Birzeit University
Faculty of Engineering
Department of Electrical Engineering
Engineering Probability and Statistics ENEE 331
Problem Set (1)
<u>Fundamental Concepts of Probability</u>

1) Let *A* and *B* denote two events defined over a sample space *S*. Suppose it is given that $P(A \cup B) = 0.76$

a. If A and B are mutually exclusive events with $P[\overline{A}] = 0.45$, then what is P[B]?
b. Suppose, instead, that A and B are independent events with $P[B/A] = 0.12$,. What is $P[A]$?
c. If it is given that $P[A] = 0.30$ and that $P[B/A] = 0.60$, what is P[B]?

2) In a certain lot of personal computers, it is known that 1 % have some minor defect as they come off the production line. They are put through a test procedure, which detects any defect 98 % of the time if a defect is really present, and indicates a defect 1 % of the time even though there is none present. What is the probability that

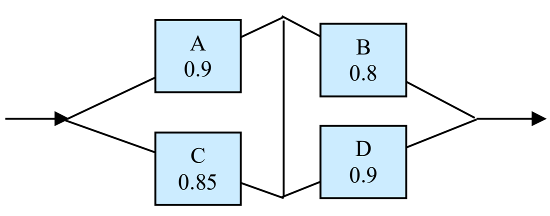
a. a computer will be classified defective as a result of the test procedure?
b. a computer is in fact defective if the test indicates that it is defective?

3) A sample space consists of three events, A, B and C. If $P(A^c) = 0.5$, and $P(A \cap B) = 0.25$, $P(B \cup C) = 0.75$. The pair of events (A and B), (B and C) are independent. Events A and C are mutually exclusive. Find the followings:

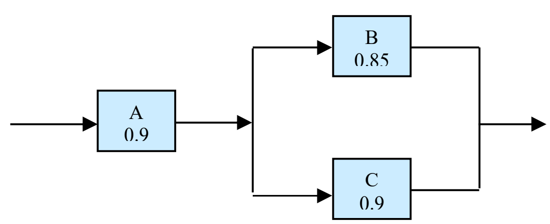
a. Probability that exactly one event will occur.
b. P(B/A)

4) Consider the following systems made up of independent components. The probability that each component functions is indicated in the figures.

a. Find the probability that the systems work properly (system reliability).
b. Is it possible to increase the reliability up to 99.5% for the system in (b) by adding more components in the parallel connection?



(a)



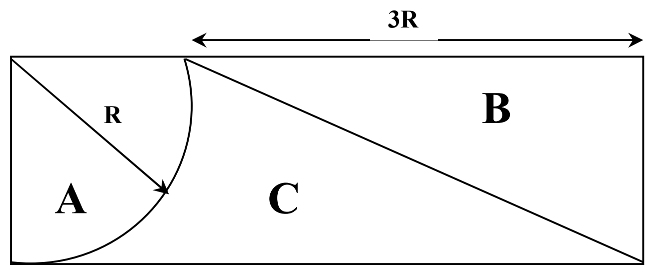
(b)

5) Items in a production line have to pass two successive quality control tests. If the probability of producing items of high quality is 0.95, the probability of misclassifying the items through the first and second tests is 0.05 and 0.02 respectively.

a. Find the percentage of items classified as high quality.
b. If an item is classified as high quality, what is the probability that it came out of the production line as high quality?

6) An irrigation well is to be drilled in the area shown in the figure. The probability of obtaining water for each sub-area A, B and C is 0.8, 0.1 and 0.3 respectively. The probability of selecting any of the three sub-areas is proportional to the area

a. Find the probability of obtaining water.
b. If no water was obtained, what is the probability that the well was drilled in area C?



7) An urn contains colored balls: 4 Red balls and 6 Green balls. Suppose that three balls are drawn from this urn.

a. If the three balls are drawn one after another without replacement, what is the probability that the colors observed will be Red, Green in this order?
b. If the three balls are drawn one after another with replacement, what is the probability that all three of the selected balls will be of the same color?
c. If the three balls are drawn simultaneously from this urn (thus without replacement), what is the probability that the selected balls will all be of the same colors?

8) Prove that a set with n elements has 2^n subsets.

9) For a storm-sewer system, estimates of annual maximum flow rates (AMFR) and their likelihood of occurrence [assuming that a maximum of 12 cubic feet per second, cfs, is possible] are given as:

Event A: ($5 \le$ AMFR ≤ 10 cfs)	P(A) = 0.6
Event B: ($8 \le$ AMFR ≤ 12 cfs)	P(B) = 0.6
Event C=$A \cup B$	P(C) = 0.7

Determine P ($8 \le$ AMFR ≤ 10 cfs), the probability that AMFR is between 8 and 10 cfs.

10) Given that P(A) = P(B) = P(C) = ¼, $P(A \cap B) = P(C \cap B) = 0$, $P(A \cap C) = 1/8$, find

a. the probability that at least one of the events A, B, or C occurs
b. P(AUBUC)
c. the probability that exactly one even occurs

11) A box contains three coins. One coin is two-headed, a second is fair, and the third is biased with p being the probability of getting a head. A coin is chosen at random from the box and flipped once

a. What is the probability that the flip results in a head?
b. Suppose that the flip yields a head, what is the probability that the chosen coin is the biased one?

12) An experiment is independently repeated five times. The probability of a success in every trial is 0.5, find the probability of obtaining three consecutive successes.

13) An experiment is independently repeated until a success is obtained for the first time. What is the probability that it will take five trials for that to happen assuming that the probability of a success in each trail is p.

14) A motor drives an electric generator. During a 30-day period, the motor needs repair with probability 8% and the generator, independently of the motor, needs repair with probability 4%. What is the probability that during the given period, the entire apparatus will need repair?

solution is missing

15) Assume that the reaction time of a driver over the age of 70 to a certain visual stimulus is described by a continuous probability function of the form $f(x) = xe^{-x}$, x>0, where x is measured in seconds. Let A be the event “Driver requires longer than 1.5 seconds to react”. Find P(A).

