Ch1 Part 3

Monday, September 20, 2021 12:19 PM

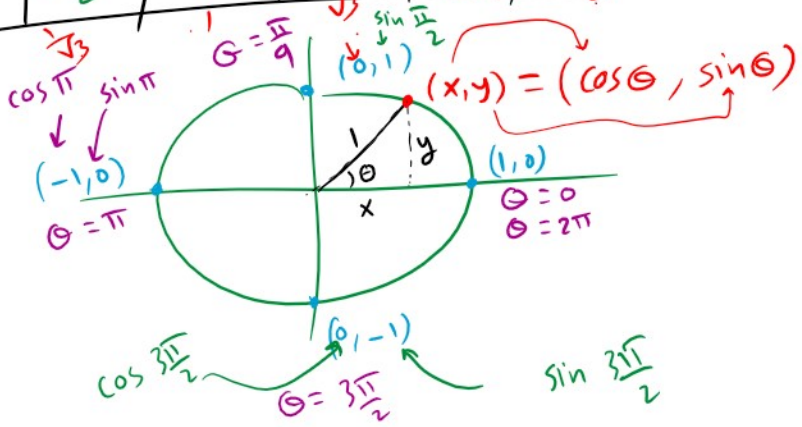
Trigonometric functions

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin\theta$	0	0.5	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0 undef	-1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undef	0

