}$$

$$\omega_y = \omega_x - 30.140 = 0$$

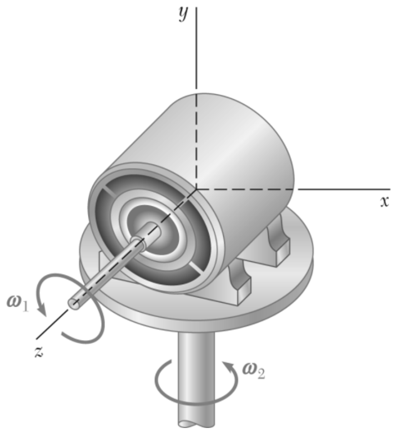
(*a*) *Angular velocity.*

$$\boldsymbol{\omega} = (30.1 \text{ rad/s})\mathbf{i} - (40.2 \text{ rad/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Velocity of Point C.*

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C = (30.140\mathbf{i} - 40.186\mathbf{k}) \times 0.35833\mathbf{j}$$
$$= 14.4\mathbf{i} + 10.8\mathbf{k}$$

$$\mathbf{v}_C = (14.4 \text{ ft/s})\mathbf{i} + (10.8 \text{ ft/s})\mathbf{k} \blacktriangleleft$$



PROBLEM 15.188

The rotor of an electric motor rotates at the constant rate $\omega_1 = 1800$ rpm. Determine the angular acceleration of the rotor as the motor is rotated about the y axis with a constant angular velocity $\boldsymbol{\omega}_2$ of 6 rpm counterclockwise when viewed from the positive y axis.

SOLUTION

$$\omega_1 = 1800 \text{ rpm}$$
$$= 60\pi \text{ rad/s}$$
$$\omega_2 = 6 \text{ rpm}$$
$$= 0.2\pi \text{ rad/s}$$

Total angular velocity.

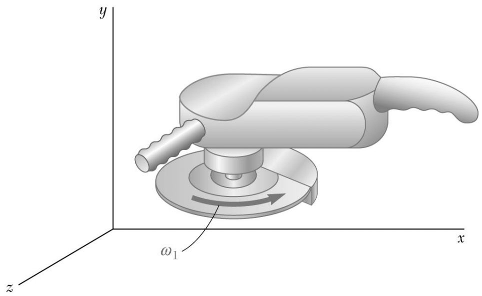
$$\boldsymbol{\omega} = \omega_2\mathbf{j} + \omega_1\mathbf{k}$$
$$\boldsymbol{\omega} = (0.2\pi \text{ rad/s})\mathbf{j} + (60\pi \text{ rad/s})\mathbf{k}$$

Angular acceleration.

Frame $Oxyz$ is rotating with angular velocity $\boldsymbol{\Omega} = \omega_2\mathbf{j}$.

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$$
$$= \dot{\boldsymbol{\omega}}_{Oxyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}$$
$$= 0 + \omega_2\mathbf{j} \times (\omega_2\mathbf{j} + \omega_1\mathbf{k})$$
$$= \omega_2\omega_1\mathbf{i}$$
$$\boldsymbol{\alpha} = (0.2\pi)(60\pi)\mathbf{i}$$
$$= (12\pi^2 \text{ rad/s}^2)\mathbf{i}$$

$$\boldsymbol{\alpha} = (118.4 \text{ rad/s}^2)\mathbf{i} \blacktriangleleft$$



PROBLEM 15.189

The disk of a portable sander rotates at the constant rate $\omega_1 = 4400$ rpm as shown. Determine the angular acceleration of the disk as a worker rotates the sander about the z axis with an angular velocity of 0.5 rad/s and an angular acceleration of 2.5 rad/s^2, both clockwise when viewed from the positive z axis.

SOLUTION

Spin rate: $\omega_1 = 4400 \text{ rpm} = 460.77 \text{ rad/s}$

Angular velocity of disk relative to the housing:

$$\boldsymbol{\omega}_1 = (460.77 \text{ rad/s})\,\mathbf{j}$$

Angular motion of the housing:

$$\boldsymbol{\omega}_2 = -(0.5 \text{ rad/s})\mathbf{k} \qquad \dot{\boldsymbol{\omega}}_2 = -(2.5 \text{ rad/s}^2)\mathbf{k}$$

Consider a frame of reference rotating with angular velocity

$$\boldsymbol{\Omega} = \omega_2\mathbf{k} = -(0.5 \text{ rad/s})\mathbf{k}$$

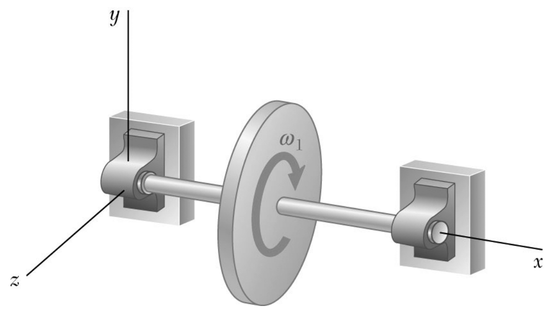
Angular velocity of the disk:

$$\begin{aligned} \boldsymbol{\omega} &= \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 \\ &= (460.77 \text{ rad/s})\mathbf{j} - (0.5 \text{ rad/s})\mathbf{k} \end{aligned}$$

Angular acceleration of the disk:

$$\begin{aligned} \boldsymbol{\alpha} &= \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2 + \boldsymbol{\Omega}\times(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \\ &= 0 - 2.5\mathbf{k} + (-0.5\mathbf{k})\times(460.77\mathbf{j} - 0.5\mathbf{k}) \\ &= (230.38 \text{ rad/s}^2)\mathbf{i} - (2.5 \text{ rad/s}^2)\mathbf{k} \end{aligned}$$

$$\boldsymbol{\alpha} = (230 \text{ rad/s}^2)\mathbf{i} - (2.5 \text{ rad/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.190

Knowing that the turbine rotor shown rotates at a constant rate $\omega_1 = 9000$ rpm, determine the angular acceleration of the rotor if the turbine housing has a constant angular velocity of 2.4 rad/s clockwise as viewed from (*a*) the positive *y* axis, (*b*) the positive *z* axis.

SOLUTION

Spin rate: $\omega_1 = 9000 \text{ rpm} = 942.48 \text{ rad/s}$

Angular velocity of the rotor relative to the axle:

$$\boldsymbol{\omega}_1 = -(942.48 \text{ rad/s})\mathbf{i}$$

(*a*) Axle rotates with angular velocity $\boldsymbol{\omega}_2 = -(2.4 \text{ rad/s})\mathbf{j}$

Consider a frame of reference rotating with angular velocity

$$\boldsymbol{\Omega} = \omega_2 \mathbf{j}$$

Angular acceleration:

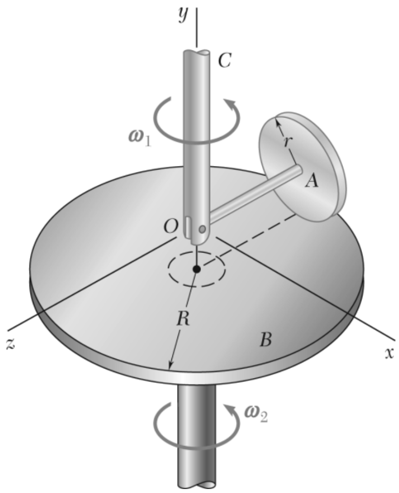
$$\begin{aligned}\boldsymbol{\alpha} &= \dot{\boldsymbol{\omega}}_1\mathbf{i} + \dot{\boldsymbol{\omega}}_2\mathbf{j} + \boldsymbol{\Omega}\times(\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \\ &= 0 + 0 + \boldsymbol{\Omega}\times\boldsymbol{\omega}_1 \\ &= (-2.4\mathbf{j})\times(-942.48\mathbf{i})\end{aligned}$$

$$\boldsymbol{\alpha} = -(2260 \text{ rad/s}^2)\mathbf{k} \blacktriangleleft$$

(*b*) Axle rotates with angular velocity $\boldsymbol{\omega}_2 = -(2.4 \text{ rad/s})\mathbf{k}$.

$$\boldsymbol{\Omega} = -(2.4 \text{ rad/s})\mathbf{k}$$
$$\boldsymbol{\alpha} = \boldsymbol{\Omega}\times\boldsymbol{\omega}_1 = (-2.4\mathbf{k})\times(-942.48\mathbf{i})$$

$$\boldsymbol{\alpha} = (2260 \text{ rad/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.191

In the system shown, disk A is free to rotate about the horizontal rod OA. Assuming that disk B is stationary $(\omega_2 = 0)$, and that shaft OC rotates with a constant angular velocity ω_1, determine (a) the angular velocity of disk A, (b) the angular acceleration of disk A.

SOLUTION

Disk A (In rotation about O):

Since
$$\omega_y = \omega_1,$$
$$\boldsymbol{\omega}_A = \omega_x \mathbf{i} + \omega_1 \mathbf{j} + \omega_z \mathbf{k}$$

Point D is point of contact of wheel and disk.

$$\mathbf{r}_{D/O} = -r\mathbf{j} - R\mathbf{k}$$
$$\mathbf{v}_D = \boldsymbol{\omega}_A \times \mathbf{r}_{D/O}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_1 & \omega_z \\ 0 & -r & -R \end{vmatrix}$$
$$\mathbf{v}_D = (-R\omega_1 + r\omega_z)\mathbf{i} + R\omega_1 \mathbf{j} - r\omega_x \mathbf{k}$$

Since $\omega_2 = 0$, $\mathbf{v}_D = 0$.

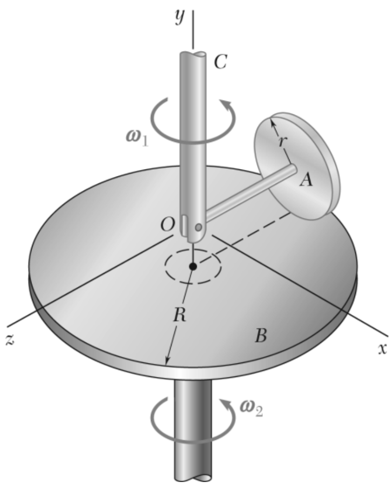
Each component of $\mathbf{v}_D$ is zero.

$$(v_D)_z = r\omega_x = 0; \quad \omega_x = 0$$
$$(v_D)_x = -R\omega_1 + r\omega_z = 0; \quad \omega_z = \left(\frac{R}{r}\right)\omega_1$$

(a) Angular velocity.
$$\boldsymbol{\omega}_A = \omega_1 \mathbf{j} + \left(\frac{R}{r}\right)\omega_1 \mathbf{k} \blacktriangleleft$$

(b) Angular acceleration. Disk A rotates about y axis at rate ω_1.

$$\boldsymbol{\alpha}_A = \frac{d\boldsymbol{\omega}_A}{dt} = \boldsymbol{\omega}_y \times \boldsymbol{\omega}_A = \omega_1 \mathbf{j} \times \left(\omega_1 \mathbf{j} + \frac{R}{r}\omega_1 \mathbf{k}\right) \qquad \boldsymbol{\alpha}_A = \frac{R}{r}\omega_1^2 \mathbf{i} \blacktriangleleft$$



PROBLEM 15.192

In the system shown, disk A is free to rotate about the horizontal rod OA. Assuming that shaft OC and disk B rotate with constant angular velocities ω_1 and ω_2, respectively, both counterclockwise, determine (*a*) the angular velocity of disk A, (*b*) the angular acceleration of disk A.

SOLUTION

Disk A (in rotation about O):

Since $\omega_y = \omega_1$, $\qquad \boldsymbol{\omega}_A = \omega_x \mathbf{i} + \omega_1 \mathbf{j} + \omega_z \mathbf{k}$

Point D is point of contact of wheel and disk.

$$\mathbf{r}_{D/O} = -r\mathbf{j} - R\mathbf{k}$$

$$\mathbf{v}_D = \boldsymbol{\omega}_A \times \mathbf{r}_{D/O} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_1 & \omega_z \\ 0 & -r & -R \end{vmatrix}$$

$$\mathbf{v}_D = (-R\omega_1 + r\omega_z)\mathbf{i} + R\omega_x \mathbf{j} - r\omega_x \mathbf{k} \qquad (1)$$

Disk B: $\qquad \boldsymbol{\omega}_B = \omega_2 \mathbf{j}$

$$\mathbf{v}_D = \boldsymbol{\omega}_B \times \mathbf{r}_{D/O} = \omega_2 \mathbf{j} \times (-r\mathbf{j} - R\mathbf{k}) = -R\omega_2 \mathbf{i} \qquad (2)$$

From Eqs. 1 and 2: $\qquad \mathbf{v}_D = \mathbf{v}_D: \quad (-R\omega_1 - r\omega_z)\mathbf{i} + R\omega_x \mathbf{j} - r\omega_x \mathbf{k} = -R\omega_2 \mathbf{i}$

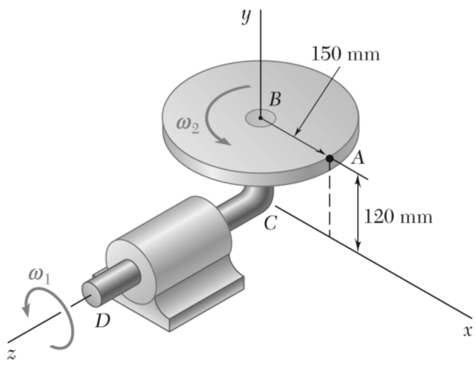
Coefficients of **k**: $\qquad -r\omega_x = 0; \qquad \omega_x = 0$

Coefficients of **i**: $\qquad (-R\omega_1 + r\omega_z) = -R\omega_2; \quad \omega_z = \dfrac{R}{r}(\omega_1 - \omega_2)$

(*a*) Angular velocity. $\qquad \boldsymbol{\omega}_A = \omega_1 \mathbf{j} + \dfrac{R}{r}(\omega_1 - \omega_2)\mathbf{k}$ ◀

(*b*) Angular acceleration. Disk A rotates about y axis at rate ω_1.

$$\boldsymbol{\alpha}_A = \frac{d\boldsymbol{\omega}_A}{dt} = \boldsymbol{\omega}_y \times \boldsymbol{\omega}_A = \omega_1 \mathbf{j} \times \left[\omega_1 \mathbf{j} + \frac{R}{r}(\omega_1 - \omega_2)\mathbf{k}\right] \qquad \boldsymbol{\alpha}_A = \frac{R}{r}\omega_1(\omega_1 - \omega_2)\mathbf{i} \text{ ◀}$$



PROBLEM 15.193

The L-shaped arm *BCD* rotates about the *z* axis with a constant angular velocity ω_1 of 5 rad/s. Knowing that the 150-mm-radius disk rotates about *BC* with a constant angular velocity ω_2 of 4 rad/s, determine (*a*) the velocity of Point *A*, (*b*) the acceleration of Point *A*.

SOLUTION

Total angular velocity. $\boldsymbol{\omega} = \omega_2 \mathbf{j} + \omega_1 \mathbf{k}$

$$\boldsymbol{\omega} = (4 \text{ rad/s})\mathbf{j} + (5 \text{ rad/s})\mathbf{k}$$

Angular acceleration.

Frame *Oxyz* is rotating with angular velocity $\boldsymbol{\Omega} = \omega_1 \mathbf{k}$.

$$\begin{aligned}
\alpha &= \dot{\boldsymbol{\omega}} \\
&= \dot{\boldsymbol{\omega}}_{Oxyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega} \\
&= 0 + \omega_1 \mathbf{k} \times (\omega_2 \mathbf{j} + \omega_1 \mathbf{k}) \\
&= -\omega_1 \omega_2 \mathbf{i} \\
\alpha &= -(5)(4)\mathbf{i} \\
&= -20\mathbf{i}
\end{aligned}$$

$$\boldsymbol{\alpha} = -(20.0 \text{ rad/s}^2)\mathbf{i}$$

(*a*) *Velocity of Point A.* $\mathbf{r}_A = (0.15 \text{ m})\mathbf{i} + (0.12 \text{ m})\mathbf{j}$

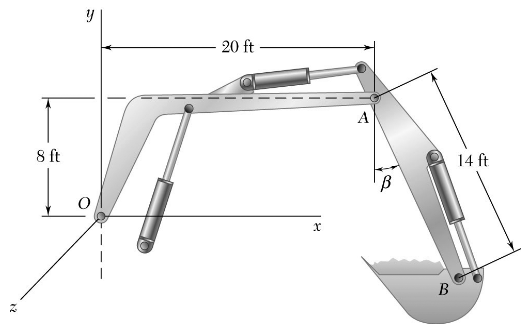
$$\begin{aligned}
\mathbf{v}_A &= \boldsymbol{\omega} \times \mathbf{r}_A \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 5 \\ 0.15 & 0.12 & 0 \end{vmatrix} \\
&= -0.6\mathbf{i} + 0.75\mathbf{j} - 0.6\mathbf{k}
\end{aligned}$$

$$\mathbf{v}_A = -(0.600 \text{ m/s})\mathbf{i} + (0.750 \text{ m/s})\mathbf{j} - (0.600 \text{ m/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Acceleration of Point A.* $\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times \mathbf{v}_A$

$$\begin{aligned}
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -20 & 0 & 0 \\ 0.15 & 0.6 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 5 \\ -0.6 & 0.75 & -0.6 \end{vmatrix} \\
&= -2.4\mathbf{k} - 6.15\mathbf{i} - 3\mathbf{j} + 2.4\mathbf{k} \\
&= -6.15\mathbf{i} - 3\mathbf{j}
\end{aligned}$$

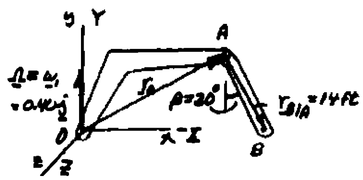
$$\mathbf{a}_A = -(6.15 \text{ m/s}^2)\mathbf{i} - (3.00 \text{ m/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.194

The cab of the backhoe shown rotates with the constant angular velocity $\boldsymbol{\omega}_1 = (0.4 \text{ rad/s})\mathbf{j}$ about the y axis. The arm OA is fixed with respect tot he cab, while the arm AB rotates about the horizontal axle A at the constant rate $\omega_2 = d\beta/dt = 0.6$ rad/s. Knowing that $\beta = 30°$, determine (a) the angular velocity and angular acceleration of AB, (b) the velocity and acceleration of Point B.

SOLUTION



$$\mathbf{r}_A = 20\mathbf{i} + 8\mathbf{j} \quad \text{(ft)}$$
$$\mathbf{r}_{B/A} = 7\mathbf{i} - 12.12\mathbf{j} \quad \text{(ft)}$$
$$\mathbf{r}_B = 27\mathbf{i} - 4.12\mathbf{j} \quad \text{(ft)}$$

O_{XYZ} is fixed; O_{xyz} rotates with $\boldsymbol{\Omega} = 0.40\mathbf{j}$

Angular velocity of AB

With respect to rotating frame: $\boldsymbol{\omega}_2 = +(0.60 \text{ rad/s})\mathbf{k}$

With respect to fixed frame: $\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = (0.40 \text{ rad/s})\mathbf{j} + (0.60 \text{ rad/s})\mathbf{k}$ ◀

Angular acceleration of AB

$$\boldsymbol{\alpha} = (\dot{\boldsymbol{\omega}})_{Oxyz} = (\dot{\boldsymbol{\omega}})_{Oxyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}$$

$$\boldsymbol{\alpha} = 0 + (0.40\mathbf{j}) \times (0.40\mathbf{j} + 0.60\mathbf{k}) \qquad \boldsymbol{\alpha} = (0.24 \text{ rad/s}^2)\mathbf{i} \blacktriangleleft$$

Motion of B relative to rotating frame O_{xyz}.

Since A does not move relative to O_{xyz},

$$(\mathbf{v}_{B/F}) = (\dot{\mathbf{r}}_B)_{Oxyz} = (\dot{\mathbf{r}}_A)_{Oxyz} + (\dot{\mathbf{r}}_{B/A})_{Oxyz} = 0 + \boldsymbol{\omega}' \times \mathbf{r}_{B/A}$$
$$= (0.60\mathbf{k}) \times (7\mathbf{i} - 12.12\mathbf{j})$$
$$\mathbf{v}_{B/F} = (7.27 \text{ ft/s})\mathbf{i} + (4.2 \text{ ft/s})\mathbf{j} \qquad (1)$$

$$(\mathbf{a}_{B/F}) = (\dot{\mathbf{r}}_A)_{Oxyz} + (\dot{\mathbf{r}}_{B/A})_{Oxyz} = 0 + \boldsymbol{\omega}' \times (\boldsymbol{\omega}' \times \mathbf{r}_{B/A})$$
$$= (0.60\mathbf{k}) \times (7.27\mathbf{i} - 4.2\mathbf{j})$$
$$\mathbf{a}_{B/F} = (2.52 \text{ ft/s}^2)\mathbf{i} + (4.36 \text{ ft/s}^2)\mathbf{j} \qquad (2)$$

Motion of B' of frame O_{xyz} which coincides with B.

$$\mathbf{v}_{B'} = \boldsymbol{\Omega} \times \mathbf{r}_B = (0.40\mathbf{j}) \times (27\mathbf{i} - 4.12\mathbf{j})$$
$$\mathbf{v}_{B'} = -(10.8 \text{ ft/s})\mathbf{k} \qquad (3)$$

$$\mathbf{a}_{B'} = \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_B) = \boldsymbol{\Omega} \times \mathbf{v}_{B'} = (0.4\mathbf{j}) \times (-10.8\mathbf{k})$$
$$\mathbf{a}_{B'} = -(4.32 \text{ ft/s}^2)\mathbf{i} \qquad (4)$$

PROBLEM 15.194 (Continued)

Velocity of B using Equations (1) and (3):

$$\mathbf{v}_{B'} = \mathbf{v}_{B'} + \mathbf{v}_{B/F} = -10.8\mathbf{k} + 7.27\mathbf{i} + 4.2\mathbf{j}$$

$$\mathbf{v}_B = (7.27 \text{ ft/s})\mathbf{i} + (4.2 \text{ ft/s})\mathbf{j} - (10.8 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

Acceleration of B

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_C$$

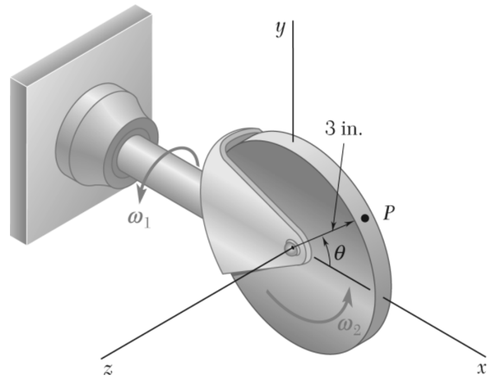
We first compute the Coriolis acceleration. $\mathbf{a}_C$

$$\mathbf{a}_C = 2\mathbf{\Omega} \times \mathbf{v}_{B/F} = 2(0.40\mathbf{j}) \times (7.27\mathbf{i} + 4.2\mathbf{j})$$

Recalling Equations (2) and (4), we now write

$$\mathbf{a}_B = -4.32\mathbf{i} - 2.52\mathbf{i} + 4.36\mathbf{j} - 5.82\mathbf{k}$$

$$\mathbf{a}_B = -(6.84 \text{ ft/s}^2)\mathbf{i} + (4.36 \text{ ft/s}^2)\mathbf{j} - (5.82 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.195

A 3-in.-radius disk spins at the constant rate $\omega_2 = 4$ rad/s about an axis held by a housing attached to a horizontal rod that rotates at the constant rate $\omega_1 = 5$ rad/s. For the position shown, determine (*a*) the angular acceleration of the disk, (*b*) the acceleration of Point *P* on the rim of the disk if $\theta = 0$, (*c*) the acceleration of Point *P* on the rim of the disk if $\theta = 90°$.

SOLUTION

Angular velocity.

$$\boldsymbol{\omega} = \omega_1\mathbf{i} + \omega_2\mathbf{k}$$
$$\boldsymbol{\omega} = (5 \text{ rad/s})\mathbf{i} + (4 \text{ rad/s})\mathbf{k}$$

(*a*) *Angular acceleration.*

Frame *Oxyz* is rotating with angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{i}$.

$$\begin{aligned}\alpha &= \dot{\boldsymbol{\omega}} \\ &= \dot{\boldsymbol{\omega}}_{Oxyz} + \boldsymbol{\Omega}\times\boldsymbol{\omega} \\ &= 0 + \omega_1\mathbf{i}\times(\omega_1\mathbf{i} + \omega_2\mathbf{k}) \\ &= -\omega_1\omega_2\mathbf{j} \\ &= -(4)(5)\mathbf{j} \\ &= -20\mathbf{j}\end{aligned}$$

$\boldsymbol{\alpha} = -(20.0 \text{ rad/s}^2)\mathbf{j}$ ◀

(*b*) $\theta = 0$. *Acceleration at Point P.*

$$\begin{aligned}\mathbf{r}_P &= (3 \text{ in.})\mathbf{i} \\ &= (0.25 \text{ ft})\mathbf{i} \\ \mathbf{v}_P &= \boldsymbol{\omega}\times\mathbf{r}_P \\ &= (5\mathbf{i} + 4\mathbf{k})\times 0.25\mathbf{i} \\ &= (1 \text{ ft/s})\mathbf{j} \\ \mathbf{a}_P &= \boldsymbol{\alpha}\times\mathbf{r}_P + \boldsymbol{\omega}\times\mathbf{v}_P \\ &= -20\mathbf{j}\times 0.25\mathbf{i} + (5\mathbf{i} + 4\mathbf{k})\times(1 \text{ ft/s})\mathbf{j} \\ &= 5\mathbf{k} + 5\mathbf{k} - 4\mathbf{i} \\ &= -4\mathbf{i} + 10\mathbf{k}\end{aligned}$$

$\mathbf{a}_P - (4.00 \text{ ft/s}^2)\mathbf{i} + (10.00 \text{ ft/s}^2)\mathbf{k}$ ◀

(*c*) $\theta = 90°$. *Acceleration at Point P.*

$$\mathbf{r}_P = (0.25 \text{ ft})\mathbf{j}$$

$$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}_P$$

$$= (5\mathbf{i} + 4\mathbf{k}) \times 0.25\mathbf{j}$$

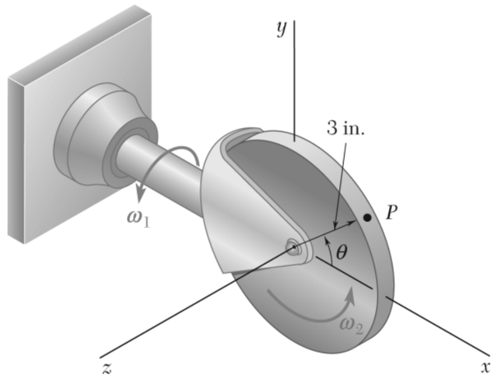
$$= -(1.25 \text{ ft/s})\mathbf{i} + (1 \text{ ft/s})\mathbf{j}$$

$$\mathbf{a}_P = \boldsymbol{\alpha} \times \mathbf{r}_P + \boldsymbol{\omega} \times \mathbf{v}_P$$

$$= -20\mathbf{j} \times 0.25\mathbf{j} + (5\mathbf{i} + 4\mathbf{k}) \times (-1.25\mathbf{i} + \mathbf{j})$$

$$= 0 + 0 - 6.25\mathbf{j} - 4\mathbf{j} + 0$$

$$= -10.25\mathbf{j} \qquad \mathbf{a}_P = -(10.25 \text{ ft/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.196

A 3-in.-radius disk spins at the constant rate $\omega_2 = 4$ rad/s about an axis held by a housing attached to a horizontal rod that rotates at the constant rate $\omega_1 = 5$ rad/s. Knowing that $\theta = 30°$, determine the acceleration of Point P on the rim of the disk.

SOLUTION

Angular velocity.

$$\boldsymbol{\omega} = \omega_1 \mathbf{i} + \omega_2 \mathbf{k}$$

$$\boldsymbol{\omega} = (5 \text{ rad/s})\mathbf{i} + (4 \text{ rad/s})\mathbf{k}$$

Angular acceleration. Frame *Oxyz* is rotating with angular velocity $\Omega = \omega_1 \mathbf{i}$.

$$\begin{aligned}\boldsymbol{\alpha} &= \dot{\boldsymbol{\omega}} = \dot{\boldsymbol{\omega}}_{Oxyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega} \\ &= 0 + \omega_1 \mathbf{i} \times (\omega_1 \mathbf{i} + \omega_2 \mathbf{k}) \\ &= -\omega_1 \omega_2 \mathbf{j} \\ &= -(4)(5)\mathbf{j} = -20\mathbf{j}\end{aligned}$$

$$\boldsymbol{\alpha} = -(20.0 \text{ rad/s}^2)\mathbf{j}$$

Geometry. $\theta = 30°$,

$$\begin{aligned}\mathbf{r}_P &= (3 \text{ in.})(\cos 30° \mathbf{i} + \sin 30° \mathbf{j}) \\ &= (0.25 \text{ ft})(\cos 30° \mathbf{i} + \sin 30° \mathbf{j})\end{aligned}$$

Velocity of Point P.

$$\mathbf{v}_P = \boldsymbol{\omega} \times \mathbf{r}_P$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 4 \\ 0.25\cos 30° & 0.25\sin 30° & 0 \end{vmatrix}$$

$$= -(0.5 \text{ ft/s})\mathbf{i} + (0.86603 \text{ ft/s})\mathbf{j} + (0.625 \text{ ft/s})\mathbf{k}$$

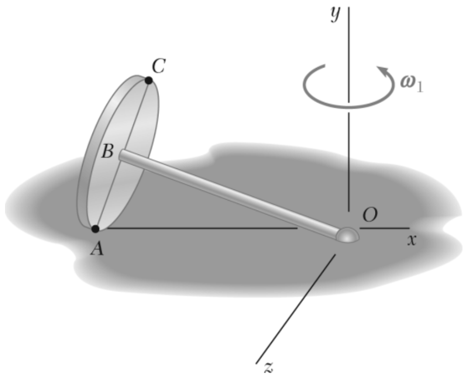
Acceleration of Point P.

$$\mathbf{a}_P = \boldsymbol{\alpha} \times \mathbf{r}_P + \boldsymbol{\omega} \times \mathbf{v}_P$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -20 & 0 \\ 0.25\cos 30° & 0.25\sin 30° & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 4 \\ -0.5 & 0.86603 & 0.625 \end{vmatrix}$$

$$= 4.3301\mathbf{k} - 3.4641\mathbf{i} - 5.125\mathbf{j} + 4.3301\mathbf{k}$$

$$\mathbf{a}_P = -(3.46 \text{ ft/s}^2)\mathbf{i} - (5.13 \text{ ft/s}^2)\mathbf{j} + (8.66 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.197

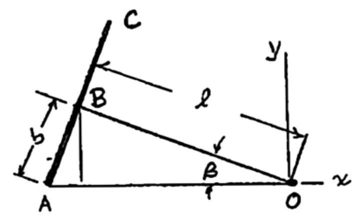
A 30 mm-radius wheel is mounted on an axle *OB* of length 100 mm. The wheel rolls without sliding on the horizontal floor, and the axle is perpendicular to the plane of the wheel. Knowing that the system rotates about the *y* axis at a constant rate $\omega_1 = 2.4$ rad/s, determine (*a*) the angular velocity of the wheel, (*b*) the angular acceleration of the wheel, (*c*) the acceleration of Point *C* located at the highest point on the rim of the wheel.

SOLUTION

Geometry.

$$l = 100 \text{ mm} = 0.1 \text{ m}$$

$$b = 30 \text{ mm} = 0.03 \text{ m}$$



$$\tan\beta = \frac{b}{l} = 0.3$$

$$\beta = 16.699°$$

$$\mathbf{r}_A = -l\sec\beta\mathbf{i}$$

$$\mathbf{r}_B = -l\cos\beta\mathbf{i} + b\cos\beta\mathbf{j}$$

(*a*) *Angular velocities.*

For the system, $\quad \mathbf{\Omega} = \omega_1\mathbf{j} = (2.4 \text{ rad/s})\mathbf{j}$

For the wheel, $\quad \boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k})\times(-l\sec\beta\mathbf{i}) = 0$$

$$-(l\omega_z\sec\beta)\mathbf{j} - (l\omega_y\sec\beta)\mathbf{k} = 0$$

$$\omega_y = 0,\quad \omega_z = 0 \qquad \boldsymbol{\omega} = \omega_x\mathbf{i}$$

$$\begin{aligned}\mathbf{v}_B &= \boldsymbol{\omega}\times\mathbf{r}_B\\ &= \omega_x\mathbf{i}\times(-l\cos\beta\mathbf{i} + b\cos\beta\mathbf{j})\\ &= (\omega_x b\cos\beta)\mathbf{k}\end{aligned}$$

For the system,

$$\begin{aligned}\mathbf{v}_B &= \mathbf{\Omega}\times\mathbf{r}_B\\ &= \omega_1\mathbf{j}\times(-l\cos\beta\mathbf{i} + b\cos\beta\mathbf{j})\\ &= (\omega_1 l\cos\beta)\mathbf{k}\end{aligned}$$

Matching the two expressions for $\mathbf{v}_B$,

$$\omega_x b\cos\beta = \omega_1 l\cos\beta$$

or $\quad \omega_x = \dfrac{\omega_1 l}{b} = \dfrac{(2.4)(30)}{100} = 8 \text{ rad/s}$ $\qquad \boldsymbol{\omega} = (8.00 \text{ rad/s})\mathbf{i}$ ◀

(*b*) *Angular acceleration.*

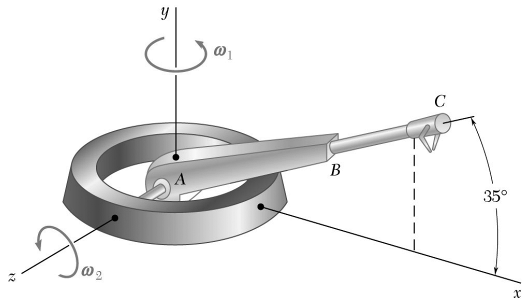
$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$$
$$= \dot{\boldsymbol{\omega}}_{Oxyz} + \boldsymbol{\Omega} \times \boldsymbol{\omega}$$
$$= (0 + 2.4\mathbf{j}) \times 8\mathbf{i}$$
$$= -(19.2 \text{ rad/s}^2)\mathbf{k}$$

$$\boldsymbol{\alpha} = -(19.20 \text{ rad/s}^2)\mathbf{k} \blacktriangleleft$$

(*c*) *Conditions at Point C.*

$$\mathbf{r}_C = -(l\cos\beta - b\sin\beta)\mathbf{i} + 2b\cos\beta\,\mathbf{j}$$
$$= (-87.162 \text{ mm})\mathbf{i} + (57.47 \text{ mm})\mathbf{j}$$
$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C$$
$$= 8\mathbf{i} \times (-87.162\mathbf{i} + 57.47\mathbf{j})$$
$$= (459.76 \text{ mm/s})\mathbf{k}$$
$$\mathbf{a}_C = \boldsymbol{\alpha} \times \mathbf{r}_C + \boldsymbol{\omega} \times \mathbf{v}_C$$
$$= -19.2\mathbf{k} \times (-87.162\mathbf{i} + 57.47\mathbf{j}) + 8\mathbf{i} \times 459.76\mathbf{k}$$
$$= (1103.4 \text{ mm/s}^2)\mathbf{i} - (2004.6 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_C = (1.103 \text{ m/s}^2)\mathbf{i} - (2.005 \text{ m/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.198

At the instant shown, the robotic arm ABC is being rotated simultaneously at the constant rate $\omega_1 = 0.15$ rad/s about the y axis, and at the constant rate $\omega_2 = 0.25$ rad/s about the z axis. Knowing that the length of arm ABC is 1 m, determine (a) the angular acceleration of the arm, (b) the velocity of Point C, (c) the acceleration of Point C.

SOLUTION

Angular velocity:

$$\boldsymbol{\omega} = \omega_1 \mathbf{j} + \omega_2 \mathbf{k}$$
$$= (0.15 \text{ rad/s})\mathbf{j} + (0.25 \text{ rad/s})\mathbf{k}$$

Consider a frame of reference rotating with angular velocity

$$\boldsymbol{\Omega} = \omega_1 \mathbf{j} = (0.15 \text{ rad/s})\mathbf{j}$$

(a) *Angular acceleration of the arm.*

$$\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}_1 + \dot{\boldsymbol{\omega}}_2 + \boldsymbol{\Omega} \times (\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2)$$
$$= 0 + 0 + (0.15\mathbf{j}) \times (0.15\mathbf{j} + 0.25\mathbf{k})$$

$$\boldsymbol{\alpha} = (0.0375 \text{ rad/s}^2)\mathbf{i} \blacktriangleleft$$

Arm ABC rotates about the fixed Point A.

$$\mathbf{r}_{C/A} = (1 \text{ m})(\cos 35°\mathbf{i} + \sin 35°)\mathbf{j}$$
$$= (0.81915 \text{ m})\mathbf{i} + (0.57358 \text{ m})\mathbf{j}$$

(b) *Velocity of Point C.*

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_{C/A}$$

$$\mathbf{v}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.15 & 0.25 \\ 0.81915 & 0.57358 & 0 \end{vmatrix}$$
$$= -0.14340\mathbf{i} + 0.20479\mathbf{j} - 0.12287\mathbf{k}$$

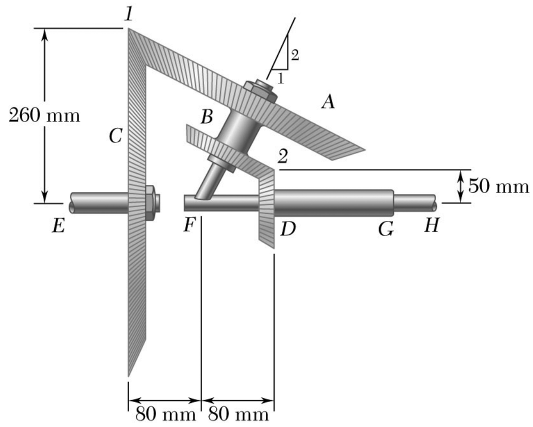
$$\mathbf{v}_C = -(0.1434 \text{ m/s})\mathbf{i} + (0.204 \text{ m/s})\mathbf{j} - (0.1229 \text{ m/s})\mathbf{k} \blacktriangleleft$$

(c) *Acceleration of C:*

$$\mathbf{a}_C = \boldsymbol{\alpha} \times \mathbf{r}_{C/A} + \boldsymbol{\omega} \times \mathbf{v}_C$$

$$\mathbf{a}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.0375 & 0 & 0 \\ 0.81915 & 0.57358 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.15 & 0.25 \\ -0.14340 & 0.20479 & -0.12287 \end{vmatrix}$$
$$= 0.021509\mathbf{k} + 0.06962\mathbf{i} + 0.03585\mathbf{j} + 0.02151\mathbf{k}$$

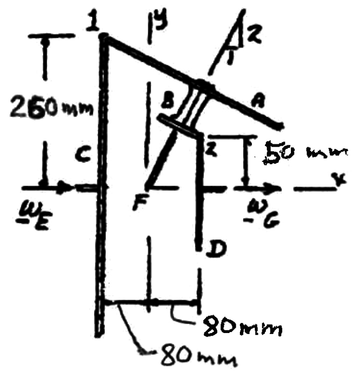
$$\mathbf{a}_C = -(0.0696 \text{ m/s}^2)\mathbf{i} + (0.0359 \text{ m/s}^2)\mathbf{j} + (0.0430 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.199

In the planetary gear system shown, gears A and B are rigidly connected to each other and rotate as a unit about the inclined shaft. Gears C and D rotate with constant angular velocities of 30 rad/s and 20 rad/s, respectively (both counterclockwise when viewed from the right). Choosing the x axis to the right, the y axis upward, and the z axis pointing out of the plane of the figure, determine (a) the common angular velocity of gears A and B, (b) the angular velocity of shaft FH, which is rigidly attached to the inclined shaft.