16) One model to describe the mortality is $f(t) = kt^2(100-t)^2$, $0 \leq t \leq 100$, where t describes the age at which a person dies.

a. Find k
b. Let A be the event “Person lives over 60”. Find P(A)
c. What is the probability that a person will die between the age of 80 and 85 given that the person has lived to be at least 70?

solution is missing

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a. If A and B are mutually exclusive events with $P[\overline{A}] = 0.45$, then what is P[B]?
b. Suppose, instead, that A and B are independent events with $P[B/A] = 0.12$,. What is $P[A]$?
c. If it is given that $P[A] = 0.30$ and that $P[B/A] = 0.60$, what is P[B]?

a) A and B are mutually exclusive $P(A \cap C) = 0$;

$$P(A \cup B) = 0.75 = P(A) + P(B) - P(A \cap B) = (1 - 0.45) + P(B) + 0$$
$$P(B) = 0.75 - 0.55 = 0.2$$

b) A and B are independent events $P(A \cap C) = P(A)P(B)$

$$P\,(B/A) = 0.12 = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)}$$
$$P(B) = 0.12$$
$$P(A \cup B) = 0.75 = P(A) + P(B) - P(A)P(B) = P(A) + 0.12 - 0.12P(A)$$
$$P(A) = \frac{0.12}{0.88} = 0.1364$$

c) $P\,(B/A) = 0.6 = \frac{P(A \cap B)}{P(A)} = \frac{0.75 - 0.3 - P(B)}{0.3}$

$P(B) = 0.27$

2) In a certain lot of personal computers, it is known that 1 % have some minor defect as they come off the production line. They are put through a test procedure, which detects any defect 98 % *(let it P(D/T))* of the time if a defect is really present, and indicates a defect 1 % *(let it P(D'/T))* of the time even though there is none present. What is the probability that

a. a computer will be classified defective as a result of the test procedure?
b. a computer is in fact defective if the test indicates that it is defective?

Solution
Suppose P(D): defect computer
P(T): indicates as defect in the test
According to the total probability theorem

a) $P(T) = P(D)P\,(T/D) + P(\overline{D})P\,(T/\overline{D}) = 0.01 * 0.98 + 0.01 * 0.99 = 0.0197$
b) $P\,(D/T) = \frac{P(D)P(T/D)}{P(T)} = \frac{0.01*0.98}{P0.0197} = 0.4975$

3) A sample space consists of three events, A, B and C. If $P(A^c) = 0.5$, and $P(A \cap B) = 0.25$, P(BUC) = 0.75. The pair of events (A and B), (B and C) are independent. Events A and C are mutually exclusive. Find the followings:

a. Probability that exactly one event will occur.
b. P(B/A)

Solution:

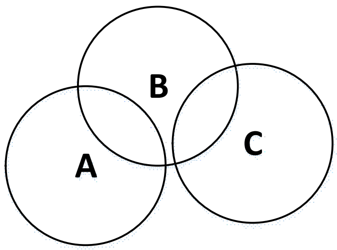
$P(A) = 1 - 0.5 = 0.5$

$P(B) = \frac{P(A \cap B)}{P(A)} = \frac{0.25}{0.5} = 0.5$

$P(C) = P(B \cup C) - P(B) + P(B) * P(C)$

$P(C) = 0.75 - 0.5 + 0.5 * P(C)$

$P(C) = \frac{0.25}{0.5} = 0.5$



a) Probability that exactly one event will occur= $\{A\overline{BC},\ \bar{A}B\bar{C}, \overline{AB}C\}$

Events A and C are mutually exclusive $A \cap \bar{C} = A \cap (1 - C) = A, and\ \bar{A} \cap C = C$

$=P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{B} \cap C)$

$= P(A \cap (1 - B)) + P((1 - A) \cap B \cap (1 - C)) + P((1 - B) \cap C)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B) - P(B \cap C) + P(A \cap B \cap C) + P(C) - P(B \cap C)$

$= 0.5 - 0.25 + 0.5 - 0.25 - 0.25 + 0 + 0.5 - 0.25 = 0.5$

4) Consider the following systems made up of independent components. The probability that each component functions is indicated in the figures.

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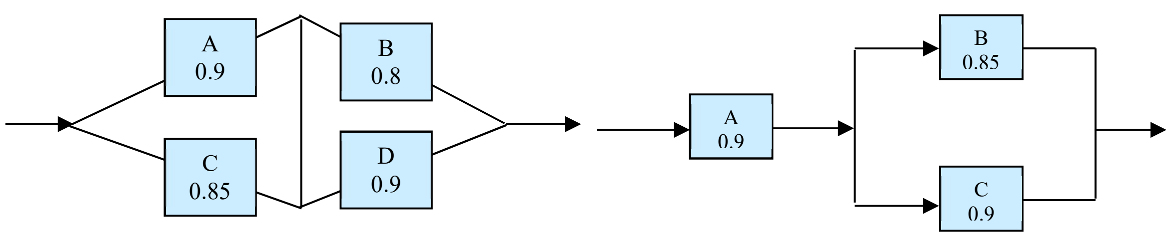


(a) **(b)**

a) $=P((A_a \cup C_a) \cap (B_a \cup D_a) \cap A_b \cap (B_b \cup C_b)$

$= P(A_a \cup C_a)P(B_a \cup D_a)P(A_b)P(B_b \cup C_b)$

$= (1 - P(\overline{A_a \cup C_a}))(1 - P(\overline{B_a \cup D_a}))(1 - P(\overline{A_b}))(1 - P(\overline{B_b \cup C_b}))$

$= (1 - P(\overline{A_a} \cap \overline{C_a}))(1 - P(\overline{B_a} \cap \overline{D_a}))(1 - P(\overline{A_b}))(1 - P(\overline{B_b} \cap \overline{C_b}))$

$= (1 - P(\overline{A_a})P(\overline{C_a}))(1 - P(\overline{B_a})P(\overline{D_a}))(1 - P(\overline{A_b}))(1 - P(\overline{B_b})P(\overline{C_b}))$

$= (1 - 0.1 * 0.15)(1 - 0.2 * 0.1)(1 - 0.1)(1 - 0.15 * 0.1)$

$= (0.985)(0.98)(0.9)(0.985) = 0.847$

b) No it is impossible because system P(system a)=0.9653

5) Items in a production line have to pass two successive quality control tests. If the probability of producing items of high quality is 0.95, the probability of misclassifying the items through the first and second tests is 0.05 and 0.02 respectively.

a. Find the percentage of items classified as high quality.
b. If an item is classified as high quality, what is the probability that it came out of the production line as high quality?

Solution

Start with 100000 items

let *H*= high quality

L= low quality

F= first test

S= second tests

T= Final test.

