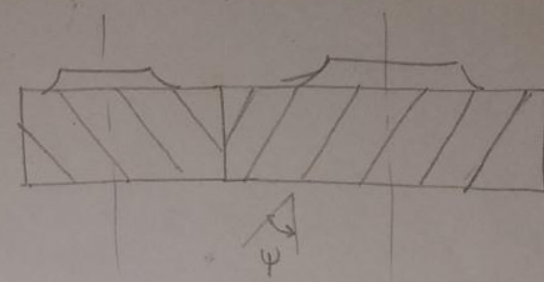
Helical and Bevel gears:

when power transmitted by helical gears the shaft is subjected to thrust loading.

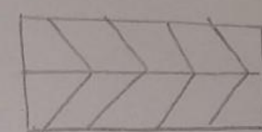
Helical gear



Thrust loading is eliminated by using Double Helical

But it is expensive to manufacture and mount.

Double helical



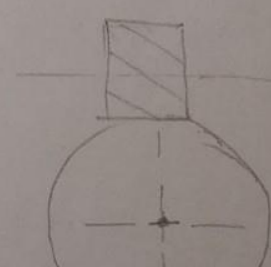
Advantage of helical gears:

- As gears rotate tooth contact spread across the tooth surface gradually
  Teeth come into engagement gradually → Smoother and quieter operation than spur gears.

  gradual engagement → lower dynamic factor (K_v) permits higher rotational speed.

  Common application of helical gear → Automotive transmission where quiet is a proirity.

Cross Helical gears: Non-parallel shafts helical gears



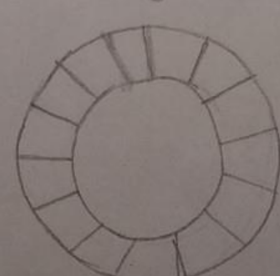
Bevel gears: a) Straight-tooth Bevel gears.

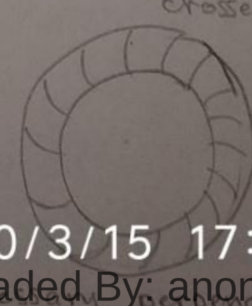
- Teeth are formed like spur gear but tooth surface are made of conical elements.

b) Spiral-tooth Bevel gears.

Teeth engage gradualy starting at one side → Smooth and quiete operation

Straight Bevel





Hypoid gears: Bevel gears mounted on shafts having non-Intersecting axis (Not necessary perpendicular

