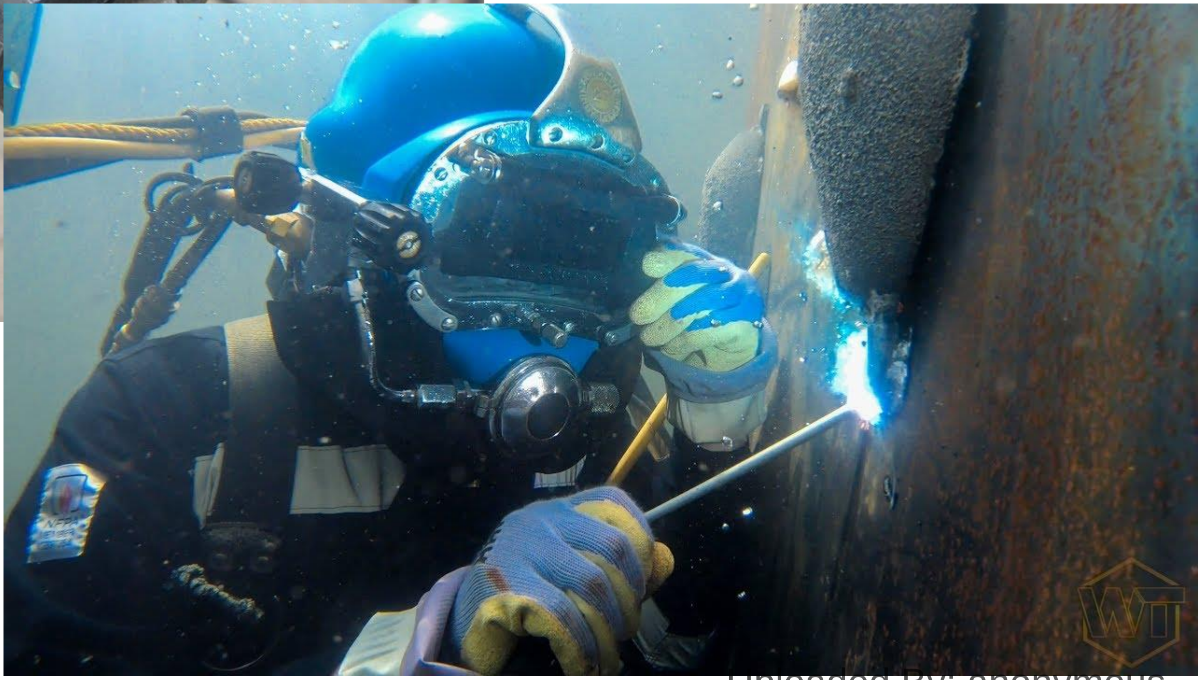
9 Introduction



9 Introduction

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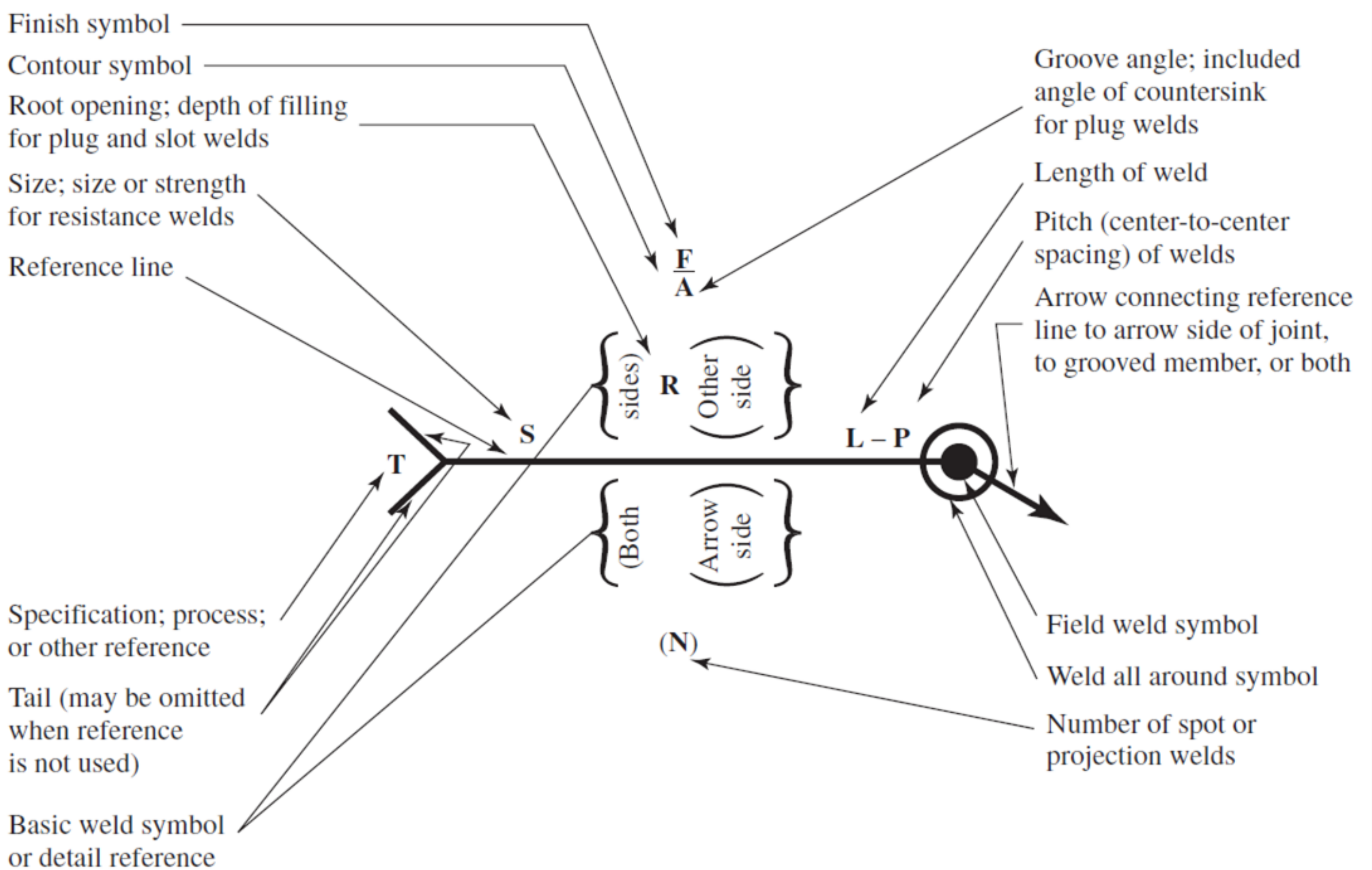
1 UNS No.	2 SAE and/or AISI No.	3 Process-ing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126
G10200	1020	HR	380 (55)	210 (30)	25	50	111
		CD	470 (68)	390 (57)	15	40	131
G10300	1030	HR	470 (68)	260 (37.5)	20	42	137
		CD	520 (76)	440 (64)	12	35	149
G10350	1035	HR	500 (72)	270 (39.5)	18	40	143
		CD	550 (80)	460 (67)	12	35	163
G10400	1040	HR	520 (76)	290 (42)	18	40	149
		CD	590 (85)	490 (71)	12	35	170

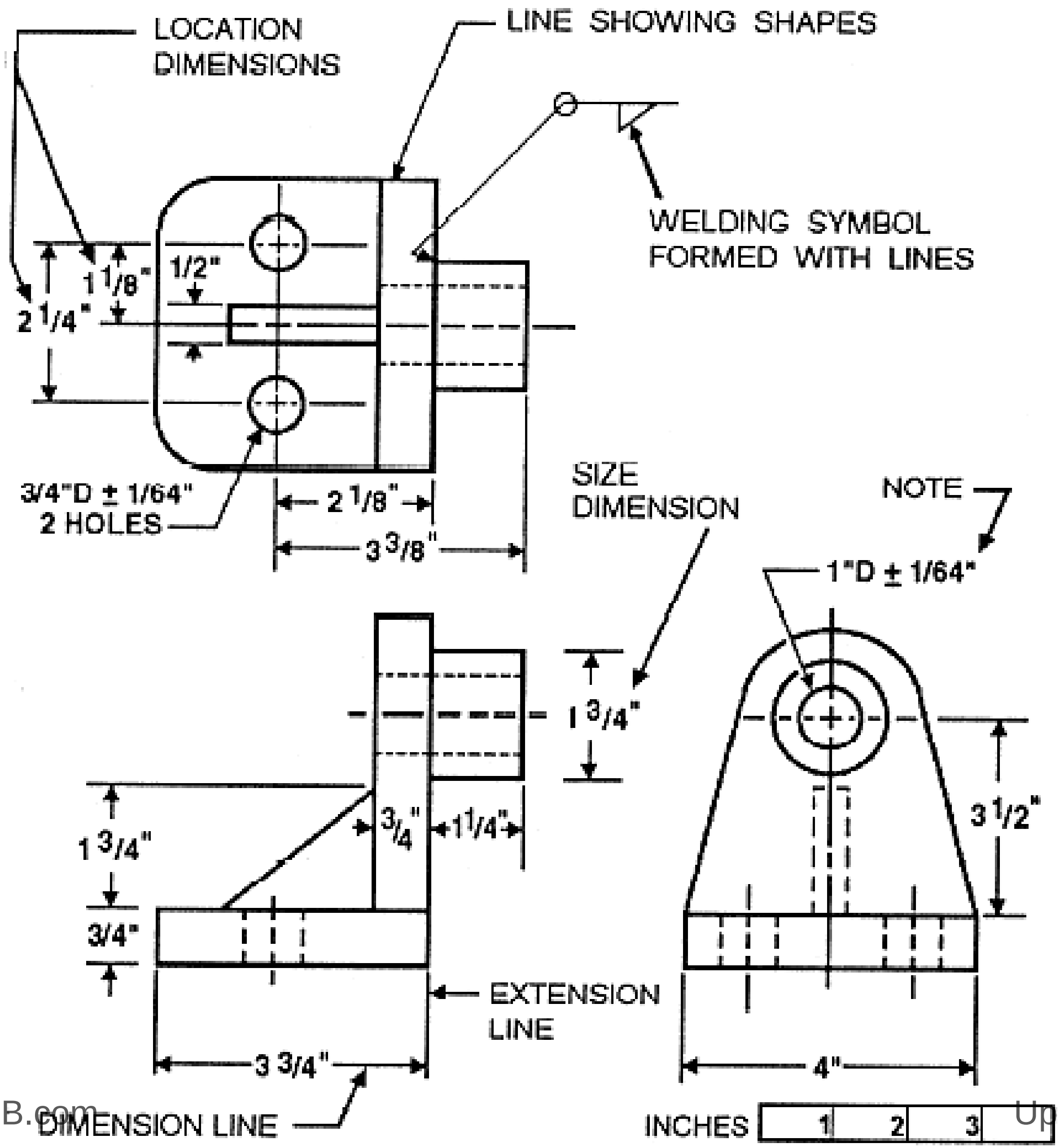
9.1 Welding Symbols

Figure 9–1

The AWS standard welding symbol showing the location of the symbol elements.



9.1 Welding Symbols



9.1 Welding Symbols

Figure 9–2

Arc- and gas-weld symbols.

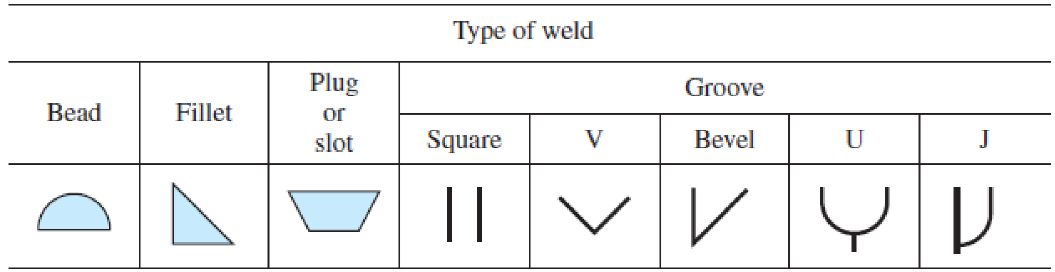


Figure 9–3

Fillet welds. (*a*) The number indicates the leg size; the arrow should point only to one weld when both sides are the same. (*b*) The symbol indicates that the welds are intermittent and staggered 60 mm along on 200-mm centers.

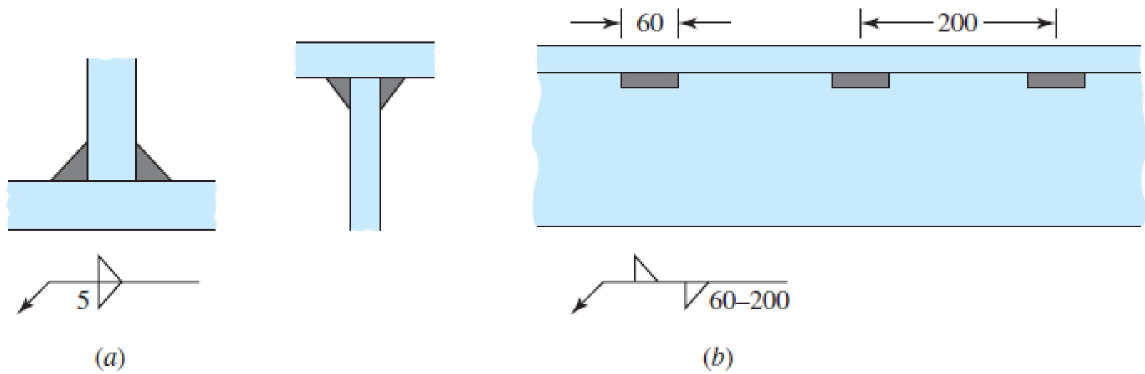
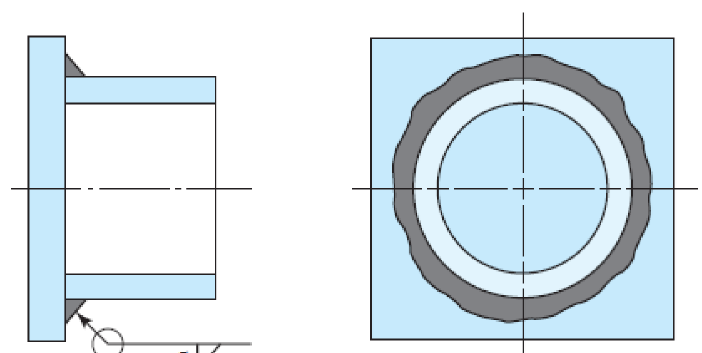


Figure 9–4

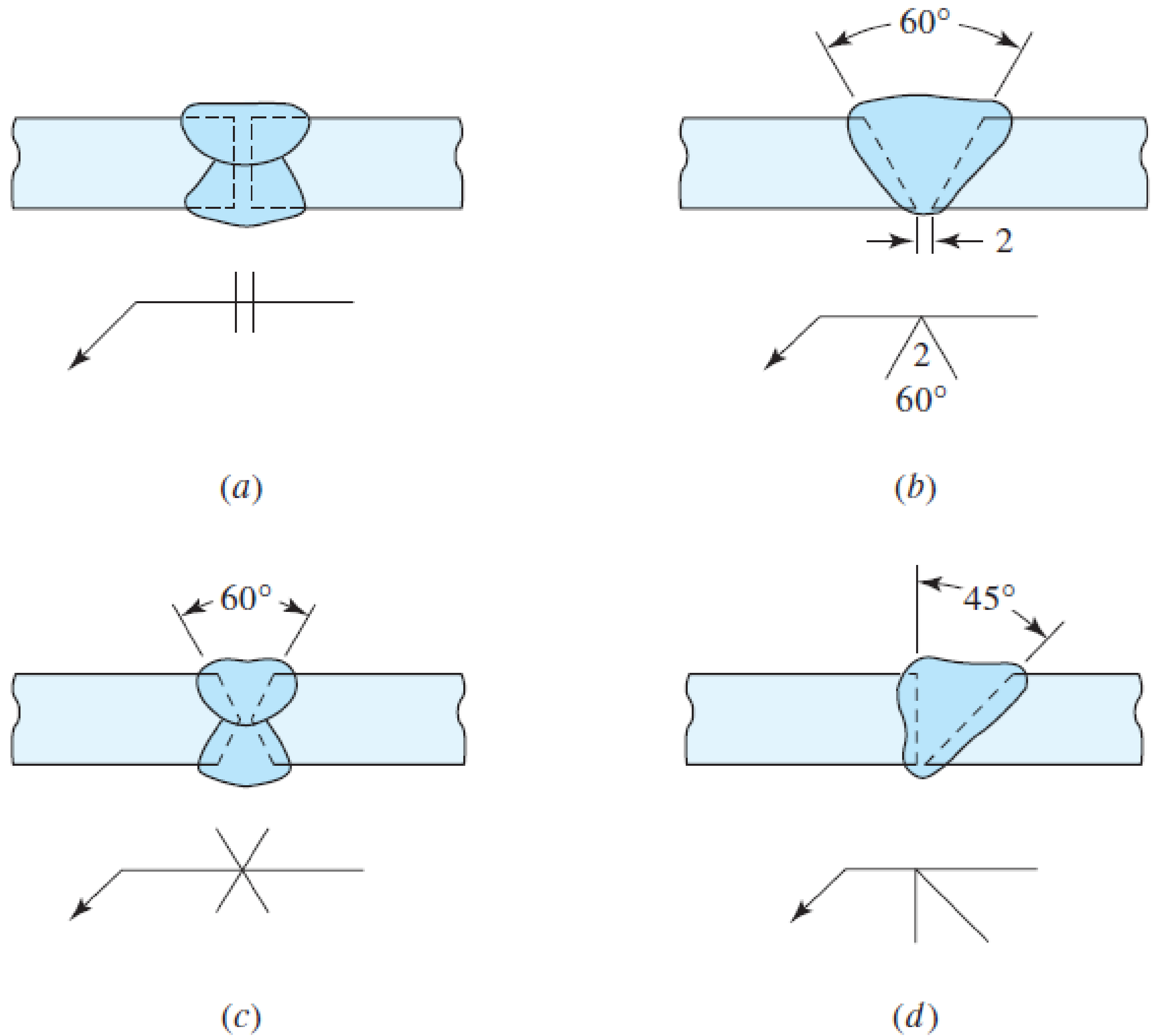
The circle on the weld symbol indicates that the welding is to go all around.



9.1 Welding Symbols

Figure 9–5

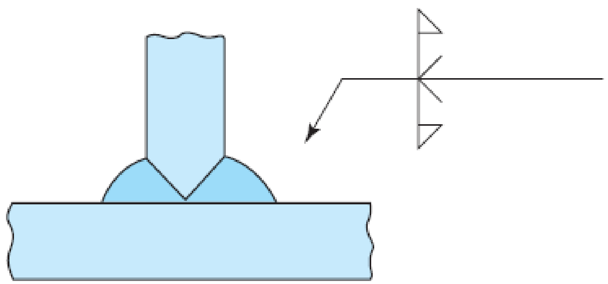
Butt or groove welds: (*a*) square butt-welded on both sides; (*b*) single V with 60° bevel and root opening of 2 mm; (*c*) double V; (*d*) single bevel.

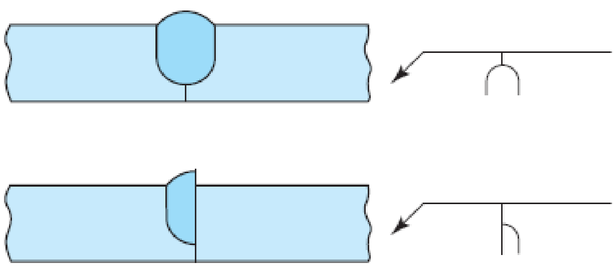


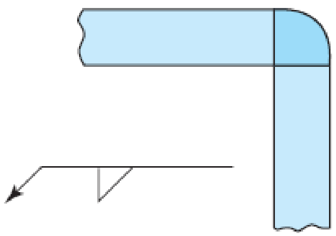
9.1 Welding Symbols

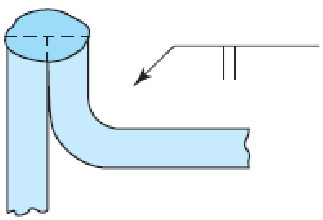
Figure 9–6

Special groove welds: (*a*) T joint for thick plates; (*b*) U and J welds for thick plates; (*c*) corner weld (may also have a bead weld on inside for greater strength but should not be used for heavy loads); (*d*) edge weld for sheet metal and light loads.


(*a*)


(*b*)


(*c*)


(*d*)

9.1 Welding Symbols

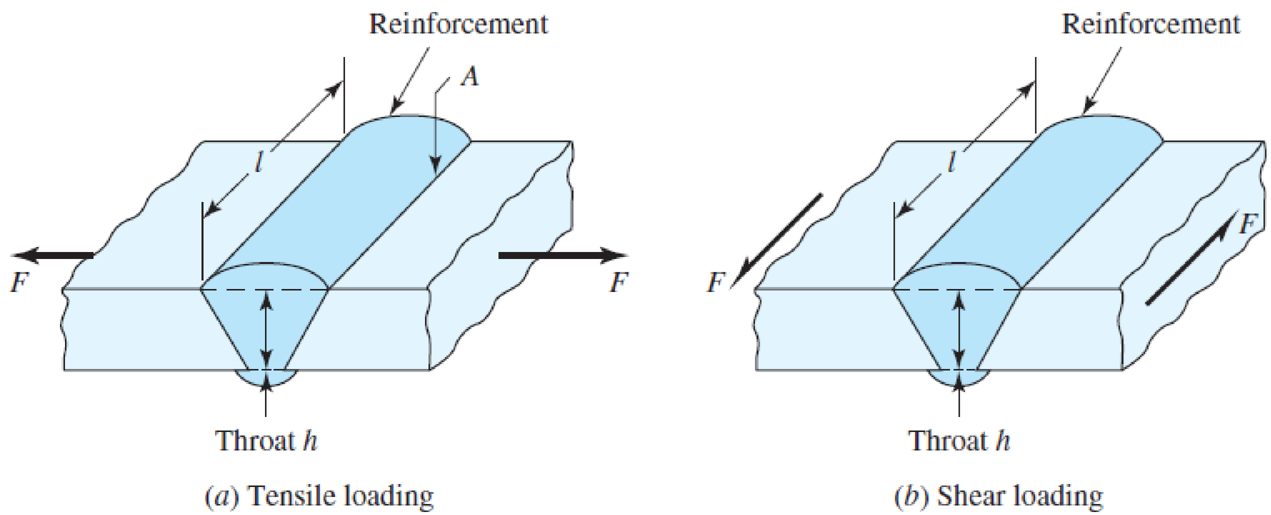
Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0

Figure 9–7

A typical butt joint.



$$\sigma = \frac{F}{hl}$$

$$\tau = \frac{F}{hl}$$

9.1 Welding Symbols

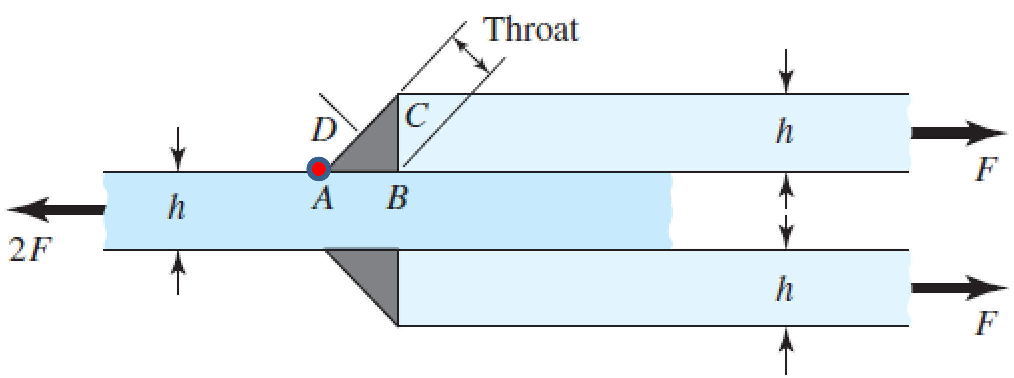
Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0

Figure 9–8

A transverse fillet weld.



9.1 Welding Symbols

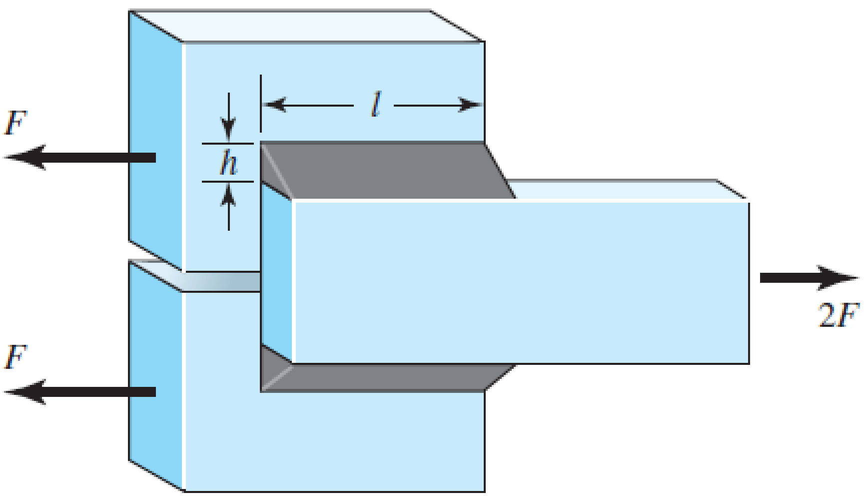
Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0

Figure 9–11

Parallel fillet welds.

