

$$y = 0$$

Tuesday, April 20, 2021 11:23 AM

$$\Sigma F_x = m \ddot{x}_2$$

Ex: Find the SSR if the cart put
The selected DOFs (x_1, θ)

$$x_2 = x_1 + L \sin \theta$$

$$\dot{x}_2 = \dot{x}_1 + L \dot{\theta} \cos \theta$$

$$\ddot{x}_2 = \ddot{x}_1 + L \ddot{\theta} \cos \theta - L \dot{\theta}^2 \sin \theta \quad \text{--- (1)}$$

$$y_2 = L \cos \theta \Rightarrow \dot{y}_2 = -L \dot{\theta} \sin \theta$$

$$\ddot{y}_2 = -L \ddot{\theta} \sin \theta - L \dot{\theta}^2 \cos \theta \quad \text{--- (2)}$$

From Fig (b)

$$\Sigma F_x = m \ddot{x}_2$$

$$H = m \ddot{x}_2 \quad \text{--- (C)}$$

sub Eq (1) into Eq (C)

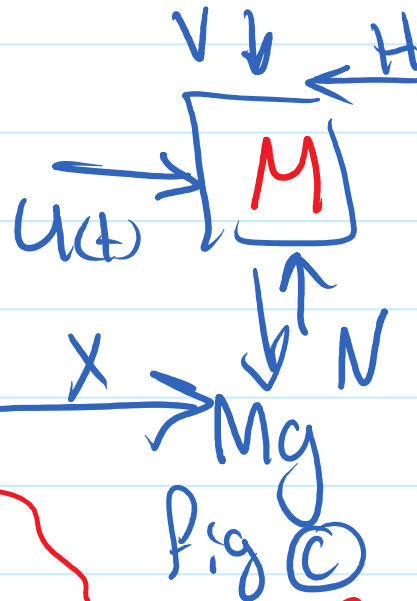
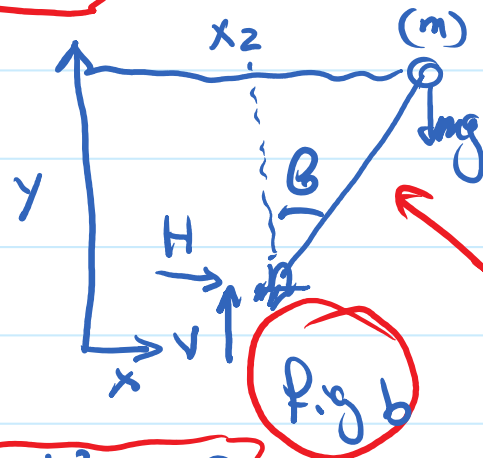
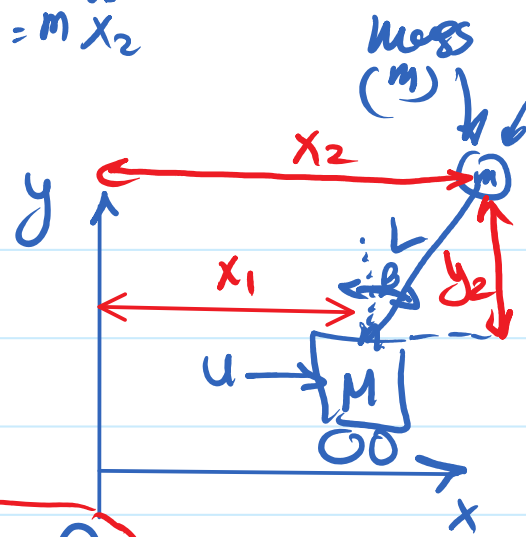
$$H = m \ddot{x}_1 + mL \ddot{\theta} \cos \theta - mL \dot{\theta}^2 \sin \theta \quad \text{--- (3)}$$

