Numbering Systems

Lecture 1

Comp 230

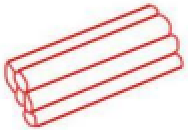
Outline

- History.
- Decimal System.
- Binary System.
- Octal System.
- Hexadecimal System.
- Converting from one System to another system and back.
- Binary Addition
- Signed Numbers
- Summary

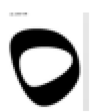
History

- Long ago, humans used sticks to count.

- Later learned how to draw pictures of sticks in the ground and eventually on paper.



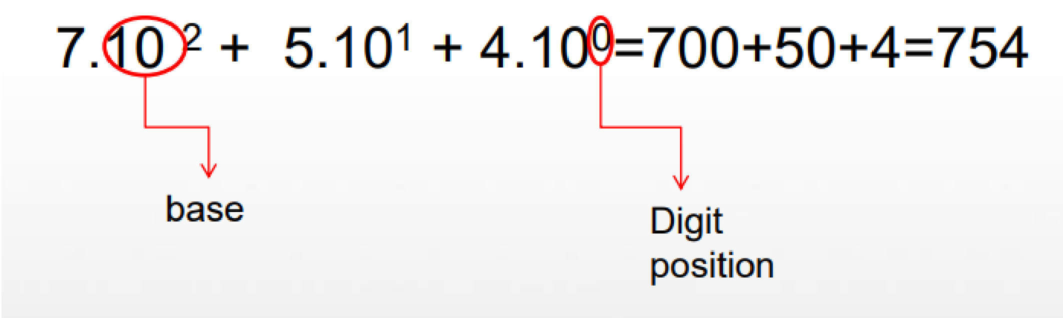
- Using symbols to represent the numbers instead of sticks.

5 V 

Decimal System

- Most People Use decimal representation to count.
- In decimal there are 10 digits

  0,1,2,3,4,5,6,7,8,9
- The base is 10
- We can Represent any value for these digits
- Ex: 754 , 123 , 889 , 345



position

123_{10} =>	3×10^0	=	3
	2×10^1	=	20
	1×10^2	=	100
			123

Base

Binary System

- Each digit in binary number called BIT.
- 1 0 1 0 , 4 digits , How many bits ?

  answer : 4 bits
- 4 bits form a NIBBLE.
- 8 bits form a byte.
- 1 0 1 0 0 0 1 1, How many Bits, Nibbles and Bytes?

  Answer :8 bits ,2 Nibbles and 1 byte
- Two bytes form a WORD and two words form a DOUBLE WORD (rarely used) .
- EX: 0000 1111 1010 1010 : 16 bits , WORD

Octal System

- Uses 8 digits

  0,1,2,3,4,5,6,7

- The base is 8

- EX: (123)8 , (156)8

Hexadecimal System

- Uses 16 digits

  0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

- The base is 16

- EX: 123h , 456h 0E120h

Number Systems

Number System	Base	Symbol
Binary	Base 2	B
Octal	Base 8	O
Decimal	Base 10	D
Hexadecimal	Base 16	H

Quantities/Counting

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

Quantities/Counting

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Binary to Decimal

- 10110_b
- $1*2^4 + 0*2^3 + 1*2^2 + 1*2^1 + 0*2^0 = 16+0+4+2 = (22)_{10}$
- 1010b =?? , 0010b = ?? , 101b=??
- Answer: $1010b=(10)_{10}$

  $0010b=(2)_{10}$

  $101b=(5)_{10}$

Decimal to Binary

$(22)_{10} = (\ )_2$

Input	Result	Remainder
22/2	11	0
11/2	5	1
5/2	2	1
2/2	1	0
1/2	0	1

$(22)_{10} = (10110)_2$

Decimal to Binary

- $(13)_{10} = (1101)_2$
- $(220)_{10} = (11011100\ )_2$
- $(21)_{10} = (\ )_2$ H.W
- $(15)_{10} = (\ )_2$ H.W

Binary to Octal

- 100101010b = ($)_8$
- 100 101 010 = $(452)_8$

- 111000111b= ($)_8$
- 111 000 111 = $(707)_8$

Binary to Octal

- $100101011_b = (453)_8$
- $101101011_b = (\ )_8$ H.W
- $100101001_b = (\ )_8$ H.W

Binary to Hexadecimal

- $10010101_b = (\ )_h$

  $1001\ 0101 = (95)_h$

- $11100011_b = (E3)_h$

Decimal to Hexadecimal

Let's convert the value $(39)_{10}$ to Hexadecimal

Input	Result	Remainder
39/16	2	7 ↑
2/16	0	2

$(39)_{10}$ = (27h)

H.W

- Covert the following numbers to decimal

  a. $(72)_8 = (58)_{10}$

  b. $(72)_{16} = (114)_{10}$

  c. $(DE1)_{16} = (3553)_{10}$

Extra Exercises

- Using pen and paper , solve the following questions :
  a. (AB) 16 =( ) 2
  b. (23) 4 =( ) 8
  c. (35) 7=( ) 8
  d. (72E) 16 =( ) 8

