

8.1 Integration By Parts:

173

If u and v are diff function of x , then

$$\int u dv = u \cdot v - \int v du$$

Evaluate the following integral:-

1 $\int x e^{3x} dx$

$$u = x$$

$$du = dx$$

$$dv = e^{3x} dx$$

$$v = \frac{e^{3x}}{3}$$

$$\begin{aligned} \int x e^{3x} dx &= \frac{x e^{3x}}{3} - \int \frac{e^{3x}}{3} dx \\ &= \frac{x e^{3x}}{3} - \frac{e^{3x}}{9} + C \end{aligned}$$

2 $\int x^7 \ln x dx$

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$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$dv = x^7 dx$$

$$v = \frac{x^8}{8}$$

$$\begin{aligned} \int x^7 \ln x dx &= \frac{x^8}{8} \ln x - \int \frac{x^7}{8} dx \\ &= \frac{x^8}{8} \ln x - \frac{1}{8} \frac{x^8}{8} + C \end{aligned}$$

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3 $\int \ln x \, dx$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$dv = dx$$

$$v = x$$

$$\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} dx$$

$$\int \ln x \, dx = x \ln x - x + C$$

4 $\int \frac{\ln x}{\sqrt{x}} dx = \int \ln x (x)^{-1/2} dx$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$dv = x^{-1/2} dx$$

$$v = 2x^{1/2}$$

$$= 2x^{1/2} \ln x - \int 2x^{1/2} \cdot \frac{1}{x} dx$$

$$= 2x^{1/2} \ln x - 4x^{1/2} + C$$

5 $\int 3x^2 \tan^{-1} x \, dx$

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$$u = \tan^{-1} x$$

$$du = \frac{1}{1+x^2} dx$$

$$dv = 3x^2 dx$$

$$v = \frac{3x^3}{3}$$

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$$\int 3x^2 \tan^{-1} x \, dx = x^3 \tan^{-1} x - \int \frac{x^3}{1+x^2} dx$$

$$= x^3 \tan^{-1} x - \int x \cdot \frac{1.2x}{1+x^2} dx$$

$$= x^3 \tan^{-1} x - \frac{x^2}{2} + \frac{1}{2} \ln|1+x^2| + C$$

$$\frac{x}{1+x^2} = \frac{x^3 + x}{x^3 + x - x} = \frac{x^3 + x}{-x}$$

6

$$\int \sin^{-1} x \, dx$$

175

$$u = \sin^{-1} x$$

$$dv = dx$$

$$du = \frac{1}{\sqrt{1-x^2}} dx$$

$$v = x$$

$$\int \sin^{-1} x \, dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$u = 1-x^2$$

$$du = -2x dx$$

$$= x \sin^{-1} x + \frac{1}{2} \int \frac{1}{\sqrt{u}} du$$

$$= x \sin^{-1} x + \frac{2}{2} u^{1/2} + C$$

$$= x \sin^{-1} x + \sqrt{1-x^2} + C$$

7

$$\int x^2 \cosh 2x \, dx$$

$$u = x^2$$

$$du = 2x dx$$

$$dv = \cosh 2x \, dx$$

$$v = \frac{\sinh(2x)}{2}$$

$$\int x^2 \cosh 2x \, dx = \frac{x^2 \sinh(2x)}{2} - \int x \sinh(2x) \, dx$$

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$$u = x$$

$$du = dx$$

$$dv = \sinh(2x) \, dx$$

$$v = \frac{\cosh(2x)}{2}$$

$$= \frac{x^2 \sinh(2x)}{2} - \frac{x \cosh(2x)}{2} + \int \frac{\cosh(2x)}{2} dx$$

$$= \frac{1}{2} x^2 \sinh(2x) - \frac{1}{2} x \cosh(2x) + \frac{1}{4} \sinh(2x) + C$$

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Remark:

176

If n is positive integer. We use the tabular integration

in the following cases:-

$$\int x^n \cosh(ax) dx$$

$$\int x^n \sinh(ax) dx$$

$$\int x^n \cos(ax) dx$$

$$\int x^n \sin(ax) dx$$

$$\int x^n e^{ax} dx$$

$$\int x^n a^x dx$$

Exp:- $\int \cosh(2x) x^2 dx$

Derivative

Integral

$$x^2$$

$$\cosh(2x)$$

$$2x$$

$$\frac{\sinh(2x)}{2}$$

$$2$$

$$\frac{\cosh(2x)}{4}$$

$$0$$

$$\frac{\sinh(2x)}{8}$$

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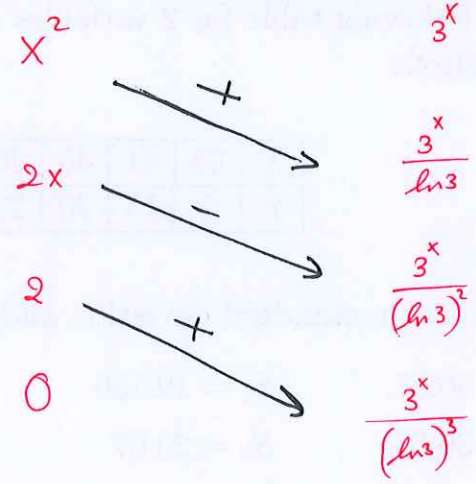
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$$\int x^2 \cosh(2x) dx = \frac{1}{2} x^2 \sinh(2x) - \frac{1}{2} x \cosh(2x) + \frac{1}{4} \sinh(2x) + C$$

