Chapter 7.1, Problem 21E

Problem

If *b* is any positive real number with $b \neq 1$ and *x* is any real number, b^{-x} is defined as follows: $b^{-x} = \frac{1}{b^x}$. Use this definition and the definition of logarithm to prove that $\log_b\left(\frac{1}{u}\right) = -\log_b(u)$ for all positive real numbers *u* and *b*, with $b \neq 1$.

Step-by-step solution

Step 1 of 1

Let *b* be a positive real number, and *x* be a real number with

$b \neq 1.$

Define the value of b^{-x}, as follows:

$$b^{-x} = \frac{-1}{b^x}.$$

The objective is to prove, $\log_b\left(\frac{1}{u}\right) = -\log_b(u).$

Let $v = \log_b\left(\frac{1}{u}\right).$

By the definition of a logarithm,

$b^v = \frac{1}{u}.$ ...... (1)

Multiply both sides of equation (1), by *u*, and dividing by b^v, obtained as,

$u = b^{-v}.$

Thus, $-v = \log_b(u)$ [by the definition of a logarithm]

Now, multiply both sides by -1, obtained as,

$v = -\log_b(u).$

Therefore,

$$\boxed{\log_b\left(\frac{1}{u}\right) = -\log_b(u)}.$$

Since, $v = \log_b\left(\frac{1}{u}\right).$