SOLUTION



Place origin at F.

Point 1: $\mathbf{r}_1 = -(80 \text{ mm})\mathbf{i} + (260 \text{ mm})\mathbf{j}$

Point 2: $\mathbf{r}_2 = +(80 \text{ mm})\mathbf{i} + (50 \text{ mm})\mathbf{j}$

$$\boldsymbol{\omega}_E = +(30 \text{ rad/s})\mathbf{i}$$

$$\boldsymbol{\omega}_G = +(20 \text{ rad/s})\mathbf{i}$$

$$\mathbf{v}_1 = \boldsymbol{\omega}_E \times \mathbf{r}_1 = (30\mathbf{i}) \times (-80\mathbf{i} + 260\mathbf{j})$$

$$\mathbf{v}_1 = (7800 \text{ mm/s})\mathbf{k} \qquad (1)$$

$$\mathbf{v}_2 = \boldsymbol{\omega}_G \times \mathbf{r}_2 = (20\mathbf{i}) \times (80\mathbf{i} + 50\mathbf{j})$$

$$\mathbf{v}_2 = (1000 \text{ mm/s})\mathbf{k} \qquad (2)$$

Motion of gear unit AB: $\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$

$$\mathbf{v}_1 = \boldsymbol{\omega} \times \mathbf{r}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & \omega_z \\ -80 & +260 & 0 \end{vmatrix}$$

$$= -260\omega_z\mathbf{i} - 80\omega_z\mathbf{j} + (260\omega_x + 80\omega_y)\mathbf{k}$$

Recall from Eq. (1) that $\mathbf{v} = 7800\mathbf{k}$.

$$7800\mathbf{k} = -260\omega_z\mathbf{i} - 80\omega_z\mathbf{j} + (260\omega_x + 80\omega_y)\mathbf{k}$$

Equate coefficients of unit vectors.

$$\omega_z = 0$$

$$7800 = 260\omega_x + 80\omega_y \qquad (3)$$

$$\mathbf{v}_2 = \boldsymbol{\omega} \times \mathbf{r}_2$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_x & \omega_y & 0 \\ 80 & 50 & 0 \end{vmatrix}$$
$$= (50\omega_x - 80\omega_y)\mathbf{k}$$

Recall from Eq. (2) that $\mathbf{v}_2 = 1000\mathbf{k}$, and write

$$1000 = 50\omega_x - 80\omega_y \tag{4}$$

Add Eqs. (3) and (4): $8800 = 310\omega_x$ $\qquad \omega_x = 28.387$ rad/s

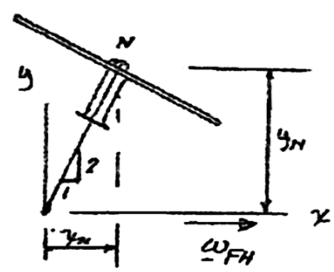
Eq. (4): $1000 = 50(28.387) - 80\omega_y$ $\qquad \omega_y = 5.242$ rad/s

(*a*) Common angular velocity of unit *AB*. $\boldsymbol{\omega} = (28.387 \text{ rad/s})\mathbf{i} + (5.242 \text{ rad/s})\mathbf{j}$

$\boldsymbol{\omega} = (28.4 \text{ rad/s})\mathbf{i} + (5.24 \text{ rad/s})\mathbf{j}$ ◀

(*b*) Angular velocity of shaft *FH*. (See figure in text.)

Point *N* is at nut, which is a part of unit *AB* and also is a part of shaft *GH*.



$$x_N = \frac{1}{2} y_N$$
$$\mathbf{r}_N = x_N \mathbf{i} + y_N \mathbf{j}$$
$$\mathbf{r}_N = \frac{1}{2} y_N \mathbf{i} + y_N \mathbf{j}$$

Nut *N* as a part of unit *AB*: $\boldsymbol{\omega} = (28.387 \text{ rad/s})\mathbf{i} + (5.242 \text{ rad/s})\mathbf{j}$

$$\mathbf{v}_N = \boldsymbol{\omega} \times \mathbf{r}_N$$
$$= (28.387\mathbf{i} + 5.242\mathbf{j}) \times \left(\frac{1}{2} y_N \mathbf{i} + y_N \mathbf{j}\right)$$
$$\mathbf{v}_N = (28.387 y_N - 2.621 y_N)\mathbf{k}$$
$$= +(25.766 y_N)\mathbf{k} \tag{5}$$

Nut *N* as a part of shaft *FH*. $\boldsymbol{\omega}_{FH} = \omega_{FH}\mathbf{i}$

$$\mathbf{v}_N = \boldsymbol{\omega}_{FH} \times \mathbf{r}_N$$
$$= (\omega_{FH}\mathbf{i}) \times \left(\frac{1}{2} y_N \mathbf{i} + y_N \mathbf{j}\right)$$
$$= \omega_{FH}\, y_N \mathbf{k} \tag{6}$$

Equating expressions for $\mathbf{v}_N$ from Eqs. (5) and (6),

$$+(25.766 y_N)\mathbf{k} = \omega_{FH}\, y_N \mathbf{k}$$
$$\boldsymbol{\omega}_{FH} = (25.766 \text{ rad/s})\mathbf{i}$$

$\boldsymbol{\omega}_{FH} = (25.8 \text{ rad/s})\mathbf{i}$ ◀

PROBLEM 15.200

In Problem 15.199, determine (*a*) the common angular acceleration of gears *A* and *B*, (*b*) the acceleration of the tooth of gear *A* which is in contact with gear *C* at Point *I*.

SOLUTION

See the solution to part (*a*) of Problem 15.199 for the calculation of the common angular velocity of unit *AB*.

$$\boldsymbol{\omega} = (28.387 \text{ rad/s})\mathbf{i} + (5.242 \text{ rad/s})\mathbf{j}$$

The angular velocity vector ω rotates about the *x*-axis with angular velocity $\boldsymbol{\omega}_{FH}$. See part (*b*) of Problem 15.199 for the calculation of $\boldsymbol{\omega}_{FH}$.

$$\boldsymbol{\omega}_{FH} = (25.776 \text{ rad/s})\mathbf{i}$$

(*a*) Common angular acceleration of unit *AB*.

$$\begin{aligned}\boldsymbol{\alpha} &= \boldsymbol{\omega}_{FH} \times \boldsymbol{\omega}\\ &= (25.776\mathbf{i}) \times (28.387\mathbf{i} + 5.242\mathbf{j})\\ &= 135.12\mathbf{k}\end{aligned}$$

$$\boldsymbol{\alpha} = 135.1 \text{ rad/s}^2\mathbf{k} \blacktriangleleft$$

The position and velocity vectors of a tooth at the contact Point 1 of gears *A* and *C* are

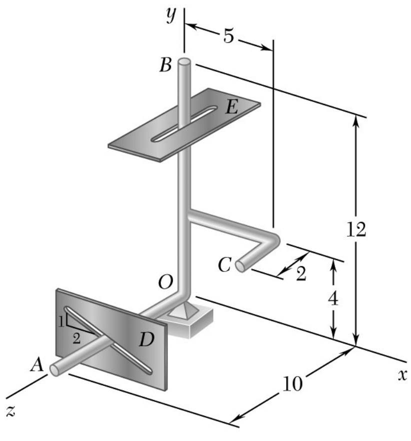
$$\mathbf{r}_1 = -(80 \text{ mm})\mathbf{i} + (260 \text{ mm})\mathbf{j}$$
$$\mathbf{v}_1 = (7800 \text{ mm/s})\mathbf{k}$$

as determined in part (*a*) of Problem 15.199.

(*b*) Acceleration of the tooth at Point 1 of gear *A*.

$$\begin{aligned}\mathbf{a}_1 &= \boldsymbol{\alpha} \times \mathbf{r}_1 + \boldsymbol{\omega} \times \mathbf{v}_1\\ &= (135.12\mathbf{k}) \times (-80\mathbf{i} + 260\mathbf{j}) + (28.387\mathbf{i} + 5.242\mathbf{j}) \times 7800\mathbf{k}\\ &= -10810\mathbf{j} - 35131\mathbf{i} - 221419\mathbf{j} + 40888\mathbf{i}\\ &= (5757 \text{ mm/s}^2)\mathbf{i} - (232229 \text{ mm/s}^2)\mathbf{j}\end{aligned}$$

$$\mathbf{a}_1 = (5.8 \text{ m/s}^2)\mathbf{i} - (232 \text{ m/s}^2)\mathbf{j} \blacktriangleleft$$

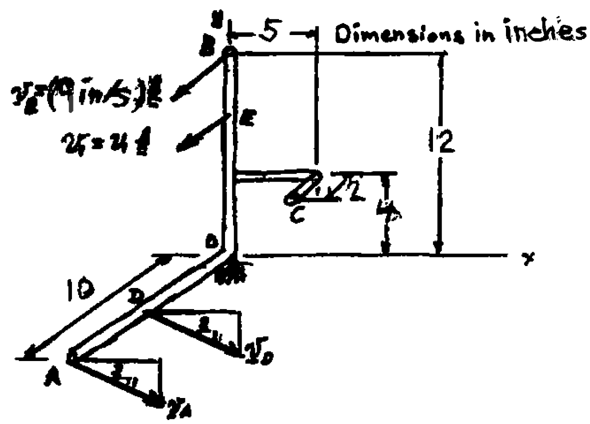


Dimensions in in.

PROBLEM 15.201

Several rods are brazed together to form the robotic guide arm shown which is attached to a ball-and-socket joint at O. Rod OA slides in a straight inclined slot while rod OB slides in a slot parallel to the z axis. Knowing that at the instant shown $\mathbf{v}_B = (9\text{ in./s})\mathbf{k}$, determine (*a*) the angular velocity of the guide arm, (*b*) the velocity of Point A, (*c*) the velocity of Point C.

SOLUTION



Since rod at D slides in slot which is of slope 1:2,

$$(v_D)_x = -2(v_D)_y$$

and

$$(v_A)_x = -2(v_A)_y$$

(*a*) Angular velocity.

$$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B: \quad (9\text{ in./s})\mathbf{k} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k})\times(12\text{ in.})\mathbf{j}$$

$$9\mathbf{k} = 12\omega_x\mathbf{k} - 12\omega_z\mathbf{i}$$

Coefficients of $\mathbf{k}$: $\quad 9 = 12\omega_x \quad \omega_x = 0.75\text{ rad/s}$

Coefficients of $\mathbf{i}$: $\quad 0 = -12\omega_z \quad \omega_z = 0$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A$$

$$\mathbf{v}_A = (0.75\mathbf{i} + \omega_y\mathbf{j})\times(10\mathbf{k})$$

$$(v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} + (v_A)_z\mathbf{k} = -7.5\mathbf{j} + 10\omega_y\mathbf{i}$$

Coefficients of **j**: $(v_A)_y = -7.5$

Coefficients of **i**: $(v_A)_x = 10\omega_y$

Coefficients of **k**: $(v_A)_z = 0$

Recall the Equations $(v_A)_x = -2(v_A)_y$

and $10\omega_y = -2(-7.5)$

So, $\omega_y = 1.5 \text{ rad/s}$ and $(v_A)_x = 15 \text{ in./s}$

$$\boldsymbol{\omega} = (0.75 \text{ rad/s})\mathbf{i} + (1.5 \text{ rad/s})\mathbf{j} \blacktriangleleft$$

(*b*) Velocity of *A*:

$$\mathbf{v}_A = (15 \text{ in./s})\mathbf{i} - (7.5 \text{ in./s})\mathbf{j} \blacktriangleleft$$

(*c*) Velocity of *C*:

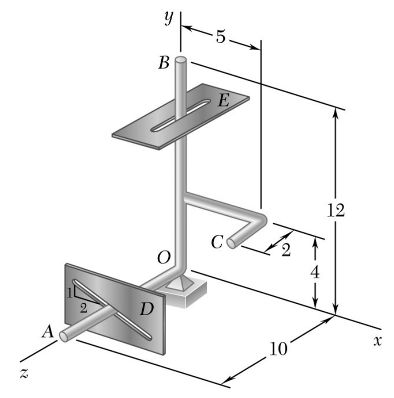
$$\mathbf{r}_C = 5\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 5 & 4 & 2 \end{vmatrix}$$

$$= 3\mathbf{i} - 1.5\mathbf{j} + (3 - 7.5)\mathbf{k}$$

$$\mathbf{v}_C = (3 \text{ in./s})\mathbf{i} - (1.5 \text{ in./s})\mathbf{j} - (4.5 \text{ in./s})\mathbf{k} \blacktriangleleft$$



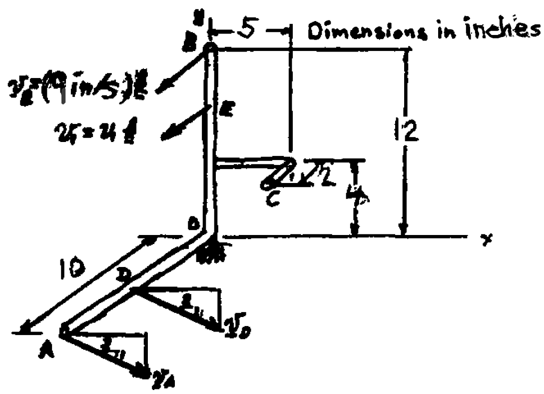
Dimensions in in.

PROBLEM 15.202

In Problem 15.201 the speed of Point B is known to be constant. For the position shown, determine (a) the angular acceleration of the guide arm, (b) the acceleration of Point C.

PROBLEM 15.201 Several rods are brazed together to form the robotic guide arm shown, which is attached to a ball-and-socket joint at O. Rod OA slides in a straight inclined slot while rod OB slides in a slot parallel to the z-axis. Knowing that at the instant shown $\mathbf{v}_B = (9 \text{ in./s})\mathbf{k}$, determine ($a$) the angular velocity of the guide arm, (b) the velocity of Point A, (c) the velocity of Point C.

SOLUTION



Since rod at D slides in slot which is of slope 1:2,

$$(v_D)_x = -2(v_D)_y$$

and

$$(v_A)_x = -2(v_A)_y$$

Angular velocity.

$$\boldsymbol{\omega} = \omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k}$$

$$\mathbf{v}_B = \boldsymbol{\omega}\times\mathbf{r}_B: \quad (9 \text{ in./s})\mathbf{k} = (\omega_x\mathbf{i} + \omega_y\mathbf{j} + \omega_z\mathbf{k})\times(12 \text{ in.})\mathbf{j}$$

$$9\mathbf{k} = 12\omega_x\mathbf{k} - 12\omega_z\mathbf{i}$$

Coefficients of **k**: $\quad 9 = 12\omega_x \quad \omega_x = 0.75 \text{ rad/s}$

Coefficients of **i**: $\quad 0 = -12\omega_z \quad \omega_z = 0$

$$\mathbf{v}_A = \boldsymbol{\omega}\times\mathbf{r}_A$$

$$\mathbf{v}_A = (0.75\mathbf{i} + \omega_y\mathbf{j})\times(10\mathbf{k})$$

$$(v_A)_x\mathbf{i} + (v_A)_y\mathbf{j} + (v_A)_z\mathbf{k} = -7.5\mathbf{j} + 10\omega_y\mathbf{i}$$

Coefficients of **j**: $\quad (v_A)_y = -7.5$

Coefficients of **i**: $\quad (v_A)_x = 10\omega_y$

Coefficients of **k**: $\quad (v_A)_z = 0$

PROBLEM 15.202 (Continued)

Recall the Equations $(v_A)_x = -2(v_A)_y$

and $10\omega_y = -2(-7.5)$

So, $\omega_y = 1.5 \text{ rad/s}$ and $(v_A)_x = 15 \text{ in./s}$

$$\boldsymbol{\omega} = (0.75 \text{ rad/s})\mathbf{i} + (1.5 \text{ rad/s})\mathbf{j}$$

Velocity of A: $\mathbf{v}_A = (15 \text{ in./s})\mathbf{i} - (7.5 \text{ in./s})\mathbf{j}$

Velocity of C: $\mathbf{r}_C = 5\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$

$$\mathbf{v}_C = \boldsymbol{\omega} \times \mathbf{r}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 5 & 4 & 2 \end{vmatrix}$$

$$= 3\mathbf{i} - 1.5\mathbf{j} + (3 - 7.5)\mathbf{k}$$

$$\mathbf{v}_C = (3 \text{ in./s})\mathbf{i} - (1.5 \text{ in./s})\mathbf{j} - (4.5 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_B = \boldsymbol{\alpha} \times \mathbf{r}_B + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_C)$$

$$= \boldsymbol{\alpha} \times \mathbf{r}_C + \boldsymbol{\omega} \times \mathbf{v}_B$$

$$\mathbf{a}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \alpha_x & \alpha_y & \alpha_z \\ 0 & 12 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 0 & 0 & 9 \end{vmatrix}$$

$$= -12\alpha_z\mathbf{i} + 12\alpha_x\mathbf{k} + 13.5\mathbf{i} - 6.75\mathbf{j}$$

$$\mathbf{a}_B = (13.5 - 12\alpha_z)\mathbf{i} - 6.75\mathbf{j} + 12\alpha_x\mathbf{k} \qquad (1)$$

$$(a_B)_x = 13.5 - 12\alpha_z = 0 \qquad \alpha_z = 1.125 \text{ rad/s}^2$$

$$(a_B)_y = -6.75 \qquad (a_B)_y = -6.75 \text{ in./s}^2$$

$$(a_B)_z = 12\alpha_x = 0 \qquad \alpha_x = 0$$

$$\mathbf{a}_A = \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_A)$$

$$= \boldsymbol{\alpha} \times \mathbf{r}_A + \boldsymbol{\omega} \times \mathbf{v}_A$$

$$\mathbf{a}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \alpha_y & 1.125 \\ 0 & 0 & 10 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 15 & -7.5 & 0 \end{vmatrix}$$

$$= 10\alpha_y\mathbf{i} + (-5.625 - 22.5)\mathbf{k}$$

$$\mathbf{a}_A = 10\alpha_y\mathbf{i} - 28.125\mathbf{k}$$

Thus, $(a_A)_x = 10\alpha_y \quad (a_A)_y = 0 \quad (a_A)_z = -28.125 \text{ in./s}^2$

But $(a_A)_x = -2(a_A)_y = 0$

Therefore, $(a_A)_x = 10\alpha_y = 0 \quad \alpha_y = 0$

PROBLEM 15.202 (Continued)

(*a*) Angular acceleration: $\boldsymbol{\alpha} = (1.125 \text{ rad/s}^2)\mathbf{k}$ ◀

(*b*) Acceleration of *C*:

$$\mathbf{a}_C = \boldsymbol{\alpha} \times \mathbf{r}_C + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_C)$$

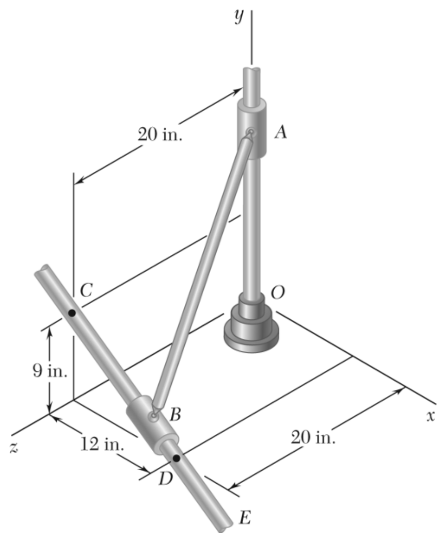
$$= \boldsymbol{\alpha} \times \mathbf{r}_C + \boldsymbol{\omega} \times \mathbf{v}_C$$

$$\mathbf{r}_C = (5 \text{ in.})\mathbf{i} + (4 \text{ in.})\mathbf{j} + (2 \text{ in.})\mathbf{k}$$

$$\mathbf{a}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 1.125 \\ 5 & 4 & 2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.75 & 1.5 & 0 \\ 3 & -1.5 & -4.5 \end{vmatrix}$$

$$= -4.5\mathbf{i} + 5.625\mathbf{j} - 6.75\mathbf{i} + 3.375\mathbf{j} + (-1.125 - 4.5)\mathbf{k}$$

$$\mathbf{a}_C = -(11.3 \text{ in./s}^2)\mathbf{i} + (9 \text{ in./s}^2)\mathbf{j} - (5.63 \text{ in./s}^2)\mathbf{k}$$ ◀



PROBLEM 15.203

Rod AB of length 25 in. is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves toward Point E at a constant speed of 20 in./s, determine the velocity of collar A as collar B passes through Point D.

SOLUTION

Geometry. $$l_{AB}^2 = x_{A/B}^2 + y_{A/B}^2 + z_{A/B}^2: \quad 25^2 = (-12)^2 + y_{A/B}^2 + (-20)^2$$

$$y_{A/B} = 9 \text{ in.}$$

$$\mathbf{r}_{A/B} = (-12 \text{ in.})\mathbf{i} + (9 \text{ in.})\mathbf{j} - (20 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{D/C} = (12 \text{ in.})\mathbf{i} - (9 \text{ in.})\mathbf{j},$$

$$l_{CD} = \sqrt{(12)^2 + (-9)^2} = 15 \text{ in.}$$

Velocity of collar B. $$\mathbf{v}_B = v_B \frac{r_{D/C}}{l_{CD}}$$

$$\mathbf{v}_B = (20)\frac{(12\mathbf{i} - 9\mathbf{j})}{15} = (16 \text{ in./s})\mathbf{i} - (12 \text{ in./s})\mathbf{j}$$

Velocity of collar A. $$\mathbf{v}_A = v_A\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where $$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0$.

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get $$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$

$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

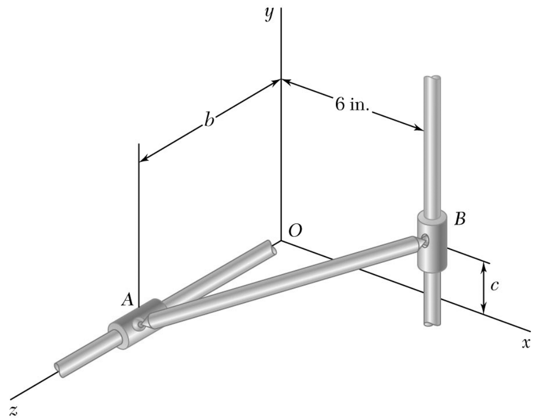
or $$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1), $$(-12\mathbf{i} + 9\mathbf{j} - 20\mathbf{k}) \cdot (v_A\mathbf{j}) = (-12\mathbf{i} + 9\mathbf{j} - 80\mathbf{k}) \cdot (16\mathbf{i} - 12\mathbf{j})$$

$$9v_A = (-12)(16) + (9)(-12)$$

or $$v_A = -33.333 \text{ in./s}$$

$$\mathbf{v}_A = -(33.3 \text{ in./s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.204

Rod AB, of length 11 in., is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves downward at a constant speed of 54 in./s, determine the velocity of collar A when $c = 2$ in.

SOLUTION

Geometry. $l_{AB}^2 = x_{A/B}^2 + y_{A/B}^2 + z_{A/B}^2$: $(11)^2 = (6)^2 + (-2)^2 + (z_{A/B})^2$

$$z_{A/B} = 9 \text{ in.} \quad \mathbf{r}_{A/B} = (-6 \text{ in.})\mathbf{i} - (2 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$$

Velocity of collar B. $\mathbf{v}_B = -v_B\mathbf{j} = -(54 \text{ in./s})\mathbf{j}$

Velocity of collar A. $\mathbf{v}_A = v_A\mathbf{k}$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B},$$

where $\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0$.

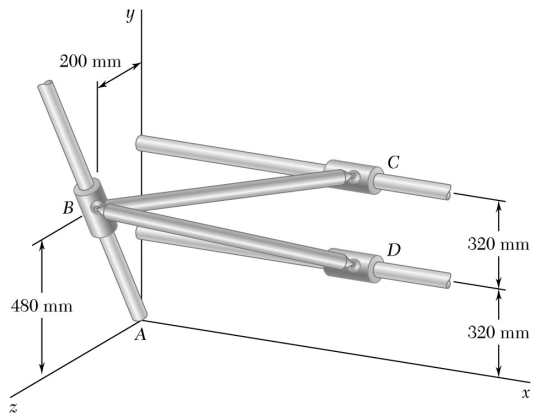
Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get $\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$

or $\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B$ (1)

From Eq. (1), $(-6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}) \cdot v_A\mathbf{k} = (-6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}) \cdot (-54\mathbf{j})$

or $9v_A = 108$ $\quad\quad \mathbf{v}_A = (12.00 \text{ in./s})\mathbf{k}$ ◀

PROBLEM 15.205



Rods BC and BD are each 840 mm long and are connected by ball-and-socket joints to collars which may slide on the fixed rods shown. Knowing that collar B moves toward A at a constant speed of 390 mm/s, determine the velocity of collar C for the position shown.

SOLUTION

Geometry.

$$l_{BC}^2 = x_{C/B}^2 + y_{C/B}^2 + z_{C/B}^2$$

$$(840)^2 = c^2 + (640-480)^2 + (200)^2 \qquad c = 800 \text{ mm}$$

$$\mathbf{r}_{C/B} = (800 \text{ mm})\mathbf{i} + (160 \text{ mm})\mathbf{j} - (200 \text{ mm})\mathbf{k}$$

Velocity of B.

$$\mathbf{v}_B = v_B\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right)$$

$$= (390 \text{ mm/s})\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right)$$

$$= -(360 \text{ mm/s})\mathbf{j} - (150 \text{ mm/s})\mathbf{k}$$

Velocity of C.

$$\mathbf{v}_C = v_C\mathbf{i}$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

where

$$\mathbf{v}_{C/B} = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

Noting that $\mathbf{v}_{C/B}$ is perpendicular to $\mathbf{r}_{C/B}$, we get $\mathbf{r}_{C/B} \cdot \mathbf{v}_{C/B} = 0$

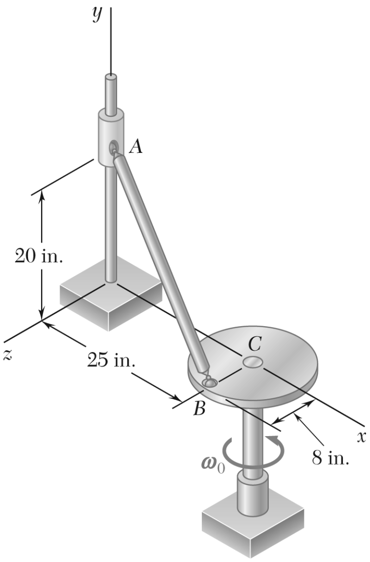
So that

$$\mathbf{r}_{C/B} \cdot \mathbf{v}_C = \mathbf{r}_{C/B} \cdot \mathbf{v}_B$$

$$(800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot (-360\mathbf{j} - 150\mathbf{k}) = (800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot (v_C\mathbf{i})$$

$$(160)(-360) + (-200)(-150) = 800\,v_C$$

$$v_C = -34.5 \text{ mm/s} \qquad \mathbf{v}_C = -(34.5 \text{ mm/s})\mathbf{i} \blacktriangleleft$$



PROBLEM 15.206

Rod AB is connected by ball-and-socket joints to collar A and to the 16-in.-diameter disk C. Knowing that disk C rotates counterclockwise at the constant rate $\omega_0 = 3$ rad/s in the zx plane, determine the velocity of collar A for the position shown.

SOLUTION

Geometry.

$$\mathbf{r}_{B/C} = (-8 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{A/B} = -(25 \text{ in.})\mathbf{i} + (20 \text{ in.})\mathbf{j} - (8 \text{ in.})\mathbf{k}$$

Velocity at B.

$$\mathbf{v}_B = \omega_0 \mathbf{j} \times \mathbf{r}_{B/C}$$

$$= 3\mathbf{j} \times (-8\mathbf{k})$$

$$= -(24 \text{ in./s})\mathbf{i}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0$.

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$

$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

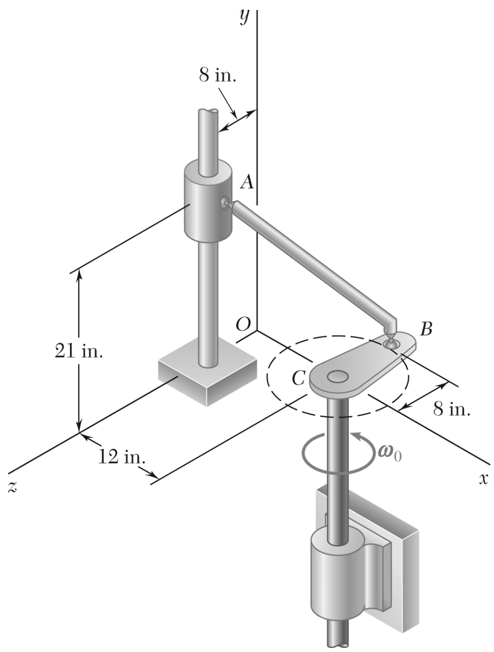
From Eq. (1),

$$(-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (v_A \mathbf{j}) = (-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (24\mathbf{i})$$

$$20 v_A = -600$$

or

$$v_A = -30 \text{ in./s} \qquad \mathbf{v}_A = -(30.0 \text{ in./s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.207

Rod AB of length 29 in. is connected by ball-and-socket joints to the rotating crank BC and to the collar A. Crank BC is of length 8 in. and rotates in the horizontal xz plane at the constant rate $\omega_0 = 10$ rad/s. At the instant shown, when crank BC is parallel to the z axis, determine the velocity of collar A.

SOLUTION

Geometry.

$$\mathbf{r}_{B/C} = (-8 \text{ in.})\mathbf{k},$$
$$\mathbf{r}_{A/B} = (-12 \text{ in.})\mathbf{i} + (21 \text{ in.})\mathbf{j} + (16 \text{ in.})\mathbf{k}$$

Velocity at B.

$$\mathbf{v}_B = \omega_0 \mathbf{j} \times \mathbf{r}_{B/C}$$
$$= 10\mathbf{j} \times (-8\mathbf{k})$$
$$= (-80 \text{ in./s})\mathbf{i}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$
$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

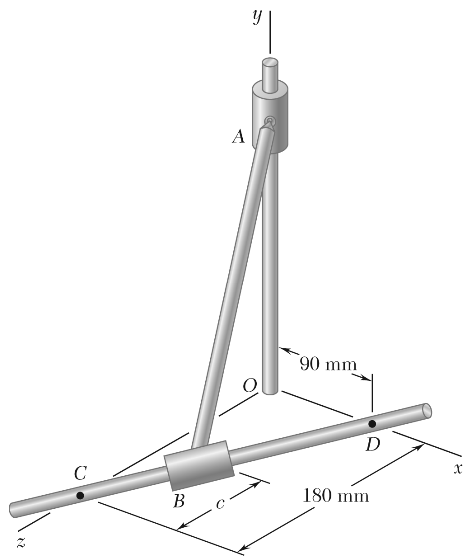
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1), $(-12\mathbf{i} + 21\mathbf{j} + 16\mathbf{k}) \cdot (v_A \mathbf{j}) = (-12\mathbf{i} + 21\mathbf{j} + 16\mathbf{k}) \cdot (-80\mathbf{i})$

$$21 v_A = 960$$

or $v_A = 45.714$ in./s

$$\mathbf{v}_A = (45.7 \text{ in./s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.208

Rod AB of length 300 mm is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves toward Point D at a constant speed of 50 mm/s, determine the velocity of collar A when $c = 80$ mm.

SOLUTION

Geometry.

$$\mathbf{r}_A = y\mathbf{j},$$
$$\mathbf{r}_D = (90 \text{ mm})\mathbf{i}$$
$$\mathbf{r}_C = (180 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C$$
$$= (40 \text{ mm})\mathbf{i} - (180 \text{ mm})\mathbf{k}$$

$$l_{CD} = \sqrt{(90)^2 + (180)^2}$$
$$= 201.246 \text{ mm}$$

$$\mathbf{r}_{B/C} = \frac{c(\mathbf{r}_{D/C})}{180}$$
$$= \frac{80(90\mathbf{i} - 180\mathbf{k})}{180}$$
$$= (40 \text{ mm})\mathbf{i} - (80 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_B = \mathbf{r}_C + \mathbf{r}_{B/C}$$
$$= 180\mathbf{k} + 40\mathbf{i} - 80\mathbf{k}$$
$$= (40 \text{ mm})\mathbf{i} + (100 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B$$
$$= -(40 \text{ mm})\mathbf{i} + (y \text{ mm})\mathbf{j} - (100 \text{ mm})\mathbf{k}$$

$$l_{AB}^2 = x_{A/B}^2 + y^2 + z_{A/B}^2: \quad 300^2 = (-40)^2 + y^2 + (-100)^2$$
$$y = 280 \text{ mm},$$
$$\mathbf{r}_{A/B} = (-40 \text{ mm})\mathbf{i} + (280 \text{ mm})\mathbf{j} - (100 \text{ mm})\mathbf{k}$$

Velocity of collar B.

$$\mathbf{v}_B = v_B \frac{\mathbf{r}_{D/C}}{l_{CD}}$$

$$\mathbf{v}_B = \frac{(50)(90\mathbf{i} - 180\mathbf{k})}{201.246}$$

$$= (22.3607 \text{ mm/s})\mathbf{i} - (44.7214 \text{ mm/s})\mathbf{k}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$

$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

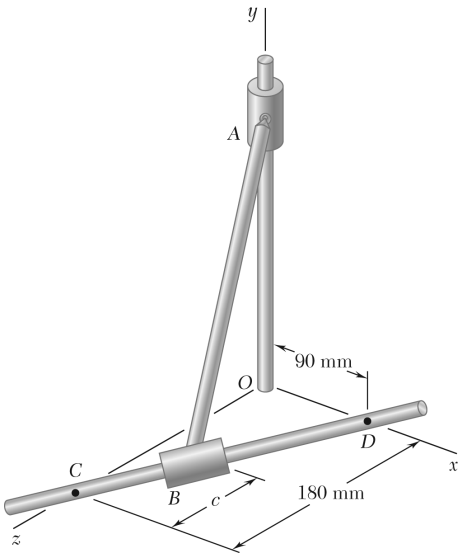
From Eq. (1),

$$(-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (v_A \mathbf{j}) = (-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (22.3607\mathbf{i} - 44.7214\mathbf{k})$$

$$280 v_A = (-40)(22.3607) + (-100)(-44.7214)$$

or

$$v_A = 12.7775 \text{ mm/s} \qquad \mathbf{v}_A = (12.78 \text{ mm/s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.209

Rod AB of length 300 mm is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves toward Point D at a constant speed of 50 mm/s, determine the velocity of collar A when $c = 120$ mm.

SOLUTION

Geometry.

$$\mathbf{r}_A = y\mathbf{j},$$
$$\mathbf{r}_D = (90\text{ mm})\mathbf{i}$$
$$\mathbf{r}_C = (180\text{ mm})\mathbf{k}$$

$$\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C$$
$$= (90\text{ mm})\mathbf{i} - (180\text{ mm})\mathbf{k}$$
$$l_{CD} = \sqrt{(90)^2 + (-180)^2}$$
$$= 201.246\text{ mm}$$
$$\mathbf{r}_{B/C} = \frac{c(\mathbf{r}_{D/C})}{180}$$
$$= \frac{120(90\mathbf{i} - 180\mathbf{k})}{180}$$
$$= (60\text{ mm})\mathbf{i} - (120\text{ mm})\mathbf{k}$$
$$\mathbf{r}_B = \mathbf{r}_C + \mathbf{r}_{B/C}$$
$$= 180\mathbf{k} + 60\mathbf{i} - 120\mathbf{k}$$
$$= (60\text{ mm})\mathbf{i} + (60\text{ mm})\mathbf{k}$$
$$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B$$
$$= -60\mathbf{i} + y\mathbf{j} - 60\mathbf{k}$$

$$l_{AB}^2 = x_{A/B}^2 + y^2 + z_{A/B}^2: \quad 300^2 = 60^2 + y^2 + 60^2$$
$$y = 287.75\text{ mm},$$
$$\mathbf{r}_{A/B} = (-60\text{ mm})\mathbf{i} + (287.75\text{ mm})\mathbf{j} - (60\text{ mm})\mathbf{k}$$

Velocity of collar B.

$$\mathbf{v}_B = v_B \frac{\mathbf{r}_{D/C}}{l_{CD}}$$

$$\mathbf{v}_B = \frac{(50)(90\mathbf{i} - 180\mathbf{k})}{201.246}$$

$$= (22.3607 \text{ mm/s})\mathbf{i} - (44.7214 \text{ mm/s})\mathbf{k}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$

$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

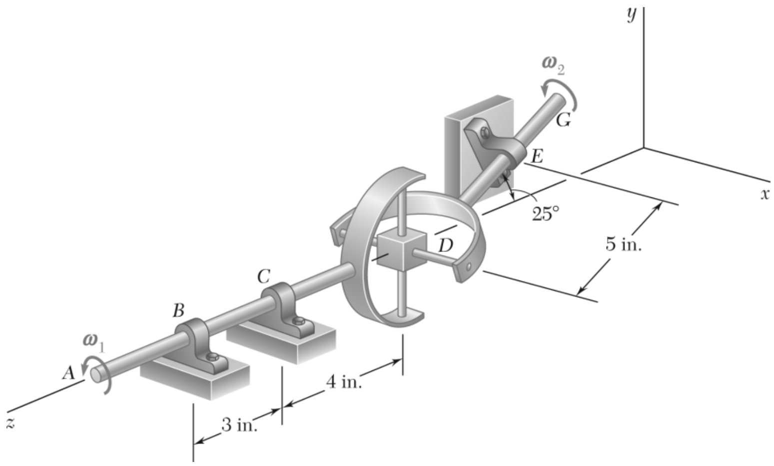
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1), $(-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (v_A \mathbf{j}) = (-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (22.3607\mathbf{i} - 44.7214\mathbf{j})$

$$287.75 v_A = (-60)(22.3607) + (-60)(-44.7214)$$

or

$$v_A = 4.6626 \text{ mm/s} \qquad \mathbf{v}_A = (4.66 \text{ mm/s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.210

Two shafts AC and EG, which lie in the vertical yz plane, are connected by a universal joint at D. Shaft AC rotates with a constant angular velocity ω_1 as shown. At a time when the arm of the crosspiece attached to shaft AC is vertical, determine the angular velocity of shaft EG.

SOLUTION

Angular velocity of shaft AC. $\quad \boldsymbol{\omega}_{AC} = \omega_1 \mathbf{k}$

Let $\omega_3 \mathbf{j}$ be the angular velocity of body D relative to shaft AD.

Angular velocity of body D. $\quad \boldsymbol{\omega}_D = \omega_1 \mathrm{k} + \omega_3 \mathbf{j}$

Angular velocity of shaft EG. $\quad \boldsymbol{\omega}_{EG} = \omega_2 (\cos 25° \mathbf{k} - \sin 25° \mathbf{j})$

Let $\omega_4 \mathbf{i}$ be the angular velocity of body D relative to shaft EG.

