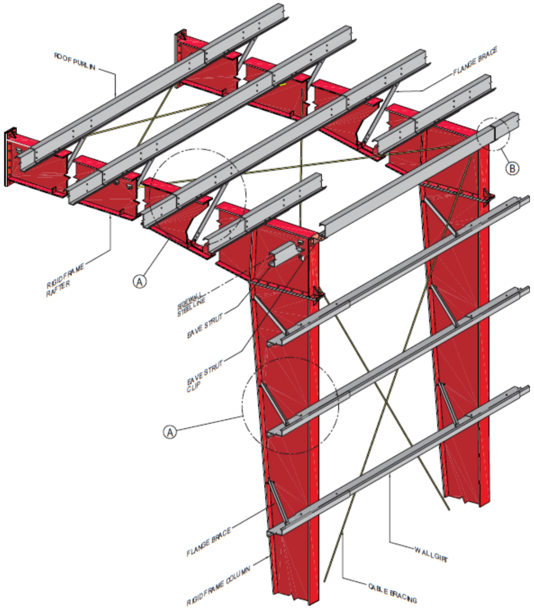
Introduction to Structural analysis



Introduction

A **structure** may be defined as *a system of connected parts used to support a load.*

For a structure to be design properly, the designer should carefully consider the following:

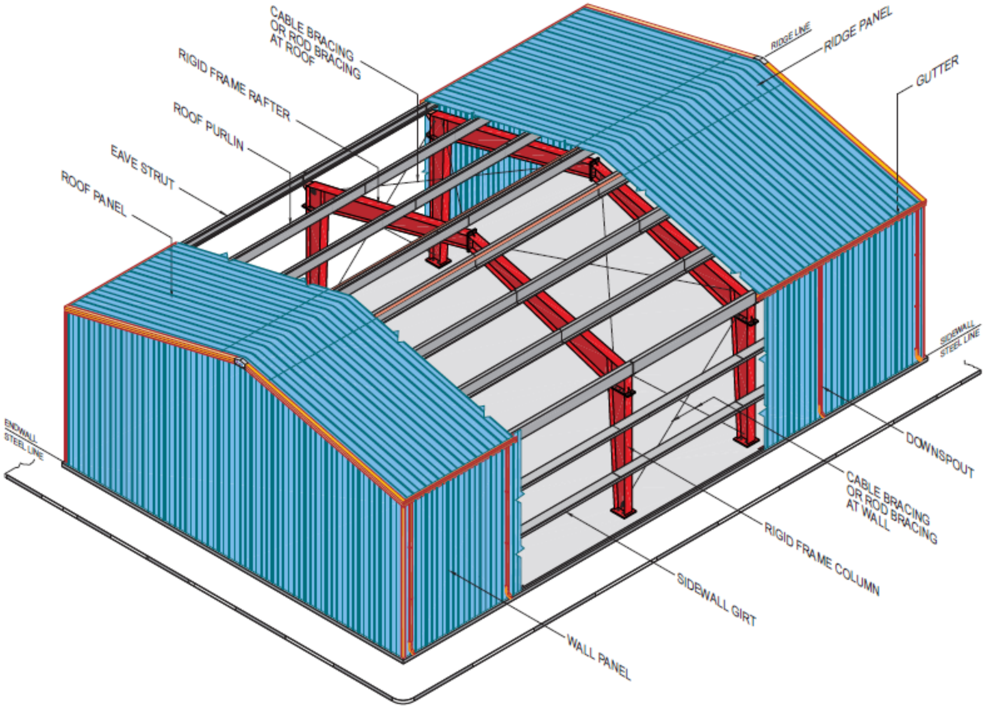
- ***safety***
- ***serviceability***
- ***economy***
- esthetics
- and the environment.

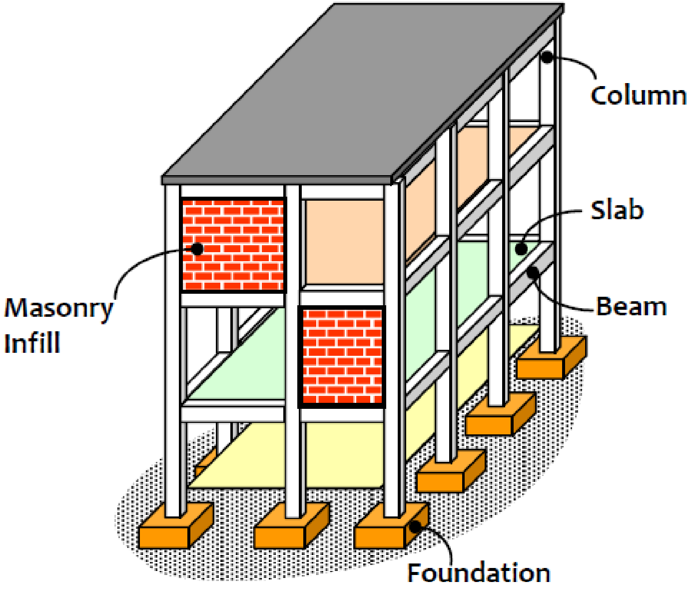
for a structure to be designed properly, the structural engineer must carefully study the following:

- The *structural loads* that the structure would be exposed to.
- *Material properties* used in the structure (*stress strain diagram* which shows how a material responds to load).
- And *type of structural system* to be used (truss, frame, arch … etc.)

Idealized Structure

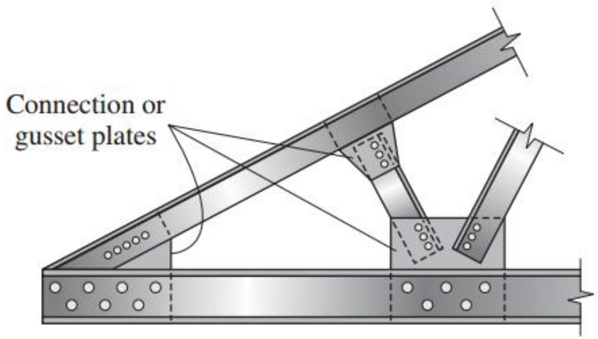
- The process of replacing an **actual (physical)** structure with **a simple (mathematical)** system conducive to analysis is called structural idealization.

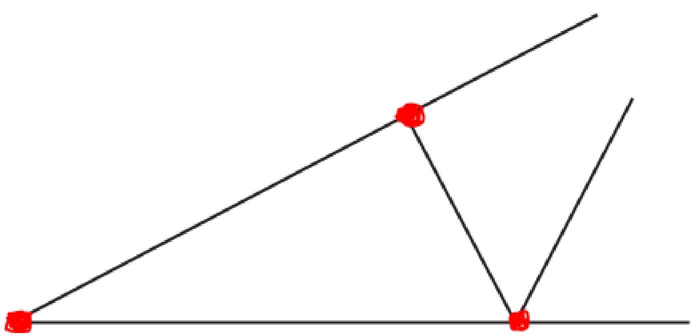


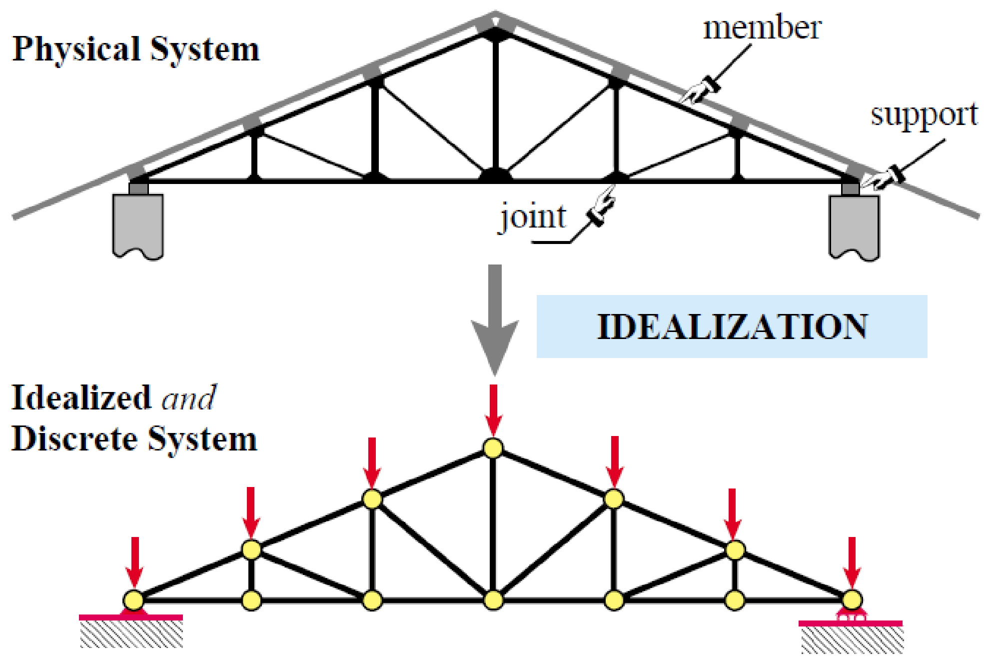


Idealized Structure

- Replacing actual **physical** structure by a **mathematical** model.



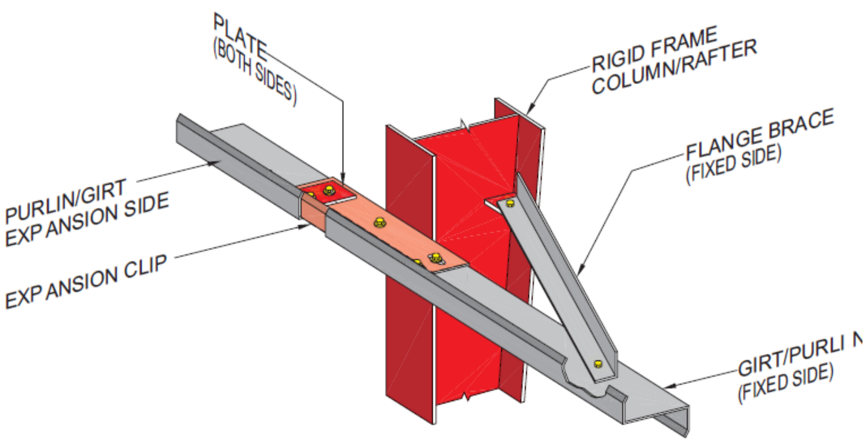


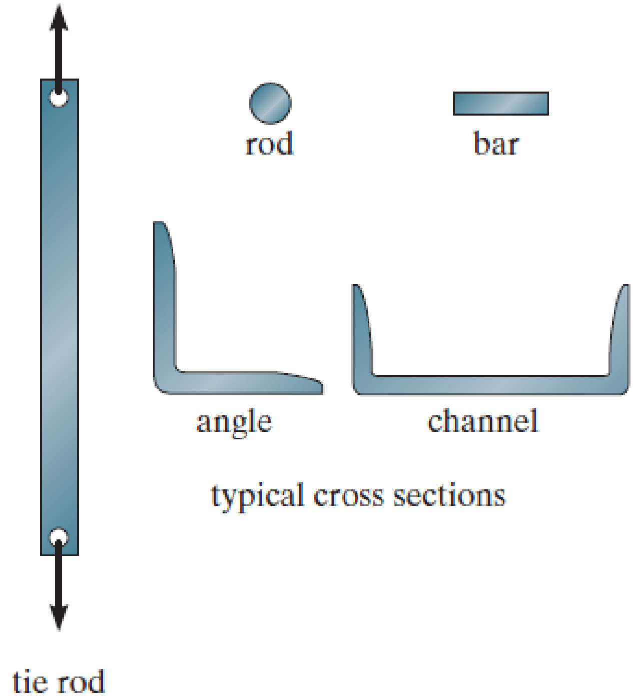


Structural elements: bars or rods

Structural Elements:

- ***Bars or Rods :*** Structural members subjected to a *axial force* are often referred to as ***tie rods (tension)*** or ***bracing struts (compression).***

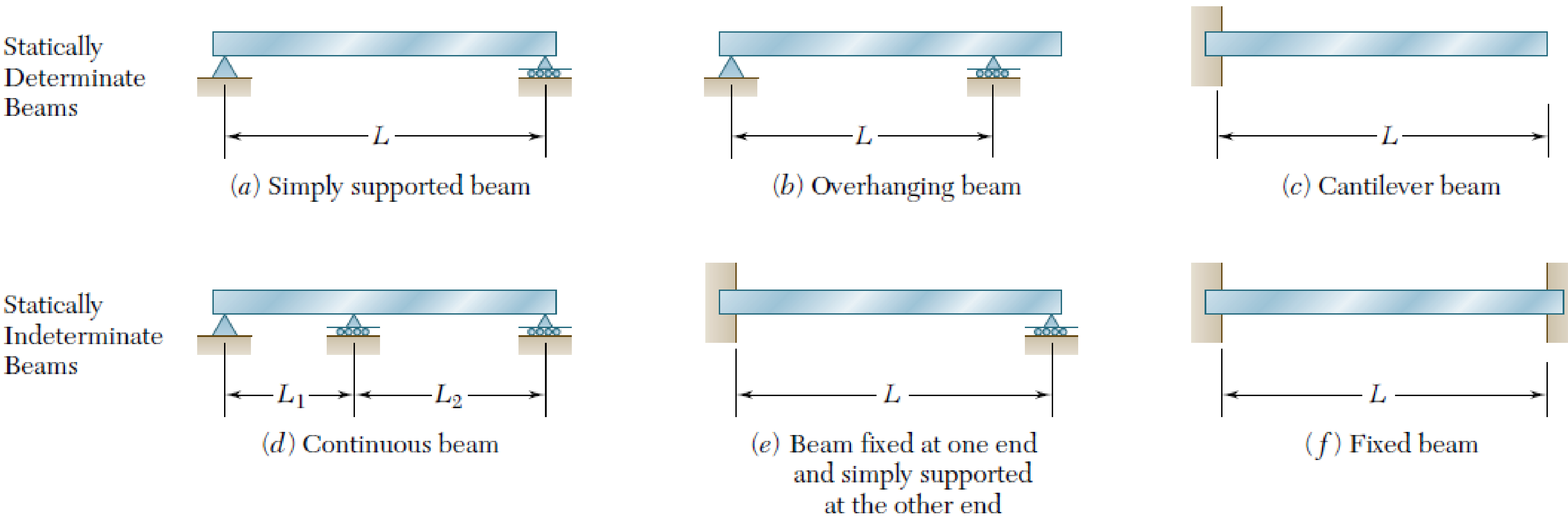




Structural Elements - Beams

Structural Elements:

- ***Beams:*** Slender structural members that are used to support load that is applied perpendicular to their longitudinal axis. They are often classified according to the way they are supported to the following:



(*a*) Simply supported beam

(*b*) Overhanging beam

(*c*) Cantilever beam

(*d*) Continuous beam

(*e*) Beam fixed at one end and simply supported at the other end

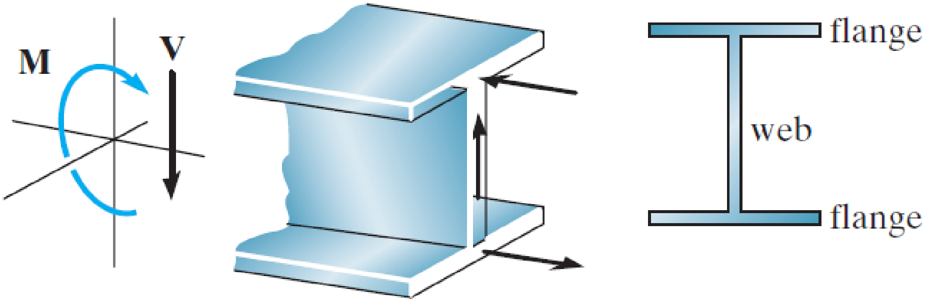
(*f*) Fixed beam

Structural Elements - Beams

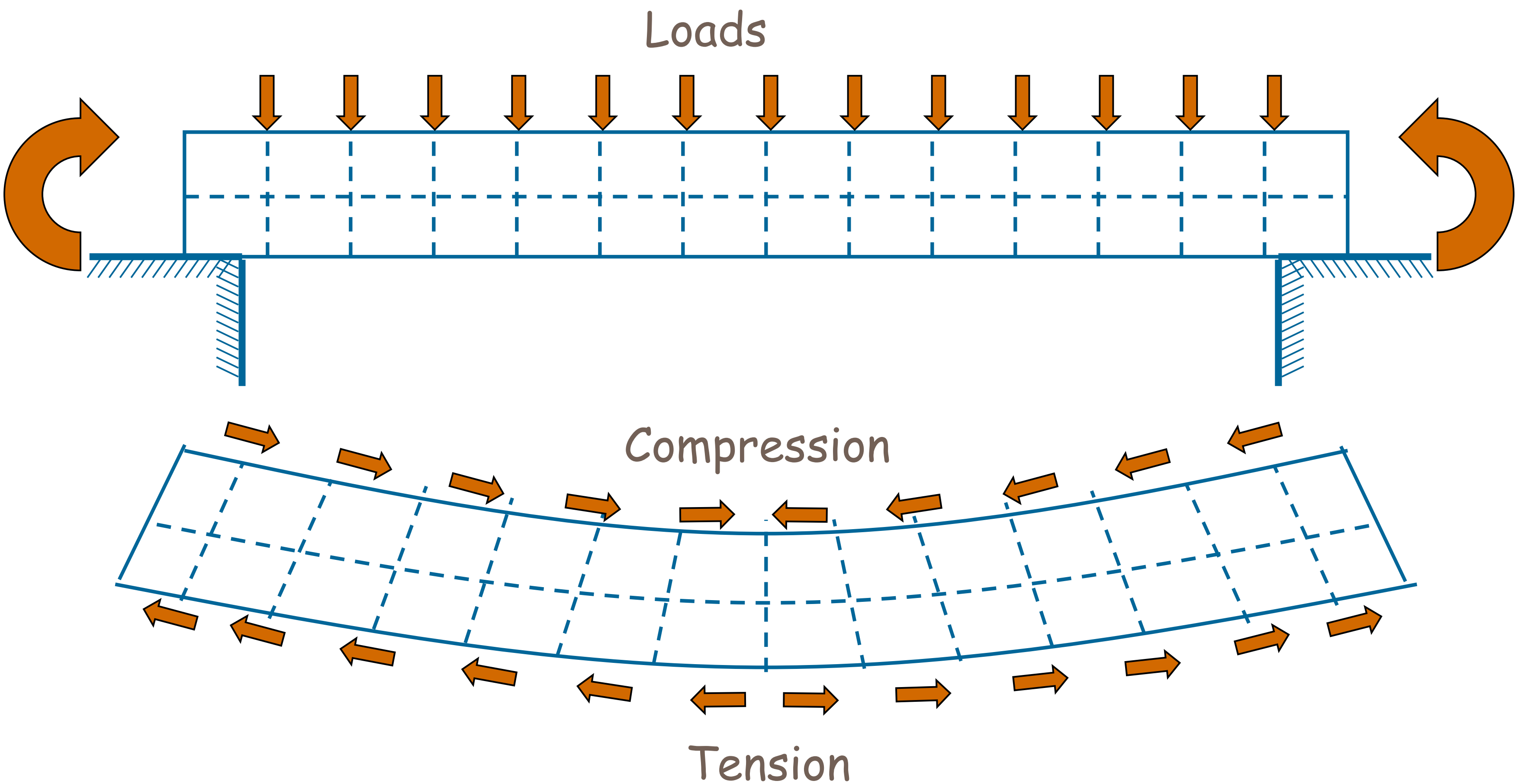
Beams are used to support loads by bending (flexure).