Binary Addition

```
   0        0       1       1
 + 0      + 1     + 0     + 1
--------  -------  ------  ------
   0        1       1      10


   1
 + 1
 + 1
---------
  11
```

Binary Addition

01111+00110 =

```
  1   1 1
  0 1 1 1 1
+ 0 0 1 1 0
--------------
  1 0 1 0 1
```

Binary Addition

11010011+01010110=

```
       1   1   1  1
       1 1 0 1  0  0  1  1      →(211)10
  +    0 1 0 1  0 1   1   0     →(86)10
       ------------------------
     1 0 0  1  0 1  0   0  1    = (297)10
```

Binary Addition

H.W

Solve Question 7 ,lab 1 ,page 9

Signed Numbers

- Our study of binary arithmetic, we have only considered positive numbers.
- What about negative numbers?

Signed Numbers

- Signed Magnitude add an extra digit to the front of our binary number to indicate whether the number is positive or negative.
- this digit called sign bit.
- o for positive
- 1 for negative

Signed Numbers

Example:

(5) 10 = (101) 2

Positive 5 is 0 1 0 1
Negative 5 is 1 1 0 1

The Problem : We need to specify how many bits in our numbers so we can be certain which bit is representing the sign !!

Signed Numbers

1 1 0 1 is 13 or -5

One’s Complement

Representing a signed number with 1’s Complement is done by changing all the bits that are 1 to 0 and all bits that are 0 to 1.

Signed Numbers

- Represent -5 in 1's complement by using 4-bit arithmetic?

  0101 -> 1010

- Represent -1 in 1's complement ?

  0001 -> 1110

Signed Numbers

- Two's Complement

  2's comp=1'comp +1

- Represent -5 in 2's complement by using 4-bit arithmetic?

  (101)1's -> 1010

  2's + 1

  ----

  1 0 1 1 =(-5)

H.W

Lab 1 . P8,9

Q.1,2,3,4,8,10

Summary

- Decimal System.
- Binary System.
- Octal System.
- Hexadecimal System.
- Converting from one System to another system and back.
- Binary Addition
- Signed Number

Octal to Decimal

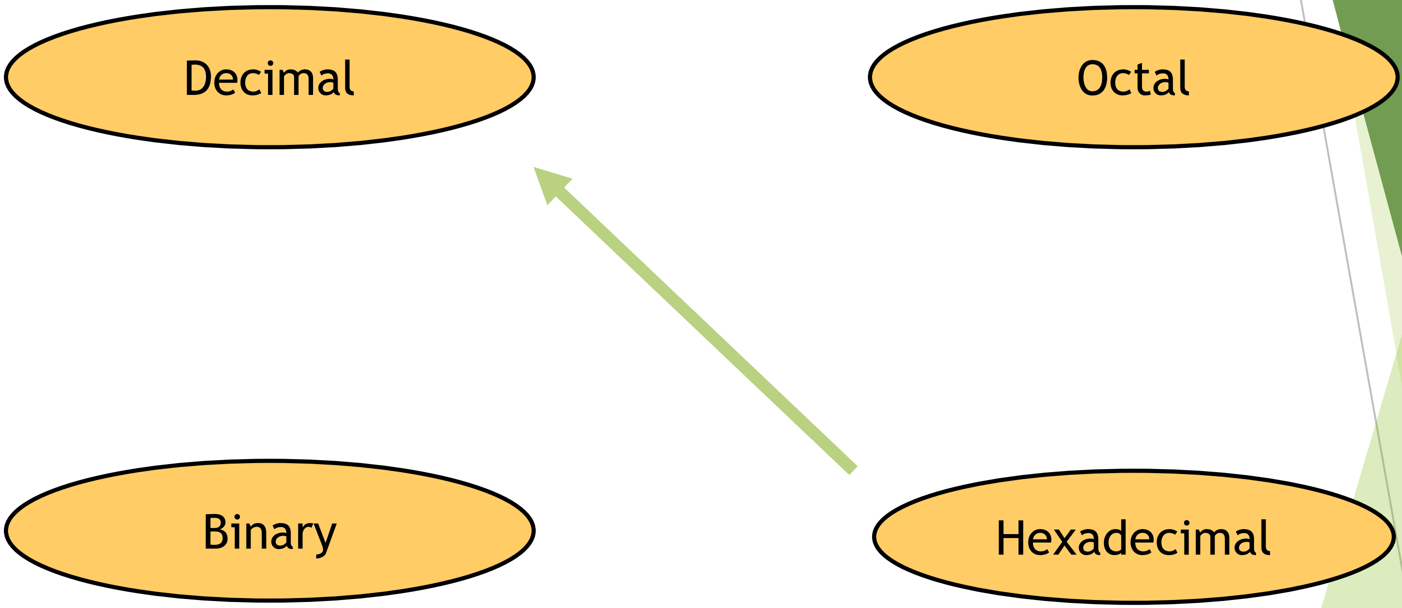
- Technique
  - Multiply each bit by 8^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

