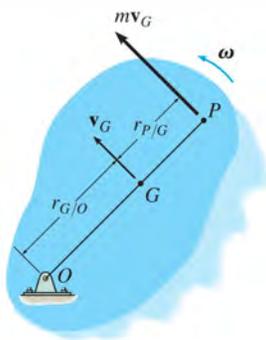


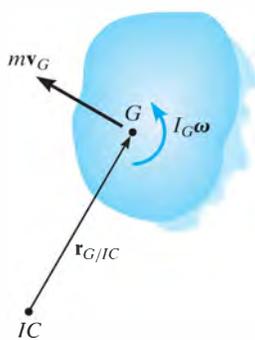
## PROBLEMS

**•19-1.** The rigid body (slab) has a mass  $m$  and rotates with an angular velocity  $\omega$  about an axis passing through the fixed point  $O$ . Show that the momenta of all the particles composing the body can be represented by a single vector having a magnitude  $m\mathbf{v}_G$  and acting through point  $P$ , called the *center of percussion*, which lies at a distance  $r_{P/G} = k_G^2/r_{G/O}$  from the mass center  $G$ . Here  $k_G$  is the radius of gyration of the body, computed about an axis perpendicular to the plane of motion and passing through  $G$ .



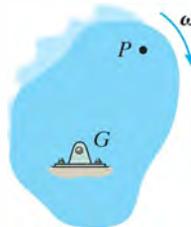
**Prob. 19-1**

**19-2.** At a given instant, the body has a linear momentum  $\mathbf{L} = m\mathbf{v}_G$  and an angular momentum  $\mathbf{H}_G = I_G\omega$  computed about its mass center. Show that the angular momentum of the body computed about the instantaneous center of zero velocity  $IC$  can be expressed as  $\mathbf{H}_{IC} = I_{IC}\omega$ , where  $I_{IC}$  represents the body's moment of inertia computed about the instantaneous axis of zero velocity. As shown, the  $IC$  is located at a distance  $r_{G/IC}$  away from the mass center  $G$ .



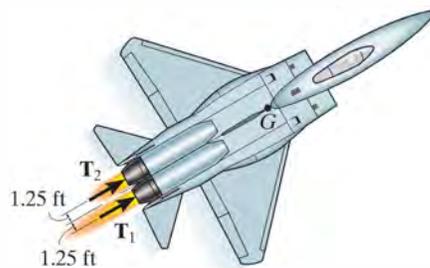
**Prob. 19-2**

**19-3.** Show that if a slab is rotating about a fixed axis perpendicular to the slab and passing through its mass center  $G$ , the angular momentum is the same when computed about any other point  $P$ .



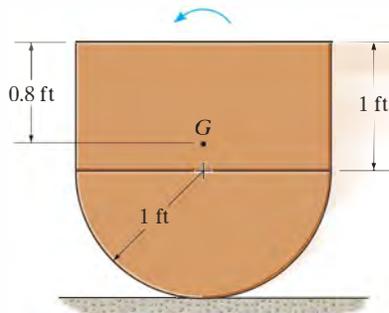
**Prob. 19-3**

**\*19-4.** The pilot of a crippled jet was able to control his plane by throttling the two engines. If the plane has a weight of 17 000 lb and a radius of gyration of  $k_G = 4.7$  ft about the mass center  $G$ , determine the angular velocity of the plane and the velocity of its mass center  $G$  in  $t = 5$  s if the thrust in each engine is altered to  $T_1 = 5000$  lb and  $T_2 = 800$  lb as shown. Originally the plane is flying straight at 1200 ft/s. Neglect the effects of drag and the loss of fuel.



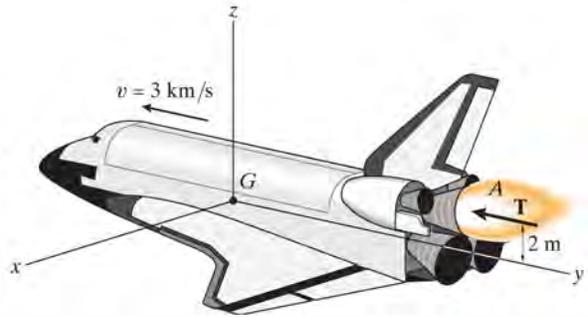
**Prob. 19-4**

•19-5. The assembly weighs 10 lb and has a radius of gyration  $k_G = 0.6$  ft about its center of mass  $G$ . The kinetic energy of the assembly is 31 ft · lb when it is in the position shown. If it rolls counterclockwise on the surface without slipping, determine its linear momentum at this instant.



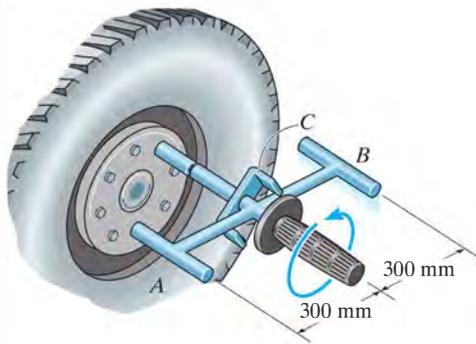
Prob. 19-5

19-7. The space shuttle is located in “deep space,” where the effects of gravity can be neglected. It has a mass of 120 Mg, a center of mass at  $G$ , and a radius of gyration  $(k_G)_x = 14$  m about the  $x$  axis. It is originally traveling forward at  $v = 3$  km/s when the pilot turns on the engine at  $A$ , creating a thrust  $T = 600(1 - e^{-0.3t})$  kN, where  $t$  is in seconds. Determine the shuttle’s angular velocity 2 s later.



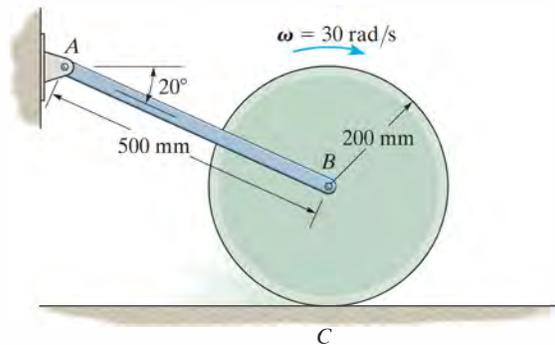
Prob. 19-7

19-6. The impact wrench consists of a slender 1-kg rod  $AB$  which is 580 mm long, and cylindrical end weights at  $A$  and  $B$  that each have a diameter of 20 mm and a mass of 1 kg. This assembly is free to rotate about the handle and socket, which are attached to the lug nut on the wheel of a car. If the rod  $AB$  is given an angular velocity of 4 rad/s and it strikes the bracket  $C$  on the handle without rebounding, determine the angular impulse imparted to the lug nut.