Angular velocity of body D. $\quad \boldsymbol{\omega}_D = \omega_2 (\cos 25° \mathbf{k} - \sin 25° \mathbf{j}) + \omega_4 \mathbf{i}$

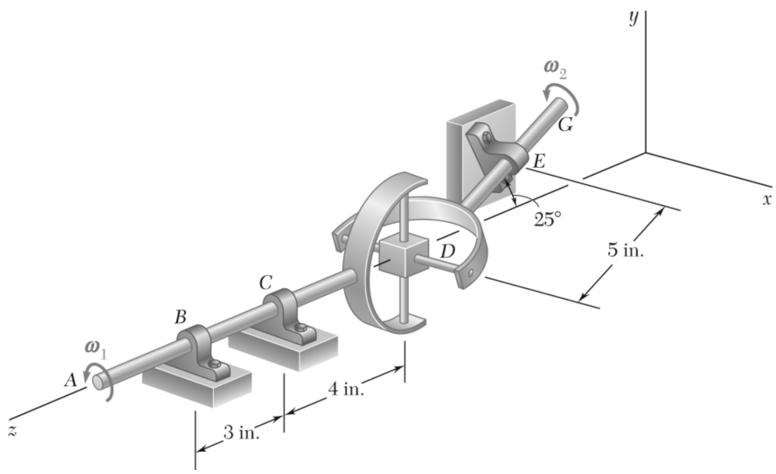
Equate the two expressions for ω_D and resolve into components.

$$\mathbf{i}: \quad 0 = \omega_4 \qquad (1)$$

$$\mathbf{j}: \quad \omega_3 = -\omega_2 \sin 25° \qquad (2)$$

$$\mathbf{k}: \quad \omega_1 = \omega_2 \cos 25° \qquad (3)$$

From Eq. (3), $\quad \omega_2 = \dfrac{\omega_1}{\cos 25°} \qquad \boldsymbol{\omega}_{EG} = \dfrac{\omega_2}{\cos 25°}(-\sin 25° \mathbf{j} + \cos 25° \mathbf{k})$ ◀



PROBLEM 15.211

Solve Problem 15.210, assuming that the arm of the crosspiece attached to the shaft *AC* is horizontal.

PROBLEM 15.210 Two shafts *AC* and *EG*, which lie in the vertical *yz* plane, are connected by a universal joint at *D*. Shaft *AC* rotates with a constant angular velocity ω_1 as shown. At a time when the arm of the crosspiece attached to shaft *AC* is vertical, determine the angular velocity of shaft *EG*.

SOLUTION

Angular velocity of shaft AC. $\qquad \boldsymbol{\omega}_{AC} = \omega_1 \mathbf{k}$

Let $\omega_3 \mathbf{i}$ be the angular velocity of body *D* relative to shaft *AD*.

Angular velocity of body D. $\qquad \boldsymbol{\omega}_D = \omega_1 \mathbf{k} + \omega_3 \mathbf{i}$

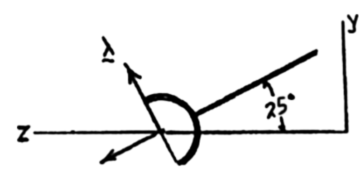
Angular velocity of shaft EG. $\qquad \boldsymbol{\omega}_{EG} = \omega_2 (\cos 25° \mathbf{k} - \sin 25° \mathbf{j})$

Let $\omega_4 \boldsymbol{\lambda}$ be the angular velocity of body *D* relative to shaft *EG*,

where $\boldsymbol{\lambda}$ is a unit vector along the clevis axle attached to shaft *EG*.

$$\boldsymbol{\lambda} = \cos 25° \mathbf{j} + \sin 25° \mathbf{k}$$

$$\omega_4 \boldsymbol{\lambda} = \omega_4 \cos 25° \mathbf{j} + \omega_4 \sin 25° \mathbf{k}$$



Angular velocity of body D.

$$\boldsymbol{\omega}_D = \boldsymbol{\omega}_{EG} + \omega_4 \boldsymbol{\lambda}$$

$$\boldsymbol{\omega}_D = (\omega_4 \cos 25° - \omega_2 \sin 25°)\mathbf{j} + (\omega_4 \sin 25° + \omega_2 \cos 25°)\mathbf{k}$$

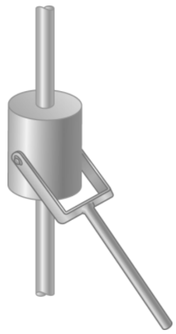
Equate the two expressions for $\boldsymbol{\omega}_D$ and resolve into components.

$$\mathbf{i}: \quad \omega_3 = 0 \tag{1}$$

$$\mathbf{j}: \quad 0 = \omega_4 \cos 25° - \omega_2 \sin 25° \tag{2}$$

$$\mathbf{k}: \quad \omega_1 = \omega_4 \sin 25° + \omega_2 \cos 25° \tag{3}$$

From Eqs. (2) and (3), $\qquad \omega_2 = \omega_1 \cos 25° \qquad \boldsymbol{\omega}_{EG} = \omega_1 \cos 25°(-\sin 25° \mathbf{j} + \cos 25° \mathbf{k})$ ◀



PROBLEM 15.212

In Problem 15.206, the ball-and-socket joint between the rod and collar A is replaced by the clevis shown. Determine (*a*) the angular velocity of the rod, (*b*) the velocity of collar A.

SOLUTION

Geometry. $\mathbf{r}_{B/C} = (8 \text{ in.})\mathbf{k} \quad \mathbf{r}_{A/B} = -(25 \text{ in.})\mathbf{i} + (20 \text{ in.})\mathbf{j} - (8 \text{ in.})\mathbf{k}$

Velocity of collar B. $\mathbf{v}_B = \omega_0 \mathbf{j} \times \mathbf{r}_{B/C} = -3\mathbf{j} \times 8\mathbf{k} = (24 \text{ in./s})\mathbf{i}$

Velocity of collar A. $\mathbf{v}_A = \mathbf{v}_A \mathbf{j}$

Angular velocity of collar A. $\boldsymbol{\omega}_A = \omega_1 \mathbf{j}$

The axle of the clevis at A is perpendicular to both the y axis and the rod AB. A vector $\mathbf{p}$ along this axle is

$$\mathbf{p} = \mathbf{j} \times \mathbf{r}_{A/B} = \mathbf{j} \times (-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) = -8\mathbf{i} + 25\mathbf{k}$$

$$p = \sqrt{8^2 + 25^2} = 26.2488$$

Unit vector $\boldsymbol{\lambda}$ along axle: $\boldsymbol{\lambda} = \dfrac{\mathbf{p}}{p} = -0.30478\mathbf{i} + 0.95242\mathbf{k}$

Let $\boldsymbol{\omega}_2$ be the angular velocity of the rod AB relative to collar A.

$$\boldsymbol{\omega}_2 = \omega_2 \boldsymbol{\lambda} = -0.30478\omega_2 \mathbf{i} + 0.95242\omega_2 \mathbf{k}$$

Angular velocity of rod AB. $\boldsymbol{\omega}_{AB} = \boldsymbol{\omega}_A + \boldsymbol{\omega}_2$

$$\boldsymbol{\omega}_{AB} = -0.30478\omega_2 \mathbf{i} + \omega_1 \mathbf{j} + 0.95242\omega_2 \mathbf{k} \tag{1}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} = \mathbf{v}_B + \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

$$\mathbf{v}_A \mathbf{j} = 24\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.30478\omega_2 & \omega_1 & 0.95242\omega_2 \\ -25 & 20 & -8 \end{vmatrix}$$

Resolving into components,

$$\mathbf{i}: \quad 0 = 24 - 8\omega_1 - 19.0484\omega_2 \tag{2}$$

$$\mathbf{j}: \quad v_A = -26.2487\omega_2 \tag{3}$$

$$\mathbf{k}: \quad 0 = 25\omega_1 - 6.0956\omega_2 \tag{4}$$

PROBLEM 15.212 (Continued)

Solving Eqs. (2), (3) and (4) simultaneously,

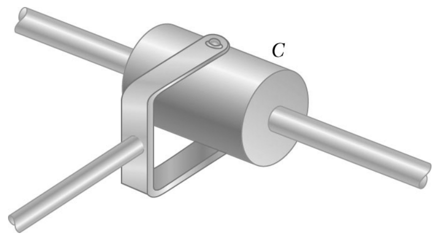
$$v_A = -30 \text{ in./s},$$
$$\omega_1 = 0.27867 \text{ rad/s},$$
$$\omega_2 = 1.1429 \text{ rad/s}$$

(a) Angular velocity of rod AB.

From Eq. (1)
$$\boldsymbol{\omega}_{AB} = -(0.30478)(1.1429)\mathbf{i} + 0.27867\mathbf{j} + (0.95242)(1.1429)\mathbf{k}$$

$$\boldsymbol{\omega}_{AB} = -(0.348 \text{ rad/s})\mathbf{i} + (0.279 \text{ rad/s})\mathbf{j} + (1.089 \text{ rad/s})\mathbf{k} \blacktriangleleft$$

(b) Velocity of A.
$$\mathbf{v}_A = -(30.0 \text{ in./s})\mathbf{j} \blacktriangleleft$$



PROBLEM 15.213

In Problem 15.205, the ball-and-socket joint between the rod and collar C is replaced by the clevis connection shown. Determine (a) the angular velocity of the rod, (b) the velocity of collar C.

SOLUTION

Geometry.

$$\mathbf{r}_C = x_C\mathbf{i} + (640\text{ mm})\mathbf{j},\ \mathbf{r}_B = (480\text{ mm})\mathbf{j} + (200\text{ mm})\mathbf{k}$$

$$l_{AB} = \sqrt{480^2 + 200^2} = 520\text{ mm}$$

$$\mathbf{r}_{C/B} = x_C\mathbf{i} + (160\text{ mm})\mathbf{j} - (200\text{ mm})\mathbf{k}$$

Length of rod BC. $\quad l_{BC}^2 = 840^2 = x_C^2 + 160^2 + 200^2$

Solving for x_C, $\quad x_C = 800\text{ mm}$

$$\mathbf{r}_{C/B} = (800\text{ mm})\mathbf{i} + (160\text{ mm})\mathbf{j} - (200\text{ mm})\mathbf{k}$$

Velocity. $\quad \mathbf{v}_B = \dfrac{390}{520}(-480\mathbf{j} - 200\mathbf{k}) = (-360\text{ mm})\mathbf{j} - (150\text{ mm/s})\mathbf{k}$

$$\mathbf{v}_C = v_C\mathbf{i}$$

Angular velocity of collar C. $\quad \boldsymbol{\omega}_C = \omega_C\mathbf{i}$

The axle of the clevis at C is perpendicular to the x-axis and to the rod BC.

A vector along this axle is $\quad \mathbf{p} = \mathbf{i} \times \mathbf{r}_{C/B}$

$$\mathbf{p} = \mathbf{i} \times (800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) = (200\text{ mm})\mathbf{j} + (160\text{ mm})\mathbf{k}$$

$$p = \sqrt{200^2 + 160^2} = 256.125\text{ mm}$$

Let $\boldsymbol{\lambda}$ be a unit vector along the axle. $\quad \boldsymbol{\lambda} = \dfrac{\mathbf{p}}{p} = 0.78087\mathbf{j} + 0.62470\mathbf{k}$

Let $\boldsymbol{\omega}_s = \omega_s\boldsymbol{\lambda}$ be the angular velocity of rod BC relative to collar C.

$$\boldsymbol{\omega}_s = 0.78087\omega_s\mathbf{j} + 0.62470\omega_s\mathbf{k}$$

Angular velocity of rod BC. $\quad \boldsymbol{\omega}_{BC} = \boldsymbol{\omega}_C + \boldsymbol{\omega}_s$

$$\boldsymbol{\omega}_{BC} = \omega_C\mathbf{i} + 0.78087\omega_s\mathbf{j} + 0.62470\omega_s\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

$$v_C\mathbf{i} = -360\mathbf{j} - 150\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \omega_C & 0.78087\omega_s & 0.62470\omega_s \\ 800 & 160 & -200 \end{vmatrix}$$

PROBLEM 15.213 (Continued)

Resolving into components,

$$\mathbf{i}: \quad v_C = -256.126\omega_s \qquad (1)$$

$$\mathbf{j}: \quad 0 = -360 + 200\omega_C + 499.76\omega_s \qquad (2)$$

$$\mathbf{k}: \quad 0 = -150 + 160\omega_C - 624.70\omega_s \qquad (3)$$

Solving the simultaneous equations (1), (2), and (3),

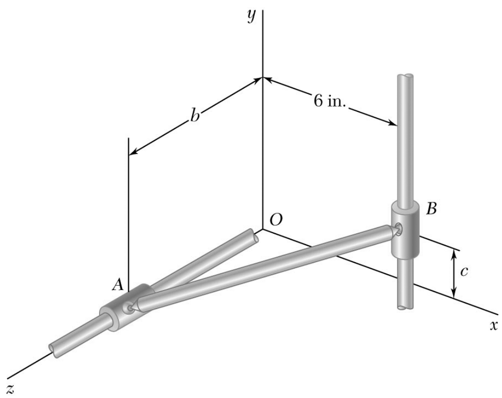
$$\omega_C = 1.4634 \text{ rad/s}, \quad \omega_s = 0.13470 \text{ rad/s}, \quad v_C = -34.50 \text{ mm/s}$$

(*a*) *Angular velocity of rod BC.*

$$\boldsymbol{\omega}_{BC} = 1.4634\mathbf{i} + (0.78087)(0.13470)\mathbf{j} + (0.62470)(0.13470)\mathbf{k}$$

$$\boldsymbol{\omega}_{BC} = (1.463 \text{ rad/s})\mathbf{i} + (0.1052 \text{ rad/s})\mathbf{j} + (0.0841 \text{ rad/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Velocity of collar C.* $\mathbf{v}_C = -(34.5 \text{ mm/s})\mathbf{i} \blacktriangleleft$



PROBLEM 15.214

In Problem 15.204, determine the acceleration of collar A when $c = 2$ in.

PROBLEM 15.204 Rod AB of length 11 in., is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves downward at a constant speed of 54 in./s, determine the velocity of collar A when $c = 2$ in.

SOLUTION

Geometry. $l_{AB}^2 = x_{A/B}^2 + y_{A/B}^2 + z_{A/B}^2: \quad (11)^2 = (6)^2 + (-2)^2 + (z_{A/B})^2$

$$z_{A/B} = 9 \text{ in.} \quad \mathbf{r}_{A/B} = (-6 \text{ in.})\mathbf{i} - (2 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$$

Velocity of collar B. $\mathbf{v}_B = -v_B\mathbf{j} = -(54 \text{ in./s})\mathbf{j}$

$$\mathbf{v}_B = \frac{(2.5)(4.5\mathbf{i} - 9\mathbf{k})}{10.0623} = (1.11803 \text{ in.})\mathbf{i} - (2.23607 \text{ in./s})\mathbf{k}$$

Velocity of collar A. $\mathbf{v}_A = v_A\mathbf{k}$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B},$$

where $\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get $\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$

or $\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$

From Eq. (1), $(-6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}) \cdot v_A\mathbf{k} = (-6\mathbf{i} - 2\mathbf{j} + 9\mathbf{k}) \cdot (-54\mathbf{j})$

or $9v_A = 108 \qquad v_A = (12.00 \text{ in./s})\mathbf{k}$

Relative velocity $\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$

$$\mathbf{v}_{A/B} = (54 \text{ in./s})\mathbf{j} + (12.00 \text{ in./s})\mathbf{k}$$

$$(v_{A/B})^2 = (54)^2 + (12.00)^2 = 3060 \text{ in}^2/\text{s}^2$$

Acceleration of collar B. $\mathbf{a}_B = 0$

Acceleration of collar A. $\mathbf{a}_A = a_A\mathbf{k}$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where $\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B}$

Noting that $\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B} \cdot \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} = 0$

PROBLEM 15.214 (Continued)

We note also that
$$\mathbf{r}_{A/B} \cdot \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B} = \mathbf{v}_{A/B} \cdot \mathbf{r}_{A/B} \times \boldsymbol{\omega}_{A/B}$$
$$= -\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = -(v_{A/B})^2$$

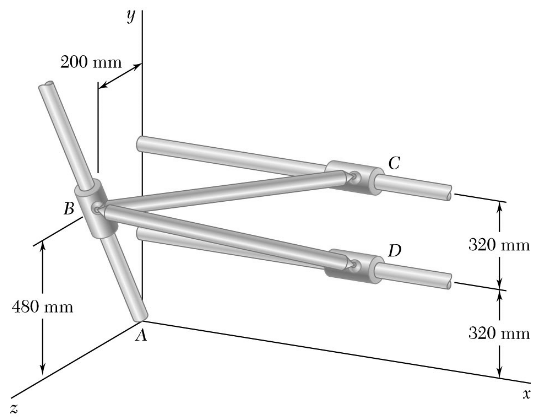
Then,
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B} = 0 - (v_{A/B})^2 = -(v_{A/B})^2$$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{a}_A$, we get
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot (\mathbf{a}_A + \mathbf{a}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{a}_B + \mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B}$$

or
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot \mathbf{a}_B - (v_{A/B})^2 \tag{2}$$

From Eq. (2)
$$(-6\mathbf{i} - 2\mathbf{i} + 9\mathbf{k}) \cdot a_A\mathbf{k} = 0 - 3060$$
$$9a_A = -3060$$

$\mathbf{a}_A = -(340 \text{ in./s}^2)\mathbf{k}$ ◀



PROBLEM 15.215

In Problem 15.205, determine the acceleration of collar *C*.

PROBLEM 15.205 Rod *BC* and *BD* are each 840 mm long and are connected by ball-and-socket joints to collars which may slide on the fixed rods shown. Knowing that collar *B* moves toward *A* at a constant speed of 390 mm/s, determine the velocity of collar *C* for the position shown.

SOLUTION

Geometry.

$$l_{BC}^2 = x_{C/B}^2 + y_{C/B}^2 + z_{C/B}^2$$

$$(840)^2 = c^2 + (640 - 480)^2 + (200)^2 \qquad c = 800m$$

$$\mathbf{r}_{C/B} = (800 \text{ mm})\mathbf{i} + (160 \text{ mm})\mathbf{j} - (200 \text{ mm})\mathbf{k}$$

Velocity of B.

$$\mathbf{v}_B = v_B\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right)$$

$$= (390 \text{ mm/s})\left(-\frac{12}{13}\mathbf{j} - \frac{5}{13}\mathbf{k}\right)$$

$$= -(360 \text{ mm/s})\mathbf{j} - (150 \text{ mm/s})\mathbf{k}$$

Velocity of C.

$$\mathbf{v}_C = v_C\mathbf{i}$$

where

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$\mathbf{v}_{C/B} = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{C/B}$$

Noting that $\mathbf{v}_{C/B}$ is perpendicular to $\mathbf{r}_{C/B}$, we get $\mathbf{r}_{C/B} \cdot \mathbf{v}_C = \mathbf{r}_{C/B} \cdot \mathbf{v}_B$

$$(800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot (-360\mathbf{j} + 150\mathbf{k}) = (800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot (v_C\mathbf{i})$$

$$(160)(-360) + (-200)(-150) = 800v_C$$

$$v_C = -34.5 \text{ mm/s} \qquad \mathbf{v}_C = -(34.5 \text{ mm/s})\mathbf{i}$$

Relative velocity:

$$\mathbf{v}_{C/B} = \mathbf{v}_C - \mathbf{v}_B$$

$$\mathbf{v}_{C/B} = -(34.5 \text{ mm/s})\mathbf{i} + (360 \text{ mm/s})\mathbf{j} + (150 \text{ mm/s})\mathbf{k}$$

$$v_{C/B}^2 = (34.5)^2 + (360)^2 + (150)^2 = 153290 \text{ mm}^2/\text{s}^2$$

Acceleration of collar B:

$$\mathbf{a}_B = 0$$

Acceleration of collar C:

$$\mathbf{a}_C = \mathbf{a}_C\mathbf{i}$$

$$\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$$

PROBLEM 15.215 (Continued)

where $$\mathbf{a}_{C/B} = \boldsymbol{\alpha}_{CB} \times \mathbf{r}_{C/B} + \boldsymbol{\omega}_{CB} \times \mathbf{v}_{C/B}$$

Noting that $\boldsymbol{\alpha}_{CB} \times \mathbf{r}_{C/B}$ is perpendicular to $\mathbf{r}_{C/B}$, we get $\mathbf{r}_{C/B} \cdot \boldsymbol{\alpha}_{CB} \times \mathbf{r}_{C/B} = 0$

We note also that
$$\mathbf{r}_{B/C} \cdot \boldsymbol{\omega}_{CB} \times \mathbf{v}_{C/B} = \mathbf{v}_{C/B} \cdot \mathbf{r}_{C/B} \times \boldsymbol{\omega}_{C/B}$$
$$= -\mathbf{v}_{C/B} \cdot \mathbf{v}_{C/B} = -(v_{C/B})^2$$

Then, $$\mathbf{r}_{C/B} \cdot \mathbf{a}_C = 0 - (v_{C/B})^2 = -(v_{C/B})^2$$

Forming $\mathbf{r}_{C/B} \cdot \mathbf{a}_C$, we get $$\mathbf{r}_{C/B} \cdot \mathbf{a}_C = \mathbf{r}_{C/B} \cdot (\mathbf{a}_B + \mathbf{a}_{C/B}) = \mathbf{r}_{B/C} \cdot \mathbf{a}_B + \mathbf{r}_{B/C} \cdot \mathbf{a}_{C/B}$$

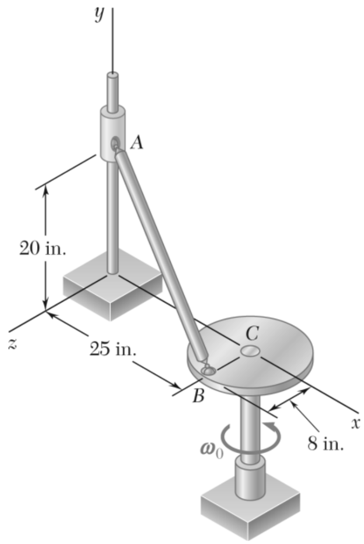
so that $$\mathbf{r}_{C/B} \cdot \mathbf{a}_C = \mathbf{r}_{C/B} \cdot \mathbf{a}_B - (v_{C/B})^2 \qquad (2)$$

From Equation (2), $$(800\mathbf{i} + 160\mathbf{j} - 200\mathbf{k}) \cdot a_C\mathbf{i} = 0 - 153290$$

$$800a_C = -153290$$

$$a_C = -191.6 \text{ mm/s}^2$$

$$\mathbf{a}_C = -(191.6 \text{ mm/s}^2)\mathbf{i} \blacktriangleleft$$



PROBLEM 15.216

In Problem 15.206, determine the acceleration of collar A.

PROBLEM 15.206 Rod AB is connected by ball-and-socket joints to collar A and to the 16-in.-diameter disk C. Knowing that disk C rotates counterclockwise at the constant rate $\omega_0 = 3$ rad/s in the zx plane, determine the velocity of collar A for the position shown.

SOLUTION

Geometry.

$$\mathbf{r}_{B/C} = (-8 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{A/B} = -(25 \text{ in.})\mathbf{i} + (20 \text{ in.})\mathbf{j} - (8 \text{ in.})\mathbf{k}$$

Velocity at B.

$$\mathbf{v}_B = \omega_0 \mathbf{j} \times \mathbf{r}_{B/C} = 3\mathbf{j} \times (-8\mathbf{k}) = (24 \text{ in./s})\mathbf{i}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1)

$$(-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (v_A \mathbf{j}) = (-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (24\mathbf{i})$$

$$20 v_A = -600$$

or

$$v_A = -30 \text{ in./s}$$

Relative velocity.

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$\mathbf{v}_{A/B} = (-30 \text{ in./s})\mathbf{j} + (24 \text{ in./s})\mathbf{i}$$

$$(v_{A/B})^2 = (-30)^2 + (24)^2 = 1476 \text{ in}^2/\text{s}^2$$

PROBLEM 15.216 (Continued)

Acceleration at B.

$$\mathbf{a}_B = \omega_0 \mathbf{j} \times \mathbf{v}_B$$
$$= 3\mathbf{j} \times 24\mathbf{i}$$
$$= -(72 \text{ in./s}^2)\mathbf{k}$$

Acceleration of collar A.

$$\mathbf{a}_A = a_A \mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where

$$\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B}$$

Noting that $\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B} \cdot \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} = 0$

We note also that

$$\mathbf{r}_{A/B} \cdot \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B} = \mathbf{v}_{A/B} \cdot \mathbf{r}_{A/B} \times \boldsymbol{\omega}_{A/B}$$
$$= -\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B}$$
$$= -(v_{A/B})^2$$

Then

$$\mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B} = 0 - (v_{A/B})^2$$
$$= -(v_{A/B})^2$$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{a}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot (\mathbf{a}_A + \mathbf{a}_{A/B})$$
$$= \mathbf{r}_{A/B} \cdot \mathbf{a}_B + \mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B}$$

or

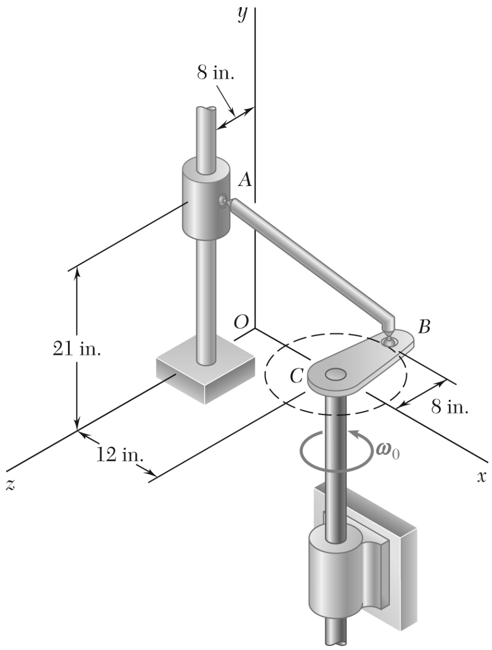
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot \mathbf{a}_B - (v_{A/B})^2 \qquad (2)$$

From Eq. (2),

$$(-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (a_A \mathbf{j}) = (-25\mathbf{i} + 20\mathbf{j} - 8\mathbf{k}) \cdot (-72\mathbf{k}) - 1476$$

$$20a_A = 576 - 1476$$
$$= -45 \text{ in./s}^2$$

$$\mathbf{a}_A = -(45.0 \text{ in./s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.217

In Problem 15.207, determine the acceleration of collar *A*.

PROBLEM 15.207 Rod *AB* of length 29 in. is connected by ball-and-socket joints to the rotating crank *BC* and to the collar *A*. Crank *BC* is of length 8 in. and rotates in the horizontal *xz* plane at the constant rate $\omega_0 = 10$ rad/s. At the instant shown, when crank *BC* is parallel to the *z* axis, determine the velocity of collar *A*.

SOLUTION

Geometry.

$$\mathbf{r}_{B/C} = (-8 \text{ in.})\mathbf{k},$$
$$\mathbf{r}_{A/B} = (-12 \text{ in.})\mathbf{i} + (21 \text{ in.})\mathbf{j} + (16 \text{ in.})\mathbf{k}$$

Velocity at B.

$$\mathbf{v}_B = \omega_0 \mathbf{j} \times \mathbf{r}_{B/C}$$
$$= 10\mathbf{j} \times (-8\mathbf{k})$$
$$= (-80 \text{ in./s})\mathbf{i}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A \mathbf{j}$$
$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where

$$\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0$.

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B})$$
$$= \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or

$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1)

$$(-12\mathbf{i} + 21\mathbf{j} + 16\mathbf{k}) \cdot (v_A \mathbf{j}) = (-12\mathbf{i} + 21\mathbf{j} - 16\mathbf{k}) \cdot (-80\mathbf{i})$$
$$21 v_A = 960$$

or

$$v_A = 45.714 \text{ in./s} \qquad \mathbf{v}_A = (45.7 \text{ in./s})\mathbf{j} \blacktriangleleft$$

Relative velocity.

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B$$

$$\mathbf{v}_{A/B} = (45.714 \text{ in./s})\mathbf{j} + (80 \text{ in./s})\mathbf{i}$$

$$(v_{A/B})^2 = (45.714)^2 + (80)^2$$

$$= 8489.8 \text{ (in./s}^2)$$

Acceleration of Point B.

$$\mathbf{a}_B = \omega_0 \mathbf{j} \times \mathbf{v}_B$$

$$= 10\mathbf{j} \times (-80)\mathbf{i}$$

$$= (800 \text{ in./s}^2)\mathbf{k}$$

Acceleration of collar A.

$$\mathbf{a}_A = a_A \mathbf{j}$$

$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where

$$\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B}$$

Noting that $\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B} \cdot \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} = 0.$

We note also that

$$\mathbf{r}_{A/B} \cdot \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B} = \mathbf{v}_{A/B} \cdot \mathbf{r}_{A/B} \times \boldsymbol{\omega}_{A/B}$$

$$= -\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B}$$

$$= -(v_{A/B})^2$$

Then

$$\mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B} = 0 - (v_{A/B})^2$$

$$= -(v_{A/B})^2$$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{a}_A$, we get

$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot (\mathbf{a}_A + \mathbf{a}_{A/B})$$

$$= \mathbf{r}_{A/B} \cdot \mathbf{a}_B + \mathbf{r}_{A/B} - \mathbf{a}_{A/B}$$

or

$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot \mathbf{a}_B - (v_{A/B})^2 \qquad (2)$$

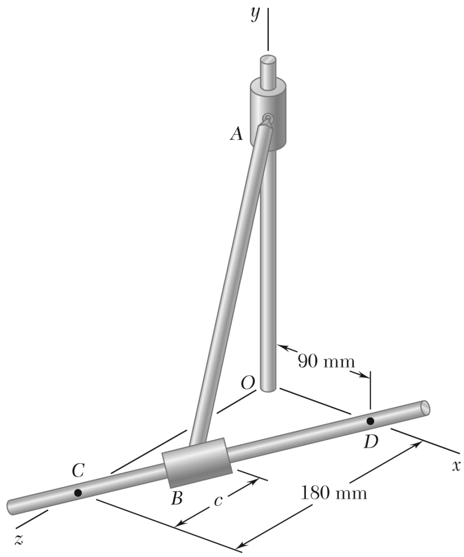
From Eq. (2)

$$(-12\mathbf{i} + 21\mathbf{j} + 16\mathbf{k}) \cdot (a_A \mathbf{j}) = (-12\mathbf{i} + 21\mathbf{j} + 16\mathbf{k}) \cdot (800\mathbf{k}) - 8489.8$$

$$21a_A = 12{,}800 - 8489.8$$

$$\mathbf{a}_A = (205 \text{ in./s}^2)\mathbf{j} \blacktriangleleft$$

PROBLEM 15.218



In Problem 15.208, determine the acceleration of collar A.

PROBLEM 15.208 Rod AB of length 300 mm is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves toward Point D at a constant speed of 50 mm/s, determine the velocity of collar A when $c = 80$ mm.

SOLUTION

Geometry. $\mathbf{r}_A = y\mathbf{j}, \quad \mathbf{r}_D = (90 \text{ mm})\mathbf{i} \quad \mathbf{r}_C = (180 \text{ mm})\mathbf{k}$

$$\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C = (90 \text{ mm})\mathbf{i} - (180 \text{ mm})\mathbf{k}$$

$$l_{CD} = \sqrt{(90)^2 + (180)^2} = 201.246 \text{ mm}$$

$$\mathbf{r}_{B/C} = \frac{c(\mathbf{r}_{D/C})}{180} = \frac{80(90\mathbf{i} - 180\mathbf{k})}{180} = (40 \text{ mm})\mathbf{i} - (80 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_B = \mathbf{r}_C + \mathbf{r}_{B/C} = 180\mathbf{k} + 40\mathbf{i} - 80\mathbf{k} = (40 \text{ mm})\mathbf{i} + (100 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B = -(40 \text{ mm})\mathbf{i} + (y \text{ mm})\mathbf{j} - (100 \text{ mm})\mathbf{k}$$

$$l_{AB}^2 = x_{A/B}^2 + y^2 + z_{A/B}^2: \quad 300^2 = (-40)^2 + y^2 + (-100)^2$$

$$y = 280 \text{ mm},$$

$$\mathbf{r}_{A/B} = (-40 \text{ mm})\mathbf{i} + (280 \text{ mm})\mathbf{j} - (100 \text{ mm})\mathbf{k}$$

Velocity of collar B.

$$\mathbf{v}_B = v_B \frac{\mathbf{r}_{D/C}}{l_{CD}}$$

$$\mathbf{v}_B = \frac{(50)(90\mathbf{i} - 180\mathbf{k})}{201.246}$$

$$= (22.3607 \text{ mm/s})\mathbf{i} - (44.7214 \text{ mm/s})\mathbf{k}$$

Velocity of collar A.

$$\mathbf{v}_A = v_A\mathbf{j}$$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where $\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0$.

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From (1)
$$(-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (v_A \mathbf{j}) = (-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (22.3607\mathbf{i} - 44.7214\mathbf{k})$$
$$280 v_A = (-40)(22.3607) + (-100)(-44.7214)$$

or
$$v_A = 12.7775 \text{ mm/s} \qquad \mathbf{v}_A = (12.7775 \text{ mm/s})\,\mathbf{j}$$

<u>Relative velocity</u>.
$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = (12.7775 \text{ mm/s})\mathbf{i} - (22.3607 \text{ mm/s})\mathbf{j} + (44.7214 \text{ mm/s})\mathbf{k}$$
$$\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = (12.7775)^2 + (22.3607)^2 + (44.7214)^2 = 2663.3 \text{ mm}^2/\text{s}^2$$

Acceleration of collar B.
$$\mathbf{a}_B = 0$$

Acceleration of collar A.
$$\mathbf{a}_A = a_A \mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where
$$\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B}$$

Noting that $\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B} \cdot \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} = 0$

We note also that
$$\mathbf{r}_{A/B} \cdot \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B} = \mathbf{v}_{A/B} \cdot \mathbf{r}_{A/B} \times \boldsymbol{\omega}_{A/B} = -\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = -(v_{A/B})^2$$

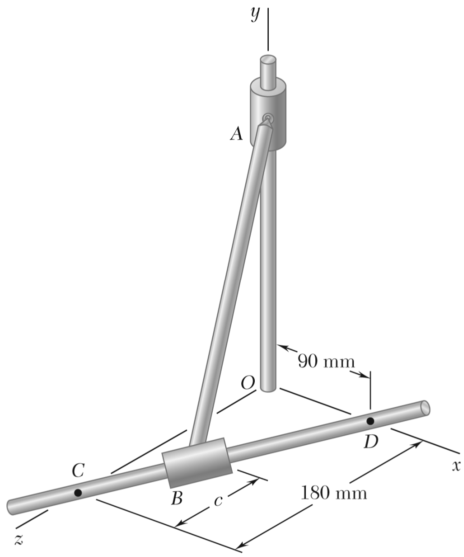
Then
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B} = 0 - (v_{A/B})^2 = -(v_{A/B})^2$$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{a}_A$, we get
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot (\mathbf{a}_A + \mathbf{a}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{a}_B + \mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B}$$

or
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot \mathbf{a}_B - (v_{A/B})^2 \qquad (2)$$

From Eq. (2)
$$(-40\mathbf{i} + 280\mathbf{j} - 100\mathbf{k}) \cdot (a_A \mathbf{j}) = 0 - 2663.3$$
$$280 a_A = -2663.3 \qquad a_A = -9.512 \text{ mm/s}^2$$

$$\mathbf{a}_A = -(9.51 \text{ mm/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.219

In Problem 15.209, determine the acceleration of collar A.

PROBLEM 15.209 Rod AB of length 300 mm is connected by ball-and-socket joints to collars A and B, which slide along the two rods shown. Knowing that collar B moves toward Point D at a constant speed of 50 mm/s, determine the velocity of collar A when $c = 120$ mm.

SOLUTION

Geometry. $\mathbf{r}_A = y\mathbf{j}, \quad \mathbf{r}_D = (90 \text{ mm})\mathbf{i} \quad \mathbf{r}_C = (180 \text{ mm})\mathbf{k}$

$$\mathbf{r}_{D/C} = \mathbf{r}_D - \mathbf{r}_C = (40 \text{ mm})\mathbf{i} - (180 \text{ mm})\mathbf{k}$$

$$l_{CD} = \sqrt{(90)^2 + (-180)^2} = 201.246$$

$$\mathbf{r}_{B/C} = \frac{c(\mathbf{r}_{D/C})}{180} = \frac{120(90\mathbf{i} - 180\mathbf{k})}{180} = (60 \text{ mm})\mathbf{i} - (120 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_B = \mathbf{r}_C + \mathbf{r}_{B/C} = 180\mathbf{k} + 60\mathbf{i} - 120\mathbf{k} = (60 \text{ mm})\mathbf{i} + (60 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/B} = \mathbf{r}_A - \mathbf{r}_B = -60\mathbf{i} + y\,\mathbf{j} - 60\mathbf{k}$$

$$l_{AB}^2 = x_{A/B}^2 + y^2 + z_{A/B}^2: \quad 300^2 = 60^2 + y^2 + 60^2$$

$$y = 287.75 \text{ mm},$$

$$\mathbf{r}_{A/B} = (-60 \text{ mm})\mathbf{i} + (287.75 \text{ mm})\mathbf{j} - (60 \text{ mm})\mathbf{k}$$

Velocity of collar B. $\mathbf{v}_B = v_B \dfrac{\mathbf{r}_{D/C}}{l_{CD}}$

$$\mathbf{v}_B = \frac{(50)(90\mathbf{i} - 180\mathbf{k})}{201.246}$$

$$= (22.3607 \text{ mm/s})\mathbf{i} - (44.7214 \text{ mm/s})\mathbf{k}$$

Velocity of collar A. $\mathbf{v}_A = v_A\mathbf{j}$

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

where $\mathbf{v}_{A/B} = \boldsymbol{\omega}_{AB} \times \mathbf{r}_{A/B}$

Noting that $\mathbf{v}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{B/A} \cdot \mathbf{v}_{B/A} = 0.$

Forming $\mathbf{r}_{A/B} \cdot \mathbf{v}_A$, we get
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot (\mathbf{v}_B + \mathbf{v}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{v}_B + \mathbf{r}_{A/B} \cdot \mathbf{v}_{A/B}$$

or
$$\mathbf{r}_{A/B} \cdot \mathbf{v}_A = \mathbf{r}_{A/B} \cdot \mathbf{v}_B \qquad (1)$$

From Eq. (1) $\quad (-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (v_A\mathbf{j}) = (-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (1.11803\mathbf{i} - 2.23607\mathbf{j})$

$$287.75 v_A = (-60)(22.3607) + (-60)(-44.7214)$$

or
$$v_A = 4.6626 \text{ mm/s} \qquad \mathbf{v}_A = (4.6626 \text{ mm/s})\mathbf{j}$$

<u>Relative velocity</u>.
$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = -(22.3607 \text{ mm/s})\mathbf{i} + (4.6626 \text{ mm/s})\mathbf{j} + (44.7214 \text{ mm/s})\mathbf{k}$$
$$\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = (22.3607)^2 + (4.6626)^2 + (44.7214)^2 = 2521.7$$

Acceleration of collar B.
$$\mathbf{a}_B = 0$$

Acceleration of collar A.
$$\mathbf{a}_A = a_A\mathbf{j}$$
$$\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

where
$$\mathbf{a}_{A/B} = \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} + \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B}$$

Noting that $\boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B}$ is perpendicular to $\mathbf{r}_{A/B}$, we get $\mathbf{r}_{A/B} \cdot \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{A/B} = 0.$

We note also that
$$\mathbf{r}_{A/B} \cdot \boldsymbol{\omega}_{AB} \times \mathbf{v}_{A/B} = \mathbf{v}_{A/B} \cdot \mathbf{r}_{A/B} \times \boldsymbol{\omega}_{A/B} = -\mathbf{v}_{A/B} \cdot \mathbf{v}_{A/B} = -(v_{A/B})^2$$

Then
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B} = 0 - (v_{A/B})^2 = -(v_{A/B})^2$$

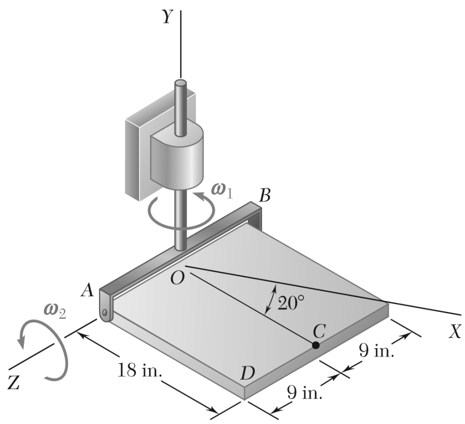
Forming $\mathbf{r}_{A/B} \cdot \mathbf{a}_A$, we get
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot (\mathbf{a}_A + \mathbf{a}_{A/B}) = \mathbf{r}_{A/B} \cdot \mathbf{a}_B + \mathbf{r}_{A/B} \cdot \mathbf{a}_{A/B}$$

or
$$\mathbf{r}_{A/B} \cdot \mathbf{a}_A = \mathbf{r}_{A/B} \cdot \mathbf{a}_B - (v_{A/B})^2 \qquad (2)$$

From Eq. (2), $\quad (-60\mathbf{i} + 287.75\mathbf{j} - 60\mathbf{k}) \cdot (a_A\mathbf{j}) = 0 - 2521.7$

$$287.75 a_A = -2521.7 \qquad a_A = -8.764 \text{ mm/s}^2$$

$$\mathbf{a}_A = -(8.76 \text{ mm/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.220

A square plate of side 18 in. is hinged at A and B to a clevis. The plate rotates at the constant rate $\omega_2 = 4$ rad/s with respect to the clevis, which itself rotates at the constant rate $\omega_1 = 3$ rad/s about the Y axis. For the position shown, determine (a) the velocity of Point C, (b) the acceleration of Point C.