$\frac{1}{2}$ $\frac{\sqrt{3}}{2}$

$\cos\theta = \frac{x}{1} = x$

$\sin\theta = \frac{y}{1} = y$



(1) $f(x) = \sin x$

$D = (-\infty, \infty) = \mathbb{R}$

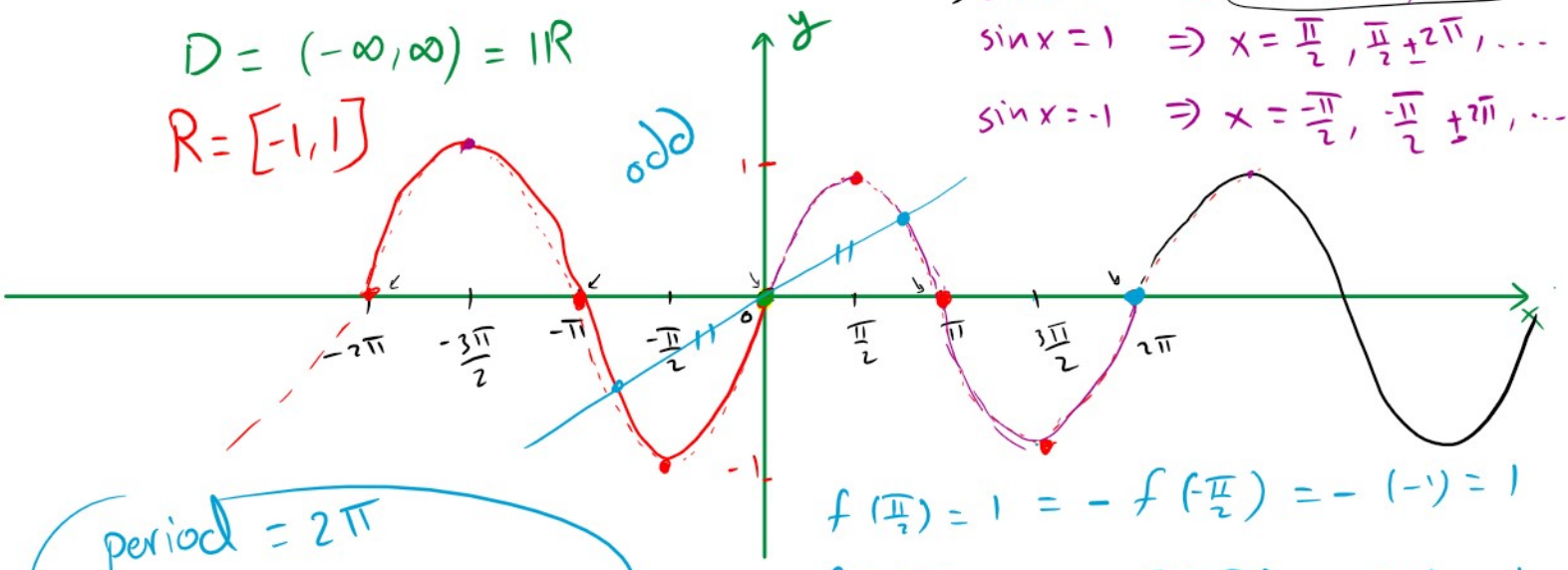
$R = [-1, 1]$

odd

$\sin x = 0 \Rightarrow x = 0, \pm\pi, \pm 2\pi, \ldots$

$\sin x = 1 \Rightarrow x = \frac{\pi}{2}, \frac{\pi}{2} \pm 2\pi, \ldots$

$\sin x = -1 \Rightarrow x = \frac{-\pi}{2}, \frac{-\pi}{2} \pm 2\pi, \ldots$



period $= 2\pi$

$\sin(x + 2\pi) = \sin x$

$f(\frac{\pi}{2}) = 1 = -f(-\frac{\pi}{2}) = -(-1) = 1$

$f(-\frac{3\pi}{2}) = 1 = -f(\frac{3\pi}{2}) = -(-1) = 1$

(2) $f(x) = \csc x$ "co secant"