- The forces developed in the top and bottom ***flanges*** of the beam form the necessary couple used to resist the applied moment **M.** The ***web*** is effective in resisting the applied shear **V**.
- When the beam is required to have a very large span and the loads applied are rather large, the cross section may take the form of a ***plate girder (built up section from steel plates)***.

Shown are typical splice plate joints used to connect the steel girders of a highway bridge.

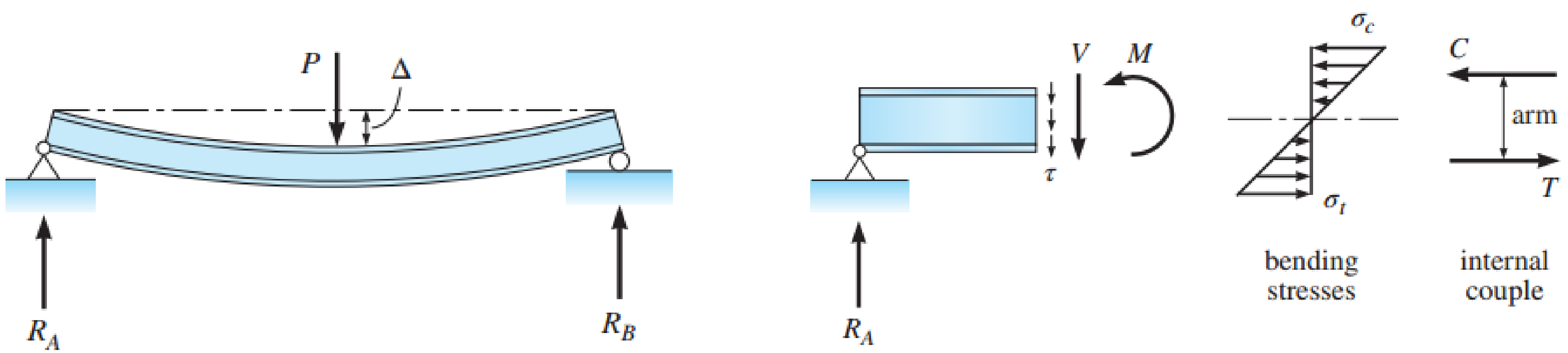


Structural Elements - Beams



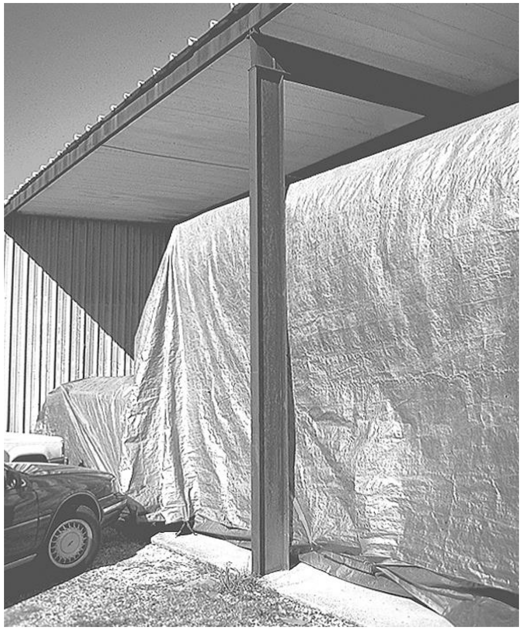
Structural Elements - Beams

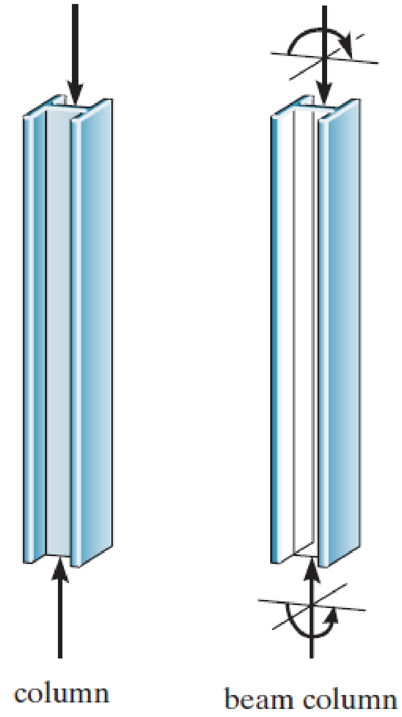
- As the *transverse* load is applied, a beam bends and deflects into a shallow curve. At a typical section of a beam, internal forces of shear V and moment M develop.
- Except in short, heavily loaded beams, the shear stresses τ produced by V are relatively small, but the longitudinal bending stresses produced by M are large.
- If the beam behaves elastically, the bending stresses on a cross section (compression on the top and tension on the bottom) vary linearly from a horizontal axis passing through the centroid of the cross section.
- The bending stresses are directly proportional to the moment, and vary in magnitude along the axis of the beam.



Structural Elements - columns

- Members that are generally vertical and resist axial compressive loads.
- columns are subjected to both an axial load and a bending moment as shown in the figure. These members are referred to as ***beam columns.***
- ***Columns*** may fail in crushing, yielding or buckling. The slenderness ratio L/r is important in compression members, where L is the length of the member and r is the radius of gyration.

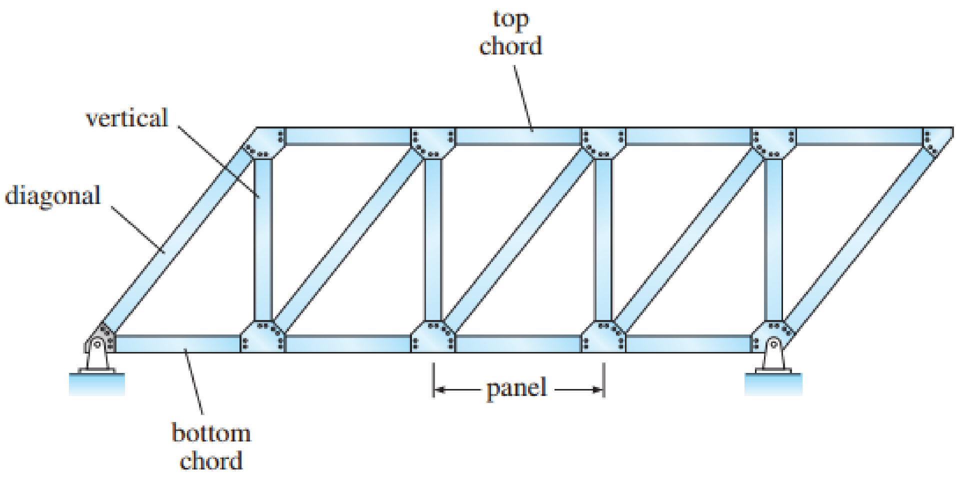


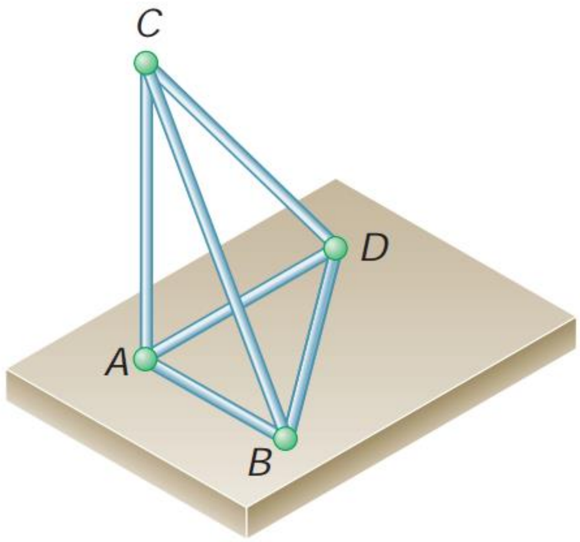


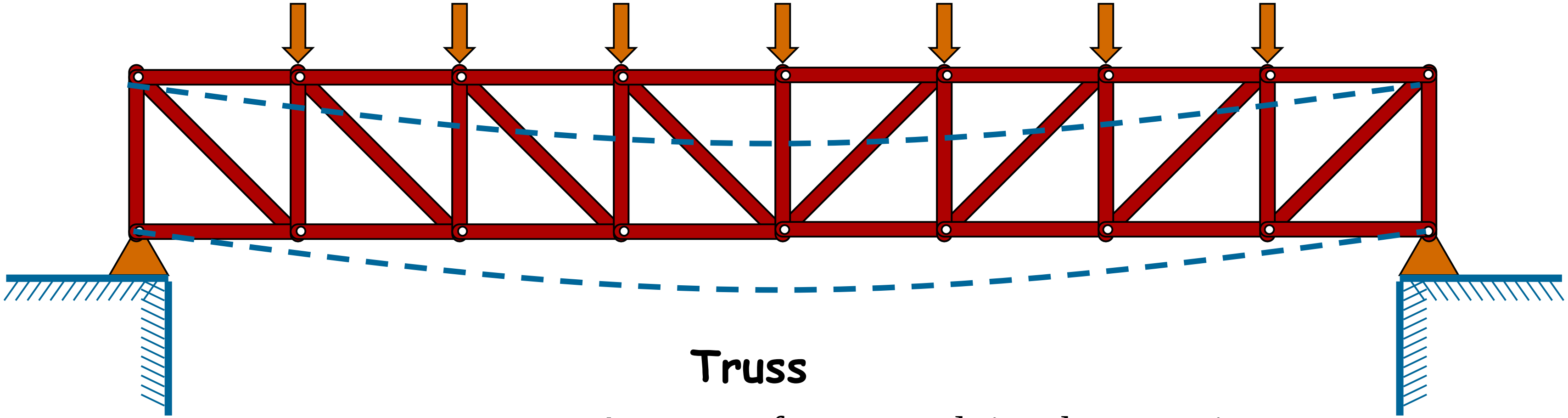
Structural Systems - Trusses

- A truss is a structural system composed of slender bars whose ends are assumed to be connected by frictionless pin joints.
- If pin-jointed trusses are loaded at the joints only, direct or axial stress develops in all bars.

- ***Planar trusses*** are composed of members that lie in the same plane and are frequently used for bridge and roofs.
- ***Space trusses*** have members extending in three dimensions and are suitable for towers.

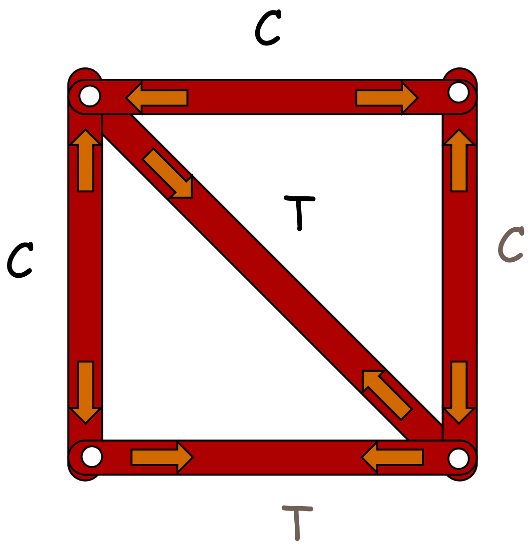






Truss

A group of struts and ties that are pin connected forms the truss system



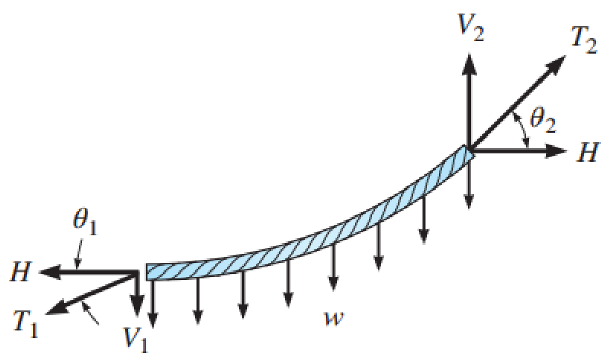
Forces in Truss Members

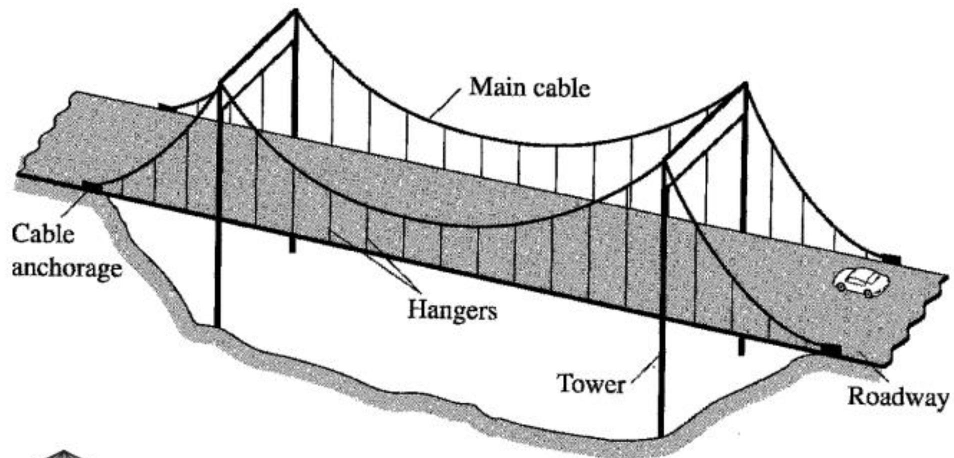
Structural Systems – Trusses

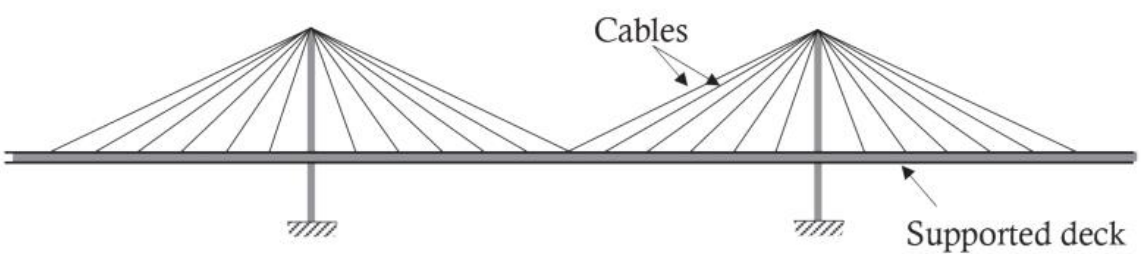
- The behavior of a truss is similar to that of a beam as:
  - Trusses develops compression in the top members (top chords), and tension in the bottom members (bottom chords), the couple of these forces resist the bending caused by applied loading
  - The solid beam web (which transmits the shear) is replaced by a series of vertical and diagonal bars. By eliminating the solid web, the designer can reduce the dead weight of the structure significantly. Trusses are much lighter than beams especially for large spans
- The above is one of the primary advantages of a truss, compared to a beam, it uses less material to support a given load

Structural Systems – Cables $ Arches

- ***Cables*** are usually flexible and carry their loads in tension. They are used to support bridges and building roofs.