SOLUTION

Geometry. $\mathbf{r}_C = (18 \text{ in.})(\cos 20°\mathbf{i} - \sin 20°\mathbf{j})$

Let frame $Oxyz$ rotate about the y axis with angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j}$ and angular acceleration $\dot{\mathbf{\Omega}} = 0$. Then the motion relative to the frame consists of rotation with angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k}$ and angular acceleration $\boldsymbol{\alpha}_2 = 0$ about the z axis.

(a)

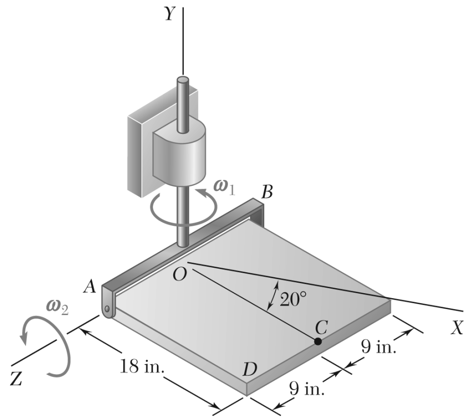
$$\begin{aligned}
\mathbf{v}_{C'} &= \mathbf{\Omega} \times \mathbf{r}_C \\
&= 3\mathbf{j} \times (18\cos 20°\mathbf{i} - 18\sin 20°\mathbf{j}) \\
&= -54\cos 20°\mathbf{k} \\
\mathbf{v}_{C/F} &= \boldsymbol{\omega}_2 \times \mathbf{r}_C \\
&= 4\mathbf{k} \times (18\cos 20°\mathbf{i} - 18\sin 20°\mathbf{j}) \\
&= 72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j} \\
\mathbf{v}_C &= \mathbf{v}_{C'} + \mathbf{v}_{C/F} \\
&= 72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j} - 54\cos 20°\mathbf{k}
\end{aligned}$$

$$\mathbf{v}_C = (24.6 \text{ in./s})\mathbf{i} + (67.7 \text{ in./s})\mathbf{j} - (50.7 \text{ in./s})\mathbf{k} \blacktriangleleft$$

(b)

$$\begin{aligned}
\mathbf{a}_{C'} &= \mathbf{\Omega} \times \mathbf{v}_{C'} \\
&= 3\mathbf{j} \times (-54\cos 20°\mathbf{k}) \\
&= -162\cos 20°\mathbf{i} \\
\mathbf{a}_{C/F} &= \boldsymbol{\omega}_2 \times \mathbf{v}_{C/F} \\
&= 4\mathbf{k} \times (72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j}) \\
&= -288\cos 20°\mathbf{i} + 288\sin 20°\mathbf{j} \\
2\mathbf{\Omega} \times \mathbf{v}_{C/F} &= (2)(3\mathbf{j}) \times (72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j}) \\
&= -432\sin 20°\mathbf{k} \\
\mathbf{a}_C &= \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\mathbf{\Omega} \times \mathbf{v}_{C/F} \\
&= -(162 + 288)\cos 20°\mathbf{i} + 288\sin 20°\mathbf{j} - 432\sin 20°\mathbf{k}
\end{aligned}$$

$$\mathbf{a}_C = -(423 \text{ in./s}^2)\mathbf{i} + (98.5 \text{ in./s}^2)\mathbf{j} - (147.8 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.221

A square plate of side 18 in. is hinged at A and B to a clevis. The plate rotates at the constant rate $\omega_2 = 4$ rad/s with respect to the clevis, which itself rotates at the constant rate $\omega_1 = 3$ rad/s about the Y axis. For the position shown, determine (*a*) the velocity of corner D, (*b*) the acceleration of corner D.

SOLUTION

Geometry. $\mathbf{r}_D = (18 \text{ in.})(\cos 20°\mathbf{i} - \sin 20°\mathbf{j}) + (9 \text{ in.})\mathbf{k}$

Let frame $Oxyz$ rotate about the y axis with angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j}$ and angular acceleration $\dot{\mathbf{\Omega}} = 0$. Then the motion relative to the frame consists of rotation with angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k}$ and angular acceleration $\boldsymbol{\alpha}_2 = 0$ about the z axis.

(*a*)

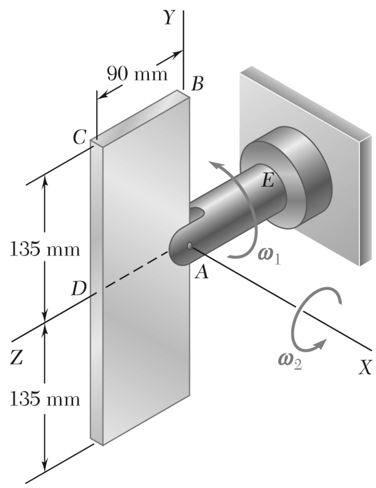
$$\begin{aligned}
\mathbf{v}_{D'} &= \mathbf{\Omega} \times \mathbf{r}_D \\
&= 3\mathbf{j} \times (18\cos 20°\mathbf{i} - 18\sin 20°\mathbf{j} + 9\mathbf{k}) \\
&= 27\mathbf{i} - 54\cos 20°\mathbf{k} \\
\mathbf{v}_{D/F} &= \boldsymbol{\omega}_2 \times \mathbf{r}_D \\
&= 4\mathbf{k} \times (18\cos 20°\mathbf{i} - 18\sin 20°\mathbf{j} + 9\mathbf{k}) \\
&= 72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j} \\
\mathbf{v}_D &= \mathbf{v}_{D'} + \mathbf{v}_{D/F} \\
&= (27 + 72\sin 20°)\mathbf{i} + 72\cos 20°\mathbf{j} - 54\cos 20°\mathbf{k}
\end{aligned}$$

$$\mathbf{v}_D = (51.6 \text{ in./s})\mathbf{i} + (67.7 \text{ in./s})\mathbf{j} - (50.7 \text{ in./s})\mathbf{k} \blacktriangleleft$$

(*b*)

$$\begin{aligned}
\mathbf{a}_{D'} &= \mathbf{\Omega} \times \mathbf{v}_{D'} \\
&= 3\mathbf{j} \times (27\mathbf{i} - 54\cos 20°\mathbf{k}) \\
&= -162\cos 20°\mathbf{i} - 81\mathbf{k} \\
\mathbf{a}_{D/F} &= \boldsymbol{\omega}_2 \times \mathbf{v}_{D/F} \\
&= 4\mathbf{k} \times (72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j}) \\
&= -288\cos 20°\mathbf{i} + 288\sin 20°\mathbf{j} \\
2\mathbf{\Omega} \times \mathbf{v}_{D/F} &= (2)(3\mathbf{j}) \times (72\sin 20°\mathbf{i} + 72\cos 20°\mathbf{j}) \\
&= -432\sin 20°\mathbf{k} \\
\mathbf{a}_D &= \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\mathbf{\Omega} \times \mathbf{v}_{C/F} \\
&= -(162 + 288)\cos 20°\mathbf{i} + 288\sin 20°\mathbf{j} - (81 + 432\sin 20°)\mathbf{k}
\end{aligned}$$

$$\mathbf{a}_D = -(423 \text{ in./s}^2)\mathbf{i} + (98.5 \text{ in./s}^2)\mathbf{j} - (229 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.222

The rectangular plate shown rotates at the constant rate $\omega_2 = 12$ rad/s with respect to arm AE, which itself rotates at the constant rate $w_1 = 9$ rad/s about the Z axis. For the position shown, determine the velocity and acceleration of the point of the plate indicated.

Corner B.

SOLUTION

Geometry. With the origin at A, $\quad \mathbf{r}_B = (0.135 \text{ m})\mathbf{j}$

Let frame $AXYZ$ rotate about the Y axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{k} = (9 \text{ rad/s})\mathbf{k}$. Then the motion relative to the frame consists of rotation about the X axis with constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{i} = (12 \text{ rad/s})\mathbf{i}$.

Motion of coinciding Point B'.

$$\mathbf{v}_{B'} = \mathbf{\Omega} \times \mathbf{r}_B$$
$$= 9\mathbf{k} \times 0.135\mathbf{j}$$
$$= -(1.215 \text{ m/s})\mathbf{i}$$
$$\mathbf{a}_{B'} = \boldsymbol{\alpha} \times \mathbf{r}_B + \mathbf{\Omega} \times \mathbf{v}_{B'}$$
$$= 0 + 9\mathbf{k} \times (-1.215\mathbf{i})$$
$$= -(10.935 \text{ m/s}^2)\mathbf{j}$$

Motion relative to the frame.

$$\mathbf{v}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_B = 12\mathbf{i} \times 0.135\mathbf{j}$$
$$= (1.62 \text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{B/F} = \boldsymbol{\alpha}_2 \times \mathbf{r}_B + \boldsymbol{\omega}_2 \times \mathbf{v}_{B/F}$$
$$= 0 + 12\mathbf{i} \times 1.62\mathbf{k}$$
$$= -(19.44 \text{ m/s}^2)\mathbf{j}$$

Velocity of Point B. $\quad \mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$

$$\mathbf{v}_B = -(1.215 \text{ m/s})\mathbf{i} + (1.620 \text{ m/s})\mathbf{k} \blacktriangleleft$$

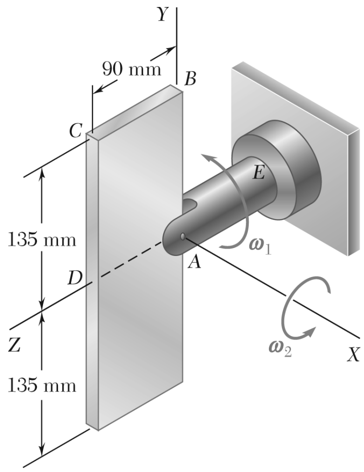
Coriolis acceleration. $\quad 2\mathbf{\Omega} \times \mathbf{v}_{B/F}$

$$2\mathbf{\Omega} \times \mathbf{v}_{B/F} = (2)(9\mathbf{k}) \times 1.62\mathbf{k} = 0$$

Acceleration of Point B.

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\mathbf{\Omega} \times \mathbf{v}_{B/F}$$
$$\mathbf{a}_B = -(30.375 \text{ in./s}^2)\mathbf{j}$$

$$\mathbf{a}_B = -(30.4 \text{ m/s}^2)\mathbf{j} \blacktriangleleft$$



PROBLEM 15.223

The rectangular plate shown rotates at the constant rate $\omega_2 = 12$ rad/s with respect to arm *AE*, which itself rotates at the constant rate $\omega_1 = 9$ rad/s about the *Z* axis. For the position shown, determine the velocity and acceleration of the point of the plate indicated.

Corner *C*.

SOLUTION

Geometry. With the origin at *A*, $\mathbf{r}_C = (0.135 \text{ m})\mathbf{j} + (0.09 \text{ m})\mathbf{k}$

Let frame *AXYZ* rotate about the *Y* axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{k} = (9 \text{ rad/s})\mathbf{k}$. Then the motion relative to the frame consists of rotation about the *X* axis with constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{i} = (12 \text{ rad/s})\mathbf{i}$.

Motion of coinciding Point C' in the frame.

$$\begin{aligned}\mathbf{v}_{C'} &= \mathbf{\Omega} \times \mathbf{r}_C \\ &= 9\mathbf{k} \times (0.135\mathbf{j} + 0.09\mathbf{k}) \\ &= -(1.215 \text{ m/s})\mathbf{i}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{C'} &= \boldsymbol{\alpha} \times \mathbf{r}_C + \mathbf{\Omega} \times \mathbf{v}_{C'} \\ &= 0 + 9\mathbf{k} \times (1.215\mathbf{i}) \\ &= -(10.935 \text{ m/s}^2)\mathbf{j}\end{aligned}$$

Motion relative to the frame.

$$\begin{aligned}\mathbf{v}_{C/F} &= \boldsymbol{\omega}_2 \times \mathbf{r}_C \\ &= 12\mathbf{i} \times (0.135\mathbf{j} + 0.09\mathbf{k}) \\ &= -(1.08 \text{ m/s})\mathbf{j} + (1.62 \text{ m/s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{C/F} &= \boldsymbol{\alpha}_2 \times \mathbf{r}_C + \boldsymbol{\omega}_2 \times \mathbf{v}_{C/F} \\ &= 0 + 12\mathbf{i} \times (-1.08\mathbf{j} + 1.62\mathbf{k}) \\ &= -(19.44 \text{ m/s}^2)\mathbf{j} - (12.96 \text{ m/s}^2)\mathbf{k}\end{aligned}$$

Velocity of Point C. $\mathbf{v}_C = \mathbf{v}_{C'} + \mathbf{v}_{C/F}$

$$\mathbf{v}_C = -(1.215 \text{ m/s})\mathbf{i} - (1.080 \text{ m/s})\mathbf{j} + (1.620 \text{ m/s})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 15.223 (Continued)

Coriolis acceleration. $2\boldsymbol{\Omega} \times \mathbf{v}_{C/F}$

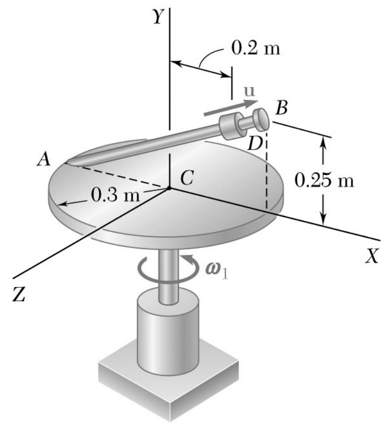
$$2\boldsymbol{\Omega} \times \mathbf{v}_{C/F} = (2)(9\mathbf{k}) \times (-1.08\mathbf{j} + 1.62\mathbf{k})$$

$$= (19.44 \text{ m/s}^2)\mathbf{i}$$

Acceleration of Point C. $\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{C/F}$

$$\mathbf{a}_C = (19.44 \text{ m/s}^2)\mathbf{i} - (30.375 \text{ m/s}^2)\mathbf{j} - (12.96 \text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_C = (19.44 \text{ m/s}^2)\mathbf{i} - (30.4 \text{ m/s}^2)\mathbf{j} - (12.96 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.224

Rod AB is welded to the 0.3-m-radius plate, which rotates at the constant rate $\omega_1 = 6$ rad/s. Knowing that collar D moves toward end B of the rod at a constant speed $u = 1.3$ m, determine, for the position shown, (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

Geometry.

$$\mathbf{r}_{B/A} = (0.6\text{ m})\mathbf{i} + (0.25\text{ m})\mathbf{j} \qquad \mathbf{r}_{C/A} = (0.3\text{ m})\mathbf{i}$$

$$\mathbf{r}_{D/A} = \frac{0.5}{0.6}\mathbf{r}_{B/A}$$

$$\mathbf{r}_{D/C} = \mathbf{r}_{D/A} - \mathbf{r}_{C/A}$$
$$= (0.2\text{ m})\mathbf{i} + (0.20833\text{ m})\mathbf{j}$$

$$l_{AB} = \sqrt{0.6^2 + 0.25^2}$$
$$= 0.65\text{ m}$$

Unit vector along AB:

$$\boldsymbol{\lambda}_{AB} = \frac{\mathbf{r}_{B/A}}{l_{AB}}$$
$$= \frac{12}{13}\mathbf{i} + \frac{5}{12}\mathbf{j}$$

Let $Oxyz$ be a frame of reference currently coinciding with $OXYZ$, but rotating with angular velocity

$$\boldsymbol{\Omega} = \omega_1\mathbf{j} = (6\text{ rad/s})\mathbf{j}$$

(*a*) *Velocity of D.*

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/AB}$$

$$\mathbf{v}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_{D/C}$$
$$= 6\mathbf{j} \times (0.2\mathbf{i} + 0.20833\mathbf{j})$$
$$= -(1.2\text{ m/s})\mathbf{k}$$

$$\mathbf{v}_{D/AB} = u\boldsymbol{\lambda}_{AB}$$
$$= 1.3\left(\frac{12}{13}\mathbf{i} + \frac{5}{13}\mathbf{j}\right)$$
$$= (1.2\text{ m/s})\mathbf{i} + (0.5\text{ m/s})\mathbf{j}$$

$$\mathbf{v}_D = (1.2\text{ m/s})\mathbf{i} + (0.5\text{ m/s})\mathbf{j} - (1.2\text{ m/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Acceleration of D.*

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/AB} + 2\mathbf{\Omega}\times\mathbf{v}_{D/AB}$$

$$\mathbf{a}_{D'} = \boldsymbol{\omega}_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_{D/C})$$

$$= 6\mathbf{j}\times(6\mathbf{j}\times(0.2\mathbf{i} + 0.20833\mathbf{j}))$$

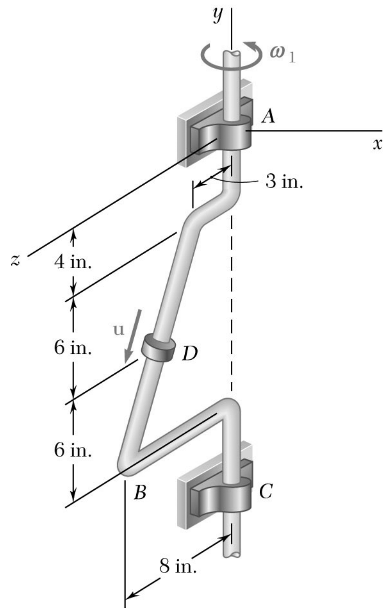
$$= -(7.2 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_{D/AB} = 0$$

$$2\mathbf{\Omega}\times\mathbf{v}_{D/F} = (2)(6\mathbf{j})\times((1.2)\mathbf{i} + (0.5)\mathbf{j})$$

$$= -(14.4 \text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_D = -(7.2 \text{ m/s}^2)\mathbf{i} - (14.4 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

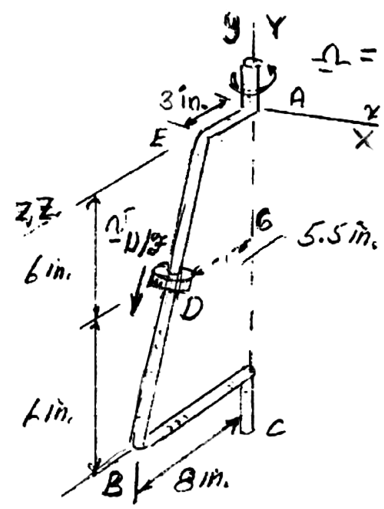


PROBLEM 15.225

The bent rod ABC rotates at the constant rate $\omega_1 = 4$ rad/s. Knowing that collar D moves downward along the rod at a constant relative speed $u = 65$ in./s, determine, for the position shown, (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

Units: inches, in./s, in./s^2



Geometry.

$$\mathbf{r}_E = 3\mathbf{k}$$

$$\mathbf{r}_B = -12\mathbf{j} + 8\mathbf{k}$$

$$\mathbf{r}_{B/E} = -12\mathbf{j} + 5\mathbf{k}$$

$$\mathbf{r}_D = \frac{1}{2}(\mathbf{r}_E + \mathbf{r}_B) = -6\mathbf{j} + 5.5\mathbf{k}$$

$$l_{EB} = \sqrt{12^2 + 5^2} = 13$$

Unit vector along EB: $\quad \boldsymbol{\lambda} = \dfrac{\mathbf{r}_{B/E}}{l_{EB}} = -\dfrac{12}{13}\mathbf{j} + \dfrac{5}{13}\mathbf{k}$

Use a rotating frame of reference that rotates with angular velocity

$$\boldsymbol{\Omega} = \boldsymbol{\omega}_1 = (4 \text{ rad/s})\mathbf{j}$$

Motion of Point D' in the frame currently at D.

$$\mathbf{v}_{D'} = \boldsymbol{\omega}_1 \times \mathbf{r}_D = 4\mathbf{j} \times (-6\mathbf{j} + 5.5\mathbf{k})$$
$$= (22 \text{ in./s})\mathbf{i}$$

$$\mathbf{a}_{D'} = \dot{\boldsymbol{\omega}}_1\mathbf{j} \times \mathbf{r}_D + \boldsymbol{\omega}_1 \times \mathbf{v}_{D'}$$
$$= 0 + (4\mathbf{j}) \times (22\mathbf{i})$$
$$= -(88 \text{ in./s}^2)\mathbf{k}$$

PROBLEM 15.225 (Continued)

Motion of collar D relative to the frame.

$$\mathbf{v}_{D/F} = u\boldsymbol{\lambda} = (65 \text{ in./s})\left(-\frac{12}{13}\mathbf{j} + \frac{5}{13}\mathbf{k}\right)$$

$$= -(60 \text{ in./s})\mathbf{j} + (25 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{D/F} = 0 \qquad \text{(Constant speed on straight path)}$$

(*a*) *Velocity of D.*

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$$\mathbf{v}_D = 22\mathbf{i} - 60\mathbf{j} + 25\mathbf{k}$$

$$\mathbf{v}_D = (22 \text{ in./s})\mathbf{i} - (60 \text{ in./s})\mathbf{j} + (25 \text{ in./s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration.

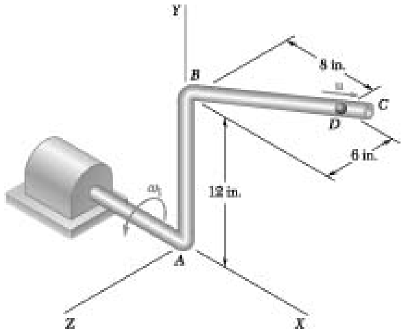
$$2\boldsymbol{\Omega} \times \mathbf{v}_{D/F} = (2)(4\mathbf{j}) \times (-60\mathbf{j} + 25\mathbf{k})$$

$$= (200 \text{ in./s}^2)\mathbf{i}$$

(*b*) *Acceleration of Point D.*

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_D$$

$$\mathbf{a}_D = (200 \text{ in./s}^2)\mathbf{i} - (88 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.226

The bent pipe shown rotates at the constant rate $\omega_1 = 10$ rad/s. Knowing that a ball bearing D moves in portion BC of the pipe toward end C at a constant relative speed $u = 2$ ft/s, determine at the instant shown (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

With the origin at Point A,
$$\mathbf{r}_D = (8\text{ in.})\mathbf{i} + (12\text{ in.})\mathbf{j} - (6\text{ in.})\mathbf{k}$$

$$\mathbf{r}_{C/B} = (8\text{ in.})\mathbf{i} - (6\text{ in.})\mathbf{k},$$

$$l_{BC} = \sqrt{8^2 + 6^2} = 10\text{ in.}$$

Let the frame $Axyz$ rotate with angular velocity $\quad \boldsymbol{\Omega} = \omega_1\mathbf{i} = (10\text{ rad/s})\mathbf{i}$

(*a*) *Velocity of D.*

$$\mathbf{v}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_D$$
$$= 10\mathbf{i} \times (8\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$$
$$= (60\text{ in./s})\mathbf{j} + (120\text{ in./s})\mathbf{k}$$

$$u = 2\text{ ft/s} = 24\text{ in./s},$$

$$\mathbf{v}_{D/F} = \frac{24}{10}(8\mathbf{i} - 6\mathbf{k})$$
$$= (19.2\text{ in./s})\mathbf{i} - (14.4\text{ in./s})\mathbf{k}$$

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$
$$= (19.2\text{ in./s})\mathbf{i} + (60\text{ in./s})\mathbf{j} + (105.6\text{ in./s})\mathbf{k}$$

$$\mathbf{v}_D = (1.600\text{ ft/s})\mathbf{i} + (5.00\text{ ft/s})\mathbf{j} + (8.80\text{ ft/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Acceleration of D.*

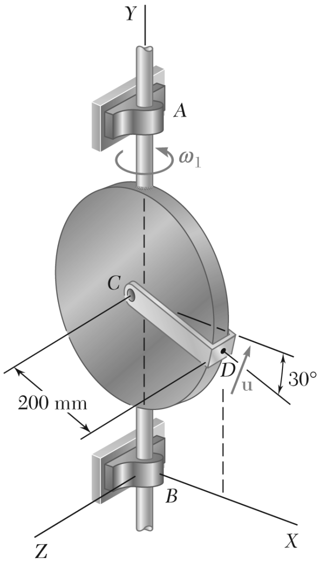
$$\mathbf{a}_{D'} = \boldsymbol{\Omega} \times \mathbf{v}_{D'} = 10\mathbf{i} \times (60\mathbf{j} + 120\mathbf{k}) = -(1200\text{ in./s}^2)\mathbf{j} + (600\text{ in./s}^2)\mathbf{k}$$
$$\mathbf{a}_{D/F} = 0$$

$$2\boldsymbol{\Omega} \times \mathbf{v}_{D/F} = (2)(10\mathbf{i}) \times (19.2\mathbf{i} - 14.4\mathbf{k}) = (288\text{ in./s}^2)\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{D/F} = -(912\text{ in./s}^2)\mathbf{j} + (600\text{ in./s}^2)\mathbf{k}$$

$$\mathbf{a}_D = -(76.0\text{ ft/s}^2)\mathbf{j} + (50.0\text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.227



The circular plate shown rotates about its vertical diameter at the constant rate $\omega_1 = 10$ rad/s. Knowing that in the position shown the disk lies in the *XY* plane and Point *D* of strap *CD* moves upward at a constant relative speed $u = 1.5$ m/s, determine (*a*) the velocity of *D*, (*b*) the acceleration of *D*.

SOLUTION

Geometry.

$$\mathbf{r}_{D/C} = (0.2 \text{ m})(\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$$
$$= (0.1\sqrt{3} \text{ m})\mathbf{i} - (0.1 \text{ m})\mathbf{j}$$

Let frame *Cxyz*, which at the instant shown coincides with *CXYZ*, rotate with angular velocity

$$\boldsymbol{\Omega} = \omega_1\mathbf{j} = (10 \text{ rad/s})\mathbf{j}.$$

Motion of coinciding Point D' in the frame.

$$\mathbf{v}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_{D/C}$$
$$= 10\mathbf{j} \times (0.1\sqrt{3}\mathbf{i} + 0.1\mathbf{j})$$
$$= -(\sqrt{3} \text{ m/s})\mathbf{k}$$

$$\mathbf{a}_{D'} = -\Omega^2(r\cos 30°)\mathbf{i}$$
$$= -10^2(0.1\sqrt{3})\mathbf{i}$$
$$= -(10\sqrt{3} \text{ m/s}^2)\mathbf{i}$$

Motion of Point D relative to the frame. $u = 1.5$ m/s

$$\mathbf{v}_{D/F} = u(\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$
$$= (0.75 \text{ m/s})\mathbf{i} + (0.75\sqrt{3} \text{ m/s})\mathbf{j}$$

$$\mathbf{a}_{D/F} = \frac{u^2}{\rho} \cdot (-\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$$
$$= \frac{1.5^2}{0.2}(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$$
$$= -(5.625\sqrt{3} \text{ m/s}^2)\mathbf{i} + (5.625 \text{ m/s}^2)\mathbf{j}$$

(*a*) *Velocity of Point D.*

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$$\mathbf{v}_D = (0.75\text{ m/s})\mathbf{i} + (0.75\sqrt{3}\text{ m/s})\mathbf{j} - (\sqrt{3}\text{ m/s})\mathbf{k}$$

$$\mathbf{v}_D = (0.750\text{ m/s})\mathbf{i} + (1.299\text{ m/s})\mathbf{j} - (1.732\text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration. $2\mathbf{\Omega} \times \mathbf{v}_{D/F}$

$$2\mathbf{\Omega} \times \mathbf{v}_{D/F} = (2)(10\mathbf{j}) \times (0.75\mathbf{i} + 0.75\sqrt{3}\mathbf{j})$$

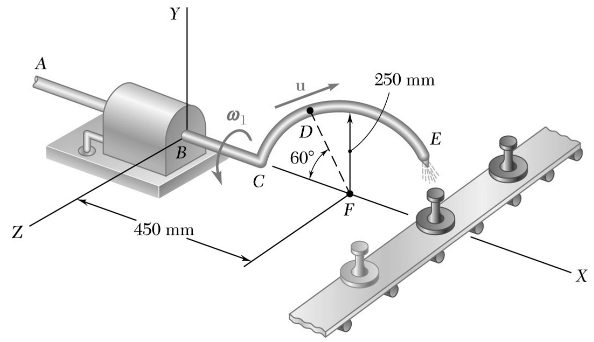
$$= -(15\text{ m/s}^2)\mathbf{k}$$

(*b*) *Acceleration of Point D.*

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\mathbf{\Omega} \times \mathbf{v}_{D/F}$$

$$\mathbf{a}_D = -(15.625\sqrt{3}\text{ m/s}^2)\mathbf{i} + (5.625\text{ m/s}^2)\mathbf{j} - (15\text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_D = (27.1\text{ m/s}^2)\mathbf{i} + (5.63\text{ m/s}^2)\mathbf{j} - (15.00\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.228

Manufactured items are spray-painted as they pass through the automated work station shown. Knowing that the bent pipe *ACE* rotates at the constant rate $\omega_1 = 0.4$ rad/s and that at Point *D* the paint moves through the pipe at a constant relative speed $u = 150$ mm/s, determine, for the position shown, (*a*) the velocity of the paint at *D*, (*b*) the acceleration of the paint at *D*.

SOLUTION

Use a frame of reference *CE* rotating about the *x*-axis with angular velocity

$$\mathbf{\Omega} = \boldsymbol{\omega}_1 = (0.4 \text{ rad/s})\mathbf{i}$$

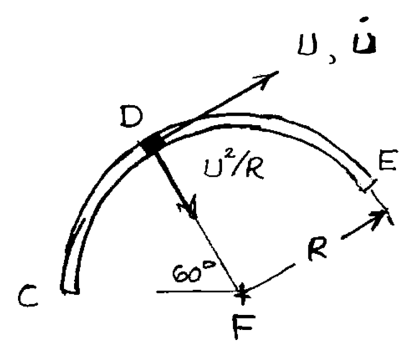
Geometry:

$$\mathbf{r}_{D/F} = (250 \text{ mm})(-\cos 60°\mathbf{i} + \sin 60°\mathbf{j})$$
$$= -(125 \text{ mm})\mathbf{i} + (216.51 \text{ mm})\mathbf{j}$$

Motion of Point *D*′ fixed in the frame *CE* but coinciding with Point *D* at the instant considered.

$$\mathbf{v}_{D'} = \Omega \times \mathbf{r}_{D/F} = (0.4\mathbf{i}) \times (-125\mathbf{i} + 216.51\mathbf{j}) = (86.603 \text{ mm/s})\mathbf{k}$$
$$\mathbf{a}_{D'} = \dot{\Omega}\mathbf{i} \times \mathbf{r}_{D/F} + \mathbf{\Omega} \times \mathbf{v}_{D'}$$
$$= 0 + (0.4\mathbf{i}) \times (86.603\mathbf{k}) = -(34.641 \text{ mm/s}^2)\mathbf{j}$$

Motion of *D* relative to the frame *CE*.



$$\mathbf{v}_{D/CE} = u(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = (150 \text{ mm/s})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$$
$$= (129.90 \text{ mm/s})\mathbf{i} + (75 \text{ mm/s})\mathbf{j}$$
$$\mathbf{a}_{D/CE} = \dot{u}(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) + \frac{u^2}{R}(\sin 30°\mathbf{i} - \cos 30°\mathbf{j})$$
$$= 0 + \frac{(150)^2}{250}(\sin 30°\mathbf{i} - \cos 30°\mathbf{j})$$
$$= (45 \text{ mm/s}^2)\mathbf{i} - (77.94 \text{ mm/s}^2)\mathbf{j}$$

(*a*) *Velocity of D.* $\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/CE}$

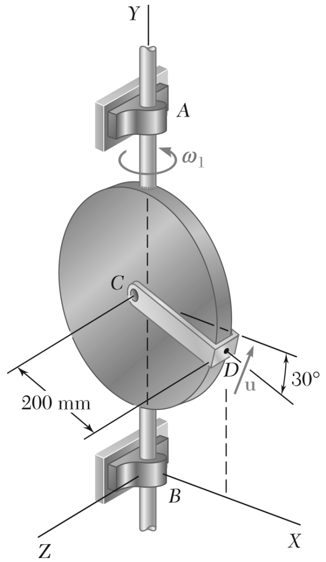
$$\mathbf{v}_D = (129.9 \text{ mm/s})\mathbf{i} + (75.0 \text{ mm/s})\mathbf{j} + (86.6 \text{ mm/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration $2\mathbf{\Omega} \times \mathbf{v}_{D/CE}$

$$2\mathbf{\Omega} \times \mathbf{v}_{D/CE} = (2)(0.4\mathbf{i}) \times (129.90\mathbf{i} + 75\mathbf{j}) = (60 \text{ mm/s}^2)\mathbf{k}$$

(*b*) *Acceleration of D.* $\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/CE} + 2\mathbf{\Omega} \times \mathbf{v}_{D/CE}$

$$\mathbf{a}_D = (45.0 \text{ mm/s}^2)\mathbf{i} - (112.6 \text{ mm/s}^2)\mathbf{j} + (60.0 \text{ mm/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.229

Solve Problem 15.227, assuming that at the instant shown the angular velocity $\boldsymbol{\omega}_1$ of the plate is 10 rad/s and is decreasing at the rate of 25 rad/s^2, while the relative speed u of Point D of strap CD is 1.5 m/s and is decreasing at the rate of 3 m/s^2.

PROBLEM 15.227 The circular plate shown rotates about its vertical diameter at the constant rate $\omega_1 = 10$ rad/s. Knowing that in the position shown the disk lies in the XY plane and Point D of strap CD moves upward at a constant relative speed $u = 1.5$ m/s, determine (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

Geometry.

$$\mathbf{r}_{D/C} = (0.2\text{ m})(\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$$
$$= (0.1\sqrt{3}\text{ m})\mathbf{i} - (0.1\text{ m})\mathbf{j}$$

Let frame $Cxyz$, which at the instant shown coincides with $CXYZ$, rotate with angular velocity and angular acceleration

$$\boldsymbol{\Omega} = \omega_1\mathbf{j} = (10\text{ rad/s})\mathbf{j}.$$

$$\dot{\boldsymbol{\Omega}} = -(25\text{ rad/s}^2)\mathbf{j}.$$

Motion of coinciding Point D' in the frame

$$\mathbf{v}_{D'} = \boldsymbol{\Omega}\times\mathbf{r}_{D/C}$$
$$= 10\mathbf{j}\times(0.1\sqrt{3}\mathbf{i} + 0.1\mathbf{j}) = -(\sqrt{3}\text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{D'} = -\Omega^2(r\cos 30°)\mathbf{i} + \dot{\boldsymbol{\Omega}}\times\mathbf{r}_{D/C}$$
$$= -10^2(0.1\sqrt{3})\mathbf{i} - 25\mathbf{j}\times(0.1\sqrt{3}\mathbf{i} + 0.1\mathbf{j})$$
$$= -(10\sqrt{3}\text{ m/s}^2)\mathbf{i} + (2.5\sqrt{3}\text{ m/s}^2)\mathbf{k}$$

Motion of Point D relative to the frame. $\quad u = 1.5$ m/s $\quad \dot{u} = -3$ m/s^2

$$\mathbf{v}_{D/F} = u(\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$
$$= (0.75\text{ m/s})\mathbf{i} + (0.75\sqrt{3}\text{ m/s})\mathbf{j}$$

PROBLEM 15.229 (Continued)

$$\mathbf{a}_{D/F} = \frac{u^2}{\rho}\cdot(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) + \dot{u}(\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$

$$= \frac{1.5^2}{0.2}(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) - 3(\sin 30°\mathbf{i} + \cos 30°\mathbf{j})$$

$$= -(5.625\sqrt{3}\text{ m/s}^2)\mathbf{i} + (5.625\text{ m/s}^2)\mathbf{j} - (1.5\text{ m/s}^2)\mathbf{i} - (1.5\sqrt{3}\text{ m/s}^2)\mathbf{j}$$

(*a*) *Velocity of Point D.* $\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$

$$\mathbf{v}_D = (0.75\text{ m/s})\mathbf{i} + (0.75\sqrt{3}\text{ m/s})\mathbf{j} - (\sqrt{3}\text{ m/s})\mathbf{k}$$

$$\mathbf{v}_D = (0.750\text{ m/s})\mathbf{i} + (1.299\text{ m/s})\mathbf{j} - (1.732\text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration. $2\mathbf{\Omega}\times\mathbf{v}_{D/F}$

$$2\mathbf{\Omega}\times\mathbf{v}_{D/F} = (2)(10\mathbf{j})\times(0.75\mathbf{i} + 0.75\sqrt{3}\mathbf{j})$$

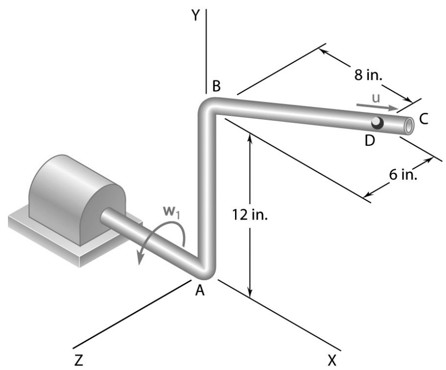
$$= -(15\text{ m/s}^2)\mathbf{k}$$

(*b*) *Acceleration of Point D.* $\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\mathbf{\Omega}\times\mathbf{v}_{D/F}$

$$\mathbf{a}_D = -(10\sqrt{3}\text{ m/s}^2)\mathbf{i} + (2.5\sqrt{3}\text{ m/s}^2)\mathbf{k}$$
$$-(5.625\sqrt{3}\text{ m/s}^2)\mathbf{i} + (5.625\text{ m/s}^2)\mathbf{j}$$
$$-(1.5\text{ m/s}^2)\mathbf{i} - (1.5\sqrt{3}\text{ m/s}^2)\mathbf{j} - (15\text{ m/s}^2)\mathbf{k}$$

$$\mathbf{a}_D = -(28.6\text{ m/s}^2)\mathbf{i} + (3.03\text{ m/s}^2)\mathbf{j} - (10.67\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.230



Solve Problem 15.226, assuming that at the instant shown the angular velocity ω_1 of the pipe is 10 rad/s and is decreasing at the rate of 15 rad/s^2, while the relative speed u of the ball bearing is 2 ft/s and is increasing at the rate of 10 ft/s^2.

