

# Motion in 2 and 3 Dimensions

## • Position and Displacement

If the position vector changes from  $\vec{r}_1$  to  $\vec{r}_2$  during a certain time interval Then the displacement  $\Delta\vec{r}$  during that time interval is:-

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta\vec{r} = \underset{\Delta x}{(x_2 - x_1)}\hat{i} + \underset{\Delta y}{(y_2 - y_1)}\hat{j} + \underset{\Delta z}{(z_2 - z_1)}\hat{k}$$

In 2D: When an object reaches Max position in one direction Then its velocity in this direction  $v=0$  component

## • Average velocity & Instantaneous velocity

If a particle moves through a displacement  $\Delta\vec{r}$  in a time interval  $\Delta t$  then its average velocity  $\vec{V}_{avg}$  is

$$\vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{V}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$\vec{V}$  is the same direction as  $\vec{r}$

$$V_{Instant} = \frac{d\vec{r}}{dt}$$

$V_{avg}$  takes the tangents (at  $\vec{r}$ ) direction

## • Average Acceleration & instantaneous Acceleration \* if the velocity

$$a_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{Instant} = \frac{d\vec{v}}{dt}$$

If velocity changes in Magnitude or direction or Both The particle must have  $\vec{a}$

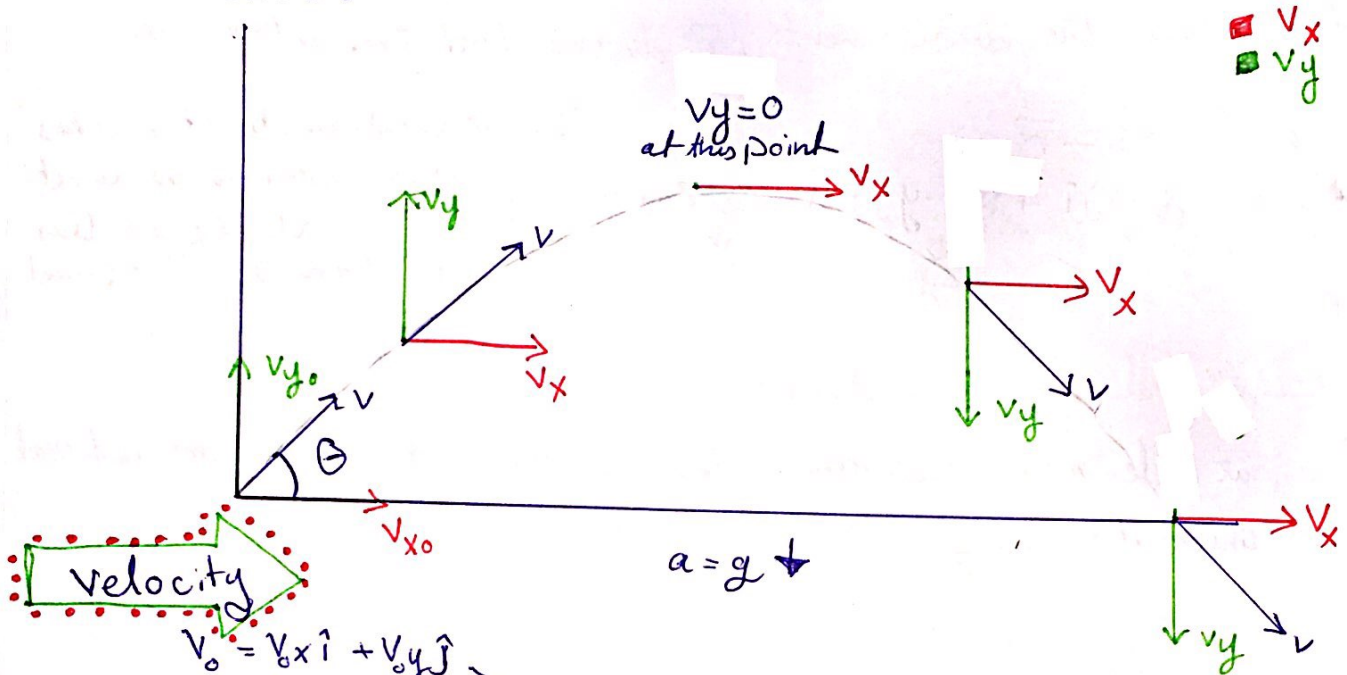
changes in direction or magnitude  
→ Particle must have an acceleration

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2D motion

Horizontal and vertical motions are independent of each other



Velocity

$$\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$$

$$v_{0x} = v_0 \cos \theta$$

Constant

$$v_{0y} = v_0 \sin \theta$$

Variable

$$V = \sqrt{v_x^2 + v_y^2}$$

$$v_x = v_{0x}$$

$$v_y = v_{0y} - gt \quad \text{or} \quad v_y^2 = v_{0y}^2 - 2gy(y - y_0)$$

$$v_{0y} = v_0 \sin \theta$$

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# Displacement

$$x = v_0 t \cos \theta$$

$$y = v_0 t \sin \theta - \frac{1}{2} g t^2$$

$$\Delta r = \sqrt{x^2 + y^2}$$

## • the Horizontal Range

$$R = v_{0x} t_p \quad \dots \textcircled{1}$$

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2$$

$\Delta y = 0$  لأنه إرتداد  
من مستوى  
للمستوى نفسه

$$0 = v_{0y} t - \frac{1}{2} g t^2$$

توجد في ① و ②

$$t_p = 2 v_{0y} / g$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$R_{\max} = v_0^2 / g \quad \text{when } \theta = 45^\circ$$