$724_8 \Rightarrow$

	$4 \times 8^0 =$	4
	$2 \times 8^1 =$	16
	$7 \times 8^2 =$	448
		468_{10}

Hexadecimal to Decimal



Hexadecimal to Decimal

- Technique
  - Multiply each bit by 16^n, where n is the "weight" of the bit
  - The weight is the position of the bit, starting from 0 on the right
  - Add the results

Example

ABC_{16} =>

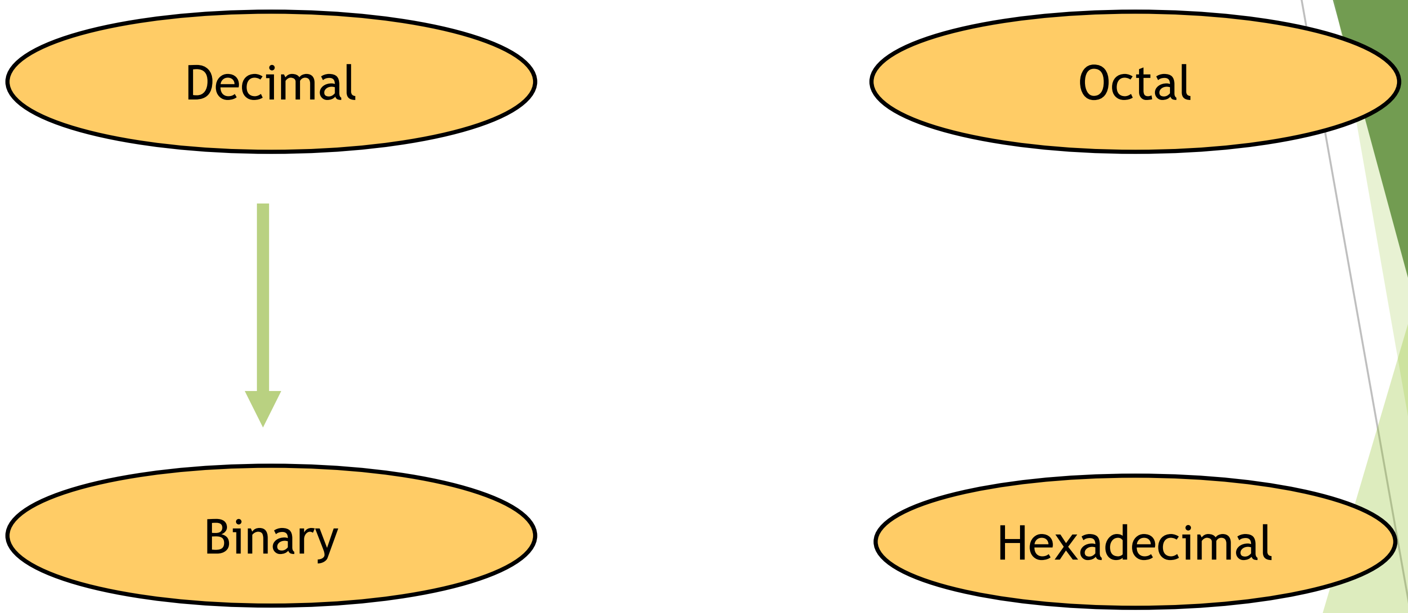
$C \times 16^0 = 12 \times 1 = 12$

$B \times 16^1 = 11 \times 16 = 176$

$A \times 16^2 = 10 \times 256 = 2560$

2748_{10}

Decimal to Binary



Decimal to Binary

- Technique
  - Divide by two, keep track of the remainder
  - First remainder is bit 0 (LSB, least-significant bit)
  - Second remainder is bit 1
  - Etc.