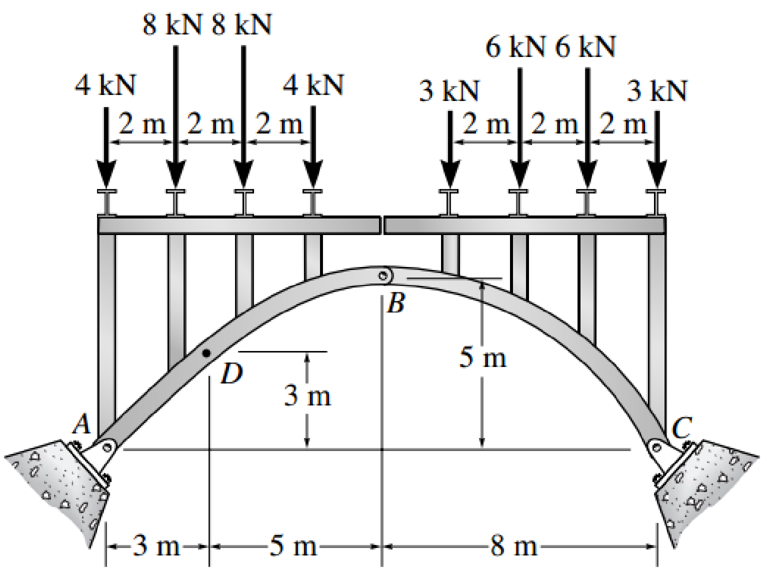






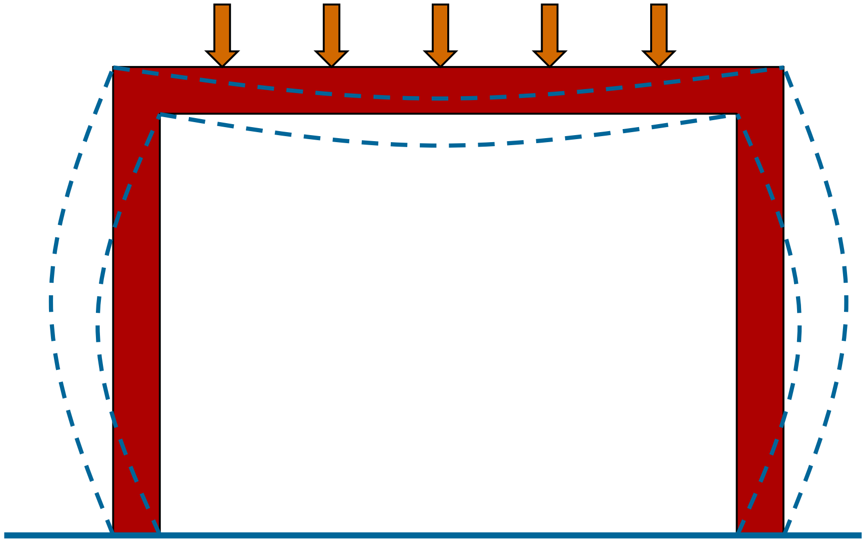
Structural Systems – Cables $ Arches

- The ***arch*** achieves its strength in compression, since it has a reverse curvature to that of the cable.
- Due to arch rigidity, it may also resist some bending and shear depending upon how it is loaded and shaped (geometry).
- If the arch has the geometry (shape) of a cable under a loading pattern, then the arch will carry only compression forces when it supports the same load pattern. For example catenary geometry (funicular- approximate parabola) under uniformly distributed load



Arches support their loads in compression.

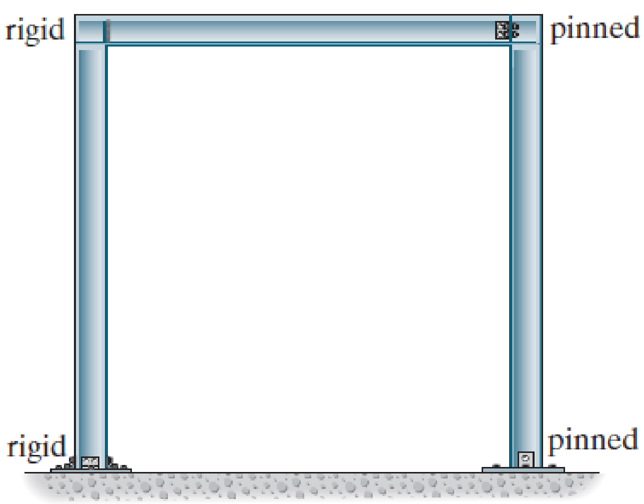
Structural Systems – Frames

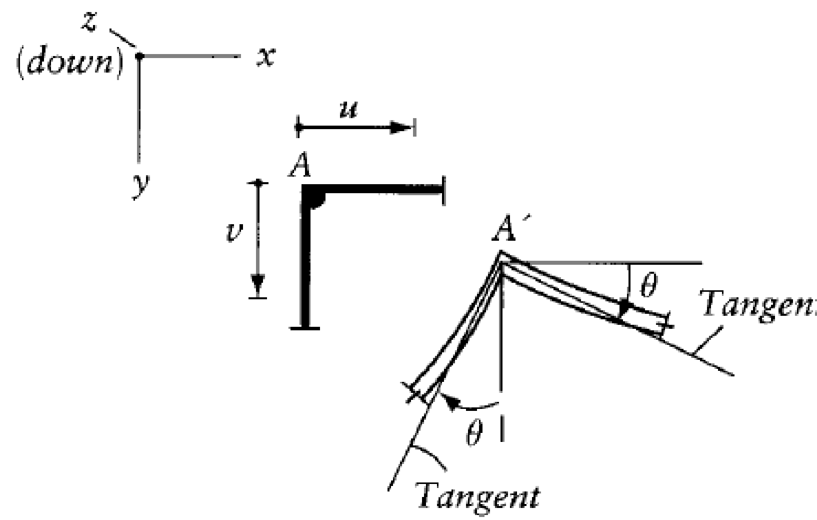


Frames are composed of beams and columns that are either pin or fixed connected. The loading on a frame (composed of slender elements) causes bending of its members

Structural Systems – Frames

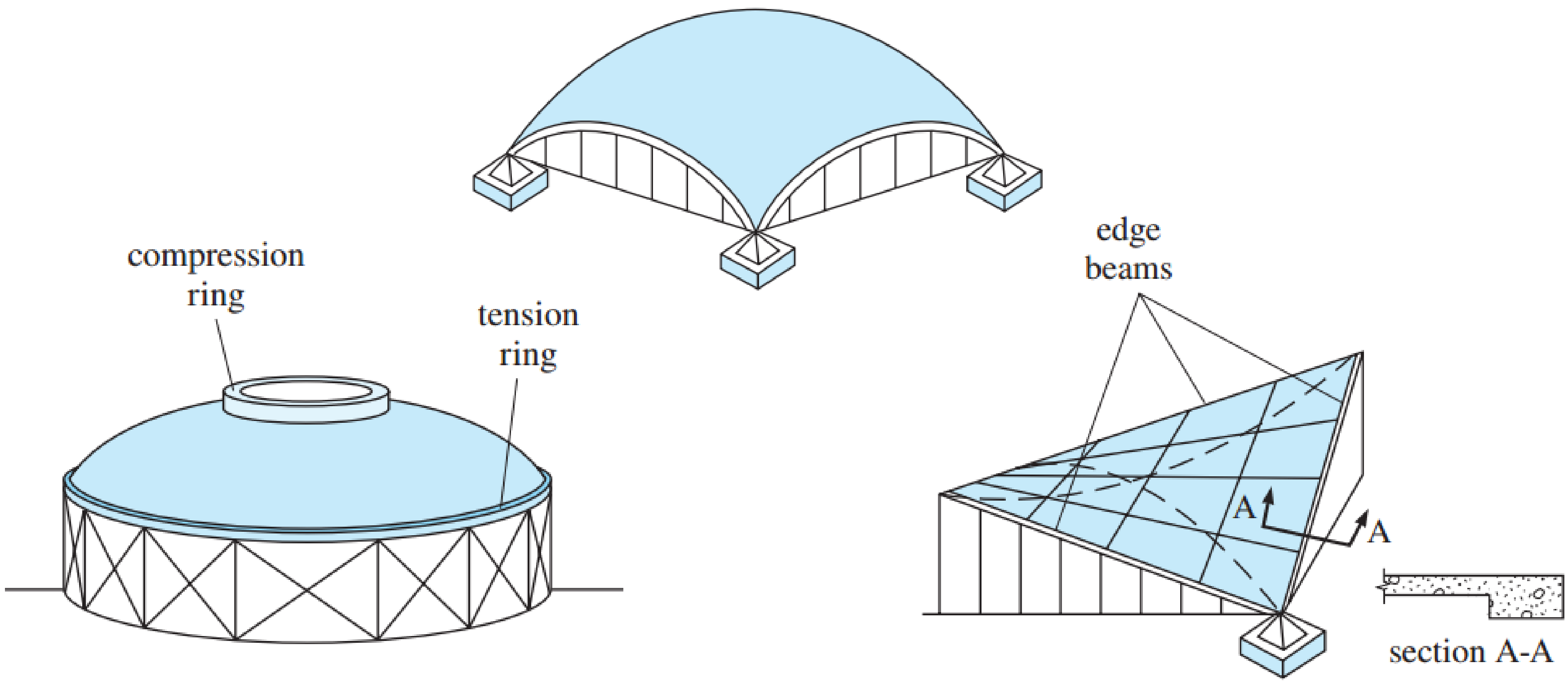
- If it has continuous (rigid) joint connections, then the internal forces (axial, shear, and moment are transferred between frame elements)
- For a joint to be continuous (rigid), the angle between the members must not change; the rotation of the beam's end is the same as the column's end at the connecting joint.



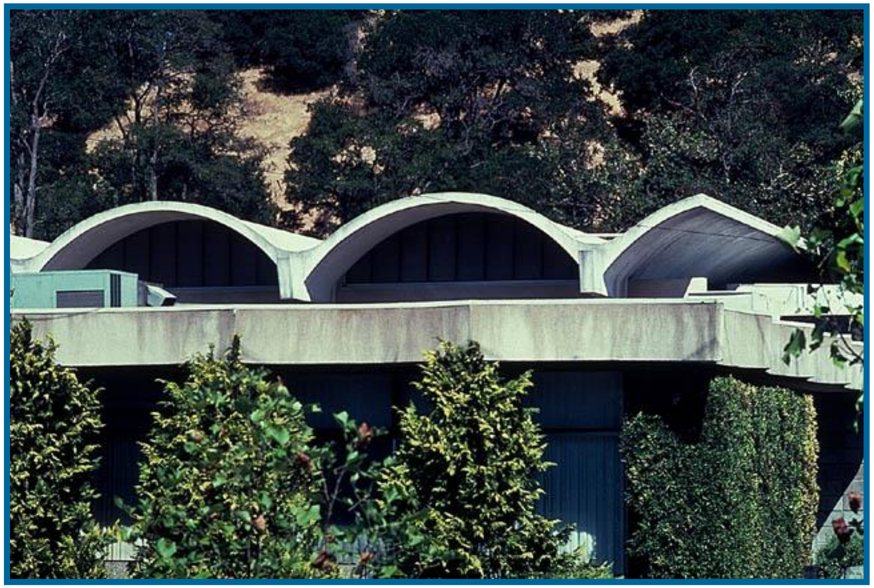


Structural Systems – Surface Structures

- ***A surface structure*** is made from a material having a very small thickness compared to its other dimensions. The are referred to as ***shells.***
- They can span large distances because of the inherent strength and stiffness of the curved shape.



Structural Systems – Surface Structures



The loading is resisted by the three dimentional surface, often through tension or compression with very little bending

Loads

- In order to design a structure, it is therefore necessary to first specify the loads that act on it.
- The design loading for a structure is often specified in codes.
- *The ultimate responsibility for the design lies with the structural engineer.*

TABLE 1.1 Codes

General Building Codes
Minimum Design Loads for Buildings and Other Structures, ASCE/SEI 7-10, American Society of Civil Engineers
International Building Code
Design Codes
Building Code Requirements for Reinforced Concrete, Am. Conc. Inst. (ACI)
Manual of Steel Construction, American Institute of Steel Construction (AISC)
Standard Specifications for Highway Bridges, American Association of State Highway and Transportation Officials (AASHTO)
National Design Specification for Wood Construction, American Forest and Paper Association (AFPA)
Manual for Railway Engineering, American Railway Engineering Association (AREA)

Dead Loads

- ***Dead loads*** consist of the weights of the various structural members and the weights of any objects that are permanently attached to the structure.
- Hence, for a building, the dead loads include the weights of the columns, beams, shear walls, floor slabs, roofing, partitions, floor finishes, windows, plumbing, electrical fixtures, and other miscellaneous attachments.
- structural dead load is calculated using simple formulas based on the densities given by building standards or/and manufactures

TABLE 1.2 Minimum Densities for Design Loads from Materials*

	lb/ft^3	kN/m^3
Aluminum	170	26.7
Concrete, plain cinder	108	17.0
Concrete, plain stone	144	22.6
Concrete, reinforced cinder	111	17.4
Concrete, reinforced stone	150	23.6
Clay, dry	63	9.9
Clay, damp	110	17.3
Sand and gravel, dry, loose	100	15.7
Sand and gravel, wet	120	18.9
Masonry, lightweight solid concrete	105	16.5
Masonry, normal weight	135	21.2
Plywood	36	5.7
Steel, cold-drawn	492	77.3
Wood, Douglas Fir	34	5.3
Wood, Southern Pine	37	5.8
Wood, spruce	29	4.5

*Minimum Densities for Design Loads from Materials, Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10. Copies of this standard may be purchaed from ASCE at www.pubs.asce.org, American Society of Civil Engineers.

Source: ASCE/SEI 7

Live Loads

- ***Live loads*** can vary both in their magnitude and location.
- They may be caused by the weights of occupants using the structure, objects temporarily placed on a structure, moving vehicles, or natural forces.
- The minimum live loads are specified in standards and categorized by the occupancy or usage of the structure.

TABLE 1.4 Minimum Live Loads*

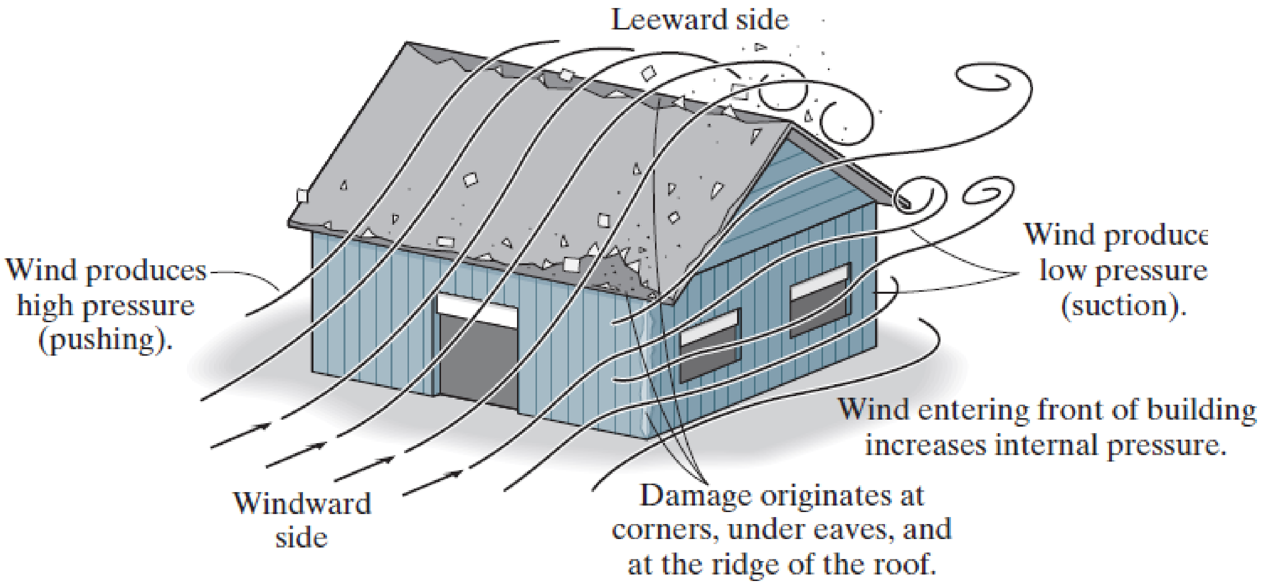
Occupancy or Use	Live Load psf	Live Load kN/m²	Occupancy or Use	Live Load psf	Live Load kN/m²
Assembly areas and theaters			Residential		
Fixed seats	60	2.87	Dwellings (one- and two-family)	40	1.92
Movable seats	100	4.79	Hotels and multifamily houses		
Garages (passenger cars only)	40	1.92	Private rooms and corridors	40	1.92
Office buildings			Public rooms and corridors	100	4.79
Lobbies	100	4.79	Schools		
Offices	50	2.40	Classrooms	40	1.92
Storage warehouse			First-floor corridors	100	4.79
Light	125	6.00	Corridors above first floor	80	3.83
Heavy	250	11.97			

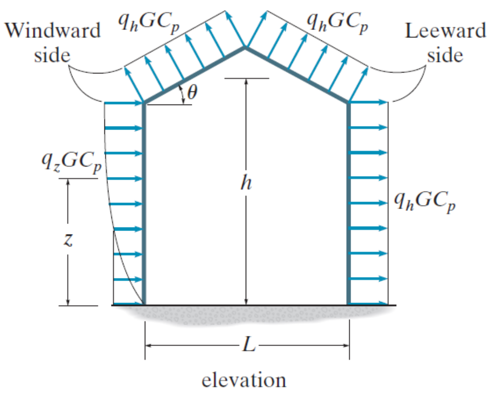
*Minimum Live Loads, Reproduced with permission from American Society of Civil Engineers *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10, American Society of Civil Engineers.

