Multiplication of Matrices

If A and B are any two matrices with any order, then the matrix product AB applies only when the

the number of columns in A = the number of rows in B

Exp Let $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 3 & 7 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 3 & 0 \\ -2 & 1 & 5 \end{bmatrix}$

Find

① $AB = \underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & 4 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 3 & 7 \end{bmatrix}} = \begin{bmatrix} (1)(0)+(2)(3) & (1)(1)+(2)(7) \\ (3)(0)+(-1)(3) & (3)(1)+(-1)(7) \\ (0)(0)+(4)(3) & (0)(1)+(4)(7) \end{bmatrix} = \underset{3\times 2}{\begin{bmatrix} 6 & 15 \\ -3 & -4 \\ 12 & 28 \end{bmatrix}}$

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② $BA = \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 3 & 7 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 3 & -1 \\ 0 & 4 \end{bmatrix}}$ not possible $\Rightarrow$ Hence, $AB \neq BA$

product of matrices is not commutative

③ $BI = \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 3 & 7 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} (0)(1)+(1)(0) & (0)(0)+(1)(1) \\ (3)(1)+(7)(0) & (3)(0)+(7)(1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & 7 \end{bmatrix} = B$

④ $IB = \underset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 0 & 1 \\ 3 & 7 \end{bmatrix}} = \begin{bmatrix} (1)(0)+(0)(3) & (1)(1)+(0)(7) \\ (0)(0)+(1)(3) & (0)(1)+(1)(7) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & 7 \end{bmatrix} = B$

Hence, $BI = IB = B_{2\times 2}$ since $I_{2\times 2}$ is the identity matrix

⑤ $ID = \underset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 2 & 3 & 0 \\ -2 & 1 & 5 \end{bmatrix}} = \begin{bmatrix} (1)(2)+(0)(-2) & (1)(3)+(0)(1) & (1)(0)+(0)(5) \\ (0)(2)+(1)(-2) & (0)(3)+(1)(1) & (0)(0)+(1)(5) \end{bmatrix}$

$= \begin{bmatrix} 2 & 3 & 0 \\ -2 & 1 & 5 \end{bmatrix} = D$

since $ID = D_{2\times 3}$

Note that DI not possible

Exp ① write the idenity matrix 3X3

$$I_{3\times3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

② If A is 3X3 matrix. Find $A^T I_{3\times3}$

$$A^T I = I A^T = A^T$$

③ Find $\begin{bmatrix} 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$

$$\underset{1\times3}{\begin{bmatrix} 2 & 0 & 3 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}} = \begin{bmatrix} (2)(0) + (0)(1) + (3)(-3) \end{bmatrix} = \begin{bmatrix} -9 \end{bmatrix}$$

Exp Assume $A = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix}$

① Find A^2

$$A^2 = AA = \underset{2\times2}{\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}} \underset{2\times2}{\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}} = \begin{bmatrix} (5)(5) + (3)(1) & (5)(3) + (3)(2) \\ (1)(5) + (2)(1) & (1)(3) + (2)(2) \end{bmatrix} = \begin{bmatrix} 28 & 21 \\ 7 & 7 \end{bmatrix}$$

② Find $(AB^T)^T$

$$B^T = \begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix} \Rightarrow AB^T = \underset{2\times2}{\begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix}} \underset{2\times2}{\begin{bmatrix} -1 & 0 \\ 3 & 2 \end{bmatrix}} = \begin{bmatrix} -5+9 & 0+6 \\ -1+6 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 5 & 4 \end{bmatrix}$$

$$(AB^T)^T = \begin{bmatrix} 4 & 6 \\ 5 & 4 \end{bmatrix}^T = \begin{bmatrix} 4 & 5 \\ 6 & 4 \end{bmatrix}$$

③ $\frac{1}{7} A^2$

$$\frac{1}{7} A^2 = \frac{1}{7} \begin{bmatrix} 28 & 21 \\ 7 & 7 \end{bmatrix} = \begin{bmatrix} \frac{28}{7} & \frac{21}{7} \\ \frac{7}{7} & \frac{7}{7} \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$$