Transmission Lines Parameters

T.L Resistance — T.L Inductance — T.L Capacitance

Transmission Line Capactance :

» Capacitance of transmission line is the result of the potential difference between the conductors, it causes them to be charged in the same manner as the plates of a capacitor, when there is a potential difference between them the capacitance between conductors is the charge per unit of the potential difference.

1)) Electric Field and Voltage Calculation
2)) Transmission Line Capacitance for:-
- [A] Single-phase Line.
- [B] 3ϕ Lines with equal spacing.
- [C] 3ϕ Lines, bundled conductor, and unequal spacing.

1)) Gauss's Law → Electric Field Strength (E) → Voltage between Conductors → Capacitance $C = q/V$

Gauss's Law :- Total electric flux leaving a closed surface = Total charge within the vollume enclosed by the closed surface.

⇓ Leads to

Normal Electric Flux density integrated over the closed surface = charge enclosed by this closed surface.

Surface integral over closed surface ← $\oiint D_{\perp}\, ds = \oiint \varepsilon E_{\perp}\, ds = Q_{enclosed}$

Where,

$\varepsilon \triangleq$ permittivity of the medium $= \varepsilon_r \varepsilon_o$

$\varepsilon_o = 8.854 * 10^{-12}$ F/m

$D_{\perp} \triangleq$ normal component of electric flux density.

$E_{\perp} \triangleq$ normal component of electric field strength.

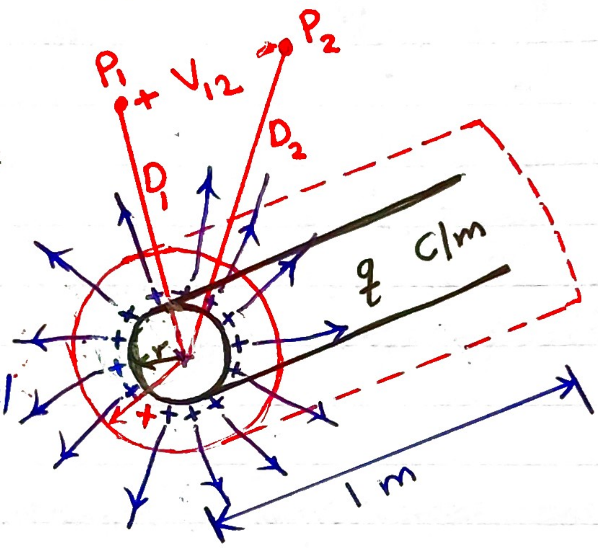
ds = the differential surface area.

Note:-

Inside the perfect conductor, ohm's law give $E_{int} = 0$

⇓

That is, the internal electric field $E_{int} = 0$



$$\oiint \varepsilon E_{\perp}\, ds = Q_{enclosed}$$

1 m length

$$\varepsilon E_x (2\pi x)(1) = q\ (1)$$

$$E_x = \frac{q}{2\pi\varepsilon x} \quad V/m$$

$$V_{12} = \int_{D_1}^{D_2} E_x\, dx = \int_{D_1}^{D_2} \frac{q}{2\pi\varepsilon x}\, dx$$

$$V_{12} = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1} \quad \text{Volts}$$