Source: ASCE/SEI 7

Wind Loads

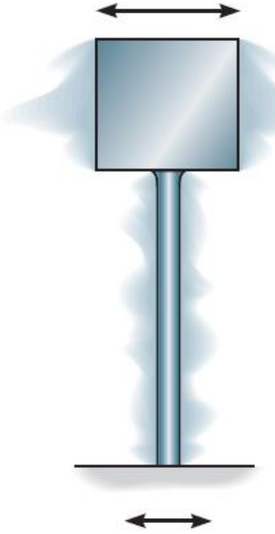
- The pressure created by the wind is proportional to the *square* of the wind speed



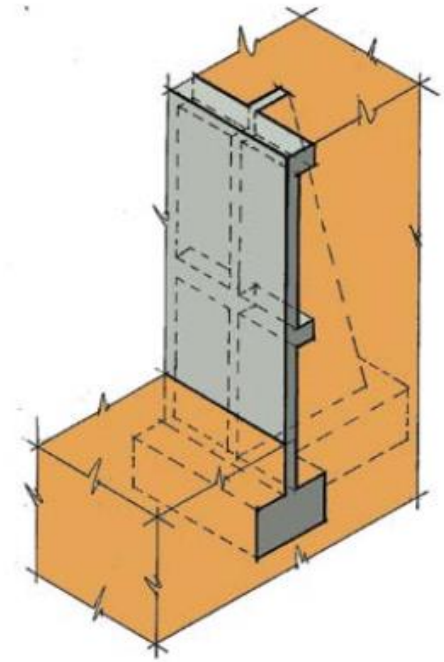


Earthquake Loads

- Earthquakes causes inertia forces at the floor levels (mass points) of the building
- The magnitude of an earthquake loads (seismic loads) depends on the severity and probability of occurrence of earthquakes in the region, and the mass, stiffness and importance of the structure



Hydrostatic and Soil Pressure



- When structures are used to retain water, soil, or granular materials, the pressure developed by these loadings becomes an important criterion for their design.

- ***Other Natural Loads:*** Several other types of live loads may also have to be considered in the design of a structure, depending on its location or use. These include the effect of **blast, temperature changes, and differential settlement of the foundation**.

Structural Design

- Load combinations are needed to define the load cases for analysis
- Building standards give the load combinations used in analysis

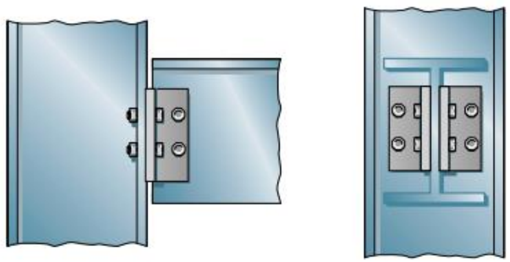
Load combinations:

- 1.4 (dead load)
- 1.2 (dead load) + 1.6 (live load) + 0.5 (snow load)
- 0.9 (dead load) + 1.0 (wind load)
- 0.9 (dead load) + 1.0 (earthquake load)

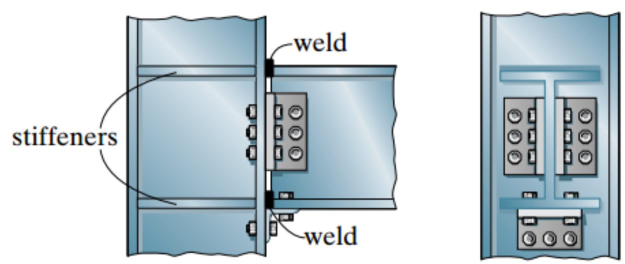
Load factors are used in the load combinations to consider material and load uncertainties.

2.1 Idealized Structures

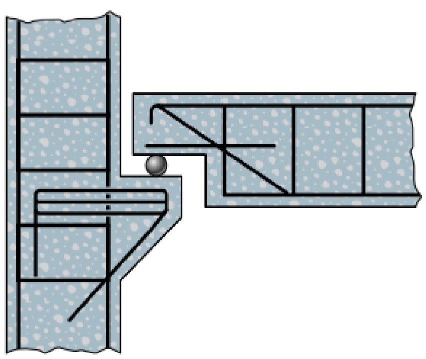
- An exact model of a structure can never be constructed!
- Connections to be modeled based on expected behavior:



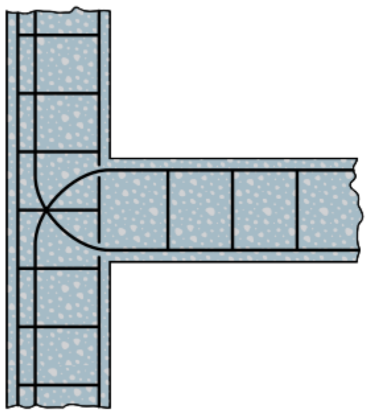
typical "pin-supported" connection (metal)



typical "fixed-supported" connection (metal)



typical "roller-supported" connection (concrete)



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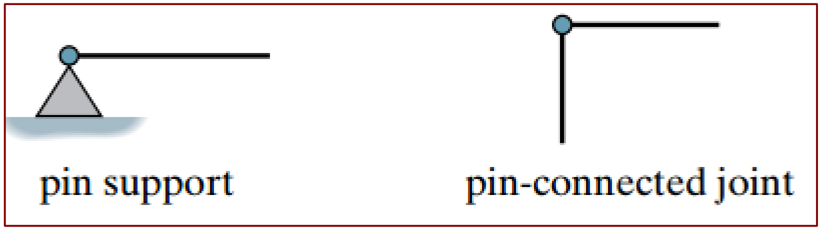
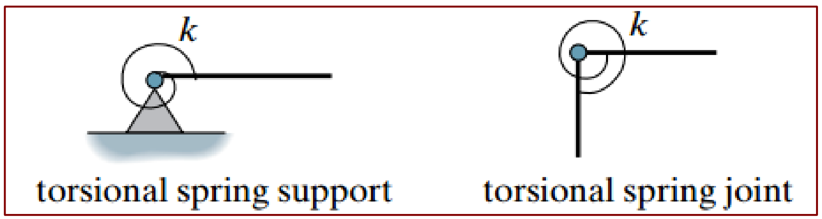
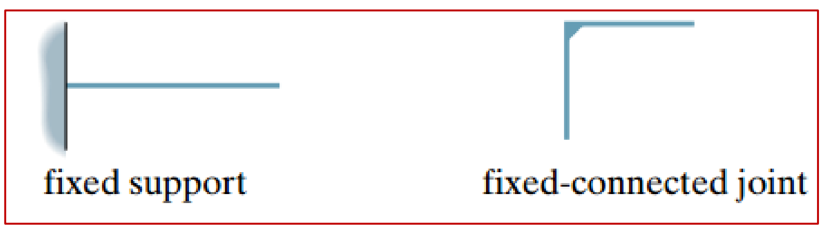
2.1 Idealized Structures







Pin and roller connections allow for **rotation** of members relative to each other.

However, **fixed** connections prevent translation and rotation.

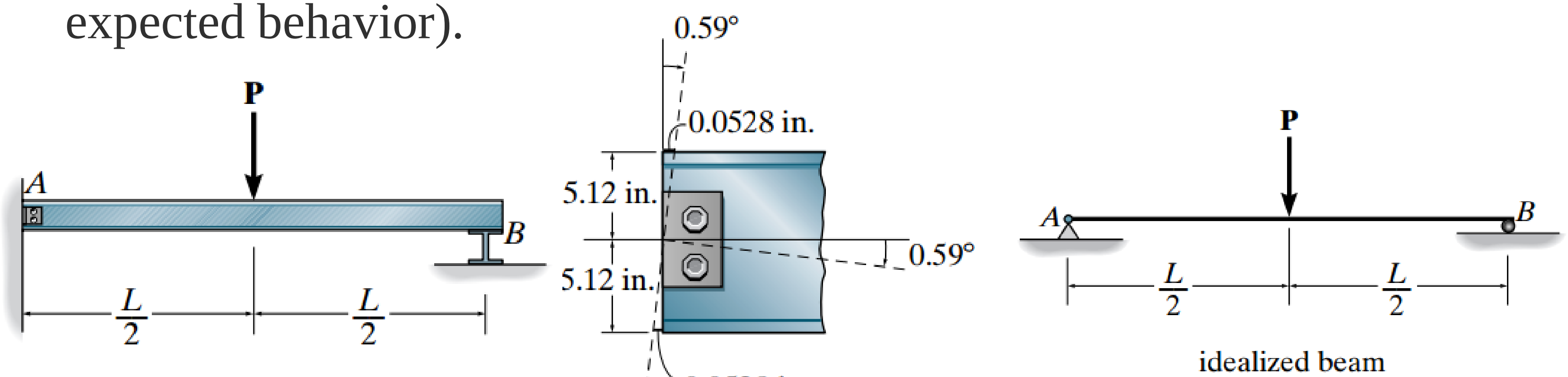
In reality, all connections exhibit some stiffness toward joint rotations, owing to friction and material behavior.

As such, a more **appropriate model would be a rotational spring.**

If $k=0$, then the joint is pinned. If $k=\infty$, then the joint is fixed.

2.1 Idealized Structures

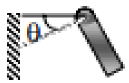
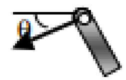
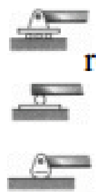
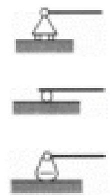
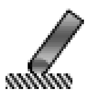
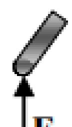
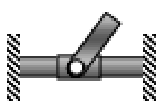
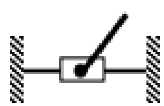
- The beam's two dimensions (width and depth) are small in comparison to the beam's length (span), so it can be modeled as a line element.
- The support at A allows rotation as shown below, therefore, it's our assumption that the support at A is a pin
- The support at *B* provides an approximate point of smooth contact and so it can be idealized as a roller (again our assumption based on expected behavior).



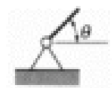
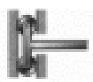
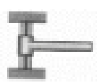
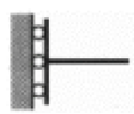
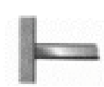
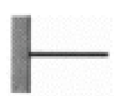
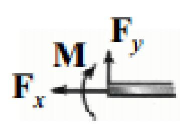
actual beam | Rotation at support A | idealized beam

Support Reactions

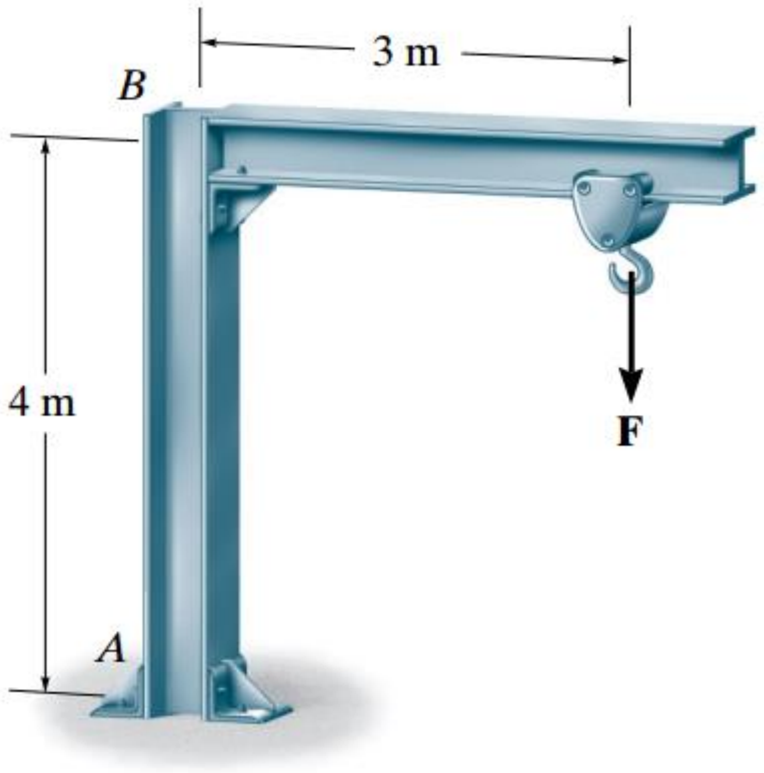
Table 2-1 Supports for Coplanar Structures

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1)  Light cable			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2)  rollers rockers			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3) 			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4) 			One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

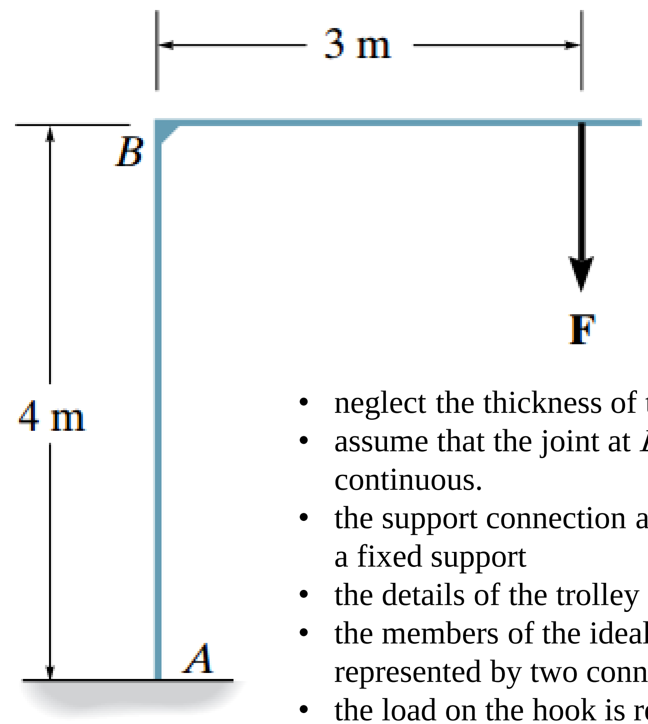
Support Reactions

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(5)  Smooth pin or hinge			Two unknowns. The reactions are two force components.
(6)  slider  fixed-connected collar			Two unknowns. The reactions are a force and moment.
(7)  fixed support			Three unknowns. The reactions are the moment and the two force components.

Idealized Structure



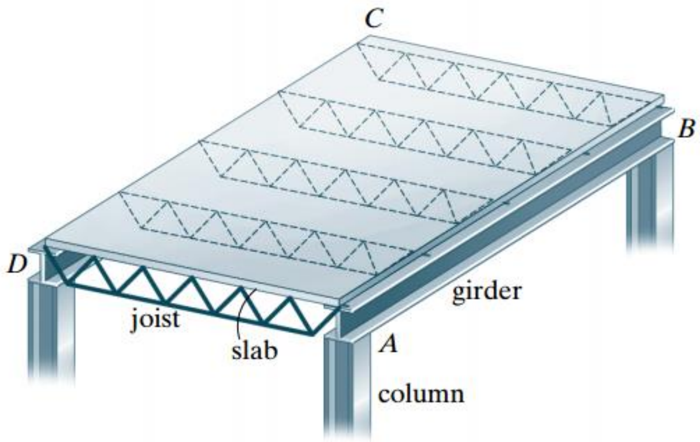
actual structure

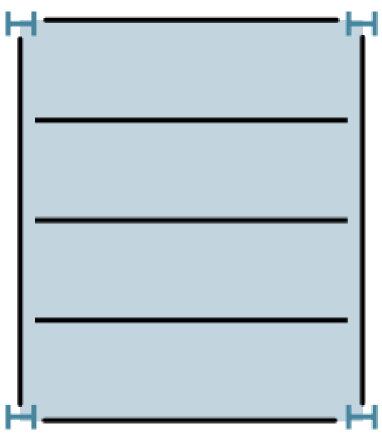


- neglect the thickness of the two main members
- assume that the joint at *B* is fabricated to be continuous.
- the support connection at *A* can be modeled as a fixed support
- the details of the trolley excluded.
- the members of the idealized structure are represented by two connected lines.
- the load on the hook is represented by a single concentrated force **F**

idealized structure

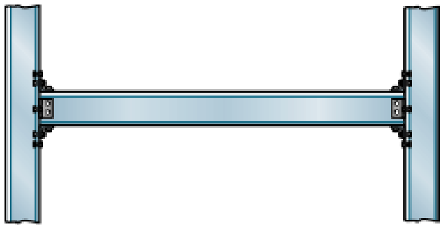
Idealized Structure – Beams and girders




idealized framing plan

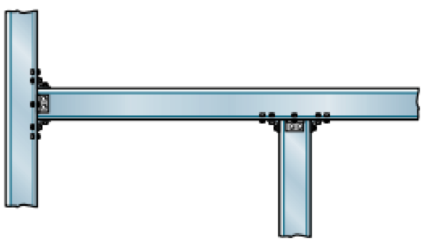
- a ***girder (main beam)*** is the main load-carrying element of the floor.
- the smaller elements having a shorter span and connected to the girders are called secondary ***beams.***
- Here the slab is supported by ***floor joists (small truss system)*** located at even intervals.
- and these in turn are supported by the two side girders AB and CD.
- assume that the joints are pin and/or roller connected to the girders and that the girders are pin and/or roller connected to the columns.