Prob. 19-6

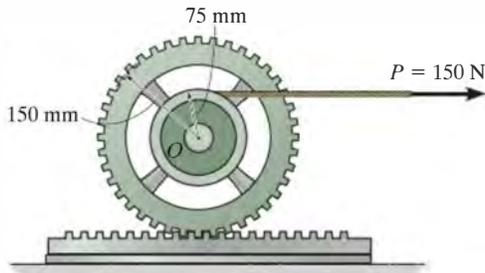
\*19-8. The 50-kg cylinder has an angular velocity of 30 rad/s when it is brought into contact with the horizontal surface at  $C$ . If the coefficient of kinetic friction is  $\mu_C = 0.2$ , determine how long it will take for the cylinder to stop spinning. What force is developed in link  $AB$  during this time? The axle through the cylinder is connected to two symmetrical links. (Only  $AB$  is shown.) For the computation, neglect the weight of the links.



Prob. 19-8

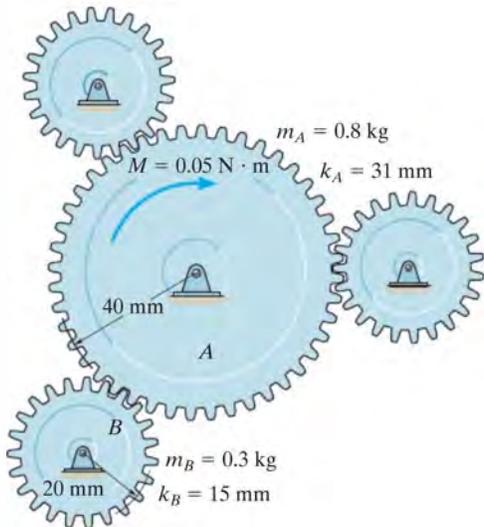
•19-9. If the cord is subjected to a horizontal force of  $P = 150$  N, and the gear rack is fixed to the horizontal plane, determine the angular velocity of the gear in 4 s, starting from rest. The mass of the gear is 50 kg, and it has a radius of gyration about its center of mass  $O$  of  $k_O = 125$  mm.

19-10. If the cord is subjected to a horizontal force of  $P = 150$  N, and gear is supported by a fixed pin at  $O$ , determine the angular velocity of the gear and the velocity of the 20-kg gear rack in 4 s, starting from rest. The mass of the gear is 50 kg and it has a radius of gyration of  $k_O = 125$  mm. Assume that the contact surface between the gear rack and the horizontal plane is smooth.



Probs. 19-9/10

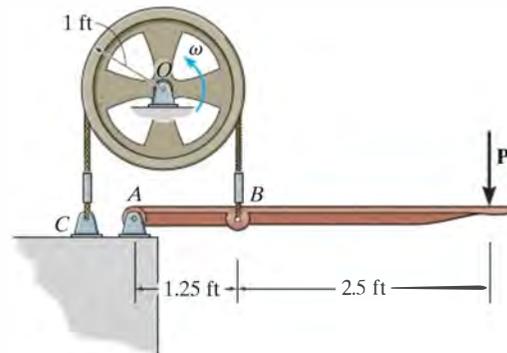
19-11. A motor transmits a torque of  $M = 0.05$  N·m to the center of gear  $A$ . Determine the angular velocity of each of the three (equal) smaller gears in 2 s starting from rest. The smaller gears ( $B$ ) are pinned at their centers, and the masses and centroidal radii of gyration of the gears are given in the figure.



Prob. 19-11

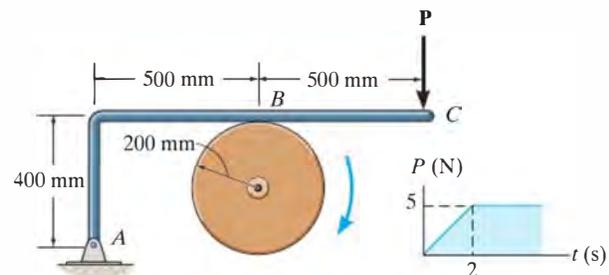
\*19-12. The 200-lb flywheel has a radius of gyration about its center of gravity  $O$  of  $k_O = 0.75$  ft. If it rotates counterclockwise with an angular velocity of 1200 rev/min before the brake is applied, determine the time required for the wheel to come to rest when a force of  $P = 200$  lb is applied to the handle. The coefficient of kinetic friction between the belt and the wheel rim is  $\mu_k = 0.3$ . (Hint: Recall from the statics text that the relation of the tension in the belt is given by  $T_B = T_C e^{\mu\beta}$ , where  $\beta$  is the angle of contact in radians.)

•19-13. The 200-lb flywheel has a radius of gyration about its center of gravity  $O$  of  $k_O = 0.75$  ft. If it rotates counterclockwise with a constant angular velocity of 1200 rev/min before the brake is applied, determine the required force  $\mathbf{P}$  that must be applied to the handle to stop the wheel in 2 s. The coefficient of kinetic friction between the belt and the wheel rim is  $\mu_k = 0.3$ . (Hint: Recall from the statics text that the relation of the tension in the belt is given by  $T_B = T_C e^{\mu\beta}$ , where  $\beta$  is the angle of contact in radians.)



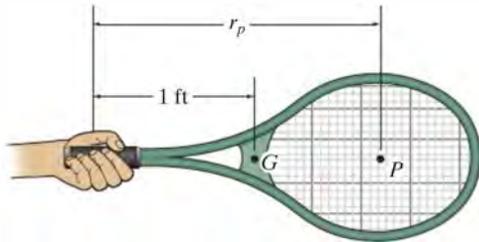
Probs. 19-12/13

19-14. The 12-kg disk has an angular velocity of  $\omega = 20$  rad/s. If the brake  $ABC$  is applied such that the magnitude of force  $\mathbf{P}$  varies with time as shown, determine the time needed to stop the disk. The coefficient of kinetic friction at  $B$  is  $\mu_k = 0.4$ . Neglect the thickness of the brake.