PROBLEM 15.226 The bent pipe shown rotates at the constant rate $\omega_1 = 10$ rad/s. Knowing that a ball bearing D moves in portion BC of the pipe toward end C at a constant relative speed $u = 2$ ft/s, determine at the instant shown (*a*) the velocity of D, (*b*) the acceleration of D.

SOLUTION

With the origin at Point A,

$$\mathbf{r}_D = (8 \text{ in.})\mathbf{i} + (12 \text{ in.})\mathbf{j} - (6 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{C/B} = (8 \text{ in.})\mathbf{i} - (6 \text{ in.})\mathbf{k},$$

$$l_{BC} = \sqrt{8^2 + 6^2} = 10 \text{ in.}$$

Let the frame $Axyz$ rotate with angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{i} = (10 \text{ rad/s})\mathbf{i}$ and angular acceleration

$\dot{\boldsymbol{\Omega}} = \dot{\omega}_1\mathbf{i} = -(15 \text{ rad/s}^2)\mathbf{i}.$

(*a*) *Velocity of D.*

$$\mathbf{v}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_D$$
$$= 10\mathbf{i} \times (8\mathbf{i} + 12\mathbf{j} - 6\mathbf{k})$$
$$= (60 \text{ in./s})\mathbf{j} + (120 \text{ in./s})\mathbf{k}$$

$$u = 2 \text{ ft/s} = 24 \text{ in./s},$$

$$\mathbf{u} = \frac{24}{10}(8\mathbf{i} - 6\mathbf{k})$$
$$= (19.2 \text{ in./s})\mathbf{i} - (14.4 \text{ in./s})\mathbf{k}$$

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{u}$$
$$= (19.2 \text{ in./s})\mathbf{i} + (60 \text{ in./s})\mathbf{j} + (105.6 \text{ in./s})\mathbf{k}$$

$$\mathbf{v}_D = (1.600 \text{ ft/s})\mathbf{i} + (5.00 \text{ ft/s})\mathbf{j} + (8.80 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

(*b*) *Acceleration of D.*

$$\mathbf{a}_{D'} = \boldsymbol{\Omega} \times \mathbf{r}_D + \boldsymbol{\Omega} \times \mathbf{v}_{D'}$$
$$= -15\mathbf{i} \times (8\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}) + 10\mathbf{i} \times (60\mathbf{j} + 120\mathbf{k})$$
$$= -90\mathbf{j} - 180\mathbf{k} - 1200\mathbf{j} + 600\mathbf{k}$$
$$= -(1290 \text{ in./s}^2)\mathbf{j} + (420 \text{ in./s}^2)\mathbf{k}$$

PROBLEM 15.230 (Continued)

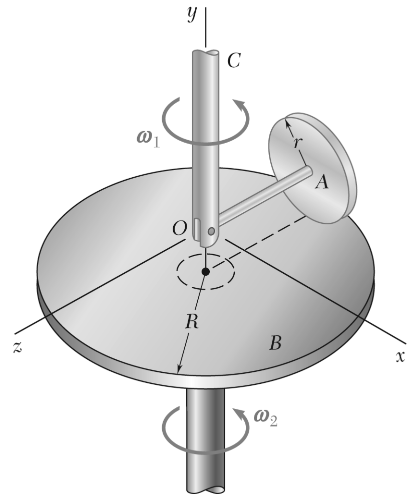
$$a_{\text{rel}} = 10 \text{ ft/s}^2 = 120 \text{ in./s}^2$$

$$\mathbf{a}_{\text{rel}} = \frac{120}{10}(8\mathbf{i} - 6\mathbf{k}) = (96 \text{ in./s}^2)\mathbf{i} - (72 \text{ in./s}^2)\mathbf{k}$$

$$2\mathbf{\Omega} \times \mathbf{u} = (2)(10\mathbf{i}) \times (19.2\mathbf{i} - 14.4\mathbf{k}) = (288 \text{ in./s}^2)\mathbf{j}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{\text{rel}} + 2\mathbf{\Omega} \times \mathbf{u} = (96 \text{ in./s})\mathbf{i} - (1002 \text{ in./s})\mathbf{j} + (348 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_D = (8.00 \text{ ft/s}^2)\mathbf{i} - (83.5 \text{ ft/s}^2)\mathbf{j} + (29.0 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$

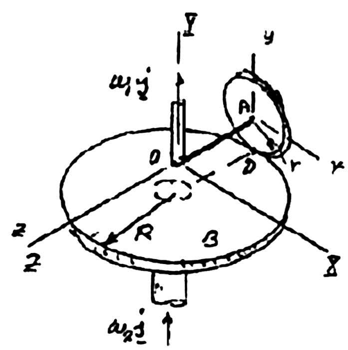


PROBLEM 15.231

Using the method of Section 15.14, solve Problem 15.192.

PROBLEM 15.192 In the system shown, disk A is free to rotate about the horizontal rod OA. Assuming that shaft OC and disk B rotate with constant angular velocities ω_1 and ω_2, respectively, both counterclockwise, determine (a) the angular velocity of A, (b) the angular acceleration of disk A.

SOLUTION



Moving frame $Axyz$ rotates with angular velocity

$$\mathbf{\Omega} = \omega_1 \mathbf{j}$$

$$\boldsymbol{\omega}_{\text{disk}/F} = \omega_x \mathbf{i} + \omega_z \mathbf{k}$$

$$\mathbf{r}_{D/A} = -r\mathbf{j} - R\mathbf{k}$$

(a) Total angular velocity of disk A:

$$\boldsymbol{\omega} = \omega_1 \mathbf{j} + \boldsymbol{\omega}_{\text{disk}/F}$$

$$= \omega_x \mathbf{i} + \omega_1 \mathbf{j} + \omega_z \mathbf{k} \qquad (1)$$

Denote by D point of contact of disks

Consider disk B: $\quad \mathbf{v}_D = \omega_2 \mathbf{j} \times (-R\mathbf{k}) = -R\omega_2 \mathbf{i} \qquad (2)$

Consider system OC, OA and disk A.

$$\mathbf{v}_{D'} = \mathbf{\Omega} \times \mathbf{r}_{B/A}$$

$$= \omega_1 \mathbf{j} \times (-r\mathbf{j} - R\mathbf{k})$$

$$= -R\omega_1 \mathbf{i}$$

$$\mathbf{v}_{B/F} = \omega_{\text{disk}/F} \times \mathbf{r}_{D/A}$$

$$= (\omega_2 \mathbf{i} + \omega_z \mathbf{k}) \times (-r\mathbf{j} - R\mathbf{k})$$

$$= -r\omega_x \mathbf{k} + R\omega_x \mathbf{j} + r\omega_z \mathbf{i}$$

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$$= -R\omega_1 \mathbf{i} - r\omega_x \mathbf{k} + R\omega_x \mathbf{j} + r\omega_2 \mathbf{i} \qquad (3)$$

Equate $\mathbf{v}_D = \mathbf{v}_D$ from Eq. (2) and Eq. (3).

$$-R\omega_2 \mathbf{i} = -R\omega_1 + r\omega_2 + R\omega_x \mathbf{j} - r\omega_x \mathbf{k}$$

PROBLEM 15.231 (Continued)

Coefficient of **j**: $0 = R\omega_x \rightarrow \omega_x = 0$

Coefficient of **i**: $-R\omega_2 = -R\omega_1 + r\omega_z;$

$$\omega_z = \frac{R}{r}(\omega_1 - \omega_2)$$

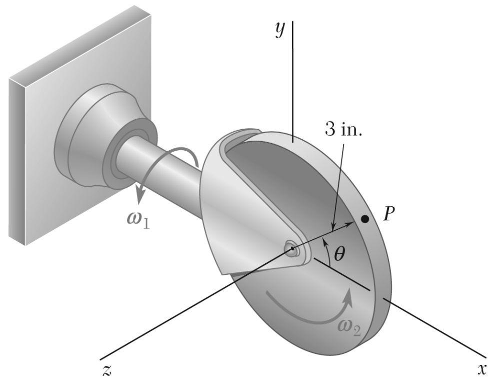
Eq. (3): $\boldsymbol{\omega} = \omega_1\mathbf{j} + \frac{R}{r}(\omega_1 - \omega_2)\mathbf{k}$ ◀

(*b*) Disk *A* rotates about *y* axis at rate $\boldsymbol{\omega}_1$.

$$\boldsymbol{\alpha} = \boldsymbol{\omega}_1 \times \boldsymbol{\omega}$$

$$= \omega_1\mathbf{j} \times \left[\omega\mathbf{j} + \frac{R}{r}(\omega_1 - \omega_2)\right]\mathbf{k}$$

$\boldsymbol{\alpha} = \omega_1(\omega_1 - \omega_2)\frac{R}{r}\mathbf{j}$ ◀



PROBLEM 15.232

Using the method of Section 15.14, solve Problem 15.196.

PROBLEM 15.196 A 3-in.-radius disk spins at the constant rate $\omega_2 = 4$ rad/s about an axis held by a housing attached to a horizontal rod that rotates at the constant rate $\omega_1 = 5$ rad/s. Knowing that $\theta = 30°$, determine the acceleration of Point P on the rim of the disk.

SOLUTION

Let frame $Oxyz$ rotate with angular velocity

$$\mathbf{\Omega} = \omega_1\mathbf{i} = (5 \text{ rad/s})\mathbf{i}$$

The motion relative to the frame is the spin

$$\omega_2\mathbf{k} = (4 \text{ rad/s})\mathbf{k}$$

$$\theta = 30°$$

$$\mathbf{r}_P = (3 \text{ in.})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$$

$$\mathbf{v}_{P'} = \omega_1\mathbf{i} \times \mathbf{r}_P$$
$$= 5\mathbf{i} \times (3\cos 30°\mathbf{i} + 3\sin 30°\mathbf{j})$$
$$= (7.5 \text{ in./s})\mathbf{k}$$

$$\mathbf{v}_{P/F} = \omega_2\mathbf{k} \times \mathbf{r}_P$$
$$= 4\mathbf{k} \times (3\cos 30°\mathbf{i} + 3\sin 30°\mathbf{j})$$
$$= -(6 \text{ in./s})\mathbf{i} + (10.392 \text{ in./s})\mathbf{j}$$

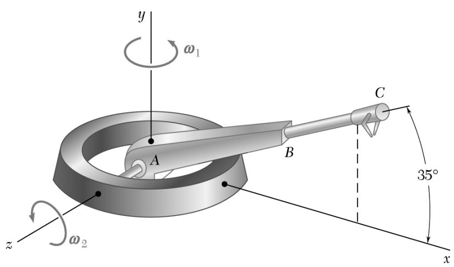
$$\mathbf{a}_{P'} = \dot{\omega}_1\mathbf{i} \times \mathbf{r}_P + \omega_1\mathbf{i} \times \mathbf{v}_{P'}$$
$$= 0 + 5\mathbf{i} \times 7.5\mathbf{k}$$
$$= -(37.5 \text{ in./s}^2)\mathbf{j}$$

$$\mathbf{a}_{P/F} = \dot{\omega}_2\mathbf{k} \times \mathbf{r}_P$$
$$= \omega_2\mathbf{k} \times \mathbf{v}_{P/F}$$
$$= 0 + 4\mathbf{k} \times (-6\mathbf{i} + 10.392\mathbf{j})$$
$$= -(41.569 \text{ in./s}^2)\mathbf{i} - (24 \text{ in./s}^2)\mathbf{j}$$

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{P/F}$$
$$= (2)(5\mathbf{i}) \times (-6\mathbf{i} + 10.392\mathbf{j})$$
$$= (103.92 \text{ in./s}^2)\mathbf{k}$$

Acceleration at Point P. $\quad \mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_c$

$$\mathbf{a}_P = -(41.6 \text{ in./s}^2)\mathbf{i} - (61.5 \text{ in./s}^2)\mathbf{j} + (103.9 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.233

Using the method of Section 15.14, solve Problem 15.198.

PROBLEM 15.198 At the instant shown, the robotic arm ABC is being rotated simultaneously at the constant rate $\omega_1 = 0.15$ rad/s about the y axis, and at the constant rate $\omega_2 = 0.25$ rad/s about the z axis. Knowing that the length of arm ABC is 1 m, determine (a) the angular acceleration of the arm, (b) the velocity of Point C, (c) the acceleration of Point C.

SOLUTION

Geometry: Dimensions in meters.

$$\mathbf{r}_{C/A} = (1.0\cos 35°)\mathbf{i} + (1.0\sin 35°)\mathbf{j} = 0.81915\mathbf{i} + 0.57358\mathbf{j}$$

Angular velocities:

$$\boldsymbol{\omega}_1 = \omega_1\mathbf{j} = (0.15 \text{ rad/s})\mathbf{j} \qquad (\dot{\omega}_1 = 0)$$

$$\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (0.15 \text{ rad/s})\mathbf{k} \qquad (\dot{\omega}_2 = 0)$$

Use a frame of reference rotating about the y-axis.

Its angular velocity is $\quad \boldsymbol{\Omega} = \omega_1\mathbf{j} = (0.15 \text{ rad/s})\mathbf{j}$

(a) *Angular acceleration*:

$$\boldsymbol{\alpha} = \dot{\omega}_1\mathbf{j} + \dot{\omega}_2\mathbf{k} + \boldsymbol{\Omega}\times\boldsymbol{\omega}_2$$
$$= 0 + 0 + (0.15\mathbf{j})(0.25\mathbf{k})$$

$$\boldsymbol{\alpha} = (0.0375 \text{ rad/s}^2)\mathbf{i} \blacktriangleleft$$

Motion of coinciding Point C.

$$\mathbf{v}_{C'} = \boldsymbol{\Omega}\times\mathbf{r}_{C/A} = 0.15\mathbf{j}\times(0.81915\mathbf{i} + 0.57358\mathbf{j})$$
$$= -(0.12287 \text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{C'} = \boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}_{C/A}) = (0.15\mathbf{j})\times(-0.12287\mathbf{k})$$
$$= -(0.018431 \text{ m/s}^2)\mathbf{i}$$

Motion of C relative to the frame.

$$\mathbf{v}_{C/F} = \omega_2\mathbf{k}\times\mathbf{r}_{C/A} = 0.25\mathbf{k}(0.81915\mathbf{i} + 0.57358\mathbf{j})$$
$$= -(0.14339 \text{ m/s})\mathbf{i} + (0.20479 \text{ m/s})\mathbf{j}$$
$$\mathbf{a}_{C/F} = \omega_1\mathbf{k}\times\mathbf{v}_{C/F} = 0.25\mathbf{k}\times(-0.14339\mathbf{i} + 0.20479\mathbf{j})$$
$$= -(0.051198 \text{ m/s}^2)\mathbf{i} - (0.035848 \text{ m/s}^2)\mathbf{j}$$

(*b*) *Velocity of C.* $\mathbf{v}_C = \mathbf{v}_{C'} + \mathbf{v}_{C/F}$

$$\mathbf{v}_C - (0.143\text{ m/s})\mathbf{i} + (0.205\text{ m/s})\mathbf{j} - (0.123\text{ m/s})\mathbf{k} \blacktriangleleft$$

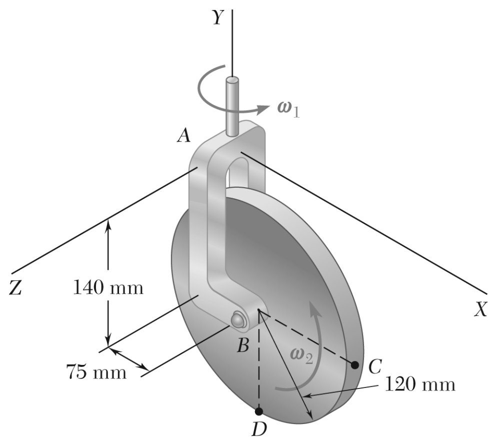
Coriolis acceleration. $2\boldsymbol{\Omega}\times\mathbf{v}_{C/F}$

$$2\boldsymbol{\Omega}\times\mathbf{v}_{C/F} = (2)(0.15\mathbf{j})\times(-0.14339\mathbf{i} + 0.20479\mathbf{j})$$
$$= (0.043017\text{ m/s}^2)\mathbf{k}$$

(*c*) *Acceleration of C.* $\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\boldsymbol{\Omega}\times\mathbf{v}_{C/F}$

$$\mathbf{a}_C = -0.01843\mathbf{i} - 0.051198\mathbf{i} - 0.035848\mathbf{j} + 0.043017\mathbf{k}$$

$$\mathbf{a}_C = -(0.0696\text{ m/s}^2)\mathbf{i} - (0.0358\text{ m/s}^2)\mathbf{j} + (0.0430\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.234

A disk of radius 120 mm rotates at the constant rate $\omega_2 = 5$ rad/s with respect to the arm AB, which itself rotates at the constant rate $\omega_1 = 3$ rad/s. For the position shown, determine the velocity and acceleration of Point C.

SOLUTION

Geometry.

$$\mathbf{r}_{C/A} = (0.195\text{ m})\mathbf{i} - (0.1)\mathbf{j}$$

$$\mathbf{r}_{C/B} = (0.12\text{ m})\mathbf{i}$$

Let frame $Axyz$, which coincides with the fixed frame $AXYZ$ at the instant shown, be rotating about the y axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j} = (3\text{ rad/s})\mathbf{j}$. Then the motion relative to the frame consists of rotation about the axle B with a constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (5\text{ rad/s})\mathbf{k}$.

Motion of the coinciding Point C' in the frame.

$$\begin{aligned}\mathbf{v}_{C'} &= \mathbf{\Omega}\times\mathbf{r}_{C/A}\\ &= 3\mathbf{j}\times(0.195\mathbf{i} - 0.14\mathbf{j})\\ &= -(0.585\text{ m/s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{C'} &= \mathbf{\Omega}\times\mathbf{v}_{C'}\\ &= 3\mathbf{j}\times(-0.585\mathbf{k})\\ &= -(1.755\text{ m/s}^2)\mathbf{i}\end{aligned}$$

Motion relative to the frame.

$$\begin{aligned}\mathbf{v}_{C/F} &= \boldsymbol{\omega}_2\times\mathbf{r}_{C/B}\\ &= 5\mathbf{k}\times 0.12\mathbf{i}\\ &= (0.6\text{ m/s})\mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{C/F} &= \boldsymbol{\omega}_2\times\mathbf{v}_{C/F}\\ &= 5\mathbf{k}\times(-0.6\mathbf{j})\\ &= -(3\text{ m/s}^2)\mathbf{i}\end{aligned}$$

Velocity of Point C. $\quad \mathbf{v}_C = \mathbf{v}_{C'} + \mathbf{v}_{C/F}$

$$\mathbf{v}_C = (0.600\text{ m/s})\mathbf{j} - (0.585\text{ m/s})\mathbf{k} \blacktriangleleft$$

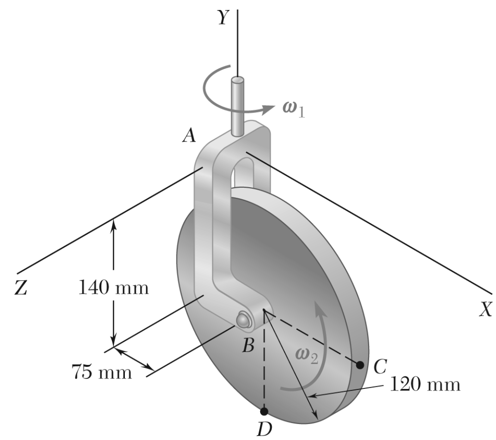
Coriolis acceleration. $\quad 2\mathbf{\Omega}\times\mathbf{v}_{C/F} = (2)(3\mathbf{j})\times(0.6\mathbf{j}) = 0$

Acceleration of Point C.

$$\mathbf{a}_C = \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\mathbf{\Omega}\times\mathbf{v}_{C/F}$$

$$\mathbf{a}_C = -1.7551\mathbf{i} - 3\mathbf{i} + 0$$

$$\mathbf{a}_C = -(4.76\text{ m/s}^2)\mathbf{i} \blacktriangleleft$$



PROBLEM 15.235

A disk of radius 120 mm rotates at the constant rate $\omega_2 = 5$ rad/s with respect to the arm AB, which itself rotates at the constant rate $\omega_1 = 3$ rad/s. For the position shown, determine the velocity and acceleration of Point D.

SOLUTION

Geometry.

$$\mathbf{r}_{D/A} = (0.075 \text{ m})\mathbf{i} - (0.26 \text{ m})\mathbf{j}$$

$$\mathbf{r}_{D/B} = -(0.12 \text{ m})\mathbf{j}$$

Let frame $AXYZ$, which coincides with the fixed frame $AXYZ$ at the instant shown, be rotating about the y axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j} = (3 \text{ rad/s})\mathbf{j}$. Then the motion relative to the frame consists of rotation about the axle B with a constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (5 \text{ rad/s})\mathbf{k}$.

Motion of the coinciding Point D' in the frame.

$$\mathbf{v}_{D'} = \mathbf{\Omega} \times \mathbf{r}_{D/A}$$

$$= 3\mathbf{j} \times (0.075\mathbf{i} - 0.26\mathbf{j})$$

$$= -(0.225 \text{ m/s})\mathbf{k}$$

$$\mathbf{a}_{D'} = \mathbf{\Omega} \times \mathbf{v}_{D'}$$

$$= 3\mathbf{j} \times (-0.225\mathbf{k})$$

$$= -(0.675 \text{ m/s}^2)\mathbf{i}$$

Motion relative to the frame.

$$\mathbf{v}_{D/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{D/B}$$

$$= 5\mathbf{k} \times (-0.12\mathbf{j})$$

$$= (0.6 \text{ m/s})\mathbf{i}$$

$$\mathbf{a}_{D/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{C/F}$$

$$= 5\mathbf{k} \times (0.6\mathbf{i})$$

$$= (3 \text{ m/s}^2)\mathbf{j}$$

Velocity of Point D.

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F} \qquad \mathbf{v}_D = (0.600 \text{ m/s})\mathbf{i} - (0.225 \text{ m/s})\mathbf{k} \blacktriangleleft$$

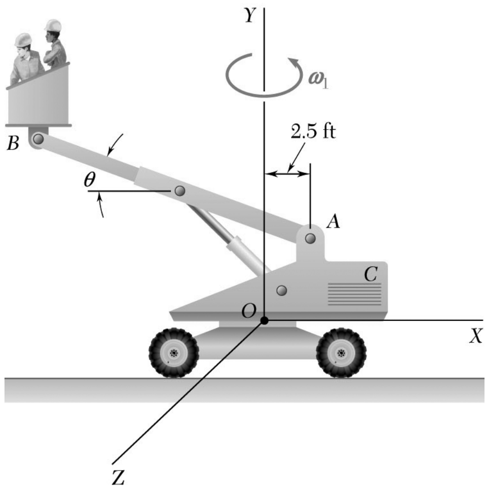
Coriolis acceleration.

$$2\mathbf{\Omega} \times \mathbf{v}_{D/F} = (2)(3\mathbf{j}) \times (0.6\mathbf{i}) = -(3.6 \text{ m/s}^2)\mathbf{k}$$

Acceleration of Point D.

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\mathbf{\Omega} \times \mathbf{v}_{D/F}$$

$$\mathbf{a}_D = -(0.675 \text{ m/s}^2)\mathbf{i} + (3.00 \text{ m/s}^2)\mathbf{j} - (3.60 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.236

The arm AB of length 16 ft is used to provide an elevated platform for construction workers. In the position shown, arm AB is being raised at the constant rate $d\theta/dt = 0.25$ rad/s; simultaneously, the unit is being rotated about the Y axis at the constant rate $\omega_1 = 0.15$ rad/s. Knowing that $\theta = 20°$, determine the velocity and acceleration of Point B.

SOLUTION

Frame of reference. Let moving frame $Axyz$ rotate about the Y axis with angular velocity

$$\begin{aligned}\mathbf{\Omega} &= \omega_1\mathbf{j}\\ &= (0.15\text{ rad/s})\mathbf{j}.\end{aligned}$$

Geometry.

$$\begin{aligned}\mathbf{r}_{B/A} &= -16\cos 20°\mathbf{i} + 16\sin 20°\mathbf{j}\\ &= -(15.035\text{ ft})\mathbf{i} + (5.4723\text{ ft})\mathbf{j}\end{aligned}$$

Place Point O on Y axis at same level as Point A.

$$\begin{aligned}\mathbf{r}_{B/O} &= \mathbf{r}_{B/A} + \mathbf{r}_{A/O}\\ &= \mathbf{r}_{B/A} + (2.5\text{ ft})\mathbf{i}\\ &= -(12.535\text{ ft})\mathbf{i} + (5.4723\text{ ft})\mathbf{j}\end{aligned}$$

Motion of corresponding Point B′ in the frame.

$$\begin{aligned}\mathbf{v}_{B'} &= \mathbf{\Omega}\times\mathbf{r}_{B/O}\\ &= (0.15\mathbf{j})\times -(12.535\mathbf{i} + 5.4723\mathbf{j})\\ &= (1.8803\text{ ft/s})\mathbf{k}\\ \mathbf{a}_{B'} &= \mathbf{\Omega}\times\mathbf{v}_{B'}\\ &= (0.15\mathbf{j})\times(1.8803\mathbf{k})\\ &= (0.28204\text{ ft/s}^2)\mathbf{i}\end{aligned}$$

Motion of Point B relative to the frame.

$$\boldsymbol{\omega}_2 = -\frac{d\theta}{dt}\mathbf{k}$$

$$= -(0.25 \text{ rad/s})\mathbf{k}$$

$$\mathbf{v}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{B/A}$$

$$= (-0.25)\mathbf{k} \times (-15.035\mathbf{i} + 5.4723\mathbf{j})$$

$$= (1.36808 \text{ ft/s})\mathbf{i} + (3.7588 \text{ ft/s})\mathbf{j}$$

$$\mathbf{a}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{B/F}$$

$$= (-0.25\mathbf{k}) \times (1.36808\mathbf{i} + 3.7588\mathbf{j})$$

$$= (0.93969 \text{ ft/s}^2)\mathbf{i} - (0.34202 \text{ ft/s}^2)\mathbf{j}$$

Velocity of Point B. $\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$

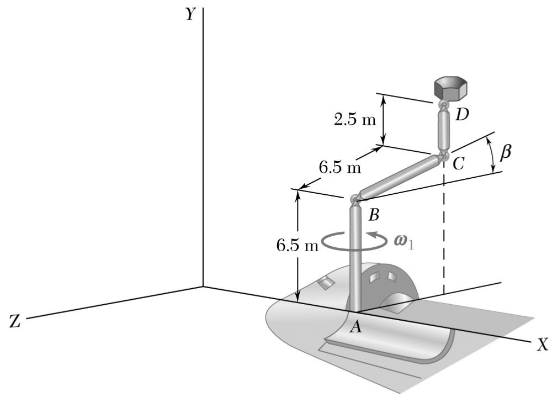
$$\mathbf{v}_B = -(1.37 \text{ ft/s})\mathbf{i} + (3.76 \text{ ft/s})\mathbf{j} + (1.88 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration. $2\boldsymbol{\Omega} \times \mathbf{v}_{B/F} = (2)(0.15\mathbf{j}) \times (1.36808\mathbf{i} + 3.7588\mathbf{j})$

$$= -(0.41042 \text{ ft/s}^2)\mathbf{k}$$

Acceleration of Point B. $\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{B/F}$

$$\mathbf{a}_B = (1.22 \text{ ft/s}^2)\mathbf{i} - (0.342 \text{ ft/s}^2)\mathbf{j} - (0.410 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.237

The remote manipulator system (RMS) shown is used to deploy payloads from the cargo bay of space shuttles. At the instant shown, the whole RMS is rotating at the constant rate $\omega_1 = 0.03$ rad/s about the axis AB. At the same time, portion BCD rotates as a rigid body at the constant rate $\omega_2 = d\beta/dt = 0.04$ rad/s about an axis through B parallel to the X axis. Knowing that $\beta = 30°$, determine (a) the angular acceleration of BCD, (b) the velocity of D, (c) the acceleration of D.

SOLUTION

At the instant given, Points A, B, C, and D lie in a plane which is parallel to the YZ plane. The plane $ABCD$ is rotating with angular velocity.

$$\mathbf{\Omega} = \omega_1\mathbf{j}$$
$$= (0.03 \text{ rad/s})\mathbf{j} \qquad (\dot{\omega}_1 = 0)$$

Body BCD is rotating about an axis through B parallel to the x-axis at angular velocity.

$$\boldsymbol{\omega}_2 = \frac{d\beta}{dt}\mathbf{i} = (0.04 \text{ rad/s})\mathbf{i} \qquad (\dot{\omega}_2 = 0)$$

(a) *Angular acceleration of BCD.*

$$\boldsymbol{\alpha}_{BCD} = \dot{\omega}_1\mathbf{j} + \dot{\omega}_2\mathbf{i} + \mathbf{\Omega}\times\boldsymbol{\omega}_2$$
$$= 0 + 0 + (0.03\mathbf{j})\times(0.04\mathbf{i})$$

$$\boldsymbol{\alpha}_{BCD} = -(0.0012 \text{ rad/s}^2)\mathbf{k} \blacktriangleleft$$

Let the plane of BCD be a rotating frame of reference rotating about AB with angular velocity $\mathbf{\Omega}$.

Geometry: $\beta = 30°$

$$\mathbf{r}_{D/B} = (6.5 \text{ m})(\sin\beta\mathbf{j} - \cos\beta\mathbf{k}) + (2.5 \text{ m})\mathbf{j} = (5.75 \text{ m})\mathbf{j} - (5.6292 \text{ m})\mathbf{k}$$

Motion of Point D' in the frame.

$$\mathbf{v}_{D'} = \mathbf{\Omega}\times\mathbf{r}_{D/B} = 0.03\mathbf{j}\times(3.25\mathbf{j} - 5.6292\mathbf{k})$$
$$= -(0.168875 \text{ m/s})\mathbf{i}$$
$$\mathbf{a}_{D'} = \mathbf{\Omega}\times\mathbf{v}_{D'} = (0.03\mathbf{j})\times(-0.168875\mathbf{i})$$
$$= (0.0050662 \text{ m/s}^2)\mathbf{k}$$

PROBLEM 15.237 (Continued)

Motion of D relative to the frame: This motion is a rotation about B with angular velocity.

$$\boldsymbol{\omega}_2 = (0.04 \text{ rad/s})\mathbf{i}$$

$$\begin{aligned}\mathbf{v}_{D/\text{frame}} &= \boldsymbol{\omega}_2 \times \mathbf{r}_{D/B}\\ &= (0.04\mathbf{i}) \times (5.75\mathbf{j} - 5.6292\mathbf{k})\\ &= (0.22517 \text{ m/s})\mathbf{j} + (0.23 \text{ m/s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{D/\text{frame}} &= \boldsymbol{\omega}_2 \times r_{D/\text{frame}}\\ &= (0.04\mathbf{i}) \times (0.22517\mathbf{j} + 0.23\mathbf{k})\\ &= -(0.0092 \text{ m/s}^2)\mathbf{j} + (0.009007 \text{ m/s}^2)\mathbf{k}\end{aligned}$$

(*b*) *Velocity of D.* $\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/\text{frame}}$

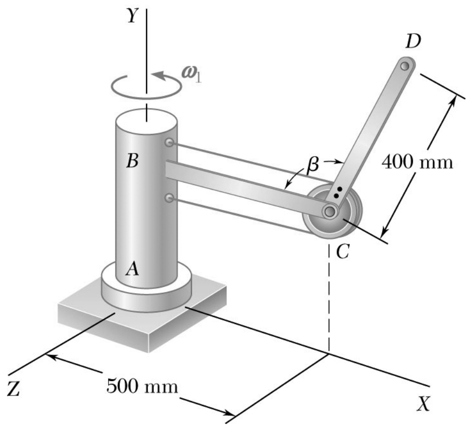
$$\mathbf{v}_D = -(0.169 \text{ m/s})\mathbf{i} + (0.225 \text{ m/s})\mathbf{j} + (0.230 \text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration: $2\boldsymbol{\Omega} \times \mathbf{v}_{D/\text{frame}}$

$$\begin{aligned}2\boldsymbol{\Omega} \times \mathbf{v}_{D/\text{frame}} &= (2)(0.03\mathbf{j}) \times (0.22517\mathbf{j} + 0.23\mathbf{k})\\ &= (0.0138 \text{ m/s}^2)\mathbf{i}\end{aligned}$$

(*c*) *Acceleration of D.* $\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/\text{frame}} + 2\boldsymbol{\Omega} \times \mathbf{v}_{D/\text{frame}}$

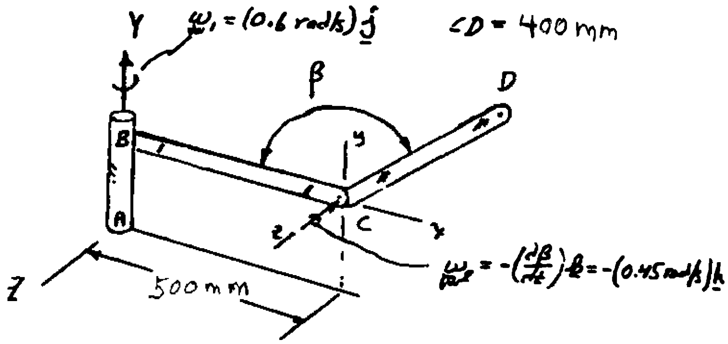
$$\mathbf{a}_D = (0.0138 \text{ m/s}^2)\mathbf{i} - (0.0092 \text{ m/s}^2)\mathbf{j} + (0.0141 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.238

The body AB and rod BC of the robotic component shown rotate at the constant rate $\omega_1 = 0.60$ rad/s about the Y axis. Simultaneously a wire-and-pulley control causes arm CD to rotate about C at the constant rate $\omega_2 = d\beta/dt = 0.45$ rad/s. Knowing that $\beta = 120°$, determine (a) the angular acceleration of arm CD, (b) the velocity of D, (c) the acceleration of D.

SOLUTION



$$\boldsymbol{\Omega} = \boldsymbol{\omega}_1$$
$$= (0.6 \text{ rad/s})\mathbf{j}$$
$$\boldsymbol{\omega}_{D/F} = \boldsymbol{\omega}_2$$
$$= -(0.45 \text{ rad/s})\mathbf{k}$$
$$\boldsymbol{\omega} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2$$
$$= (0.6 \text{ rad/s})\mathbf{j} - (0.45 \text{ rad/s})\mathbf{k}$$

(a) Angular acceleration of CD.

$$\boldsymbol{\alpha} = \boldsymbol{\Omega} \times \boldsymbol{\omega}$$
$$= (0.6 \text{ rad/s})\mathbf{j} \times [(0.6 \text{ rad/s})\mathbf{j} - (0.45 \text{ rad/s})\mathbf{k}]$$

$$\boldsymbol{\alpha} = -(0.27 \text{ rad/s}^2)\mathbf{i} \blacktriangleleft$$

For $\beta = 120°$:

$$\mathbf{r}_{D/C} = (400 \text{ mm})\sin 30°\mathbf{i} + (400 \text{ mm})\cos 30°\mathbf{j}$$
$$= (200 \text{ mm})\mathbf{i} + (346.41 \text{ mm})\mathbf{j}$$
$$\mathbf{r}_{D/B} = (500 \text{ mm})\mathbf{i} + \mathbf{r}_{D/C}$$
$$= (700 \text{ mm})\mathbf{i} + (346.41 \text{ mm})\mathbf{j}$$

PROBLEM 15.238 (Continued)

(*b*) Velocity of *D*.

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$$\mathbf{v}_D = \mathbf{\Omega} \times \mathbf{r}_{D/B}$$
$$= (0.6 \text{ rad/s})\mathbf{j} \times [(700 \text{ mm})\mathbf{i} + (346.41 \text{ mm})\mathbf{j}]$$
$$= -(420 \text{ mm/s})\mathbf{k}$$

$$\mathbf{v}_{D/F} = \boldsymbol{\omega}_{D/F} \times \mathbf{r}_{D/C}$$
$$= -(0.45 \text{ rad/s})\mathbf{k} \times [(200 \text{ mm})\mathbf{i} + (346.41 \text{ mm})\mathbf{j}]$$
$$= -(90 \text{ mm/s})\mathbf{j} + (155.88 \text{ mm/s})\mathbf{i}$$

$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}:$ $\quad \mathbf{v}_D = (156 \text{ mm/s})\mathbf{i} - (90 \text{ mm/s})\mathbf{j} - (420 \text{ mm/s})\mathbf{k}$ ◀

(*c*) Acceleration of *D*.

