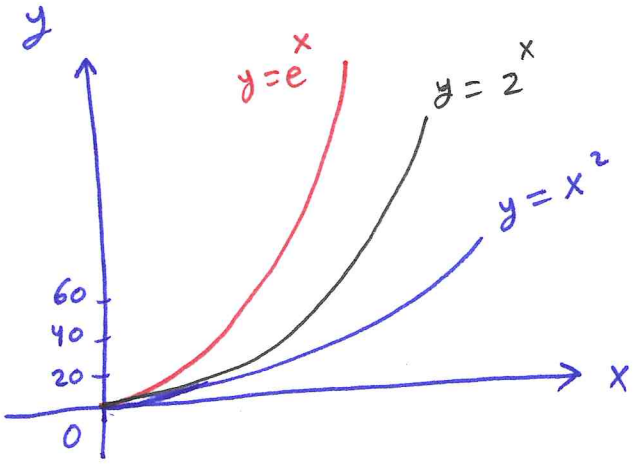
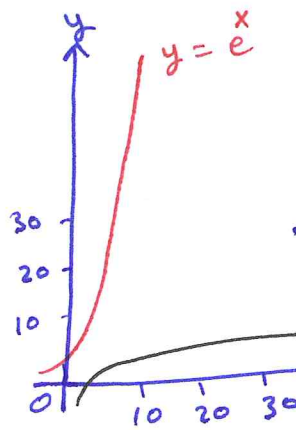
7.8 Relative Rates of Growth





- at $x = 24$ cm e^x is halfway to the moon
- at $x = 43$ cm, e^x is close to sun.

$y = \ln x$

we need ≈ 5 light-years on x-axis to find point where $y = \ln x = 43$ cm

Def: Let $f(x)$ and $g(x)$ be positive for large x:

- If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$, then f grows faster than g as $x \to \infty$

III

- If $\lim_{x\to\infty} \frac{g(x)}{f(x)} = 0$, then g grows slower than f as $x \to \infty$.

- If $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$ "$0 < L < \infty$", then f and g grow at the same rate as $x \to \infty$

Exp. ① 4^x grows faster than e^x as $x \to \infty$ since

$$\lim_{x\to\infty} \frac{4^x}{e^x} = \left(\frac{4}{e}\right)^x = \infty \qquad e \approx 2.718$$

② $\left(\frac{3}{2}\right)^x$ grows slower than e^x as $x \to \infty$ since

$$\lim_{x\to\infty} \frac{\left(\frac{3}{2}\right)^x}{e^x} = \lim_{x\to\infty} \left(\frac{3}{2e}\right)^x = 0$$

③ $\log_3 x$ grows same as $\ln x$ as $x \to \infty$ since

$$\lim_{x\to\infty} \frac{\log_3 x}{\ln x} = \lim_{x\to\infty} \frac{\frac{\ln x}{\ln 3}}{\ln x} = \lim_{x\to\infty} \frac{1}{\ln 3} = \frac{1}{\ln 3}$$

④ $\ln x$ grows slower than e^x as $x \to \infty$ since

$$\lim_{x\to\infty} \frac{\ln x}{e^x} = \lim_{x\to\infty} \frac{\frac{1}{x}}{e^x} = \lim_{x\to\infty} \frac{1}{xe^x} = 0.$$