D_2 → P_2 −, D_1 → P_1 +

where,

$\varepsilon = \varepsilon_r \varepsilon_o$

$\varepsilon_o = 8.854 * 10^{-12}$ F/m

note ⇓

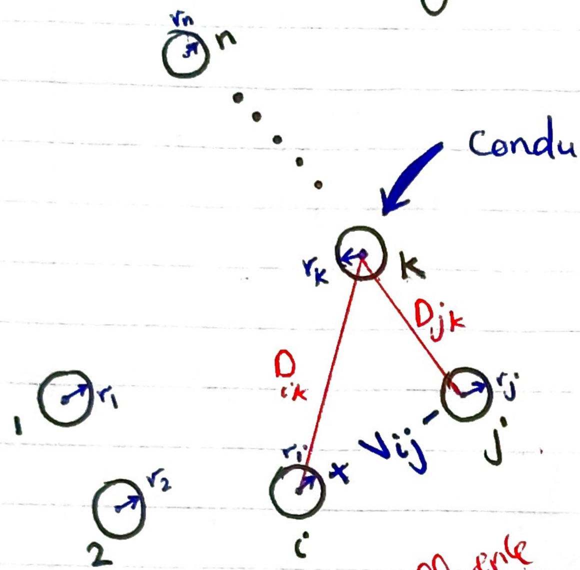
− P_2

V_{12}

+ P_1

$$\boxed{V_{12} = \frac{q}{2\pi\varepsilon} \ln \frac{D_2}{D_1} \text{ Volts}}$$

Multi-Conductor System:



Conductor k has radius r_k and charge q_k ((per meter length of the conductor))

$$V_{ijk} = \frac{q_k}{2\pi\varepsilon} \ln \frac{D_{jk}}{D_{ik}} \quad \text{volts}$$

$$\boxed{V_{ij} = \sum_{k=1}^{n} \frac{q_k}{2\pi\varepsilon} \ln \frac{D_{jk}}{D_{ik}} \quad \text{volts}}$$

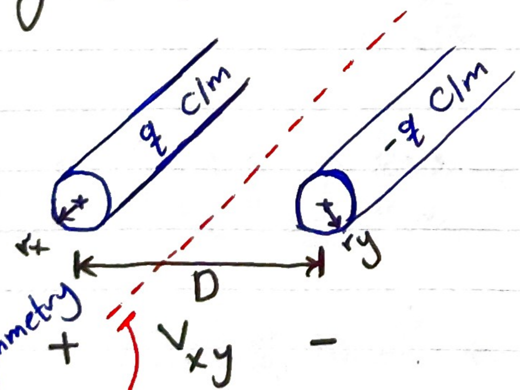
voltage difference due to charges in all conductors

Super-position Theorem

Transmission Line Capacitance

Single-Phase Line [A] | Three-Phase Lines [B]

[A] Single-Phase Line



due to Symmetry

→ zero-voltage
→ zero-potential
→ potential of neutral

$$V_{xy} = \frac{1}{2\pi\varepsilon}\left[q \ln \frac{D_{yx}}{D_{xx}} - q \ln \frac{D_{yy}}{D_{xy}}\right]$$

$$= \frac{q}{2\pi\varepsilon} \ln \frac{D_{yx} D_{xy}}{D_{xx} D_{yy}}$$

$$= \frac{q}{\pi\varepsilon} \ln \frac{D}{\sqrt{r_x r_y}} \quad \text{Volts}$$

$$C_{xy} = \frac{q}{V_{xy}} = \frac{\pi\varepsilon}{\ln\left(\frac{D}{\sqrt{r_x r_y}}\right)} \quad F/m$$

Notes

» $V_{12}(q_1) = \frac{q_1}{2\pi\varepsilon} \ln \frac{D}{r}$

» $V_{12}(q_2) = \frac{q_2}{2\pi\varepsilon} \ln \frac{r}{D}$

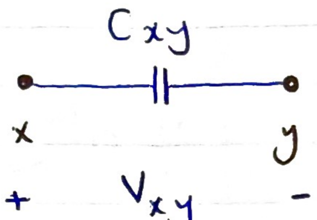
» $V_{21}(q_2) = \frac{q_2}{2\pi\varepsilon} \ln \frac{D}{r} = -V_{12}$