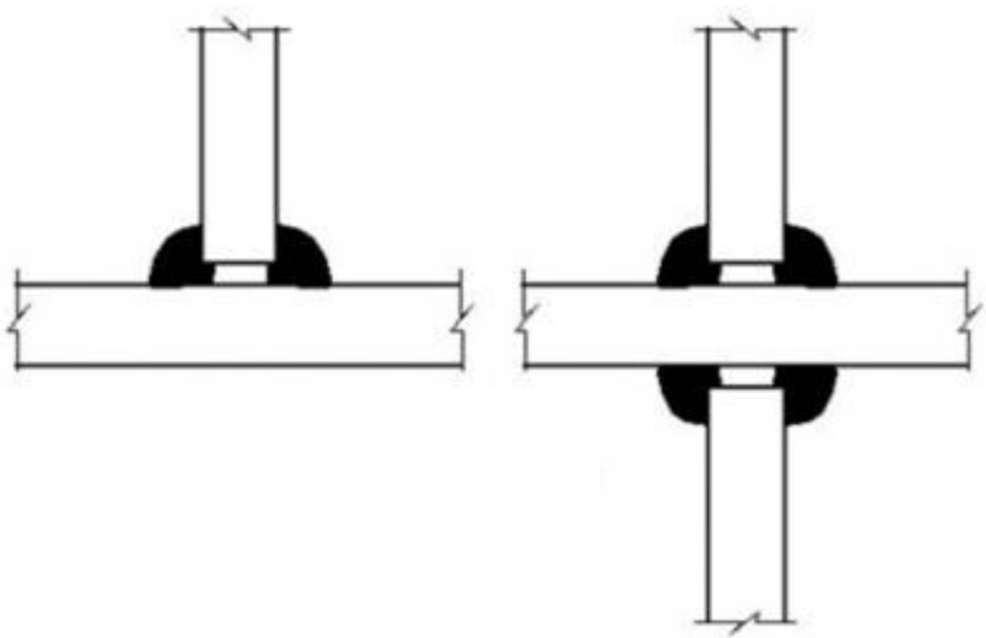


9.1 Welding Symbols

Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0



9.2 Butt and Fillet Welds

Figure 9–8

A transverse fillet weld.

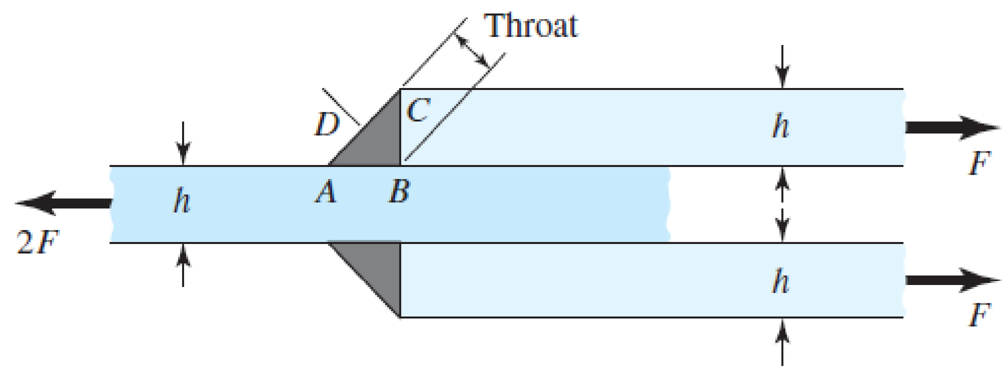
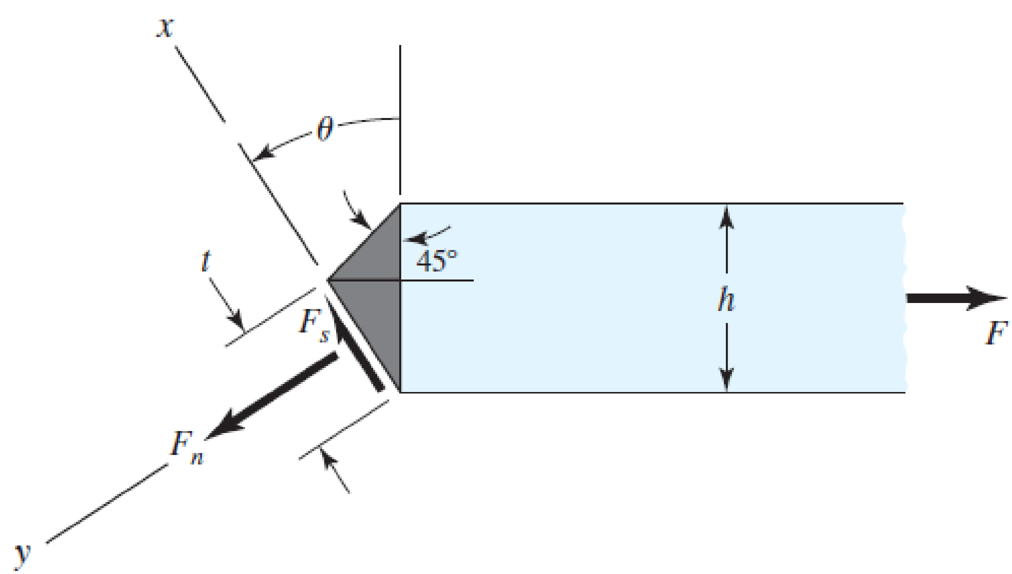


Figure 9–9

Free body from Fig. 9–8.



Maximum shear stress *is* at the minimum throat area

$$\tau = \frac{F}{0.707hl}$$

9.3 Stresses in Welded Joints in Torsion (9.4 in Bending)

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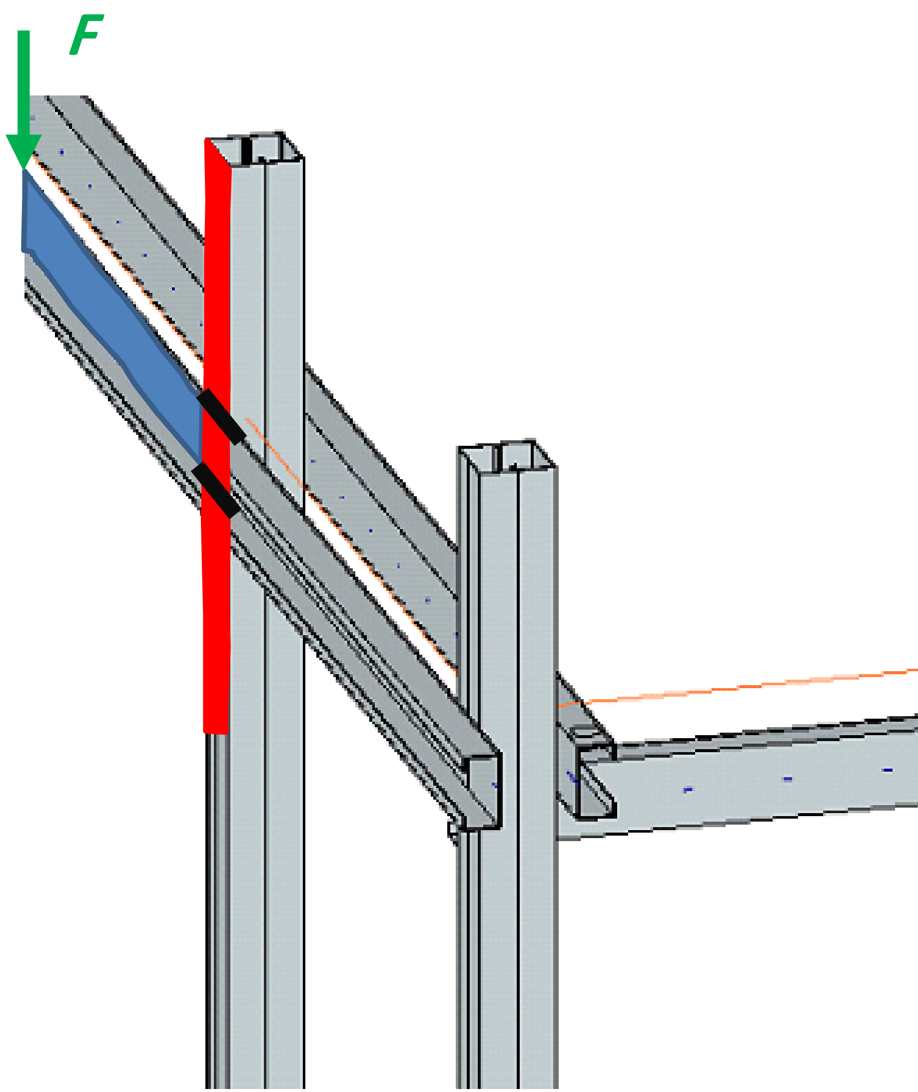
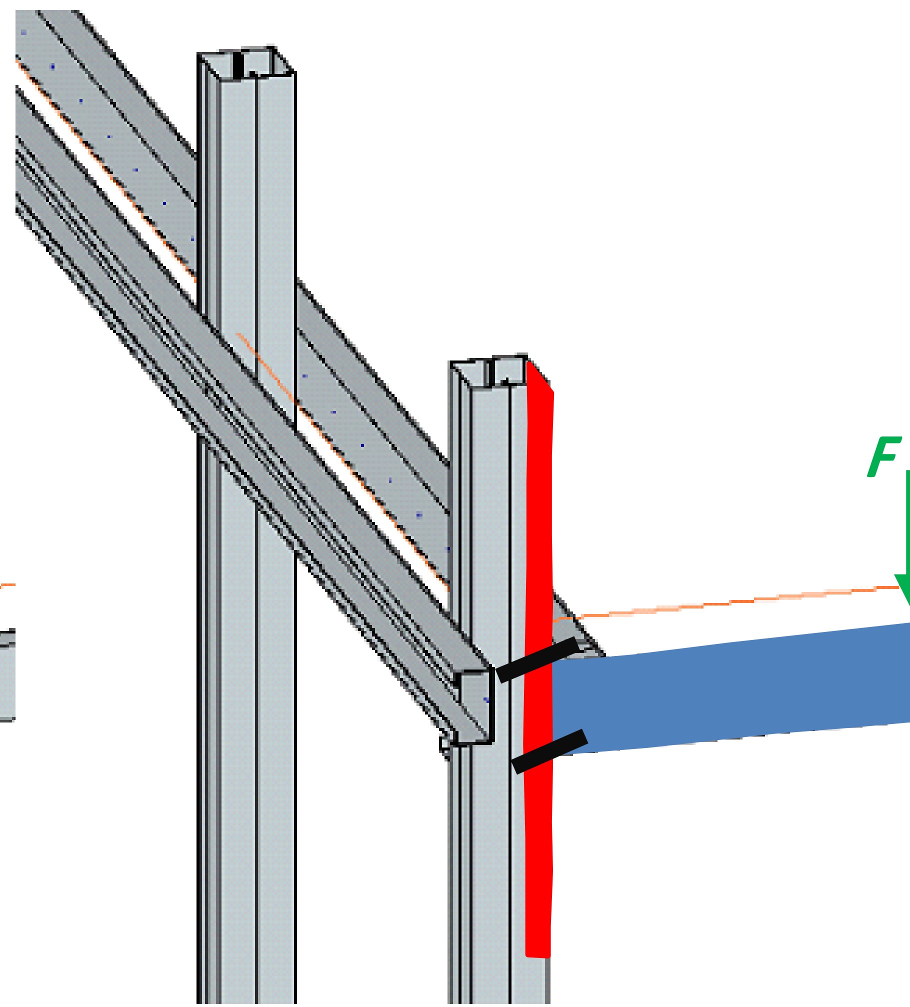
9.3 Stresses in Welded Joints in Torsion



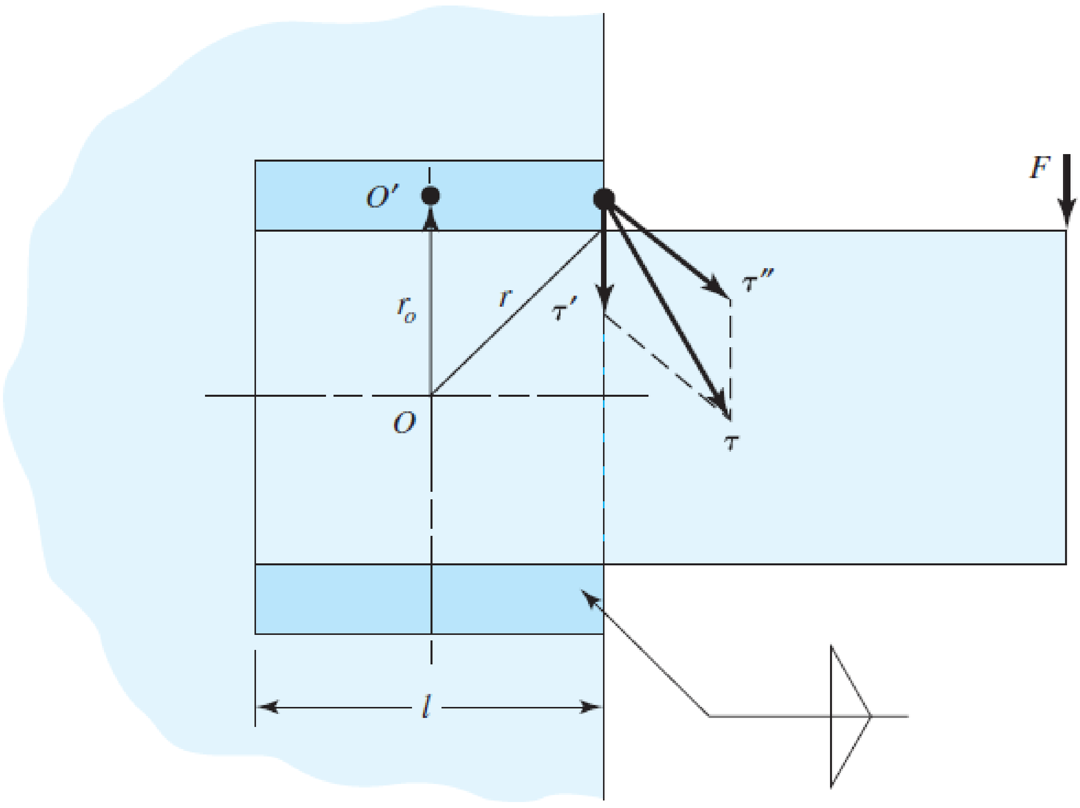
9.4 Stresses in Welded Joints in Bending



9.3 Stresses in Welded Joints in Torsion

Figure 9–12

This is a *moment connection;* such a connection produces *torsion* in the welds. The shear stresses shown are resultant stresses.

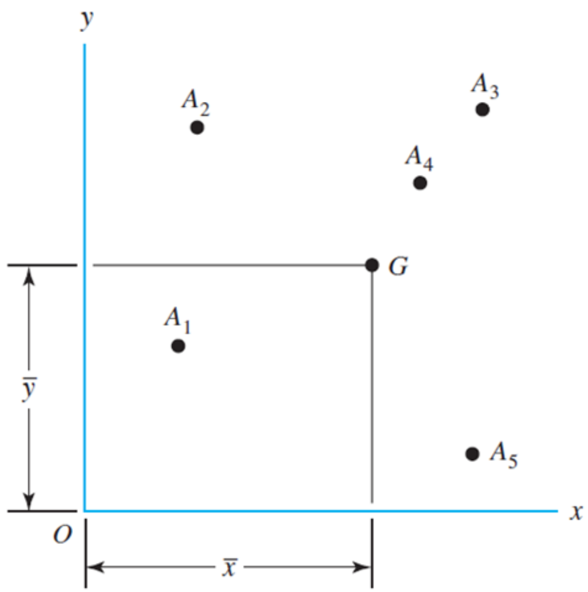


8.12 Bolted and Riveted Joints Loaded in Shear

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_i x_i}{\sum_1^n A_i}$$
$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4 + A_5y_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_i y_i}{\sum_1^n A_i} \qquad (8\text{–}56)$$

Figure 8–26

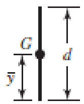
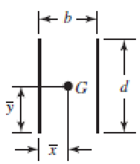
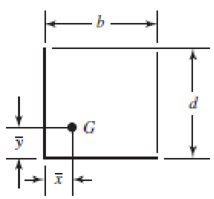
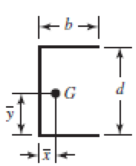
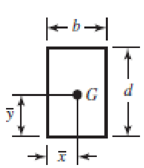
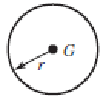
Centroid of pins, rivets, or bolts.



9.3 Stresses in Welded Joints in Torsion

Table 9–1

Torsional Properties of Fillet Welds*

Weld	Throat Area	Location of G	Unit Second Polar Moment of Area
1. 	$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$J_u = d^3/12$
2. 	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \dfrac{d(3b^2 + d^2)}{6}$
3. 	$A = 0.707h(b + d)$	$\bar{x} = \dfrac{b^2}{2(b + d)}$ $\bar{y} = \dfrac{d^2}{2(b + d)}$	$J_u = \dfrac{(b + d)^4 - 6b^2d^2}{12(b + d)}$
4. 	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2b + d}$ $\bar{y} = d/2$	$J_u = \dfrac{8b^3 + 6bd^2 + d^3}{12} - \dfrac{b^4}{2b + d}$
5. 	$A = 1.414h(b + d)$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \dfrac{(b + d)^3}{6}$
6. 	$A = 1.414\pi hr$		$J_u = 2\pi r^3$

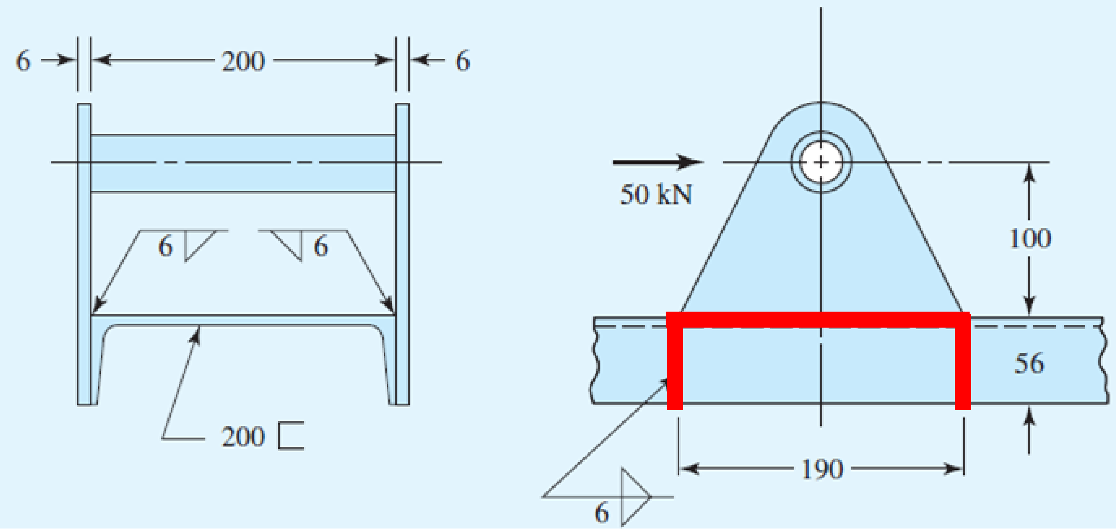
*G is the centroid of weld group; h is weld size; plane of torque couple is in the plane of the paper; all welds are of unit width.

9.3 Stresses in Welded Joints in Torsion

EXAMPLE 9–1 A 50-kN load is transferred from a welded fitting into a 200-mm steel channel as illustrated in Fig. 9–14. Estimate the maximum stress in the weld.

Figure 9–14

Dimensions in millimeters.



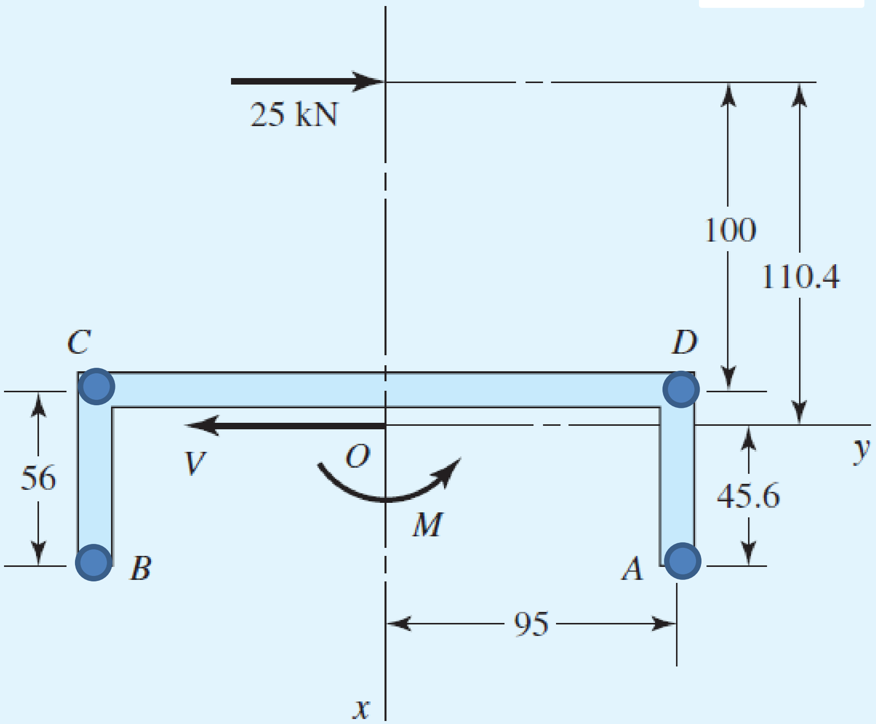
9.3 Stresses in Welded Joints in Torsion

EXAMPLE 9–1 A 50-kN load is transferred from a welded fitting into a 200-mm steel channel as illustrated in Fig. 9–14. Estimate the maximum stress in the weld.

(*a*) Label the ends and corners of each weld by letter. See Fig. 9–15. Sometimes it is desirable to label each weld of a set by number.

Figure 9–15

Diagram showing the weld geometry on a single plate; all dimensions in millimeters. Note that *V* and *M* represent the reaction loads applied by the welds *to the plate.*



9.3 Stresses in Welded Joints in Torsion

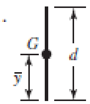
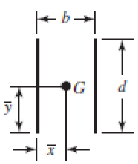
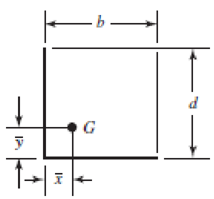
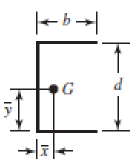
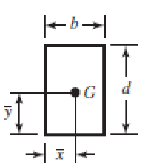
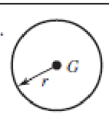
Then the primary shear stress is

$$\tau' = \frac{V}{A} = \frac{25(10)^3}{1280} = 19.5 \text{ MPa}$$

9.3 Stresses in Welded Joints in Torsion

Table 9–1

Torsional Properties of Fillet Welds*

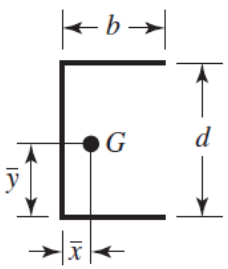
Weld	Throat Area	Location of G	Unit Second Polar Moment of Area
1. 	$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$J_u = d^3/12$
2. 	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \dfrac{d(3b^2 + d^2)}{6}$
3. 	$A = 0.707h(b + d)$	$\bar{x} = \dfrac{b^2}{2(b + d)}$ $\bar{y} = \dfrac{d^2}{2(b + d)}$	$J_u = \dfrac{(b + d)^4 - 6b^2d^2}{12(b + d)}$
4. 	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2b + d}$ $\bar{y} = d/2$	$J_u = \dfrac{8b^3 + 6bd^2 + d^3}{12} - \dfrac{b^4}{2b + d}$
5. 	$A = 1.414h(b + d)$	$\bar{x} = b/2$ $\bar{y} = d/2$	$J_u = \dfrac{(b + d)^3}{6}$
6. 	$A = 1.414\pi hr$		$J_u = 2\pi r^3$

*G is the centroid of weld group; h is weld size; plane of torque couple is in the plane of the paper; all welds are of unit width.

9.3 Stresses in Welded Joints in Torsion

Then the primary shear stress is

$$\tau' = \frac{V}{A} = \frac{25(10)^3}{1280} = 19.5 \text{ MPa}$$

4. 

