Exp Given $f(x) = \dfrac{x^2}{x+1}$, $f'(x) = \dfrac{x^2+2x}{(x+1)^2}$, $f''(x) = \dfrac{2}{(x+1)^3}$

Find the following:

1. Domain : $D = \mathbb{R} \setminus \{-1\}$

2. H. Asym. None

3. O. Asym. $y = x - 1$

$$\begin{array}{r} x - 1 \\ x+1 \,\overline{)\, x^2 \qquad\quad} \\ \underline{-x^2 \mp x} \\ -x \quad \\ \underline{+x + 1} \\ 1 \end{array}$$

4. V. Asym. check $x = -1$

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} \frac{x^2}{x+1} = \frac{1}{\text{small}+} = +\infty$$

$\Rightarrow x = -1$ is V. Asy.

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \frac{x^2}{x+1} = \frac{1}{\text{small}-} = -\infty \checkmark$$

5. $\displaystyle \lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^2}{x+1} = \lim_{x \to \infty} \frac{2x}{1} = \infty$ $\left(\frac{\infty}{\infty}\right)$

6. $\displaystyle \lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^2}{x+1} = \lim_{x \to -\infty} \frac{2x}{1} = -\infty$

7. Critical Points: $f'(x) = 0 \iff x^2 + 2x = 0$

$x(x+2) = 0$

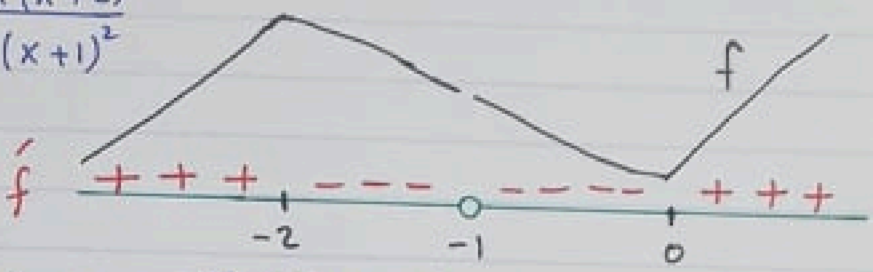
$\boxed{x = 0}$ or $\boxed{x = -2}$
$\in D$ $\quad\in D$

$(0, f(0)) = (0, 0)$

$(-2, f(-2)) = (-2, -4)$

8 Increasing and Decreasing intervals

$$f'(x) = \frac{x^2+2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$$



f is increasing on $(-\infty,-2] \cup [0,\infty)$

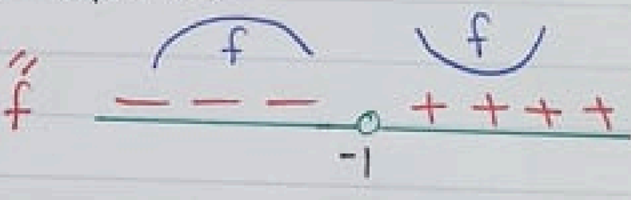
f is decreasing on $[-2,-1) \cup (-1,0]$

9 Extreme Values

- f has local max of $f(-2)=-4$ at $x=-2$
  - ↳ not Absolute since $f(0) > f(-2)$
- f has local min of $f(0)=0$ at $x=0$
  - ↳ not Abs. min since $f(-2) < f(0)$
- f has no Abs. Max and has no Abs. Min