$$\Sigma F_x = M \ddot{x}_1$$

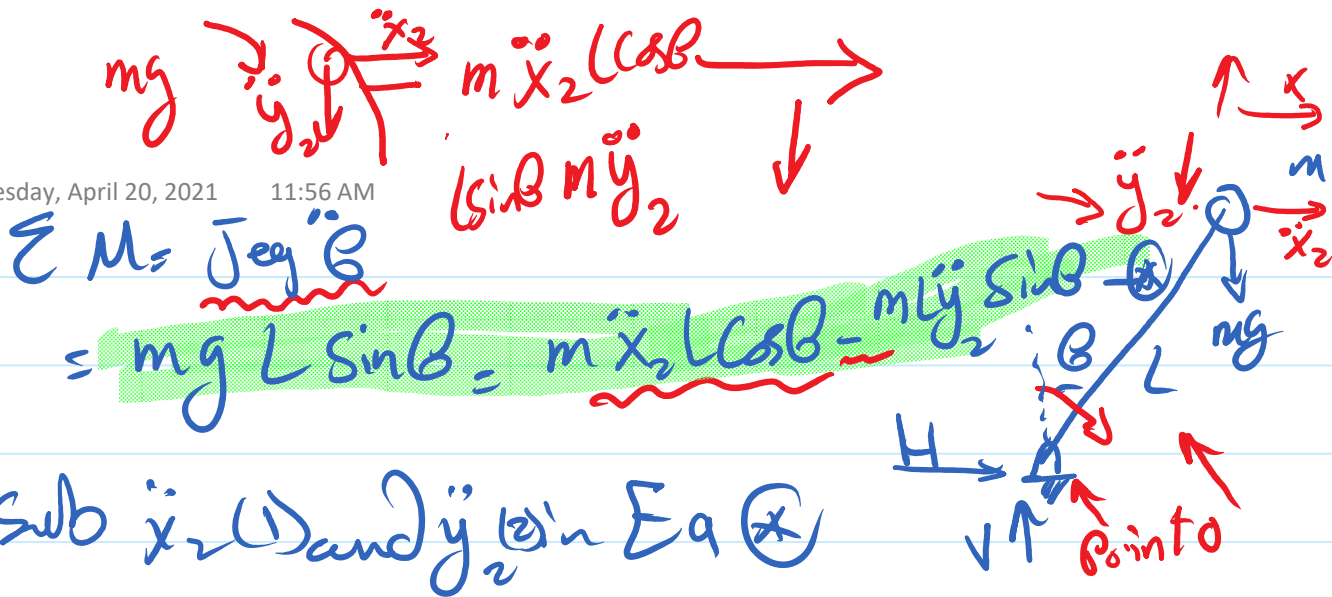
$$U - H = M \ddot{x}_1 \quad \text{--- (d)}$$

sub Eq (3) into Eq (d)

$$U(t) = (M+m) \ddot{x}_1 + mL \ddot{\theta} \cos \theta - mL \dot{\theta}^2 \sin \theta \quad \text{--- (i)}$$



States
inputs
outputs



$$\sum M = \sum \tau = I \ddot{\theta}$$

$$= mgL \sin \theta = m \ddot{x}_2 L \cos \theta - m \ddot{y}_2 L \sin \theta$$

Sub $\ddot{x}_2(t)$ and $\ddot{y}_2(t)$ in Eq (*)

$$mgL \sin \theta = mL(\ddot{x}_1 \cos \theta + L \ddot{\theta} \cos^2 \theta - L \dot{\theta}^2 \sin \theta \cos \theta + L \sin^2 \theta + L \dot{\theta}^2 \sin \theta \cos \theta)$$

$$mgL \sin \theta = mL \ddot{x}_1 \cos \theta + mL^2 \ddot{\theta} \cos^2 \theta + mL^2 \sin^2 \theta$$

$$m \ddot{x}_1 \cos \theta + mL \ddot{\theta} \cos^2 \theta - mg \sin \theta + mL \dot{\theta}^2 \sin \theta = 0$$

The model is nonlinear (Eq (i) and Eq (ii)) based on that. It is recommended to linearize the model when $\theta(0) = 0$

$$\cos \theta = 1$$

$$\sin \theta = \theta$$

$$(M+m) \ddot{x}_1 + mL \ddot{\theta} = u(t)$$

$$m \ddot{x}_1 + mL \ddot{\theta} - mg \theta = 0$$

$$\begin{bmatrix} (m+M) & (mL) \\ m & (mL) \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} u(t) \\ mg\theta \end{bmatrix}$$

Linear model when $\theta \approx 0$

$$\Rightarrow \begin{bmatrix} (m+M) & (mL) \\ m & (mL) \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} u(t) \\ mg\theta \end{bmatrix}$$

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{\theta} \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} mL & -mL \\ -m & (M+m) \end{bmatrix} \begin{bmatrix} u(t) \\ mg\theta \end{bmatrix}$$

$$\Delta = (M+m)mL - m^2L = \underline{\underline{mML}}$$

$$\ddot{x}_1 = \frac{mL u(t) - mL * mg\theta}{mML}$$

$$\ddot{x}_1 = \frac{1}{M} [u(t) - mg\theta] \quad \text{--- (i)}$$

$$\ddot{\theta} = \frac{-m u(t) + (M+m)mg\theta}{MmL}$$

$$\ddot{\theta} = \frac{[-u(t) + (M+m)g\theta]}{ML} \quad \text{--- (ii)}$$

Now let the linear SSR

$$q_1 = x_1 \Rightarrow \dot{q}_1 = \dot{x}_1 = q_2 \Rightarrow \boxed{\dot{q}_1 = q_2} \quad \text{--- (I)}$$

$$q_2 = \dot{x}_1$$

$$q_3 = \theta \Rightarrow \dot{q}_3 = \dot{\theta} = q_4 \Rightarrow \boxed{\dot{q}_3 = q_4} \quad \text{--- (II)}$$

$$q_4 = \dot{\theta}$$

Sub the states in Eq (i) and (ii)

$$\dot{q}_2 = \frac{1}{M} [u(t) - mg q_3] \quad \text{--- (III)}$$

$$\dot{q}_4 = \frac{1}{ML} [-u(t) + (M+m)g q_3] \quad \text{--- (IV)}$$

$$\dot{q} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{M+m}{ML} & 0 \end{bmatrix} q + \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ -\frac{1}{ML} \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} q$$