8.4 Integration by Partial fraction 18

Saturday, June 5, 2021 7:28 AM

8.4	**20, 29, 33**	**12, 14, 17, 25, 30, 34, 42, 47**

12 $\displaystyle\int \frac{2x+1}{x^2-7x+12}\,dx = \int \frac{2x+1}{(x-4)(x-3)}\,dx$

$\displaystyle = \int \left(\frac{\overset{9}{A}}{x-4} + \frac{\overset{-7}{B}}{x-3} \right) dx$

4 3

Different linear factors

Cover Method

$A = \frac{2\boxed{4}+1}{\boxed{4}-3} = 9$

$B = \frac{2\boxed{3}+1}{\boxed{3}-4} = -7$

$\displaystyle = \int \frac{9}{x-4}\,dx + \int \frac{-7}{x-3}\,dx$

$= 9\ln|x-4| - 7\ln|x-3| + C$

$= \ln|x-4|^9 - \ln|x-3|^7 + C$

$= \ln\left|\frac{(x-4)^9}{(x-3)^7}\right| + C$

$$\boxed{18}\quad \int_{-1}^{0} \frac{x^3}{x^2-2x+1}\,dx$$

درجة البسط ≥ درجة المقام

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قسمة طويلة

Long Division

$$\begin{array}{r} x+2 \\ x^2-2x+1 \overline{)\; x^3 \qquad\qquad} \\ -x^3+2x^2 \mp x \\ \hline 2x^2-x \\ -2x^2+4x\mp 2 \\ \hline 3x-2 \end{array}$$

$$\int_{-1}^{0}\left(x+2+\frac{3x-2}{x^2-2x+1}\right)dx$$

$$\int_{-1}^{0}(x+2)\,dx+\int_{-1}^{0}\frac{3x-2}{(x-1)^2}\,dx$$

$$\left(\frac{x^2}{2}+2x\right)\Big|_{-1}^{0}+\int_{-1}^{0}\left(\frac{\overset{3}{A}}{x-1}+\frac{\overset{1}{B}}{(x-1)^2}\right)dx$$

تكرار

$$(0+0)-\left(\frac{1}{2}-2\right)$$

$$0-\frac{-3}{2}$$

$$\frac{3}{2}$$

$$\frac{3x-2}{(x-1)^2}=\frac{A(x-1)}{(x-1)^2}+\frac{B}{(x-1)^2}$$

$$=\frac{A(x-1)+B}{(x-1)^2}$$

$$3x-2=A(x-1)+B=Ax-A+B$$

$$\boxed{A=3},\quad -2=-A+B$$

$$-2=-3+B$$

$$\boxed{1=B}$$

$$\int\frac{3x-2}{(2x^2-1)^2}$$ تكرار

$$=\int\frac{Ax+B}{2x^2-1}+\frac{Cx+D}{(2x^2-1)^2}$$

$$\boxed{1 = B}$$

$$\frac{3}{2} + \int_{-1}^{0} \frac{3}{x-1}\,dx + \int_{-1}^{0} \frac{1}{(x-1)^2}\,dx$$

$u = x-1$

$du = dx$

$x = -1 \Rightarrow u = -2$

$x = 0 \Rightarrow u = -1$

$$\frac{3}{2} + 3\ln|x-1| \Big|_{-1}^{0} + \int_{-2}^{-1} \frac{du}{u^2}$$

$\int u^{-2}\,du$

$\frac{u^{-1}}{-1}$

$\frac{-1}{u}$

$$\frac{3}{2} + 3\ln 1 - 3\ln 2 + \frac{-1}{u}\Big|_{-2}^{-1}$$

$\frac{-1}{-1} - \left(\frac{-1}{-2}\right)$

$1 - \frac{1}{2}$

$\frac{1}{2}$

$$\frac{3}{2} + \frac{1}{2} + 0 - 3\ln 2$$

$$2 - 3\ln 2$$

4/2 $\quad \int \frac{\sin\theta\, d\theta}{\cos^2\theta + \cos\theta - 2}$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$-\int \frac{du}{u^2 + u - 2} = -\int \frac{du}{(u+2)(u-1)}$$

$u^2 + u - 2 \qquad (u+2)(u-1)$

$$A = \frac{-1}{\boxed{-2}-1} = \frac{1}{3}$$

$$B = \frac{-1}{\boxed{1}+2} = -\frac{1}{3}$$

$$= -\int \left(\frac{A}{u+2} + \frac{B}{u-1} \right) du$$

$$= -\left[\int \frac{\frac{1}{3}}{u+2} du + \int \frac{-\frac{1}{3}}{u-1} du \right]$$

$$= -\frac{1}{3} \ln |u+2| + \frac{1}{3} \ln |u-1| + C$$

$$= -\frac{1}{3} \ln |\cos\theta + 2| + \frac{1}{3} \ln |\cos\theta - 1| + C$$

$$= \frac{1}{3} \left[\ln|\cos\theta - 1| - \ln|\cos\theta + 2| \right] + C$$

$$= \frac{1}{3} \ln \left| \frac{\cos\theta - 1}{\cos\theta + 2} \right| + C$$