Idealized Structure – Beams and girders



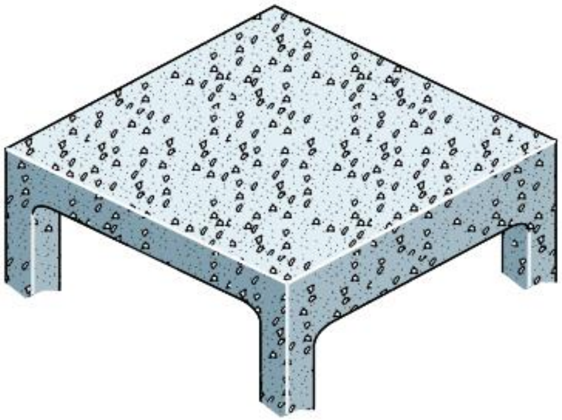
fixed-connected beam

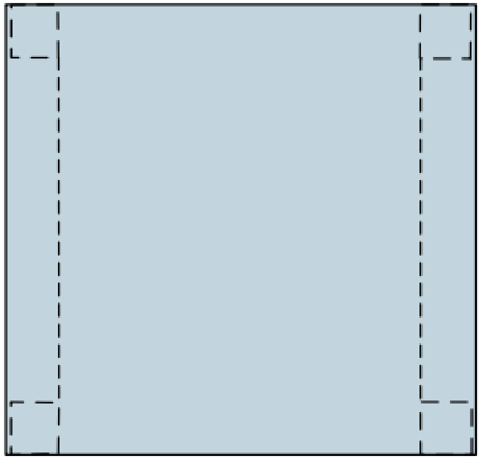
idealized beam



fixed-connected overhanging beam

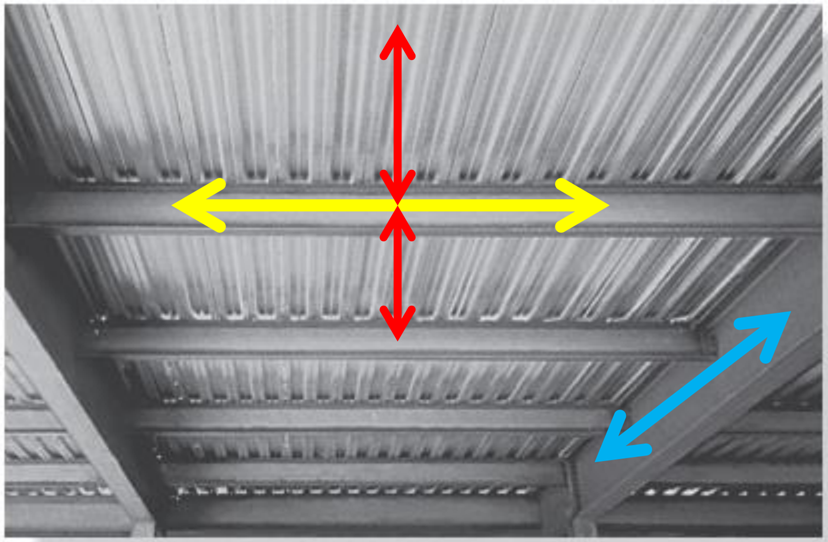
idealized beam





idealized framing plan

Tributary Loadings



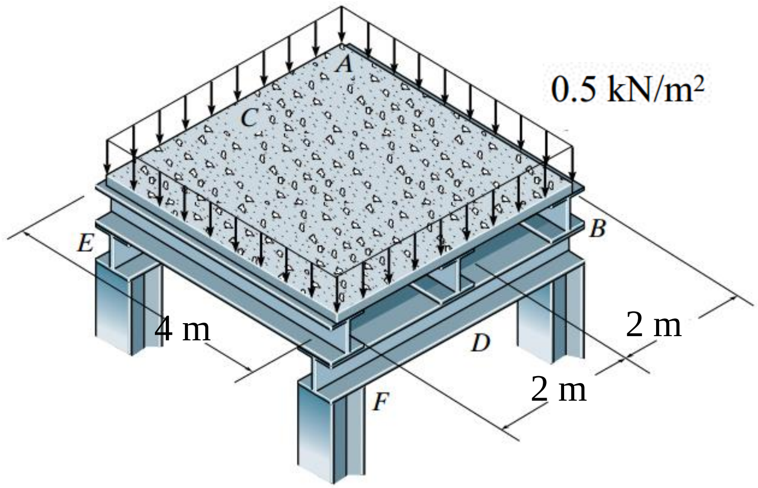
An example of one-way slab construction of a steel frame building having a poured concrete floor on a corrugated metal deck. The load on the floor is considered to be transmitted to the beams, not the girders.

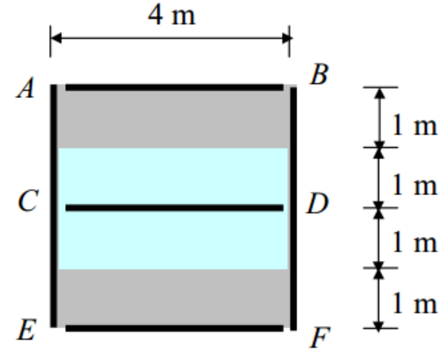
- Many floor systems consist of a reinforced concrete slab supported on a rectangular grid of beams.
- The supporting beams reduce the span of the slab and permit the designer to reduce the depth and weight of the floor system.
- The distribution of dead loads to a floor beam depends on the geometric configuration of the beams forming the grid.
- determine how the load on these surfaces is transmitted to the various structural elements used for their support !!!
- There are generally two ways in which this can be done :

❖ One Way system.

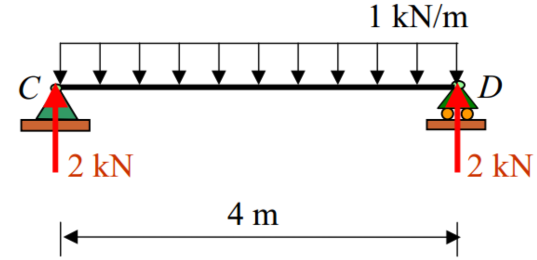
❖ Two Way system.

One Way System

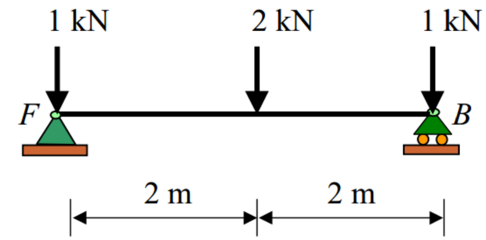




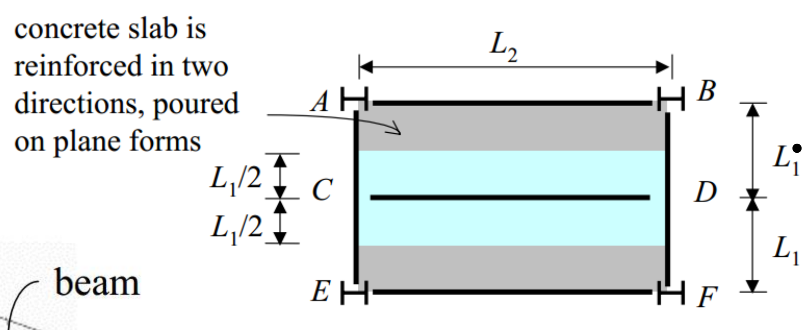
idealized framing plan



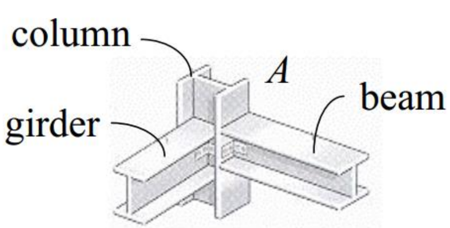
idealized beam



idealized girder

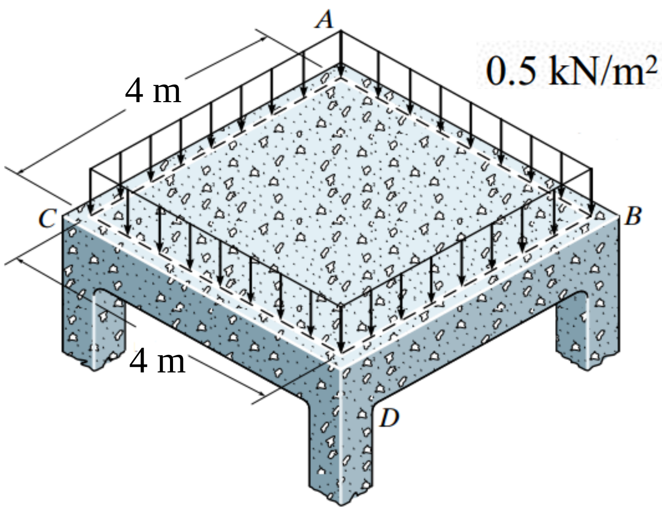


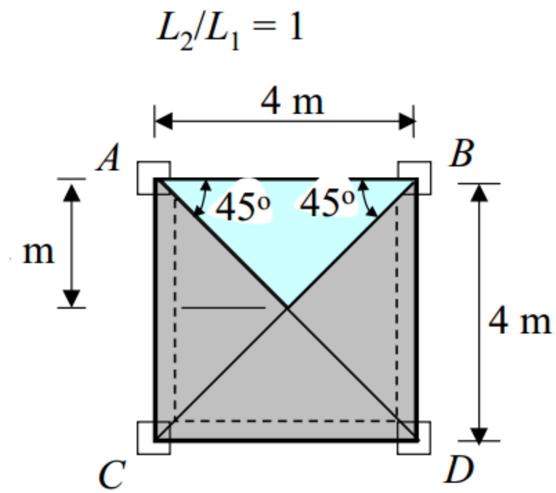
Idealized framing plan for one-way slab action requires $L_2 / L_1 \geq 2$



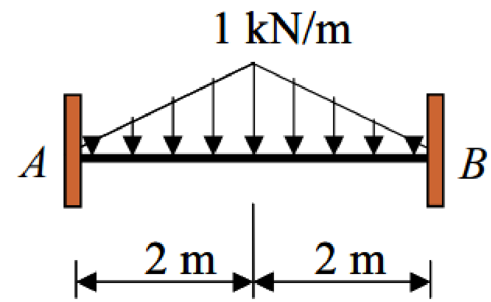
According to the ASCE-7, if $L2>L1$ and the support ratio $(L2/L1) > 2$, then the load is assumed to be transferred to the supporting beams and girders in *one direction.*

Two Way System

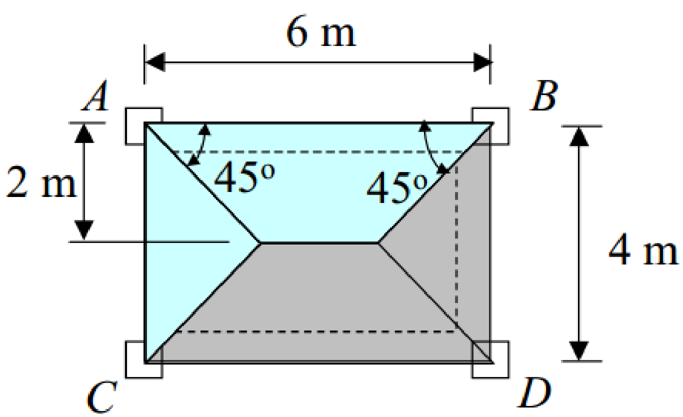




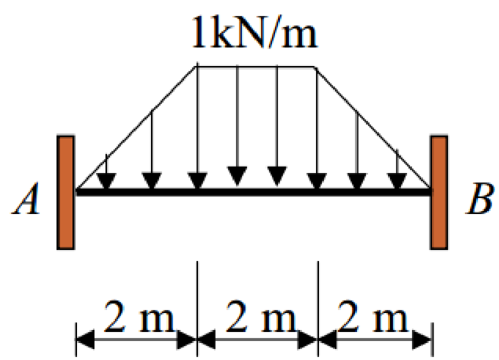
idealized framing plan

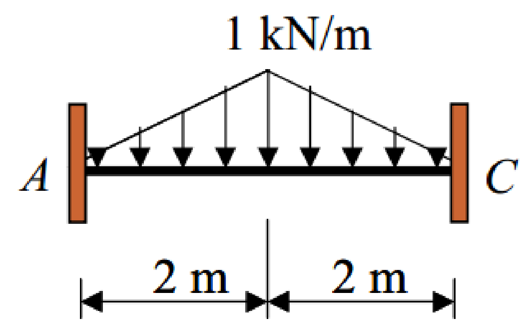


idealized beam, all



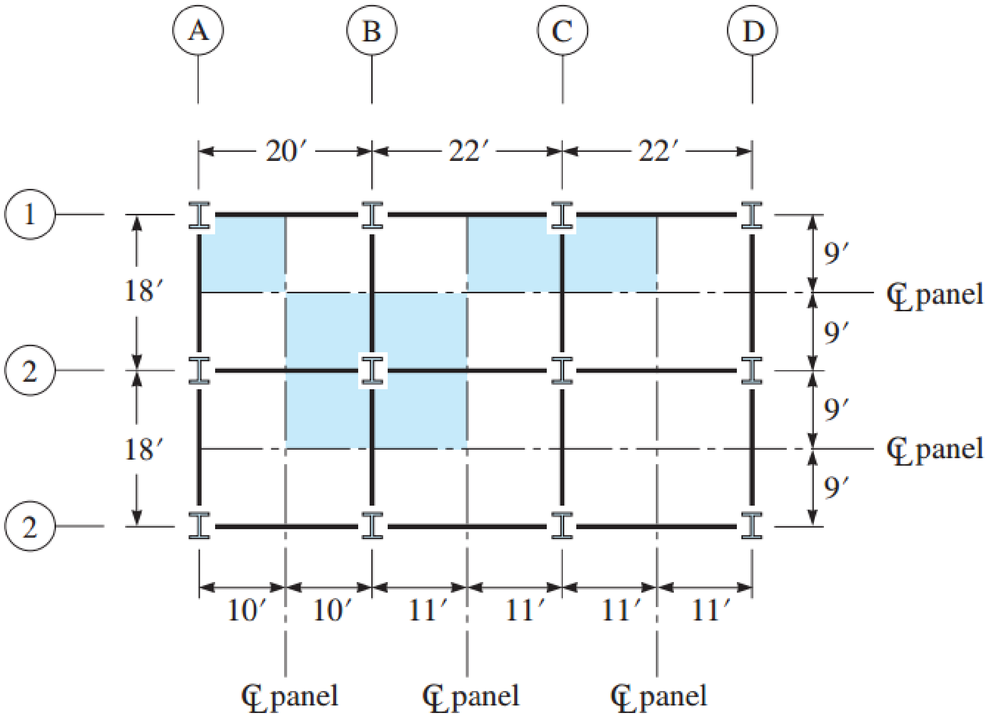
idealized framing plan





idealized beam

Tributary Areas of Columns



To determine the gravity loads transmitted into a column from a floor slab, the designer can either

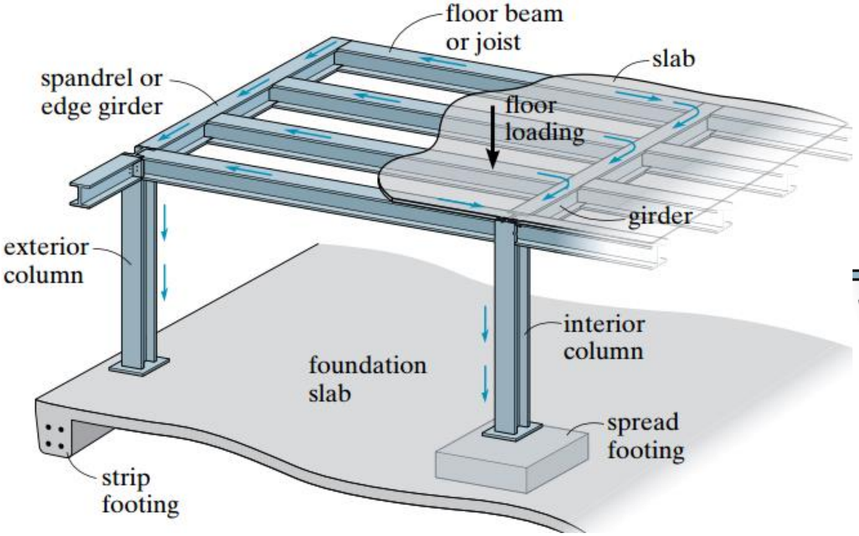
(1) determine the reactions of the beams framing into the column or

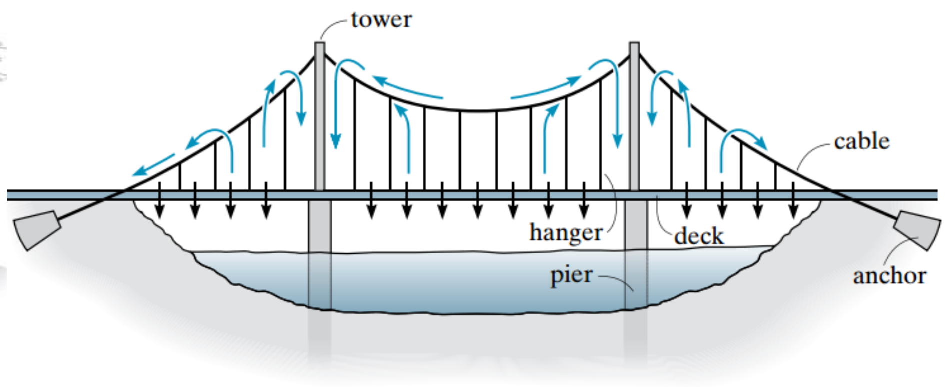
(2) multiply the tributary area of the floor surrounding the column by the magnitude of the load per unit area acting on the floor.

The *tributary area* of a column is defined as *the area surrounding the column that is bounded by the panel centerlines*.