$A = 0.707h(2b + d)$

$\bar{x} = \dfrac{b^2}{2b + d}$

$\bar{y} = d/2$

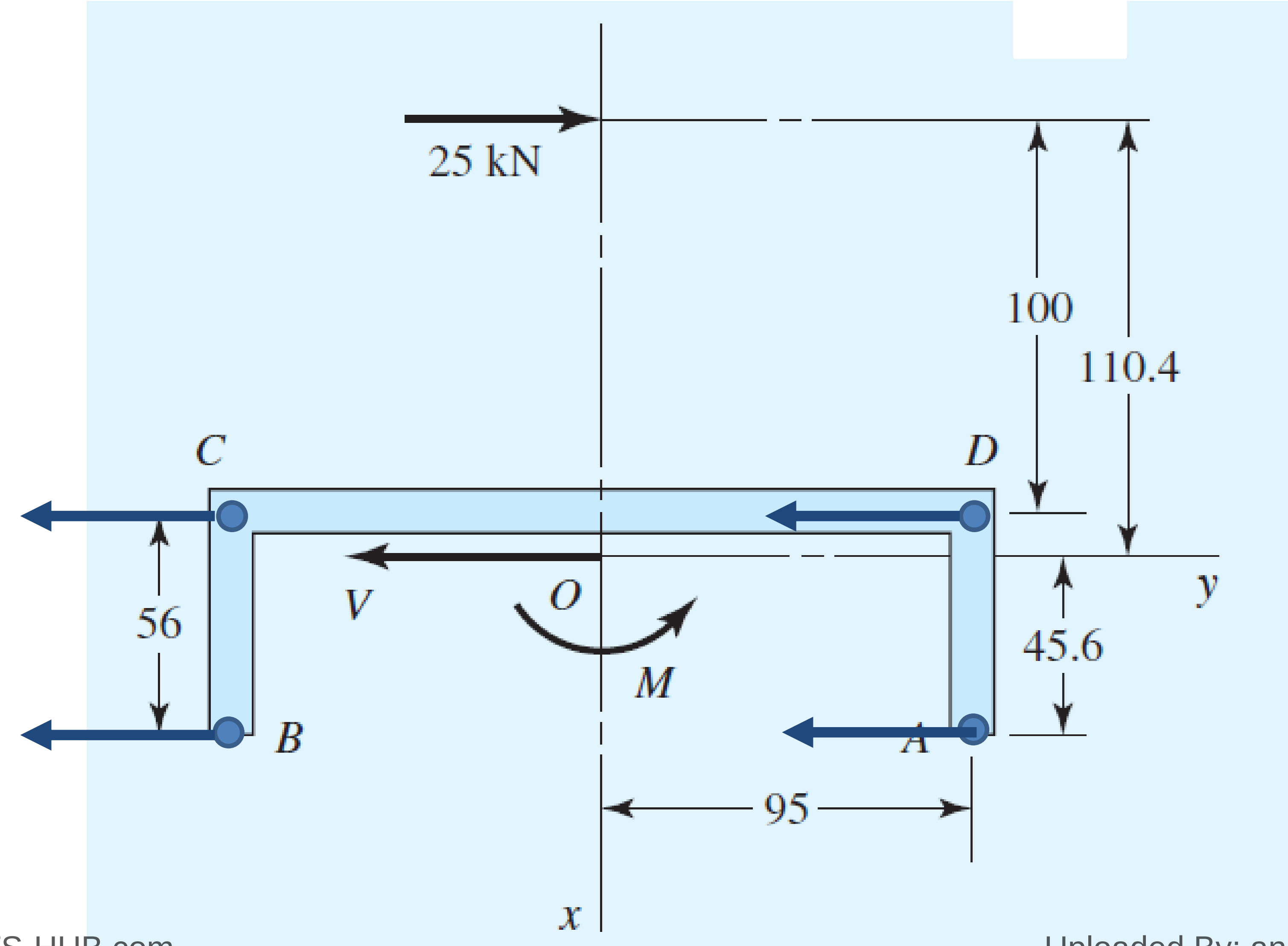
$J_u = \dfrac{8b^3 + 6bd^2 + d^3}{12} - \dfrac{b^4}{2b + d}$

(*b*) Estimate the primary shear stress τ'. As shown in Fig. 9–14, each plate is welded to the channel by means of three 6-mm fillet welds. Figure 9–15 shows that we have divided the load in half and are considering only a single plate. From case 4 of Table 9–1 we find the throat area as

$$A = 0.707(6)[2(56) + 190] = 1280 \text{ mm}^2$$

9.3 Stresses in Welded Joints in Torsion

(c) Draw the τ' stress, to scale, at each lettered corner or end.

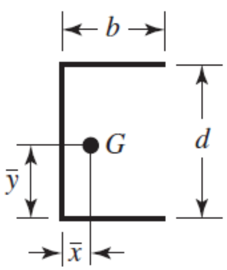


9.3 Stresses in Welded Joints in Torsion

(*d*) Locate the centroid of the weld pattern. Using case 4 of Table 9–1, we find

$$\bar{x} = \frac{(56)^2}{2(56) + 190} = 10.4 \text{ mm}$$

4.

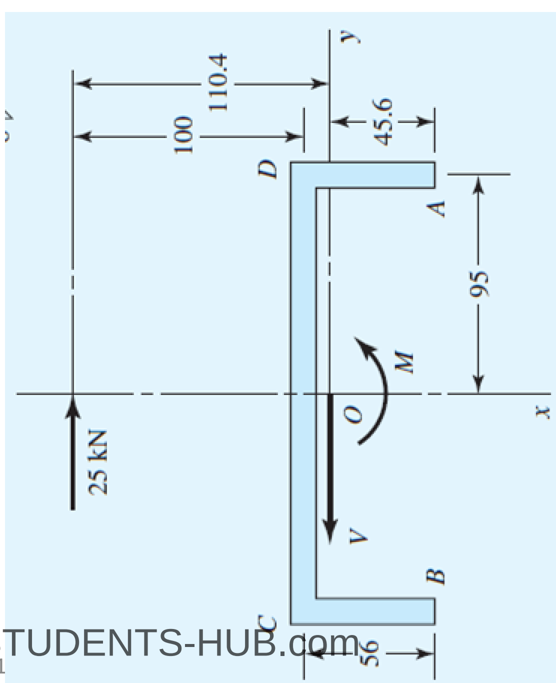


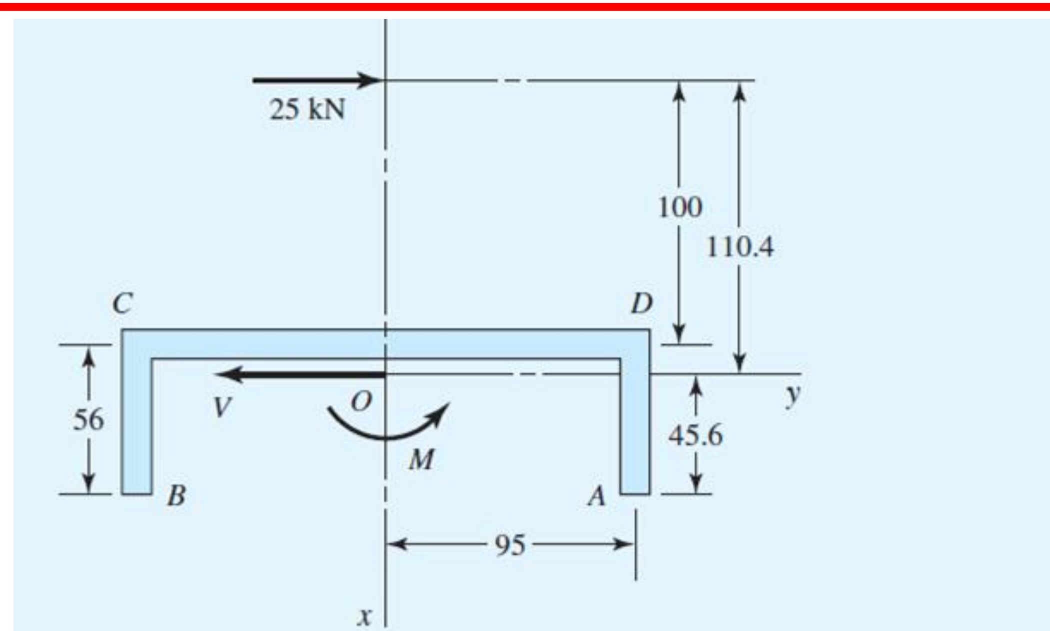
$$A = 0.707h(2b + d)$$

$$\bar{x} = \frac{b^2}{2b + d}$$

$$\bar{y} = d/2$$

$$J_u = \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b + d}$$



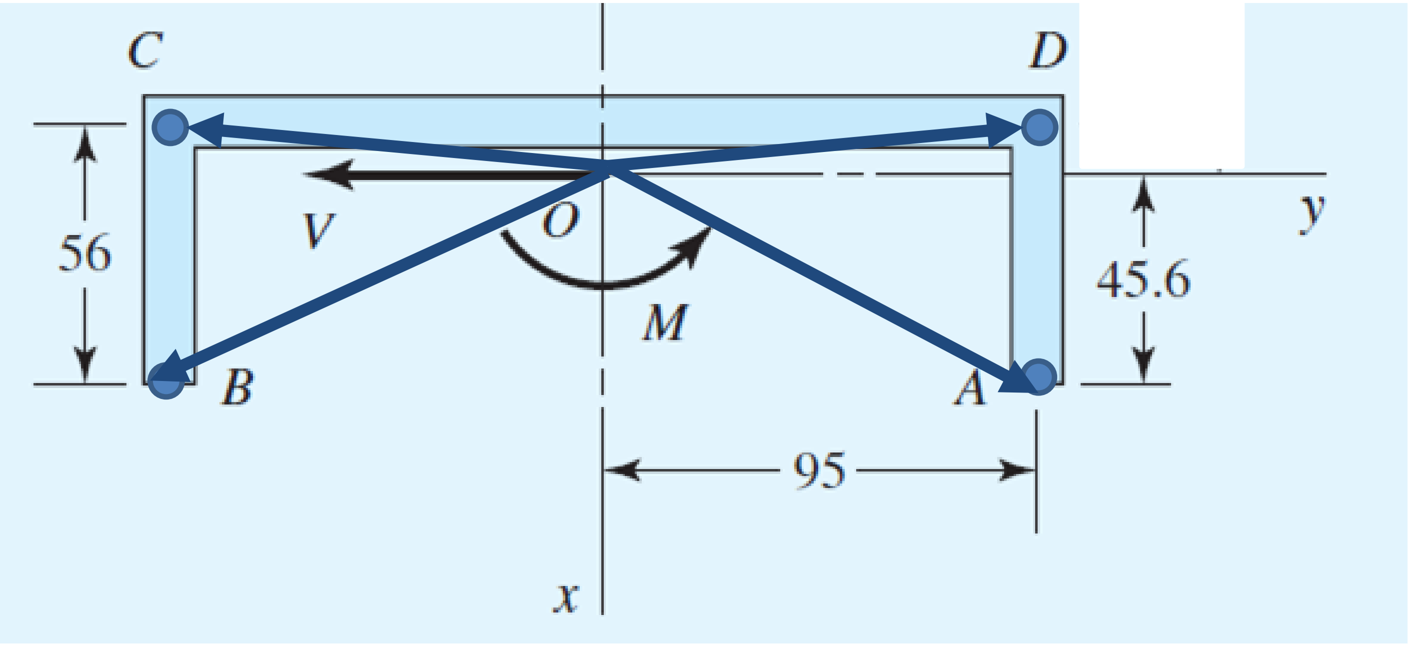


9.3 Stresses in Welded Joints in Torsion

(*e*) Find the distances r_i (see Fig. 9–16):

$$r_A = r_B = [(190/2)^2 + (56 - 10.4)^2]^{1/2} = 105 \text{ mm}$$

$$r_C = r_D = [(190/2)^2 + (10.4)^2]^{1/2} = 95.6 \text{ mm}$$

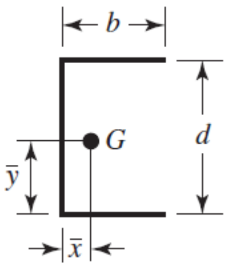


9.3 Stresses in Welded Joints in Torsion

(*f*) Find J. Using case 4 of Table 9–1 again, with Eq. (9–6), we get

$$J = 0.707(6)\left[\frac{8(56)^3 + 6(56)(190)^2 + (190)^3}{12} - \frac{(56)^4}{2(56) + 190}\right]$$
$$= 7.07(10)^6 \text{ mm}^4$$

4.



$A = 0.707h(2b + d)$

$\bar{x} = \dfrac{b^2}{2b + d}$

$\bar{y} = d/2$

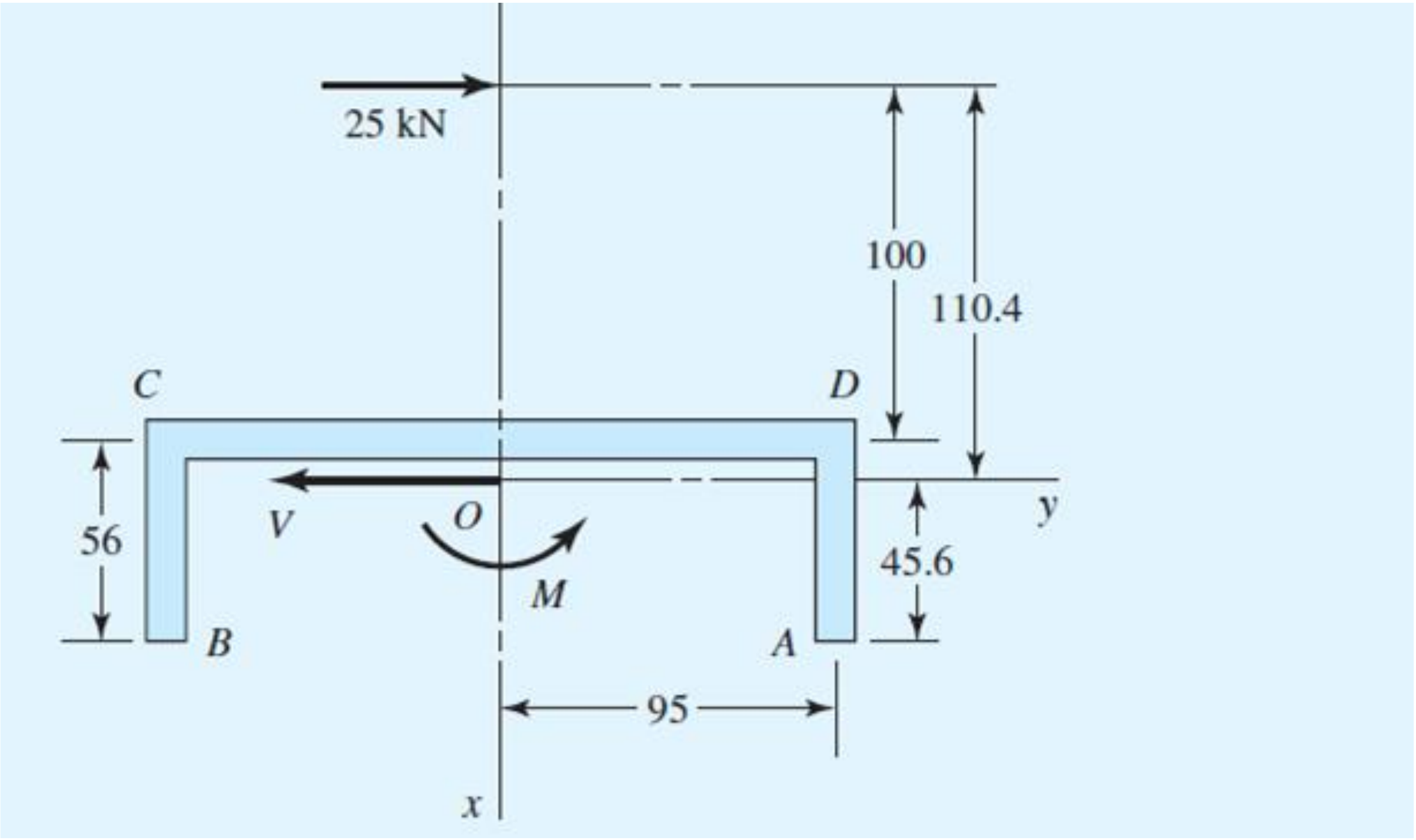
$J_u = \dfrac{8b^3 + 6bd^2 + d^3}{12} - \dfrac{b^4}{2b + d}$

$$J = 0.707hJ_u \qquad (9\text{–}6)$$

9.3 Stresses in Welded Joints in Torsion

(*g*) Find M:

$$M = Fl = 25(100 + 10.4) = 2760\ \text{N} \cdot \text{m}$$

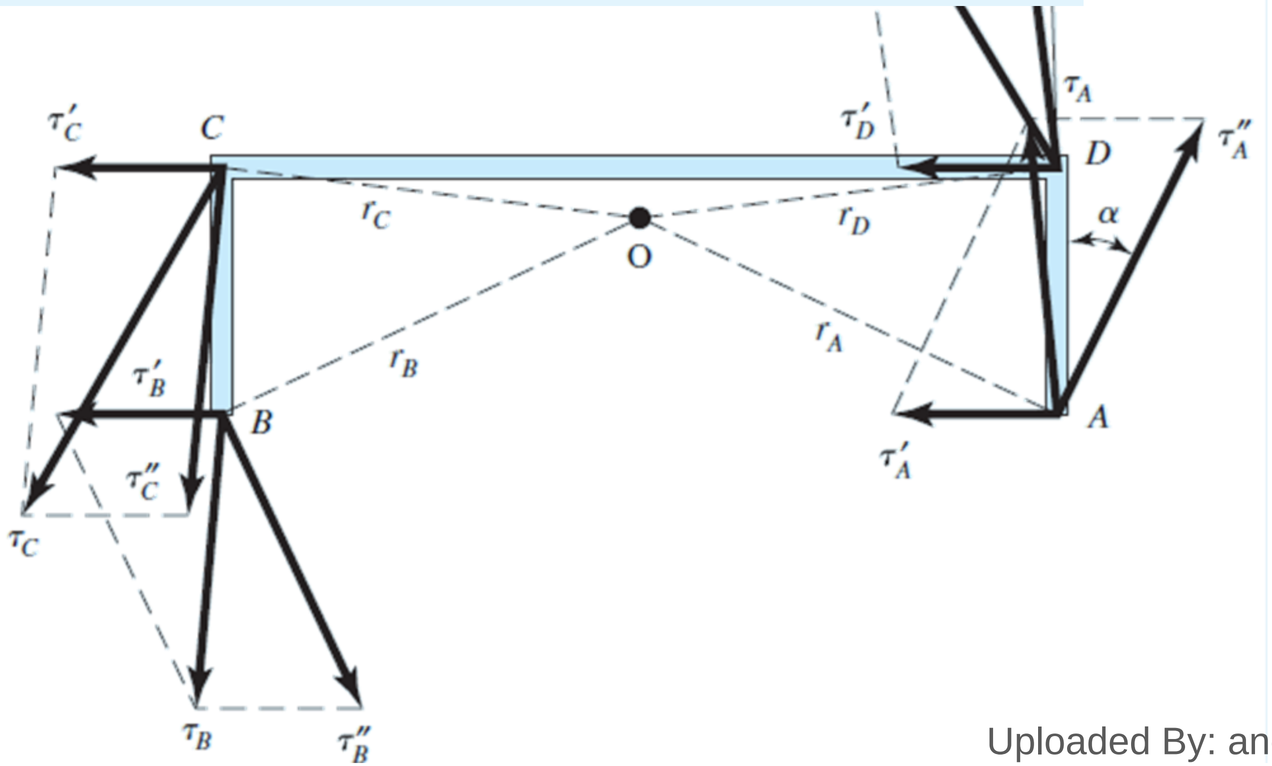


9.3 Stresses in Welded Joints in Torsion

(*h*) Estimate the secondary shear stresses τ'' at each lettered end or corner:

$$\tau''_A = \tau''_B = \frac{Mr}{J} = \frac{2760(10)^3(105)}{7.07(10)^6} = 41.0\ \text{MPa}$$

$$\tau''_C = \tau''_D = \frac{2760(10)^3(95.6)}{7.07(10)^6} = 37.3\ \text{MPa}$$



9.3 Stresses in Welded Joints in Torsion

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(j) At each point labeled, combine the two stress components as vectors (since they apply to the same area). At point A, the angle that τ_A'' makes with the vertical, α, is also the angle r_A makes with the horizontal, which is $\alpha = \tan^{-1}(45.6/95) = 25.64°$. This angle also applies to point B. Thus

$$\tau_A = \tau_B = \sqrt{(19.5 - 41.0 \sin 25.64°)^2 + (41.0 \cos 25.64°)^2} = 37.0 \text{ MPa}$$

Similarly, for C and D, $\beta = \tan^{-1}(10.4/95) = 6.25°$. Thus

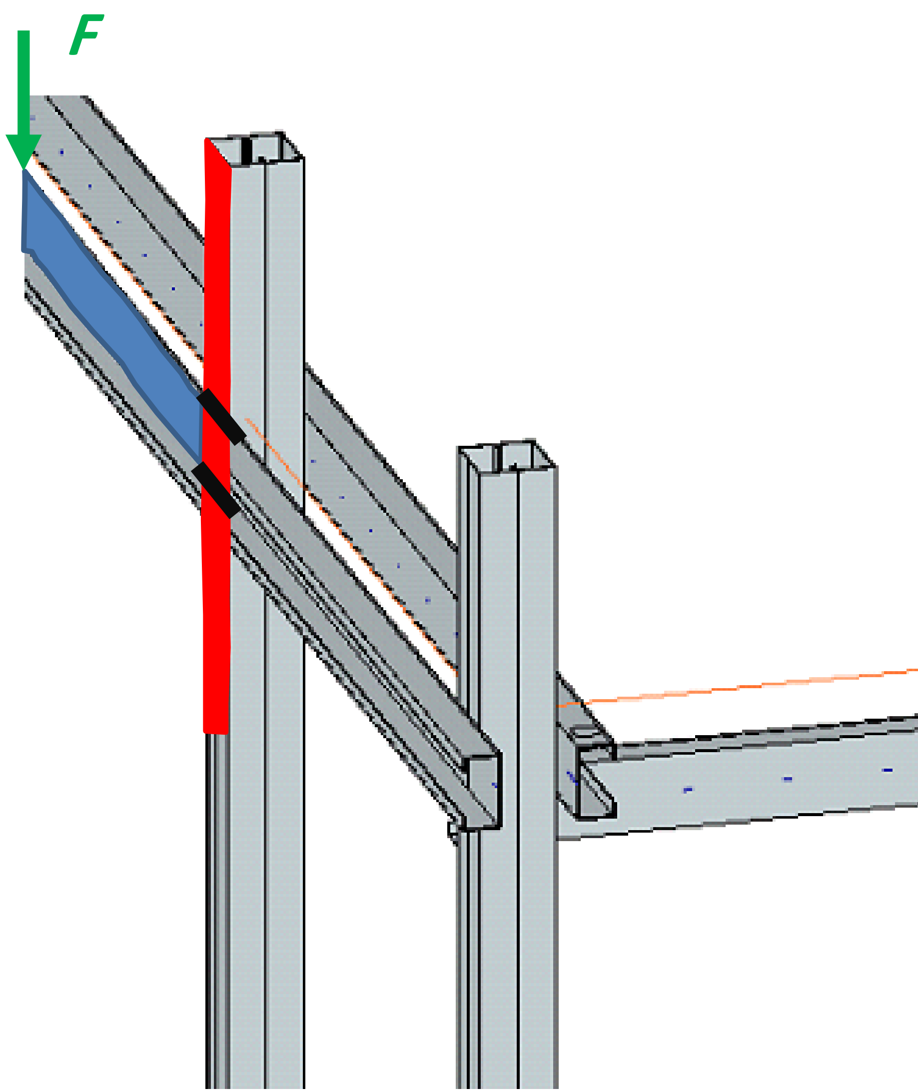
$$\tau_C = \tau_D = \sqrt{(19.5 + 37.3 \sin 6.25°)^2 + (37.3 \cos 6.25°)^2} = 43.9 \text{ MPa}$$

(k) Identify the most highly stressed point:

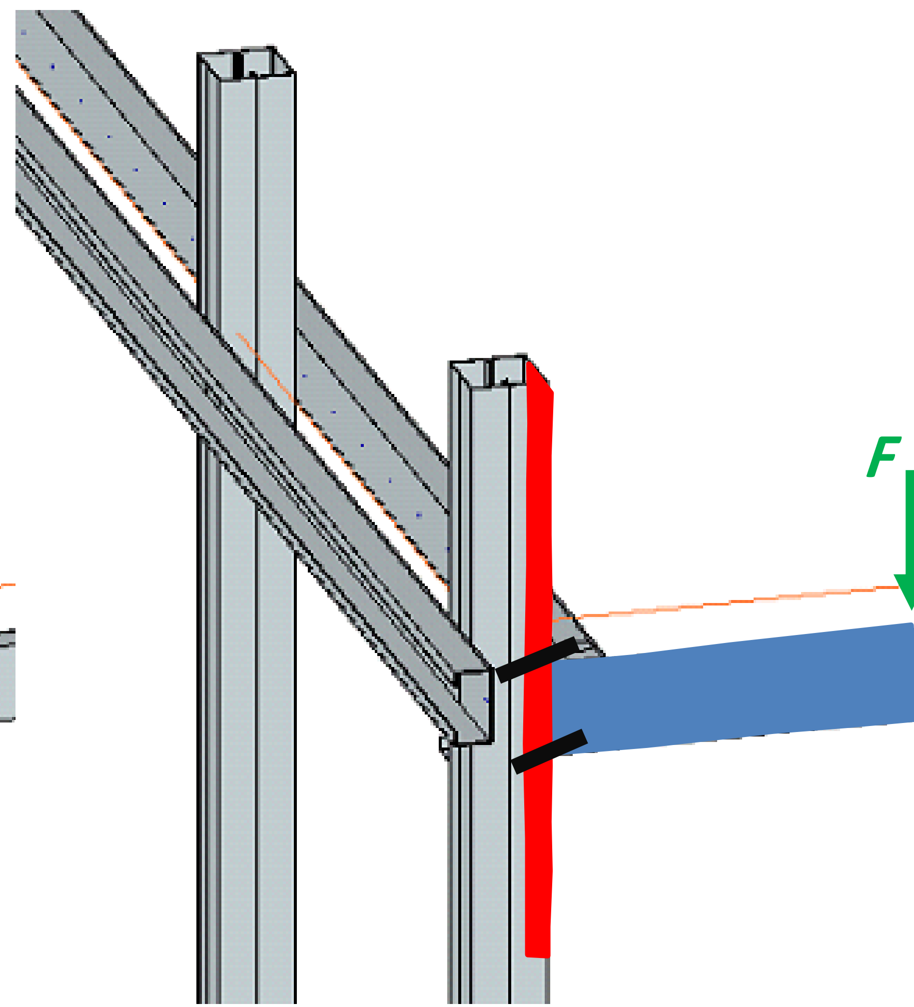
$$\tau_{\max} = \tau_C = \tau_D = 43.9 \text{ MPa}$$

9.3 Stresses in Welded Joints in Torsion (9.4 in Bending)

9.3 Stresses in Welded Joints in Torsion



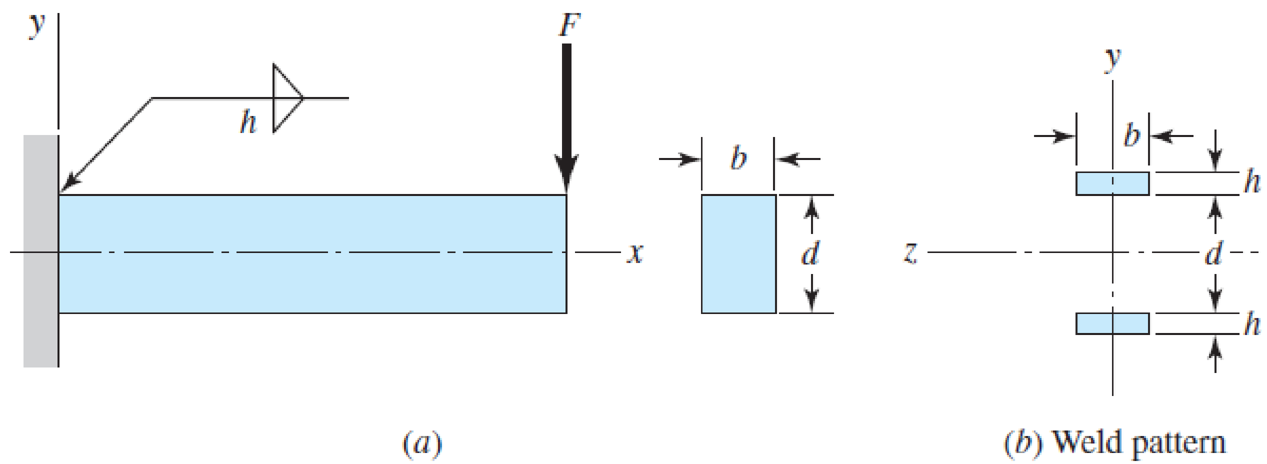
9.4 Stresses in Welded Joints in Bending



9.4 Stresses in Welded Joints in Bending

Figure 9–17

A rectangular cross-section cantilever welded to a support at the top and bottom edges.