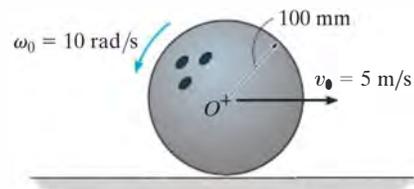
Prob. 19-14

**19–15.** The 1.25-lb tennis racket has a center of gravity at  $G$  and a radius of gyration about  $G$  of  $k_G = 0.625$  ft. Determine the position  $P$  where the ball must be hit so that ‘no sting’ is felt by the hand holding the racket, i.e., the horizontal force exerted by the racket on the hand is zero.



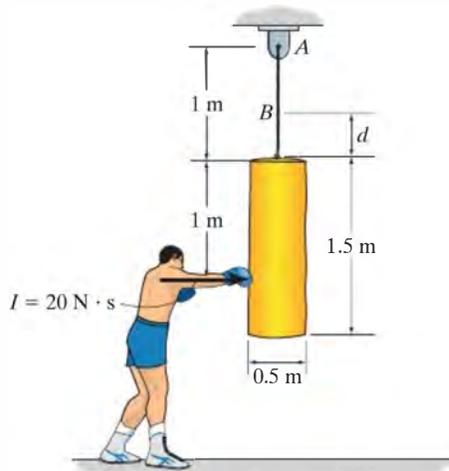
Prob. 19–15

**•19–17.** The 5-kg ball is cast on the alley with a backspin of  $\omega_0 = 10$  rad/s, and the velocity of its center of mass  $O$  is  $v_0 = 5$  m/s. Determine the time for the ball to stop back spinning, and the velocity of its center of mass at this instant. The coefficient of kinetic friction between the ball and the alley is  $\mu_k = 0.08$ .



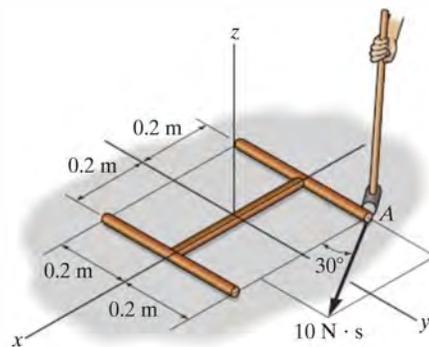
Prob. 19–17

**\*19–16.** If the boxer hits the 75-kg punching bag with an impulse of  $I = 20$  N·s, determine the angular velocity of the bag immediately after it has been hit. Also, find the location  $d$  of point  $B$ , about which the bag appears to rotate. Treat the bag as a uniform cylinder.



Prob. 19–16

**19–18.** The smooth rod assembly shown is at rest when it is struck by a hammer at  $A$  with an impulse of  $10$  N·s. Determine the angular velocity of the assembly and the magnitude of velocity of its mass center immediately after it has been struck. The rods have a mass per unit length of  $6$  kg/m.



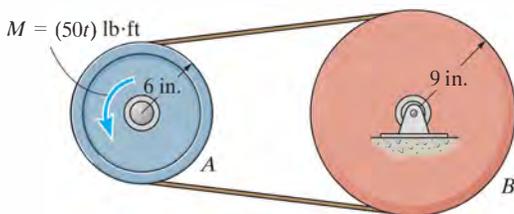
Prob. 19–18

**19–19.** The flywheel  $A$  has a mass of 30 kg and a radius of gyration of  $k_C = 95$  mm. Disk  $B$  has a mass of 25 kg, is pinned at  $D$ , and is coupled to the flywheel using a belt which is subjected to a tension such that it does not slip at its contacting surfaces. If a motor supplies a counterclockwise torque or twist to the flywheel, having a magnitude of  $M = (12t)$  N·m, where  $t$  is in seconds, determine the angular velocity of the disk 3 s after the motor is turned on. Initially, the flywheel is at rest.



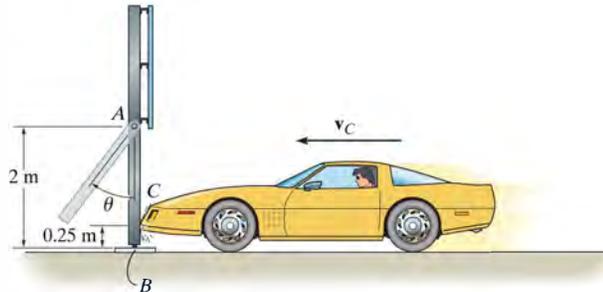
**Prob. 19–19**

**\*19–20.** The 30-lb flywheel  $A$  has a radius of gyration about its center of 4 in. Disk  $B$  weighs 50 lb and is coupled to the flywheel by means of a belt which does not slip at its contacting surfaces. If a motor supplies a counterclockwise torque to the flywheel of  $M = (50t)$  lb·ft, where  $t$  is in seconds, determine the time required for the disk to attain an angular velocity of 60 rad/s starting from rest.



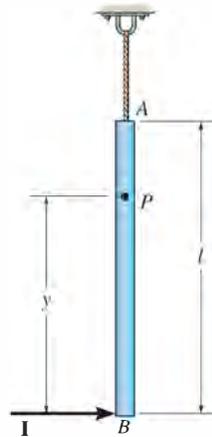
**Prob. 19–20**

**•19–21.** For safety reasons, the 20-kg supporting leg of a sign is designed to break away with negligible resistance at  $B$  when the leg is subjected to the impact of a car. Assuming that the leg is pinned at  $A$  and approximates a thin rod, determine the impulse the car bumper exerts on it, if after the impact the leg appears to rotate clockwise to a maximum angle of  $\theta_{\max} = 150^\circ$ .