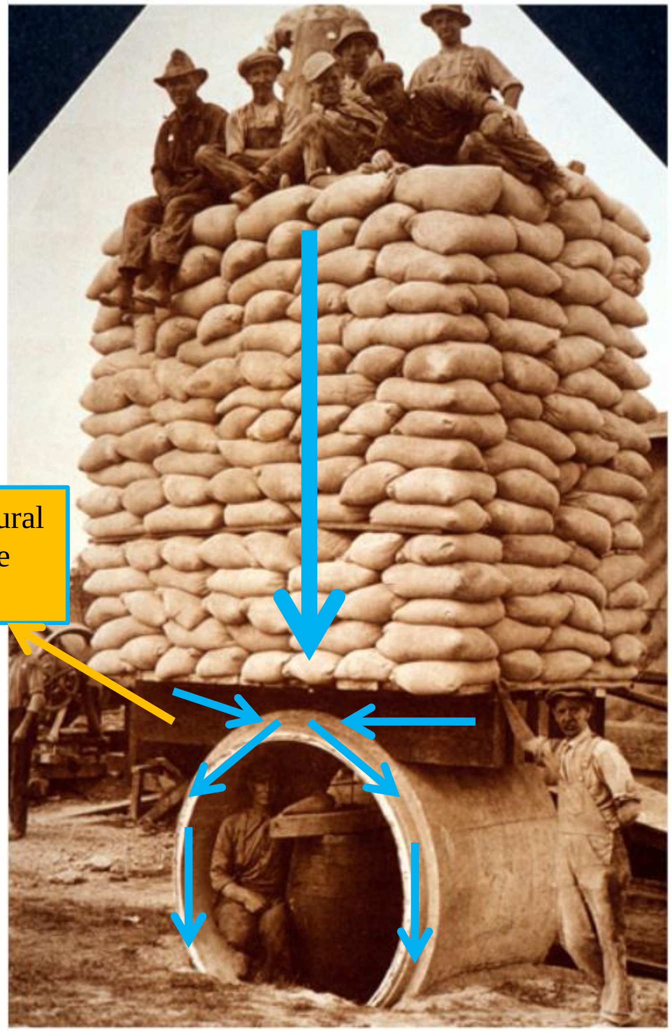
2.2 Load Path

- how the loads are transmitted through various structural members from point of application to the foundation!
- Like a chain, which is "as strong as its weakest link", so a structure is only as strong as the weakest part along its load path

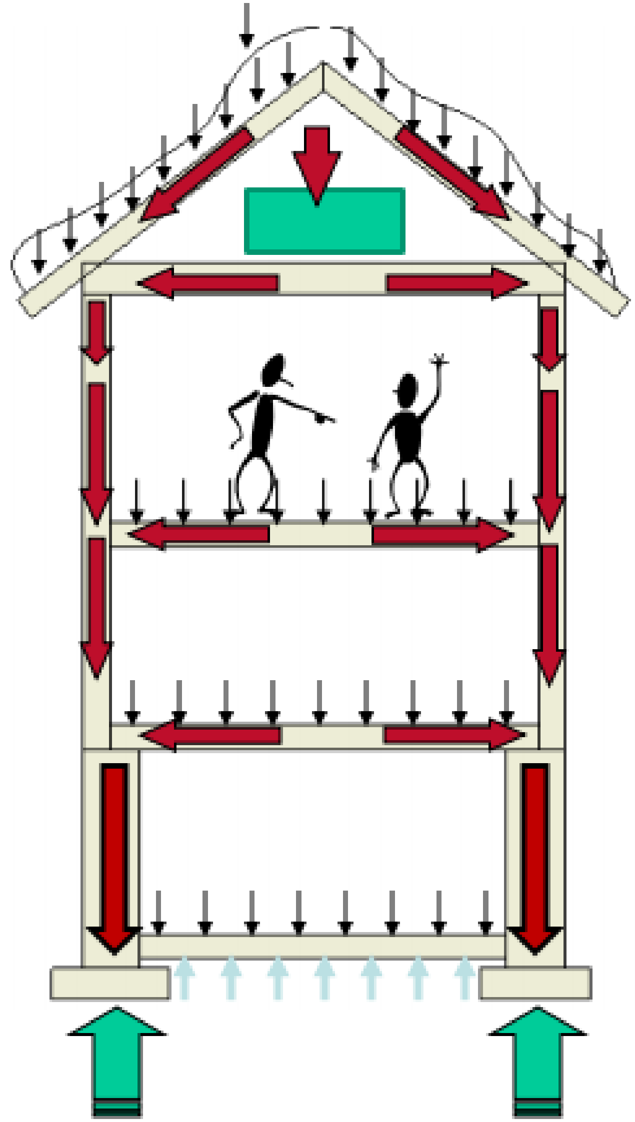




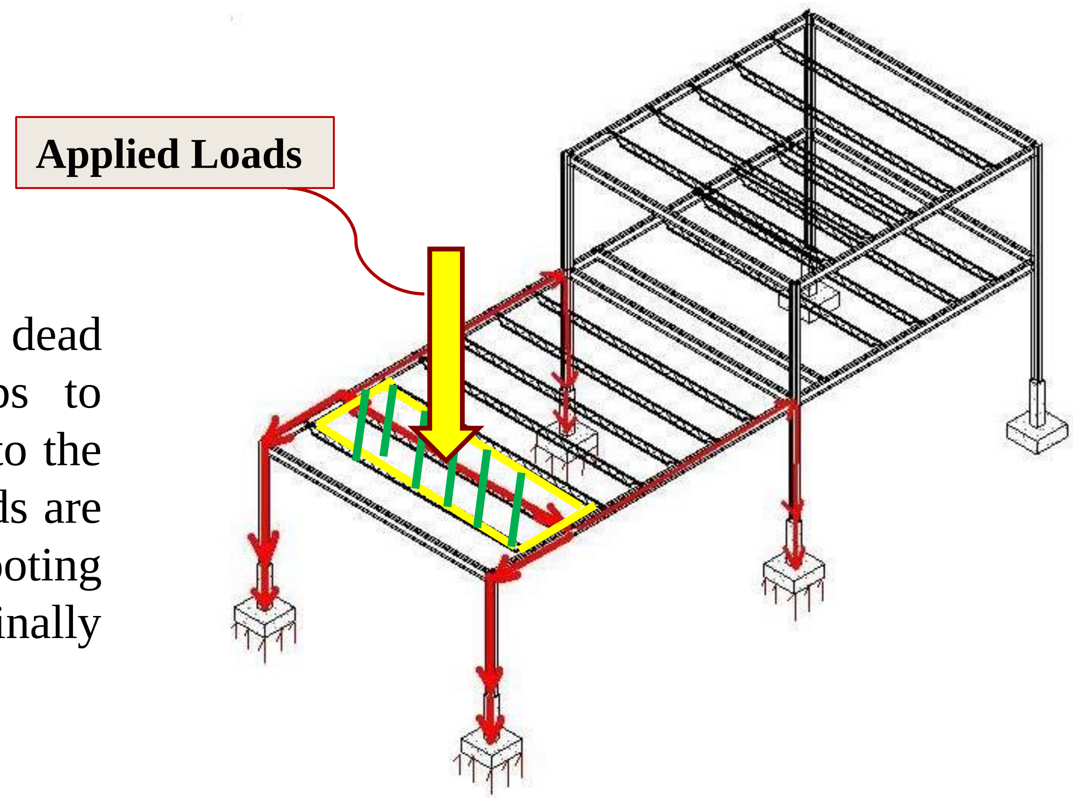
2.2 Load Path



Draw structural Model of the element?!!

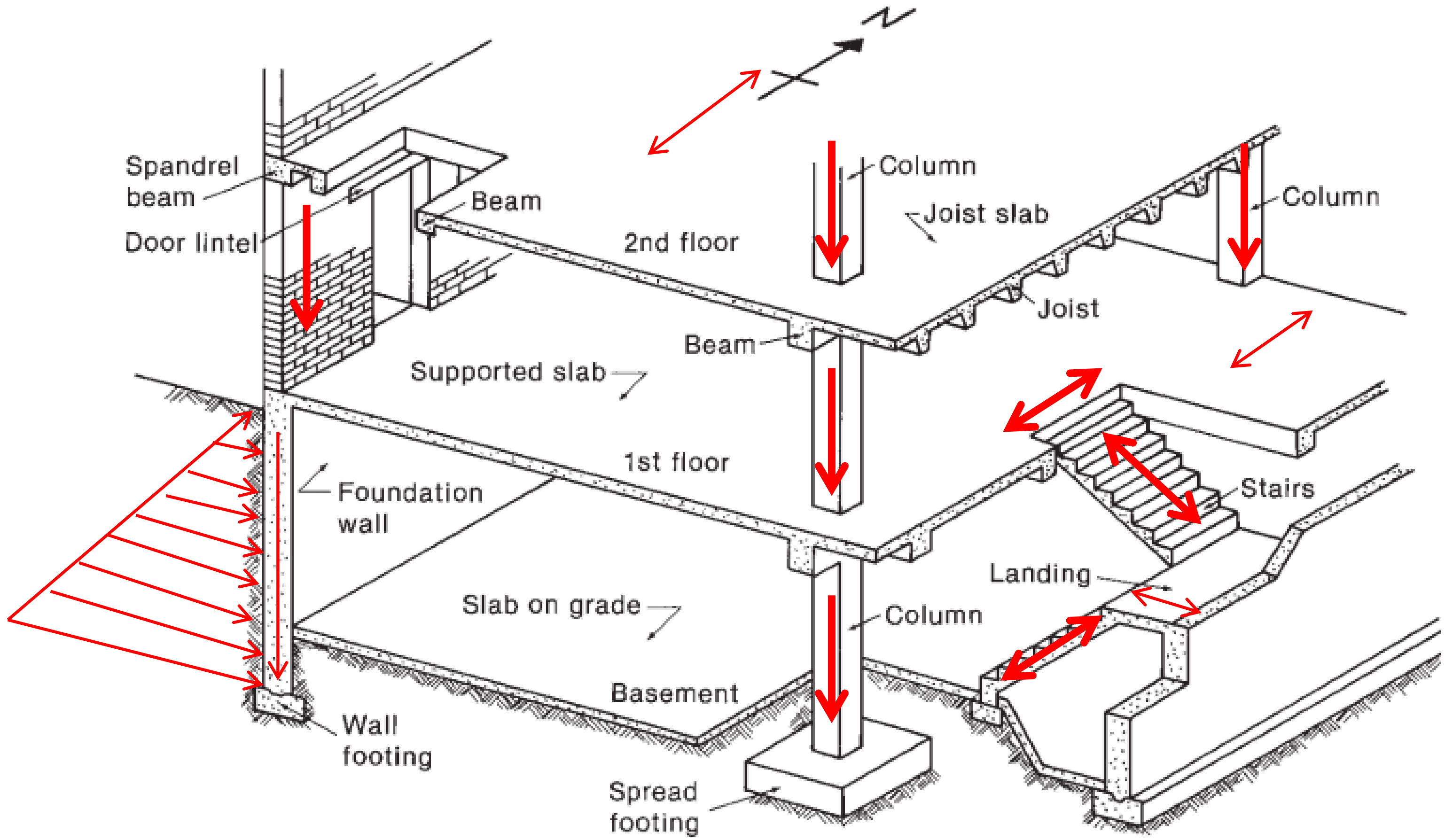


2.2 Load Path

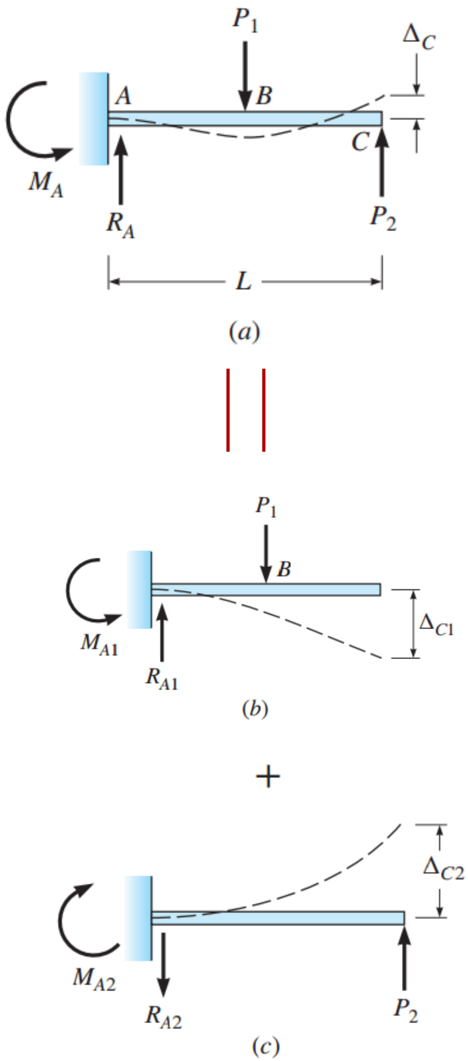


"Chasing the loads" of the dead and live load from slabs to beams to girders, then on to the columns or walls. The loads are then carried down to the footing or foundation walls and finally to the earth below.

2.2 Load Path: chase the load in a building



2.3 Principle of Superposition



- *The total displacement or internal loadings (stress) at a point in a structure subjected to several external loadings can be determined by adding together the displacements or internal loadings (stress) caused by each of the external loads acting separately*

- Two requirements must be imposed for the principle of superposition to apply:

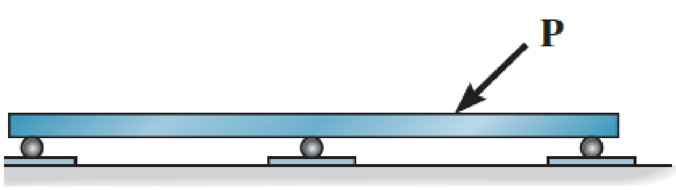
1. The material must behave in a linear-elastic manner, so that Hooke's law is valid, and therefore the load will be proportional to displacement.

2. The geometry of the structure must not undergo significant change when the loads are applied, i.e., small displacement theory applies. Large displacements will significantly change the position and orientation of the loads. An example would be a cantilevered thin rod subjected to a force at its end, causing it to bend.

Determinacy and Stability of Structural systems

Determinacy & Stability

- Criteria for the static classification of structures can be stated as
  - If there are more equations than there are unknowns, the structure is statically unstable
  - If there is the same number of equations as unknowns, the structure is statically determinate
  - If there are fewer equations than unknowns, the structure is statically indeterminate
- The first criterion is absolute, the second and third are conditions



Determinacy & Stability

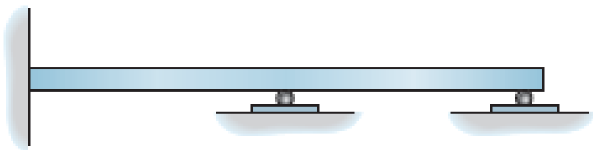
Structure is **statically indeterminate** when there are more reaction components available and/or member forces present than are necessary for the stability of the structure. The indeterminacy can be **externally or internally**.

Redundant reaction/force: reaction or force that is an excess one, in other words unnecessary for the stability of the structure

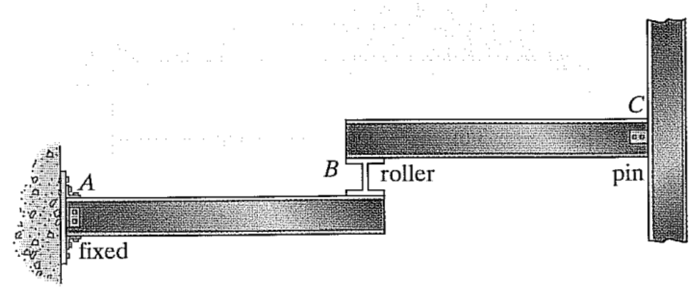
Static indeterminacy refers the number of force quantities that must be determined in order to render the equilibrium solution complete

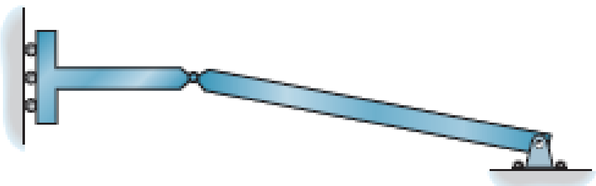
kinematic indeterminacy refers to the number of displacement quantities (kinematic degrees of freedom) that are necessary to define the deformational response of the structure

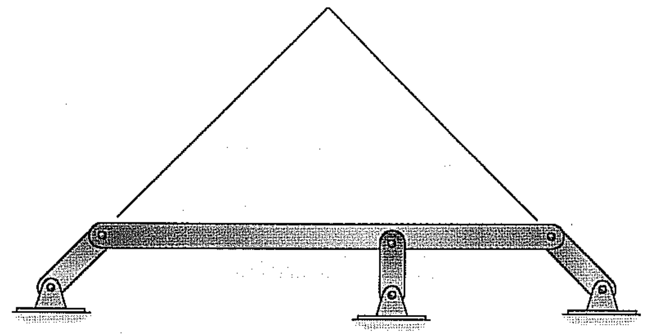
Examples: state whether the systems shown below are stable or unstable, and state whether the stable systems are statically determinate or indeterminate

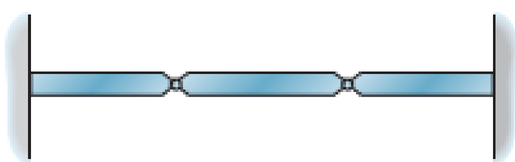


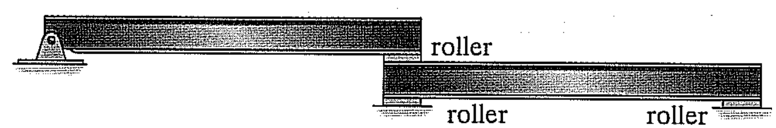
(b)











$r = 3n$, statically determinate

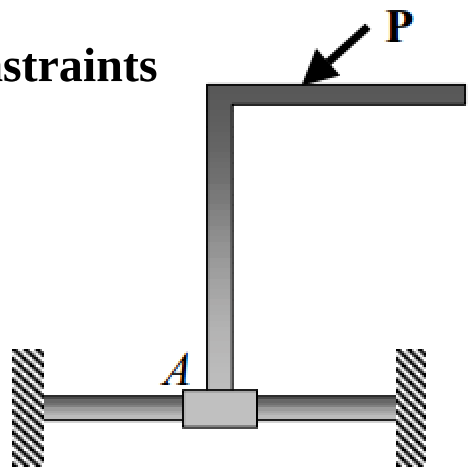
$r > 3n$, statically indeterminate

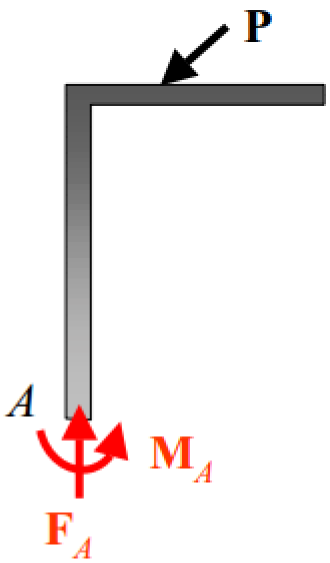
Stability:

To ensure the equilibrium of a structure or its members, it is not only necessary to satisfy the equations of equilibrium, but the members must also be properly held or constrained by their supports.