9.4 Stresses in Welded Joints in Bending

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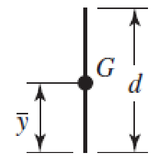
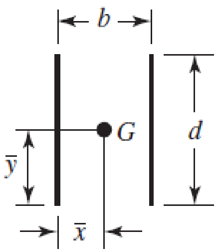
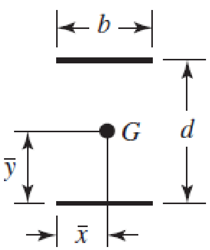
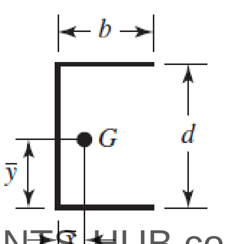
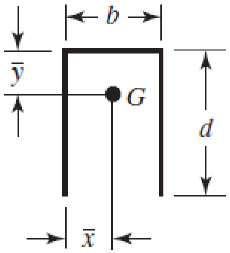
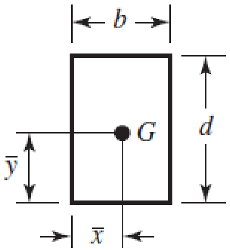
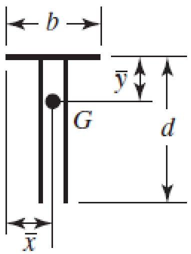
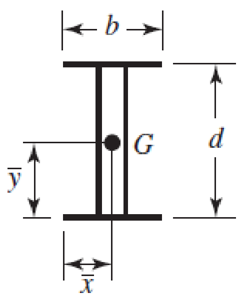
Table 9–2

Bending Properties of Fillet Welds*

Weld	Throat Area	Location of G	Unit Second Moment of Area
1. 	$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{12}$
2. 	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{6}$
3. 	$A = 1.414hb$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{bd^2}{2}$
4. 	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2b + d}$ $\bar{y} = d/2$	$I_u = \dfrac{d^2}{12}(6b + d)$

5.		$A = 0.707h(b + 2d)$	$\bar{x} = b/2$ $\bar{y} = \dfrac{d^2}{b + 2d}$	$I_u = \dfrac{2d^3}{3} - 2d^2\bar{y} + (b + 2d)\bar{y}^2$
6.		$A = 1.414h(b + d)$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^2}{6}(3b + d)$
7.		$A = 0.707h(b + 2d)$	$\bar{x} = b/2$ $\bar{y} = \dfrac{d^2}{b + 2d}$	$I_u = \dfrac{2d^3}{3} - 2d^2\bar{y} + (b + 2d)\bar{y}^2$
8.		$A = 1.414h(b + d)$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^2}{6}(3b + d)$
9.		$A = 1.414\pi hr$		$I_u = \pi r^3$

9.5 The Strength of Welded Joints

Table 9–3

Minimum Weld-Metal Properties

AWS Electrode Number*	Tensile Strength kpsi (MPa)	Yield Strength, kpsi (MPa)	Percent Elongation
E60xx	62 (427)	50 (345)	17–25
E70xx	70 (482)	57 (393)	22
E80xx	80 (551)	67 (462)	19
E90xx	90 (620)	77 (531)	14–17
E100xx	100 (689)	87 (600)	13–16
E120xx	120 (827)	107 (737)	14

*The American Welding Society (AWS) specification code numbering system for electrodes. This system uses an E prefixed to a four- or five-digit numbering system in which the first two or three digits designate the approximate tensile strength. The last digit includes variables in the welding technique, such as current supply. The next-to-last digit indicates the welding position, as, for example, flat, or vertical, or overhead. The complete set of specifications may be obtained from the AWS upon request.

9.5 The Strength of Welded Joints

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Table 9–4

Stresses Permitted by the AISC Code for Weld Metal

Type of Loading	Type of Weld	Permissible Stress	n*
Tension	Butt	$0.60S_y$	1.67
Bearing	Butt	$0.90S_y$	1.11
Bending	Butt	$0.60–0.66S_y$	1.52–1.67
Simple compression	Butt	$0.60S_y$	1.67
Shear	Butt or fillet	$0.30S_{ut}^{\dagger}$	

*The factor of safety n has been computed by using the distortion-energy theory.

†Shear stress on base metal should not exceed $0.40S_y$ of base metal.

Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0

Table 9–6

Allowable Steady Loads and Minimum Fillet Weld Sizes

Schedule A: Allowable Load for Various Sizes of Fillet Welds

	Strength Level of Weld Metal (EXX)						
	60*	70*	80	90*	100	110*	120
Allowable shear stress on throat, ksi (1000 psi) of fillet weld or partial penetration groove weld							
$\tau =$	18.0	21.0	24.0	27.0	30.0	33.0	36.0
Allowable Unit Force on Fillet Weld, kip/linear in							
$^{\dagger}f =$	$12.73h$	$14.85h$	$16.97h$	$19.09h$	$21.21h$	$23.33h$	$25.45h$
Leg Size h, in	Allowable Unit Force for Various Sizes of Fillet Welds kip/linear in						
1	12.73	14.85	16.97	19.09	21.21	23.33	25.45
7/8	11.14	12.99	14.85	16.70	18.57	20.41	22.27
3/4	9.55	11.14	12.73	14.32	15.92	17.50	19.09
5/8	7.96	9.28	10.61	11.93	13.27	14.58	15.91
1/2	6.37	7.42	8.48	9.54	10.61	11.67	12.73
7/16	5.57	6.50	7.42	8.35	9.28	10.21	11.14
3/8	4.77	5.57	6.36	7.16	7.95	8.75	9.54
5/16	3.98	4.64	5.30	5.97	6.63	7.29	7.95
1/4	3.18	3.71	4.24	4.77	5.30	5.83	6.36
3/16	2.39	2.78	3.18	3.58	3.98	4.38	4.77
1/8	1.59	1.86	2.12	2.39	2.65	2.92	3.18
1/16	0.795	0.930	1.06	1.19	1.33	1.46	1.59

*Fillet welds actually tested by the joint AISC-AWS Task Committee.

$^{\dagger}f = 0.707h\,\tau_{\text{all}}$.

Schedule B: Minimum Fillet Weld Size, h

Material Thickness of Thicker Part Joined, in		Weld Size, in
*To $\frac{1}{4}$ incl.		$\frac{1}{8}$
Over $\frac{1}{4}$	To $\frac{1}{2}$	$\frac{3}{16}$
Over $\frac{1}{2}$	To $\frac{3}{4}$	$\frac{1}{4}$
†Over $\frac{3}{4}$	To $1\frac{1}{2}$	$\frac{5}{16}$
Over $1\frac{1}{2}$	To $2\frac{1}{4}$	$\frac{3}{8}$
Over $2\frac{1}{4}$	To 6	$\frac{1}{2}$
Over 6		$\frac{5}{8}$

Not to exceed the thickness of the thinner part.

*Minimum size for bridge application does not go below $\frac{3}{16}$ in.

†For minimum fillet weld size, schedule does not go above $\frac{5}{16}$ in fillet weld for every $\frac{3}{4}$ in material.

9.6 Static Loading

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EXAMPLE 9–4

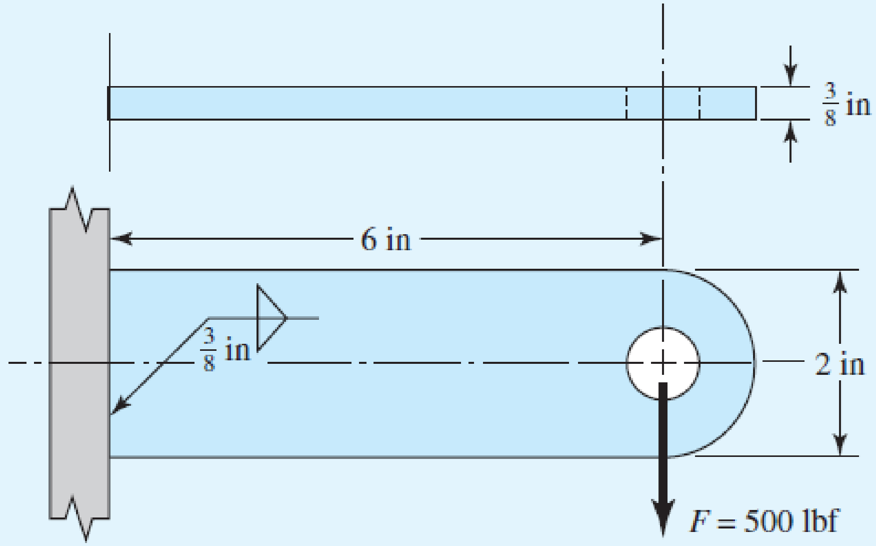
Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. 9–20. The cantilever is made of AISI 1018 HR steel and welded with a $\frac{3}{8}$-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.

(*a*) Use the conventional method for the weld metal.

(*b*) Use the conventional method for the attachment (cantilever) metal.

(*c*) Use a welding code for the weld metal.

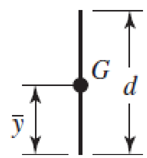
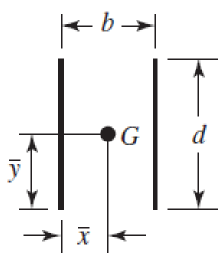
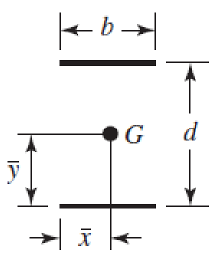
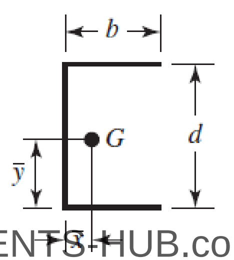
Figure 9–20



9.4 Stresses in Welded Joints in Bending

Table 9–2

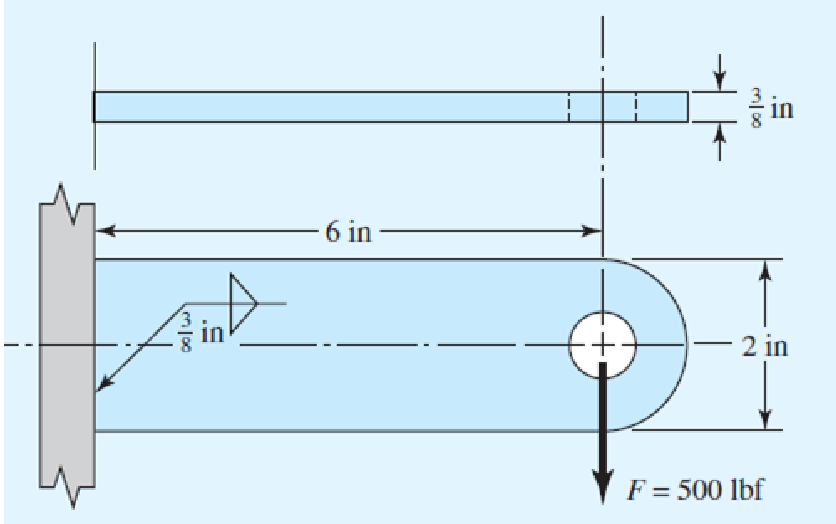
Bending Properties of Fillet Welds*

Weld	Throat Area	Location of G	Unit Second Moment of Area
1. 	$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{12}$
2. 	$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{6}$
3. 	$A = 1.414hb$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{bd^2}{2}$
4. 	$A = 0.707h(2b + d)$	$\bar{x} = \dfrac{b^2}{2b + d}$ $\bar{y} = d/2$	$I_u = \dfrac{d^2}{12}(6b + d)$

9.6 Static Loading

EXAMPLE 9–4 Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. 9–20. The cantilever is made of AISI 1018 HR steel and welded with a $\frac{3}{8}$-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.

(*a*) Use the conventional method for the weld metal.



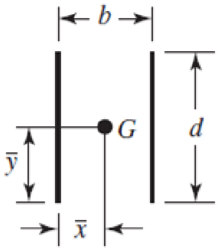
$b = 0.375$ in, $d = 2$ in, so

$$A = 1.414hd = 1.414(0.375)2 = 1.06 \text{ in}^2$$

$$I_u = d^3/6 = 2^3/6 = 1.33 \text{ in}^3$$

$$I = 0.707hI_u = 0.707(0.375)1.33 = 0.353 \text{ in}^4$$

2.



$A = 1.414hd$

$\bar{x} = b/2$

$\bar{y} = d/2$

$I_u = \dfrac{d^3}{6}$

9.6 Static Loading

EXAMPLE 9–4 Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. 9–20. The cantilever is made of AISI 1018 HR steel and welded with a $\frac{3}{8}$-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.

(*a*) Use the conventional method for the weld metal.

Table 9–3

Minimum Weld-Metal Properties

AWS Electrode Number*	Tensile Strength kpsi (MPa)	Yield Strength, kpsi (MPa)
E60xx	62 (427)	50 (345)
E70xx	70 (482)	57 (393)
E80xx	80 (551)	67 (462)
E90xx	90 (620)	77 (531)
E100xx	100 (689)	87 (600)
E120xx	120 (827)	107 (737)

From Table 9–3, $S_y = 50$ kpsi, $S_{ut} = 62$ kpsi.

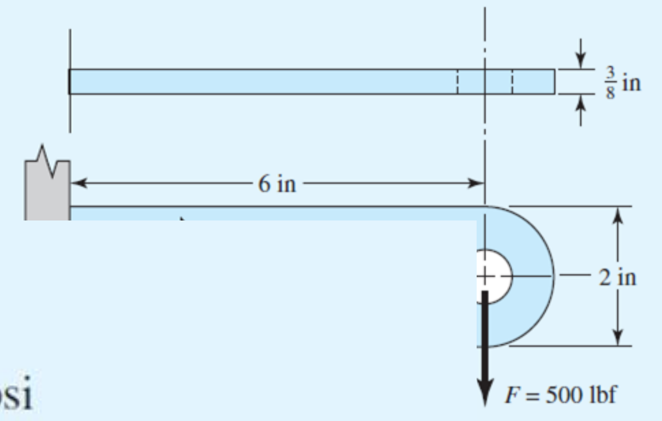
9.6 Static Loading

EXAMPLE 9–4 Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. 9–20. The cantilever is made of AISI 1018 HR steel and welded with a $\frac{3}{8}$-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.

(*a*) Use the conventional method for the weld

Primary shear:

$$\tau' = \frac{F}{A} = \frac{500(10^{-3})}{1.06} = 0.472 \text{ kpsi}$$



Secondary shear:

$$\tau'' = \frac{Mr}{I} = \frac{500(10^{-3})(6)(1)}{0.353} = 8.50 \text{ kpsi}$$

The shear magnitude τ is from the vector addition

$$\tau = (\tau'^2 + \tau''^2)^{1/2} = (0.472^2 + 8.50^2)^{1/2} = 8.51 \text{ kpsi}$$

The factor of safety based on a minimum strength and the distortion-energy criterion is

$$n = \frac{S_{sy}}{\tau} = \frac{0.577(50)}{8.51} = 3.39$$

Since $n \geq n_d$, that is, $3.39 \geq 3.0$, the weld metal has satisfactory strength.

9.6 Static Loading

EXAMPLE 9–4 Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. 9–20. The cantilever is made of AISI 1018 HR steel and welded with a $\frac{3}{8}$-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.

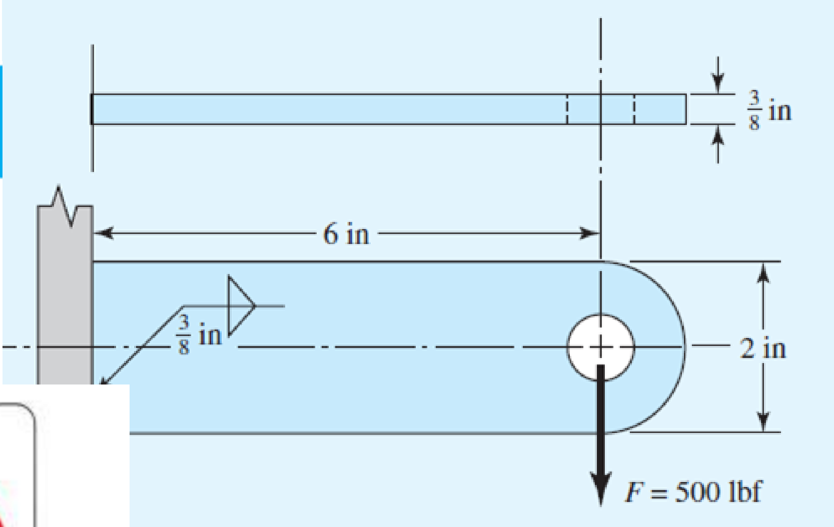
(*a*) Use the conventional method for the weld metal.

b) Use the conventional method for the attachment (cantilever) metal.

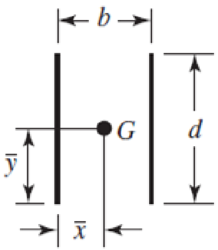
Table 9–3

Minimum W

AWS Electr Numbe	Yield Strength, kpsi (MPa)
E60xx	50 (345)
E70xx	57 (393)
E80xx	67 (462)
E90xx	77 (531)
E100xx	
E120xx	



2.



$A = 1.41$

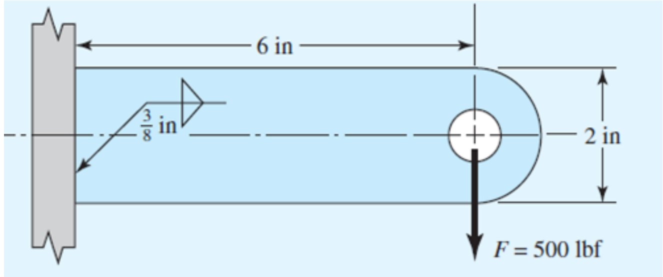
$I_u = \frac{d^3}{6}$

9.6 Static Loading

EXAMPLE 9–4 Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. 9–20. The cantilever is made of AISI 1018 HR steel and welded with a $\frac{3}{8}$-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.

(*a*) Use the conventional method for the weld metal.

(*b*) Use the conventional method for the attachment (cantilever) metal.



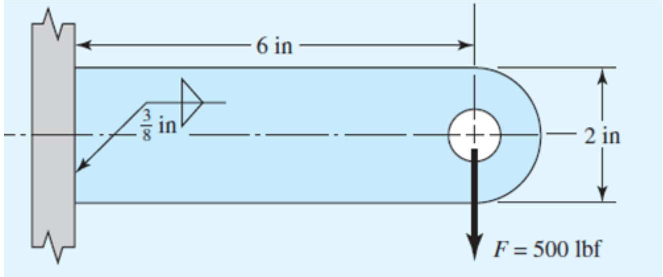
1 UNS No.	2 SAE and/or AISI No.	3 Process-ing	4 Tensile Strength, MPa (kpsi)	5 Yield Strength, MPa (kpsi)	6 Elongation in 2 in, %	7 Reduction in Area, %	8 Brinell Hardness
G10060	1006	HR	300 (43)	170 (24)	30	55	86
		CD	330 (48)	280 (41)	20	45	95
G10100	1010	HR	320 (47)	180 (26)	28	50	95
		CD	370 (53)	300 (44)	20	40	105
G10150	1015	HR	340 (50)	190 (27.5)	28	50	101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126

9.6 Static Loading

EXAMPLE 9–4 Perform an adequacy assessment of the statically loaded welded cantilever carrying 500 lbf depicted in Fig. 9–20. The cantilever is made of AISI 1018 HR steel and welded with a $\frac{3}{8}$-in fillet weld as shown in the figure. An E6010 electrode was used, and the design factor was 3.0.

(*a*) Use the conventional method for the weld metal.

(*b*) Use the conventional method for the attachment (cantilever) metal.



(*b*) From Table A–20, minimum strengths are $S_{ut} = 58$ kpsi and $S_y = 32$ kpsi. Then

$$\sigma = \frac{M}{I/c} = \frac{M}{bd^2/6} = \frac{500(10^{-3})6}{0.375(2^2)/6} = 12 \text{ kpsi}$$

$$n = \frac{S_y}{\sigma} = \frac{32}{12} = 2.67$$

Since $n < n_d$, that is, $2.67 < 3.0$, the joint is unsatisfactory as to the attachment strength.

9.7 Fatigue Loading

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- **In fatigue, the Gerber criterion is best:**

$$n_f = \frac{1}{2}\left(\frac{0.67S_{ut}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{0.67S_{ut}\tau_a}\right)^2}\,\right]$$

- **For the surface factor, an as-forged surface should always be assumed for weldments unless a superior finish is specified and obtained.**

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

9.7 Fatigue Loading

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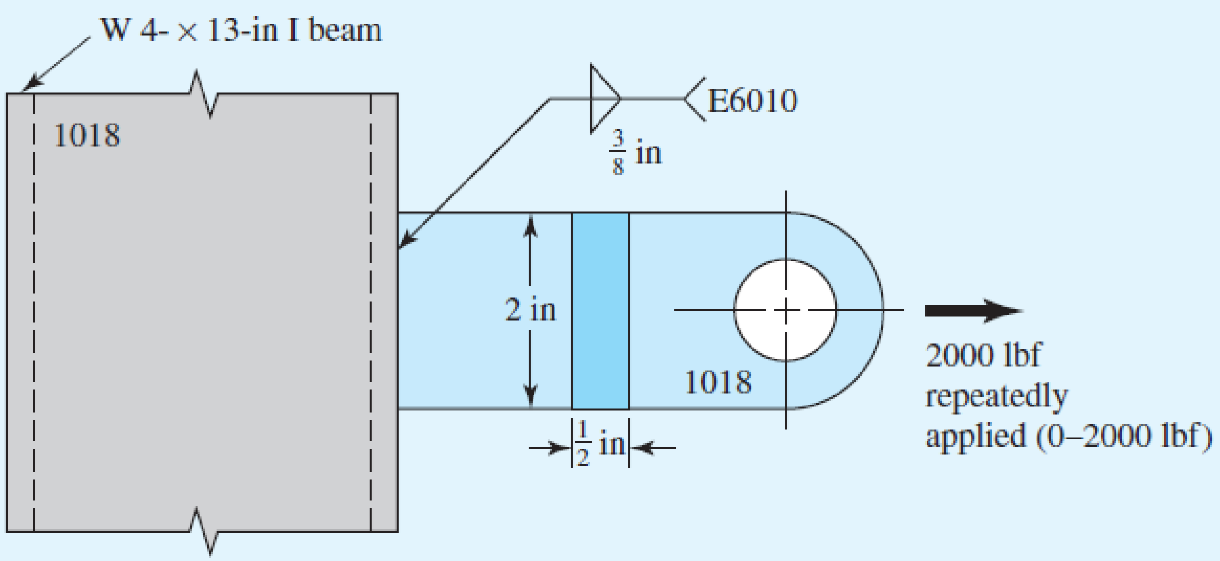
EXAMPLE 9–6 The AISI 1018 HR steel strap of Fig. 9–22 has a repeatedly applied load of 2000 lbf ($F_a = F_m = 1000$ lbf). Determine the fatigue factor of safety fatigue strength of the weldment.