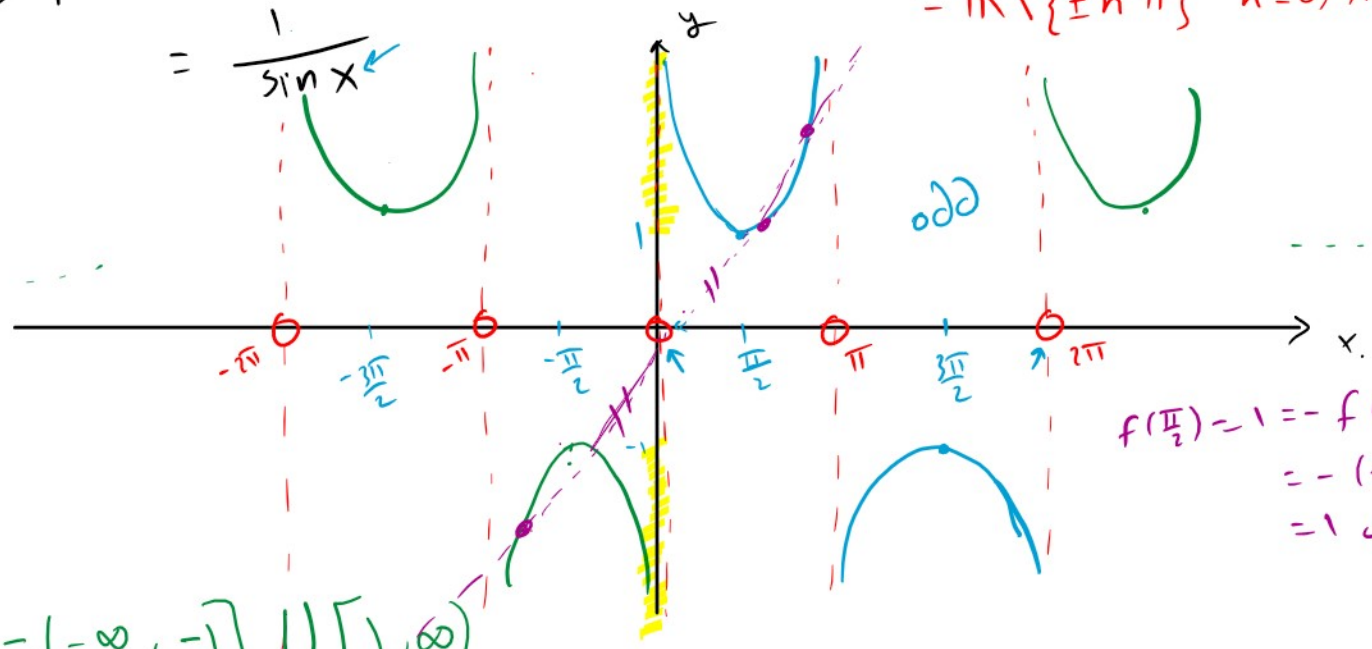
$= \frac{1}{\quad}$

$D = \mathbb{R} \setminus \{0, \pm\pi, \pm 2\pi, \ldots\}$

$= \mathbb{R} \setminus \{\pm n\pi\}$ $n = 0, 1, 2, 3, \ldots$

(2) $f(x) = \dfrac{1}{\sin x}$

$= \mathbb{R} \setminus \{\pm n\pi\}$ $n = 0, 1, 2, 3 \ldots$



$f(\frac{\pi}{2}) = 1 = -f(\frac{-\pi}{2})$

$= -(-1)$

$= 1$ ✓

$R = (-\infty, -1] \cup [1, \infty)$

period $= 2\pi \Rightarrow \csc(x + 2\pi) = \csc x$ ✓

(3) $f(x) = \cos x$

$\cos x = 0 \Rightarrow x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$