» $V_{12} = V_{12}(q_1) + V_{12}(q_2)$

$q_2 = -q_1$

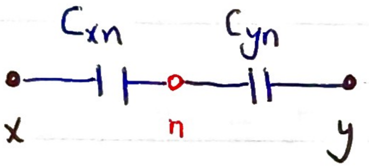
$$C_{xy} = \frac{\pi \varepsilon}{\ln\left(\frac{D}{\sqrt{r_x r_y}}\right)} \qquad \text{if } r_x = r_y$$

$$C_{xy} = \frac{\pi \varepsilon}{\ln\left(\frac{D}{r}\right)}$$

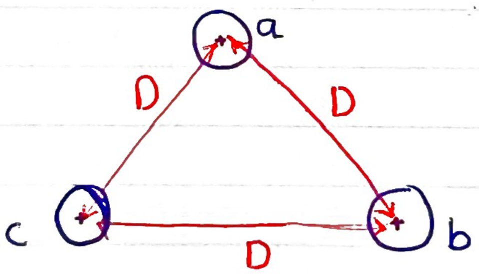


$$V_{xn} = V_{yn} = \frac{V_{xy}}{2}$$

$$C_n = C_{xn} = C_{yn} = \frac{q}{V_{xn}} = 2\,C_{xy} = \frac{2\pi\varepsilon}{\ln\left(\frac{D}{r}\right)} \quad F/m$$



$\boxed{B}$ Three-Phase Line with Equilateral Spacing:



$$q_a + q_b + q_c = 0$$

$$\Rightarrow V_{ab} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{ba}}{D_{aa}} + q_b \ln\frac{D_{bb}}{D_{ab}} + q_c \ln\frac{D_{bc}}{D_{ac}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D} + \cancel{q_c \ln\frac{D}{D}}^{\,0}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{r}{D}\right] \quad \text{Volts}$$

$$\Rightarrow V_{ac} = \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D_{ca}}{D_{aa}} + q_b \ln\frac{D_{cb}}{D_{ab}} + q_c \ln\frac{D_{cc}}{D_{ac}}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_b \ln\frac{D}{D} + q_c \ln\frac{r}{D}\right]$$

$$= \frac{1}{2\pi\varepsilon}\left[q_a \ln\frac{D}{r} + q_c \ln\frac{r}{D}\right]$$

$V_{ab} + V_{ac}$

$$V_{ab}+V_{ac} = \left(\frac{1}{2\pi\varepsilon}\right)\left[2q_a \ln\frac{D}{r} + \overbrace{(q_b+q_c)}^{-q_a}\ln\frac{r}{D}\right]$$

$$\boxed{V_{an} = \frac{1}{3}\left(V_{ab}+V_{ac}\right)}$$

$$= \frac{1}{3}\left(\frac{1}{2\pi\varepsilon}\right)\left[2q_a \ln\frac{D}{r} + q_a \ln\frac{D}{r}\right]$$

$$= \frac{q_a}{2\pi\varepsilon}\ln\frac{D}{r}$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln\frac{D}{r}} \quad F/m \quad \text{Line to neutral}$$

Notes:

$$V_{ab} = \sqrt{3}\,V_{an}\angle{+30} = \sqrt{3}\,V_{an}\left[\frac{\sqrt{3}}{2}+j\frac{1}{2}\right]$$

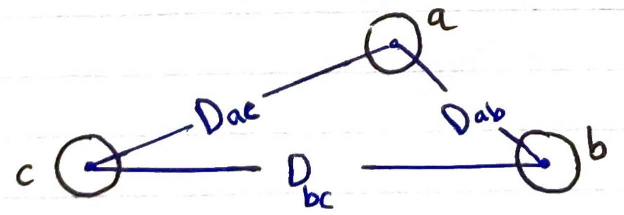
$$V_{ac} = -V_{ca} = \sqrt{3}\,V_{an}\angle{-30} = \sqrt{3}\,V_{an}\left[\frac{\sqrt{3}}{2}-j\frac{1}{2}\right]$$