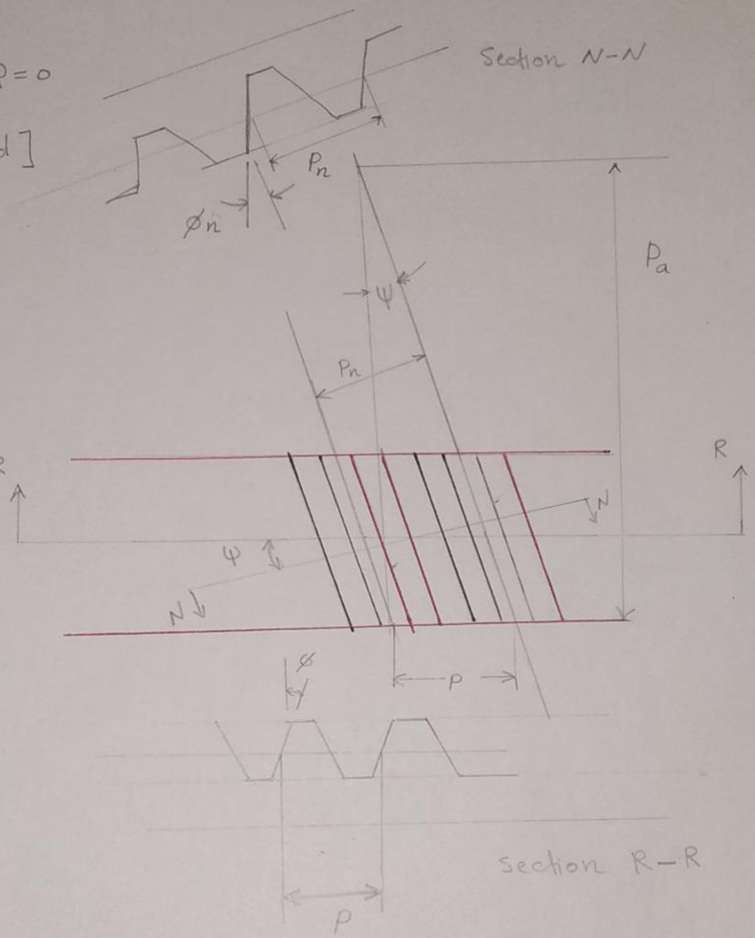
Helical gear geometry

Spur gear → Helical gear with $\psi = 0$

$15^\circ < \psi < 30^\circ$ [Non standard]

$\psi < 15^\circ$ → less thrust load.

$\psi > 30^\circ$ → quieter operation.



Helical gears:

Right hand or left hand

Mating gears must have the same ψ but opposite hand.

Circular pitch (P):

measured in the plane of rotation.

P_n = circular pitch measured in a plane normal to teeth.

$P_n = p \cos\psi$

$a = \frac{1}{P_n}$, $b = \frac{1.25}{P_n}$

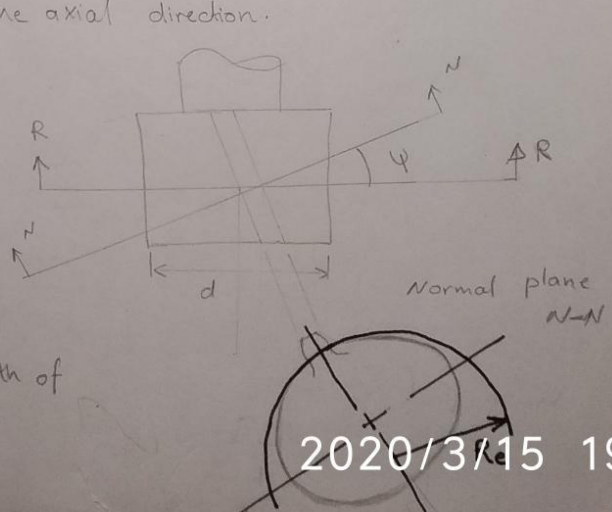
Since $pP = \pi \Rightarrow P_n = P / \cos\psi$

pitch dia. of helical gear: $d = \frac{N}{P} = \frac{N}{P_n \cos\psi}$

Axial pitch (P_a): distance between corresponding points on adjacent teeth measured on the pitch surface in the axial direction.

$P_a = p / \tan\psi$

To achieve axial overlap between adjacent teeth → $b \geq (1.15 - 2) P_a$



Normal plane (N-N) intersects the pitch cylinder in an ellipse

The shape of the tooth in the normal plane is nearly the same as the shape of spur gear tooth of pitch radius of R_e of the ellipse.

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$$R_e = \left(\frac{d}{2}\right)\frac{1}{\cos^2\psi}$$

Equivalent number of teeth (N_e) : Number of teeth in gear of radius R_e.

$$N_e = \frac{2\pi R_e}{P_n} = \frac{\pi d}{P_n \cos^2\psi}$$

$$P_n = p\cos\psi = \frac{\pi d}{N}\cos\psi \;, \quad \text{or} \quad \frac{\pi d}{P_n} = \frac{N}{\cos\psi}$$

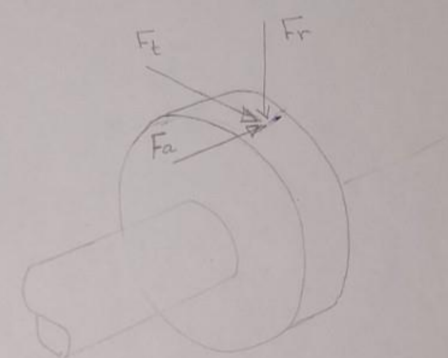
$$\Rightarrow N_e = \frac{N}{\cos^3\psi}$$