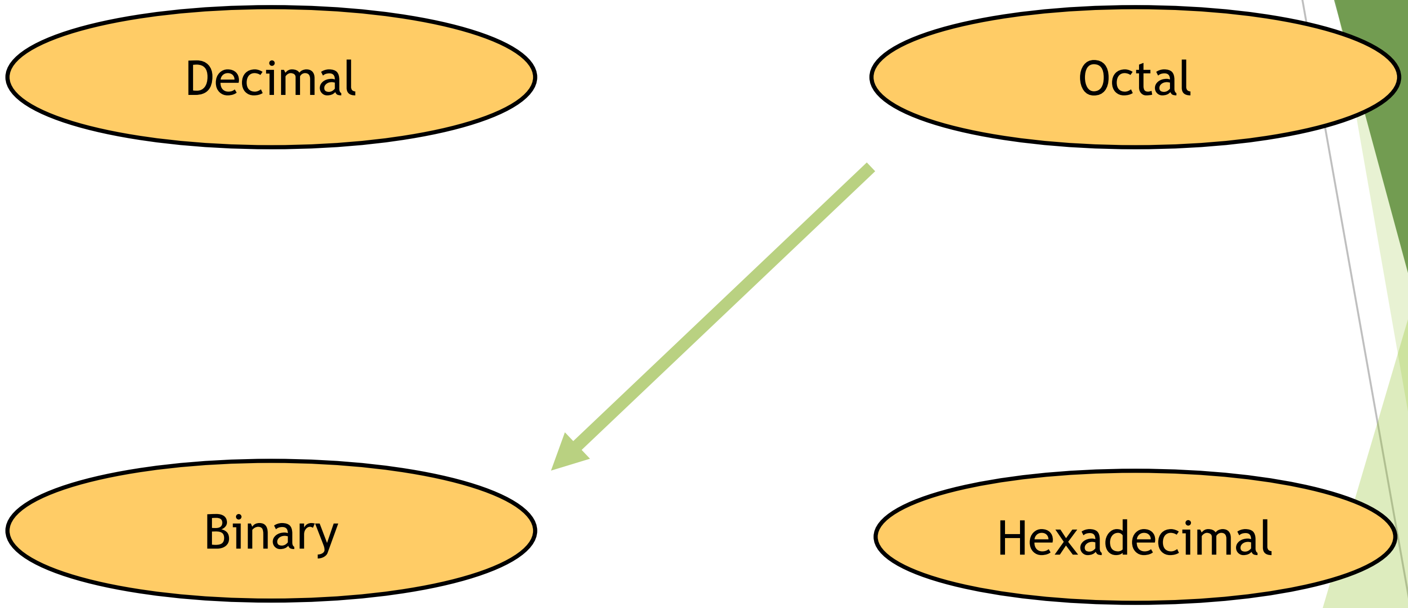
Example

$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$125_{10} = 1111101_2$

Octal to Binary



Octal to Binary

- Technique
  - Convert each octal digit to a 3-bit equivalent binary representation

Example

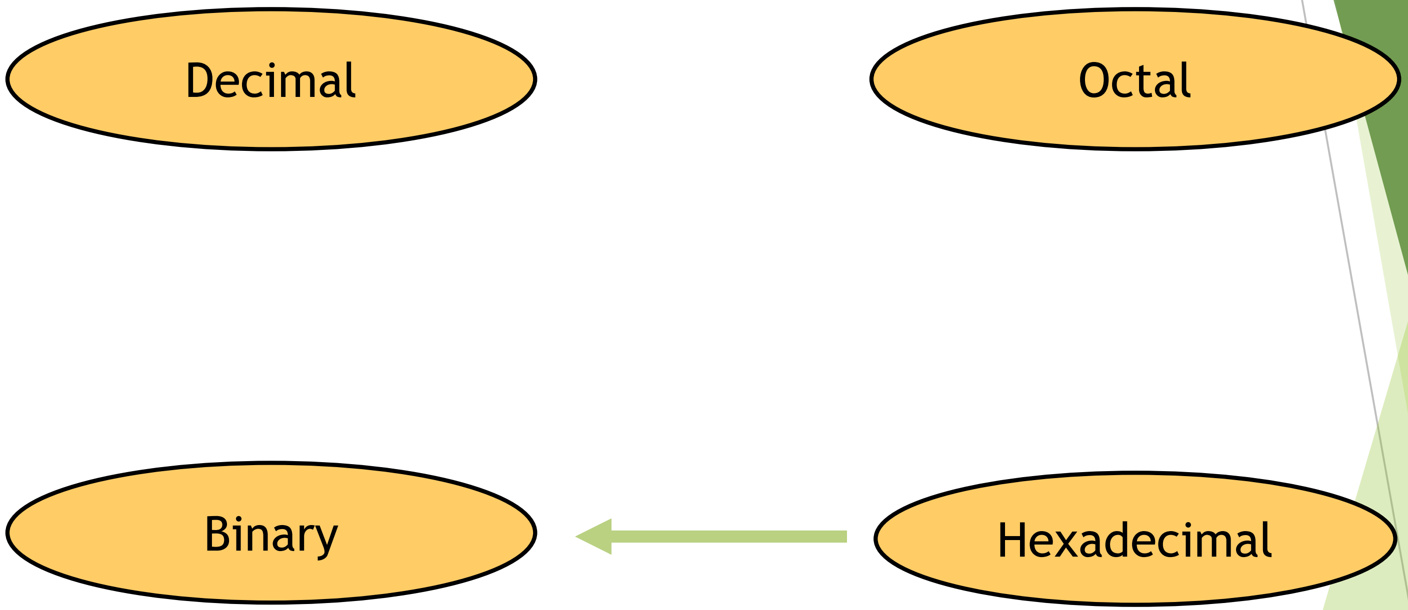
$705_8 = ?_2$

7 0 5

111 000 101

$705_8 = 111000101_2$

Hexadecimal to Binary



Hexadecimal to Binary

- Technique
  - Convert each hexadecimal digit to a 4-bit equivalent binary representation