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$

$$\mathbf{a}_{D'} = \mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{r}_{D/B})$$
$$= \mathbf{\Omega} \times \mathbf{v}_{D'}$$
$$= (0.6 \text{ rad/s})\mathbf{j} \times (-420 \text{ mm/s})\mathbf{k}$$
$$= -(252 \text{ mm/s}^2)\mathbf{i}$$

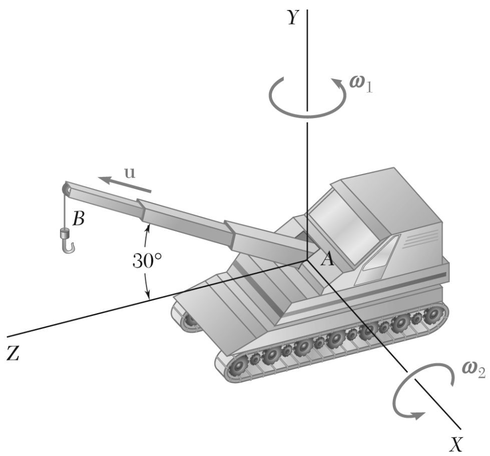
$$\mathbf{a}_{D/F} = \boldsymbol{\omega}_{D/F} \times (\boldsymbol{\omega}_{D/F} \times \mathbf{r}_{D/C})$$
$$= \boldsymbol{\omega}_{D/F} \times \mathbf{v}_{D/F}$$
$$= -(0.45 \text{ rad/s})\mathbf{k} \times [-(90)\mathbf{j} + (155.88)\mathbf{i}]$$
$$= -(40.5 \text{ mm/s}^2)\mathbf{i} - (70.148 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{D/F}$$
$$= 2(0.6 \text{ rad/s})\mathbf{j} \times [-(90)\mathbf{j} + (155.88)\mathbf{i}]$$
$$= -(187.06 \text{ mm/s}^2)\mathbf{k}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$
$$= -(252 \text{ mm/s}^2)\mathbf{i} - (40.5 \text{ mm/s}^2)\mathbf{i}$$
$$-(70.148 \text{ mm/s}^2)\mathbf{j} - (187.06 \text{ mm/s}^2)\mathbf{k}$$

$\mathbf{a}_D = -(293 \text{ mm/s}^2)\mathbf{i} - (70.1 \text{ mm/s}^2)\mathbf{j} - (187 \text{ mm/s}^2)\mathbf{k}$ ◀

PROBLEM 15.239



The crane shown rotates at the constant rate $\omega_1 = 0.25$ rad/s; simultaneously, the telescoping boom is being lowered at the constant rate $\omega_2 = 0.40$ rad/s. Knowing that at the instant shown the length of the boom is 20 ft and is increasing at the constant rate $u = 1.5$ ft/s, determine the velocity and acceleration of Point B.

SOLUTION

Geometry.

$$\mathbf{r}_{B/A} = \mathbf{r}_B$$
$$= (20\text{ ft})(\sin 30°\mathbf{j} + \cos 30°\mathbf{k})$$
$$= (10\text{ ft})\mathbf{j} + (10\sqrt{3}\text{ ft})\mathbf{k}$$

Method 1

Let the unextending portion of the boom AB be a rotating frame of reference.

Its angular velocity is

$$\boldsymbol{\Omega} = \omega_2\mathbf{i} + \omega_1\mathbf{j}$$
$$= (0.40\text{ rad/s})\mathbf{i} + (0.25\text{ rad/s})\mathbf{j}.$$

Its angular acceleration is

$$\boldsymbol{\alpha} = \omega_1\mathbf{j} \times \omega_2\mathbf{i}$$
$$= -\omega_1\omega_2\mathbf{k}$$
$$= -(0.10\text{ rad/s}^2)\mathbf{k}.$$

Motion of the coinciding Point B' in the frame.

$$\mathbf{v}_{B'} = \boldsymbol{\Omega} \times \mathbf{r}_B$$
$$= (0.40\mathbf{i} + 0.25\mathbf{j}) \times (10\mathbf{j} + 10\sqrt{3}\mathbf{k})$$
$$= (2.5\sqrt{3}\text{ ft/s})\mathbf{i} - (4\sqrt{3}\text{ ft/s})\mathbf{j} + (4\text{ ft/s})\mathbf{k}$$

$$\mathbf{a}_{B'} = \boldsymbol{\alpha} \times \mathbf{r}_B + \boldsymbol{\Omega} \times \mathbf{v}_{B'}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.10 \\ 0 & 10 & 10\sqrt{3} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.40 & 0.25 & 0 \\ 2.5\sqrt{3} & -4\sqrt{3} & 4 \end{vmatrix}$$

$$\mathbf{i} + \mathbf{i} - 1.6\mathbf{j} - 3.8538\mathbf{k} = (2\text{ ft/s}^2)\mathbf{i} - (1.6\text{ ft/s}^2)\mathbf{j} - (3.8538\text{ ft/s}^2)\mathbf{k}$$

Motion relative to the frame.

$$\mathbf{v}_{B/F} = u(\sin 30°\mathbf{j} + \cos 30°\mathbf{k})$$
$$= (1.5\text{ ft/s})\sin 30°\mathbf{j} + (1.5\text{ ft/s})\cos 30°\mathbf{k}$$
$$\mathbf{a}_{B/F} = 0$$

Velocity of Point B.

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$

$$\mathbf{v}_B = 2.5\sqrt{3}\mathbf{i} - 4\sqrt{3}\mathbf{j} + 4\mathbf{k} + 1.5\sin 30°\mathbf{j} + 1.5\cos 30°\mathbf{k}$$

$$\mathbf{v}_B = (4.33 \text{ ft/s})\mathbf{i} - (6.18 \text{ ft/s})\mathbf{j} + (5.30 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration. $2\mathbf{\Omega} \times \mathbf{v}_{B/F}$

$$2\mathbf{\Omega} \times \mathbf{v}_{B/F} = (2)(0.40\mathbf{i} + 0.25\mathbf{j}) \times (1.5\sin 30°\mathbf{j} + 1.5\cos 30°\mathbf{k})$$

$$= (0.64952 \text{ ft/s}^2)\mathbf{i} - (1.03923 \text{ ft/s}^2)\mathbf{j} + (0.6 \text{ ft/s}^2)\mathbf{k}$$

Acceleration of Point B.

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\mathbf{\Omega} \times \mathbf{v}_{B/F}$$

$$\mathbf{a}_B = (2 + 0.64952)\mathbf{i} - (1.6 + 1.03923\mathbf{j}) + (-3.8538 + 0.6)\mathbf{k}$$

$$\mathbf{a}_B = (2.65 \text{ ft/s}^2)\mathbf{i} - (2.64 \text{ ft/s}^2)\mathbf{j} - (3.25 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$

Method 2

Let frame $Axyz$, which at the instant shown coincides with $AXYZ$, rotate with an angular velocity $\mathbf{\Omega}_1 = \omega_1\mathbf{j} = (0.25 \text{ rad/s})\mathbf{j}$. Then the motion relative to this frame consists of turning the boom relative to the cab and extending the boom.

Motion of the coinciding Point B' in the frame.

$$\mathbf{v}_{B'} = \mathbf{\Omega} \times \mathbf{r}_B$$
$$= 0.25\mathbf{j} \times (10\mathbf{j} + 10\sqrt{3}\mathbf{k})$$
$$= (2.5\sqrt{3} \text{ m/s})\mathbf{i}$$
$$\mathbf{a}_{B'} = \mathbf{\Omega} \times \mathbf{v}_{B'}$$
$$= 0.25\mathbf{j} \times (2.5\sqrt{3}\mathbf{i})$$
$$= -(0.625\sqrt{3} \text{ m/s}^2)\mathbf{k}$$

Motion of Point B relative to the frame.

Let the unextending portion of the boom be a rotating frame with constant angular velocity $\mathbf{\Omega}_2 = \omega_2\mathbf{i} = (0.40 \text{ rad/s})\mathbf{i}$. The motion relative to this frame is the extensional motion with speed u.

$$\mathbf{v}_{B''} = \mathbf{\Omega}_2 \times \mathbf{r}_B$$
$$= 0.40\mathbf{i} \times (10\mathbf{j} + 10\sqrt{3}\mathbf{k})$$
$$= -(4\sqrt{3} \text{ ft/s})\mathbf{j} + (4 \text{ ft/s})\mathbf{k}$$
$$\mathbf{a}_{B''} = \mathbf{\Omega}_2 \times \mathbf{v}_{B''}$$
$$= 0.40\mathbf{i} \times (-4\sqrt{3}\mathbf{j} + 4\mathbf{k})$$
$$= -(1.6 \text{ ft/s}^2)\mathbf{j} - (1.6\sqrt{3} \text{ ft/s}^2)\mathbf{k}$$
$$\mathbf{v}_{B/\text{boom}} = u(\sin 30°\mathbf{j} + \cos 30°\mathbf{k})$$
$$= (1.5 \text{ ft/s})\sin 30°\mathbf{j} + (1.5 \text{ ft/s})\cos 30°\mathbf{k}$$
$$\mathbf{a}_{B/\text{boom}} = 0$$

PROBLEM 15.239 (Continued)

$$2\boldsymbol{\Omega}_2 \times \mathbf{v}_{B/\text{boom}} = (2)(0.40\mathbf{i}) \times (1.5\sin 30°\mathbf{j} + 1.5\cos 30°\mathbf{k})$$

$$= -(1.03923 \text{ ft/s}^2)\mathbf{j} + (0.6 \text{ ft/s}^2)\mathbf{k}$$

$$\mathbf{v}_{B/F} = \mathbf{v}_{B''} + \mathbf{v}_{B/\text{boom}}$$

$$= -4\sqrt{3}\mathbf{j} + 4\mathbf{k} + 1.5\sin 30°\mathbf{j} + 1.5\cos 30°\mathbf{k}$$

$$= -(6.1782 \text{ ft/s})\mathbf{j} + (5.299 \text{ ft/s})\mathbf{k}$$

$$\mathbf{a}_{B/F} = \mathbf{a}_{B''} + \mathbf{a}_{B/\text{boom}} + 2\boldsymbol{\Omega}_2 \times \mathbf{v}_{B/\text{boom}}$$

$$= -1.6\mathbf{j} - 1.6\sqrt{3}\mathbf{k} + 0 - 1.03923\mathbf{j} + 0.6\mathbf{k}$$

$$= -(2.6392 \text{ ft/s}^2)\mathbf{j} - (2.1713 \text{ ft/s}^2)\mathbf{k}$$

Velocity of Point B.

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$

$$\mathbf{v}_B = 2.5\sqrt{3}\mathbf{i} - 6.1782\mathbf{j} + 5.299\mathbf{k}$$

$$\mathbf{v}_B = (4.33 \text{ ft/s})\mathbf{i} - (6.18 \text{ ft/s})\mathbf{j} + (5.30 \text{ ft/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration.

$$2\boldsymbol{\Omega}_1 \times \mathbf{v}_{B/F}$$

$$2\boldsymbol{\Omega}_1 \times \mathbf{v}_{B/F} = (2)(0.25\mathbf{j}) \times (-6.1782\mathbf{j} + 5.299\mathbf{k})$$

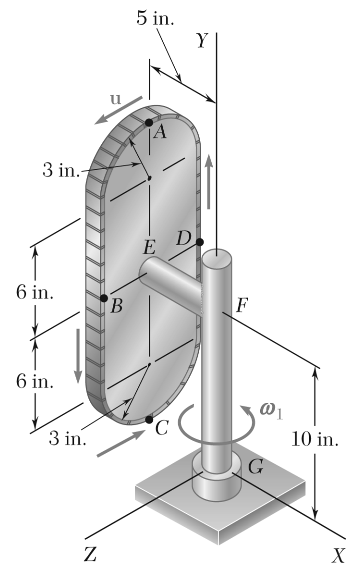
$$= (2.6495 \text{ ft/s}^2)\mathbf{i}$$

Acceleration of Point B.

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\boldsymbol{\Omega}_1 \times \mathbf{v}_{B/F}$$

$$\mathbf{a}_B = -0.625\sqrt{3}\mathbf{k} - 2.6392\mathbf{j} - 2.1713\mathbf{k} + 2.6495\mathbf{i}$$

$$\mathbf{a}_B = (2.65 \text{ ft/s}^2)\mathbf{i} - (2.64 \text{ ft/s}^2)\mathbf{j} - (3.25 \text{ ft/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.240

The vertical plate shown is welded to arm *EFG*, and the entire unit rotates at the constant rate $\omega_1 = 1.6$ rad/s about the *Y* axis. At the same time, a continuous link belt moves around the perimeter of the plate at a constant speed $u = 4.5$ in./s. For the position shown, determine the acceleration of the link of the belt located (*a*) at Point *A*, (*b*) at Point *B*.

SOLUTION

Let the moving frame of reference be the unit, less the pulleys and belt. It rotates about the *Y* axis with constant angular velocity $\mathbf{\Omega} = \omega_1 \mathbf{j} = (1.6 \text{ rad/s})\mathbf{j}.$ The relative motion is that of the pulleys and belt with speed $u = 90$ mm/s.

(*a*) *Acceleration at Point A.*

$$\mathbf{r}_A = -(5 \text{ in.})\mathbf{i} + (19 \text{ in.})\mathbf{j}$$

$$\mathbf{v}_{A'} = \mathbf{\Omega} \times \mathbf{r}_A$$

$$= 1.6\mathbf{j} \times (-5\mathbf{i} + 19\mathbf{j})$$

$$= (8 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{A'} = \mathbf{\Omega} \times \mathbf{v}_{A'}$$

$$= 1.6\mathbf{j} \times 8\mathbf{k}$$

$$= (12.8 \text{ in./s}^2)\mathbf{i}$$

$$\mathbf{v}_{A/F} = u\mathbf{k} = (4.5 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{A/F} = -\left(\frac{u^2}{\rho}\right)\mathbf{j}$$

$$= -\left(\frac{4.5^2}{3}\right)\mathbf{j}$$

$$= -\left(6.75 \text{ in./s}^2\right)\mathbf{j}$$

$$2\mathbf{\Omega} \times \mathbf{v}_{A/F} = (2)(1.6\mathbf{j}) \times (4.5\mathbf{k})$$

$$= (14.4 \text{ in./s}^2)\mathbf{i}$$

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + 2\mathbf{\Omega} \times \mathbf{v}_{A/F}$$

$$= 12.8\mathbf{i} - 6.75\mathbf{j} + 14.4\mathbf{i} \qquad \mathbf{a}_A = (27.2 \text{ in./s}^2)\mathbf{i} - (6.75 \text{ in./s}^2)\mathbf{j} \blacktriangleleft$$

PROBLEM 15.240 (Continued)

(*b*) *Acceleration of Point B.*

$$\mathbf{r}_B = -(5\text{ in.})\mathbf{i} + (10\text{ in.})\mathbf{j} + (3\text{ in.})\mathbf{k}$$

$$\mathbf{v}_{B'} = \boldsymbol{\Omega} \times \mathbf{r}_B$$

$$= 1.6\mathbf{j} \times (-5\mathbf{i} + 10\mathbf{j} + 3\mathbf{k})$$

$$= (4.8\text{ in./s})\mathbf{i} + (8\text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{B'} = \boldsymbol{\Omega} \times \mathbf{v}_{B'}$$

$$= 1.6\mathbf{j} \times (4.8\mathbf{i} + 8\mathbf{k})$$

$$= (12.8\text{ in./s}^2)\mathbf{i} - (7.68\text{ in./s}^2)\mathbf{k}$$

$$\mathbf{v}_{B/F} = -u\mathbf{j}$$

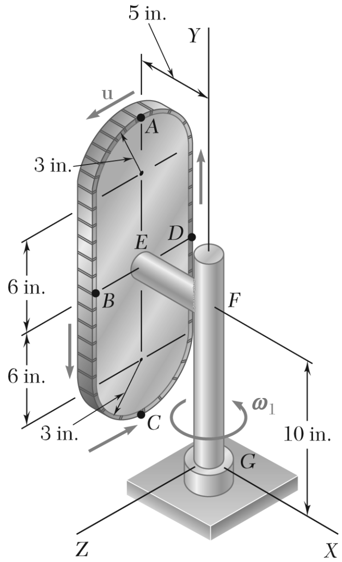
$$= -(3\text{ in./s})\mathbf{j}$$

$$\mathbf{a}_{B/F} = 0$$

$$2\boldsymbol{\Omega} \times \mathbf{v}_{B/F} = (2)(1.6\mathbf{j}) \times (4.5\mathbf{j}) = 0$$

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\Omega \times \mathbf{v}_{B/F}$$

$$= 12.8\mathbf{i} - 7.68\mathbf{k} + 0 + 0 \qquad \mathbf{a}_B = (12.80\text{ in./s}^2)\mathbf{i} - (7.68\text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.241

The vertical plate shown is welded to arm *EFG*, and the entire unit rotates at the constant rate $\omega_1 = 1.6$ rad/s about the *Y* axis. At the same time, a continuous link belt moves around the perimeter of the plate at a constant speed $u = 4.5$ in./s. For the position shown, determine the acceleration of the link of the belt located (*a*) at Point *C*, (*b*) at Point *D*.

SOLUTION

Let the moving frame of reference be the unit, less the pulleys and belt. It rotates about the *Y* axis with constant angular velocity $\mathbf{\Omega} = \omega_1 \mathbf{j} = (1.6 \text{ rad/s})\mathbf{j}$. The relative motion is that of the pulleys and belt with speed $u = 90$ mm/s.

(*a*) *Acceleration at Point C.*

$$\mathbf{r}_C = -(5 \text{ in.})\mathbf{i} + (4 \text{ in.})\mathbf{j}$$

$$\begin{aligned}\mathbf{v}_{C'} &= \mathbf{\Omega} \times \mathbf{r}_A \\ &= 1.6\mathbf{j} \times (-5\mathbf{i} + 4\mathbf{j}) \\ &= (8 \text{ in./s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{C'} &= \mathbf{\Omega} \times \mathbf{v}_{A'} \\ &= 1.6\mathbf{j} \times 8\mathbf{k} \\ &= (12.8 \text{ in./s}^2)\mathbf{i}\end{aligned}$$

$$\mathbf{v}_{C/F} = -u\mathbf{k} = -(4.5 \text{ in./s})\mathbf{k}$$

$$\begin{aligned}\mathbf{a}_{C/F} &= \left(\frac{u^2}{\rho}\right)\mathbf{j} \\ &= \left(\frac{4.5^2}{3}\right)\mathbf{j} \\ &= (6.75 \text{ in./s}^2)\mathbf{j}\end{aligned}$$

$$\begin{aligned}2\mathbf{\Omega} \times \mathbf{v}_{C/F} &= (2)(1.6\mathbf{j}) \times (-4.5\mathbf{k}) \\ &= -(14.4 \text{ in./s}^2)\mathbf{i}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_C &= \mathbf{a}_{C'} + \mathbf{a}_{C/F} + 2\mathbf{\Omega} \times \mathbf{v}_{C/F} \\ &= 12.8\mathbf{i} + 6.75\mathbf{j} - 14.4\mathbf{i}\end{aligned}$$

$$\mathbf{a}_C = -(1.600 \text{ in./s}^2)\mathbf{i} + (6.75 \text{ in./s}^2)\mathbf{j} \blacktriangleleft$$

PROBLEM 15.241 (Continued)

(*b*) *Acceleration at Point D.*

$$\mathbf{r}_D = -(5 \text{ in.})\mathbf{i} + (10 \text{ in.})\mathbf{j} - (3 \text{ in.})\mathbf{k}$$

$$\begin{aligned}\mathbf{v}_{D'} &= \mathbf{\Omega} \times \mathbf{r}_B \\ &= 1.6\mathbf{j} \times (-5\mathbf{i} + 10\mathbf{j} - 3\mathbf{k}) \\ &= -(4.8 \text{ in./s})\mathbf{i} + (8 \text{ in./s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{D'} &= \mathbf{\Omega} \times \mathbf{v}_{B'} \\ &= 1.6\mathbf{j} \times (-4.8\mathbf{i} + 8\mathbf{k}) \\ &= (12.8 \text{ in./s}^2)\mathbf{i} + (7.68 \text{ in./s}^2)\mathbf{k}\end{aligned}$$

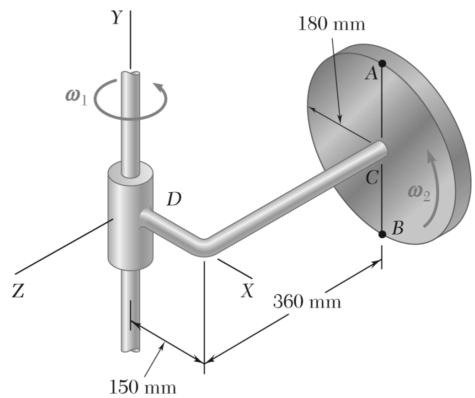
$$\mathbf{v}_{D/F} = u\mathbf{j} = (4.5 \text{ in./s})\mathbf{j}$$

$$\mathbf{a}_{D/F} = 0$$

$$2\mathbf{\Omega} \times \mathbf{v}_{D/F} = (2)(1.6) \times (-4.5\mathbf{j}) = 0$$

$$\begin{aligned}\mathbf{a}_D &= \mathbf{a}_{D'} + \mathbf{a}_{D/F} + 2\mathbf{\Omega} \times \mathbf{v}_{D/F} \\ &= 12.8\mathbf{i} + 7.68\mathbf{k} + 0 + 0\end{aligned}$$

$$\mathbf{a}_D = (12.80 \text{ in./s}^2)\mathbf{i} + (7.68 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.242

A disk of 180-mm radius rotates at the constant rate $\omega_2 = 12$ rad/s with respect to arm *CD*, which itself rotates at the constant rate $\omega_1 = 8$ rad/s about the *Y* axis. Determine at the instant shown the velocity and acceleration of Point *A* on the rim of the disk.

SOLUTION

Geometry.
$$\mathbf{r}_{A/D} = (0.15 \text{ m})\mathbf{i} + (0.18 \text{ m})\mathbf{j} - (0.36 \text{ m})\mathbf{k}$$
$$\mathbf{r}_{A/C} = (0.18 \text{ m})\mathbf{j}$$

Let frame *Dxyz*, which coincides with the fixed frame *DXYZ* at the instant shown, be rotating about the *y* axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j} = (8 \text{ rad/s})\mathbf{j}$. Then the motion relative to the frame consists of a rotation of the disk *AB* about the bent axle *CD* with constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (12 \text{ rad/s})\mathbf{k}$.

Motion of the coinciding Point A′ in the frame.

$$\mathbf{v}_{A'} = \mathbf{\Omega} \times \mathbf{r}_{A/D}$$
$$= 8\mathbf{j} \times (0.15\mathbf{i} + 0.18\mathbf{j} - 0.36\mathbf{k})$$
$$= -(2.88 \text{ m/s})\mathbf{i} - (1.2 \text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{A'} = \mathbf{\Omega} \times \mathbf{v}_{A'}$$
$$= 8\mathbf{j} \times (-2.88\mathbf{i} - 1.2\mathbf{k})$$
$$= -(9.6 \text{ m/s}^2)\mathbf{i} + (23.04 \text{ m/s}^2)\mathbf{k}$$

Motion of Point A relative to the frame.

$$\mathbf{v}_{A/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{A/D}$$
$$= 12\mathbf{k} \times 0.18\mathbf{j}$$
$$= -(2.16 \text{ m/s})\mathbf{i}$$
$$\mathbf{a}_{A/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{A/F}$$
$$= 12\mathbf{k} \times (-2.16\mathbf{i})$$
$$= -(25.92 \text{ m/s}^2)\mathbf{j}$$

Velocity of Point A.
$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$$
$$\mathbf{v}_A = -2.88\mathbf{i} - 1.2\mathbf{k} - 2.16\mathbf{i}$$

$$\mathbf{v}_A = -(5.04 \text{ m/s})\mathbf{i} - (1.200 \text{ m/s})\mathbf{k} \blacktriangleleft$$

PROBLEM 15.242 (Continued)

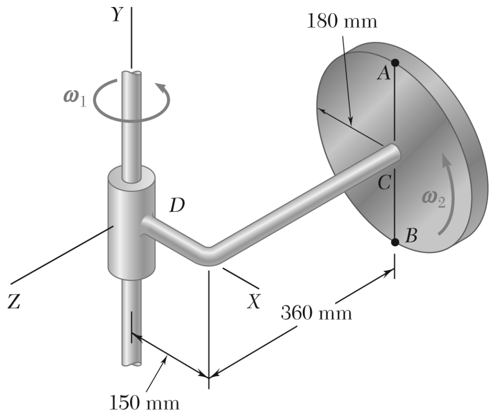
Coriolis acceleration. $2\mathbf{\Omega}\times\mathbf{v}_{A/F}$

$$2\mathbf{\Omega}\times\mathbf{v}_{A/F} = (2)(8\mathbf{j})\times(-2.16\mathbf{i})$$
$$= (34.56\text{ m/s}^2)\mathbf{k}$$

Acceleration of Point A.

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + 2\mathbf{\Omega}\times\mathbf{v}_{A/F}$$
$$\mathbf{a}_A = -9.6\mathbf{i} + 23.04\mathbf{k} - 25.92\mathbf{j} + 34.56\mathbf{k}$$

$$\mathbf{a}_A = -(9.60\text{ m/s}^2)\mathbf{i} - (25.9\text{ m/s}^2)\mathbf{j} + (57.6\text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.243

A disk of 180-mm radius rotates at the constant rate $\omega_2 = 12$ rad/s with respect to arm CD, which itself rotates at the constant rate $\omega_1 = 8$ rad/s about the Y axis. Determine at the instant shown the velocity and acceleration of Point B on the rim of the disk.

SOLUTION

Geometry.

$$\mathbf{r}_{B/D} = (0.15 \text{ m})\mathbf{i} - (0.18 \text{ m})\mathbf{j} - (0.36 \text{ m})\mathbf{k}$$

$$\mathbf{r}_{B/C} = -(0.18 \text{ m})\mathbf{j}$$

Let frame $Dxyz$, which coincides with the fixed frame $DXYZ$ at the instant shown, be rotating about the Y axis with constant angular velocity $\mathbf{\Omega} = \omega_1\mathbf{j} = (8 \text{ rad/s})\mathbf{j}$. Then the motion relative to the frame consists of a rotation of the disk AB about the bent axle CD with constant angular velocity $\boldsymbol{\omega}_2 = \omega_2\mathbf{k} = (12 \text{ rad/s})\mathbf{k}$.

Motion of the coinciding Point B' in the frame.

$$\begin{aligned}
\mathbf{v}_{B'} &= \mathbf{\Omega} \times \mathbf{r}_{B/D} \\
&= 8\mathbf{j} \times (0.15\mathbf{i} - 0.18\mathbf{j} - 0.36\mathbf{k}) \\
&= -(2.88 \text{ m/s})\mathbf{i} - (1.2 \text{ m/s})\mathbf{k} \\
\mathbf{a}_{B'} &= \mathbf{\Omega} \times \mathbf{v}_{B'} \\
&= 8\mathbf{j} \times (-2.88\mathbf{i} - 1.2\mathbf{k}) \\
&= -(9.6 \text{ m/s}^2)\mathbf{i} + (23.04 \text{ m/s}^2)\mathbf{k}
\end{aligned}$$

Motion of Point B relative to the frame.

$$\begin{aligned}
\mathbf{v}_{B/F} &= \boldsymbol{\omega}_2 \times \mathbf{r}_{B/D} \\
&= 12\mathbf{k} \times (-0.18\mathbf{j}) \\
&= (2.16 \text{ m/s})\mathbf{i} \\
\mathbf{a}_{B/F} &= \boldsymbol{\omega}_2 \times \mathbf{v}_{B/F} \\
&= 12\mathbf{k} \times 2.16\mathbf{i} \\
&= (25.92 \text{ m/s}^2)\mathbf{j}
\end{aligned}$$

Velocity of Point B.

$$\mathbf{v}_B = \mathbf{v}_{B'} + \mathbf{v}_{B/F}$$

$$\mathbf{v}_B = -2.88\mathbf{i} - 1.2\mathbf{k} + 2.16\mathbf{i}$$

$$\mathbf{v}_B = -(0.720 \text{ m/s})\mathbf{i} - (1.200 \text{ m/s})\mathbf{k} \blacktriangleleft$$

PROBLEM 15.243 (Continued)

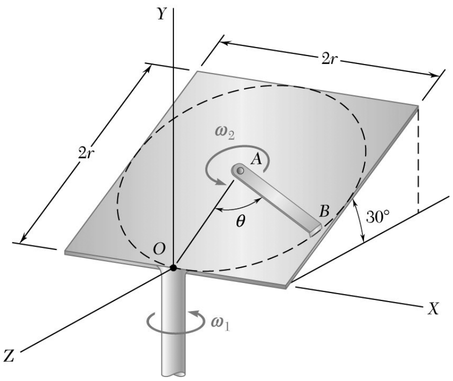
Coriolis acceleration. $2\mathbf{\Omega} \times \mathbf{v}_{B/F}$

$$2\mathbf{\Omega} \times \mathbf{v}_{B/F} = (2)(8\mathbf{j}) \times (2.16\mathbf{i})$$
$$= -(34.56)\mathbf{k}$$

Acceleration of Point B.

$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\mathbf{\Omega} \times \mathbf{v}_{B/F}$$
$$\mathbf{a}_B = -9.6\mathbf{i} + 23.04\mathbf{k} + 25.92\mathbf{j} - 34.56\mathbf{k}$$

$$\mathbf{a}_B = -(9.60 \text{ m/s}^2)\mathbf{i} + (25.9 \text{ m/s}^2)\mathbf{j} - (11.52 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.244

A square plate of side $2r$ is welded to a vertical shaft which rotates with a constant angular velocity $\boldsymbol{\omega}_1$. At the same time, rod AB of length r rotates about the center of the plate with a constant angular velocity $\boldsymbol{\omega}_2$ with respect to the plate. For the position of the plate shown, determine the acceleration of end B of the rod if (*a*) $\theta = 0$, (*b*) $\theta = 90°$, (*c*) $\theta = 180°$.

SOLUTION

Use a frame of reference moving with the plate.

Its angular velocity is $\boldsymbol{\Omega} = \omega_1 \mathbf{j} \qquad (\dot{\boldsymbol{\Omega}} = 0)$

Geometry: $\quad \mathbf{r}_{A/O} = r(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$

$$\mathbf{r}_{B/O} = \mathbf{r}_{A/O} + \mathbf{r}_{B/A}$$

Acceleration of coinciding Point B′ in the frame.

$$\mathbf{a}_{B'} = \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/O})$$

Motion relative to the frame. (Rotation about A with angular velocity $\boldsymbol{\omega}_2$).

$$\boldsymbol{\omega}_2 = \omega_2(\cos 30°\mathbf{j} + \sin 30°\mathbf{k}) \qquad (\dot{\omega}_2 = 0)$$

$$\mathbf{v}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{B/A}$$

$$\mathbf{a}_{B/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{B/F} = -\omega_2^2 \mathbf{r}_{B/A}$$

Coriolis acceleration: $\quad 2\boldsymbol{\Omega} \times \mathbf{v}_{B/F}$

Acceleration of B. $\quad \mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{B/F}$

(*a*) $\quad \theta = 0 \qquad \mathbf{r}_{B/A} = r(-\sin 30°\mathbf{j} + \cos 30°\mathbf{k})$

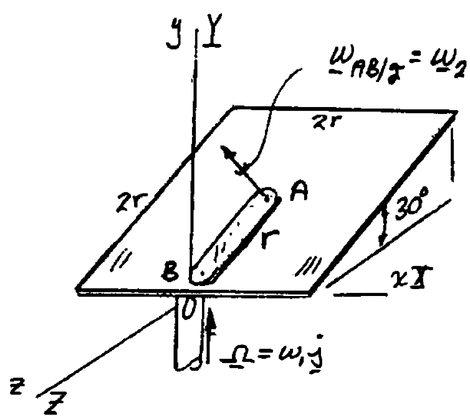
$$\mathbf{r}_{B/O} = 0$$

$$\mathbf{a}_{B'} = 0$$

$$\mathbf{v}_{B/F} = r\omega_2 \mathbf{i}$$

$$\mathbf{a}_{B/F} = -\omega_2^2 \mathbf{r}_{B/A} = r\omega_2^2(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$

$$2\boldsymbol{\Omega} \times \mathbf{v}_{B/F} = 2(\omega_1 \mathbf{j}) \times (r\omega_2 \mathbf{i}) = -2r\omega_1\omega_2 \mathbf{k}$$



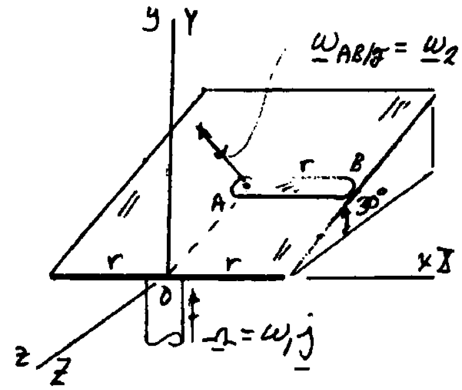
$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_C$$

$$= 0 + r\omega_2^2(\sin 30°\mathbf{j} - \cos 30°\mathbf{k}) - 2r\omega_1\omega_2\mathbf{k}$$

$$\mathbf{a}_B = r\omega_2^2 \sin 30°\mathbf{j} - (r\omega_2^2 \cos 30° + 2r\omega_1\omega_2)\mathbf{k} \blacktriangleleft$$

(*b*) $\theta = 90°$

$$\mathbf{r}_{B/A} = r\mathbf{i}$$
$$\mathbf{r}_{B/O} = r\mathbf{i} + r\sin 30°\mathbf{j} - r\cos 30°\mathbf{k}$$
$$a_{B'} = \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r}_{B/O})$$
$$= \omega_1\mathbf{j}\times[\omega_1\mathbf{j}\times(r\mathbf{i} + r\sin 30°\mathbf{j} - r\cos 30°\mathbf{k})]$$
$$= -r\omega_1^2\mathbf{i} + r\omega_1^2\cos 30°\mathbf{k}$$
$$\mathbf{v}_{B/F} = r\omega_2(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$
$$\mathbf{a}_{B/F} = -r\omega_2^2\mathbf{i}$$
$$2\mathbf{\Omega}\times\mathbf{v}_{B/F} = 2(\omega_1\mathbf{j})\times(r\omega_2)(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$
$$= -2r\omega_1\omega_2\cos 30°\mathbf{i}$$

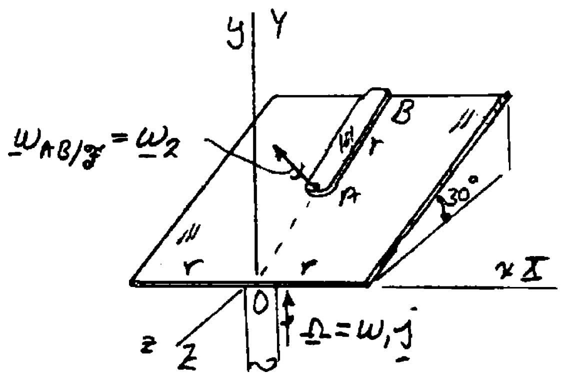


$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + \mathbf{a}_C$$
$$= [-r\omega_1^2\mathbf{i} + r\omega_1^2\cos 30°\mathbf{k}] - r\omega_2^2\mathbf{i} - 2r\omega_1\omega_2\cos 30°\mathbf{i}$$

$$\mathbf{a}_B = -r(\omega_1^2 + \omega_2^2 + 2\omega_1\omega_2\cos 30°)\mathbf{i} + r\omega_1^2\cos 30°\mathbf{k} \blacktriangleleft$$

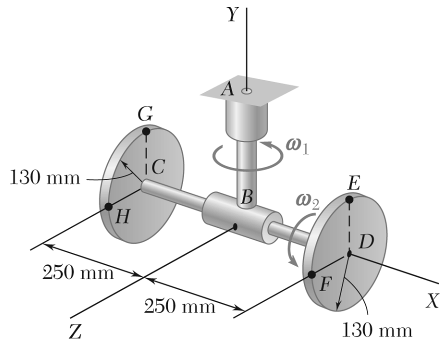
(*c*) $\theta = 180°$

$$\mathbf{r}_{B/A} = r(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$
$$\mathbf{r}_{B/O} = 2r(\sin 30°\mathbf{j} - \cos 30°\mathbf{k})$$
$$\mathbf{a}_{B'} = +(2r\cos 30°)\omega_1^2\mathbf{k} = +2r\omega_1^2\cos 30°\mathbf{k}$$
$$\mathbf{v}_{B/F} = -r\omega_2\mathbf{i}$$
$$\mathbf{a}_{B/F} = r\omega_2^2(-\sin 30°\mathbf{j} + \cos 30°\mathbf{k})$$
$$2\mathbf{\Omega}\times\mathbf{v}_{B/F} = 2(\omega_1\mathbf{j})\times(-r\omega_2\mathbf{i}) = 2r\omega_1\omega_2\mathbf{k}$$



$$\mathbf{a}_B = \mathbf{a}_{B'} + \mathbf{a}_{B/F} + 2\mathbf{\Omega}\times\mathbf{v}_{B/F}$$
$$= 2r\omega_1^2\cos 30°\mathbf{k} + r\omega_2^2(-\sin 30°\mathbf{j} + \cos 30°\mathbf{k}) + 2r\omega_1\omega_2\mathbf{k}$$

$$\mathbf{a}_B = -r\omega_2^2\sin 30°\mathbf{j} + r(2\omega_1^2\cos 30° + \omega_2^2\cos 30° + 2\omega_1\omega_2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.245

Two disks, each of 130-mm radius, are welded to the 500-mm rod *CD*. The rod-and-disks unit rotates at the constant rate $\omega_2 = 3$ rad/s with respect to arm *AB*. Knowing that at the instant shown $\omega_1 = 4$ rad/s, determine the velocity and acceleration of (*a*) Point *E*, (*b*) Point *F*.

SOLUTION

Let the frame of reference *BXYZ* be rotating about the *Y* axis with angular velocity $\mathbf{\Omega} = \omega_2 \mathbf{j} = (4 \text{ rad/s})\mathbf{j}$. The motion relative to this frame is a rotation about the *X* axis with angular velocity $\omega_x \mathbf{i} = (3 \text{ rad/s})\mathbf{i}$.

(*a*) *Point E.*

$$\mathbf{r}_{E/B} = (0.25 \text{ m})\mathbf{i} + (0.13 \text{ m})\mathbf{j}$$

$$\mathbf{r}_{E/D} = (0.13 \text{ m})\mathbf{j}$$

Motion of Point E′ in the frame.

$$\begin{aligned}\mathbf{v}_{E'} &= \mathbf{\Omega} \times \mathbf{r}_{E/B}\\ &= 4\mathbf{j} \times (0.25\mathbf{i} + 0.13\mathbf{j})\\ &= -(1 \text{ m/s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{E'} &= \mathbf{\Omega} \times \mathbf{v}_{E'}\\ &= 4\mathbf{j} \times (-\mathbf{k})\\ &= -(4 \text{ m/s}^2)\mathbf{i}\end{aligned}$$

Motion of Point E relative to the frame.

$$\begin{aligned}\mathbf{v}_{E/F} &= \omega_x \mathbf{i} \times \mathbf{r}_{E/D}\\ &= 3\mathbf{i} \times 0.13\mathbf{j}\\ &= (0.39 \text{ m/s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{E/F} &= \omega_x \mathbf{i} \times \mathbf{v}_{E/F}\\ &= 3\mathbf{i} \times (0.39\mathbf{k})\\ &= -(1.17 \text{ m/s}^2)\mathbf{j}\end{aligned}$$

Coriolis acceleration.