10 Concave up and down intervals

$$f''(x) = \frac{2}{(x+1)^3}$$



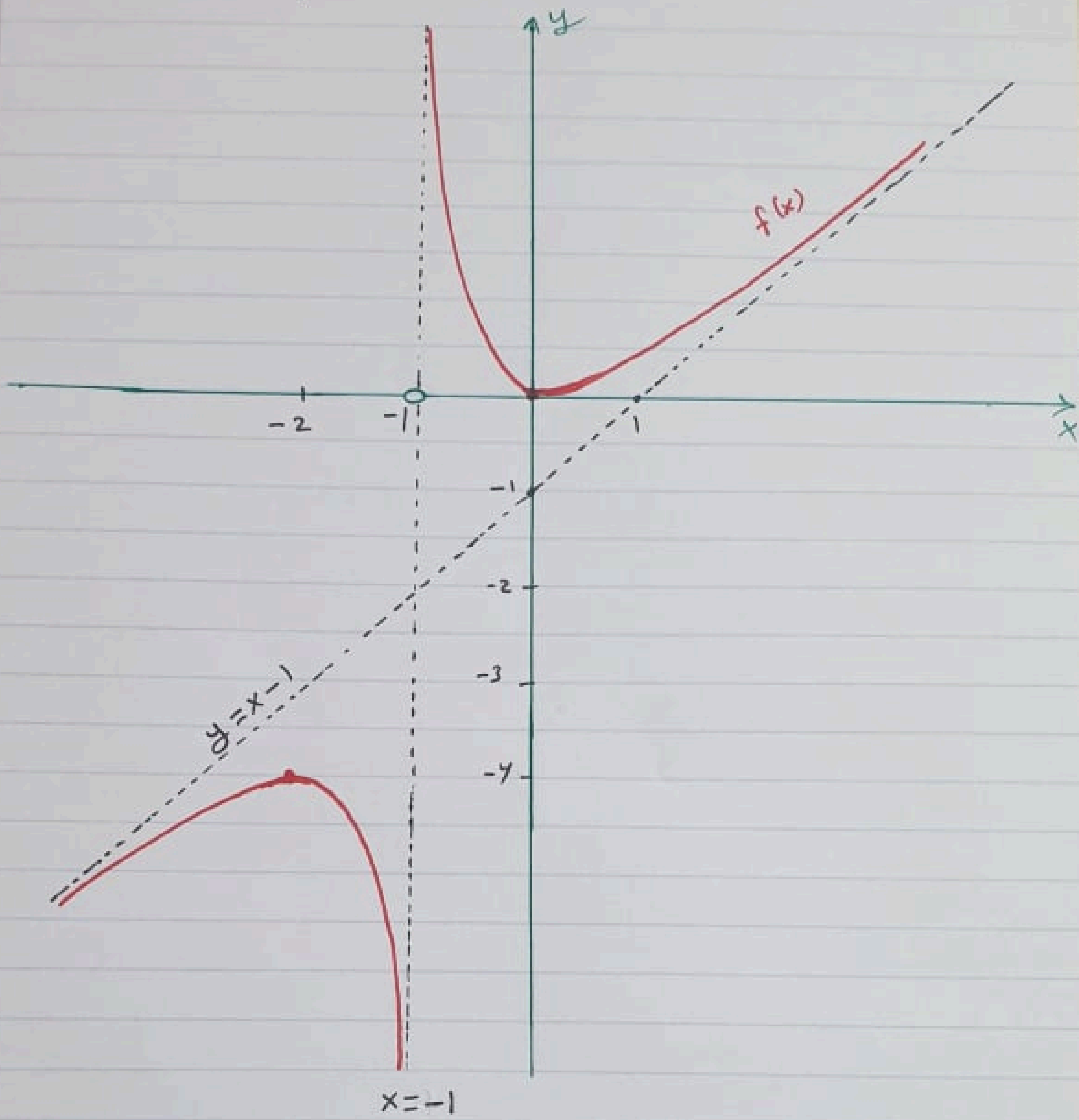
f is concave up on $(-1,\infty)$
f is concave down on $(-\infty,-1)$

11 Inflection points

None since $f''(x)$ never zero

$\boxed{53.3}$

$\boxed{12}$ Sketch $f(x)$



$\boxed{13}$ Range : $(-\infty, -4] \cup [0, \infty)$

$\boxed{14}$ Even/odd : None since $f(-x) = \dfrac{(-x)^2}{(-x)+1} = \dfrac{x^2}{-x+1} \neq f(x)$

$\neq -f(x)$