8 $\int x^2 3^x dx$

Derivative

Integral



$$\int x^2 3^x dx = \frac{3^x x^2}{\ln 3} - \frac{2x 3^x}{(\ln 3)^2} + \frac{2 \cdot 3^x}{(\ln 3)^3}$$

Remark

Some times we use the Substitution and then we use integration by Parts.

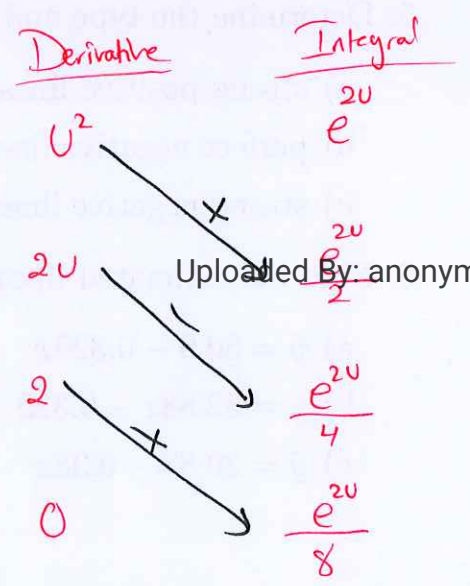
9 $\int x (\ln x)^2 dx$

$u = \ln x$
 $du = \frac{1}{x} dx$

$$\int x^2 u^2 du = \int e^{2u} u^2 du$$

STUDENTS-HUB.COM $= \frac{1}{2} u^2 e^{2u} - \frac{1}{2} u e^{2u} + \frac{1}{4} e^{2u} + C$

$$= \frac{1}{2} x^2 (\ln x)^2 - \frac{1}{2} x^2 \ln x + \frac{1}{4} x^2 + C$$



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10

$$\int \cos(\sqrt[3]{x}) dx$$

78

$$y = \sqrt[3]{x}$$

$$dy = \frac{1}{3} x^{-2/3} dx$$

$$3 \int \cos(y) y^2 dy$$

$$3 \int y^2 \cos(y) dy$$

$$= 3 y^2 \sin(y) + 6 y \cos(y) - 6 \sin(y)$$

$$= 3 x^{2/3} \sin(x^{1/3}) + 6 x^{1/3} \cos(x^{1/3}) - 6 \sin(x^{1/3}) + C$$

$$\begin{array}{rcl} y^2 & \xrightarrow{+} & \cos(y) \\ & & \sin(y) \\ 2y & \xrightarrow{-} & -\cos(y) \\ & & -\sin(y) \\ 2 & \xrightarrow{+} & \end{array}$$

11

$$\int (\sin^{-1} x)^2 dx$$

$$y = \sin^{-1} x$$

$$\sin(y) = x$$

$$\int y^2 \cos(y) dy$$

$$\cos(y) dy = dx$$

$$= y^2 \sin y + 2y \cos y - 2 \sin y + C$$

$$= (\sin^{-1} x)^2 x + 2 \sin^{-1} x \cos(\sin^{-1} x) - 2x + C$$

Derivative

Integral

$$\begin{array}{rcl} y^2 & \xrightarrow{+} & \cos(y) \\ & & \sin(y) \\ 2y & \xrightarrow{-} & -\cos(y) \\ & & -\sin(y) \\ 2 & \xrightarrow{+} & \end{array}$$

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$$\int \tan^{-1} y \, dy$$

$$u = \tan^{-1}(y)$$

$$du = \frac{1}{1+y^2} dy$$

$$v = y$$

$$\int \tan^{-1} y \, dy = y \tan^{-1}(y) - \frac{1}{2} \int \frac{2y}{1+y^2} dy$$

$$= y \tan^{-1}(y) - \frac{1}{2} \ln |1+y^2| + C.$$

$$\int e^{\sqrt{3s+9}} ds$$

$$u = \sqrt{3s+9}$$

$$du = \frac{3}{2\sqrt{3s+9}} ds$$

$$\frac{2}{3} \int e^u u \, du$$

$$\frac{2}{3} [u e^u - e^u] + C$$

$$\frac{2}{3} \sqrt{3s+9} e^{\sqrt{3s+9}} - \frac{2}{3} e^{\sqrt{3s+9}} + C$$

Derivative

Integral

u

e^u

1

e^u

0

e