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{E/F}$$

$$\mathbf{a}_c = (2)(4\mathbf{j}) \times (0.39\mathbf{k}) = (3.12 \text{ m/s}^2)\mathbf{i}$$

Velocity of Point E.

$$\mathbf{v}_E = \mathbf{v}_{E'} + \mathbf{v}_{E/F}$$

$$\mathbf{v}_E = -\mathbf{k} + 0.39\mathbf{k} = 0.61\mathbf{k}$$

$$\mathbf{v}_E = (0.610 \text{ m/s})\mathbf{k} \blacktriangleleft$$

Acceleration of Point E.

$$\mathbf{a}_E = \mathbf{a}_{E'} + \mathbf{a}_{E/F} + \mathbf{a}_c$$

$$\mathbf{a}_E = -4\mathbf{i} - 1.17\mathbf{j} + 3.12\mathbf{i}$$

$$\mathbf{a}_E = -(0.880 \text{ m/s}^2)\mathbf{i} + (1.170 \text{ m/s}^2)\mathbf{j} \blacktriangleleft$$

(*b*) *Point F.*

$$\mathbf{r}_{F/B} = (0.25 \text{ m})\mathbf{i} + (0.13 \text{ m})\mathbf{k}$$
$$\mathbf{r}_{F/D} = (0.13 \text{ m})\mathbf{k}$$

Motion of Point F′ in the frame.

$$\begin{aligned}\mathbf{v}_{F'} &= \mathbf{\Omega} \times \mathbf{r}_{F/B}\\ &= 4\mathbf{j} \times (0.25\mathbf{i} + 0.13\mathbf{k})\\ &= (0.52 \text{ m/s})\mathbf{i} - (1 \text{ m/s})\mathbf{k}\\ \mathbf{a}_{F'} &= \mathbf{\Omega} \times \mathbf{v}_{F'}\\ &= (4\mathbf{j}) \times (0.52\mathbf{i} - \mathbf{k})\\ &= -(4 \text{ m/s}^2)\mathbf{i} - (2.08 \text{ m/s}^2)\mathbf{k}\end{aligned}$$

Motion of Point F relative to the frame.

$$\begin{aligned}\mathbf{v}_{F/F} &= \omega_x \mathbf{i} \times \mathbf{r}_{F/D}\\ &= 3\mathbf{i} \times (0.13\mathbf{k})\\ &= -(0.39 \text{ m/s})\mathbf{j}\\ \mathbf{a}_{F/F} &= \omega_x \mathbf{i} \times \mathbf{v}_{F/F}\\ &= 3\mathbf{i} \times (-0.39\mathbf{j})\\ &= -(1.17 \text{ m/s}^2)\mathbf{k}\end{aligned}$$

Coriolis acceleration.

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{F/F}$$
$$\mathbf{a}_c = (2)(4\mathbf{j}) \times (-0.39\mathbf{j}) = 0$$

Velocity of Point F.

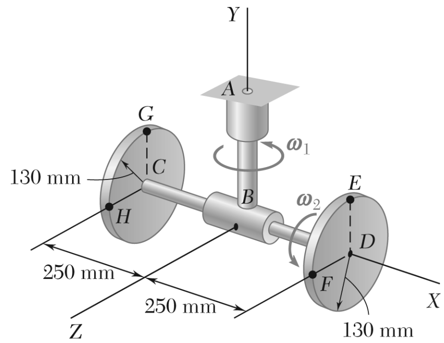
$$\mathbf{v}_F = \mathbf{v}_{F'} + \mathbf{v}_{F/F}$$
$$\mathbf{v}_F = 0.52\mathbf{i} - \mathbf{k} - 0.39\mathbf{j}$$

$$\mathbf{v}_F = (0.520 \text{ m/s})\mathbf{i} - (0.390 \text{ m/s})\mathbf{j} - (1.000 \text{ m/s})\mathbf{k} \blacktriangleleft$$

Acceleration of Point F.

$$\mathbf{a}_F = \mathbf{a}_{F'} + \mathbf{a}_{F/F} + \mathbf{a}_c$$
$$\mathbf{a}_F = -4\mathbf{i} - 2.08\mathbf{k} - 1.17\mathbf{k} + 0$$

$$\mathbf{a}_F = -(4.00 \text{ m/s}^2)\mathbf{i} - (3.25 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.246

In Problem 15.245, determine the velocity and acceleration of (*a*) Point *G*, (*b*) Point *H*.

PROBLEM 15.245 Two disks, each of 130-mm radius, are welded to the 500-mm rod *CD*. The rod-and-disks unit rotates at the constant rate $\omega_2 = 3$ rad/s with respect to arm *AB*. Knowing that at the instant shown $\omega_1 = 4$ rad/s, determine the velocity and acceleration of (*a*) Point *E*, (*b*) Point *F*.

SOLUTION

Let the frame of reference *BXYZ* be rotating about the *Y* axis with angular velocity $\mathbf{\Omega} = \omega_2\mathbf{j} = (4 \text{ rad/s})\mathbf{j}$.

The motion relative to this frame is a rotation about the *X* axis with angular velocity $\omega_x\mathbf{i} = (3 \text{ rad/s})\mathbf{i}$.

(*a*) *Point G.*

$$\mathbf{r}_{G/B} = -(0.25 \text{ m})\mathbf{i} + (0.13 \text{ m})\mathbf{j}$$
$$\mathbf{r}_{G/C} = (0.13 \text{ m})\mathbf{j}$$

Motion of Point G′ in the frame.

$$\mathbf{v}_{G'} = \mathbf{\Omega} \times \mathbf{r}_{G/B}$$
$$= 4\mathbf{j} \times (-0.25\mathbf{i} + 0.13\mathbf{j})$$
$$= (1 \text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{G'} = \mathbf{\Omega} \times \mathbf{v}_{G'}$$
$$= 4\mathbf{j} \times \mathbf{k}$$
$$= (4 \text{ m/s}^2)\mathbf{i}$$

Motion of Point G relative to the frame.

$$\mathbf{v}_{G/F} = \omega_x\mathbf{i} \times \mathbf{r}_{G/C}$$
$$= 3\mathbf{i} \times 0.13\mathbf{j}$$
$$= (0.39 \text{ m/s})\mathbf{k}$$
$$\mathbf{a}_{G/F} = \omega_x\mathbf{i} \times \mathbf{v}_{G/F}$$
$$= 3\mathbf{i} \times (0.39\mathbf{k})$$
$$= -(1.17 \text{ m/s}^2)\mathbf{j}$$

Coriolis acceleration.

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{G/F}$$
$$\mathbf{a}_c = (2)(4\mathbf{j}) \times (0.39\mathbf{k})$$
$$= (3.12 \text{ m/s}^2)\mathbf{i}$$

Velocity of Point G.

$$\mathbf{v}_G = \mathbf{v}_{G'} + \mathbf{v}_{G/F}$$
$$\mathbf{v}_G = \mathbf{k} + 0.39\mathbf{k}$$

$\mathbf{v}_G = (1.390 \text{ m/s})\mathbf{k}$ ◀

PROBLEM 15.246 (Continued)

Acceleration of Point G.

$$\mathbf{a}_G = \mathbf{a}_{G'} + \mathbf{a}_{G/F} + \mathbf{a}_c$$

$$\mathbf{a}_G = 4\mathbf{i} - 1.17\mathbf{j} + 3.12\mathbf{i} \qquad \mathbf{a}_G = (7.12 \text{ m/s}^2)\mathbf{i} - (1.170 \text{ m/s}^2)\mathbf{j} \blacktriangleleft$$

(*b*) *Point H.*

$$\mathbf{r}_{H/B} = -(0.25 \text{ m})\mathbf{i} + (0.13 \text{ m})\mathbf{k}$$

$$\mathbf{r}_{H/C} = (0.13 \text{ m})\mathbf{k}$$

Motion of Point H′ in the frame.

$$\begin{aligned}\mathbf{v}_{H'} &= \mathbf{\Omega} \times \mathbf{r}_{H/B} \\ &= 4\mathbf{j} \times (-0.25\mathbf{i} + 0.13\mathbf{k}) \\ &= (0.52 \text{ m/s})\mathbf{i} + (1 \text{ m/s})\mathbf{k}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{H'} &= \mathbf{\Omega} \times \mathbf{v}_{H'} \\ &= 4\mathbf{j} \times (0.52\mathbf{i} + \mathbf{k}) \\ &= (4 \text{ m/s}^2)\mathbf{i} - (2.08 \text{ m/s}^2)\mathbf{k}\end{aligned}$$

Motion of Point H relative to the frame.

$$\begin{aligned}\mathbf{v}_{H/F} &= \omega_x \mathbf{i} \times \mathbf{r}_{H/C} \\ &= 3\mathbf{i} \times (0.13\mathbf{k}) \\ &= -(0.39 \text{ m/s})\mathbf{j}\end{aligned}$$

$$\begin{aligned}\mathbf{a}_{H/F} &= \omega_x \mathbf{i} \times \mathbf{v}_{H/F} \\ &= 3\mathbf{i} \times (-0.39\mathbf{j}) \\ &= -(1.17 \text{ m/s}^2)\mathbf{k}\end{aligned}$$

Coriolis acceleration.

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{H/F}$$

$$\mathbf{a}_c = (2)(4\mathbf{j}) \times (0.39\mathbf{j}) = 0$$

Velocity of Point H.

$$\mathbf{v}_H = \mathbf{v}_{H'} + \mathbf{v}_{H/F}$$

$$\mathbf{v}_H = 0.52\mathbf{i} + \mathbf{k} - 0.39\mathbf{j}$$

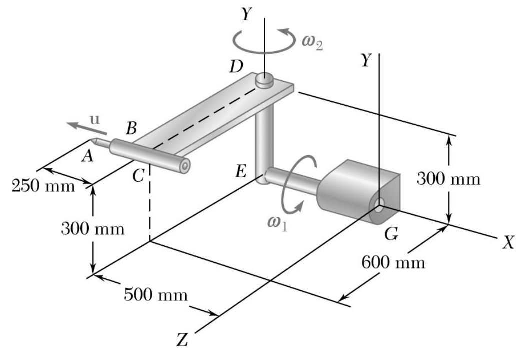
$$\mathbf{v}_H = (0.520 \text{ m/s})\mathbf{i} - (0.390 \text{ m/s})\mathbf{j} + (1.000 \text{ m/s})\mathbf{k} \blacktriangleleft$$

Acceleration of Point H.

$$\mathbf{a}_H = \mathbf{a}_{H'} + \mathbf{a}_{H/F} + \mathbf{a}_c$$

$$\mathbf{a}_H = 4\mathbf{i} - 2.08\mathbf{k} - 1.17\mathbf{k} + 0 \qquad \mathbf{a}_H = (4.00 \text{ m/s}^2)\mathbf{i} - (3.25 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$

PROBLEM 15.247



The position of the stylus tip A is controlled by the robot shown. In the position shown the stylus moves at a constant speed $u = 180$ mm/s relative to the solenoid BC. At the same time, arm CD rotates at the constant rate $\omega_2 = 1.6$ rad/s with respect to component DEG. Knowing that the entire robot rotates about the X axis at the constant rate $\omega_1 = 1.2$ rad/s, determine (*a*) the velocity of A, (*b*) the acceleration of A.

SOLUTION

Geometry:

$$\mathbf{r}_{D/G} = -(500 \text{ mm})\mathbf{i} + (300 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/D} = -(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}$$

$$\mathbf{r}_{A/G} = \mathbf{r}_{A/D} + \mathbf{r}_{D/G} = -(750 \text{ mm})\mathbf{i} + (300 \text{ mm})\mathbf{j} + (600 \text{ mm})\mathbf{k}$$

Angular velocities:

$$\boldsymbol{\omega}_1 = \omega_1\mathbf{i} = (1.2 \text{ rad/s})\mathbf{i} \qquad (\dot{\omega}_1 = 0)$$

$$\boldsymbol{\omega}_2 = \omega_2\mathbf{j} = (1.6 \text{ rad/s})\mathbf{j} \qquad (\dot{\omega}_2 = 0)$$

Stylus motion:

$$\mathbf{u} = -u\mathbf{i} = -(180 \text{ mm/s})\mathbf{i} \qquad (\dot{u} = 0)$$

Method 1

Let the rigid body BCD be a rotating frame of reference.

Its angular velocity is $\quad \boldsymbol{\omega}_{CD} = \boldsymbol{\omega}_1 + \boldsymbol{\omega}_2 = (1.2 \text{ rad/s})\mathbf{i} + (1.6 \text{ rad/s})\mathbf{j}$

Its angular acceleration is

$$\boldsymbol{\alpha}_{CD} = \boldsymbol{\omega}_1 \times \boldsymbol{\omega}_{CD} = (1.2 \text{ rad/s})\mathbf{i} \times [(1.2 \text{ rad/s})\mathbf{i} + (1.6 \text{ rad/s})\mathbf{j}]$$
$$= (1.92 \text{ rad/s}^2)\mathbf{k}$$

Motion of the coinciding Point A' in the frame.

$$\mathbf{v}_{A'} = \mathbf{v}_D + \mathbf{v}_{A'/D}$$

$$\mathbf{v}_{A'} = \boldsymbol{\omega}_1 \times \mathbf{r}_{D/G} + (\boldsymbol{\omega}_1 + \boldsymbol{\omega}_2) \times \mathbf{r}_{A/D}$$
$$= (1.2 \text{ rad/s})\mathbf{i} \times [-(500 \text{ mm})\mathbf{i} + (300 \text{ mm})\mathbf{j}]$$
$$+ [(1.2 \text{ rad/s})\mathbf{i} + (1.6 \text{ rad/s})\mathbf{j}] \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}]$$
$$= (360 \text{ mm/s})\mathbf{k} - (720 \text{ mm/s})\mathbf{j} + (400 \text{ mm/s})\mathbf{k} + (960 \text{ mm/s})\mathbf{i}$$

$$\mathbf{v}_{A'} = (960 \text{ mm/s})\mathbf{i} - (720 \text{ mm/s})\mathbf{j} + (760 \text{ mm/s})\mathbf{k}$$

$$\mathbf{a}_D = \omega_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_{D/G})$$
$$= (1.2 \text{ rad/s})\mathbf{i} \times \{(1.2 \text{ rad/s})\mathbf{i} \times [-(500 \text{ mm})\mathbf{i} + (300 \text{ mm})\mathbf{j}]\}$$
$$= -(432 \text{ mm/s}^2)\mathbf{j}$$

$$\mathbf{a}_{A'/D} = \boldsymbol{\alpha}_{CD} \times \mathbf{r}_{A/D} + \boldsymbol{\omega}_{CD} \times (\boldsymbol{\omega}_{CD} \times \mathbf{r}_{A/D})$$
$$= (1.92 \text{ rad/s}^2)\mathbf{k} \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}]$$
$$+ \boldsymbol{\omega}_{CD} \times \{[(1.2 \text{ rad/s})\mathbf{i} + (1.6 \text{ rad/s})\mathbf{j}] \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}]\}$$

$$\mathbf{a}_{A'/D} = -(480\ \text{mm/s}^2)\mathbf{j} + \omega_{CD} \times [-(720\ \text{mm/s})\mathbf{j} + (400\ \text{mm/s})\mathbf{k} + (960\ \text{mm/s})\mathbf{i}]$$

$$= -480\mathbf{j} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 1.6 & 0 \\ 960 & -720 & 400 \end{vmatrix}$$

$$= -480\mathbf{j} + 640\mathbf{i} - 480\mathbf{j} - 864\mathbf{k} - 1536\mathbf{k}$$

$$\mathbf{a}_{A'/D} = (640\ \text{mm/s}^2)\mathbf{i} - (960\ \text{mm/s}^2)\mathbf{j} - (2400\ \text{mm/s}^2)\mathbf{k}$$

$$\mathbf{a}_{A'} = \mathbf{a}_D + \mathbf{a}_{A'/D}$$

$$\mathbf{a}_{A'} = (640\ \text{mm/s}^2)\mathbf{i} - (1392\ \text{mm/s}^2)\mathbf{j} - (2400\ \text{mm/s}^2)\mathbf{k}$$

Motion of Point A relative to the frame.

$$\mathbf{v}_{A/F} = \mathbf{u} = -(180\ \text{mm/s})\mathbf{i}$$

$$\mathbf{a}_{A/F} = 0$$

(*a*) *Velocity of A.* $\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$

$$\mathbf{v}_A = (960\ \text{mm/s})\mathbf{i} - (720\ \text{mm/s})\mathbf{j} + (760\ \text{mm/s})\mathbf{k} - (180\ \text{mm/s})\mathbf{i}$$

$$\mathbf{v}_A = (0.78\ \text{m/s})\mathbf{i} - (0.72\ \text{m/s})\mathbf{j} + (0.76\ \text{m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration: $\mathbf{a}_c = 2\boldsymbol{\omega}_{CD} \times \mathbf{v}_{A/F}$

$$\mathbf{a}_c = 2[(1.2\ \text{rad/s})\mathbf{i} + (1.6\ \text{rad/s})\mathbf{j}] \times (-180\ \text{mm/s})\mathbf{i}$$

$$= +(576\ \text{mm/s}^2)\mathbf{k}$$

(*b*) *Acceleration of A.* $\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + \mathbf{a}_c$

$$\mathbf{a}_A = (640\ \text{mm/s}^2)\mathbf{i} - (1392\ \text{mm/s}^2)\mathbf{j} - (2400\ \text{mm/s}^2)\mathbf{k} + (576\ \text{mm/s}^2)\mathbf{k}$$

$$\mathbf{a}_A = (0.64\ \text{m/s}^2)\mathbf{i} - (1.392\ \text{m/s}^2)\mathbf{j} - (1.824\ \text{m/s}^2)\mathbf{k} \blacktriangleleft$$

Method 2

Use a frame of reference rotating about the x axis with angular velocity.

$$\boldsymbol{\omega}_1 = \omega_1\mathbf{i} = (1.2\ \text{rad/s})\mathbf{i} \qquad (\dot{\omega}_1 = 0)$$

Motion of coinciding Point A′ in the frame.

$$\mathbf{v}_{A'} = \boldsymbol{\omega}_1 \times \mathbf{r}_{A/G} = (1.2\ \text{rad/s})\mathbf{i} \times [-(500\ \text{mm})\mathbf{i} + (300\ \text{mm})\mathbf{j} + (600\ \text{mm})\mathbf{k}]$$

$$= (360\ \text{mm/s})\mathbf{k} - (720\ \text{mm/s})\mathbf{j}$$

$$\mathbf{a}_{A'} = \omega_1 \times (\boldsymbol{\omega}_1 \times \mathbf{r}_{A/G}) = \boldsymbol{\omega}_1 \times \mathbf{v}_{A'}$$

$$= (1.2\ \text{rad/s})\mathbf{i} \times [(360\ \text{mm/s})\mathbf{k} - (720\ \text{mm/s})\mathbf{j}]$$

$$= -(432\ \text{mm/s}^2)\mathbf{j} - (864\ \text{mm/s}^2)\mathbf{k}$$

PROBLEM 15.247 (Continued)

Motion of Point A relative to the frame.

$$\begin{aligned}\mathbf{v}_{A/F} &= \boldsymbol{\omega}_2 \times \mathbf{r}_{A/D} + \mathbf{u}\\ &= (1.6 \text{ rad/s})\mathbf{j} \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}] - (180 \text{ mm/s})\mathbf{i}\\ &= (400 \text{ mm/s})\mathbf{k} + (960 \text{ mm/s})\mathbf{i} - (180 \text{ mm/s})\mathbf{i}\end{aligned}$$

$\mathbf{a}_{A/F}$: (Since A moves on CD, which rotates at rate $\boldsymbol{\omega}_2$, we have a Coriolis term here).

$$\begin{aligned}\mathbf{a}_{A/F} &= \boldsymbol{\omega}_2 \times (\boldsymbol{\omega}_2 \times \mathbf{r}_{A/D}) + 2\boldsymbol{\omega}_2 \times \mathbf{u}\\ &= \boldsymbol{\omega}_2 \times \{(1.6 \text{ rad/s})\mathbf{j} \times [-(250 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{k}]\} + 2\boldsymbol{\omega}_2 \times \mathbf{u}\\ &= (1.6 \text{ rad/s})\mathbf{j} \times [(400 \text{ mm/s})\mathbf{k} + (960 \text{ mm/s})\mathbf{i}] + 2(1.6 \text{ rad/s})\mathbf{j} \times (-180 \text{ mm})\mathbf{i}\\ &= (640 \text{ mm/s}^2)\mathbf{i} - (1536 \text{ mm/s}^2)\mathbf{k} + (576 \text{ mm/s}^2)\mathbf{k}\\ &= (640 \text{ mm/s}^2)\mathbf{i} - (960 \text{ mm/s}^2)\mathbf{k}\end{aligned}$$

(*a*) *Velocity of A.*

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$$

$$\mathbf{v}_A = 360\mathbf{k} - 720\mathbf{j} + 400\mathbf{k} + 960\mathbf{i} - 180\mathbf{i}$$

$$\mathbf{v}_A = (0.78 \text{ m/s})\mathbf{i} - (0.72 \text{ m/s})\mathbf{j} + (0.76 \text{ m/s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration:

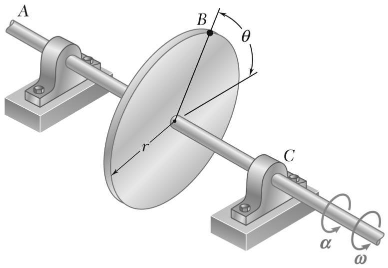
$$\begin{aligned}\mathbf{a}_c &= 2\boldsymbol{\omega}_1 \times \mathbf{v}_{A/F}\\ \mathbf{a}_c &= 2(1.2 \text{ rad/s})\mathbf{i} \times [(400 \text{ mm})\mathbf{k} + (780 \text{ mm/s})\mathbf{i}]\\ &= -(960 \text{ mm/s}^2)\mathbf{j}\end{aligned}$$

(*b*) *Acceleration of A.*

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + \mathbf{a}_c$$

$$\mathbf{a}_A = -432\mathbf{j} - 864\mathbf{k} + 640\mathbf{i} - 960\mathbf{k} - 960\mathbf{j}$$

$$\mathbf{a}_A = (0.64 \text{ m/s}^2)\mathbf{i} - (1.392 \text{ m/s}^2)\mathbf{j} - (1.824 \text{ m/s}^2)\mathbf{k} \blacktriangleleft$$



PROBLEM 15.248

The angular acceleration of the 600-mm-radius circular plate shown is defined by the relation $\alpha = \alpha_0 e^{-t}$. Knowing that the plate is at rest when $t = 0$ and that $\alpha_0 = 10 \text{ rad/s}^2$, determine the magnitude of the total acceleration of Point B when (*a*) $t = 0$, (*b*) $t = 0.5$ s, (*c*) $t = \infty$.

SOLUTION

$$\alpha = \frac{d\omega}{dt} = \alpha_0 e^{-t}; \quad \int_0^\omega d\omega = \int_0^t \alpha_0 e^{-t}\, dt$$

$$\omega = \alpha_0 \left| -e^{-t} \right|_0^t \qquad \omega = \alpha_0 (1 - e^{-t})$$

$$a_t = r\alpha = r\alpha_0 e^{-t} = (0.6 \text{ m})(10 \text{ rad/s}^2) e^{-t} = 6e^{-t}$$

$$a_n = r\omega^2 = r\alpha_0^2 (1 - e^{-t})^2 = (0.6)(10)^2 (1 - e^{-t})^2 = 60(1 - e^{-t})^2$$

(*a*) $t = 0$:

$$a_t = 6e^0 = 6 \text{ m/s}^2$$

$$a_n = 60(1 - e^0)^2 = 0$$

$$a_B^2 = a_t^2 + a_n^2 = 6^2$$

$a_B = 6.00 \text{ m/s}^2$ ◀

(*b*) $t = 0.5$ s:

$$a_t = 6e^{-0.5} = 6(0.6065) = 3.639 \text{ m/s}^2$$

$$a_n = 60(1 - e^{-0.5})^2$$

$$= 60(1 - 0.6065)^2$$

$$= 9.289 \text{ m/s}^2$$

$$a_B^2 = a_t^2 + a_n^2 = (3.639)^2 + (9.289)^2$$

$a_B = 9.98 \text{ m/s}^2$ ◀

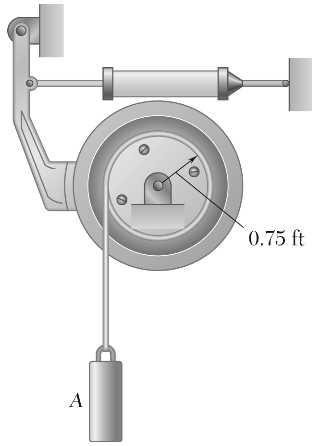
(*c*) $t = \infty$:

$$a_t = 6e^{-\infty} = 0$$

$$a_n = 60(1 - e^{-\infty})^2 = 60 \text{ m/s}^2$$

$$a_B^2 = a_t^2 + a_n^2 = 0 + 60^2$$

$a_B = 60.0 \text{ m/s}^2$ ◀



PROBLEM 15.249

Cylinder A is moving downward with a velocity of 9 ft/s when the brake is suddenly applied to the drum. Knowing that the cylinder moves 18 ft downward before coming to rest and assuming uniformly accelerated motion, determine (a) the angular acceleration of the drum, (b) the time required for the cylinder to come to rest.

SOLUTION

Block A:

$$v^2 - v_0^2 = 2as$$

$$0 - (9 \text{ ft/s})^2 = 2a\,(18 \text{ ft})$$

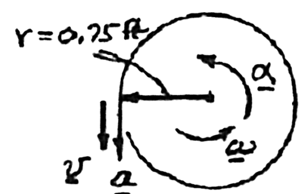
$$a = -2.25 \text{ ft/s}^2 \qquad \mathbf{a} = 2.25 \text{ ft/s}^2 \uparrow$$

Drum:

$$v_A = r\omega_0$$

$$9 \text{ ft/s} = (0.75 \text{ ft})\omega$$

$$\omega_0 = 12 \text{ rad/s}$$



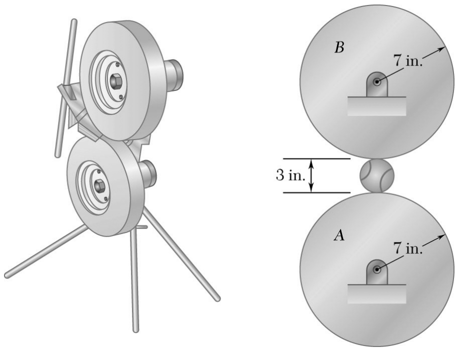
(a) $a = r\alpha$

$$-(2.25 \text{ ft/s}^2) = (0.75 \text{ ft})\alpha$$

$$\alpha = -3 \text{ rad/s}^2 \qquad \boldsymbol{\alpha} = 3.00 \text{ rad/s}^2 \circlearrowright \blacktriangleleft$$

(b) Uniformly accelerated motion. $\omega = 0$ when $t = t_1$

$$\omega = \omega_0 + \alpha t: \quad 0 = (12 \text{ rad/s}) - (3 \text{ rad/s}^2)t_1 \qquad t_1 = 4.00 \text{ s} \blacktriangleleft$$



PROBLEM 15.250

A baseball pitching machine is designed to deliver a baseball with a ball speed of 70 mph and a ball rotation of 300 rpm clockwise. Knowing that there is no slipping between the wheels and the baseball during the ball launch, determine the angular velocities of wheels A and B.

SOLUTION

Let Point G be the center of the ball, A its contact point with wheel A, and B its contact point with wheel B.

Given:
$$v_G = \left(70\frac{\text{m}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{5280\text{ ft}}{\text{mi}}\right) = 102.667\text{ ft/s} = 1232\text{ in./s}$$

$$\omega_{\text{ball}} = (300\text{ rpm})\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 10\pi\text{ rad/s}$$

$$\mathbf{v}_G = 1232\text{ in./s} \longrightarrow \qquad \boldsymbol{\omega}_{\text{ball}} = 10\pi\text{ rad/s} \circlearrowright$$

Unit vectors: $\mathbf{i} = 1\longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \mathbf{k} = 1\circlearrowleft$

Relative positions:
$$\mathbf{r}_{A/G} = \frac{1}{2}(-3\mathbf{j}) = -(1.5\text{ in.})\mathbf{j}$$

$$\mathbf{r}_{B/G} = \frac{1}{2}(3\mathbf{j}) = (1.5\text{ in.})\mathbf{j}$$

Velocities at A and B.
$$\mathbf{v}_A = \mathbf{v}_G + \boldsymbol{\omega}_{\text{ball}} \times \mathbf{r}_{A/G} = 1232\mathbf{i} + (-10\pi\mathbf{k}) \times (1.5\mathbf{j})$$
$$= (1232 + 47.12)\mathbf{i} = 1279.12\text{ in./s} \longrightarrow$$

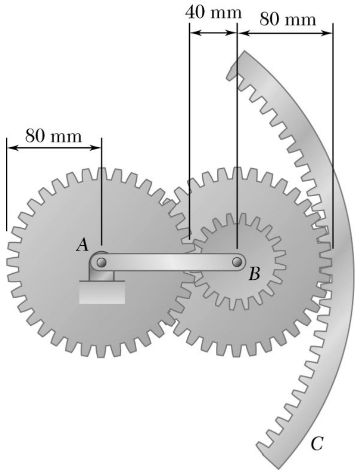
$$\mathbf{v}_B = \mathbf{v}_G + \boldsymbol{\omega}_{\text{ball}} \times \mathbf{r}_{B/G} = 1232\mathbf{i} + (-10\pi\mathbf{k}) \times (1.5\pi\mathbf{j})$$
$$= (1232 - 47.12)\mathbf{i} = 1184.88\text{ in./s} \longrightarrow$$

Angular velocity of A.
$$\omega_A = \frac{v_A}{r_A} = \frac{1184.88}{7} = 169.27\text{ rad/s}$$

$$\boldsymbol{\omega}_A = 1616\text{ rpm} \circlearrowright \blacktriangleleft$$

Angular velocity of B.
$$\omega_B = \frac{v_B}{r_B} = \frac{1279.12}{7} = 182.73\text{ rad/s} \circlearrowleft$$

$$\boldsymbol{\omega}_B = 1745\text{ rpm} \circlearrowleft \blacktriangleleft$$



PROBLEM 15.251

Knowing that inner gear A is stationary and outer gear C starts from rest and has a constant angular acceleration of 4 rad/s^2 clockwise, determine at $t = 5$ s (a) the angular velocity of arm AB, (b) the angular velocity of gear B, (c) the acceleration of the point on gear B that is in contact with gear A.

SOLUTION

Angular velocity of gear C at $t = 5$ s.

$$\omega_c = \alpha_c t = (4 \text{ rad/s}^2)(5 \text{ s}) = 20 \text{ rad/s} \qquad \boldsymbol{\omega}_c = 20 \text{ rad/s} \circlearrowright$$

Let Point 1 be the contact point between gears A and B. Let Point 2 be the contact point between gears B and C. Points A, B, and C are the centers, respectively, of gears A, B, and C.

Positions: Take x axis along the straight line A1 B2.

$$x_A = x_C = 0$$
$$x_1 = r_A = 80 \text{ mm}$$
$$x_B = (80 \text{ mm} + 40 \text{ mm}) = 120 \text{ mm}$$
$$x_2 = 120 \text{ mm} + 80 \text{ mm} = 200 \text{ mm}$$

Velocity at 1. Since gear A is stationary, $v_1 = 0$.

Velocity at 2. $v_2 = x_2 \omega_C = (200)(20)$

$$\mathbf{v}_2 = 4000 \text{ mm/s} \downarrow$$

Point 1 is the instantaneous center of gear B.

$$v_2 = (x_2 - x_1)\omega_B = (120 \text{ mm})\omega_B$$
$$\omega_B = \frac{v_2}{120} = \frac{4000}{120} = 33.333 \text{ rad/s}$$
$$v_B = (x_B - x_1)\omega_B = (40 \text{ mm})(33.333 \text{ rad/s}) = 1333.33 \text{ mm/s}$$
$$\omega_{AB} = \frac{v_B}{x_B} = \frac{1333.33}{120} = 11.111 \text{ rad/s}$$

(a) *Angular velocity of arm AB*: $\boldsymbol{\omega}_{AB} = 11.11 \text{ rad/s} \circlearrowright$ ◀

(b) *Angular velocity of gear B*: $\boldsymbol{\omega}_B = 33.3 \text{ rad/s} \circlearrowright$ ◀

PROBLEM 15.251 (Continued)

Calculate tangential accelerations:

$$(a_1)_t = 0$$

$$(a_2)_t = (a_C)_t = r_C\alpha_2 = (200\text{ mm})(4\text{ rad/s}^2)$$

$$(a_2)_t = 800\text{ mm/s}^2 \downarrow$$

$$(a_2)_t = (x_2 - x_1)\alpha_B = (120\text{ mm})\alpha_B$$

$$\alpha_B = \frac{(a_2)_t}{120} = \frac{800}{120} = 6.667\text{ rad/s}^2$$

$$(a_B)_t = (x_B - x_1)\alpha_B = (40\text{ mm})(6.667\text{ rad/s}^2)$$

$$= 266.67\text{ mm/s}^2$$

$$\alpha_{AB} = \frac{(a_B)_t}{r_B} = \frac{266.67}{120} = 2.2222\text{ rad/s}^2$$

Angular accelerations: $\boldsymbol{\alpha}_{AB} = 2.2222\text{ rad/s}^2 \circlearrowright$ $\quad\boldsymbol{\alpha}_B = 6.667\text{ rad/s} \circlearrowright$

Acceleration of Point B.

$$\mathbf{a}_B = [\alpha_{AB}\, x_B \downarrow] + [\omega_{AB}^2\, x_B \longleftarrow]$$

$$= [(2.2222)(120) \downarrow] + [(11.11)^2(120) \longleftarrow]$$

$$= (266.67\text{ mm/s}^2) \downarrow + (14815\text{ mm/s}^2) \longleftarrow$$

(*c*) *Acceleration of Point 1 on gear B.*

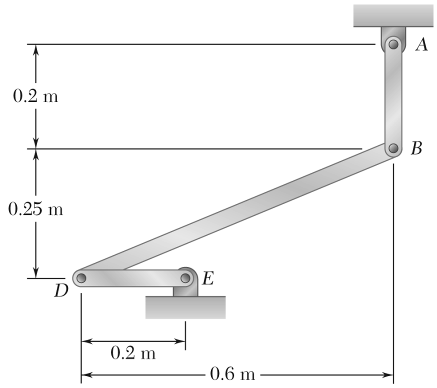
$$\mathbf{a}_1 = \mathbf{a}_B + [\alpha_B(x_B - x_1) \uparrow] + [\omega_B^2(x_B - x_1) \longrightarrow]$$

$$= [266.67 \downarrow] + [14815 \longleftarrow] + [(6.667)(40) \uparrow] + [(33.333)^2(40) \longrightarrow]$$

$$= [266.67 \downarrow] + [14815 \longleftarrow] + [266.67 \uparrow] + [44444 \longrightarrow]$$

$$= 29629\text{ mm/s} \longrightarrow \qquad \mathbf{a}_1 = 29.6\text{ m/s}^2 \longrightarrow \blacktriangleleft$$

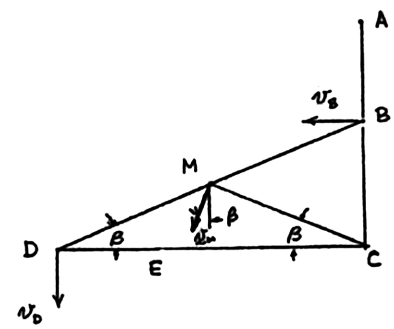
Note that the tangential component of acceleration is zero as expected.



PROBLEM 15.252

Knowing that at the instant shown bar AB has an angular velocity of 10 rad/s clockwise and it is slowing down at a rate of 2 rad/s^2, determine the angular accelerations of bar BD and bar DE.

SOLUTION



Velocity Analysis $\quad \boldsymbol{\omega}_{AB} = 10 \text{ rad/s} \circlearrowright$

$$v_B = (AB)\omega_{AB} = (0.200)(10) = 2 \text{ m/s}$$

$$\mathbf{v}_B = v_B \longleftarrow$$

$$\mathbf{v}_D = v_D \downarrow$$

Locate the instantaneous center (Point C) of bar BD by noting that velocity directions at Points B and D are known. Draw BC perpendicular to $\mathbf{v}_B$ and DC perpendicular to $\mathbf{v}_D$.

$$\omega_{BD} = \frac{v_B}{BC} = \frac{2}{0.25} = 8 \text{ rad/s} \qquad \boldsymbol{\omega}_{BD} = 8.00 \text{ rad/s} \circlearrowright$$

$$v_D = (CE)\omega_{BD} = (0.6)(8) = 4.8 \text{ m/s} \qquad \mathbf{v}_D = 4.8 \text{ m/s} \downarrow$$

$$\omega_{DE} = \frac{v_D}{DE} = \frac{4.8}{0.2} = 24 \text{ rad/s} \qquad \boldsymbol{\omega}_{DE} = 24 \text{ rad/s} \circlearrowright$$

Acceleration Analysis: $\quad \alpha_{AB} = 2 \text{ rad/s}^2 \circlearrowleft, \quad \omega_{AB} = 10 \text{ rad/s} \circlearrowright$

Unit vectors: $\quad \mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1 \uparrow, \quad \mathbf{k} = 1 \circlearrowleft$

$$\begin{aligned}\mathbf{a}_B &= \mathbf{a}_A + \boldsymbol{\alpha}_{AB} \times \mathbf{r}_{B/A} - \omega_{AB}^2 \mathbf{r}_{B/A} \\ &= 0 + (2\mathbf{k}) \times (-0.2)\mathbf{j} - (10)^2(-0.2\mathbf{j}) \\ &= (0.4 \text{ m/s}^2)\mathbf{i} + (20 \text{ m/s}^2)\mathbf{j}\end{aligned}$$

PROBLEM 15.252 (Continued)

$$\mathbf{a}_D = \mathbf{a}_B + \boldsymbol{\alpha}_{BD} \times \mathbf{r}_{D/B} - \omega_{BD}^2 \mathbf{r}_{D/B}$$
$$= 0.4\mathbf{i} + 20\mathbf{j} + \alpha_{BD}\mathbf{k} \times (-0.6\mathbf{i} - 0.25\mathbf{j}) - (8)^2(-0.6\mathbf{i} - 0.25\mathbf{j})$$
$$\mathbf{a}_D = (38.8 + 0.25\alpha_{BD})\mathbf{i} + (36 - 0.6\alpha_{BD})\mathbf{j} \quad (1)$$

$$\mathbf{a}_D = \mathbf{a}_E + \boldsymbol{\alpha}_{DE} \times \mathbf{r}_{D/E} - \omega_{DE}^2 \mathbf{r}_{D/E}$$
$$= 0 + \alpha_{DE}\mathbf{k} \times (-0.2\mathbf{i}) - (24)^2(-0.2\mathbf{i})$$
$$= 115.2\mathbf{i} - 0.2\alpha_{DE}\mathbf{j} \quad (2)$$

Equate like components of $\mathbf{a}_D$ from Equations (1) and (2).

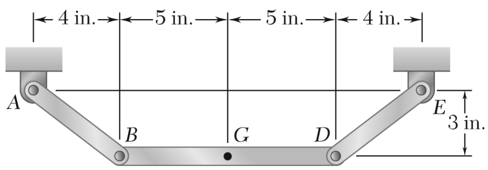
i: $\quad 38.8 + 0.25\alpha_{BD} = 115.2 \qquad \alpha_{BD} = 305.6 \text{ rad/s}^2$

j: $\quad 36 - 0.6\alpha_{BD} = -0.2\alpha_{DE}$

$$\alpha_{DE} = 3\alpha_{BD} - 180 = 736.8 \text{ rad/s}^2$$

Angular acceleration: $\quad \boldsymbol{\alpha}_{BD} = 306 \text{ rad/s}^2 \circlearrowright$ ◀

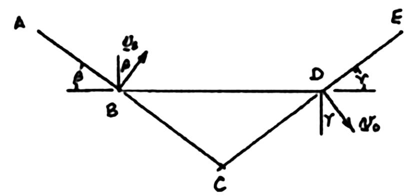
$\boldsymbol{\alpha}_{DE} = 737 \text{ rad/s}^2 \circlearrowright$ ◀



PROBLEM 15.253

Knowing that at the instant shown rod AB has zero angular acceleration and an angular velocity of 15 rad/s counterclockwise, determine (*a*) the angular acceleration of arm DE, (*b*) the acceleration of Point D.