Figure 9–22



S_{us}

$$n_f = \frac{1}{2}\left(\frac{0.67S_{ut}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{0.67S_{ut}\tau_a}\right)^2}\right]$$

9.7 Fatigue Loading

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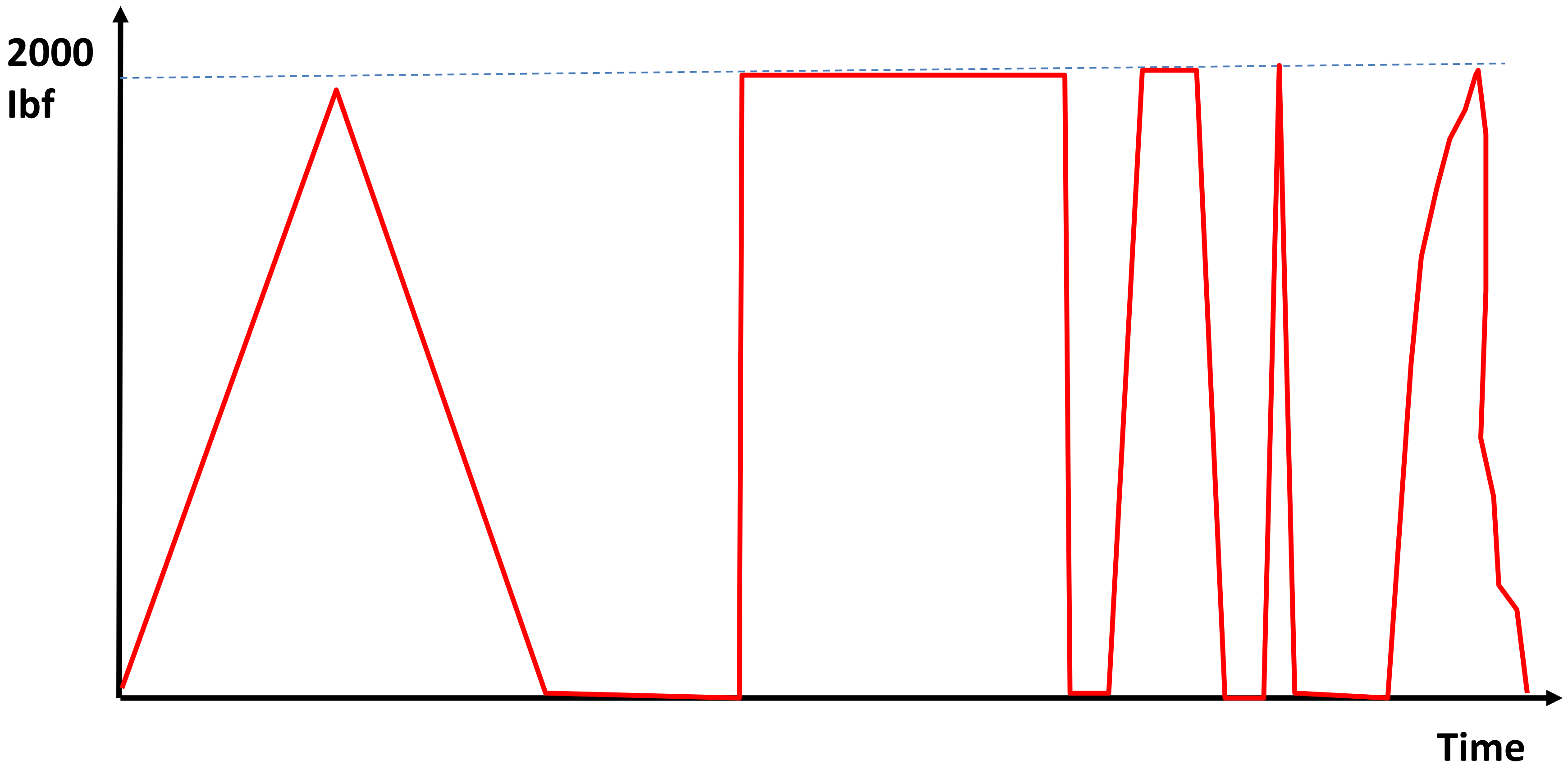
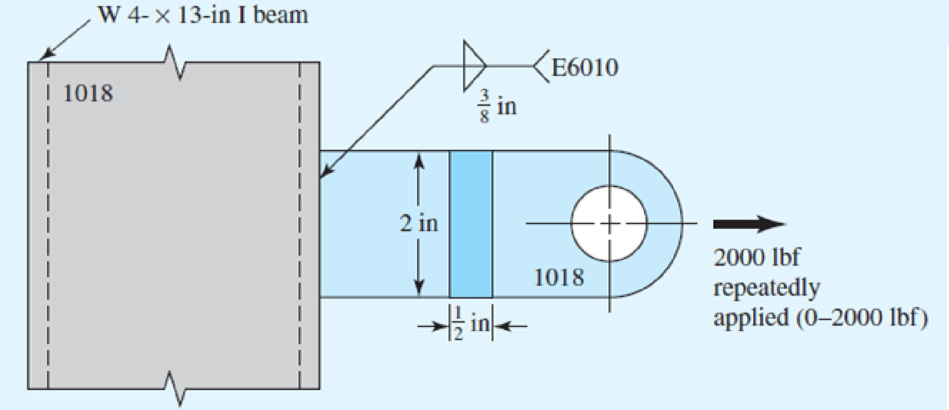
EXAMPLE 9–6 The AISI 1018 HR steel strap of Fig. 9–22 has a repeatedly applied load of 2000 lbf ($F_a = F_m = 1000$ lbf). Determine the fatigue factor of safety fatigue strength of the weldment.



9.7 Fatigue Loading

EXAMPLE 9–6 The AISI 1018 HR steel strap of Fig. 9–22 has a repeatedly applied load of 2000 lbf ($F_a = F_m = 1000$ lbf). Determine the fatigue factor of safety fatigue strength of the weldment.



AWS Electrode Number*	Tensile Strength kpsi (MPa)	Yield Strength, kpsi (MPa)
E60xx	62 (427)	50 (345)
E70xx	70 (482)	57 (393)
E80xx	80 (551)	67 (462)
E90xx	90 (620)	77 (531)
E100xx	100 (689)	87 (600)
E120xx	120 (827)	107 (737)

1 UNS No.	2 SAE and/or AISI No.	3 Processing					8 Brinell Hardness
G10060	1006	HR					86
		CD					95
G10100	1010	HR					95
		CD					105
G10150	1015	HR					101
		CD	390 (56)	320 (47)	18	40	111
G10180	1018	HR	400 (58)	220 (32)	25	50	116
		CD	440 (64)	370 (54)	15	40	126

9.7 Fatigue Loading – Example 9-6

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Surface Factor k_a $$k_a = aS_{ut}^b$$ $$k_a = 39.9(58)^{-0.995} = 0.702.$$

Table 6–2

Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

Size Factor k_b For axial loading there is no size effect, so $k_b = 1$

Load Factor k_c

$$k_c = \begin{cases} 1 & \text{bending} \\ 0.85 & \text{axial} \\ 0.59 & \text{torsion} \end{cases}$$

$$k_c = 0.59.$$

$$S_{se} = 0.702(1)0.59(1)(1)(1)0.5(58) = 12.0 \text{ kpsi}$$

9.7 Fatigue Loading

EXAMPLE 9–6 The AISI 1018 HR steel strap of Fig. 9–22 has a repeatedly applied load of 2000 lbf ($F_a = F_m = 1000$ lbf). Determine the fatigue factor of safety fatigue strength of the weldment.

From Table 9–5, $K_{fs} = 2$. Only primary shear is present:

$$\tau'_a = \tau'_m = \frac{K_{fs}F_a}{A} = \frac{\text{[redacted]}(1000)}{1.061} = 1885 \text{ psi}$$

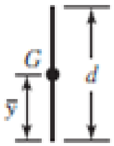
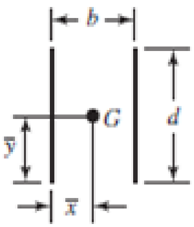
Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0

Table 9–1

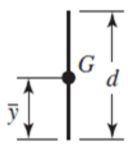
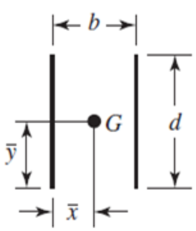
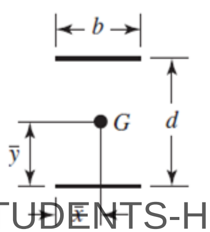
Torsional Properties of Fillet Welds*

Weld		Throat Area	Location of G	Unit Second Polar Moment of Area
1.		$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$J_u = d^3/12$
2.		$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	

$$A = 1.414(0.375)(2) = 1.061 \text{ in}^2$$

Table 9–2

Bending Properties of Fillet Welds*

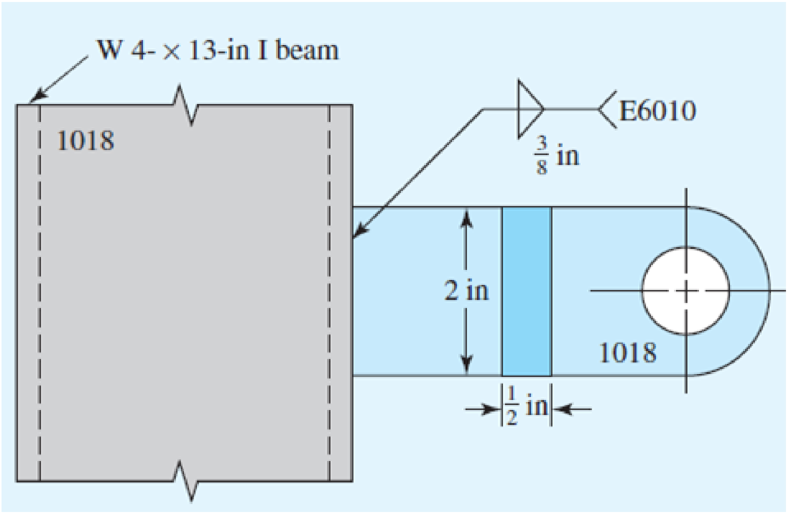
Weld		Throat Area	Location of G	Unit Second Moment of Area
1.		$A = 0.707hd$	$\bar{x} = 0$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{12}$
2.		$A = 1.414hd$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{d^3}{6}$
3.		$A = 1.414hb$	$\bar{x} = b/2$ $\bar{y} = d/2$	$I_u = \dfrac{bd^2}{2}$

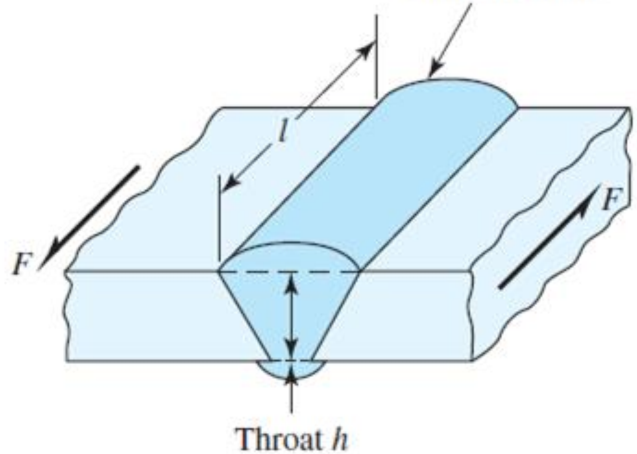
9.1 Welding Symbols

Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0





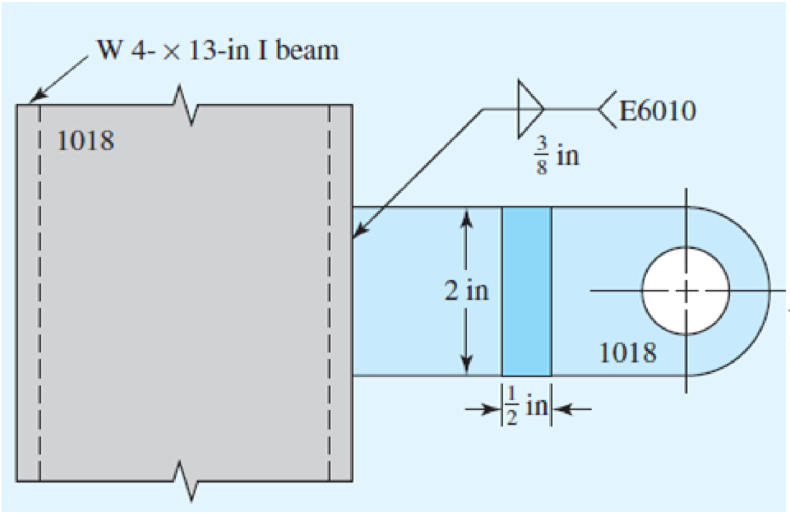
(b) Shear loading

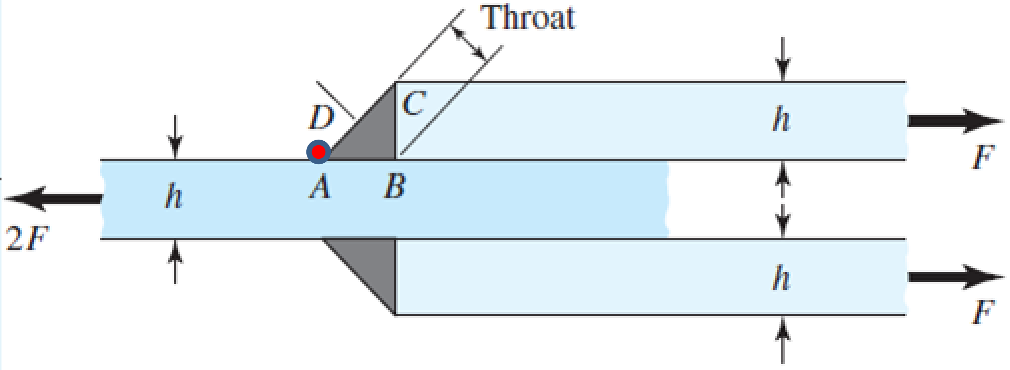
9.1 Welding Symbols

Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0





9.1 Welding Symbols

Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
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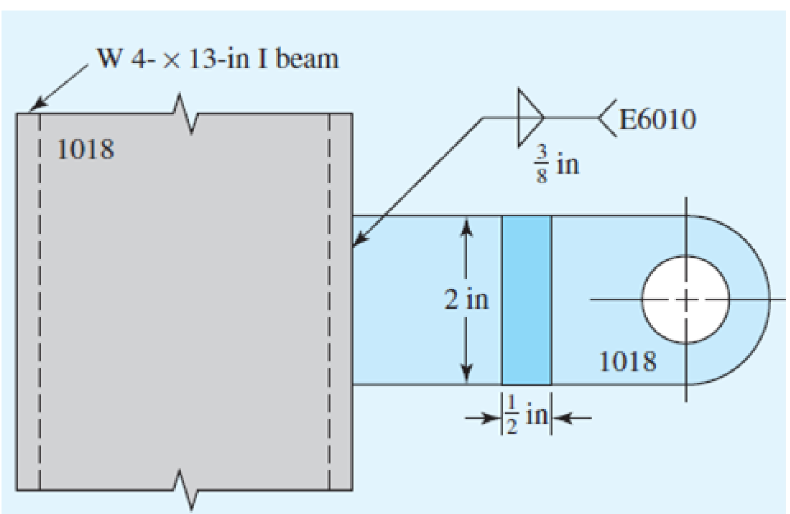
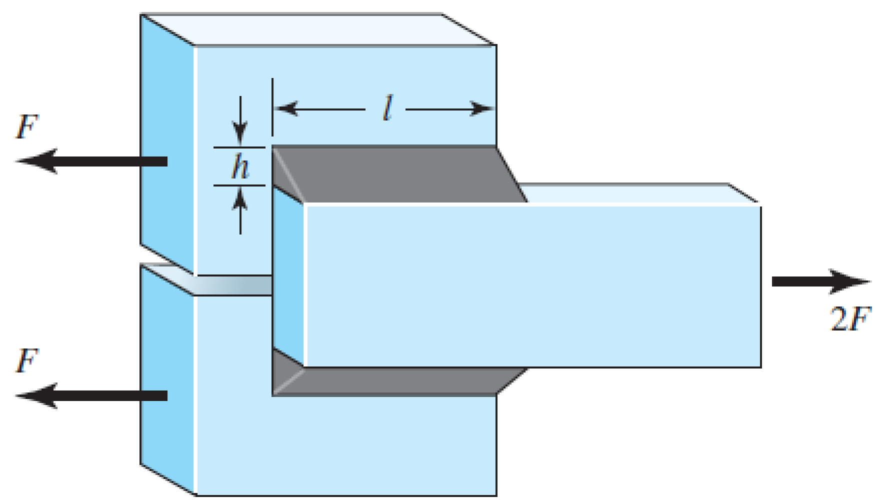


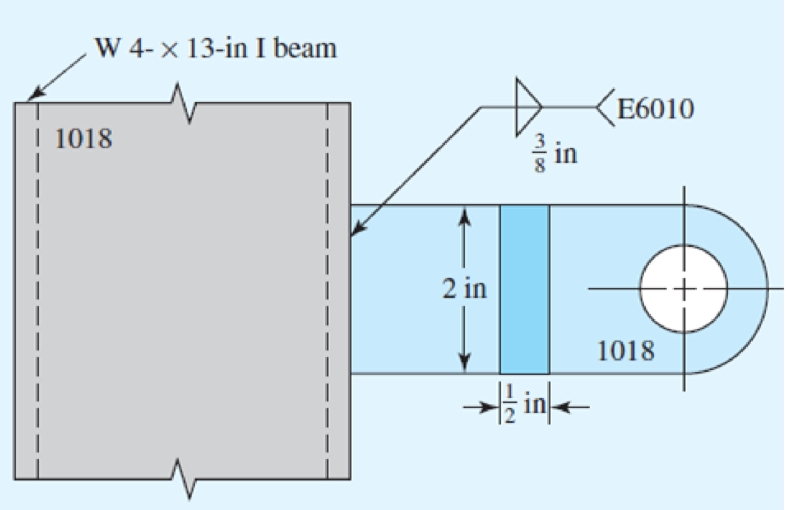


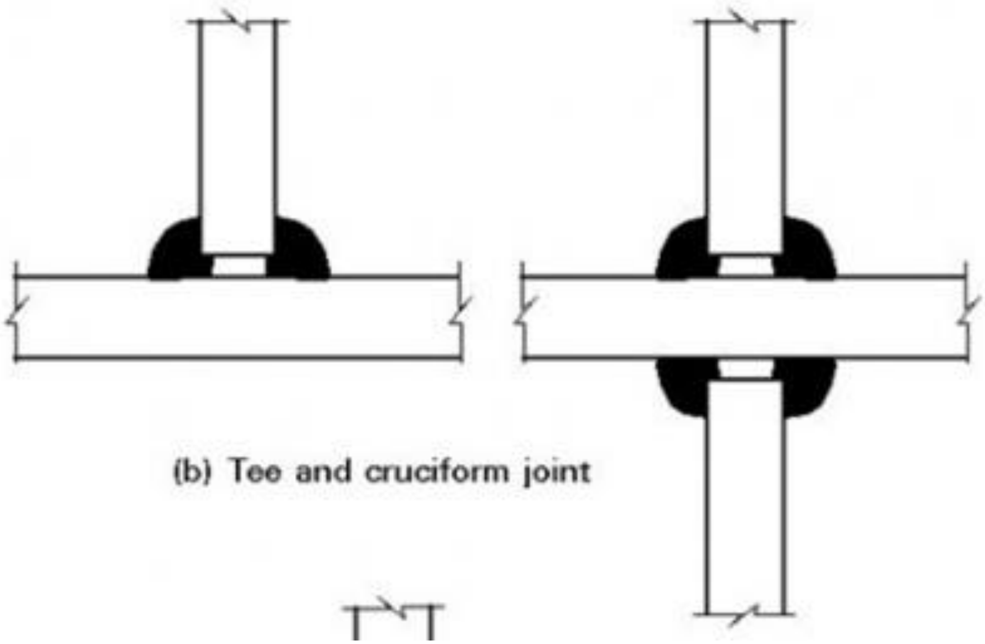
9.1 Welding Symbols

Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0





9.7 Fatigue Loading

EXAMPLE 9–6 The AISI 1018 HR steel strap of Fig. 9–22 has a repeatedly applied load of 2000 lbf ($F_a = F_m = 1000$ lbf). Determine the fatigue factor of safety fatigue strength of the weldment.

From Table 9–5, $K_{fs} = 2$. Only primary shear is present:

$$\tau'_a = \tau'_m = \frac{K_{fs} F_a}{A} = \frac{2(1000)}{1.061} = 1885 \text{ psi}$$

Table 9–5

Fatigue Stress-Concentration Factors, K_{fs}

Type of Weld	K_{fs}
Reinforced butt weld	1.2
Toe of transverse fillet weld	1.5
End of parallel fillet weld	2.7
T-butt joint with sharp corners	2.0

9.7 Fatigue Loading

EXAMPLE 9–6 The AISI 1018 HR steel strap of Fig. 9–22 has a repeatedly applied load of 2000 lbf ($F_a = F_m =$ 1000 lbf). Determine the fatigue factor of safety fatigue strength of the weldment.

$$n_f = \frac{1}{2}\left(\frac{0.67S_{ut}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{0.67S_{ut}\tau_a}\right)^2}\right]$$

$$n_f = \frac{1}{2}\left(\frac{0.67S_{ut}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{0.67S_{ut}\tau_a}\right)^2}\right]$$

$$n_f = \frac{1}{2}\left[\frac{0.67(58)}{1.885}\right]^2 \frac{1.885}{12.0}\left\{-1 + \sqrt{1 + \left[\frac{2(1.885)12.0}{0.67(58)1.885}\right]^2}\right\} = 5.85$$

10.0 Introduction

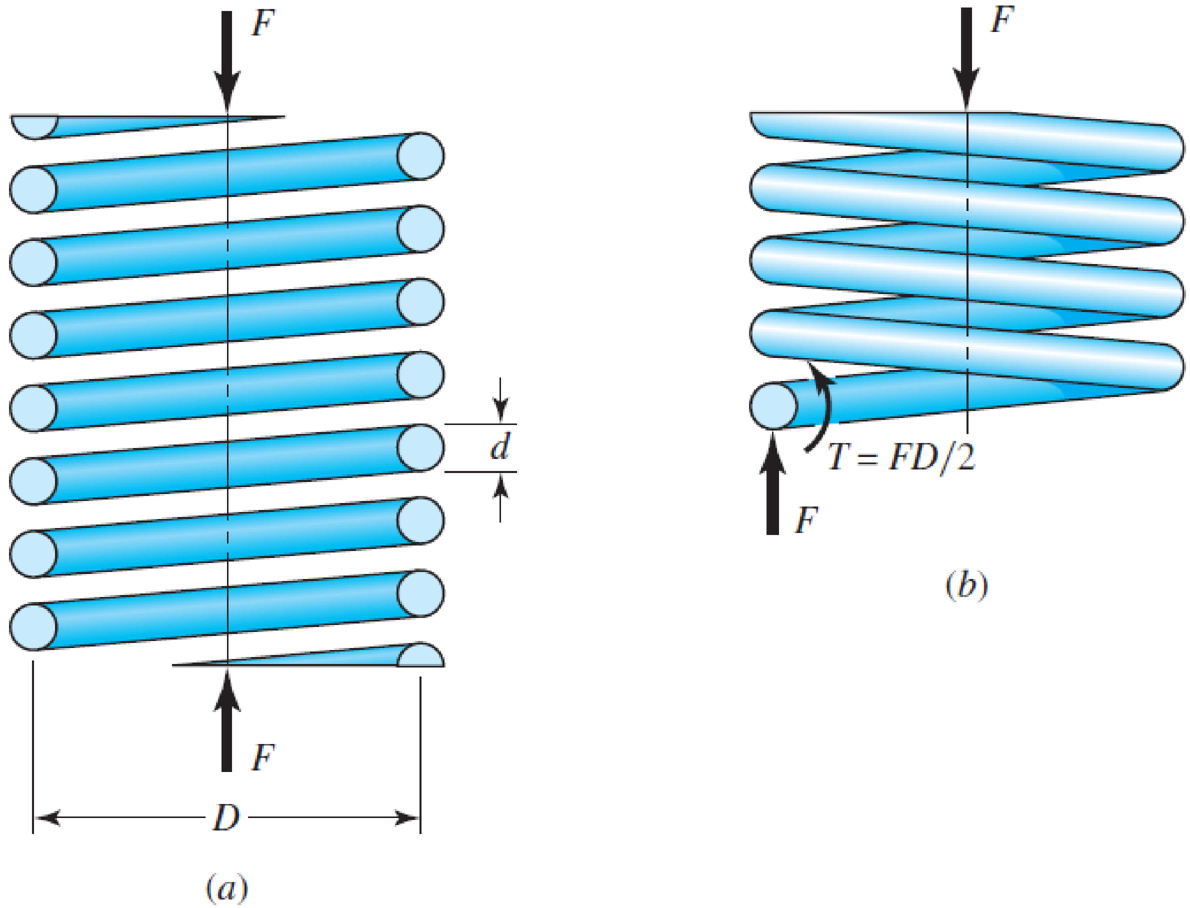
Wire Spring

Flat Spring

10.1 Stresses in Helical Springs

Figure 10–1

(*a*) Axially loaded helical spring; (*b*) free-body diagram showing that the wire is subjected to a direct shear and a torsional shear.