**Prob. 19–21**

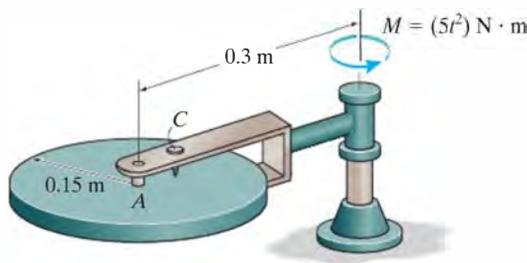
**19–22.** The slender rod has a mass  $m$  and is suspended at its end  $A$  by a cord. If the rod receives a horizontal blow giving it an impulse  $\mathbf{I}$  at its bottom  $B$ , determine the location  $y$  of the point  $P$  about which the rod appears to rotate during the impact.



**Prob. 19–22**

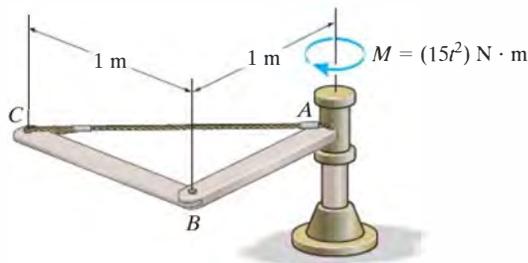
**19–23.** The 25-kg circular disk is attached to the yoke by means of a smooth axle  $A$ . Screw  $C$  is used to lock the disk to the yoke. If the yoke is subjected to a torque of  $M = (5t^2) \text{ N} \cdot \text{m}$ , where  $t$  is in seconds, and the disk is unlocked, determine the angular velocity of the yoke when  $t = 3 \text{ s}$ , starting from rest. Neglect the mass of the yoke.

**\*19–24.** The 25-kg circular disk is attached to the yoke by means of a smooth axle  $A$ . Screw  $C$  is used to lock the disk to the yoke. If the yoke is subjected to a torque of  $M = (5t^2) \text{ N} \cdot \text{m}$ , where  $t$  is in seconds, and the disk is locked, determine the angular velocity of the yoke when  $t = 3 \text{ s}$ , starting from rest. Neglect the mass of the yoke.



Probs. 19–23/24

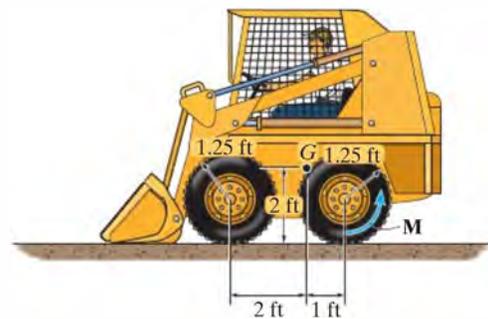
**•19–25.** If the shaft is subjected to a torque of  $M = (15t^2) \text{ N} \cdot \text{m}$ , where  $t$  is in seconds, determine the angular velocity of the assembly when  $t = 3 \text{ s}$ , starting from rest. Rods  $AB$  and  $BC$  each have a mass of  $9 \text{ kg}$ .



Prob. 19–25

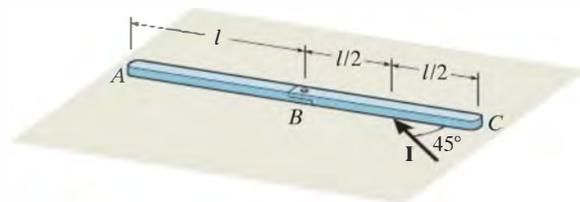
**19–26.** The body and bucket of a skid steer loader has a weight of  $2000 \text{ lb}$ , and its center of gravity is located at  $G$ . Each of the four wheels has a weight of  $100 \text{ lb}$  and a radius of gyration about its center of gravity of  $1 \text{ ft}$ . If the engine supplies a torque of  $M = 100 \text{ lb} \cdot \text{ft}$  to each of the rear drive wheels, determine the speed of the loader in  $t = 10 \text{ s}$ , starting from rest. The wheels roll without slipping.

**19–27.** The body and bucket of a skid steer loader has a weight of  $2000 \text{ lb}$ , and its center of gravity is located at  $G$ . Each of the four wheels has a weight of  $100 \text{ lb}$  and a radius of gyration about its center of gravity of  $1 \text{ ft}$ . If the loader attains a speed of  $20 \text{ ft/s}$  in  $10 \text{ s}$ , starting from rest, determine the torque  $M$  supplied to each of the rear drive wheels. The wheels roll without slipping.



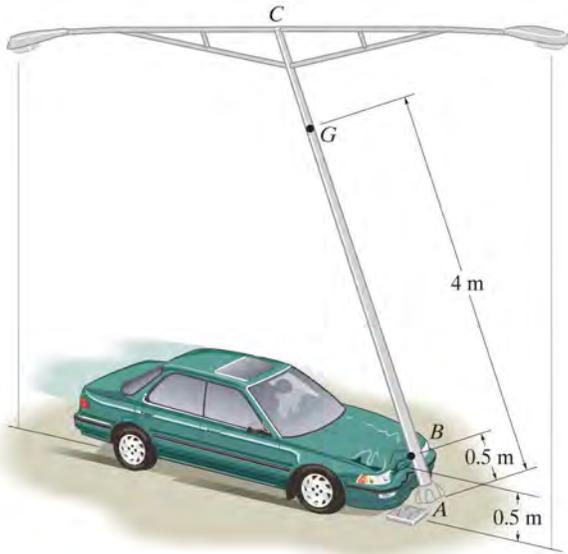
Probs. 19–26/27

**\*19–28.** The two rods each have a mass  $m$  and a length  $l$ , and lie on the smooth horizontal plane. If an impulse  $I$  is applied at an angle of  $45^\circ$  to one of the rods at midlength as shown, determine the angular velocity of each rod just after the impact. The rods are pin connected at  $B$ .