Example

$10AF_{16} = ?_2$

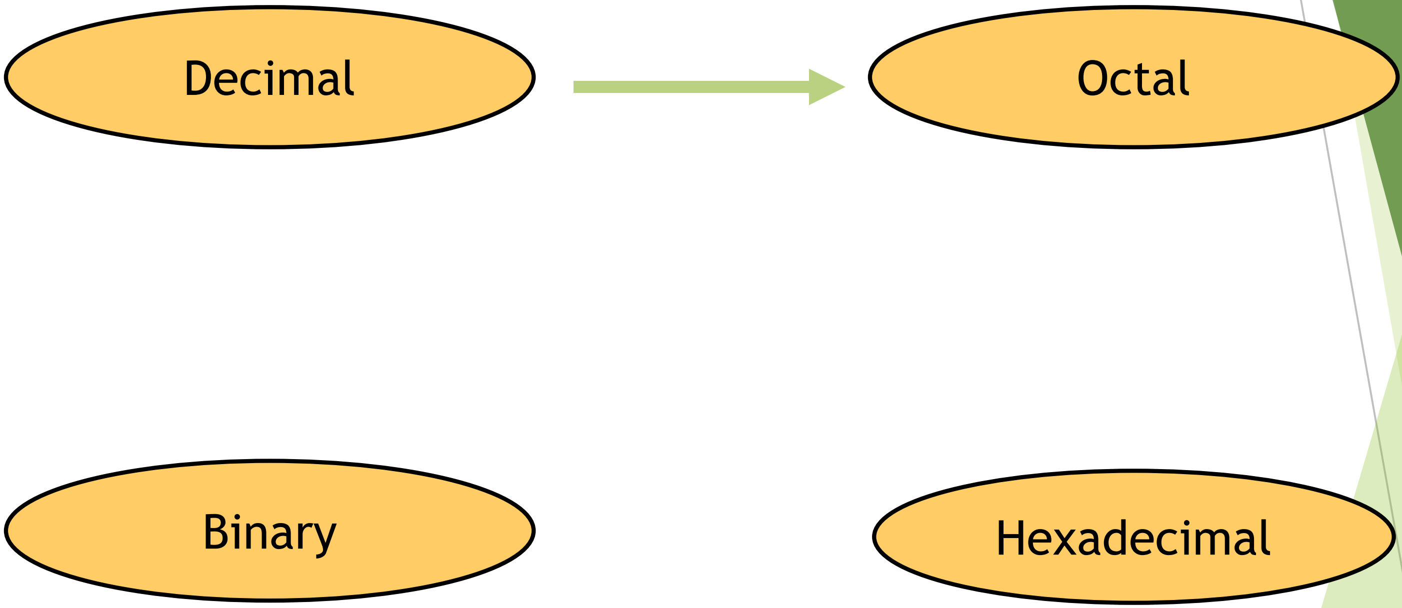
1 0 A F

↓ ↓ ↓ ↓

0001 0000 1010 1111

$10AF_{16} = 0001000010101111_2$

Decimal to Octal



Decimal to Octal

- Technique
  - Divide by 8
  - Keep track of the remainder

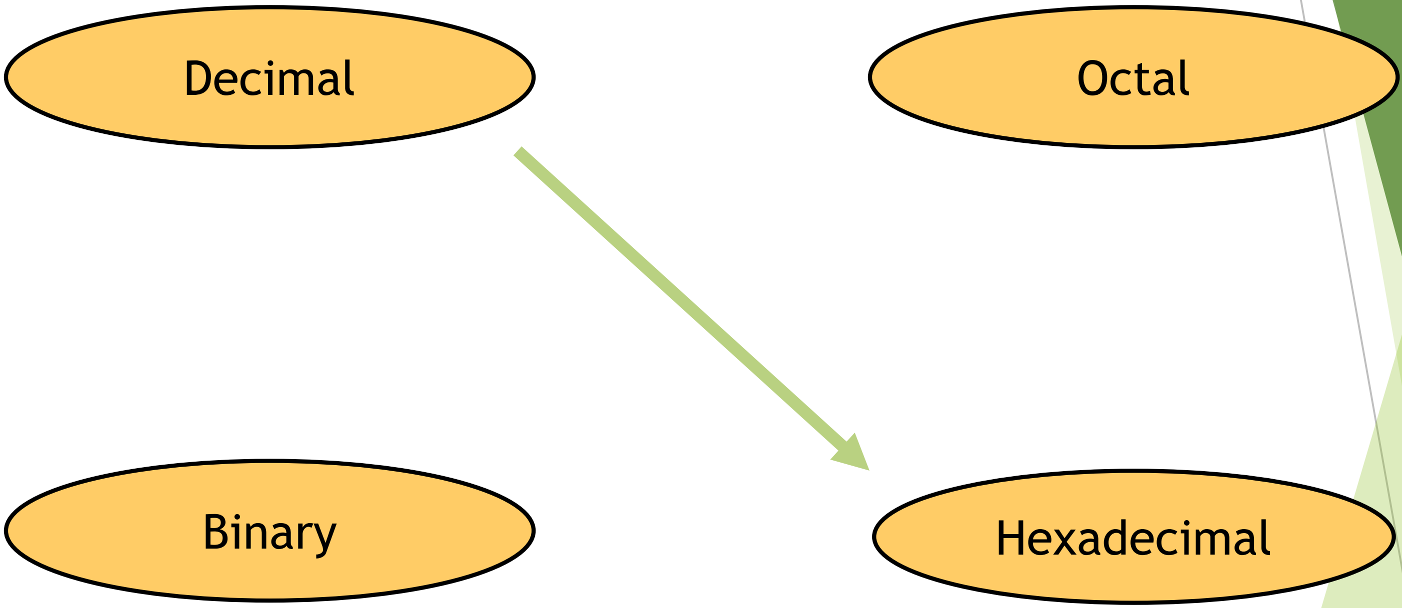
Example

$1234_{10} = ?_8$

8	1234	
8	154	2
8	19	2
8	2	3
	0	2

$1234_{10} = 2322_8$