P(H)=0.95; P(L)=0.05 ; P(L/F)=0.05 ; P(L/S)=0.02

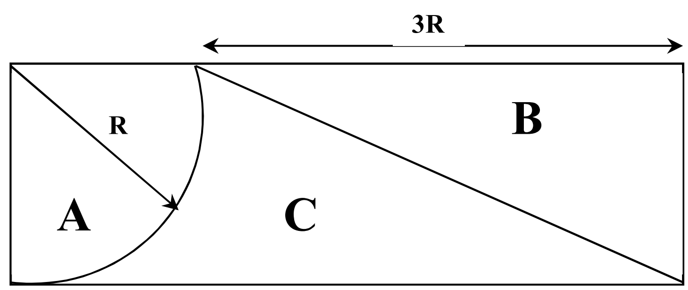
a) *P(H/F)ՈP(H/S)= P(H) P(H/F)* P(H) P(H/S)=0.95*0.95*0.98= 0.8844*

b) *P(H/F)Ո P(H/S)) /((P(H/F)Ո P(H/S))U(P(L/F)Ո P(L/S)))*

*= 0.8844+0.05*0.02=0.8844/0.8854=0.9989*

6) An irrigation well is to be drilled in the area shown in the figure. The probability of obtaining water for each sub-area A, B and C is 0.8, 0.1 and 0.3 respectively. The probability of selecting any of the three sub-areas is proportional to the area

a. Find the probability of obtaining water.
b. If no water was obtained, what is the probability that the well was drilled in area C?



$A=1/4*\pi R^2$; $B=3/2R^2$; $C=4R^2-1/4*\pi R^2+3/2R^2 =2.5R^2-1/4*\pi R^2$

$S= 4R^2$

$P(A)=1/16\ \pi$; $P(B)=3/8$; $P(C)= 0.625-1/16*\pi$

a. The probability of obtaining water.

$P(W)=1/16* \pi *0.8+3/8*0.1 +(0.625-1/16*\pi)*0.3= 0.3232$

If no water was obtained, The probability that the well was drilled in area C

$$P(C/\overline{W}) = \frac{P(C \cap \overline{W})}{P(\overline{W})} = \frac{(0.625 - 1/16 * \pi)0.7}{1 - 0.3588} = 0.4801$$

7) An urn contains colored balls: 4 Red balls and 6 Green balls. Suppose that three balls are drawn from this urn.

a. If the three balls are drawn one after another without replacement, what is the probability that the colors observed will be Red, Green in this order?

$$P(RGG) + P(RRG) = \frac{4}{10} * \frac{6}{9} * \frac{5}{8} + \frac{4}{10} * \frac{3}{9} * \frac{6}{8} = 0.267$$

b. If the three balls are drawn one after another with replacement, what is the probability that all three of the selected balls will be of the same color?

$$P(G \cap G \cap G) \cup P(R \cap R \cap R) = \left(\frac{6}{10}\right)^3 + \left(\frac{4}{10}\right)^3 = 0.28$$

c. If the three balls are drawn simultaneously from this urn (thus without replacement), what is the probability that the selected balls will all be of the same colors?

$$P(G \cap G \cap G) \cup P(R \cap R \cap R) = \frac{6}{10} * \frac{5}{9} * \frac{4}{8} + \frac{4}{10} * \frac{3}{9} * \frac{2}{8} = 0.2$$

8) Prove that a set with n elements has 2^n subsets.

Proof: *We will prove by induction that, for all $n \in \mathbb{Z}_+$, the following holds:*

$P(n)$ *Any set of n elements has 2^n subsets.*

Base case: *Since any 1-element set has 2 subsets, namely the empty set and the set itself, and $2^1 = 2$, the statement $P(n)$ is true for $n = 1$.*

Induction step:

- *Let $k \in \mathbb{Z}_+$ be given and suppose $P(k)$ is true, i.e., that any k-element set has 2^k subsets.*
- *We seek to show that $P(k+1)$ is true as well, i.e., that any $(k+1)$-element set has 2^{k+1} subsets.*
- *Let A be a set with $(k+1)$ elements.*
- *Let a be an element of A, and let $A' = A - \{a\}$ (so that A' is a set with k elements).*
- *We classify the subsets of A into two types: (I) subsets that do not contain a, and (II) subsets that do contain a.*
- *The subsets of type (I) are exactly the subsets of the set A'. Since A' has k elements, the induction hypothesis can be applied to this set and we get that there are 2^k subsets of type (I).*
- *The subsets of type (II) are exactly the sets of the form $B = B' \cup \{a\}$, where B' is a subset of A'. By the induction hypothesis there are 2^k such sets B', and hence 2^k subsets of type (II).*
- *Since there are 2^k subsets of each of the two types, the total number of subsets of A is $2^k + 2^k = 2^{k+1}$.*
- *Since A was an arbitrary $(k+1)$-element set, we have proved that any $(k+1)$-element set has 2^{k+1} subsets. Thus $P(k+1)$ is true, completing the induction step.*

Conclusion: *By the principle of induction, $P(n)$ is true for all $n \in \mathbb{Z}_+$.*

9) For a storm-sewer system, estimates of annual maximum flow rates (AMFR) and their likelihood of occurrence [assuming that a maximum of 12 cubic feet per second, cfs, is possible] are given as:

Event A: ($5 \le$ AMFR ≤ 10 cfs)	P(A) = 0.6
Event B: ($8 \le$ AMFR ≤ 12 cfs)	P(B) = 0.6
Event C=A∪B	P(C) = 0.7

Determine P ($8 \le$ AMFR ≤ 10 cfs), the probability that AMFR is between 8 and 10 cfs.

Solution P ($8 \le$ AMFR ≤ 10 cfs)= $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.6 - 0.7 = 0.5$

10) Given that P(A) = P(B) = P(C) = ¼, P(A∩B) = P(C∩B) = 0, P(A∩C) = 1/8, find

a. the probability that at least one of the events A, B, or C occurs

b. P(AUBUC)
c. the probability that exactly one even occurs

Solution:

a. The probability that at least one of the events A, B, or C occurs=
b. $P(A \cup B \cup C) =$
$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
$= 0.25 + 0.25 + 0.25 - 0 - 0.125 - 0 + 0 = 0.625$
c. $P(A \cup B \cup C) =$
$= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$
$= 0.25 + 0.25 + 0.25 - 0 - 0.125 - 0 + 0 = 0.625$
d. the probability that exactly one even occurs=
$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$
$= P(A \cup B \cup C) - P(A \cap B) - P(A \cap C) - P(B \cap C)$
$= 0.25 + 0.25 + 0.25 - 0 - 0.125 - 0 + 0 - 0 - 0.125 - 0.125 = 0.375$

11) A box contains three coins. One coin is two-headed, a second is fair, and the third is biased with p being the probability of getting a head. A coin is chosen at random from the box and flipped once

a. What is the probability that the flip results in a head?
b. Suppose that the flip yields a head, what is the probability that the chosen coin is the biased one?