Helical gear bending:

Lewis form factor Y is the same of spur gear with $N = N_e$ and $\phi = \phi_n$

where $\tan\phi_n = \tan\phi \cos\psi$

Helical gear force analysis:



Tangential component : F_t

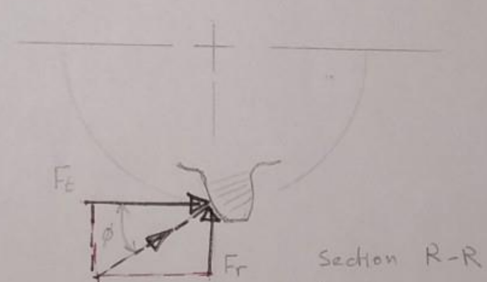
Normal component : F_r

axial component : F_a

F_b = bending for helical gear

$F_b \perp$ tooth surface

Transmitted power

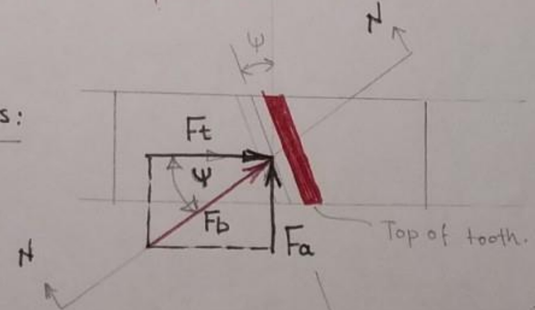


$$H = \frac{F_t\,(lb)\; V\,(ft/min)}{33,000}$$

$$w\,(watt) = F_t\,(Newton)\; V\,(m/s)$$

Force componat that will transmit power. is (F_t)

Force component in Spur and helical gears:



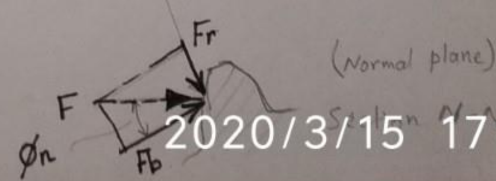
$F_r = F_t \tan\phi$ —— (a)

$F_a = F_t \tan\psi$ —— (b)

$F_b = F_t / \cos\psi$ —— (c)

$F = F_b / \cos\phi_n = F_t / \cos\psi \cos\phi_n$ —— (d)

$F_r = F_b \tan\phi_n \;, \; F_r = \dfrac{F_t \tan\phi_n}{\cos\psi}$ —— (e)



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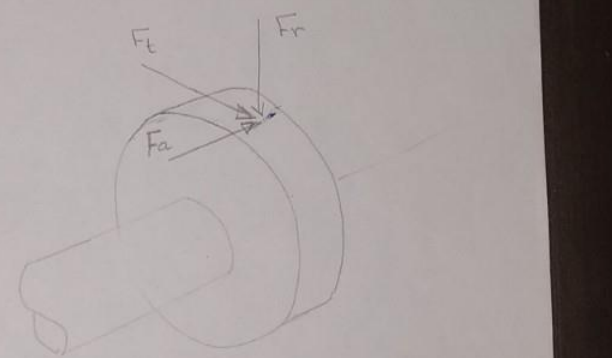

Helical gear force analysis:

Tangential component : F_t
Normal component : F_r
axial component : F_a

F_b = bending for helical gear

$F_b \perp$ tooth surface

Transmitted power

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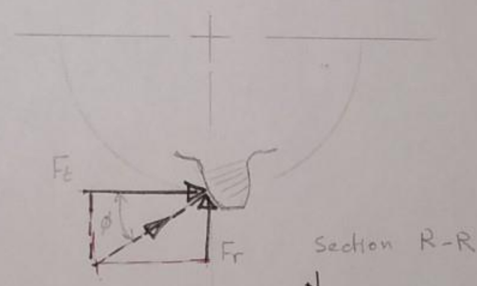
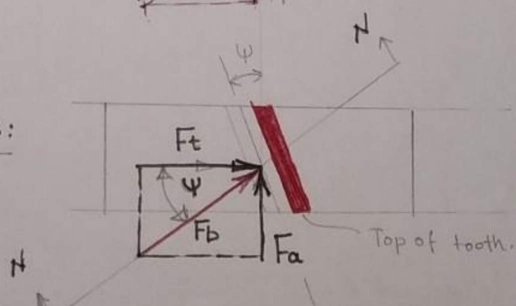


Force componat that will transmit power is (F_t)



orce component in Spur and helical gears:

$r = F_t \tan\phi$ —— (a)

$a = F_t \tan\psi$ —— (b)

$= F_t/\cos\psi$ —— (c)

$= F_b/\cos\phi_n = F_t/\cos\psi\cos\phi_n$ —— (d)

$F_r = F_b\tan\phi_n \,, \quad F_r = \dfrac{F_t\tan\phi_n}{\cos\psi}$ —— (e)

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Combining eq (e) and (a) : $\boxed{\tan\phi_n = \tan\phi \cos\psi.}$

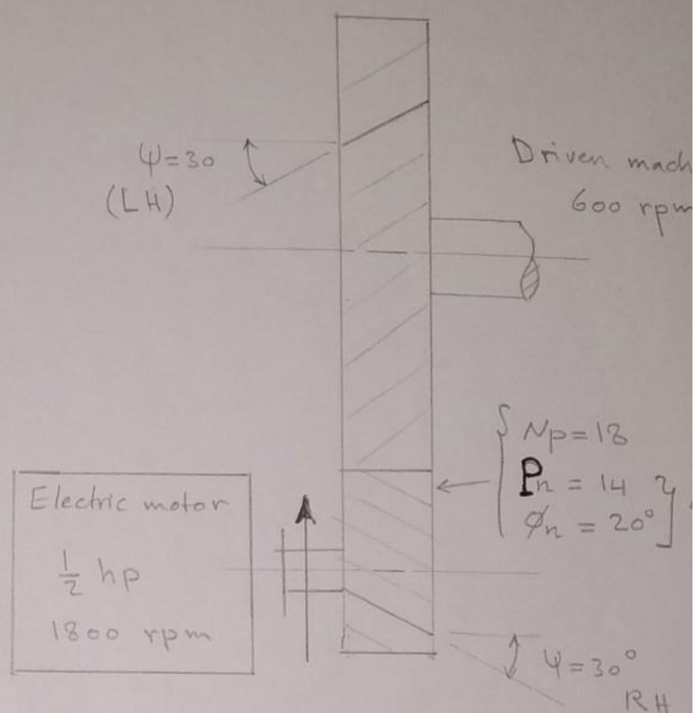
Example:

Helical – gear speed reducer.

Determine: ϕ, P, d_p, d_g, and N_g.

V, F_r, F_t, F_a and

face width (b) to give , $b = 1.5\,P_a$



Solution:

$$\phi = \tan^{-1}\left(\frac{\tan\phi_n}{\cos\psi}\right)$$

$$= \tan^{-1}\left(\frac{\tan 20}{\cos 30}\right) = 22.8^\circ$$

$$P = P_n \cos\psi = 14 \cos 30^\circ = 12.12 \text{ teeth/in}$$

$$d_p = \frac{N_p}{P} = \frac{18}{12.12} = 1.48.\text{ in}$$

$$N_g = N_p\,(n_p/n_g) = 18\,(1800/600) = 54 \text{ teeth.}$$

$$d_g = \frac{N_g}{P} = \frac{54}{12.12} = 4.45 \text{ in}$$

$$V = \frac{\pi\, d_p\, n_p}{12} = \frac{\pi\,(1.49)(1800)}{12} = 702 \text{ f/min.}$$