Decimal to Hexadecimal



Decimal to Hexadecimal

- Technique
  - Divide by 16
  - Keep track of the remainder

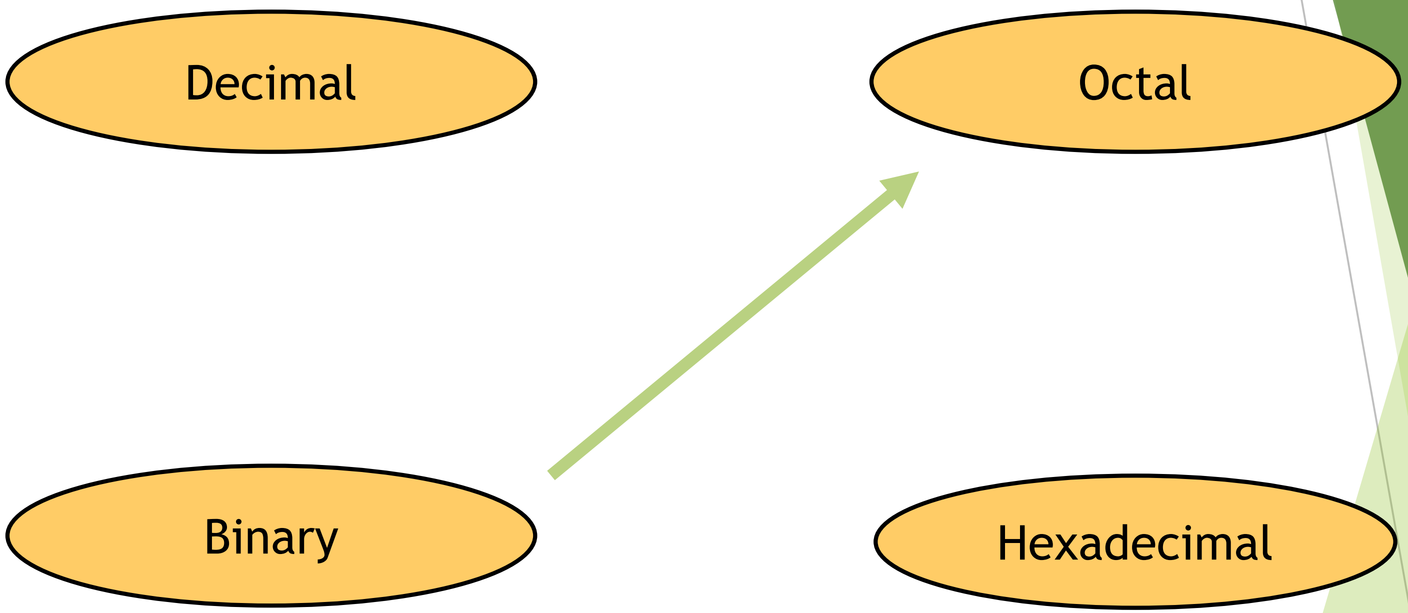
Example

$1234_{10} = ?_{16}$

16 | 1234
16 | 77   2
16 | 4   13 = D
       0   4

$1234_{10} = 4D2_{16}$

Binary to Octal



Binary to Octal

- Technique
  - Group bits in threes, starting on right
  - Convert to octal digits