Solution

a. the probability that the flip results in a head

$=P(H) = \frac{1}{3} * \frac{2}{2} + \frac{1}{3} * \frac{1}{2} + \frac{1}{3} * P = 0.5 + \frac{P}{3}$

$=P(based/H) = \frac{P(B \cap H)}{P(H)} = \frac{P}{0.5 + \frac{P}{3}} = \frac{3P}{1.5 + P}$

12) An experiment is independently repeated five times. The probability of a success in every trial is 0.5, find the probability of obtaining three consecutive successes.

$P(SSSFF) + P(FSSSF) + P(FFSS\ ) = 0.5^3 + 0.5^3 + 0.5^3 = 0.375$

13) An experiment is independently repeated until a success is obtained for the first time. What is the probability that it will take five trials for that to happen assuming that the probability of a success in each trail is p.

Geometric $P(X = x) = P(1-P)^{x-1} = P(1-P)^{5-1}$

solution to {14) is missing

15) Assume that the reaction time of a driver over the age of 70 to a certain visual stimulus is described by a continuous probability function of the form $f(x) = xe^{-x}$, x>0, where x is measured in seconds. Let A be the event "Driver requires longer than 1.5 seconds to react". Find P(A).

Exponential $\lambda e^{-\lambda x}$

$P(x \le 1.5) = 1 - e^{-1*1.5}$

16) One model to describe the mortality is $f(t) = kt^2(100-t)^2$, $0 \le t \le 100$, where t describes the age at which a person dies.

solution is missing

a. Find k

b. Let A be the event “Person lives over 60”. Find P(A)
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Fundamental Concepts of Probability

1) Three people, A, B, and C, are running for the same office, and we assume that one and only one of them wins. Suppose that A and B have the same chance of winning, but that C has only 1/2 the chance of A or B. Let E be the event that either A or C wins. Find P(E)?
2) The student body is re-electing the class President, Vice-President, Treasurer, and Secretary from a group of 28 students consisting of 10 seniors, 8 juniors, 6 sophomores, and 4 freshmen. This time each class must be represented. What is the probability of electing one officer from each class?
3) Suppose you are taking a multiple-choice test with c choices for each question. In answering a question on a test, the probability that you know the answer is p. If you don't know the answer, you choose one at random. What is the probability that you knew the answer to a question, given that you answered it correctly?
4) A sample space consists of three events, A, B and C. If $P(A^c) = 0.5$, and $P(A \cap B) = 0.25$, $P(B \cup C) = 0.75$. The pair of events (A and B), (B and C) are independent. Events A and C are mutually exclusive. Find the followings:
   a. Probability that exactly one event will occur.
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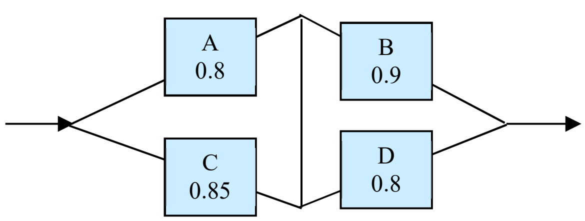

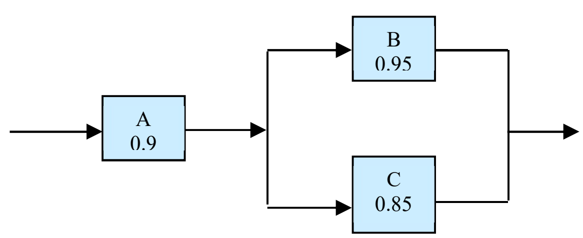

6) In a factory of Plastic tow machines produce 1000 pieces, the first produce 70% of these pieces, its produce 350 cups, and 250 plates, the second machine produce 150 cups and 120 bowls and the rest is plates. If one of these tow machines product is selected at random, find the probability that

   Let M1 :First Machine; M2 :Second Machine; C :Cups, L :Plates; B:Bowls.

a. The product is plate?
b. The product is either cups, plates, either bowls?
c. The product is produced by the first machine if it was cups?

7) A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows: $P(D \mid P1) = 0.01$, $P(D \mid P2) = 0.03$, $P(D \mid P3) = 0.02$, where $P(D \mid Pj)$ is the probability of a defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

8) A box contains colored balls: 5 Red balls and 7 Green balls. Suppose that three balls are drawn from this box.

a. If the three balls are drawn one after another without replacement, what is the probability that the colors observed will be Red, Green in this order?
b. If the three balls are drawn one after another with replacement, what is the probability that all three of the selected balls will be of the same color?
c. If the three balls are drawn simultaneously from this urn (thus without replacement), what is the probability that the selected balls will all be of the same colors?

9) For a storm-sewer system, estimates of annual maximum flow rates (AMFR) and their likelihood of occurrence [assuming that a maximum of 12 cubic feet per second, cfs, is possible] are given as:

Event A: ($5 \le AMFR \le 10$ cfs) P(A)
Event B: ($8 \le AMFR \le 12$ cfs) P(B) = 0.6
Event $C=A\cup B$ P(C) = 0.7
Determine P ($8 \le AMFR \le 10$ cfs), the probability that AMFR is between 8 and 10 cfs.

Good Luck Ahmad Alyan

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   Solution $P[A] = P[B] = 2P[C]$
   $P[A] + P[B] + P[C] = 1$
   $p + p + 0.5p = 1$
   $p = \frac{2}{5} = 0.4$
   $P[E] = P[A] + P[C] = 0.4 + 0.2 = 0.6$
2) The student body is re-electing the class President, Vice-President, Treasurer, and Secretary from a group of 28 students consisting of 10 seniors, 8 juniors, 6 sophomores, and 4 freshmen. This time each class must be represented. What is the probability of electing one officer from each class?