The following situations may occur and they are examples of instability of structural systems

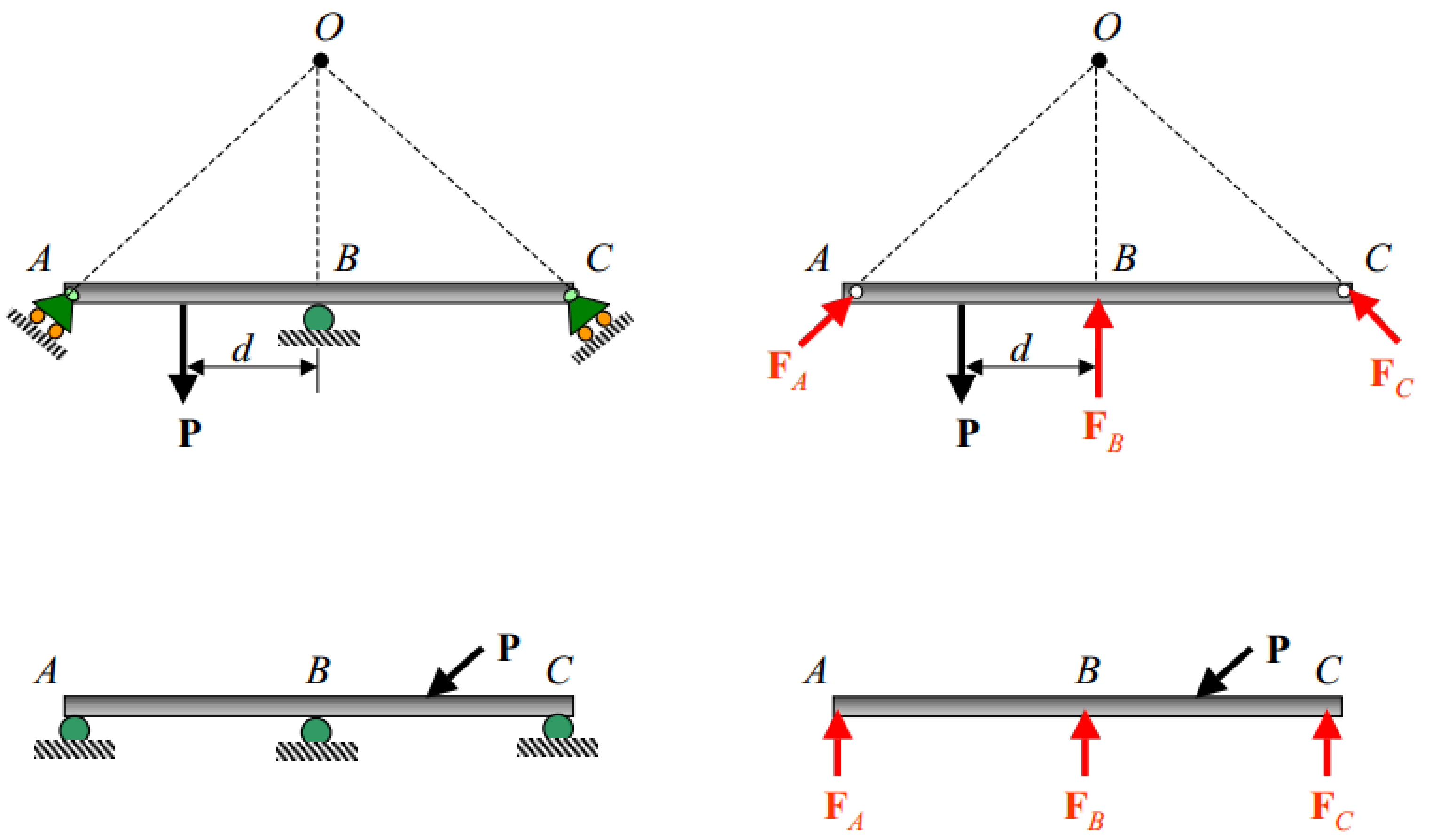
Partial Constraints



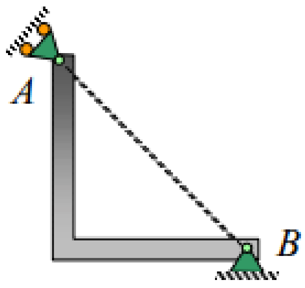


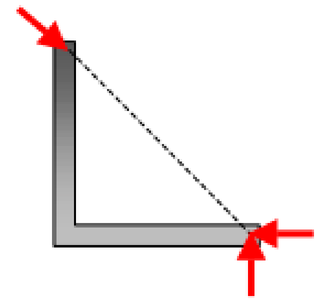
Determinacy & Stability

Improper Constraint

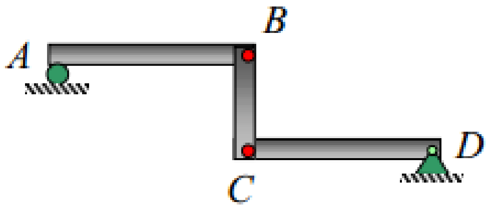


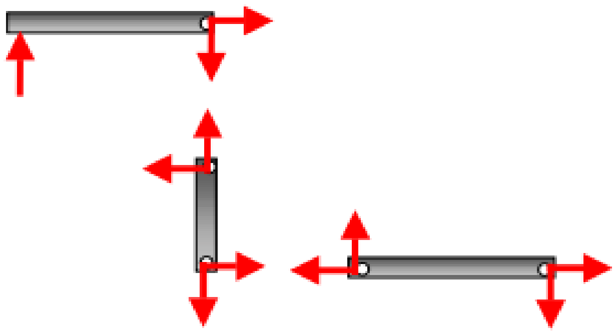
Determinacy & Stability





The member is ***unstable*** since the three reactions are concurrent at *B*.





The structure is ***unstable*** since $r = 7$, $n = 3$, so that, $r < 3n$, $7 < 9$. Also, this can be seen by inspection, since *AB* can move horizontally without restraint.

Pros and Cons of Indetermined structures

Pros

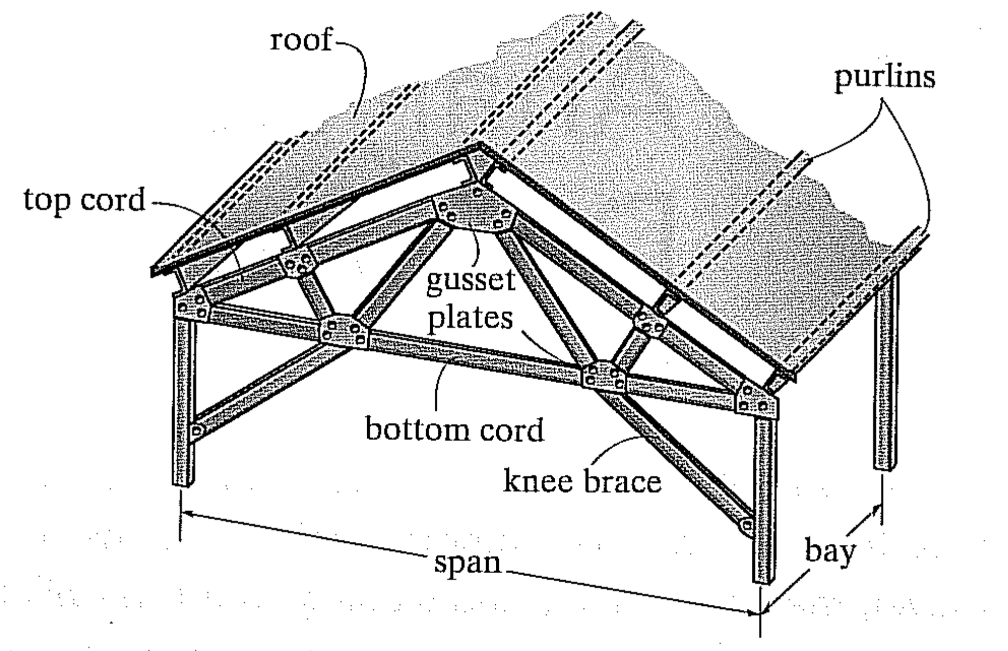
- Saving in material in the design stage, as maximum stresses and deflections of indeterminate structures are generally smaller than those of statically determinate structures
- Statically indeterminate structure has a tendency to redistribute its load to its redundant supports or members in cases of overloading or failure.

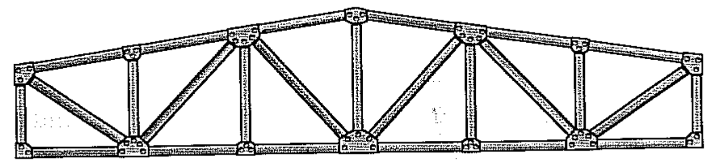
Cons

- Difficulty in analysis; it needs equilibrium and compatibilty equations and force-displacement relations to solve an indeterminate system
- Differential settlements and thermal action cause addititonal internal stresses in the structural members