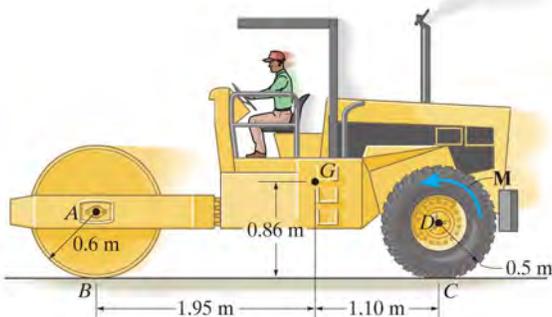
Prob. 19–28

**•19–29.** The car strikes the side of a light pole, which is designed to break away from its base with negligible resistance. From a video taken of the collision it is observed that the pole was given an angular velocity of 60 rad/s when  $AC$  was vertical. The pole has a mass of 175 kg, a center of mass at  $G$ , and a radius of gyration about an axis perpendicular to the plane of the pole assembly and passing through  $G$  of  $k_G = 2.25$  m. Determine the horizontal impulse which the car exerts on the pole at the instant  $AC$  is essentially vertical.



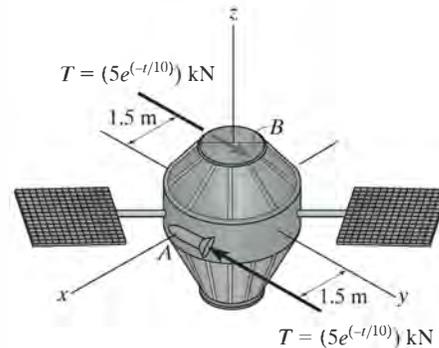
**Prob. 19–29**

**19–30.** The frame of the roller has a mass of 5.5 Mg and a center of mass at  $G$ . The roller has a mass of 2 Mg and a radius of gyration about its mass center of  $k_A = 0.45$  m. If a torque of  $M = 600$  N·m is applied to the rear wheels, determine the speed of the compactor in  $t = 4$  s, starting from rest. No slipping occurs. Neglect the mass of the driving wheels.



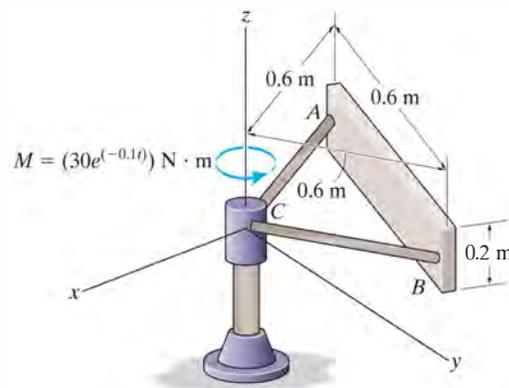
**Prob. 19–30**

**19–31.** The 200-kg satellite has a radius of gyration about the centroidal  $z$  axis of  $k_z = 1.25$  m. Initially it is rotating with a constant angular velocity of  $\omega_0 = \{1500 \mathbf{k}\}$  rev/min. If the two jets  $A$  and  $B$  are fired simultaneously and produce a thrust of  $T = (5e^{-0.1t})$  kN, where  $t$  is in seconds, determine the angular velocity of the satellite, five seconds after firing.



**Prob. 19–31**

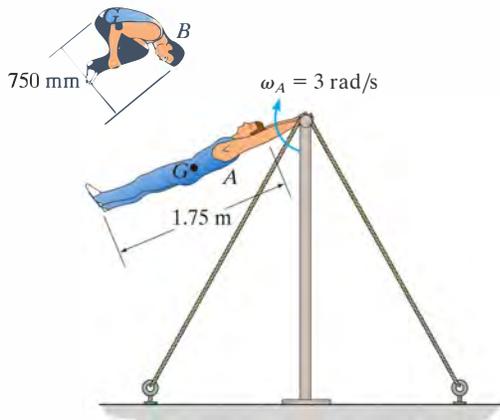
**\*19–32.** If the shaft is subjected to a torque of  $M = (30e^{-0.1t})$  N·m, where  $t$  is in seconds, determine the angular velocity of the assembly when  $t = 5$  s, starting from rest. The rectangular plate has a mass of 25 kg. Rods  $AC$  and  $BC$  have the same mass of 5 kg.



**Prob. 19–32**

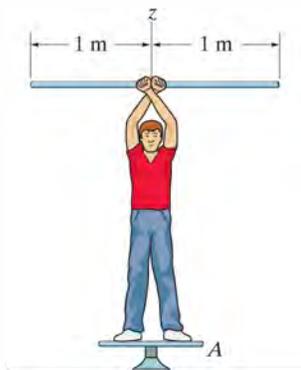
PROBLEMS

•19–33. The 75-kg gymnast lets go of the horizontal bar in a fully stretched position *A*, rotating with an angular velocity of  $\omega_A = 3 \text{ rad/s}$ . Estimate his angular velocity when he assumes a tucked position *B*. Assume the gymnast at positions *A* and *B* as a uniform slender rod and a uniform circular disk, respectively.



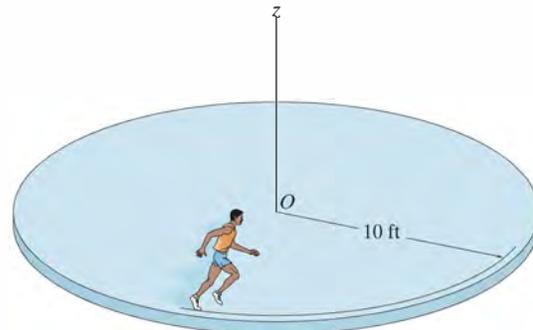
Prob. 19–33

19–34. A 75-kg man stands on the turntable *A* and rotates a 6-kg slender rod over his head. If the angular velocity of the rod is  $\omega_r = 5 \text{ rad/s}$  measured relative to the man and the turntable is observed to be rotating in the opposite direction with an angular velocity of  $\omega_t = 3 \text{ rad/s}$ , determine the radius of gyration of the man about the *z* axis. Consider the turntable as a thin circular disk of 300-mm radius and 5-kg mass.