Example

$1011010111_2 = ?_8$

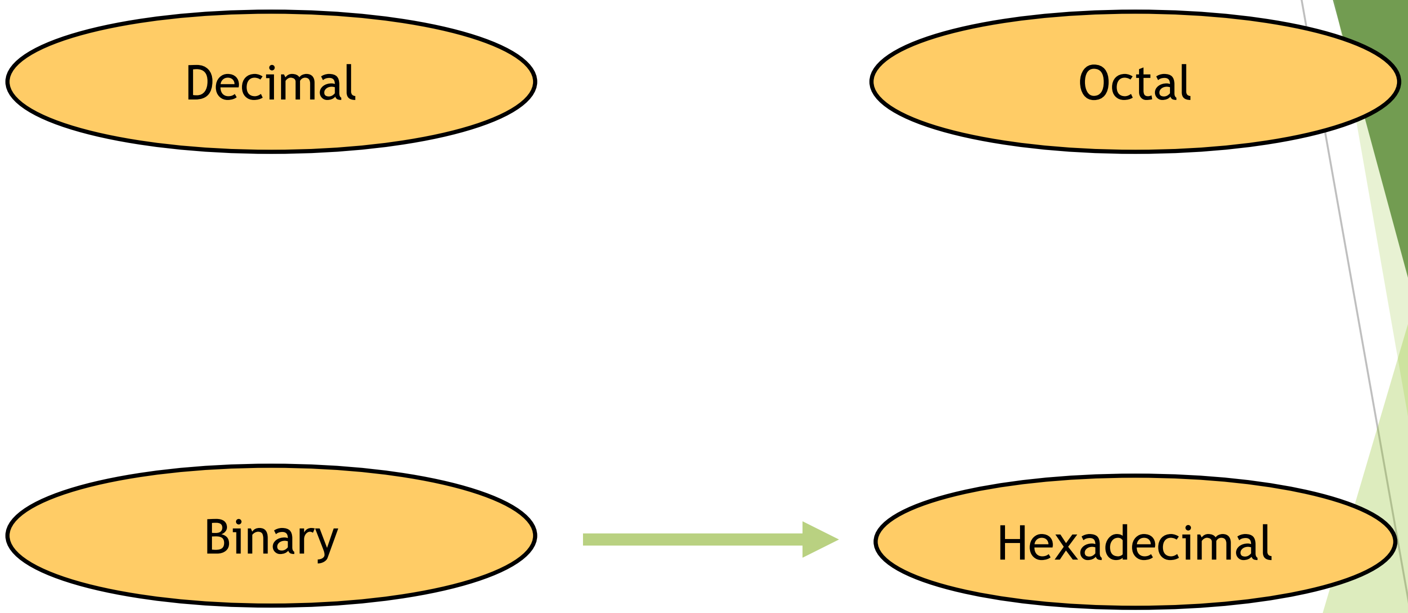
1 011 010 111

↓ ↓ ↓ ↓

1 3 2 7

$1011010111_2 = 1327_8$

Binary to Hexadecimal



Binary to Hexadecimal

- Technique
  - Group bits in fours, starting on right
  - Convert to hexadecimal digits