$\cos x = -1 \Rightarrow x = \pm\pi, \pm 3\pi, \pm 5\pi$

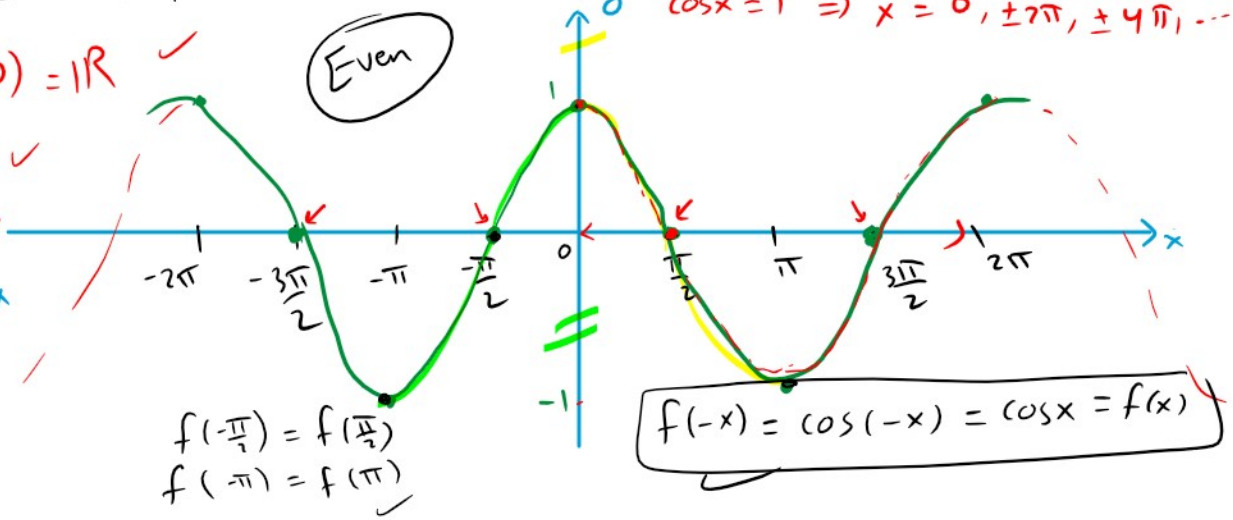
$\cos x = 1 \Rightarrow x = 0, \pm 2\pi, \pm 4\pi, \ldots$

$D = (-\infty, \infty) = \mathbb{R}$ ✓

$R = [-1, 1]$ ✓

period $= 2\pi$ ✓

$\cos(x + 2\pi) = \cos x$



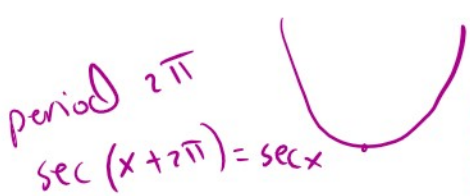
$f(-\frac{\pi}{2}) = f(\frac{\pi}{2})$

$f(-\pi) = f(\pi)$ ✓

$f(-x) = \cos(-x) = \cos x = f(x)$

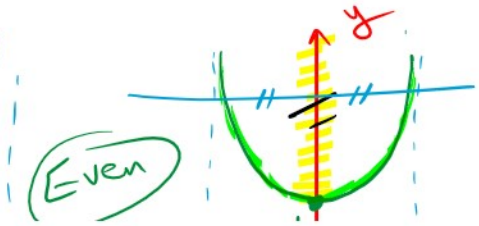
(4) $f(x) = \sec x = \dfrac{1}{\cos x}$ "secant"