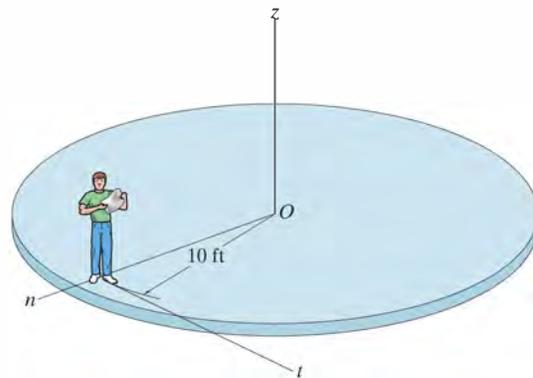
Prob. 19–34

19–35. A horizontal circular platform has a weight of 300 lb and a radius of gyration  $k_z = 8 \text{ ft}$  about the *z* axis passing through its center *O*. The platform is free to rotate about the *z* axis and is initially at rest. A man having a weight of 150 lb begins to run along the edge in a circular path of radius 10 ft. If he maintains a speed of 4 ft/s relative to the platform, determine the angular velocity of the platform. Neglect friction.



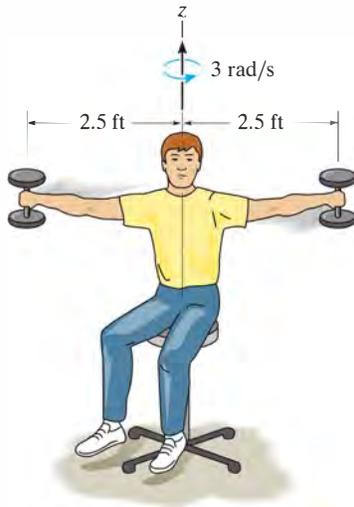
Prob. 19–35

\*19–36. A horizontal circular platform has a weight of 300 lb and a radius of gyration  $k_z = 8 \text{ ft}$  about the *z* axis passing through its center *O*. The platform is free to rotate about the *z* axis and is initially at rest. A man having a weight of 150 lb throws a 15-lb block off the edge of the platform with a horizontal velocity of 5 ft/s, measured relative to the platform. Determine the angular velocity of the platform if the block is thrown (a) tangent to the platform, along the  $+t$  axis, and (b) outward along a radial line, or  $+n$  axis. Neglect the size of the man.



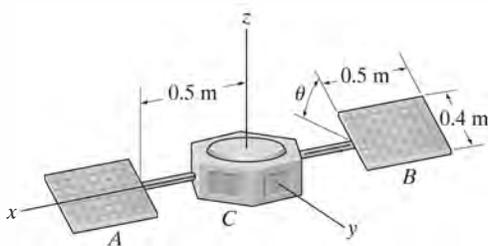
Prob. 19–36

**•19–37.** The man sits on the swivel chair holding two 5-lb weights with his arms outstretched. If he is rotating at 3 rad/s in this position, determine his angular velocity when the weights are drawn in and held 0.3 ft from the axis of rotation. Assume he weighs 160 lb and has a radius of gyration  $k_z = 0.55$  ft about the  $z$  axis. Neglect the mass of his arms and the size of the weights for the calculation.



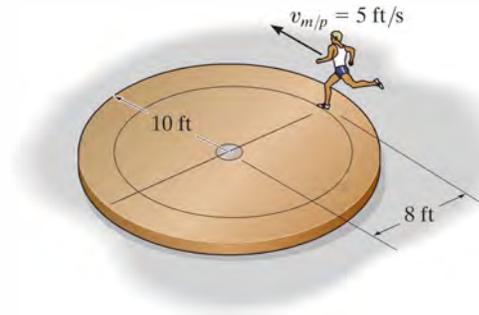
**Prob. 19–37**

**19–38.** The satellite's body  $C$  has a mass of 200 kg and a radius of gyration about the  $z$  axis of  $k_z = 0.2$  m. If the satellite rotates about the  $z$  axis with an angular velocity of 5 rev/s, when the solar panels  $A$  and  $B$  are in a position of  $\theta = 0^\circ$ , determine the angular velocity of the satellite when the solar panels are rotated to a position of  $\theta = 90^\circ$ . Consider each solar panel to be a thin plate having a mass of 30 kg. Neglect the mass of the rods.



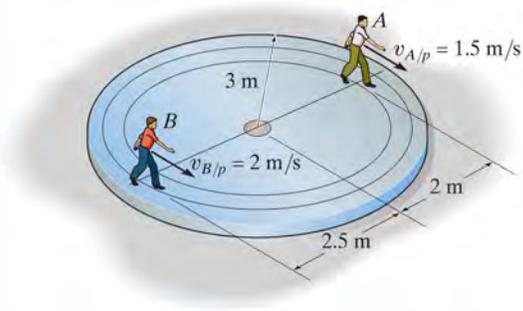
**Prob. 19–38**

**19–39.** A 150-lb man leaps off the circular platform with a velocity of  $v_{m/p} = 5$  ft/s, relative to the platform. Determine the angular velocity of the platform afterwards. Initially the man and platform are at rest. The platform weighs 300 lb and can be treated as a uniform circular disk.



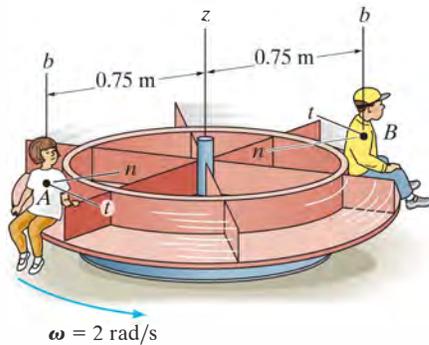
**Prob. 19–39**

**\*19–40.** The 150-kg platform can be considered as a circular disk. Two men,  $A$  and  $B$ , of 60-kg and 75-kg mass, respectively, stand on the platform when it is at rest. If they start to walk around the circular paths with speeds of  $v_{A/p} = 1.5$  m/s and  $v_{B/p} = 2$  m/s, measured relative to the platform, determine the angular velocity of the platform.



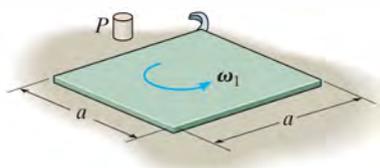
**Prob. 19–40**

•19-41. Two children *A* and *B*, each having a mass of 30 kg, sit at the edge of the merry-go-round which rotates at  $\omega = 2$  rad/s. Excluding the children, the merry-go-round has a mass of 180 kg and a radius of gyration  $k_z = 0.6$  m. Determine the angular velocity of the merry-go-round if *A* jumps off horizontally in the  $-n$  direction with a speed of 2 m/s, measured relative to the merry-go-round. What is the merry-go-round's angular velocity if *B* then jumps off horizontally in the  $-t$  direction with a speed of 2 m/s, measured relative to the merry-go-round? Neglect friction and the size of each child.