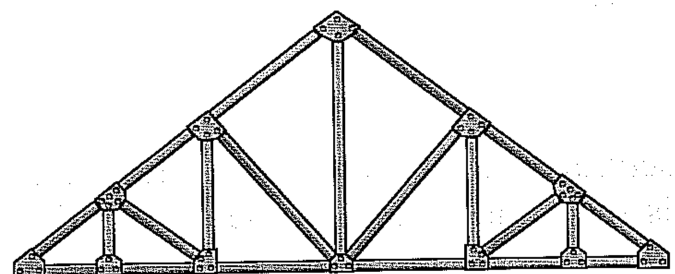
Trusses

Types of trusses





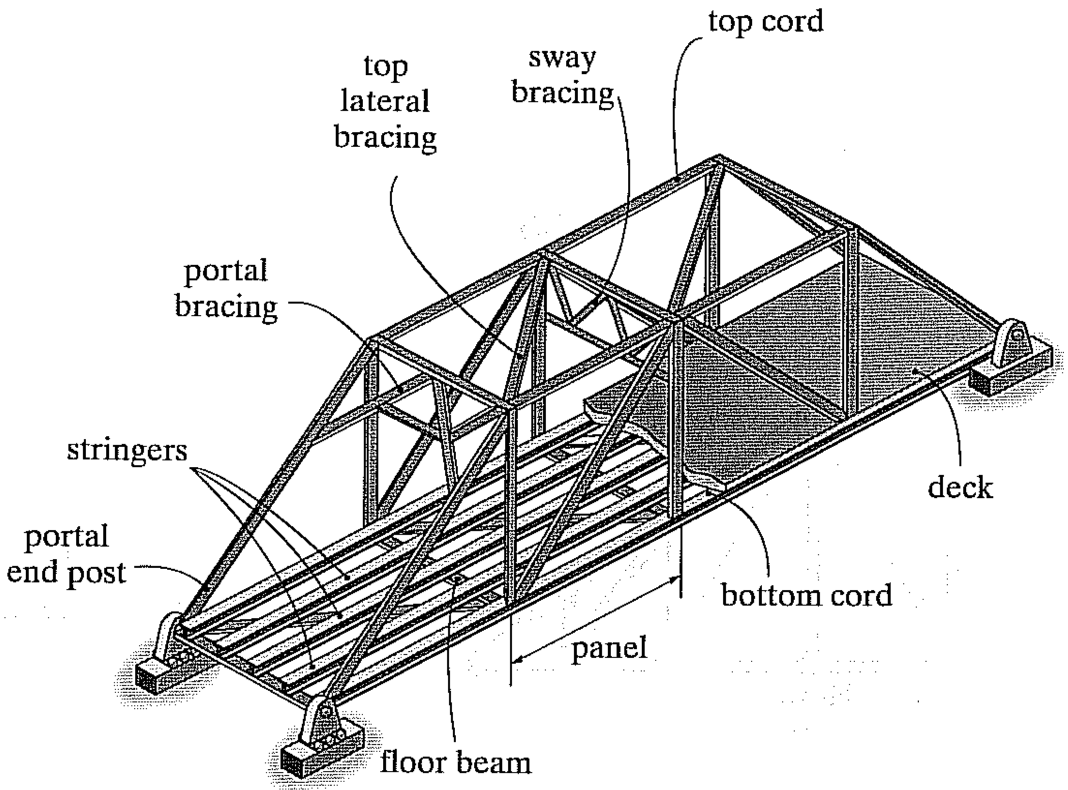
Warren

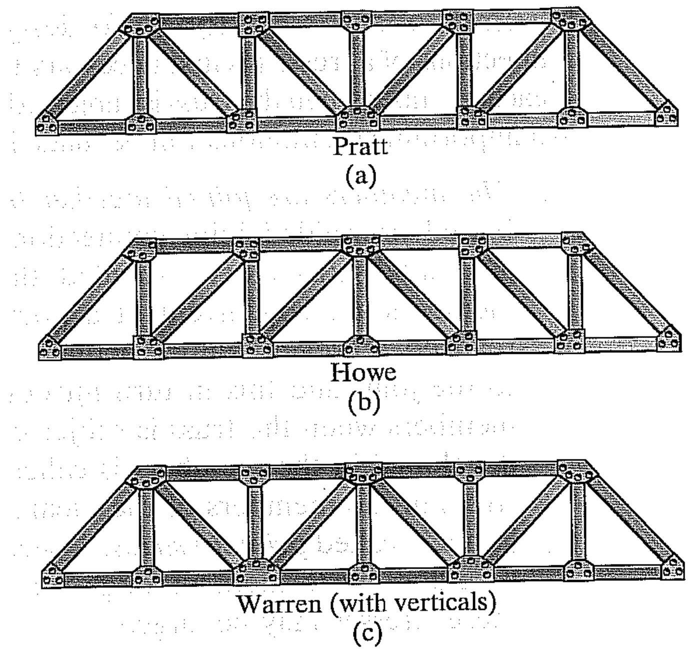


Pratt

- Roof trusses

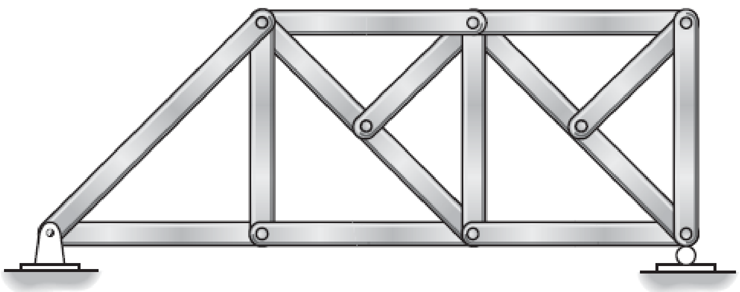
Types of trusses



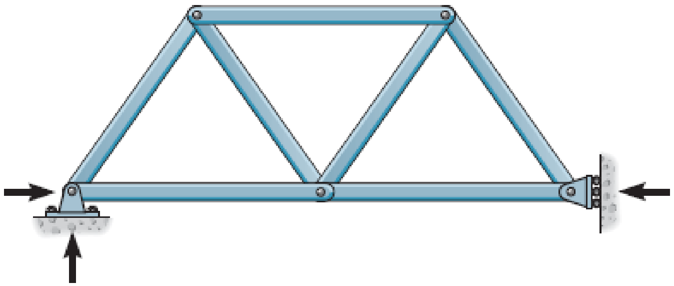


- Bridge trusses

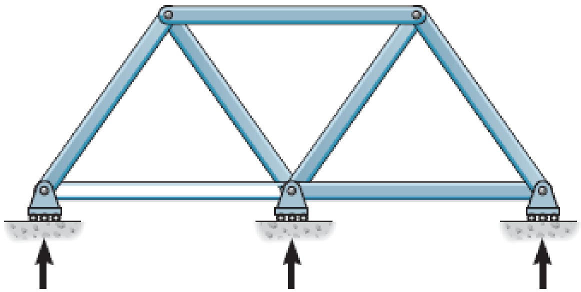
Determinacy and stability of trusses



$b + r = 2j$	statically determinate
$b + r > 2j$	statically indeterminate



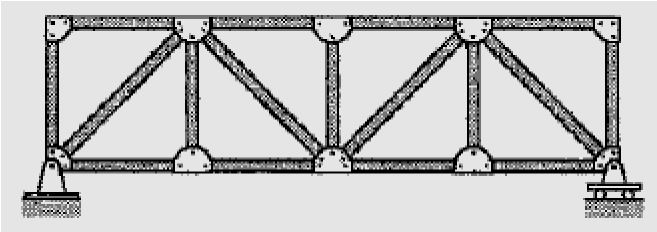
unstable concurrent reactions

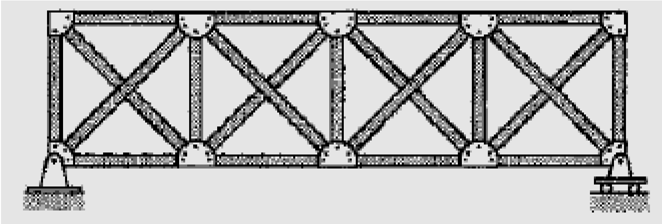


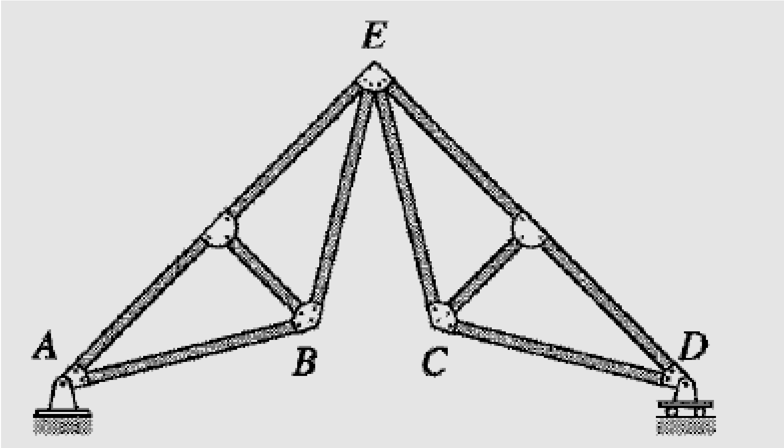
unstable parallel reactions

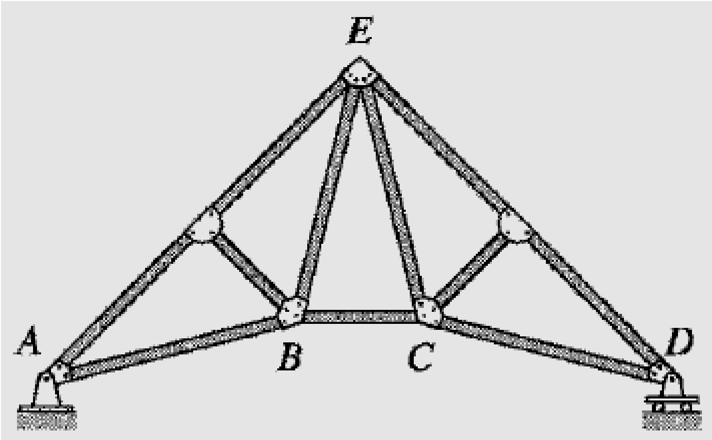
$b + r < 2j$	unstable
$b + r \geq 2j$	unstable if truss support reactions are concurrent or parallel or if some of the components of the truss form a collapsible mechanism

Examples:









- State if truss is unstable, statically determinate or statically indeterminate

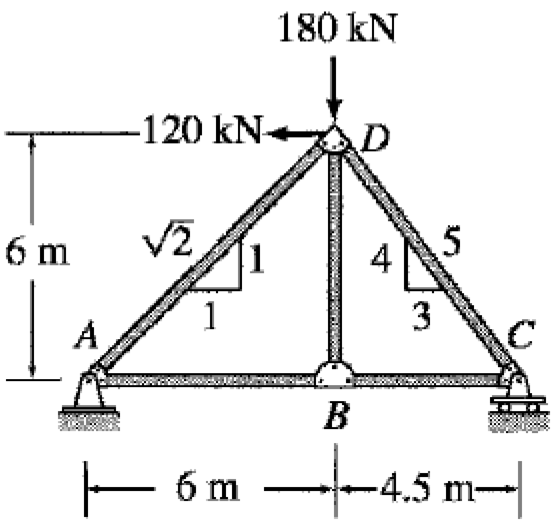
Analysis of trusses

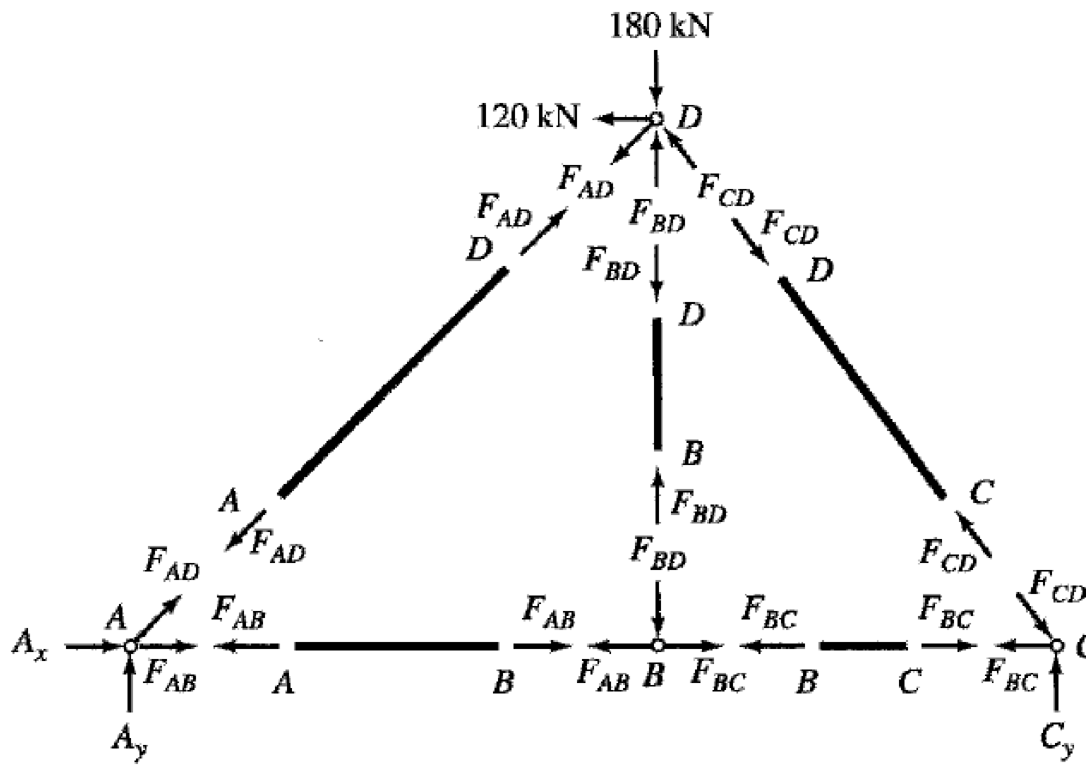
- There are two main methods to analyze stattically determinate trusses

- Method of joints, helpful to identify zero force members

- Methods of sections

Analysis of plan trusses: method of joints

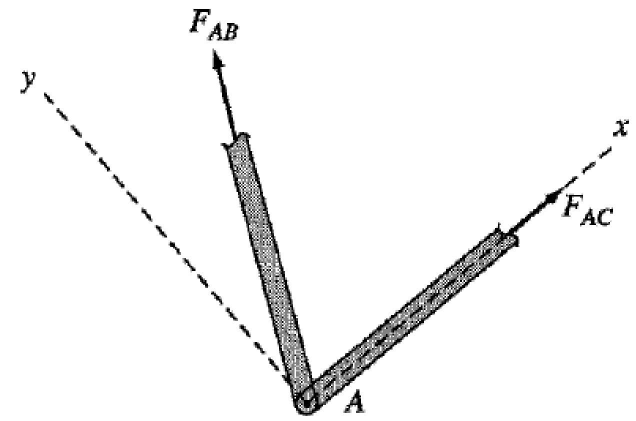




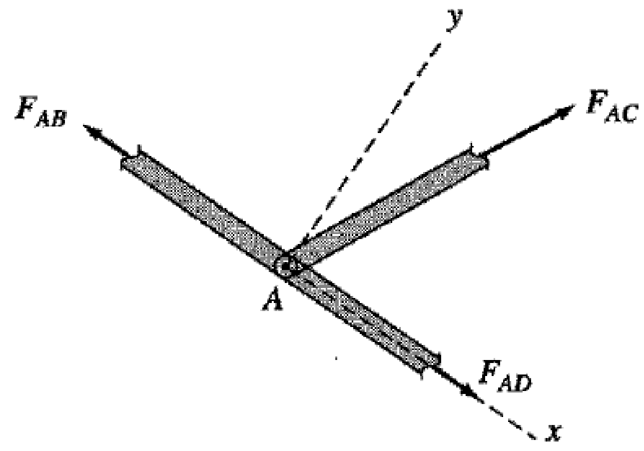
- A joint of two or fewer unknowns is chosen, equilibrium equations for the forces are applied

Zero force members

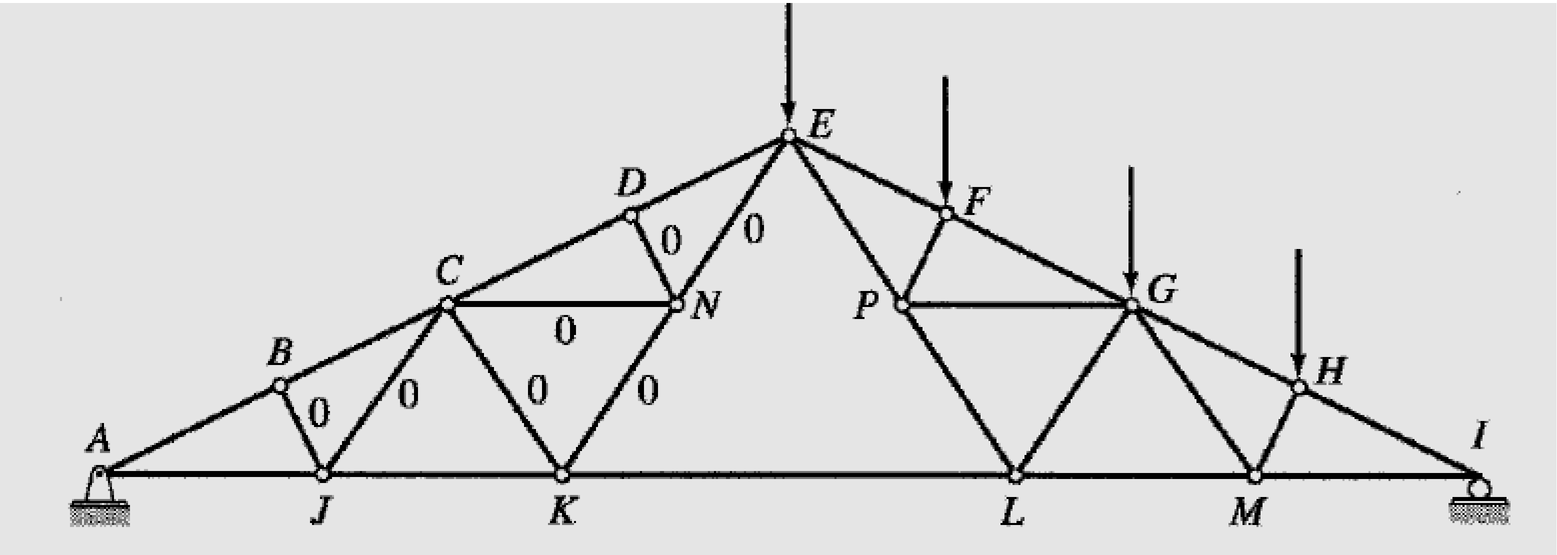
- Some tips to identify zero force members in plane trusses:
  - If only two noncollinear members are connected to a joint that has no external loading or reactions applied to it, then the force in both members is zero

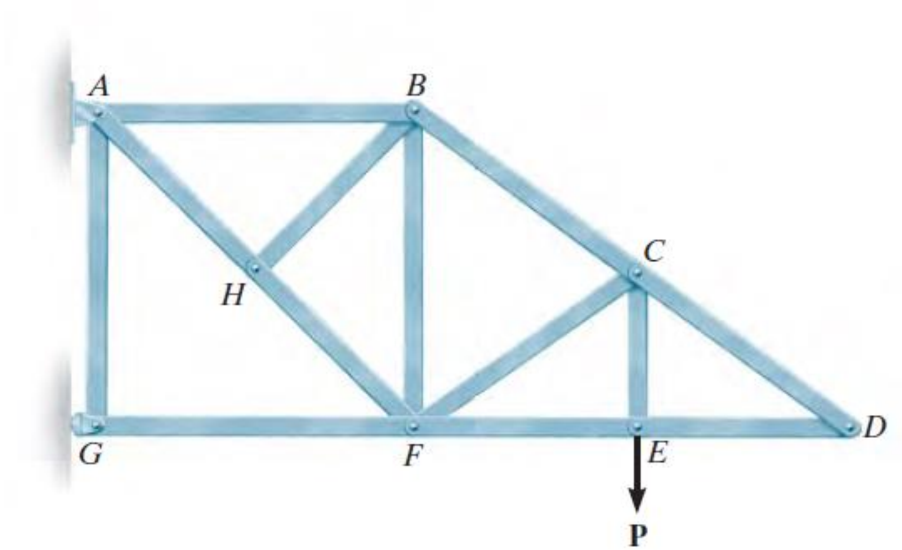


  - If three members, two of which are collinear, are connected to a joint that has no external loads or reactions applied to it, then the force in the member that is not collinear is zero

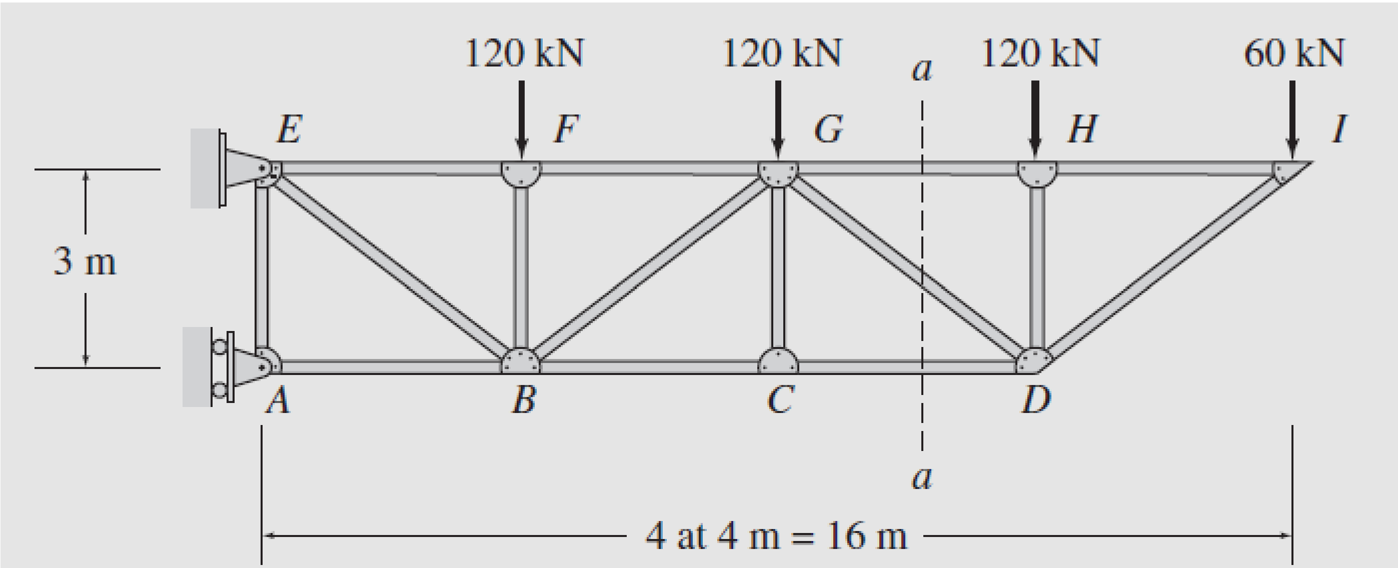


Zero force member: examples

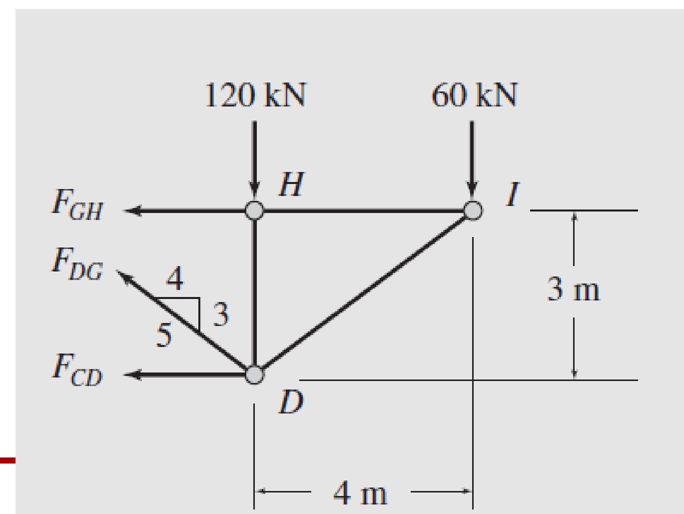




Analysis of trusses: method of sections



- Method of sections is used when forces in specific members are required



Analysis of trusses: method of sections