$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A} \Rightarrow \tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2} \Rightarrow \tau = K_s \frac{8FD}{\pi d^3}$$

10.1 Stresses in Helical Springs

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A} \quad \Rightarrow \quad \tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2} \quad \Rightarrow \quad \tau = K_s \frac{8FD}{\pi d^3}$$

Now we define the *spring index*

$$C = \frac{D}{d}$$

where K_s is a *shear stress-correction factor* and is defined by the equation

$$K_s = \frac{2C + 1}{2C}$$

10.2 The Curvature Effect

Without Curvature Effect

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A} \quad \Rightarrow \quad \tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2} \quad \Rightarrow \quad \tau = K_s \frac{8FD}{\pi d^3}$$

Shear stress correction factor $$K_s = \frac{2C + 1}{2C}$$

With Curvature Effect

$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A} \quad \Rightarrow \quad \tau = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2} \quad \Rightarrow \quad \tau = K_B \frac{8FD}{\pi d^3}$$

$$\tau = K_W \frac{8FD}{\pi d^3}$$

Bergsträsser factor $$K_B = \frac{4C + 2}{4C - 3}$$

Wahl factor $$K_W = \frac{4C - 1}{4C - 4} + \frac{0.615}{C}$$

10.3 Deflection of Helical Springs

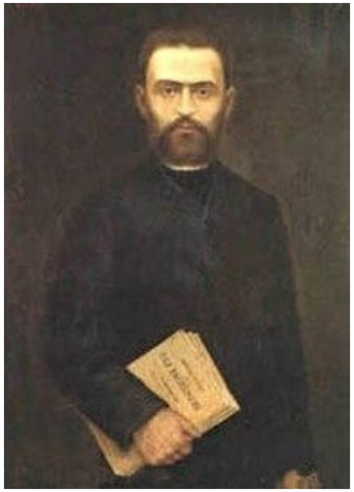
Total Strain Energy

$$U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$$

$$U = \frac{4F^2 D^3 N}{d^4 G} + \frac{2F^2 DN}{d^2 G}$$

Castigliano's Theorem

$$y = \frac{\partial U}{\partial F} = \frac{8FD^3 N}{d^4 G} + \frac{4FDN}{d^2 G} \approx \frac{8FD^3 N}{d^4 G}$$

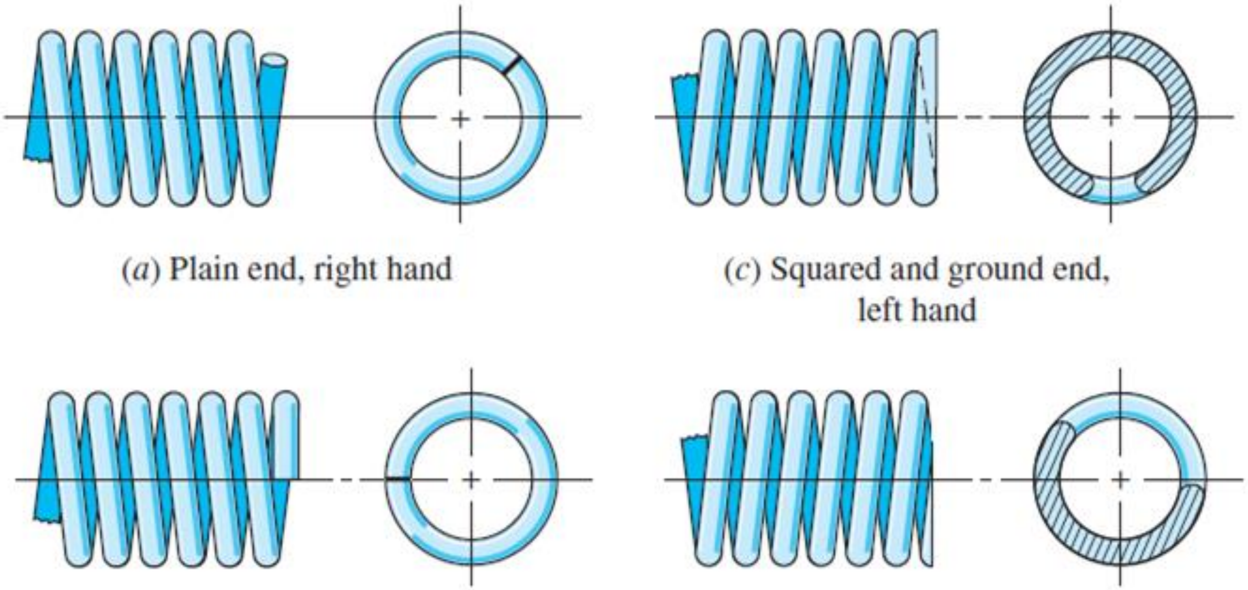


$$k \approx \frac{d^4 G}{8D^3 N}$$

10.4 Compression Springs

Figure 10–2

Types of ends for compression springs: (*a*) both ends plain; (*b*) both ends squared; (*c*) both ends squared and ground; (*d*) both ends plain and ground.



(*a*) Plain end, right hand

(*c*) Squared and ground end, left hand

(*b*) Squared or closed end, right hand

(*d*) Plain end, ground, left hand

Table 10–1

Formulas for the Dimensional Characteristics of Compression-Springs. (N_a = Number of Active Coils)

Source: From *Design Handbook*, 1987, p. 32. Courtesy of Associated Spring.

	Type of Spring Ends			
Term	**Plain**	**Plain and Ground**	**Squared or Closed**	**Squared and Ground**
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

10.5 Stability

Critical Deflection

$$y_{cr} = L_0 C_1' \left[1 - \left(1 - \frac{C_2'}{\lambda_{eff}^2} \right)^{1/2} \right] \qquad (10\text{–}10)$$

Effective Slenderness ratio

$$\lambda_{eff} = \frac{\alpha L_0}{D} \qquad (10\text{–}11)$$

Table 10–2

End-Condition Constants α for Helical Compression Springs*

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

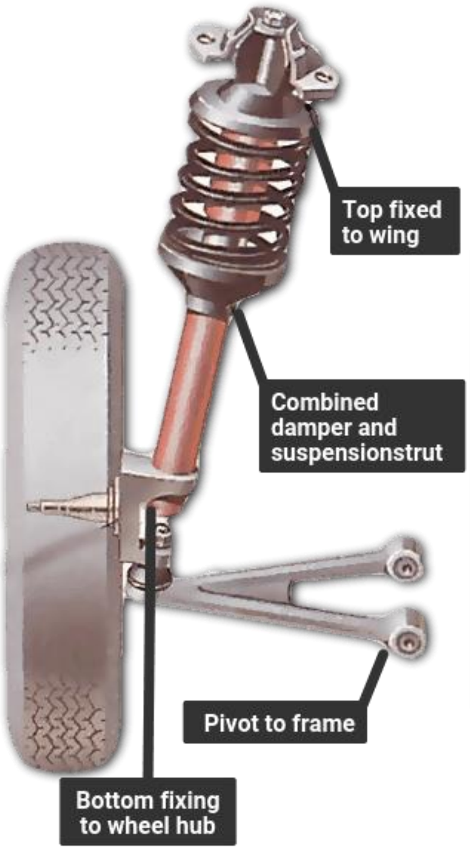
*Ends supported by flat surfaces must be squared and ground.

Dimensionless Elastic Constants

$$C_1' = \frac{E}{2(E - G)}$$

$$C_2' = \frac{2\pi^2(E - G)}{2G + E}$$

10.5 Stability



End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.

10.5 Stability

Critical Deflection

$$y_{cr} = L_0 C'_1 \left[1 - \left(1 - \frac{C'_2}{\lambda_{eff}^2} \right)^{1/2} \right] \qquad (10\text{–}10)$$

To ensure stability y *(Deflection)* must be < y_{cr} ⇨ $\frac{C'_2}{\lambda_{eff}^2} < 1$

$$L_0 < \frac{\pi D}{\alpha} \left[\frac{2(E - G)}{2G + E} \right]^{1/2} \qquad (10\text{–}12)$$

For steels, this turns out to be (No Bucking occurs):

$$L_0 < 2.63 \frac{D}{\alpha}$$

10.6 Spring Materials

Table 10–3

High-Carbon and Alloy Spring Steels

Source: From Harold C. R. Carlson, "Selection and Application of Spring Materials," *Mechanical Engineering*, vol. 78, 1956, pp. 331–334.

Name of Material	Similar Specifications	Description
Music wire, 0.80–0.95*C*	UNS G10850 AISI 1085 ASTM A228-51	This is the best, toughest, and most widely used of all spring materials for small springs. It has the highest tensile strength and can withstand higher stresses under repeated loading than any other spring material. Available in diameters 0.12 to 3 mm (0.005 to 0.125 in). Do not use above 120°C (250°F) or at subzero temperatures.
Oil-tempered wire, 0.60–0.70*C*	UNS G10650 AISI 1065 ASTM 229-41	This general-purpose spring steel is used for many types of coil springs where the cost of music wire is prohibitive and in sizes larger than available in music wire. Not for shock or impact loading. Available in diameters 3 to 12 mm (0.125 to 0.5000 in), but larger and smaller sizes may be obtained. Not for use above 180°C (350°F) or at subzero temperatures.
Hard-drawn wire, 0.60–0.70*C*	UNS G10660 AISI 1066 ASTM A227-47	This is the cheapest general-purpose spring steel and should be used only where life, accuracy, and deflection are not too important. Available in diameters 0.8 to 12 mm (0.031 to 0.500 in). Not for use above 120°C (250°F) or at subzero temperatures.

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-
-

10.6 Spring Materials

Table 10–4

Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

Source: From *Design Handbook,* 1987, p. 19. Courtesy of Associated Spring.

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

*Surface is smooth, free of defects, and has a bright, lustrous finish.

†Has a slight heat-treating scale which must be removed before plating.

‡Surface is smooth and bright with no visible marks.

§Aircraft-quality tempered wire, can also be obtained annealed.

‖Tempered to Rockwell C49, but may be obtained untempered.

#Type 302 stainless steel.

**Temper CA510.

$$S_{ut} = \frac{A}{d^m} \qquad (10\text{–}14)$$

10.6 Spring Materials

Table 10–5

Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of S_{ut} Tension	Elastic Limit, Percent of S_{ut} Torsion	Diameter d, in	E Mpsi	E GPa	G Mpsi	G GPa
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60–70	45–55	<0.032	28.8	198.6	11.7	80.7
			0.033–0.063	28.7	197.9	11.6	80.0
			0.064–0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85–90	45–50		28.5	196.5	11.2	77.2
Valve spring A230	85–90	50–60		29.5	203.4	11.2	77.2
Chrome-vanadium A231	88–93	65–75		29.5	203.4	11.2	77.2
A232	88–93			29.5	203.4	11.2	77.2
Chrome-silicon A401	85–93	65–75		29.5	203.4	11.2	77.2
Stainless steel							
A313*	65–75	45–55		28	193	10	69.0
17-7PH	75–80	55–60		29.5	208.4	11	75.8
414	65–70	42–55		29	200	11.2	77.2
420	65–75	45–55		29	200	11.2	77.2
431	72–76	50–55		30	206	11.5	79.3
Phosphor-bronze B159	75–80	45–50		15	103.4	6	41.4
Beryllium-copper B197	70	50		17	117.2	6.5	44.8
	75	50–55		19	131	7.3	50.3
Inconel alloy X-750	65–70	40–45		31	213.7	11.2	77.2

*Also includes 302, 304, and 316.

Note: See Table 10–6 for allowable torsional stress design values.

10.6 Spring Materials

Table 10–6

Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications

Source: Robert E. Joerres, "Springs," Chap. 6 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design,* 3rd ed., McGraw-Hill, New York, 2004.

	Maximum Percent of Tensile Strength	
Material	Before Set Removed (includes K_W or K_B)	After Set Removed (includes K_s)
Music wire and cold-drawn carbon steel	45	60–70
Hardened and tempered carbon and low-alloy steel	50	65–75
Austenitic stainless steels	35	55–65
Nonferrous alloys	35	55–65

Yield Shear Strength = Ultimate Tensile Strength x Maximum Percent of Tensile Strength

$$S_{sy} = \text{Maximum Percent of Tensile Strength} \times S_{ut}$$

Example 10-1

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EXAMPLE 10–1

A helical compression spring is made of no. 16 music wire. The outside coil diameter of the spring is $\frac{7}{16}$ in. The ends are squared and there are $12\frac{1}{2}$ total turns.

(*a*) Estimate the torsional yield strength of the wire.

(*b*) Estimate the static load corresponding to the yield strength.

(*c*) Estimate the scale of the spring.

(*d*) Estimate the deflection that would be caused by the load in part (*b*).

(*e*) Estimate the solid length of the spring.

(*f*) What length should the spring be to ensure that when it is compressed solid and then released, there will be no permanent change in the free length?

(*g*) Given the length found in part (*f*), is buckling a possibility?

(*h*) What is the pitch of the body coil?

Example 10-1

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EXAMPLE 10–1 A helical compression spring is made of no. 16 music wire. The outside coil diameter of the spring is $\frac{7}{16}$ in. The ends are squared and there are $12\frac{1}{2}$ total turns.
(*a*) Estimate the torsional yield strength of the wire.

$$S_{sy} = \text{Maximum Percent of Tensile Strength x } S_{ut}$$

Table 10–6

Maximum Allowable Torsional Stresses for Helical Compression Springs in Static Applications

Source: Robert E. Joerres, "Springs," Chap. 6 in Joseph E. Shigley, Charles R. Mischke, and Thomas H. Brown, Jr. (eds.), *Standard Handbook of Machine Design,* 3rd ed., McGraw-Hill, New York, 2004.

	Maximum Percent of Tensile Strength	
Material	Before Set Removed (includes K_W or K_B)	After Set Removed (includes K_s)
Music wire and cold-drawn carbon steel	$\boxed{45}$	60–70
Hardened and tempered carbon and low-alloy steel	50	65–75
Austenitic stainless steels	35	55–65
Nonferrous alloys	35	55–65

10.6 Spring Materials

Table 10–4

Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

Source: From *Design Handbook*, 1987, p. 19. Courtesy of Associated Spring.

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

*Surface is smooth, free of defects, and has a bright, lustrous finish.

†Has a slight heat-treating scale which must be removed before plating.

‡Surface is smooth and bright with no visible marks.

§Aircraft-quality tempered wire, can also be obtained annealed.

‖Tempered to Rockwell C49, but may be obtained untempered.

#Type 302 stainless steel.

**Temper CA510.

$$S_{ut} = \frac{A}{d^m} \qquad (10\text{–}14)$$

Example 10-1

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Table A–28

Decimal Equivalents of Wire and Sheet-Metal Gauges* (All Sizes Are Given in Inches)

Name of Gauge:	American or Brown & Sharpe	Birmingham or Stubs Iron Wire	United States Standard†	Manufacturers Standard	Steel Wire or Washburn & Moen	Music Wire	Stubs Steel Wire	Twist Drill
Principal Use:	Nonferrous Sheet, Wire, and Rod	Tubing, Ferrous Strip, Flat Wire, and Spring Steel	Ferrous Sheet and Plate, 480 lbf/ft^3	Ferrous Sheet	Ferrous Wire Except Music Wire	Music Wire	Steel Drill Rod	Twist Drills and Drill Steel
7/0			0.500		0.490			
6/0	0.580 0		0.468 75		0.461 5	0.004		
5/0	0.516 5		0.437 5		0.430 5	0.005		
4/0	0.460 0	0.454	0.406 25		0.393 8	0.006		
3/0	0.409 6	0.425	0.375		0.362 5	0.007		
2/0	0.364 8	0.380	0.343 75		0.331 0	0.008		
0	0.324 9	0.340	0.312 5		0.306 5	0.009		
1	0.289 3	0.300	0.281 25		0.283 0	0.010	0.227	0.228 0
2	0.257 6	0.284	0.265 625		0.262 5	0.011	0.219	0.221 0
3	0.229 4	0.259	0.25	0.239 1	0.243 7	0.012	0.212	0.213 0
4	0.204 3	0.238	0.234 375	0.224 2	0.225 3	0.013	0.207	0.209 0
5	0.181 9	0.220	0.218 75	0.209 2	0.207 0	0.014	0.204	0.205 5
6	0.162 0	0.203	0.203 125	0.194 3	0.192 0	0.016	0.201	0.204 0
7	0.144 3	0.180	0.187 5	0.179 3	0.177 0	0.018	0.199	0.201 0
8	0.128 5	0.165	0.171 875	0.164 4	0.162 0	0.020	0.197	0.199 0
9	0.114 4	0.148	0.156 25	0.149 5	0.148 3	0.022	0.194	0.196 0
10	0.101 9	0.134	0.140 625	0.134 5	0.135 0	0.024	0.191	0.193 5
11	0.090 74	0.120	0.125	0.119 6	0.120 5	0.026	0.188	0.191 0
12	0.080 81	0.109	0.109 357	0.104 6	0.105 5	0.029	0.185	0.189 0
13	0.071 96	0.095	0.093 75	0.089 7	0.091 5	0.031	0.182	0.185 0
14	0.064 08	0.083	0.078 125	0.074 7	0.080 0	0.033	0.180	0.182 0
15	0.057 07	0.072	0.070 312 5	0.067 3	0.072 0	0.035	0.178	0.180 0
16	0.050 82	0.065	0.062 5	0.059 8	0.062 5	0.037	0.175	0.177 0
17	0.045 26	0.058	0.056 25	0.053 8	0.054 0	0.039	0.172	0.173 0

Example 10-1

(a) From Table A–28, the wire diameter is $d = 0.037$ in. From Table 10–4, we find $A = 201$ kpsi · in^m and $m = 0.145$. Therefore, from Eq. (10–14)

$$S_{ut} = \frac{A}{d^m} = \frac{201}{0.037^{0.145}} = 324 \text{ kpsi}$$

Then, from Table 10–6,

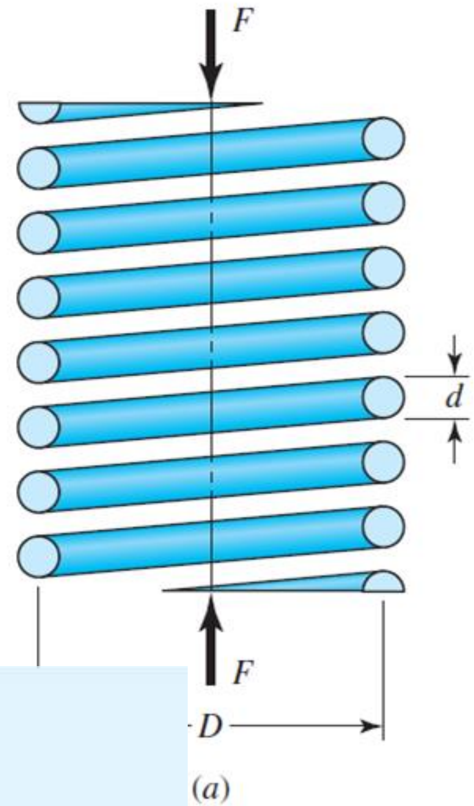
$$S_{sy} = 0.45 S_{ut} = 0.45(324) = 146 \text{ kpsi}$$

Example 10-1

EXAMPLE 10–1 A helical compression spring is made of no. 16 music wire. The outside coil diameter of the spring is $\frac{7}{16}$ in. The ends are squared and there are $12\frac{1}{2}$ total turns.

(*b*) Estimate the static load corresponding to the yield strength.

$$\tau = K_B \frac{8FD}{\pi d^3} \Rightarrow F = \frac{\pi d^3 S_{sy}}{8 K_B D}$$



$$K_B = \frac{4C+2}{4C-3} = \frac{4(10.8)+2}{4(10.8)-3} = 1.124$$

$$C = \frac{D}{d} \Rightarrow C = 0.400/0.037 = 10.8.$$

$$F = \frac{\pi d^3 S_{sy}}{8K_B D} = \frac{\pi(0.037^3)146(10^3)}{8(1.124)0.400} = 6.46 \text{ lbf}$$

7 = 0.400 in,

Example 10-1

EXAMPLE 10–1 A helical compression spring is made of no. 16 music wire. The outside coil diameter of the spring is $\frac{7}{16}$ in. The ends are squared and there are $12\frac{1}{2}$ total turns.

(*c*) Estimate the scale of the spring.

$$k = \frac{d^4 G}{8 D^3 N_a}$$

A helical compression spring is made of no. 16 music wire. The outside coil diameter of the spring is $\frac{7}{16}$ in. The ends are squared and there are $12\frac{1}{2}$ total turns.

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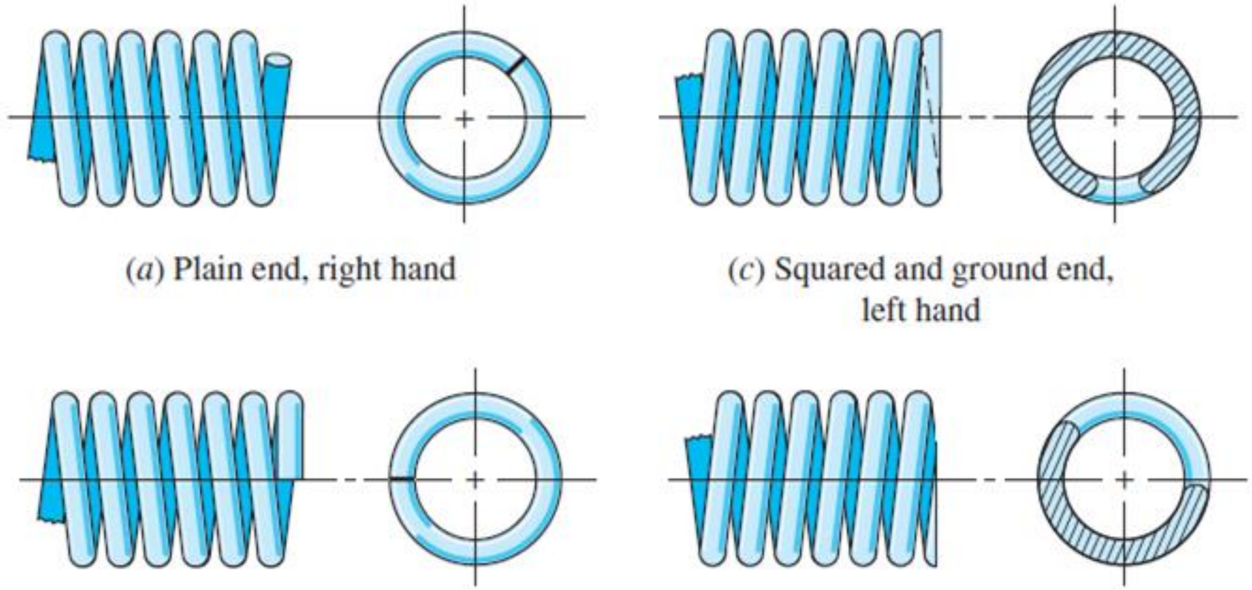
Figure 10–2

Types of ends for compression springs: (*a*) both ends plain; (*b*) both ends squared; (*c*) both ends squared and ground; (*d*) both ends plain and ground.



(*a*) Plain end, right hand

(*c*) Squared and ground end, left hand

(*b*) Squared or closed end, right hand

(*d*) Plain end, ground, left hand

Table 10–1

Formulas for the Dimensional Characteristics of Compression-Springs. (N_a = Number of Active Coils)

Source: From *Design Handbook*, 1987, p. 32. Courtesy of Associated Spring.