+

$$V_{ab}+V_{ac} = 3V_{an}$$

$$\boxed{V_{an} = \frac{1}{3}\left(V_{ab}+V_{ac}\right)}$$

C) 3ϕ with asymmetrical Spacing

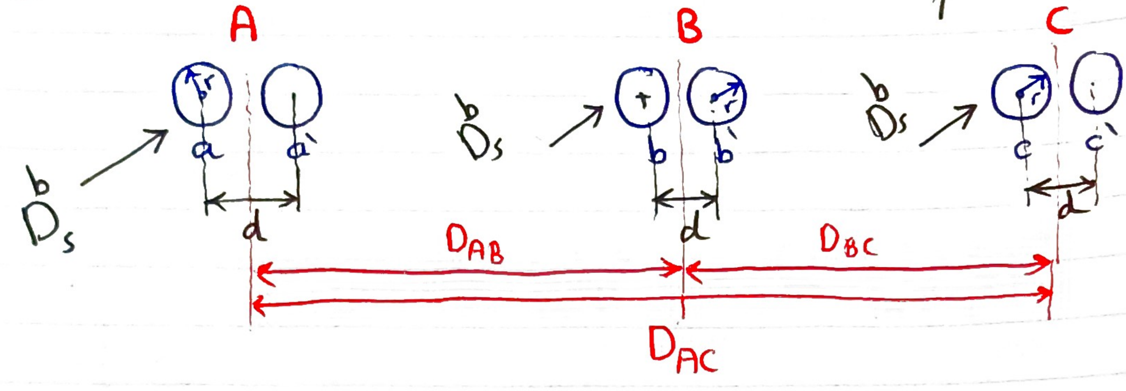


$$C_{an} = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{r}\right)}, \quad D_{eq} = \sqrt[3]{D_{ab}\,D_{ac}\,D_{bc}}$$

(r) solid

$\left(\frac{\text{outside diameter}}{2}\right)$ stranded

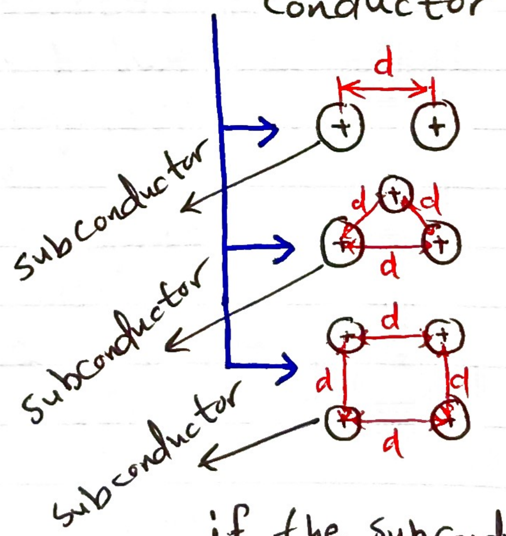
D 3ϕ Bundled Conductor with unequal spacing



$D_{AB} = GMD_{A,B}$, $D_{BC} = GMD_{B,C}$, $D_{AC} = GMD_{A,C}$

$$C_{an} = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{D_s^b}\right)}$$

$D_{eq} = \sqrt[3]{D_{AB}\, D_{BC}\, D_{AC}}$, $D_s^b \triangleq$ GMR for the bundled conductor



$D_s^b = \sqrt[2]{r\,d}$

$D_s^b = \sqrt[3]{r\,d^2}$

$D_s^b = 1.091\sqrt[4]{r\,d^3}$

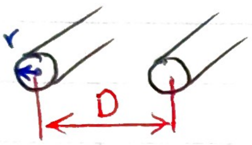
if the subconductor is stranded

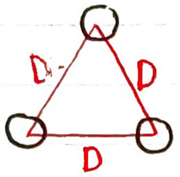
$r \longrightarrow \left[\frac{\text{outside diameter}}{2}\right]$

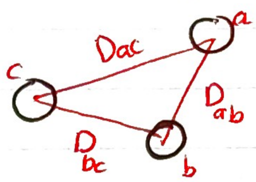
from manufacture's data (Tables)

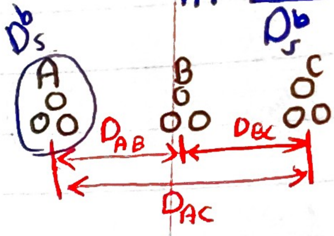
$$C$$

1ϕ	3ϕ		3ϕ Bundel
$C_{an} = \dfrac{2\pi\varepsilon}{\ln \frac{D}{r}}$	$C_{an} = \dfrac{2\pi\varepsilon}{\ln \frac{D}{r}}$	$C_{an} = \dfrac{2\pi\varepsilon}{\ln \frac{D_{eq}}{r}}$	$C_{an} = \dfrac{2\pi\varepsilon}{\ln \frac{D_{eq}}{D_s^b}}$