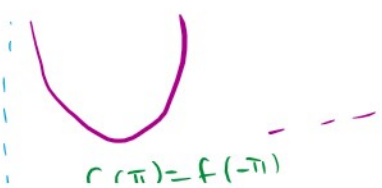
$D = \mathbb{R} \setminus \{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots\} = \mathbb{R} \setminus \{\frac{\pi}{2} \pm n\pi\}$ ✓ , $n = 0, 1, 2, 3 \ldots$



period 2π

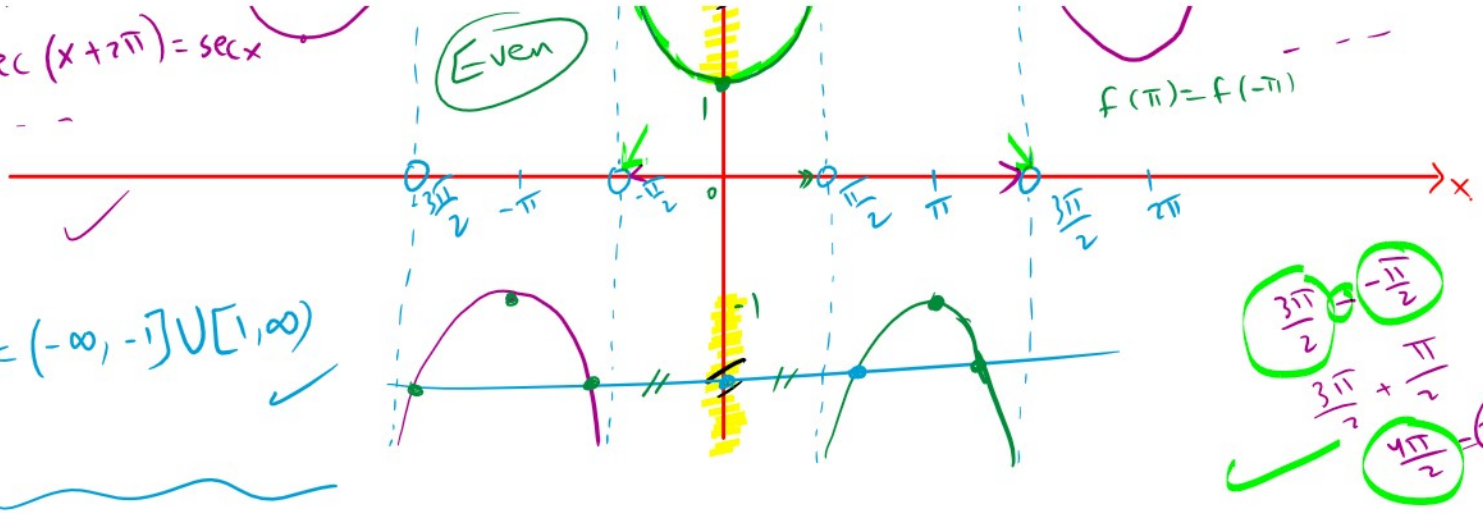
$\sec(x + 2\pi) = \sec x$





$f(\pi) = f(-\pi)$

$\sec(x+2\pi) = \sec x$



$R = (-\infty, -1] \cup [1, \infty)$

$\sin x$, $\csc x$, $\cos x$, $\sec x$ $\Longrightarrow$ period 2π

$\sin(x+2\pi) = \sin x$

$\csc(x+2\pi) = \csc x$

$\cos(x+2\pi) = \cos x$

$\sec(x+2\pi) = \sec x$

$\tan x$, $\cot x$ $\longrightarrow$ period π

$\tan(x+\pi) = \tan x$

$\cot(x+\pi) = \cot x$

$\sec x = \frac{1}{\cos x} \Rightarrow D$

صفر أصغار المقام

?? $\cos x = 0$

$x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2} \ldots$

(5) $f(x) = \tan x = \dfrac{\sin x}{\cos x}$

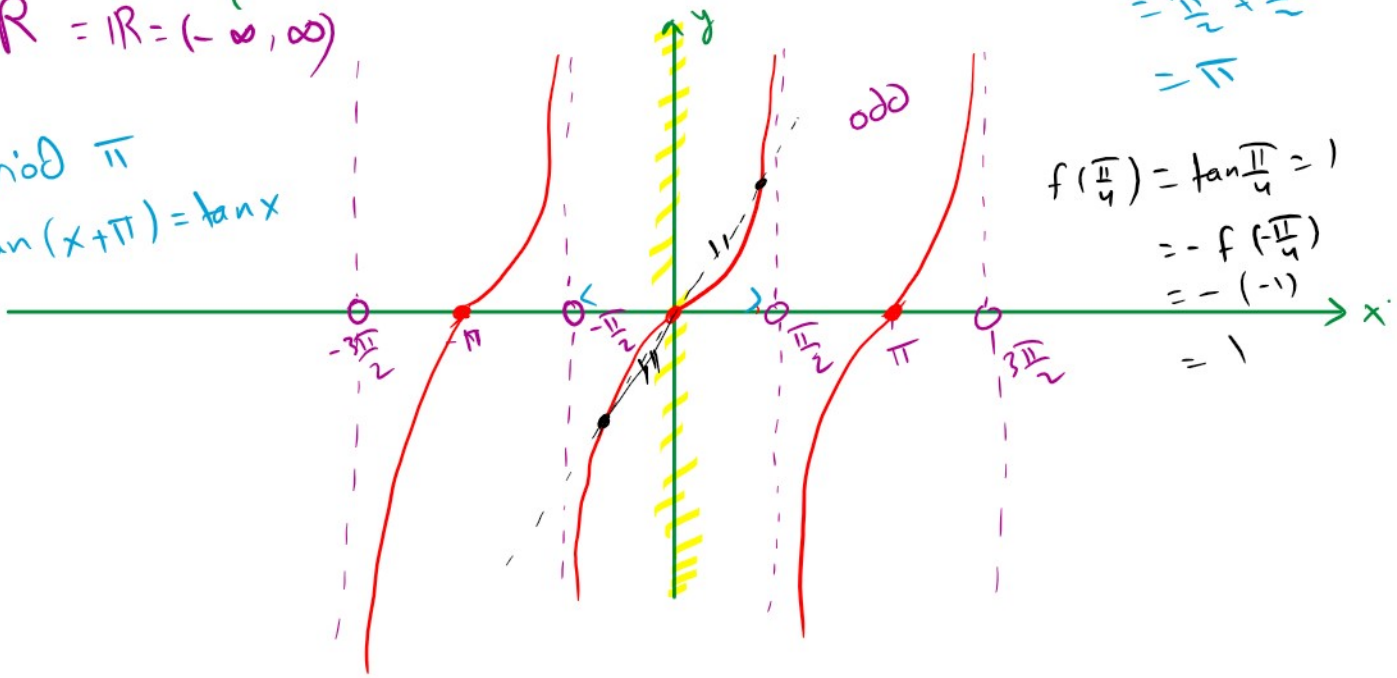
$D = \mathbb{R} \setminus \{ \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}, \dots \}$