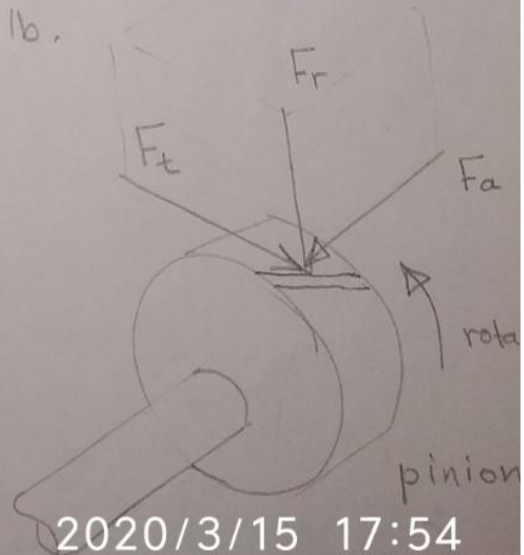
$$F_t = \frac{33{,}000 \text{ hp}}{V} = \frac{33{,}000 \times 0.5}{702} = 23.6 \text{ lb.}$$

$$F_r = F_t \tan\phi = 23.5 \tan 22.8^\circ = 9.9 \text{ lb.}$$

$$F_a = F_t \tan\psi = 23.5 \tan 30^\circ = 13.6 \text{ lb.}$$

$$= p/\tan\psi = \frac{\pi}{P\tan\psi} = \frac{\pi}{12.2 \tan 30} = 0.45 \text{ in}$$

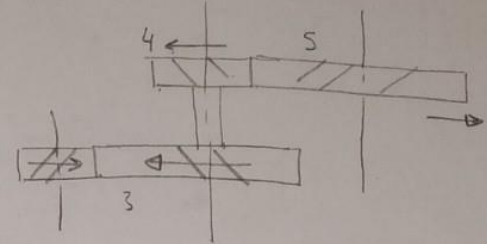
$$b = 1.5\,P_a \Rightarrow b = 1.5(0.45) = 0.67 \text{ in}$$



13-36 | $\phi_n = 20°$, $\psi = 30°$

F $P_n = 7$

$N_3 = 54$, $N_4 = 14$



$P = P_n \cos\psi = 7 \cos 30 = 6.062 \left(\frac{\text{teeth}}{\text{in}}\right)$

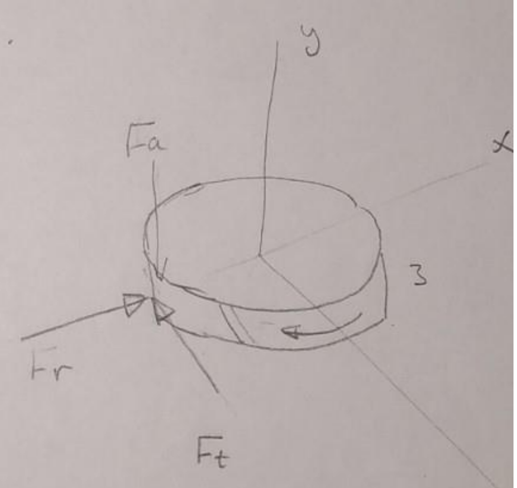
$d_3 = \frac{N_3}{P} = \frac{54}{6.062} = 8.9$ in , $\tan\phi_n = \tan\phi \cos\psi$

$\phi = \tan^{-1}(0.42) \Rightarrow \phi = 22.$

$F_t = 500$ lb.

$F_a = F_t \tan\psi = 500 \tan 30 = 288.7$ lb.

$F_r = F_t \tan\phi = 500 \tan(22.8) = 210.2$ lb.