Solution: the four elected members President, Vice-President, Treasurer, and Secretary can be
senior, junior, sophomore, and freshman respectively
Or junior, senior, sophomore, and freshman respectively
Or …

We have four position from for groups so $N_G = \frac{4!}{(4-4)!} = 24 ways$

Where we have
10 ways to select one from 10 seniors,
8 ways to select one from juniors,
6 ways to select one from sophomores,
and 4 ways to select one from freshmen.
So the number of total ways that we can select in is
$N_A = 24*10*8*6*4 = 46080 ways$
Where we have N different ways four the four elected members regardless of the number from each

$$N = \binom{28}{4} = \frac{28!}{(28-4)!} = 491400 ways$$

The probability of electing one officer from each class is

$$p = \frac{46080}{491400} = 0.0938$$

3) Suppose you are taking a multiple-choice test with c choices for each question. In answering a question on a test, the probability that you know the answer is p. If

you don't know the answer, you choose one at random. What is the probability that you knew the answer to a question, given that you answered it correctly?

Solution:

p: knew the answer to a question

A: correct answer.

$$P(A) = p + \frac{1}{c}(1-p)$$

The probability that you knew the answer to a question, given that you answered it correctly

$$P(p/A) = \frac{P(p \cap A)}{P(A)} = \frac{p}{p + \frac{1}{c}(1-p)}$$

4) A sample space consists of three events, A, B and C. If $P(A^c) = 0.5$, and $P(A \cap B) = 0.25$, $P(B \cup C) = 0.75$. The pair of events (A and B), (B and C) are independent. Events A and C are mutually exclusive. Find the followings:

a. Probability that exactly one event will occur.

b. P(B/A)

Solution:

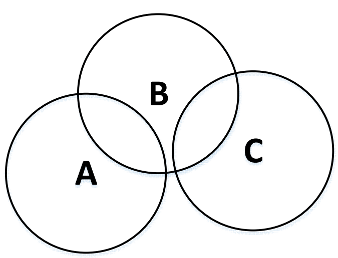
$P(A) = 1 - 0.5 = 0.5$

$$P(B) = \frac{P(A \cap B)}{P(A)} = \frac{0.25}{0.5} = 0.5$$

$P(C) = P(B \cup C) - P(B) + P(B) * P(C)$

$P(C) = 0.75 - 0.5 + 0.5 * P(C)$

$$P(C) = \frac{0.25}{0.5} = 0.5$$



a) Probability that exactly one event will occur= $\{A\overline{BC},\ \bar{A}B\bar{C}, \overline{AB}C\}$

Events A and C are mutually exclusive $A \cap \bar{C} = A \cap (1 - C) = A, and\ \bar{A} \cap C = C$

$= P(A \cap \bar{B} \cap \bar{C}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{A} \cap \bar{B} \cap C)$

$= P(A \cap \bar{B}) + P(\bar{A} \cap B \cap \bar{C}) + P(\bar{B} \cap C)$

$= P\big(A \cap (1 - B)\big) + P\big((1 - A) \cap B \cap (1 - C)\big) + P\big((1 - B) \cap C\big)$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B) - P(B \cap C) + P(A \cap B \cap C)$
$\quad + P(C) - P(B \cap C)$

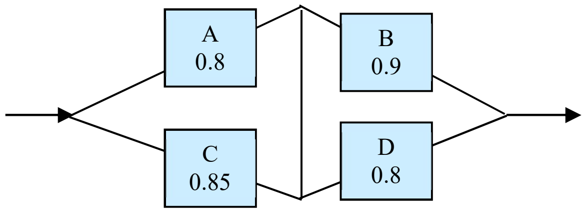
$= 0.5 - 0.25 + 0.5 - 0.25 - 0.25 + 0 + 0.5 - 0.25 = 0.5$

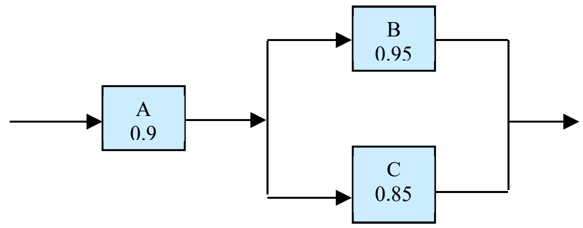
a.

5) Consider the following systems made up of independent components. The probability that each component functions is indicated in the figures.

a. Find the probability that the systems work properly (system reliability).

b. Is it possible to increase the reliability up to 99.5% for the system in (b) by adding more components in the parallel connection?





a) $=P((A_a \cup C_a) \cap (B_a \cup D_a) \cap A_b \cap (B_b \cup C_b)$

$= P(A_a \cup C_a)P(B_a \cup D_a)P(A_b)P(B_b \cup C_b)$

$=[P(A_a) + P(C_a) - P(A_a)P(C_a)][P(B_a) + P(D_a) - P(B_a)P(D_a)]P(A_b)[P(B_b) + P(C_b) - P(B_b)P(C_b)]$

$=[0.8 + 0.85 - 0.8 * 0.85][0.9 + 0.8 - 0.9 * 0.8]0.9[0.95 + 0.85 - 0.95 * 0.85]$

=0.97*0.98*0.9*0.9925=0.85

b) No it is impossible because system P(system Ab)=0.9 and it is series, so the output will be less or equal 0.9.

6) In a factory of Plastic tow machines produce 1000 pieces, the first produce 70% of these pieces, its produce 350 cups, and 250 plates, the second machine produce 150 cups and 120 bowls and the rest is plates. If one of these tow machines product is selected at random, find the probability that

Let M1 :First Machine; M2 :Second Machine; C :Cups, L :Plates; B:Bowls.

a. The product is plate?
b. The product is either cups, plates, either bowls?
c. The product is produced by the first machine if it was cups?

Let M1 :First Machine; M2 :Second Machine; C :Cups, L :Plates; B:Bowls.

Solution

$$P(M1) = 0.7; P(M2) = 0.3; P(C/M1) = \frac{0.35}{0.7} = 0.5\ ;\ P(C/M2) = \frac{0.12}{0.3} = 0.4;$$

$$P(L/M1) = \frac{0.25}{0.7} = 0.357\ ;\ P(L/M2) = \frac{0.03}{0.3} = 0.1; P(B) = 0.12\ ;$$

a- The product is plate is $P(L) = 0.25 + 0.03 = 0.28$;

Total probabilty theorm is the same.