$R = \mathbb{R} = (-\infty, \infty)$

$p = \frac{\pi}{2} - -\frac{\pi}{2}$

$= \frac{\pi}{2} + \frac{\pi}{2}$

$= \pi$



period π

$\tan(x+\pi) = \tan x$

$f(\frac{\pi}{4}) = \tan\frac{\pi}{4} = 1$

$= -f(-\frac{\pi}{4})$

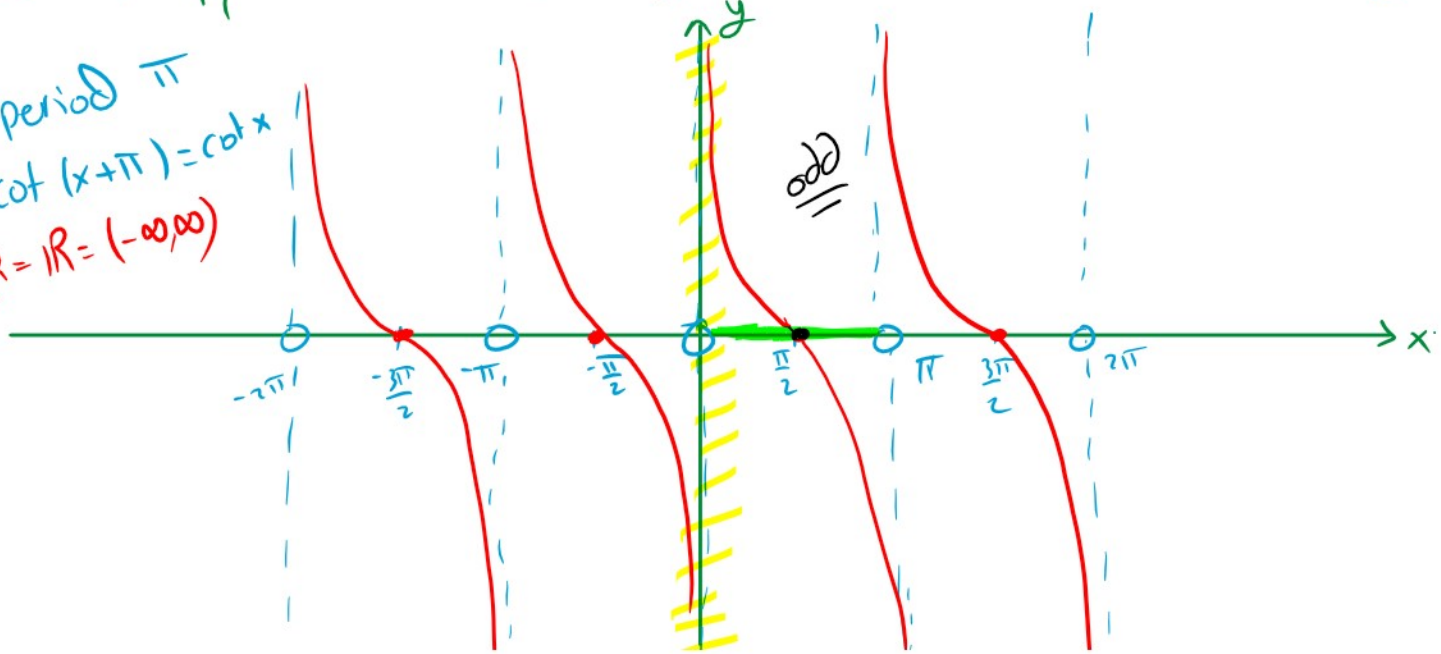
$= -(-1)$

$= 1$

(6) $f(x) = \cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$ "cotan"

$D = \mathbb{R} \setminus \{ 0, \pm\pi, \pm 2\pi, \pm 3\pi \}$

$\cot\frac{\pi}{2} = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{0}{1} = 0$

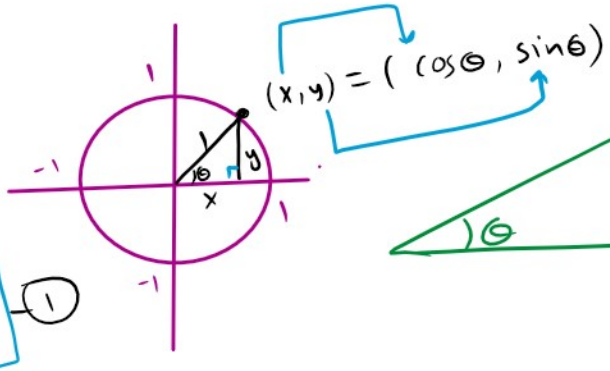


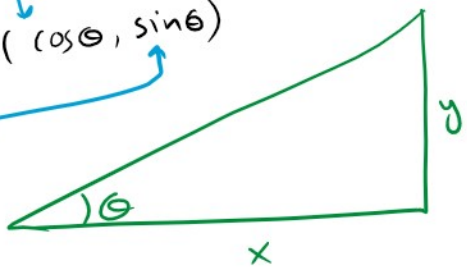
period π

$\cot(x+\pi) = \cot x$

$R = \mathbb{R} = (-\infty, \infty)$

$x^2 + y^2 = 1$





$\cos^2 x + \sin^2 x = 1$ — (1)

- divide (1) by $\cos^2 x$

$$1 + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \Rightarrow 1 + \tan^2 x = \sec^2 x \quad (2)$$

- Divide (1) by $\sin^2 x$

$$\frac{\cos^2 x}{\sin^2 x} + 1 = \frac{1}{\sin^2 x} \Rightarrow \cot^2 x + 1 = \csc^2 x \quad (3)$$

$$\sin 2x = 2 \sin x \cos x \quad (3)$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= \dots$$

$$\Rightarrow \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$= 1 - 2\sin^2 x$ ⇒ $\sin^2 x = \frac{1-\cos 2x}{2}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\sin(A+B) = \sin A \cos B + \sin B \cos A$

$\sin(x+2\pi) = \sin x \overset{??}{=} \sin x \cos 2\pi + \sin 2\pi \cos x = \sin x$ ($\sin 2\pi = 0$)

$\sin(x+\frac{\pi}{2}) = \sin x \cos\frac{\pi}{2} + \sin\frac{\pi}{2}\cos x$ ($\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$)