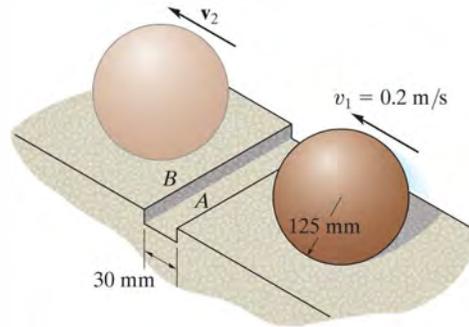
Prob. 19-41

19-42. A thin square plate of mass *m* rotates on the smooth surface with an angular velocity  $\omega_1$ . Determine its new angular velocity just after the hook at its corner strikes the peg *P* and the plate starts to rotate about *P* without rebounding.



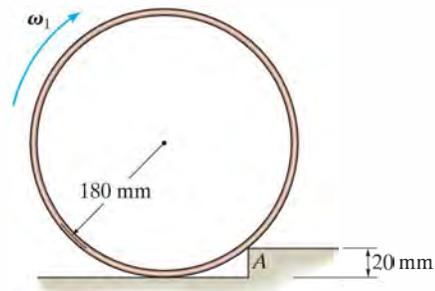
Prob. 19-42

19-43. A ball having a mass of 8 kg and initial speed of  $v_1 = 0.2$  m/s rolls over a 30-mm-long depression. Assuming that the ball rolls off the edges of contact first *A*, then *B*, without slipping, determine its final velocity  $v_2$  when it reaches the other side.



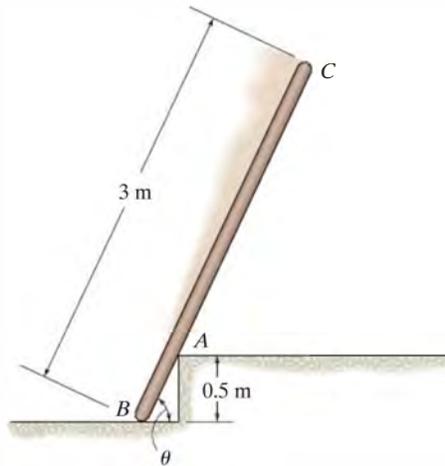
Prob. 19-43

\*19-44. The 15-kg thin ring strikes the 20-mm-high step. Determine the smallest angular velocity  $\omega_1$  the ring can have so that it will just roll over the step at *A* without slipping



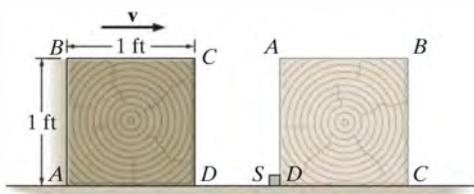
Prob. 19-44

•19–45. The uniform pole has a mass of 15 kg and falls from rest when  $\theta = 90^\circ$ . It strikes the edge at  $A$  when  $\theta = 60^\circ$ . If the pole then begins to pivot about this point after contact, determine the pole's angular velocity just after the impact. Assume that the pole does not slip at  $B$  as it falls until it strikes  $A$ .



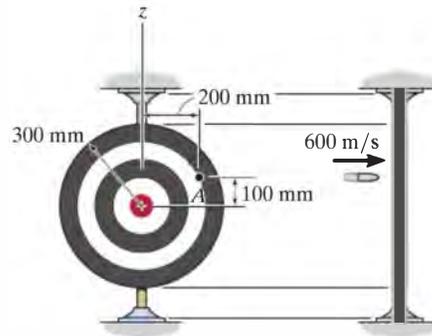
Prob. 19–45

19–46. The 10-lb block slides on the smooth surface when the corner  $D$  hits a stop block  $S$ . Determine the minimum velocity  $v$  the block should have which would allow it to tip over on its side and land in the position shown. Neglect the size of  $S$ . *Hint:* During impact consider the weight of the block to be nonimpulsive.



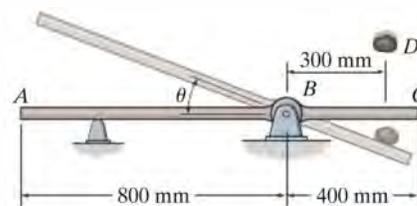
Prob. 19–46

19–47. The target is a thin 5-kg circular disk that can rotate freely about the  $z$  axis. A 25-g bullet, traveling at 600 m/s, strikes the target at  $A$  and becomes embedded in it. Determine the angular velocity of the target after the impact. Initially, it is at rest.



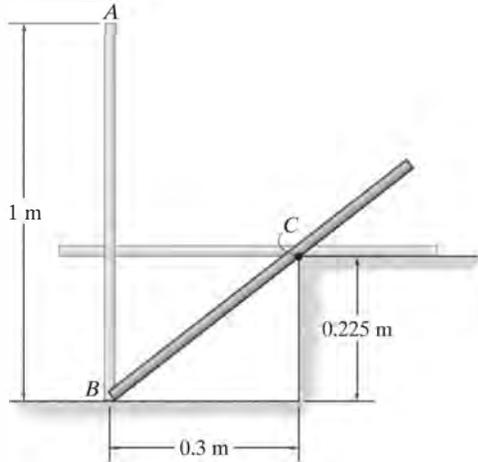
Prob. 19–47

\*19–48. A 2-kg mass of putty  $D$  strikes the uniform 10-kg plank  $ABC$  with a velocity of 10 m/s. If the putty remains attached to the plank, determine the maximum angle  $\theta$  of swing before the plank momentarily stops. Neglect the size of the putty.