## • the height

$$h = \frac{v_0^2 \sin^2(\theta)}{2g}$$

## • Relation between R and h

$$\frac{h}{R} = \frac{\tan \theta}{4} \Rightarrow h = \frac{R \tan \theta}{4}$$

## • The equation of the path

$$y = \tan \theta \cdot x - \frac{g \cdot x^2}{2 v_0^2 \cos^2 \theta}$$

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\* through the path  $a_x = 0$   
 $a_y = g$

\* path is parabolic

\* air has a large effect on R  
on the path in general

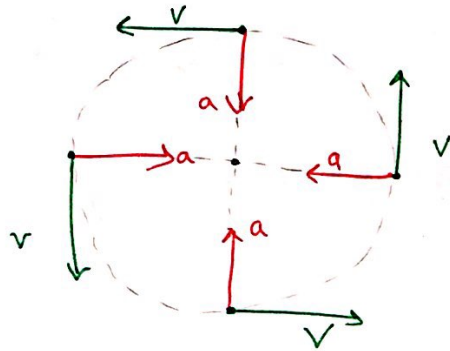
تَقْطِعُ عَلَى تَيْسٍ لِّلْمَسِيِّ: Range

## Uniform Circular Motion

- a particle travelling around a circle in a constant speed
- acceleration: centripetal (uniform circular motion)

$$\vec{a}_r = \frac{v^2}{r} \quad \begin{array}{l} v \rightarrow \text{speed of the particle} \\ r \rightarrow \text{radius} \end{array} \quad \hat{r}: \text{vector}$$

$$T = \frac{2\pi r}{v} \quad \begin{array}{l} T: \text{period} \\ \pi = 3.14 \end{array}$$



$$a_t = 0$$

$a_{\text{tangential}} = \frac{dv}{dt}$  (non-uniform circular motion)

$$a_r = \frac{v^2}{2\pi r}$$

- $v$  is not constant
- so accelerating with a tangential acceleration

$$a_t \neq 0$$

• Net force: محاذي الحركة

$$\sum \vec{F} = m(\vec{a})$$

$$\vec{a} = \vec{a}_t + \vec{a}_c \quad (\text{perpendicular})$$

$$F = |\vec{F}| = |m\vec{a}|$$

$$= m|\vec{a}|$$

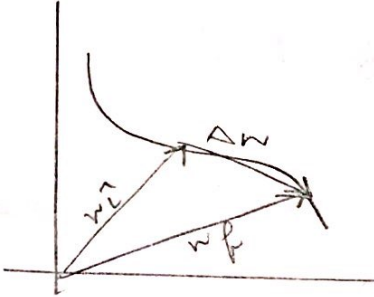
$$= m\sqrt{a_t^2 + a_c^2}$$

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# vectors

## Displacement vector

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$



## velocity vectors

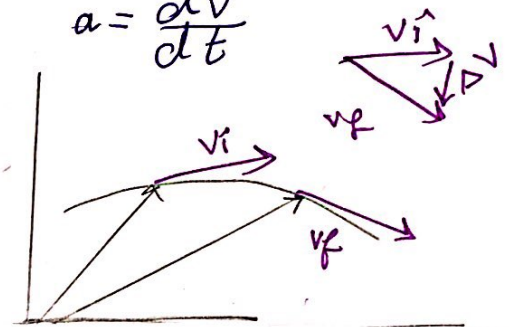
$$\text{avg: } \bar{V} = \frac{\Delta \vec{r}}{\Delta t}$$

$$\text{Ins: } V = \frac{d\vec{r}}{dt}$$

## Acceleration vectors

$$\bar{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$a = \frac{d\vec{v}}{dt}$$



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## Motion with Constant Acceleration

$$r = x\hat{i} + y\hat{j}$$

$$v = v_x\hat{i} + v_y\hat{j}$$

$$v_f = v_{x_f}\hat{i} + v_{y_f}\hat{j} \longrightarrow \underline{v_f = v_i + at}$$

$$v_f = x_{f_i}\hat{i} + y_{f_j}\hat{j} \longrightarrow \underline{v_f^2 = v_i^2 + v_i^2 + 2a_b^2}$$

## • Difference between Motion in 1D, 2D and 3D

[1D]: The Motion is in a straight line

[2D]: The Motion is in a curved path but in a single plane

[3D]: The Motion is throughout the space / not in a plane but in a complete space

Ex: a paper moving freely in the air

Note: • speed is the Magnitude of velocity

• if we have  $\Delta \vec{V} \neq 0$  Then There  
is a  $\left\{ \begin{array}{l} \text{change in speed} \\ \text{and/or} \\ \text{Change in direction} \end{array} \right.$

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