$= P(M1) * P\left(\frac{L}{M1}\right) + P(M2) * P\left(\frac{L}{M2}\right) = 0.7 * 0.357 + 0.3 * 0.1$=0.28

d. The product is either cups, plates, either bowls

$$P(C \cup L \cup B) = (0.35 + 0.25 + 0.15 + 0.12 + 0.03) = 0.9;$$

The rest is produced by machine one 100 gives the same. The product is produced by the first machine if it was cups.

$$P(M1/C) = \frac{P(M1 \cap C)}{P(C)} = \frac{0.35}{0.35 + 0.15} = 0.7$$

7) A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at

varying times. In fact, plans 1, 2, and 3 are used for 30%, 20%, and 50% of the products, respectively. The defect rate is different for the three procedures as follows: P(D | P1) = 0.01, P(D | P2) = 0.03, P(D | P3) = 0.02, where P(D | Pj) is the probability of a defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

Solution:
$$P(P1/D) = P(P1)P(D/P1) = 0.3 * 0.01 = 0.003$$
$$P(P2/D) = P(P2)P(D/P2) = 0.2 * 0.03 = 0.006$$
$$P(P3/D) = P(P3)P(D/P3) = 0.5 * 0.02 = 0.01$$

So the third plan was most likely used and thus responsible since
$$P(P3/D) > Max(P(P1/D), P(P2/D))$$

8) A box contains colored balls: 5 Red balls and 7 Green balls. Suppose that three balls are drawn from this box.
   a. If the three balls are drawn one after another without replacement, what is the probability that the colors observed will be Red, Green in this order?
   b. If the three balls are drawn one after another with replacement, what is the probability that all three of the selected balls will be of the same color?
   c. If the three balls are drawn simultaneously from this urn (thus without replacement), what is the probability that the selected balls will all be of the same colors?

Solution: The probability that the three balls color will be in order Red, Green without replacement,
$$p = P(RGG \cup RRG) = \frac{5}{12} * \frac{7}{11} * \frac{6}{10} + \frac{5}{12} * \frac{4}{11} * \frac{7}{10} = 0.265$$
The probability that the three balls have the same color with replacement,
$$p = P(RRR \cup GGG) = \frac{5}{12} * \frac{5}{12} * \frac{5}{12} + \frac{7}{12} * \frac{7}{12} * \frac{7}{12} = \frac{13}{48}$$
The probability that the three balls have the same color are drawn simultaneously,
$$p = P(RRR \cup GGG) = \frac{5}{12} * \frac{4}{11} * \frac{3}{10} + \frac{7}{12} * \frac{6}{11} * \frac{5}{10} = \frac{9}{44}$$

9) For a storm-sewer system, estimates of annual maximum flow rates (AMFR) and their likelihood of occurrence [assuming that a maximum of 12 cubic feet per second, cfs, is possible] are given as:

   Event A: (5 ≤ AMFR ≤ 10 cfs) P(A) = 0.6
   Event B: (8 ≤ AMFR ≤ 12 cfs) P(B) = 0.6
   Event C=A∪B P(C) = 0.7

   Determine P (8 ≤ AMFR ≤ 10 cfs), the probability that AMFR is between 8 and 10 cfs.

Solution P (8 ≤ AMFR ≤ 10 cfs)=
$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.6 - 0.7 = 0.5$$

In a certain lot of personal computers, it is known that 1 % have some minor defect as they come off the production line. They are put through a test procedure, which detects any defect 98 % *(let it P(D/T))* of the time if a defect is really present, and indicates a defect 1 % *(let it P(D'/T))* of the time even though there is none present. What is the probability that

a. a computer will be classified defective as a result of the test procedure?
b. a computer is in fact defective if the test indicates that it is defective?

Solution

Suppose P(D): defect computer

P(T): indicates as defect in the test

According to the total probability theorem

a)

$$P(T) = P(D)P\,(T/D) + P(\bar{D})P\,(T/\bar{D}) = 0.01 * 0.98 + 0.01 * 0.99 = 0.0197$$

b)

$$P\,(D/T) = \frac{P(D)P\,(T/D)}{P(T)} = \frac{0.01 * 0.98}{P0.0197} = 0.4975$$

Suppose you are taking a multiple-choice test with c choices for each question. In answering a question on a test, the probability that you know the answer is p. If you don't know the answer, you choose one at random. What is the probability that you knew the answer to a question, given that you answered it correctly?

Solution:

p: knew the answer to a question

A: correct answer.

$$P(A) = p + \frac{1}{c}(1-p)$$

The probability that you knew the answer to a question, given that you answered it correctly

$$P(p/A) = \frac{P(p \cap A)}{P(A)} = \frac{p}{p + \frac{1}{c}(1-p)}$$

1) A sample space consists of three events, A, B and C. If $P(A^c) = 0.5$, and $P(A \cap B) = 0.25$, $P(B \cup C) = 0.75$. The pair of events (A and B), (B and C) are independent. Events A and C are mutually exclusive. Find the followings:

a. Probability that exactly one event will occur.

b. P(B/A)

Solution:

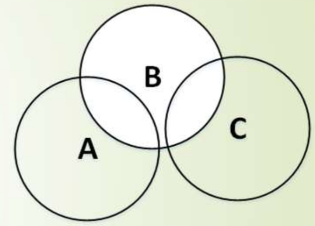
P(A)=1-0.5=0.5

P(B)=P(A∩B)/P(A)=0.25/0.5=0.5

*P(C)=P(B∪C)-P(B)+P(B)*P(C)*

*=0.75-0.5+0.5*PC*

=0.250.5=0.5



a) Probability that exactly one event will occur=

$$(A \cap \bar{B} \cap \bar{C}) \cup (B \cap \bar{A} \cap \bar{C}) \cup (C \cap \bar{B} \cap \bar{A})$$

Events A and C are mutually exclusive *A∩C'=A∩(1-C)=A,and A∩C=C*

=P(A∩B'∩C')+P(A'∩B∩C')+P(A'∩B'∩C)

= P(A∩B')+ P(A'∩B∩C')+P(B'∩C)

=P(A∩1-B)+P(1-A∩B∩1-C)+P(1-B∩C)

=P(A)-P(A∩B)+P(B)-P(A∩B)-P(B∩C)+P(A∩B∩C)+P(C)-P(B∩C)

= 0.5-0.25+0.5-0.25-0.25+0+0.5-0.25=0.5

b)

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{0.25}{0.5} = 0.5$$

1) Consider the following systems made up of independent components. The probability that each component functions is indicated in the figures.