SOLUTION



$$\tan\beta = \frac{3}{4}, \quad \beta = 36.87°$$

$$AB = \frac{4}{\cos\beta} = 5 \text{ in.}$$

$$DE = \frac{4}{\cos\beta} = 5 \text{ in.}$$

$$v_B = (AB)\omega_{AB} = (5)(15)$$

$$= 75 \text{ in./s}$$

$$\mathbf{v}_B = v_B \nearrow \beta, \quad \mathbf{v}_D = v_D \searrow \beta$$

Point C is the instantaneous center of bar BD.

$$CB = \frac{5}{\cos\beta} = 6.25 \text{ in.} \quad \omega_{BD} = \frac{v_B}{CB} = \frac{75}{6.25} = 12 \text{ rad/s} \circlearrowright$$

$$CD = \frac{5}{\cos\beta} = 6.25 \text{ in.} \quad v_D = (CD)\omega_{BD} = (6.25)(12) = 75 \text{ in./s}$$

$$\omega_{DE} = \frac{v_D}{DE} = \frac{75}{5} = 15 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\qquad \alpha_{AB} = 0$

$$\mathbf{a}_B = [(AB)\alpha_{AB} \nearrow \beta] + [(AB)\omega_{AB}^2 \searrow \beta]$$

$$= 0 + [(5)(15)^2 \searrow \beta] = 1125 \text{ in./s}^2 \searrow \beta$$

$$\mathbf{a}_{D/B} = [(BD)\alpha_{BD} \uparrow] + [(BD)\omega_{BD}^2 \longleftarrow]$$

$$= [10\alpha_{BD} \uparrow] + [(10)(12)^2 \longleftarrow]$$

$$= [10\alpha_{BD} \uparrow] + [1440 \text{ in./s}^2 \longleftarrow]$$

$$\mathbf{a}_D = [(DE)\alpha_{DE} \searrow \beta] + [(DE)\omega_{DE}^2 \nearrow \beta]$$

$$= [5\alpha_{DE} \searrow \beta] + [(5)(15)^2 \nearrow \beta]$$

$$= [5\alpha_{DE} \searrow \beta] + [1125 \text{ in./s}^2 \nearrow \beta]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \qquad \text{Resolve into components.}$$

PROBLEM 15.253 (Continued)

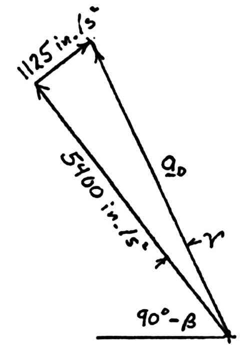
(*a*) $\xrightarrow{+}$: $5\alpha_{DE}\sin\beta + 1125\cos\beta = -1125\cos\beta - 1440$

$$\alpha_{DE} = -1080 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_{DE} = 1080 \text{ rad/s}^2$ ◀

(*b*) $\mathbf{a}_D = [(5)(-1080) \; \beta] + [1125 \text{ in./s}^2 \; \beta]$

$= [5400 \text{ in./s}^2 \; \beta] + [1125 \text{ in./s}^2 \; \beta]$



$$\tan\gamma = \frac{1125}{5400}$$

$$\gamma = 11.77°$$

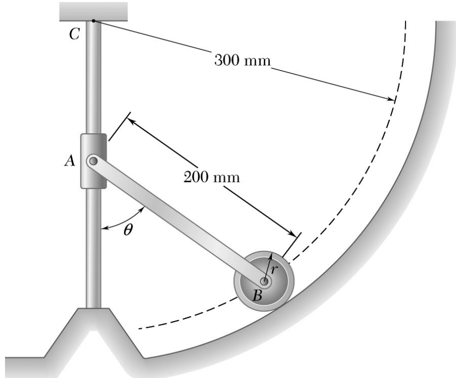
$$a_D = \sqrt{5400^2 + 1125^2}$$

$$= 5516 \text{ in./s}^2$$

$$= 460 \text{ ft/s}^2$$

$$90° - \beta + \gamma = 64.9°$$

$\mathbf{a}_D = 460 \text{ ft/s}^2 \; 64.9°$ ◀

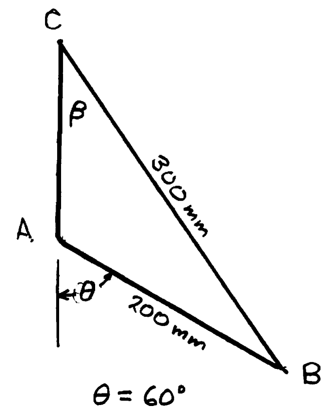


PROBLEM 15.254

Rod AB is attached to a collar at A and is fitted with a wheel at B that has a radius r = 15 mm. Knowing that when $\theta = 60°$ the colar has velocity of 250 mm/s upward and the speed of the collar is decreasing at a rate of 150 mm/s^2, determine (*a*) the angular acceleration of rod AB, (*b*) the angular acceleration of the wheel.

SOLUTION

Geometry. We note that Point B moves on a circle of radius R = 300 mm centered at C. It is useful to use the angle β, angle ACB of the triangle ABC, which indicates the motion of B along this curve. Applying the law of sines to the triangle ABC,



$$\frac{\overrightarrow{AB}}{\sin(\pi-\theta)} = \frac{\overrightarrow{BC}}{\sin\beta} \qquad (\theta = 60°)$$

or $$\sin\beta = \frac{\overrightarrow{AB}}{\overrightarrow{BC}}\sin\theta = \frac{200 \text{ mm}}{300 \text{ mm}}\sin 60°$$

$$\beta = 35.264°$$

With meters as the length unit and the unit vectors defined as

$$\mathbf{i} = 1 \longrightarrow, \quad \mathbf{j} = 1\uparrow, \quad \text{and } \mathbf{k} = 1 \circlearrowleft$$

The relative position vectors are

$$\mathbf{r}_{B/A} = (0.2 \text{ m})(\sin 60°\mathbf{i} - \cos 60°\mathbf{j})$$
$$\mathbf{r}_{B/C} = (0.3 \text{ m})(\sin\beta\mathbf{i} - \cos\beta\mathbf{j})$$

Let Point P be the contact point where the wheel rolls on the fixed surface.

$$\mathbf{r}_{P/B} = (0.015 \text{ m})(\sin\beta\mathbf{i} - \cos\beta\mathbf{j})$$

Velocity analysis.
$$\mathbf{v}_A = 0.250 \text{ m/s}\uparrow \qquad \mathbf{v}_P = 0$$
$$\mathbf{v}_B = v_B \measuredangle\, \beta \qquad \boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k} \qquad \boldsymbol{\omega}_{BP} = \omega_{BP}\mathbf{k}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \boldsymbol{\omega}_{AB}\times\mathbf{r}_{B/A}$$
$$= 0.250\mathbf{j} + (\omega_{AB})\mathbf{k}\times(0.2\sin 60°\mathbf{i} - 0.2\cos 60°\mathbf{j})$$
$$v_B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j}) = 0.25\mathbf{j} + 0.1\omega_{AB}\mathbf{i} + 0.173205\omega_{AB}\mathbf{j}$$

Resolve into components:

i: $v_B \cos\beta = +0.1\omega_{AB}$ (1)

j: $v_B \sin\beta = 0.25 - 0.173205\omega_{AB}$ (2)

Solving the simultaneous Equations (1) and (2),

$$v_B = -0.29873 \text{ m/s} \qquad \omega_{AB} = -2.43913$$

$$\mathbf{v}_B = 0.29873 \text{ m/s} \nearrow \beta \qquad \boldsymbol{\omega}_{AB} = 2.43913 \text{ rad/s} \circlearrowright$$

Since the wheel rolls without slipping, Point P is its instantaneous center.

$$|v_B| = r\omega_{BP} \qquad \omega_{PB} = \frac{0.29873}{0.15} \qquad \boldsymbol{\omega}_{BP} = 1.992 \text{ rad/s} \circlearrowleft$$

Acceleration analysis $\mathbf{a}_A = 150 \text{ mm/s}^2 \downarrow$

$$(\mathbf{a}_P)_t = 0$$

$$\begin{aligned}
\mathbf{a}_B &= \mathbf{a}_A + \mathbf{a}_{B/A} = \mathbf{a}_A + \boldsymbol{\alpha}_{AB}\mathbf{k}\times\mathbf{r}_{B/A} - \omega_{AB}^2\mathbf{r}_{B/A} \\
&= -0.150\mathbf{j} + \alpha_{AB}\mathbf{k}\times(0.2\sin 60°\mathbf{i} - 0.2\cos 60°\mathbf{j}) \\
&\quad - (2.43913)^2(0.2\sin 60°\mathbf{i} - 0.2\cos 60°\mathbf{j}) \\
&= -0.150\mathbf{j} + (0.2\sin 60°)\alpha_{AB}\mathbf{j} + (0.2\cos 60°)\alpha_{AB}\mathbf{i} \\
&\quad - 1.03046\mathbf{i} + 0.59494\mathbf{j} \\
&= (0.2\cos 60°)\alpha_{AB}\mathbf{i} + (0.2\sin 60°)\alpha_{AB}\mathbf{j} - 1.03046\mathbf{i} + 0.44494\mathbf{j}
\end{aligned}$$

Consider the motion of B along its circular path.

$$\begin{aligned}
\mathbf{a}_B &= [(a_B)_t \measuredangle \beta] + \left[\frac{v_B^2}{R} \searrow \beta\right] \\
&= (a_B)_t(\cos\beta\mathbf{i} + \sin\beta\mathbf{j}) + \frac{v_B^2}{R}(-\sin\beta\mathbf{i} + \cos\beta\mathbf{j}) \\
&= (a_B)_t\cos\beta\mathbf{i} + (a_B)_t\sin\beta\mathbf{j} - \frac{(0.29873)^2}{0.3}(-\sin\beta\mathbf{i} + \cos\beta\mathbf{j}) \\
&= (a_B)_t\cos\beta\mathbf{i} + (a_B)_t\sin\beta\mathbf{j} - 0.17174\mathbf{i} + 0.24288\mathbf{j}
\end{aligned}$$

Equate the two expression for $\mathbf{a}_B$ and resolve into components.

i: $(a_B)_t\cos\beta - 0.17174 = (0.2\cos 60°)\alpha_{AB} - 1.03046$ (3)

j: $(a_B)_t\sin\beta + 0.24288 = 0.2\sin 60°\alpha_{AB} + 0.44494$ (4)

Solving Eqs. (3) and (4) simultaneously,

$$(a_B)_t = -2.0187 \text{ m/s}^2 \qquad \alpha_{AB} = -7.8956 \text{ rad/s}^2$$

PROBLEM 15.254 (Continued)

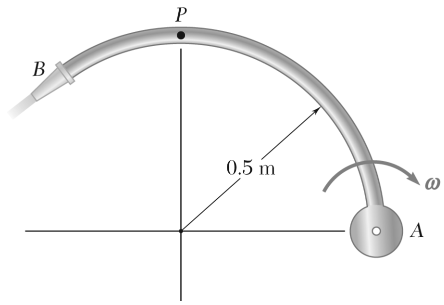
(*a*) *Angular acceleration of AB.* $\alpha_{AB} = 7.90 \text{ rad/s}^2$ ◀

(*b*) *Angular acceleration of the wheel.*

$$(a_P)_t = (a_B)_t + r\alpha_{BP} = 0$$

$$\alpha_{BP} = \frac{(a_B)_t}{r} = -\frac{(-2.0187)}{0.015} = 134.6 \text{ rad/s}^2$$

$\boldsymbol{\alpha}_{BP} = 134.6 \text{ rad/s}^2$ ◀



PROBLEM 15.255

Water flows through a curved pipe AB that rotates with a constant clockwise angular velocity of 90 rpm. If the velocity of the water relative to the pipe is 8 m/s, determine the total acceleration of a particle of water at Point P.

SOLUTION

Let the curved pipe be a rotating frame of reference. Its angular velocity is 90 rpm ↻ or $\boldsymbol{\omega} = 9.4248$ rad/s ↻.

Motion of the frame of reference at Point P′.

$$\mathbf{v}_{P'} = (AP)\omega \measuredangle 45° = (0.5\sqrt{2})(9.4248) \measuredangle 45° = 6.6643 \text{ m/s} \measuredangle 45°$$

$$\mathbf{a}_{P'} = (AP)\omega^2 \searrow 45° = (0.5\sqrt{2})(9.4248)^2 \searrow 45° = 62.81 \text{ m/s}^2 \searrow 45°$$

Motion of water relative to the frame at Point P.

$$\mathbf{v}_{P/F} = 8 \text{ m/s} \leftarrow$$

$$\mathbf{a}_{P/F} = \frac{(v_{P/F})^2}{\rho} \downarrow = \frac{(8 \text{ m/s})^2}{0.5 \text{ m}} \downarrow = 128 \text{ m/s}^2 \downarrow$$

Coriolis acceleration.

$$a_c = 2\omega v_{P/F} = (2)(9.4248)(8) = 150.797 \text{ m/s}^2$$

$$\mathbf{a}_c = 150.797 \text{ m/s}^2 \uparrow$$

Acceleration of water at Point P.

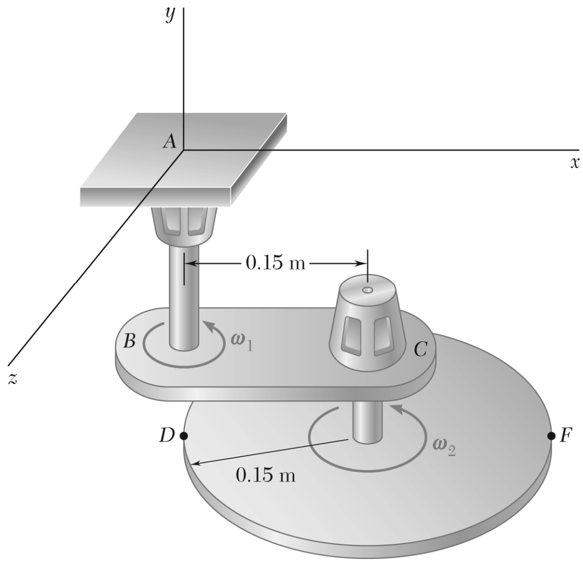
$$\mathbf{a}_P = \mathbf{a}_{P'} + \mathbf{a}_{P/F} + \mathbf{a}_c$$

$$\mathbf{a}_P = [62.81 \searrow 45°] + [128 \downarrow] + [150.797 \uparrow]$$

$$= [44.413 \text{ m/s}^2 \rightarrow] + [21.616 \text{ m/s}^2 \downarrow]$$

$$\mathbf{a}_P = 49.4 \text{ m/s}^2 \searrow 26.0° \blacktriangleleft$$

PROBLEM 15.256



A disk of 0.15-m radius rotates at the constant rate ω_2 with respect to plate BC, which itself rotates at the constant rate ω_1 about the y axis. Knowing that $\omega_1 = \omega_2 = 3$ rad/s, determine, for the position shown, the velocity and acceleration (*a*) of Point D, (*b*) of Point F.

SOLUTION

Frame *AXYZ* is fixed.

Moving frame, *Exyz*, rotates about $\overline{y}$ axis at

$$\mathbf{\Omega} = \omega_1 \mathbf{j} = (3 \text{ rad/s})\mathbf{j}$$

(*a*) *Point D*:

$$\boldsymbol{\omega}_2 = \omega_2 \mathbf{j} = (3 \text{ rad/s})\mathbf{j}$$

$$\mathbf{r}_{D/A} = 0$$

$$\mathbf{r}_{D/E} = -(0.15 \text{ m})\mathbf{i}$$

$$\mathbf{v}_{D'} = \mathbf{\Omega} \times \mathbf{r}_{D/A} = 0$$

$$\mathbf{v}_{D/F} = \boldsymbol{\omega}_A \times \mathbf{r}_{D/E}$$

$$= (3 \text{ rad/s})\mathbf{j} \times (-0.15 \text{ m})\mathbf{i}$$

$$= (0.45 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_D = \mathbf{v}_{D'} + \mathbf{v}_{D/F}$$

$\mathbf{v}_D = (0.45 \text{ m/s})\mathbf{k}$ ◀

$$\mathbf{a}_{D'} = \mathbf{\Omega} \times \mathbf{v}_{D'} = 0$$

$$\mathbf{a}_{D/F} = \boldsymbol{\omega}_1 \times \mathbf{v}_{D/F}$$

$$= (3 \text{ rad/s})\mathbf{j} \times (0.45 \text{ m/s})\mathbf{k}$$

$$= (1.35 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_c = 2\mathbf{\Omega} \times \mathbf{v}_{D/F}$$

$$= 2(3 \text{ rad/s})\mathbf{j} \times (0.45 \text{ m/s})\mathbf{k} = (2.70 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_D = \mathbf{a}_{D'} + \mathbf{a}_{D/F} + \mathbf{a}_c$$

$$= 0 + (1.35 \text{ m/s}^2)\mathbf{i} + (2.70 \text{ m/s}^2)\mathbf{i}$$

$\mathbf{a}_D = (4.05 \text{ m/s}^2)\mathbf{i}$ ◀

(*b*) *Point F*:

$$\boldsymbol{\omega}_2 = \omega_2 \mathbf{j} = (3 \text{ rad/s})\mathbf{j}$$

$$\mathbf{r}_{F/A} = (0.3 \text{ m})\mathbf{i};$$

$$\mathbf{r}_{F/E} = (0.15 \text{ m})\mathbf{i}$$

$$\mathbf{v}_{F'} = \boldsymbol{\Omega} \times \mathbf{r}_{F/A}$$
$$= (3 \text{ rad/s})\mathbf{j} \times (0.3 \text{ m})\mathbf{i}$$
$$= -(0.9 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_{F/F} = \boldsymbol{\omega}_2 \times \mathbf{r}_{F/E}$$
$$= (3 \text{ rad/s})\mathbf{j} \times (0.15 \text{ m})\mathbf{i}$$
$$= -(0.45 \text{ m/s})\mathbf{k}$$

$$\mathbf{v}_F = \mathbf{v}_{F'} + \mathbf{v}_{F/F}$$
$$= -(0.9 \text{ m/s})\mathbf{k} - (0.45 \text{ m/s})\mathbf{k}$$

$\mathbf{v}_F = -(1.35 \text{ m/s})\mathbf{k}$ ◀

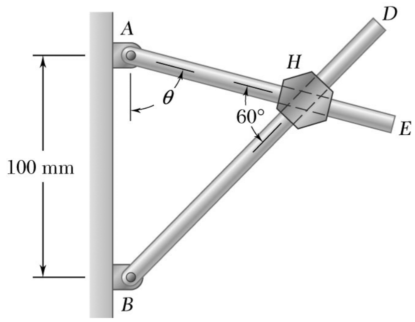
$$\mathbf{a}_{F'} = \boldsymbol{\Omega} \times \mathbf{v}_{F'}$$
$$= (3 \text{ rad/s})\mathbf{j} \times (-0.9 \text{ m/s})\mathbf{k}$$
$$= -(2.7 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_{F/F} = \boldsymbol{\omega}_2 \times \mathbf{v}_{F/F}$$
$$= (3 \text{ rad/s})\mathbf{j} \times (-0.45 \text{ m/s})\mathbf{k}$$
$$= -(1.35 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_c = 2\boldsymbol{\Omega} \times \mathbf{v}_{F/F}$$
$$= 2(3 \text{ rad/s})\mathbf{j} \times (-0.45 \text{ m/s})\mathbf{k}$$
$$= -(2.7 \text{ m/s}^2)\mathbf{i}$$

$$\mathbf{a}_F = \mathbf{a}_{F'} + \mathbf{a}_{F/F} + \mathbf{a}_c$$
$$= -(2.7 \text{ m/s}^2)\mathbf{i} - (1.35 \text{ m/s}^2)\mathbf{i} - (2.7 \text{ m/s}^2)\mathbf{i}$$

$\mathbf{a}_F = -(6.75 \text{ m/s}^2)\mathbf{i}$ ◀



PROBLEM 15.257

Two rods AE and BD pass through holes drilled into a hexagonal block. (The holes are drilled in different planes so that the rods will not touch each other.) Knowing that rod AE has an angular velocity of 20 rad/s clockwise and an angular acceleration of 4 rad/s^2 counterclockwise when $\theta = 90°$, determine, (*a*) the relative velocity of the block with respect to each rod, (*b*) the relative acceleration of the block with respect to each rod.

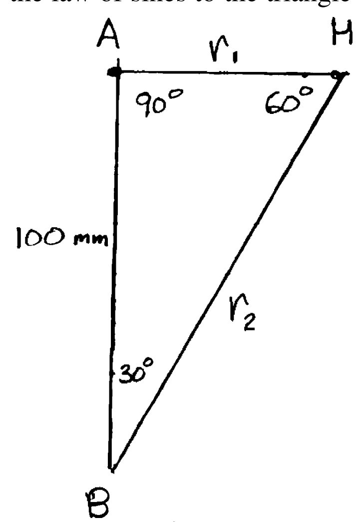
SOLUTION

Geometry: When $\theta = 90°$, Point H is located as shown in the sketch. Apply the law of sines to the triangle ABH.

$$\frac{100 \text{ mm}}{\sin 60°} = \frac{r_1}{\sin 30°} = \frac{r_2}{\sin 90°}$$

$$r_1 = 57.735 \text{ mm}$$

$$r_2 = 115.470 \text{ mm}$$



The angle at H remains at 60° so that rods AE and BD have a common angular velocity $\boldsymbol{\omega} = \omega$ ↺ and a common angular acceleration $\boldsymbol{\alpha} = \alpha$ ↺, where

$$\omega = -20 \text{ rad/s}$$

and $$\alpha = 4 \text{ rad/s}$$

Consider the double slider H as a particle sliding along the rotating rod AH with relative velocity $u_1 \longrightarrow$ and relative acceleration $\dot{u}_1 \longrightarrow$.

Let H' be the point on rod AE that coincides with H.

$$\mathbf{v}_{H'} = [r_1\omega \uparrow] = [(57.735 \text{ mm})(-20 \text{ rad/s}) \uparrow] = [1154.7 \text{ mm/s} \downarrow]$$

$$\mathbf{a}_{H'} = [r_1\alpha \uparrow] + [r_1\omega^2 \longleftarrow]$$

$$= [(57.735 \text{ mm})(4 \text{ rad/s}^2) \uparrow] + [(57.735 \text{ mm})(20 \text{ rad/s})^2 \longleftarrow]$$

$$= [230.9 \text{ mm/s}^2 \uparrow] + [23094 \text{ mm/s}^2 \longleftarrow]$$

PROBLEM 15.257 (Continued)

The corresponding Coriolis acceleration is

$$\mathbf{a}_1 = [2\omega u_1 \uparrow] = [(2)(-20)u_1 \uparrow] = 40u_1 \downarrow$$

$$\mathbf{v}_H = \mathbf{v}_{H'} + u_1 \longrightarrow = [1154.7 \text{ mm} \downarrow] + [u_1 \longrightarrow] \quad (1)$$

$$\mathbf{a}_H = a_{H'} + [\dot{u}_1 \longrightarrow] + [40u_1 \downarrow]$$
$$= [230.9 \text{ mm/s} \uparrow] + [23094 \text{ mm/s}^2 \longleftarrow] + [\dot{u}_1 \longrightarrow] + [40u_1 \downarrow] \quad (2)$$

Now consider the double slider H as a particle sliding along the rotating rod BD with relative velocity $u_2 \nearrow 60^\circ$ and relative acceleration $\dot{u}_2 \nearrow 60^\circ$.

Let H'' be the point on rod BD that coincides with H.

$$\mathbf{v}_{H''} = r_2\omega \searrow 30^\circ = (115.47 \text{ mm})(-20 \text{ rad/s}) \searrow 30^\circ = 2309.4 \text{ mm/s} \searrow 30^\circ$$

$$\mathbf{a}_{H''} = r_2\alpha \searrow 30^\circ + r_2\omega^2 \swarrow 60^\circ$$
$$= (115.47 \text{ mm})(4 \text{ rad/s}^2) \searrow 30^\circ + (115.47 \text{ mm})(20 \text{ rad/s})^2 \swarrow 60^\circ$$
$$= 461.9 \text{ mm/s}^2 \searrow 30^\circ + 46188 \text{ mm/s}^2 \swarrow 60^\circ$$

The corresponding Coriolis acceleration is

$$\mathbf{a}_2 = 2\omega u_2 \searrow 30^\circ = (2)(-20 \text{ rad/s})u_2 \searrow 60^\circ = 40u_2 \searrow 30^\circ$$

$$\mathbf{v}_H = \mathbf{v}_{H''} + u_2 \nearrow 60^\circ = 2309.4 \text{ mm/s} \searrow 30^\circ + u_2 \nearrow 60^\circ \quad (3)$$

$$\mathbf{a}_H = \mathbf{a}_{H''} + \dot{u}_2 \nearrow 60^\circ + 40u_2 \searrow 30^\circ$$
$$= [(461.9 \text{ mm/s}^2) \searrow 30^\circ] + [(46188 \text{ mm/s}^2) \swarrow 60^\circ] + [\dot{u}_2 \nearrow 60^\circ] + [40u_2 \searrow 30^\circ] \quad (4)$$

Equate expression (1) and (3) for $\mathbf{v}_H$ and resolve into components.

$+\uparrow$: $\quad 1154.7 \text{ mm/s} = -(2309.4 \text{ mm/s})\sin 30^\circ + u_2 \sin 60^\circ$

$$u_2 = \frac{2309.4 \sin 30^\circ - 1154.7}{\sin 60^\circ} \qquad u_2 = 0$$

$\xrightarrow{+}$: $\quad u_1 = 2309.4 \cos 30^\circ + 0 \qquad u_1 = 2000 \text{ mm/s}$

Substitute the values for u_1 and u_2 into the Coriolis acceleration terms, and equate expressions (2) and (4) for $\mathbf{a}_H$, and resolve into components.

$+\uparrow$: $\quad 230.9 \text{ mm/s}^2 - (40 \text{ rad/s})(2000 \text{ mm/s})$

$$= (461.9 \text{ mm/s}^2)\sin 30^\circ + (46188 \text{ mm/s}^2)\sin 60^\circ$$
$$+ \dot{u}_2 \sin 60^\circ - (40)(0)\sin 30^\circ$$

$$\dot{u}_2 = \frac{230.9 - 80000 - 230.9 + 40000}{\sin 60^\circ} \qquad \dot{u}_2 = -46188 \text{ mm/s}^2$$

PROBLEM 15.257 (Continued)

$\xrightarrow{+}$: $$\dot{u}_1 - 23094 \text{ mm/s}^2 = -(461.4 \text{ mm/s}^2)\cos 30° - (46188 \text{ mm/s}^2)\cos 60° - (46188 \text{ mm/s}^2)\cos 60° + 0$$

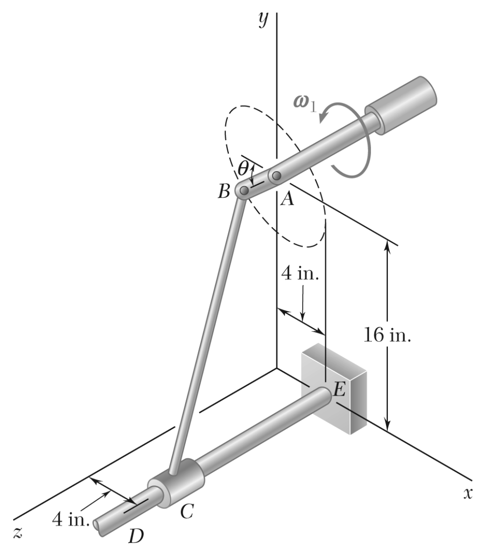
$$\dot{u}_1 = -23494 \text{ mm/s}^2$$

(*a*) *Relative velocities*: AE: $u_1 \longrightarrow = 2.00 \text{ m/s} \longrightarrow$ ◀

BD: $u_2 \measuredangle 60° = 0$ ◀

(*b*) *Relative accelerations*: AE: $\dot{u}_1 = 23.5 \text{ m/s}^2 \longleftarrow$ ◀

BD: $\dot{u}_2 \measuredangle 60° = 46.2 \text{ m/s}^2 \measuredangle 60°$ ◀



PROBLEM 15.258

Rod BC of length 24 in. is connected by ball-and-socket joints to a rotating arm AB and to a collar C that slides on the fixed rod DE. Knowing that the length of arm AB is 4 in. and that it rotates at the constant rate $\omega_1 = 10$ rad/s, determine the velocity of collar C when $\theta = 0$.

SOLUTION

Geometry.

$$l_{BC} = 24 \text{ in.}$$

$$\mathbf{r}_{C/B} = (8 \text{ in.})\mathbf{i} - (16 \text{ in.})\mathbf{j} + z_{C/B}\mathbf{k}$$

$$24^2 = 8^2 + 16^2 + z_{C/B}^2$$

$$z_{C/B} = 16 \text{ in.}$$

$$\mathbf{r}_{C/B} = (8 \text{ in.})\mathbf{i} - (16 \text{ in.})\mathbf{j} + (16 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{B/A} = -(4 \text{ in.})\mathbf{i}$$

Velocity at B.

$$\mathbf{v}_B = \omega_1\mathbf{k} \times \mathbf{r}_{B/A}$$

$$= 10\mathbf{k} \times (-4\mathbf{i})$$

$$= -(40 \text{ in./s})\mathbf{j}$$

Velocity of collar C.

$$\mathbf{v}_C = v_C\mathbf{k}$$

$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

where

$$\mathbf{v}_{C/B} = \boldsymbol{\omega}_{BC} \times \mathbf{r}_{B/C}$$

Noting that $\mathbf{v}_{C/B}$ is perpendicular to $\mathbf{r}_{C/B}$, we get $\mathbf{r}_{C/B} \cdot \mathbf{v}_{C/B} = 0$

Forming $\mathbf{r}_{C/B} \cdot \mathbf{v}_C$, we get

$$\mathbf{r}_{C/B} \cdot \mathbf{v}_C = \mathbf{r}_{C/B} \cdot (\mathbf{v}_B + \mathbf{v}_{C/B})$$

$$= \mathbf{r}_{C/B} \cdot \mathbf{v}_B + \mathbf{r}_{C/B} \cdot \mathbf{v}_{C/B}$$

or

$$\mathbf{r}_{C/B} \cdot \mathbf{v}_C = \mathbf{r}_{C/B}\mathbf{v}_B \qquad (1)$$

From Eq. (1)

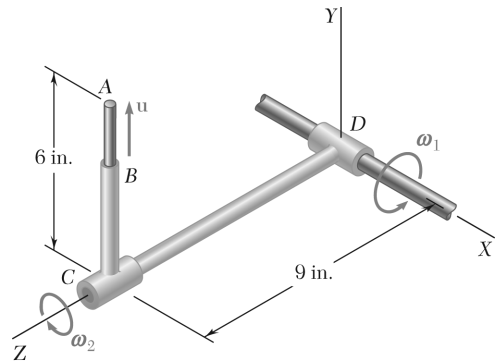
$$(8\mathbf{i} - 16\mathbf{j} + 16\mathbf{k}) \cdot (v_C\mathbf{k}) = (8\mathbf{i} - 16\mathbf{j} + 16\mathbf{k}) \cdot (-40\mathbf{j})$$

$$16v_C = (-16)(-40)$$

or

$$v_C = 40 \text{ in./s}$$

$$\mathbf{v}_C = (40.0 \text{ in./s})\mathbf{k} \blacktriangleleft$$

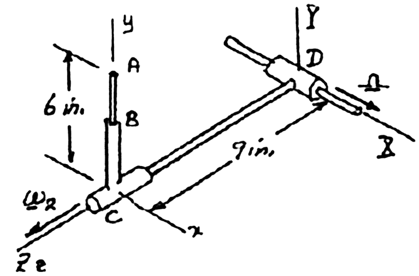


PROBLEM 15.259

In the position shown, the thin rod moves at a constant speed $u = 3$ in./s out of the tube BC. At the same time, tube BC rotates at the constant rate $\omega_2 = 1.5$ rad/s with respect to arm CD. Knowing that the entire assembly rotates about the X axis at the constant rate $\omega_1 = 1.2$ rad/s, determine the velocity and acceleration of end A of the rod.

SOLUTION

Geometry. $\mathbf{r}_A = (6 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$



Method 1

Let the rigid body DCB be a rotating frame of reference.

Its angular velocity is
$$\mathbf{\Omega} = \omega_1\mathbf{i} + \omega_2\mathbf{k}$$
$$= (1.2 \text{ rad/s})\mathbf{i} - (1.5 \text{ rad/s})\mathbf{k}.$$

Its angular acceleration is
$$\boldsymbol{\alpha} = \omega_1\mathbf{i} \times \omega_2\mathbf{k}$$
$$= -\omega_1\omega_2\mathbf{j}$$
$$= (1.8 \text{ rad/s}^2)\mathbf{j}.$$

Motion of the coinciding Point A' in the frame.

$$\mathbf{v}_{A'} = \mathbf{\Omega} \times \mathbf{r}_A$$
$$= (1.2\mathbf{i} - 1.5\mathbf{k}) \times (6\mathbf{j} + 9\mathbf{k})$$
$$= 7.2\mathbf{k} - 10.8\mathbf{j} + 9\mathbf{i}$$
$$= (9 \text{ in./s})\mathbf{i} - (10.8 \text{ in./s})\mathbf{j} + (7.2 \text{ in./s})\mathbf{k}$$

$$\mathbf{a}_{A'} = \boldsymbol{\alpha} \times \mathbf{r}_A + \mathbf{\Omega} \times \mathbf{v}_{A'}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.8 & 0 \\ 0 & 6 & 9 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 0 & -1.5 \\ 9 & -10.8 & 7.2 \end{vmatrix}$$
$$= 16.2\mathbf{i} - 16.2\mathbf{i} - 22.14\mathbf{j} - 12.96\mathbf{k}$$
$$= -(22.14 \text{ in./s}^2)\mathbf{j} - (12.96 \text{ in./s}^2)\mathbf{k}$$

Motion of Point A relative to the frame.

$$\mathbf{v}_{A/F} = u\mathbf{j} = (3 \text{ in./s})\mathbf{j},$$
$$\mathbf{a}_{A/F} = 0$$

Velocity of Point A.

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$$
$$\mathbf{v}_A = 9\mathbf{i} - 10.8\mathbf{j} + 7.2\mathbf{k} + 3\mathbf{j}$$

$$\mathbf{v}_A = (9.00 \text{ in./s})\mathbf{i} - (7.80 \text{ in./s})\mathbf{j} + (7.20 \text{ in./s})\mathbf{k} \blacktriangleleft$$

Coriolis acceleration.

$$2\boldsymbol{\Omega} \times \mathbf{v}_{A/F} = (2)(1.2\mathbf{i} - 1.5\mathbf{k}) \times 3\mathbf{j}$$
$$= (9 \text{ in./s}^2)\mathbf{i} + (7.2 \text{ in./s}^2)\mathbf{k}$$

Acceleration of Point A.

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + 2\boldsymbol{\Omega} \times \mathbf{v}_{A/F}$$
$$\mathbf{a}_A = -22.14\mathbf{j} - 12.92\mathbf{k} + 9\mathbf{i} + 7.2\mathbf{k}$$

$$\mathbf{a}_A = (9.00 \text{ in./s}^2)\mathbf{i} - (22.1 \text{ in./s}^2)\mathbf{j} - (5.76 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$

Method 2

Let frame *Dxyz*, which at instant shown coincides with *DXYZ*, rotate with an angular velocity $\boldsymbol{\Omega} = \omega_1\mathbf{i} = 1.2\mathbf{i}$ rad/s. Then the motion relative to the frame consists of the rotation of body *DCB* about the *Z* axis with angular velocity $\omega_2\mathbf{k} = -(1.5 \text{ rad/s})\mathbf{k}$ plus the sliding motion $\mathbf{u} = u\mathbf{i} = (3 \text{ in./s})\mathbf{j}$ of the rod *AB* relative to the body *DCB*.

Motion of the coinciding Point A′ in the frame.

$$\mathbf{v}_{A'} = \boldsymbol{\Omega} \times \mathbf{r}_A$$
$$= 1.2\mathbf{i} \times (6\mathbf{j} + 9\mathbf{k})$$
$$= -(10.8 \text{ in./s})\mathbf{j} + (7.2 \text{ in./s})\mathbf{k}$$
$$\mathbf{a}_{A'} = \boldsymbol{\Omega} \times \mathbf{v}_{A'}$$
$$= 1.2\mathbf{i} \times (-10.8\mathbf{j} + 7.2\mathbf{k})$$
$$= -(8.64 \text{ in./s}^2)\mathbf{j} - (12.96 \text{ in./s}^2)\mathbf{k}$$

Motion of Point A relative to the frame.

$$\mathbf{v}_{A/F} = \omega_2\mathbf{k} \times \mathbf{r}_A + u\mathbf{j}$$
$$= (-1.5\mathbf{k}) \times (6\mathbf{j} + 9\mathbf{k}) + 3\mathbf{j}$$
$$= (9 \text{ in./s})\mathbf{i} + (3 \text{ in./s})\mathbf{j}$$
$$\mathbf{a}_{A/F} = \alpha_2\mathbf{k} \times \mathbf{r}_A + \omega_2\mathbf{k} \times (\omega_2\mathbf{k} \times \mathbf{r}_A) + \dot{u}\mathbf{j} + 2\omega_2\mathbf{k} \times (u\mathbf{j})$$
$$= 0 + (-1.5\mathbf{k}) \times (9\mathbf{i}) + 0 + (2)(-1.5\mathbf{k}) \times (3\mathbf{j})$$
$$= -13.5\mathbf{j} + 9\mathbf{i}$$
$$= (9 \text{ in./s}^2)\mathbf{i} - (13.5 \text{ in./s}^2)\mathbf{j}$$

Velocity of Point A.

$$\mathbf{v}_A = \mathbf{v}_{A'} + \mathbf{v}_{A/F}$$
$$\mathbf{v}_A = -10.8\mathbf{j} + 7.2\mathbf{k} + 9\mathbf{i} + 3\mathbf{j}$$

$$\mathbf{v}_A = (9.00 \text{ in./s})\mathbf{i} - (7.80 \text{ in./s})\mathbf{j} + (7.20 \text{ in./s})\mathbf{k} \blacktriangleleft$$

PROBLEM 15.259 (Continued)

Coriolis acceleration.

$$2\mathbf{\Omega}\times\mathbf{v}_{A/F} = (2)(1.2\mathbf{i})\times(-9\mathbf{i}+3\mathbf{j})$$

$$= (7.2 \text{ in./s}^2)\mathbf{k}$$

Acceleration of Point A.

$$\mathbf{a}_A = \mathbf{a}_{A'} + \mathbf{a}_{A/F} + 2\mathbf{\Omega}\times\mathbf{v}_{A/F}$$

$$\mathbf{a}_A = -8.64\mathbf{j} - 12.96\mathbf{k} + 9\mathbf{i} - 13.5\mathbf{j} + 7.2\mathbf{k}$$

$$\mathbf{a}_A = (9.00 \text{ in./s}^2)\mathbf{i} - (22.1 \text{ in./s}^2)\mathbf{j} - (5.76 \text{ in./s}^2)\mathbf{k} \blacktriangleleft$$