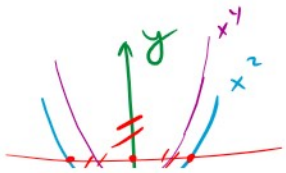
$= 0 + \cos x$

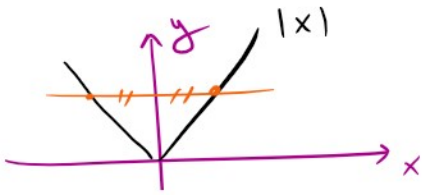
$\sin(x+\frac{\pi}{2}) = \cos x$

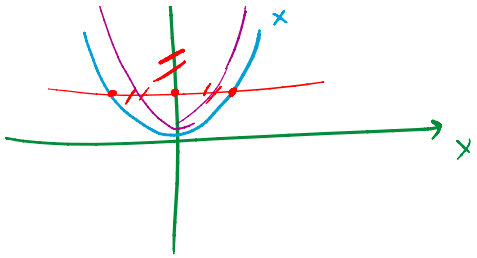
Even function دالة زوجية

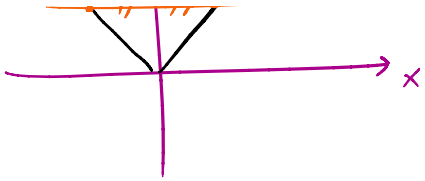
- $f(-x) = f(x)$ $\forall x \in D(f)$
- symmetric about y-axis

- Exp: $f(x) = x^2, x^4, \cos x, x^6, |x|, \sec x, \cdots$







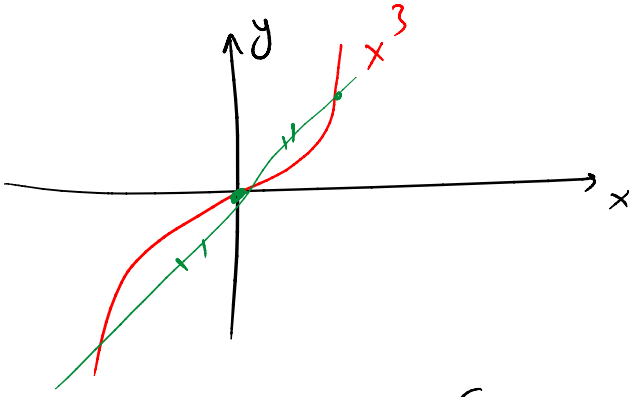


Odd function الدوال الفردية

→ $f(-x) = -f(x) \quad \forall x \in D(f)$

→ f is symmetric about origin (0,0)

→ Exp $f(x) = x, x^3, x^5, \sin x, \csc x, \tan x, \cot x, \frac{1}{x}$



Exp show that $f(x) = \frac{x}{x^2-1}$ is odd

$f(-x) \overset{??}{=} -f(x)$

$\frac{(-x)}{(-x)^2-1} \overset{??}{=} -\frac{x}{x^2-1}$

$\frac{-x}{} = -\frac{x}{}$ ✓

$$\frac{-x}{x^2-1} = -\frac{x}{x^2-1}$$ ✓

Exp Show that $g(x) = \frac{1}{x^2-1}$ is even

$$f(-x) \overset{??}{=} f(x)$$

$$\frac{1}{(-x)^2-1} \overset{??}{=} \frac{1}{x^2-1}$$

$$\frac{1}{x^2-1} = \frac{1}{x^2-1}$$ ✓

Exp Is $f(x) = \frac{x}{x^2-1}$ even/odd ?

Even : $f(-x) \overset{??}{=} f(x)$

$$\frac{(-x)}{(-x)^2-1} \overset{??}{=} \frac{x}{x^2-1}$$

$$-\frac{x}{x^2-1} \neq \frac{x}{x^2-1}$$ not even

odd $f(-x) \stackrel{?}{=} -f(x)$

$$\frac{(-x)}{(-x)^2-1} \stackrel{?}{=} -\frac{x}{x^2-1}$$

$$-\frac{x}{x^2-1} = -\frac{x}{x^2-1}$$

odd

Exp Is $f(x) = \frac{1}{x+1}$ even / odd?

Even? $f(-x) \stackrel{?}{=} f(x)$

$$\frac{1}{(-x)+1} \neq \frac{1}{x+1}$$

not even

odd? $f(-x) \stackrel{?}{=} -f(x)$

$$\frac{1}{-x+1} \stackrel{?}{\neq} -\frac{1}{x+1}$$

not odd