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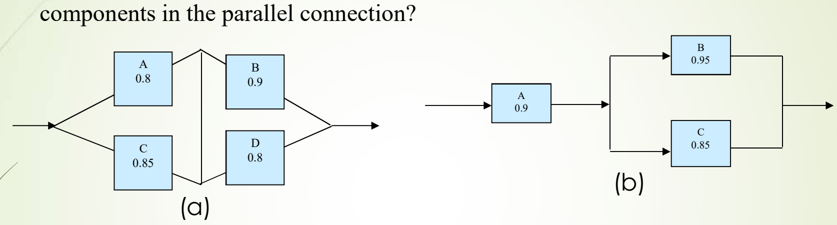

a) =P((Aa∪Ca)∩(Ba∪Da)∩Ab∩(Bb ∪Cb)

=P Aa∪CaP Ba∪DaPAbP(Bb ∪Cb)

= P Aa+P Ca-P AaP Ca P Ba+P Da-PBaP DaP AbP Bb+P Cb-P BbP Cb

= 0.8+0.85-0.8*0.85 0.9+0.8-0.9*0.80.90.95+0.85-0.95*0.85

=0.97*0.98*0.9*0.9925=0.85

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The probability that the three balls have the same color with replacement,

$$p = P(RRR \cup GGG) = \frac{5}{12} * \frac{5}{12} * \frac{5}{12} + \frac{7}{12} * \frac{7}{12} * \frac{7}{12} = \frac{13}{48}$$

The probability that the three balls have the same color are drawn simultaneously,

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Event C=A∪B P(C) = 0.7
Determine P ($8 \leq$ AMFR ≤ 10 cfs), the probability that AMFR is between 8 and 10 cfs.

Solution P ($8 \leq$ AMFR ≤ 10 cfs)=

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.6 + 0.6 - 0.7 = 0.5$$

Suppose that for married couples to vote in an election, the probability that a husband will vote is 0.7, the probability that a wife will vote is 0.5, and the probability a wife votes given her husband votes is 0.6. Find the probability that for randomly selected couple,

a. Both will vote.
b. Neither will vote.
c. Only one of them will vote.

H: Husband will vote , P(H)= 0.7
W: Wife will vote, P(W)= 0.5

$$P(W/H) = 0.6$$

Solution

a) Both will vote

$$P(W/H) = \frac{P(W \cap H)}{P(H)} = 0.6 = \frac{P(W \cap H)}{0.7}$$

$$P(W \cap H) = 0.42$$

b) Neither will vote.

$$P(\overline{W \cup H}) = 1 - P(W \cup H) = 1 - [P(W) + P(H) - P(W \cap H)]$$
$$1 - [0.7 + 0.5 - 0.42] = 0.22$$

c) Only one of them will vote.

$$P(W \cap \bar{H}) + P(\bar{W} \cap H) = P(W) + P(H) - 2P(W \cap H)$$
$$= 0.7 + 0.5 - 2 * 0.42 = 0.36$$

2.106 The probabilities that a service station will pump gas into 0, 1, 2, 3, 4, or 5 or more cars during a certain 30-minute period are 0.03, 0.18, 0.24, 0.28, 0.10, and 0.17, respectively. Find the probability that in this 30-minute period
(a) more than 2 cars receive gas;
(b) at most 4 cars receive gas;
(c) 4 or more cars receive gas.

Solution
(a) $0.28 + 0.10 + 0.17 = 0.55$.
(b) $1 - 0.17 = 0.83$.
(c) $0.10 + 0.17 = 0.27$.

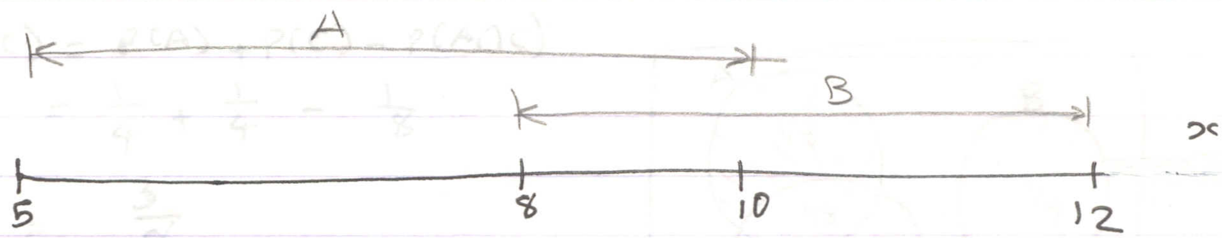
2.108 If the probability is 0.1 that a person will make a mistake on his or her state income tax return, find the probability that
(a) four totally unrelated persons each make a mistake;
(b) Mr. Jones and Ms. Clark both make mistakes,
and Mr. Roberts and Ms. Williams do not make a mistake.

(a) $P(M_1 \cap M_2 \cap M_3 \cap M_4) = (0.1)^4 = 0.0001$, where M_i represents that ith person make a mistake.
(b) $P(J \cap C \cap R' \cap W') = (0.1)(0.1)(0.9)(0.9) = 0.0081$.

Dr. Wael
Hashlamoun

Problem Set (1)
H.W #1

9)



$A = \{ 5 \le x \le 10 \}$ $\qquad P(A) = 0.6$

$B = \{ 8 \le x \le 12 \}$ $\qquad P(B) = 0.6$

$C = A \cup B$ $\qquad P(C) = 0.7$

$C = \{ 5 \le x \le 12 \}$

Note that $8 \le x \le 10 = A \cap B$

$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.6 + 0.6 - P(A \cap B)$

$P(A \cap B) = 1.2 - 0.7 = 0.5$

15) $f(x) = x e^{-x} \qquad x > 0$

$P(A) = P(x > 1.5)$

$$= \int_{1.5}^{\infty} x e^{-x} \, dx$$

$$= 2.5 e^{-1.5} = 0.556$$

10) $P(A) = P(B) = P(C) = 1/4$

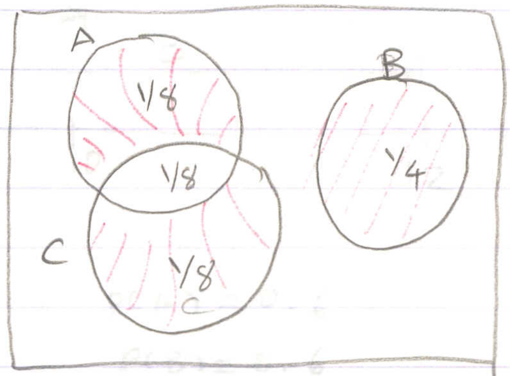
$P(A \cap B) = P(C \cap B) = 0$

$P(A \cap C) = 1/8$

$P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$= \frac{1}{4} + \frac{1}{4} - \frac{1}{8}$