Term	Plain	Plain and Ground	Squared or Closed	Squared and Ground
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

10.6 Spring Materials

Table 10–5

Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of S_{ut} Tension	Torsion	Diameter d, in	E Mpsi	GPa	G Mpsi	GPa
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60–70	45–55	<0.032	28.8	198.6	11.7	80.7
			0.033–0.063	28.7	197.9	11.6	80.0
			0.064–0.125	28.6	197.2	11.5	79.3
			>0.125	28.5	196.5	11.4	78.6
Oil tempered A239	85–90	45–50		28.5	196.5	11.2	77.2
Valve spring A230	85–90	50–60		29.5	203.4	11.2	77.2
Chrome-vanadium A231	88–93	65–75		29.5	203.4	11.2	77.2
A232	88–93			29.5	203.4	11.2	77.2
Chrome-silicon A401	85–93	65–75		29.5	203.4	11.2	77.2
Stainless steel							
A313*	65–75	45–55		28	193	10	69.0
17-7PH	75–80	55–60		29.5	208.4	11	75.8
414	65–70	42–55		29	200	11.2	77.2
420	65–75	45–55		29	200	11.2	77.2
431	72–76	50–55		30	206	11.5	79.3
Phosphor-bronze B159	75–80	45–50		15	103.4	6	41.4
Beryllium-copper B197	70	50		17	117.2	6.5	44.8
	75	50–55		19	131	7.3	50.3
Inconel alloy X-750	65–70	40–45		31	213.7	11.2	77.2

*Also includes 302, 304, and 316.

Note: See Table 10–6 for allowable torsional stress design values.

10.9 Fatigue Loading of Helical Compression Springs

Zimmerli Data

Unpeened:

$$S_{sa} = 35 \text{ kpsi } (241 \text{ MPa}) \qquad S_{sm} = 55 \text{ kpsi } (379 \text{ MPa}) \tag{10–28}$$

Peened:

$$S_{sa} = 57.5 \text{ kpsi } (398 \text{ MPa}) \qquad S_{sm} = 77.5 \text{ kpsi } (534 \text{ MPa}) \tag{10–29}$$

For example, given an unpeened spring with $S_{su} = 211.5$ kpsi, the Gerber ordinate intercept for shear, from Eq. (6–42), p. 314, is

$$S_{se} = \frac{S_{sa}}{1 - \left(\dfrac{S_{sm}}{S_{su}}\right)^2} = \frac{35}{1 - \left(\dfrac{55}{211.5}\right)^2} = 37.5 \text{ kpsi}$$

10.9 Fatigue Loading of Helical Compression Springs

$$S_{su} = 0.67 S_{ut} \tag{10–30}$$

$$F_a = \frac{F_{\max} - F_{\min}}{2} \tag{10–31a}$$

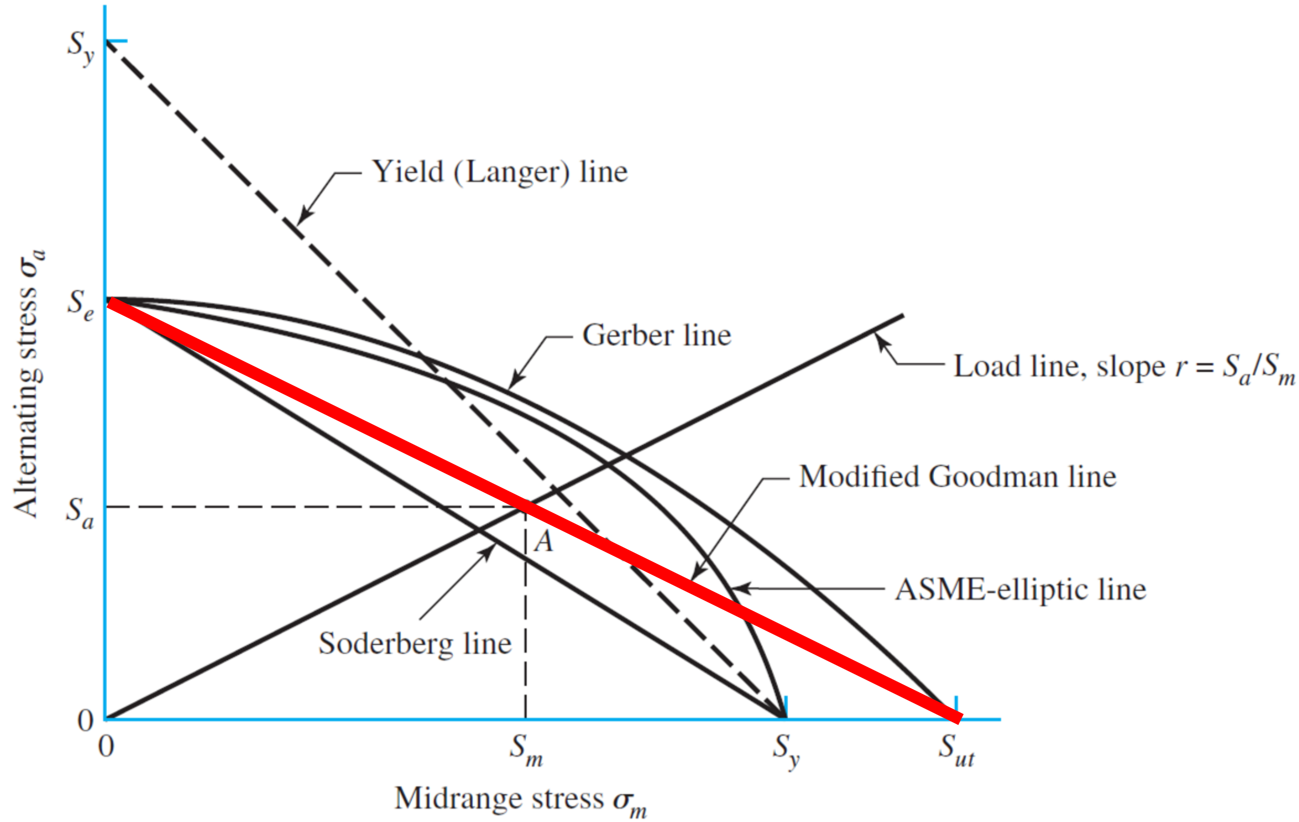
$$F_m = \frac{F_{\max} + F_{\min}}{2} \tag{10–31b}$$

$$\tau_a = K_B \frac{8 F_a D}{\pi d^3} \tag{10–32}$$

$$\tau_m = K_B \frac{8 F_m D}{\pi d^3} \tag{10–33}$$

Fatigue Failure Criteria for Fluctuating Stress

Modified Goodman line

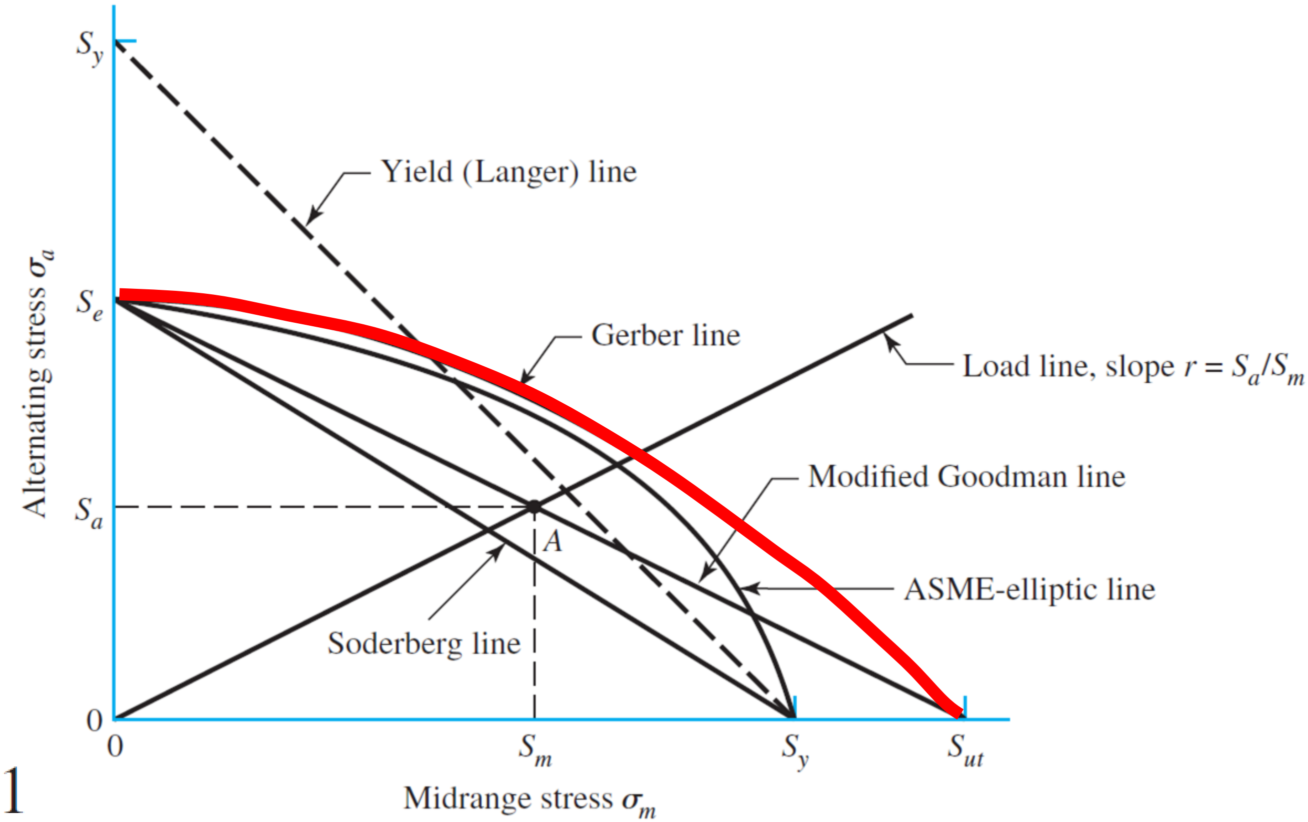


$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

Fatigue Failure Criteria for Fluctuating Stress

Gerber line



$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

10.9 Fatigue Loading of Helical Compression Springs

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EXAMPLE 10–4 An as-wound helical compression spring, made of music wire, has a wire size of 0.092 in, an outside coil diameter of $\frac{9}{16}$ in, a free length of $4\frac{3}{8}$ in, 21 active coils, and both ends squared and ground. The spring is unpeened. This spring is to be assembled with a preload of 5 lbf and will operate with a maximum load of 35 lbf during use. (*a*) Estimate the factor of safety guarding against fatigue-failure using a torsional Gerber fatigue-failure criterion with Zimmerli data.

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

$$n_f = \frac{1}{2}\left(\frac{0.67S_{ut}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{0.67S_{ut}\tau_a}\right)^2}\right]$$

$$S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2S_{se}}{rS_{su}}\right)^2}\right]$$

$$n_f = \frac{S_{sa}}{\tau_a}$$

10.9 Fatigue Loading of Helical Compression Springs

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Table 6–7

Amplitude and Steady Coordinates of Strength and Important Intersections in First Quadrant for Gerber and Langer Failure Criteria

Intersecting Equations	Intersection Coordinates
$\dfrac{S_a}{S_e} + \left(\dfrac{S_m}{S_{ut}}\right)^2 = 1$	$S_a = \dfrac{r^2 S_{ut}^2}{2S_e}\left[-1 + \sqrt{1 + \left(\dfrac{2S_e}{rS_{ut}}\right)^2}\right]$
Load line $r = \dfrac{S_a}{S_m}$	$S_m = \dfrac{S_a}{r}$
$\dfrac{S_a}{S_y} + \dfrac{S_m}{S_y} = 1$	$S_a = \dfrac{rS_y}{1 + r}$
Load line $r = \dfrac{S_a}{S_m}$	$S_m = \dfrac{S_y}{1 + r}$
$\dfrac{S_a}{S_e} + \left(\dfrac{S_m}{S_{ut}}\right)^2 = 1$	$S_m = \dfrac{S_{ut}^2}{2S_e}\left[1 - \sqrt{1 + \left(\dfrac{2S_e}{S_{ut}}\right)^2\left(1 - \dfrac{S_y}{S_e}\right)}\right]$
$\dfrac{S_a}{S_y} + \dfrac{S_m}{S_y} = 1$	$S_a = S_y - S_m,\ r_{\text{crit}} = S_a/S_m$

Fatigue factor of safety

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

An as-wound helical compression spring, made of music wire, has a wire size of 0.092 in, an outside coil diameter of $\frac{9}{16}$ in, a free length of $4\frac{3}{8}$ in, 21 active coils, and both ends squared and ground. The spring is unpeened. This spring is to be assembled

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Table 10–4

Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

Source: From *Design Handbook*, 1987, p. 19. Courtesy of Associated Spring.

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of Wire
Music wire*	A228	$\boxed{0.145}$	0.004–0.256	$\boxed{201}$	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

$$S_{ut} = \frac{A}{d^m} = \frac{201}{0.092^{0.145}} = 284.1 \text{ kpsi}$$

10.9 Fatigue Loading of Helical Compression Springs

EXAMPLE 10–4 An as-wound helical compression spring, made of music wire, has a wire size of 0.092 in, an outside coil diameter of $\frac{9}{16}$ in, a free length of $4\frac{3}{8}$ in, 21 active coils, and both ends squared and ground. The spring is unpeened. This spring is to be assembled with a preload of 5 lbf and will operate with a maximum load of 35 lbf during use. (*a*) Estimate the factor of safety guarding against fatigue-failure using a torsional Gerber fatigue-failure criterion with Zimmerli data.

$$F_a = \frac{35 - 5}{2} = 15 \text{ lbf} \qquad F_m = \frac{35 + 5}{2} = 20 \text{ lbf}$$

The alternating shear-stress component is found from Eq. (10–32) to be

$$\tau_a = K_B \frac{8F_a D}{\pi d^3} = (1.287)\frac{8(15)0.4705}{\pi(0.092)^3}(10^{-3}) = 29.7 \text{ kpsi}$$

Equation (10–33) gives the midrange shear-stress component

$$\tau_m = K_B \frac{8F_m D}{\pi d^3} = 1.287\frac{8(20)0.4705}{\pi(0.092)^3}(10^{-3}) = 39.6 \text{ kpsi}$$

10.9 Fatigue Loading of Helical Compression Springs

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EXAMPLE 10–4 An as-wound helical compression spring, made of music wire, has a wire size of 0.092 in, an outside coil diameter of $\frac{9}{16}$ in, a free length of $4\frac{3}{8}$ in, 21 active coils, and both ends squared and ground. The spring is unpeened. This spring is to be assembled with a preload of 5 lbf and will operate with a maximum load of 35 lbf during use.
(*a*) Estimate the factor of safety guarding against fatigue-failure using a torsional Gerber fatigue-failure criterion with Zimmerli data.

$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\,\right] \qquad \sigma_m > 0$$

$$\boxed{n_f = \frac{1}{2}\left(\frac{0.67S_{ut}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_{se}}{0.67S_{ut}\tau_a}\right)^2}\,\right]}$$

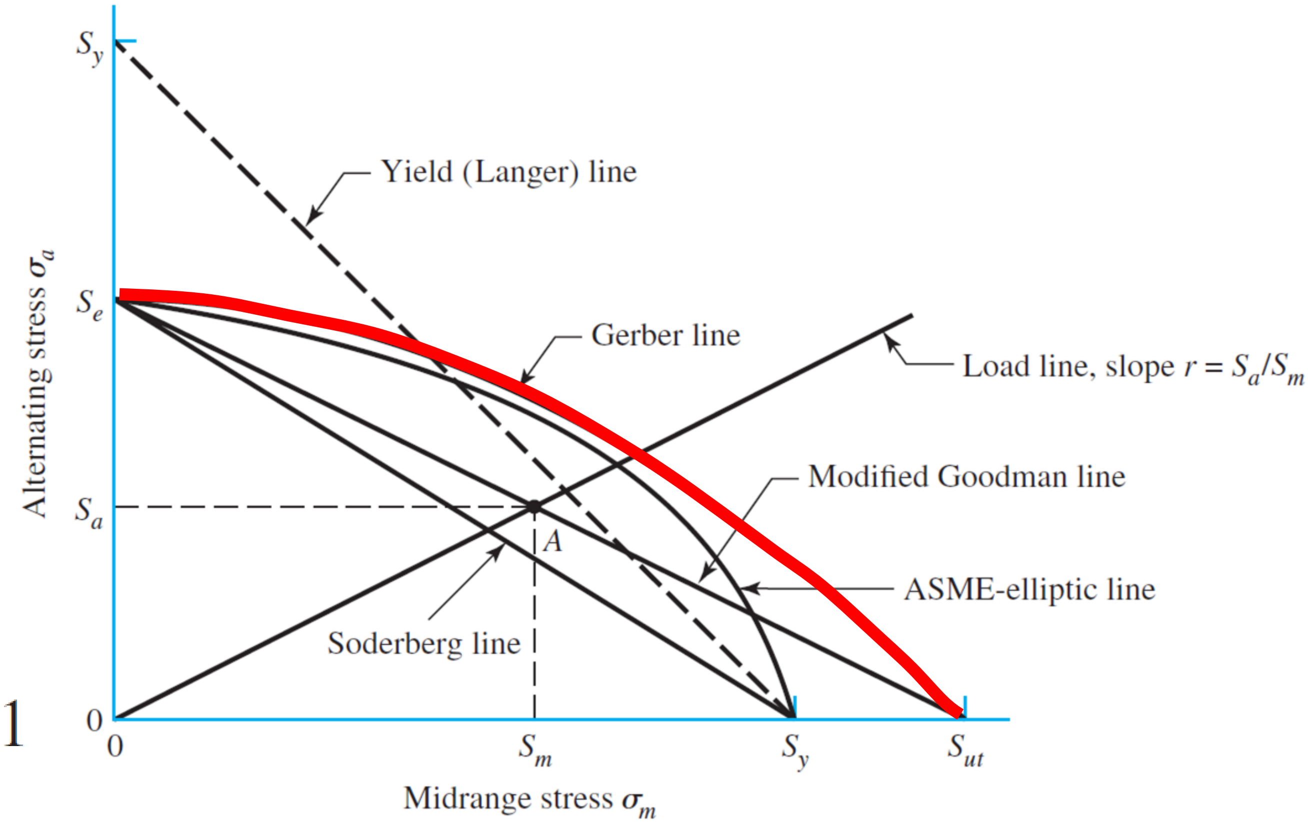
$$\boxed{S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2S_{se}}{rS_{su}}\right)^2}\,\right]} \qquad \boxed{n_f = \frac{S_{sa}}{\tau_a}}$$

Fatigue Failure Criteria for Fluctuating Stress

Gerber line

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1$$

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$



$$n_f = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2 \frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(\frac{2\sigma_m S_e}{S_{ut}\sigma_a}\right)^2}\right] \qquad \sigma_m > 0$$

The load-line slope $r = \tau_a/\tau_m = 29.7/39.6 = 0.75.$

10.9 Fatigue Loading of Helical Compression Springs

(a) The Gerber ordinate intercept for the Zimmerli data, Eq. (10–28), is

$$S_{se} = \frac{S_{sa}}{1 - (S_{sm}/S_{su})^2} = \frac{35}{1 - (55/190.3)^2} = 38.2 \text{ kpsi}$$

The amplitude component of strength S_{sa}, from Table 6–7, p. 315, is

$$S_{sa} = \frac{r^2 S_{su}^2}{2S_{se}}\left[-1 + \sqrt{1 + \left(\frac{2S_{se}}{rS_{su}}\right)^2}\right]$$

$$= \frac{0.75^2 190.3^2}{2(38.2)}\left\{-1 + \sqrt{1 + \left[\frac{2(38.2)}{0.75(190.3)}\right]^2}\right\} = 35.8 \text{ kpsi}$$

and the fatigue factor of safety n_f is given by

$$n_f = \frac{S_{sa}}{\tau_a} = \frac{35.8}{29.7} = 1.21$$

10.9 Fatigue Loading of Helical Compression Springs

EXAMPLE 10–4

An as-wound helical compression spring, made of music wire, has a wire size of 0.092 in, an outside coil diameter of $\frac{9}{16}$ in, a free length of $4\frac{3}{8}$ in, 21 active coils, and both ends squared and ground. The spring is unpeened. This spring is to be assembled with a preload of 5 lbf and will operate with a maximum load of 35 lbf during use.

(*a*) Estimate the factor of safety guarding against fatigue-failure using a torsional Gerber fatigue-failure criterion with Zimmerli data.

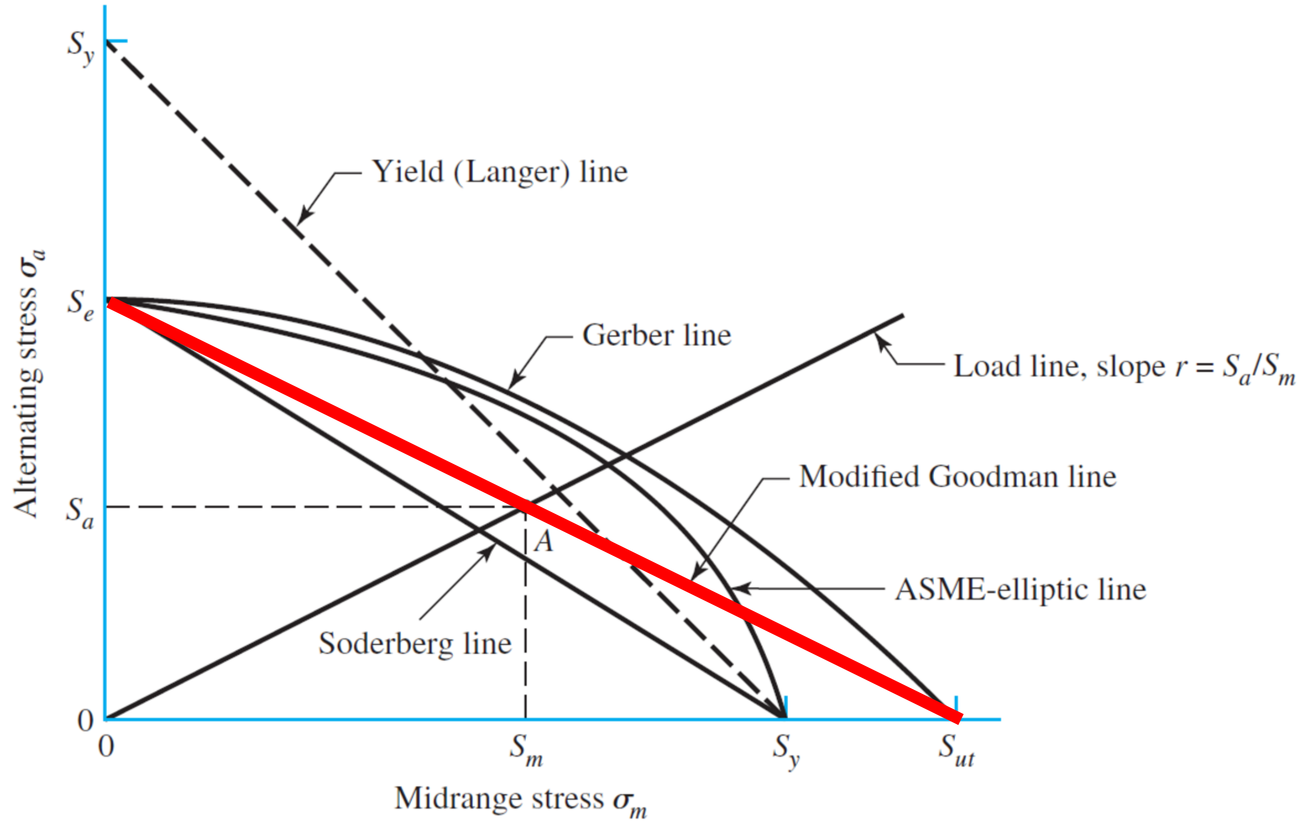
(*b*) Repeat part (*a*) using the Sines torsional fatigue criterion (steady stress component has no effect), with Zimmerli data.

(*c*) Repeat using a torsional Goodman failure criterion with Zimmerli data.

(*d*) Estimate the critical frequency of the spring.