Example

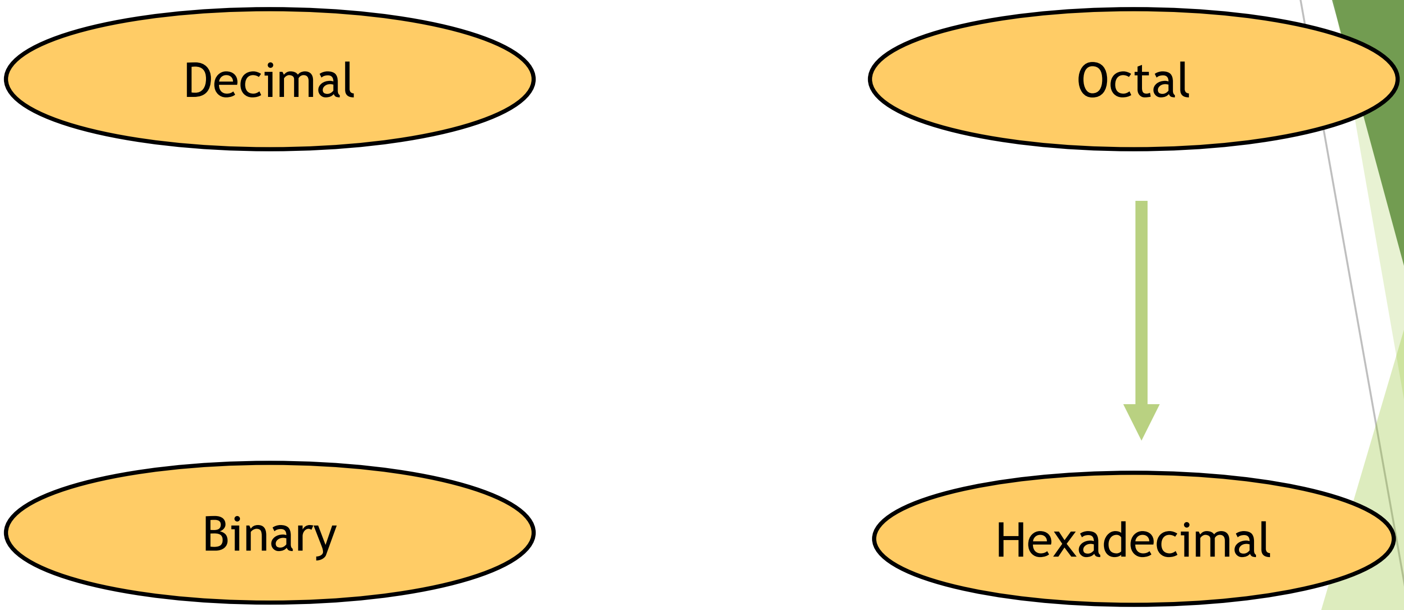
$1010111011_2 = ?_{16}$

10 1011 1011

2 B B

$1010111011_2 = 2BB_{16}$

Octal to Hexadecimal



Octal to Hexadecimal

- Technique
  - Use binary as an intermediary

Example

$1076_8 = ?_{16}$

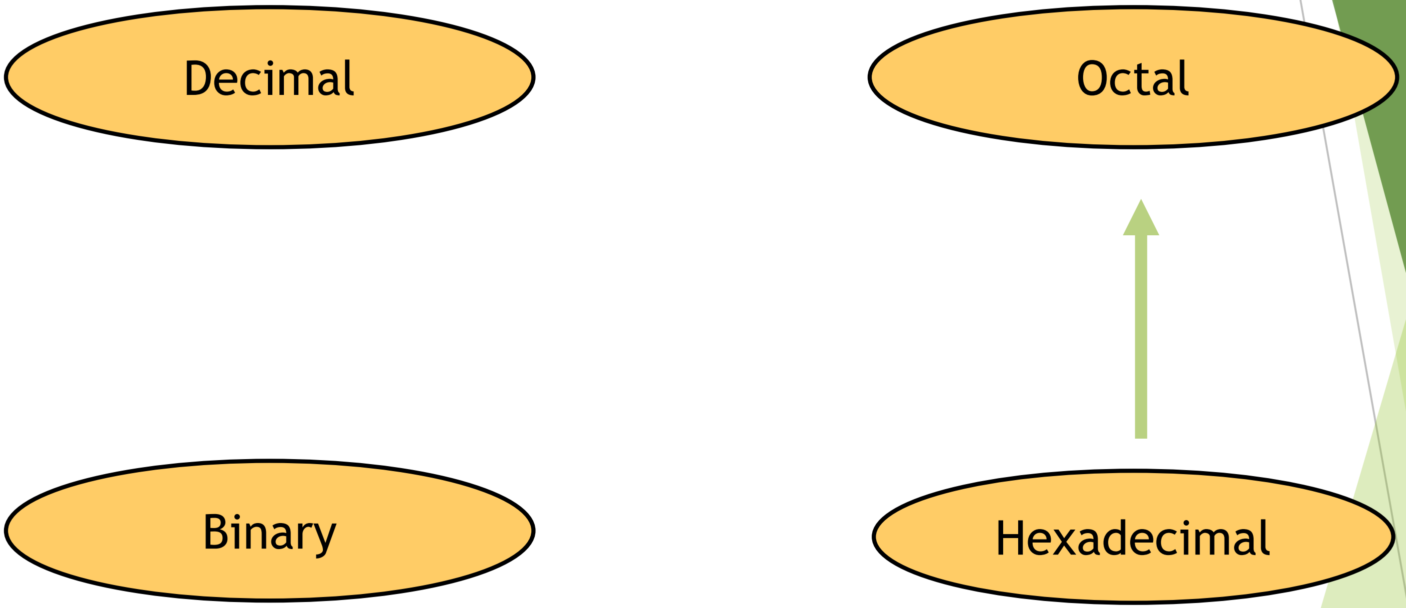
1 0 7 6

001 000 111 110

2 | 3 | E

$1076_8 = 23E_{16}$

Hexadecimal to Octal



Hexadecimal to Octal

- Technique
  - Use binary as an intermediary

Example

$1F0C_{16} = ?_8$

1	F	0	C
0001	1111	0000	1100

1	7	4	1	4

$1F0C_{16} = 17414_8$

Exercise – Convert ...

Decimal	Binary	Octal	Hexa-decimal
33			
	1110101		
		703	
			1AF

Exercise – Convert …

Decimal	Binary	Octal	Hexa-decimal
33	100001	41	21
117	1110101	165	75
451	111000011	703	1C3
431	110101111	657	1AF

Common Powers (1 of 2)

- Base 10

Power	Preface	Symbol	Value
10^{-12}	pico	p	.000000000001
10^{-9}	nano	n	.000000001
10^{-6}	micro	µ	.000001
10^{-3}	milli	m	.001
10^{3}	kilo	k	1000
10^{6}	mega	M	1000000
10^{9}	giga	G	1000000000
10^{12}	tera	T	1000000000000

Common Powers (2 of 2)

- Base 2

Power	Preface	Symbol	Value
2^{10}	kilo	k	1024
2^{20}	mega	M	1048576
2^{30}	Giga	G	1073741824

- In computing, particularly w.r.t. <u>memory</u>, the base-2 interpretation generally applies

The End