$= \frac{3}{8}$



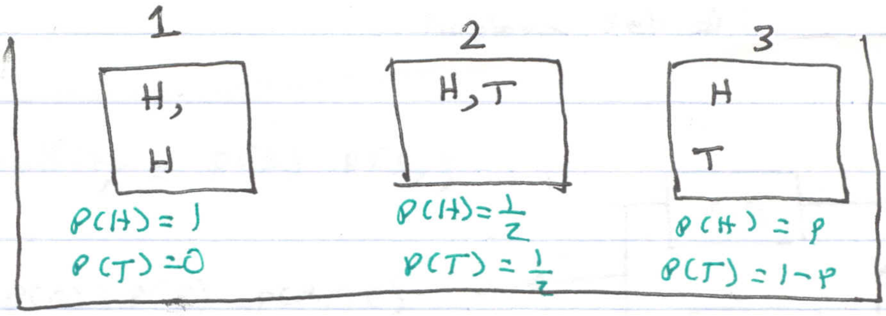
a. $P(\text{at least } 1) = P(A \cup C) + P(B)$

$= \frac{3}{8} + \frac{1}{4} = \frac{5}{8}$

b. $P(A \cup B \cup C) = \frac{5}{8}$

c. $P(\text{exactly one event}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$

11)



a. $P(H) = P(1)\,P(H/1) + P(2)\,P(H/2) + P(3)\,P(H/3)$

$= \frac{1}{3} \times 1 + \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times p$

$P(H) = \frac{1}{3}\left(1 + \frac{1}{2} + p\right) = \frac{1}{2} + \frac{1}{3}p$

b. $P(3/H) = \frac{P(3 \cap H)}{P(H)} = \frac{P(3)\,P(H/3)}{P(H)}$

$P(3/H) = \frac{\frac{1}{3} \times p}{\frac{1}{3}\left(1 + \frac{1}{2} + p\right)} = \frac{p}{1 + \frac{1}{2} + p}$

$\frac{p}{\frac{3}{2} + p}$

13) $P(S) = p$

$P(F) = 1 - p$

$P(5 \text{ trials}) = P(FFFFS)$; experiment is terminated when a success is obtained

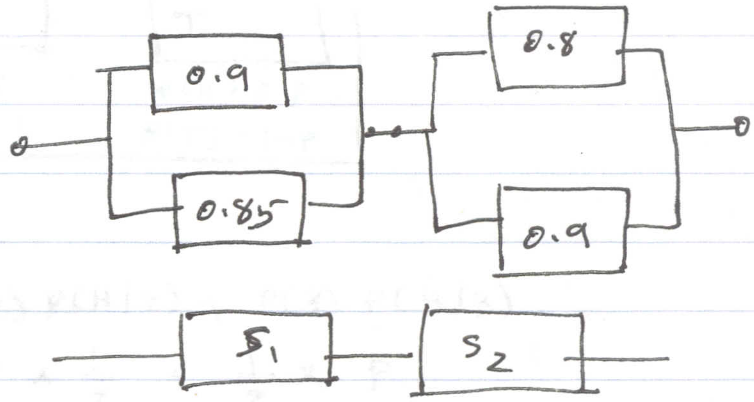
$= P(F)^4\,P(S)$; because of independence

$= [1 - p]^4\,p$

H.W #2

Problem set #1

a. Reliability = $P(S_1)\,P(S_2)$



$P(S_1) = P(A) + P(C) - P(A \wedge C)$

$P(S_1) = 0.9 + 0.85 - 0.9 \times 0.85$

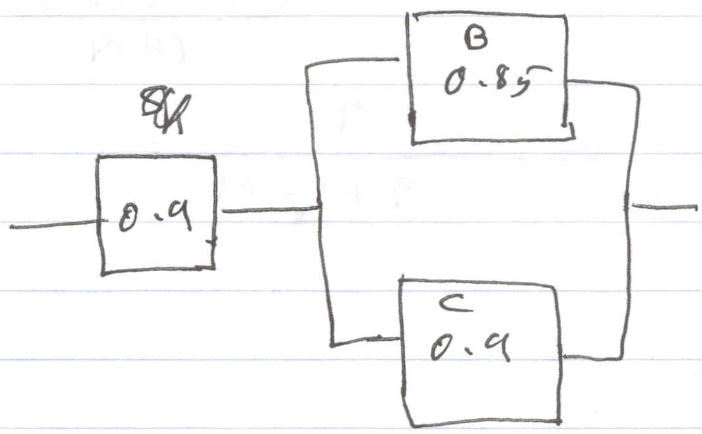
$P(S_1) = 0.985$

$P(S_2) = P(B) + P(D) - P(B \wedge D)$

$P(S_2) = 0.8 + 0.9 - 0.8 \times 0.9$

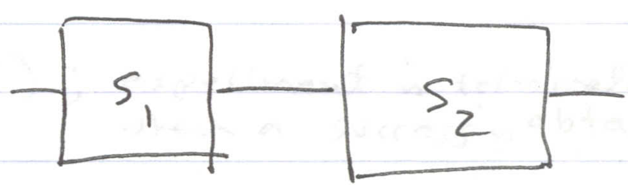
$P(S_2) = 0.98$

$R = 0.9653$



b. Reliability

$R = P(S_1)\,P(S_2)$



$P(S_1) = 0.9$

$P(S_2) = P(B) + P(C) - P(B \wedge C)$

$= 0.85 + 0.9 - 0.85 * 0.9$

$= 0.985$

$R = 0.9 \times P(S_2)$ cannot be > 0.995

If so, then $P(S_2) = \dfrac{0.995}{0.9} > 1$ Impossible

$\Rightarrow$ Impossible to increase R up to 0.995.