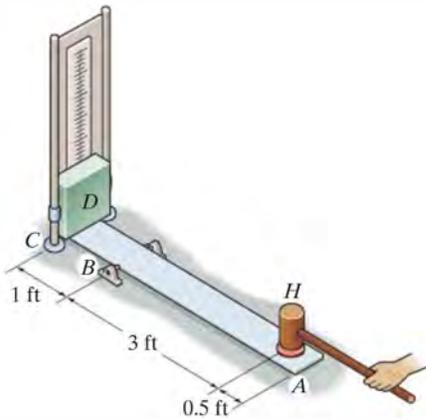
Prob. 19–48

•19–49. The uniform 6-kg slender rod  $AB$  is given a slight horizontal disturbance when it is in the vertical position and rotates about  $B$  without slipping. Subsequently, it strikes the step at  $C$ . The impact is perfectly plastic and so the rod rotates about  $C$  without slipping after the impact. Determine the angular velocity of the rod when it is in the horizontal position shown.



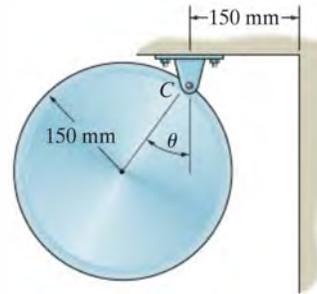
Prob. 19–49

19–50. The rigid 30-lb plank is struck by the 15-lb hammer head  $H$ . Just before the impact the hammer is gripped loosely and has a vertical velocity of 75 ft/s. If the coefficient of restitution between the hammer head and the plank is  $e = 0.5$ , determine the maximum height attained by the 50-lb block  $D$ . The block can slide freely along the two vertical guide rods. The plank is initially in a horizontal position.



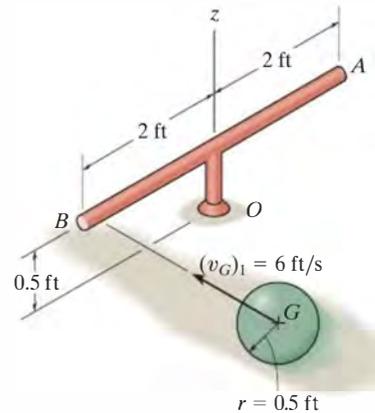
Prob. 19–50

19–51. The disk has a mass of 15 kg. If it is released from rest when  $\theta = 30^\circ$ , determine the maximum angle  $\theta$  of rebound after it collides with the wall. The coefficient of restitution between the disk and the wall is  $e = 0.6$ . When  $\theta = 0^\circ$ , the disk hangs such that it just touches the wall. Neglect friction at the pin  $C$ .



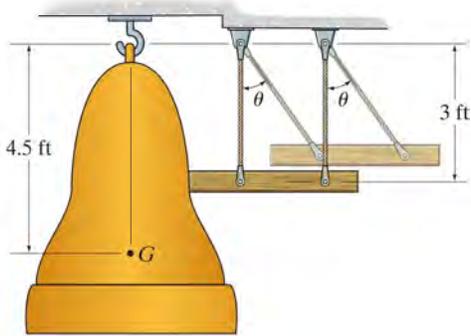
Prob. 19–51

\*19–52. The mass center of the 3-lb ball has a velocity of  $(v_G)_1 = 6$  ft/s when it strikes the end of the smooth 5-lb slender bar which is at rest. Determine the angular velocity of the bar about the  $z$  axis just after impact if  $e = 0.8$ .



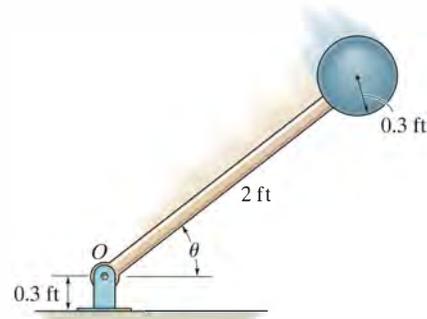
Prob. 19–52

•19–53. The 300-lb bell is at rest in the vertical position before it is struck by a 75-lb wooden post suspended from two equal-length ropes. If the post is released from rest at  $\theta = 45^\circ$ , determine the angular velocity of the bell and the velocity of the post immediately after the impact. The coefficient of restitution between the bell and the post is  $e = 0.6$ . The center of gravity of the bell is located at point  $G$  and its radius of gyration about  $G$  is  $k_G = 1.5$  ft.



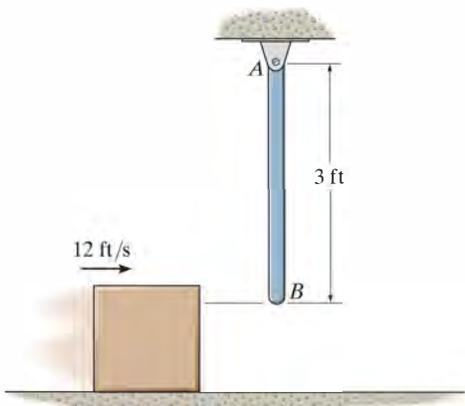
Prob. 19–53

19–55. The pendulum consists of a 10-lb sphere and 4-lb rod. If it is released from rest when  $\theta = 90^\circ$ , determine the angle  $\theta$  of rebound after the sphere strikes the floor. Take  $e = 0.8$ .



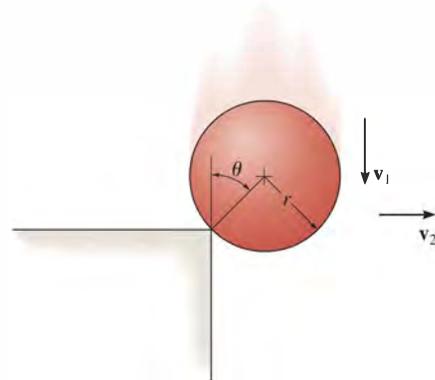
Prob. 19–55

19–54. The 4-lb rod  $AB$  hangs in the vertical position. A 2-lb block, sliding on a smooth horizontal surface with a velocity of 12 ft/s, strikes the rod at its end  $B$ . Determine the velocity of the block immediately after the collision. The coefficient of restitution between the block and the rod at  $B$  is  $e = 0.8$ .



Prob. 19–54

\*19–56. The solid ball of mass  $m$  is dropped with a velocity  $\mathbf{v}_1$  onto the edge of the rough step. If it rebounds horizontally off the step with a velocity  $\mathbf{v}_2$ , determine the angle  $\theta$  at which contact occurs. Assume no slipping when the ball strikes the step. The coefficient of restitution is  $e$ .



Prob. 19–56