$F_3 = F_r i - F_a j - F_t z$

Gear 4

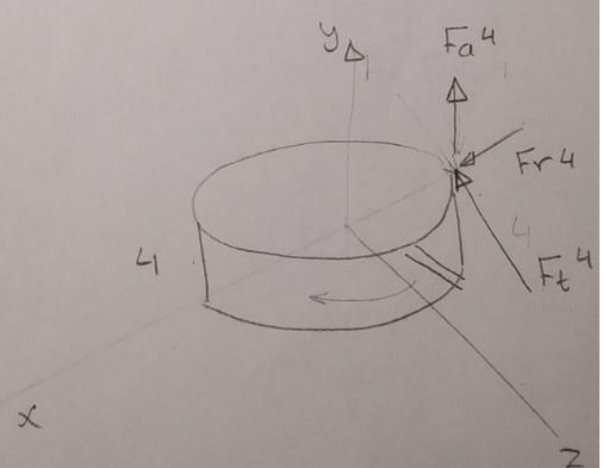
$d_4 = \frac{N_4}{P} = \frac{14}{6.062} = 2.309''$

$F_{t4} = F_{t3} \frac{d_3}{d_4} = 1929$ lb.

$F_{a4} = F_{t4} \tan 30 = 1113$ lb

$F_{r4} = F_{t4} \tan 22.8 = 811$ lb

$F_4 = -F_{r4}\, i + F_a^4\, i - F_{t4}\, k$



Helical gear – Tooth Bending and surface fatigue.

Bending stress:

$$\sigma = \frac{F_t P}{b J} K_o K_m K_v \quad , \quad \sigma = \frac{F_t K_o K_m K_v}{b J m} \quad [SI]$$

J' = geometry Factor of helical gear with $\phi_n = 20°$, when used with 75-teeth mating gear

Fig. [14-7]

For mating gears other than 75-teeth → modification factor

K_J → Fig [14-8]

$\Rightarrow J = k_J J'$

K_o, K_m, K_v → as spur gears

Bending strength

$$S_e = S_t \frac{K_L K_{ms} K_t}{K_r}$$

as spur gear.

Contact stress

Fundamental difference between spur and helical gears is encountered at pitch surface [zero sliding velocity], oil film gets squeezed out hence surface pitting is likely to occur.

Theoretical length of contact for spur gear $= b$

For helical gear ; Length of contact per tooth $= \frac{b}{\cos\psi}$

AGMA: recomends to take 95% of C.R. for contact stress

Length of contact $= 0.95 \frac{b\, C.R.}{\cos\psi}$

Helical gear contact stress:

$$S_{fe} = S_c \frac{C_L C_s C_H}{C_r}$$

as spur gear

$$\sigma_c = C_p \sqrt{\frac{F_t C_o C_m C_v}{b I d_p}} \quad , \quad I = \frac{\sin\phi \cos\phi}{2 m_n} \frac{R}{R+1}$$

$m_n = \frac{P_{bn}}{\ldots}$

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$$m_n = \frac{b}{L}$$, m_n function of C.R in plane of rotatio

, m_F , tooth deflection or modification.

L = min. total length of line of contact.

$$L_{ab} = \sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - C \sin\phi$$

For spur gear $m_N = 1.0$

For helical gear:

If $m_F > 2.0$

$$m_N = \frac{P_{bn}}{0.95\, L_{ab}}$$

P_{bn} = base pitch in normal plane.

$$P_{bn} = P_n \cos\phi_n.$$

$$r_{ap} = \frac{N_P}{2P} \qquad r_{ag} = \frac{N_g}{2P}$$

$$r_{bp} = r_p \cos\phi \qquad a = \frac{1}{P_n}$$

$$r_{bg} = r_g \cos\phi$$

$$C = r_p + r_g$$

Strength geometry factor:

$$I = \frac{\sin\phi \cos\phi}{2\, m_n} \frac{R}{R+1}$$

For spur gear → $m_n = 1.0$

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