Fatigue Failure Criteria for Fluctuating Stress

Modified Goodman line



$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1$$

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

10.9 Fatigue Loading of Helical Compression Springs

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Zimmerli Data

Unpeened:

$$S_{sa} = 35 \text{ kpsi } (241 \text{ MPa}) \qquad S_{sm} = 55 \text{ kpsi } (379 \text{ MPa}) \qquad (10\text{–}28)$$

Peened:

$$S_{sa} = 57.5 \text{ kpsi } (398 \text{ MPa}) \qquad S_{sm} = 77.5 \text{ kpsi } (534 \text{ MPa}) \qquad (10\text{–}29)$$

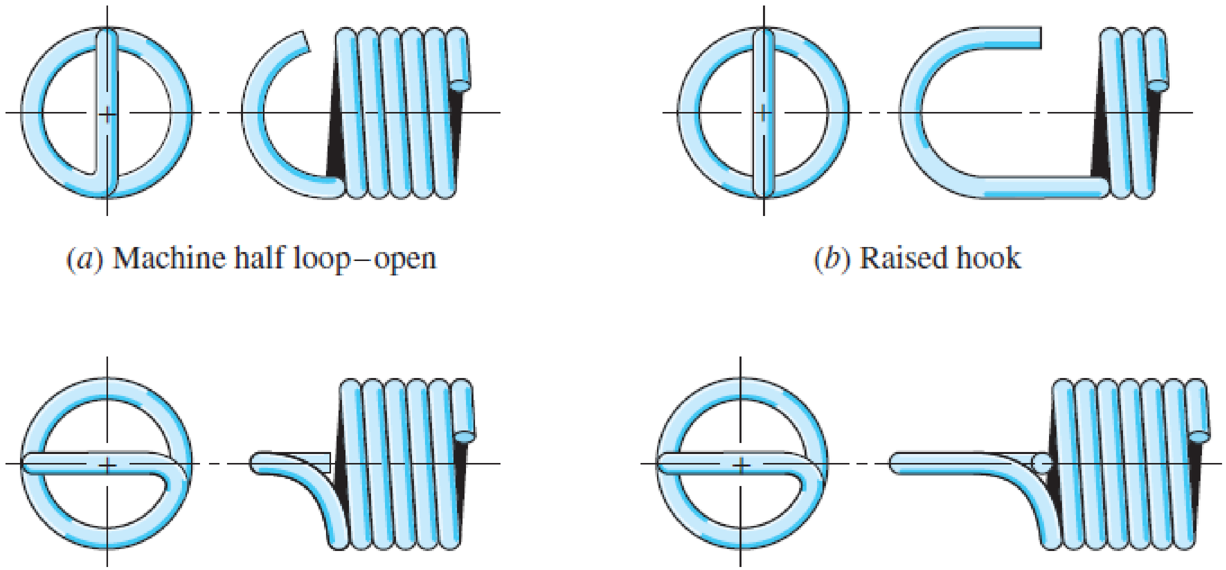
For example, given an unpeened spring with $S_{su} = 211.5$ kpsi, the Gerber ordinate intercept for shear, from Eq. (6–42), p. 314, is

$$S_{se} = \frac{S_{sa}}{1 - \left(\dfrac{S_{sm}}{S_{su}}\right)^2} = \frac{35}{1 - \left(\dfrac{55}{211.5}\right)^2} = 37.5 \text{ kpsi}$$

10.11 Extension Spring

Figure 10–5

Types of ends used on extension springs. *(Courtesy of Associated Spring.)*



(*a*) Machine half loop–open

(*b*) Raised hook

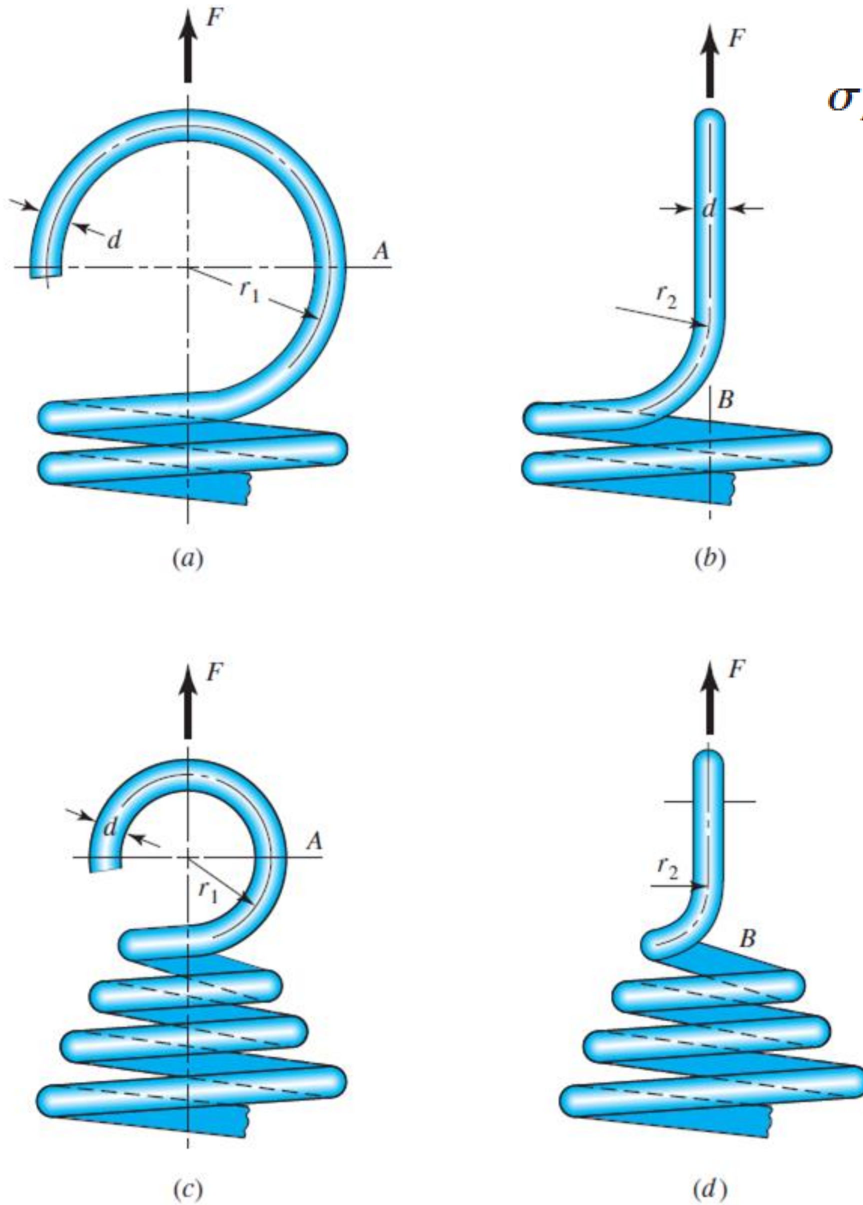
(*c*) Short twisted loop

(*d*) Full twisted loop

10.11 Extension Spring

Figure 10–6

Ends for extension springs. (*a*) Usual design; stress at *A* is due to combined axial force and bending moment. (*b*) Side view of part *a*; stress is mostly torsion at *B*. (*c*) Improved design; stress at *A* is due to combined axial force and bending moment. (*d*) Side view of part *c*; stress at *B* is mostly torsion.



$$\sigma_A = F\left[(K)_A \frac{16D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}$$

$$C_1 = \frac{2r_1}{d}$$

10.11 Extension Spring

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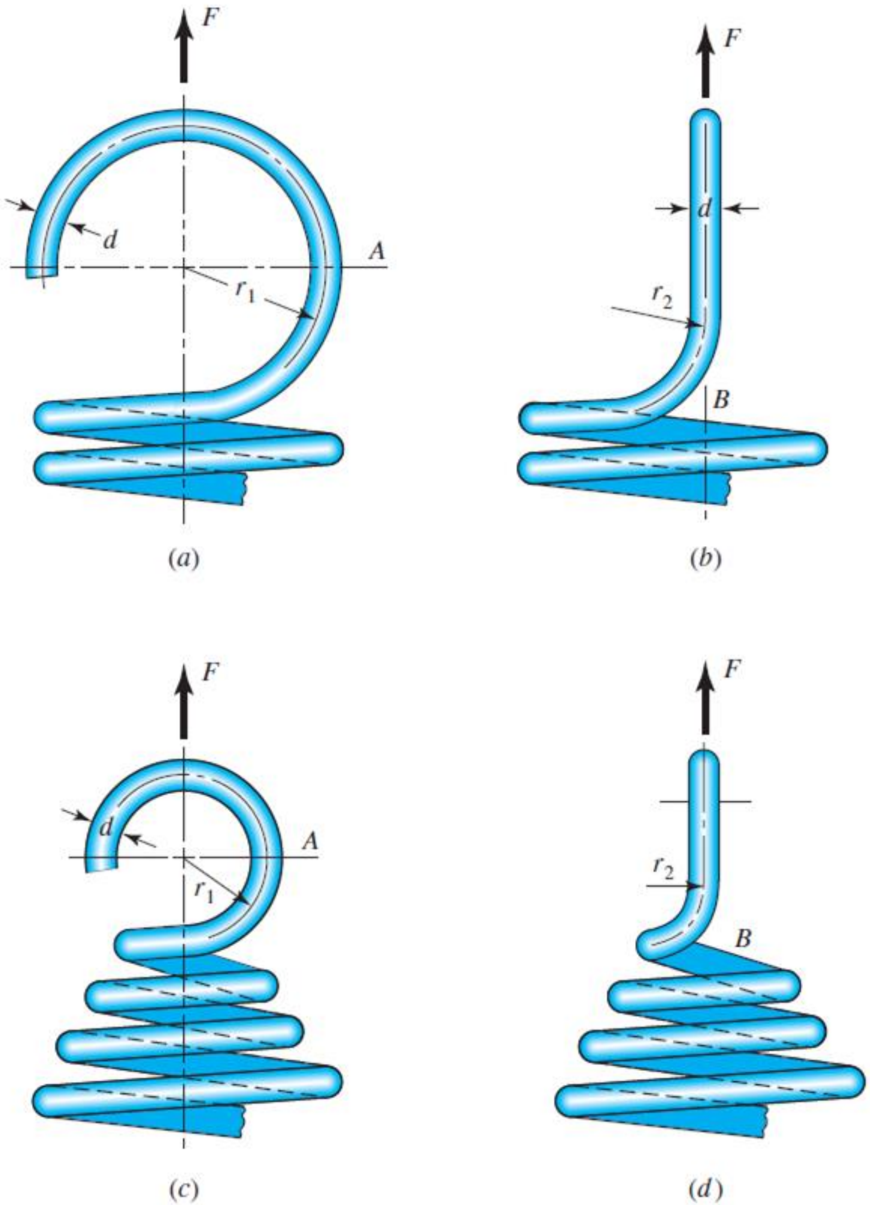
Figure 10–6

Ends for extension springs. (*a*) Usual design; stress at *A* is due to combined axial force and bending moment. (*b*) Side view of part *a*; stress is mostly torsion at *B*. (*c*) Improved design; stress at *A* is due to combined axial force and bending moment. (*d*) Side view of part *c*; stress at *B* is mostly torsion.



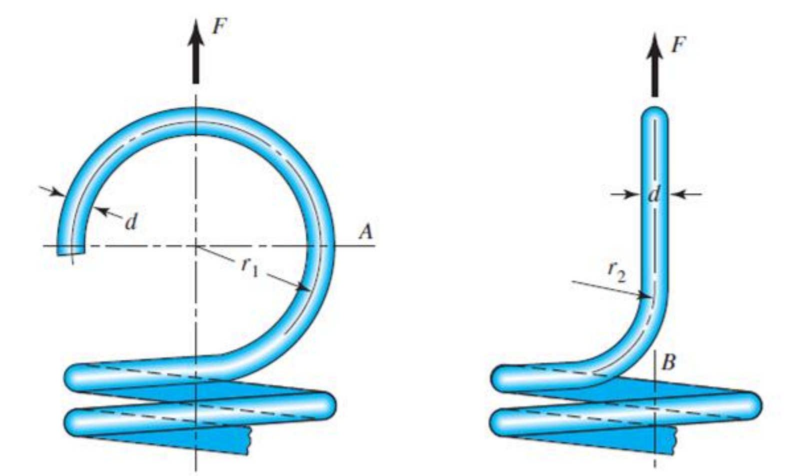
$$\tau_B = (K)_B \frac{8FD}{\pi d^3}$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4}$$

$$C_2 = \frac{2r_2}{d}$$

10.11 Extension Spring

EXAMPLE 10–7 The helical coil extension spring of Ex. 10–6 is subjected to a dynamic loading from 1.5 to 5 lbf. Estimate the factors of safety using the Gerber failure criterion for (*a*) coil fatigue, (*b*) coil yielding, (*c*) end-hook bending fatigue at point *A* of Fig. 10–6*a*, and (*d*) end-hook torsional fatigue at point *B* of Fig. 10–6*b*.



Solution A number of quantities are the same as in Ex. 10–6: $d = 0.035$ in, $S_{ut} = 264.7$ kpsi, $D = 0.213$ in, $r_1 = 0.106$ in, $C = 6.086$, $K_B = 1.234$, $(K)_A = 1.14$, $(K)_B = 1.18$, $N_b = 12.17$ turns, $L_0 = 0.817$ in, $k = 17.91$ lbf/in, $F_i = 1.19$ lbf, and $(\tau_i)_{\text{uncorr}} = 15.1$ kpsi. Then

$$F_a = (F_{\max} - F_{\min})/2 = (5 - 1.5)/2 = 1.75 \text{ lbf}$$

$$F_m = (F_{\max} + F_{\min})/2 = (5 + 1.5)/2 = 3.25 \text{ lbf}$$

The strengths from Ex. 10–6 include $S_{ut} = 264.7$ kpsi, $S_y = 198.5$ kpsi, and $S_{sy} = 119.1$ kpsi. The ultimate shear strength is estimated from Eq. (10–30) as

$$S_{su} = 0.67 S_{ut} = 0.67(264.7) = 177.3 \text{ kpsi}$$

10.11 Extension Spring

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(*a*) Body-coil fatigue:

$$\tau_a = \frac{8K_B F_a D}{\pi d^3} = \frac{8(1.234)1.75(0.213)}{\pi(0.035^3)}(10^{-3}) = 27.3 \text{ kpsi}$$

$$\tau_m = \frac{F_m}{F_a}\tau_a = \frac{3.25}{1.75}27.3 = 50.7 \text{ kpsi}$$

Using the Zimmerli data of Eq. (10–28) gives

$$S_{se} = \frac{S_{sa}}{1 - \left(\dfrac{S_{sm}}{S_{su}}\right)^2} = \frac{35}{1 - \left(\dfrac{55}{177.3}\right)^2} = 38.7 \text{ kpsi}$$

From Table 6–7, p. 315, the Gerber fatigue criterion for shear is

$$(n_f)_{\text{body}} = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(2\frac{\tau_m}{S_{su}}\frac{S_{se}}{\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{177.3}{50.7}\right)^2 \frac{27.3}{38.7}\left[-1 + \sqrt{1 + \left(2\frac{50.7}{177.3}\frac{38.7}{27.3}\right)^2}\right] = 1.24$$

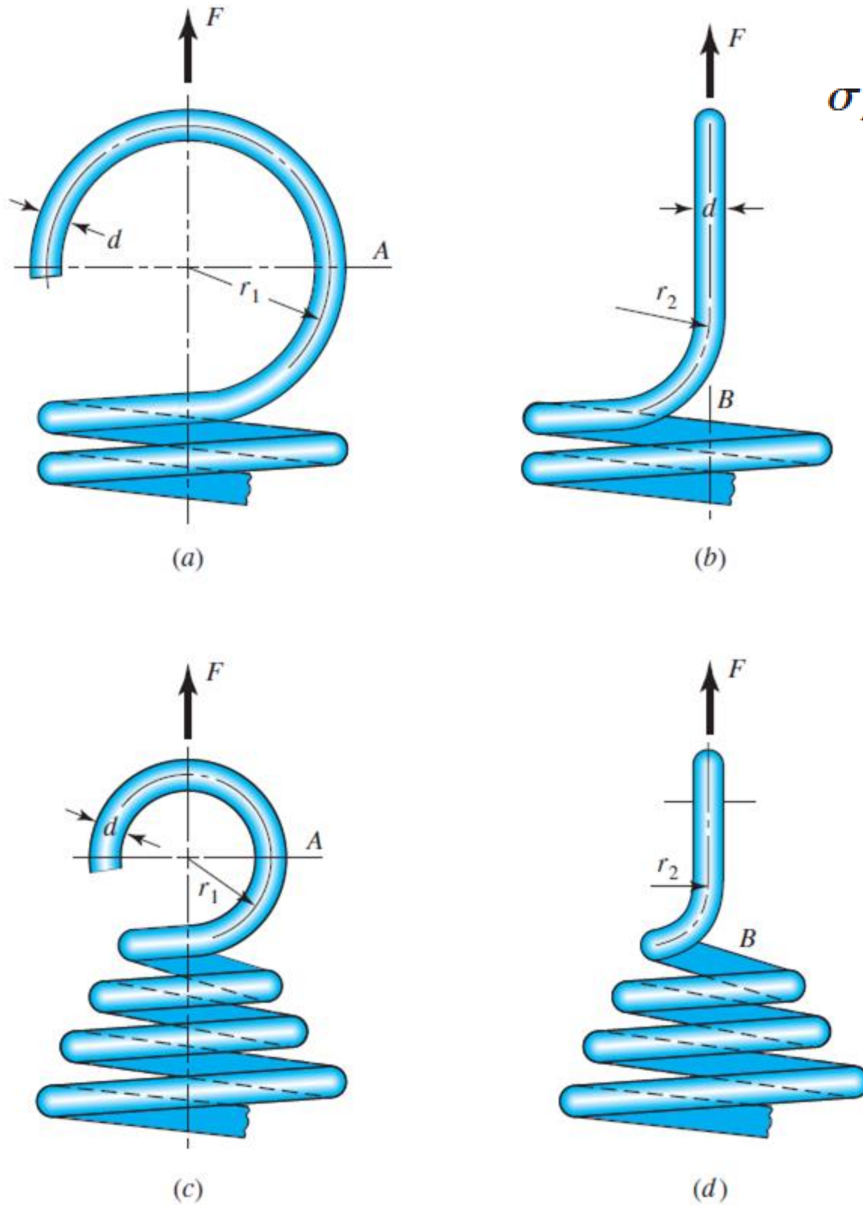
10.11 Extension Spring

EXAMPLE 10–7 The helical coil extension spring of Ex. 10–6 is subjected to a dynamic loading from 1.5 to 5 lbf. Estimate the factors of safety using the Gerber failure criterion for (*a*) coil fatigue, (*b*) coil yielding, (*c*) end-hook bending fatigue at point *A* of Fig. 10–6*a*, and (*d*) end-hook torsional fatigue at point *B* of Fig. 10–6*b*.

10.11 Extension Spring

Figure 10–6

Ends for extension springs. (*a*) Usual design; stress at *A* is due to combined axial force and bending moment. (*b*) Side view of part *a*; stress is mostly torsion at *B*. (*c*) Improved design; stress at *A* is due to combined axial force and bending moment. (*d*) Side view of part *c*; stress at *B* is mostly torsion.



$$\sigma_A = F\left[(K)_A \frac{16D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$(K)_A = \frac{4C_1^2 - C_1 - 1}{4C_1(C_1 - 1)}$$

$$C_1 = \frac{2r_1}{d}$$

10.11 Extension Spring

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EXAMPLE 10–7 The helical coil extension spring of Ex. 10–6 is subjected to a dynamic loading from 1.5 to 5 lbf. Estimate the factors of safety using the Gerber failure criterion for (*a*) coil fatigue, (*b*) coil yielding, (*c*) end-hook bending fatigue at point *A* of Fig. 10–6*a*, and (*d*) end-hook torsional fatigue at point *B* of Fig. 10–6*b*.

(*c*) End-hook bending fatigue: using Eqs. (10–34) and (10–35) gives

$$\sigma_a = F_a\left[(K)_A\frac{16D}{\pi d^3} + \frac{4}{\pi d^2}\right]$$

$$= 1.75\left[1.14\frac{16(0.213)}{\pi(0.035^3)} + \frac{4}{\pi(0.035^2)}\right](10^{-3}) = 52.3 \text{ kpsi}$$

$$\sigma_m = \frac{F_m}{F_a}\sigma_a = \frac{3.25}{1.75}52.3 = 97.1 \text{ kpsi}$$

To estimate the tensile endurance limit using the distortion-energy theory,

$$S_e = S_{se}/0.577 = 38.7/0.577 = 67.1 \text{ kpsi}$$

Using the Gerber criterion for tension gives

$$(n_f)_A = \frac{1}{2}\left(\frac{S_{ut}}{\sigma_m}\right)^2\frac{\sigma_a}{S_e}\left[-1 + \sqrt{1 + \left(2\frac{\sigma_m}{S_{ut}}\frac{S_e}{\sigma_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{264.7}{97.1}\right)^2\frac{52.3}{67.1}\left[-1 + \sqrt{1 + \left(2\frac{97.1}{264.7}\frac{67.1}{52.3}\right)^2}\right] = 1.08$$

10.11 Extension Spring

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10.11 Extension Spring

Figure 10–6

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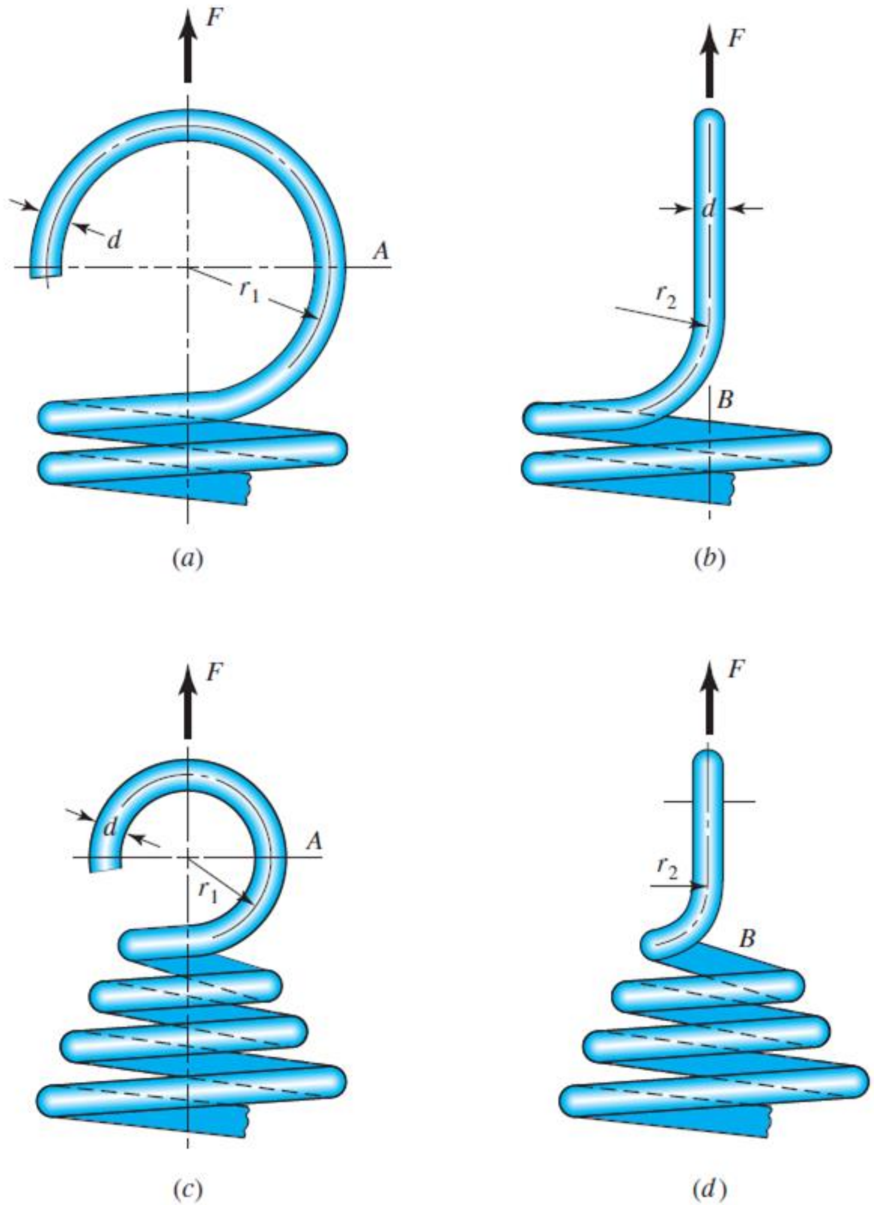

$$\tau_B = (K)_B \frac{8FD}{\pi d^3}$$

$$(K)_B = \frac{4C_2 - 1}{4C_2 - 4}$$

$$C_2 = \frac{2r_2}{d}$$

10.11 Extension Spring

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EXAMPLE 10–7 The helical coil extension spring of Ex. 10–6 is subjected to a dynamic loading from 1.5 to 5 lbf. Estimate the factors of safety using the Gerber failure criterion for (*a*) coil fatigue, (*b*) coil yielding, (*c*) end-hook bending fatigue at point *A* of Fig. 10–6*a*, and (*d*) end-hook torsional fatigue at point *B* of Fig. 10–6*b*.

(*d*) End-hook torsional fatigue: from Eq. (10–36)

$$(\tau_a)_B = (K)_B \frac{8F_a D}{\pi d^3} = 1.18 \frac{8(1.75)0.213}{\pi(0.035^3)}(10^{-3}) = 26.1 \text{ kpsi}$$

$$(\tau_m)_B = \frac{F_m}{F_a}(\tau_a)_B = \frac{3.25}{1.75} 26.1 = 48.5 \text{ kpsi}$$

Then, again using the Gerber criterion, we obtain

$$(n_f)_B = \frac{1}{2}\left(\frac{S_{su}}{\tau_m}\right)^2 \frac{\tau_a}{S_{se}}\left[-1 + \sqrt{1 + \left(2\frac{\tau_m}{S_{su}}\frac{S_{se}}{\tau_a}\right)^2}\right]$$

$$= \frac{1}{2}\left(\frac{177.3}{48.5}\right)^2 \frac{26.1}{38.7}\left[-1 + \sqrt{1 + \left(2\frac{48.5}{177.3}\frac{38.7}{26.1}\right)^2}\right] = 1.30$$

10.12 Helical Coil Torsion Springs

Figure 10–8

Torsion springs. *(Courtesy of Associated Spring.)*