[if stranded conductor exist instead of solid $r \longrightarrow \dfrac{\text{outside diameter}}{2}$]

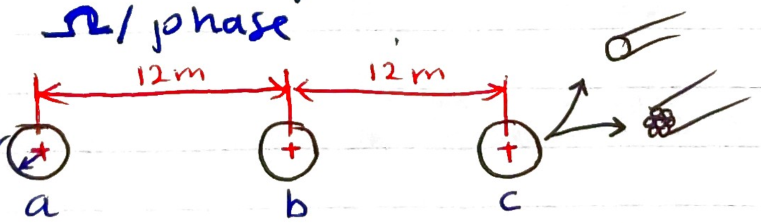
example

A three-phase, 400 kV, 50 Hz, 350 km overhead T.L. has flat horizontal spacing with three identical conductors. The conductors have an outside diameter of 3.28 cm with 12 m between adjacent conductors.

>> Determine the capacitive reactance-to-neutral in Ω/m/phase

>> Determine the capacitive reactance for the line in Ω/phase

Solution



$$D_{eq} = \sqrt[3]{D_{ab} D_{ac} D_{bc}} = \sqrt[3]{(12)(24)(12)} = 15.119 \text{ m}$$

$$C_{an} = \frac{2\pi\varepsilon}{\ln\left(\frac{D_{eq}}{r}\right)} = 8.163 * 10^{-6} \ \mu F/m$$

note: $Z_c = \dfrac{1}{j\omega C}$

$Y_c = \omega C$

$$Y_n = 2\pi * 50 * C_n = 2.565 * 10^{-9} \ ℧/m/phase$$

Length = 350 km

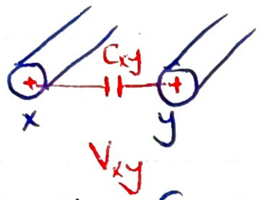
$$Y_n = 8.978 * 10^{-4} \ ℧/phase$$

$$\text{Reactance} = X_n = \frac{1}{Y_n} = 1.1138 * 10^{3} \ \Omega/phase$$

Line charging current :-

The current supplied to the transmission line capacitance is called charging current.

For a single-phase circuit operating at line-to-line voltage $V_{xy} = V_{xy} \angle 0°$.



» The charging current is

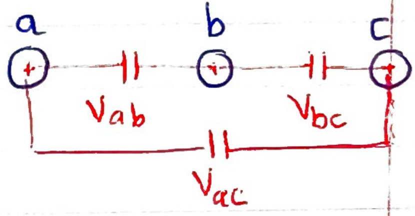
$$I_{chg} = Y_{xy} V_{xy} = j\omega C_{xy} V_{xy} \quad \text{Amp}$$

» The capacitor delivers reactive power, the reactive power delivered by this line-to-line capacitance is

$$Q_c = \frac{V_{xy}^2}{X_c} = Y_{xy} V_{xy}^2$$

$$= \omega C_{xy} V_{xy}^2 \quad \text{var}$$

For a completely transposed 3ϕ line that has $V_{an} = V_{LN} \angle 0°$



» The phase a charging current i

$$I_{chg} = Y_{an} V_{an} = j\omega C_{an} V_{LN}$$

» The reactive power delivered by phase a is

$$Q_{C1\phi} = Y_{an} V_{an}^2 = \omega C_{an} V_{LN}^2 \quad \text{Var}$$

» The total reactive power supplied by the 3ϕ line is

$$Q_{C3\phi} = 3 Q_{C1\phi} = 3\omega C_{an} V_{LN}^2$$

$$= \sqrt{3}\sqrt{3}\, \omega C_{an} V_{LN} V_{LN}$$

$$Q_{C3\phi} = \omega C_{an} V_{LL}^2 \quad \text{Var}$$