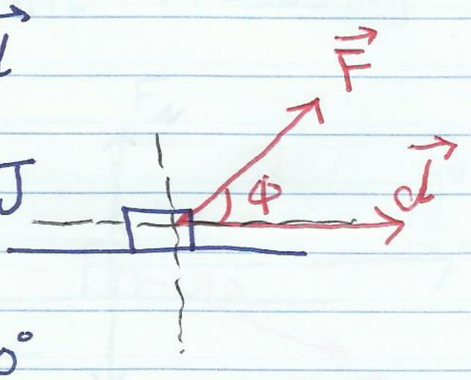
Chapter 7: Lecture Problems

Work done by a Constant $\vec{F} = \vec{F} \cdot \vec{d}$



$$W = \vec{F} \cdot \vec{d} = Fd\cos\phi \quad N.m = J$$

We have three cases:

1) $W = 0$, for $\phi = 90°$
2) W is positive for $\phi < 90°$
   Positive Work means energy transferred to the object
3) W is negative for $180° \geq \phi > 90°$
   negative Work means Energy transferred from the object

$$W_{\text{done by resultan force}} = \Delta K \quad J$$

$$W_{net} = K_f - K_i \quad , K = \tfrac{1}{2}mv^2 \quad J$$

Work done by variable Force $= \int_{r_i}^{r_f} \vec{F} \cdot \vec{dr}$

$$W = \int_{r_i}^{r_f} \vec{F} \cdot \vec{dr}$$

$$= \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

W = The Area under the curve of (F vs. d) from $d_1 \longrightarrow d_2$

Sample Problem 7.02:

$m = 22{,}5\,kg$ 2 Workers are acting on it $m = 225\,kg$

by $\vec{F_1} = 12N$, $30°$ under the x-axis

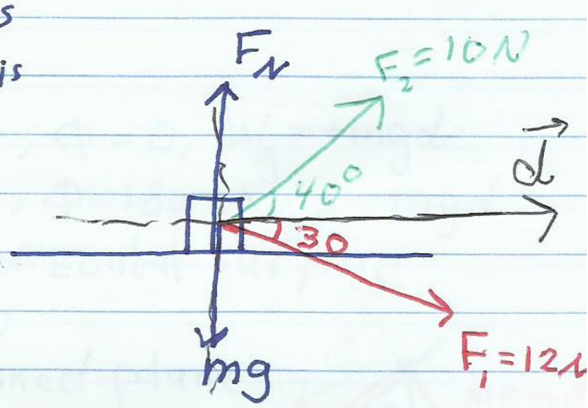
$\vec{F_2} = 10N$, $40°$ above the x-axis

$d = 8.5\,m$ along x-axis

a) Find W_1? W_2?

b) Find W_g? W_{F_N}?

c) Find V_f? $V_i = 0$



4 Forces are acting on m

$\vec{F_1}, \vec{F_2}, m\vec{g}, \vec{F_N}$

$$W = \vec{F} \cdot \vec{d} = F\cos\phi\, d$$

$$W_1 = (12)(\cos 30)(8.5) = 88.3\,J$$

$$W_2 = (10)(\cos 40)(8.5) = 65.11\,J$$

b) $W_g = (mg)(\cos 90)(8.5) = 0$

$W_{F_N} = F_N(\cos 90)(8.5) = 0$

c) $\vec{F}_{net} = (10\cos 40 + 12\cos 30)\hat{i} = 18.05\hat{i}\ N$

$$W_{net} = K_f - K_i \quad , \quad K_i = 0$$

$$W_1 + W_2 + W_g + W_{F_N} = \frac{1}{2}mV_f^2$$

$$153.4 = \frac{1}{2}(22{,}5)V_f^2$$

$$V_f = \sqrt{\frac{2(153.4)}{22{,}5}} =$$

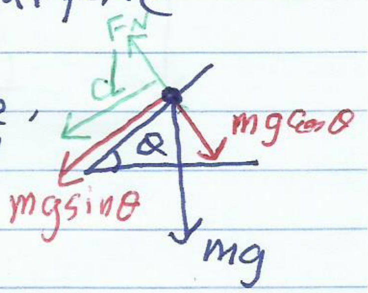
$$= 1.17\ m/s$$

Work done by gravitational Force (mg):

$$W_g = m\vec{g}\cdot\vec{d}$$
$$= mg\cos\phi\, d$$

We have different caseses

1) m moves downward , $\phi = 0$, $W_g = +mgd$
2) m moves upward , $\phi = 180$, $W_g = -mgd$
3) m moves along ahorizontal surface
   $\phi = 90$, $W_g = 0$
4) m moves on inclined plane ,

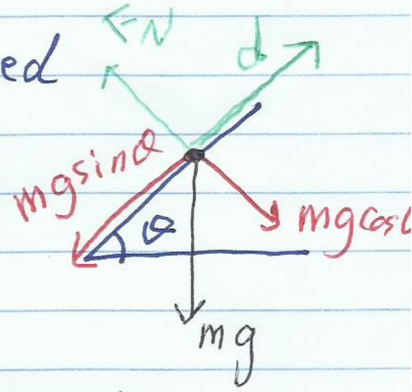


(m) moves downward the inclined

$$W_g = (mg\sin\theta)(d)\cos 0$$
$$= (mg\sin\theta)\, d$$

(m) m moves upward the inclined

$$W_g = (mg\sin\theta)(d)(\cos 180)$$
$$= (-)\, mg\sin\theta \cdot d$$



Sample Problem 7.05:

Work done on an accelerating elevator cab.

$m = 500\,kg$, descending ( ) with $V_i = 4\,m/s$

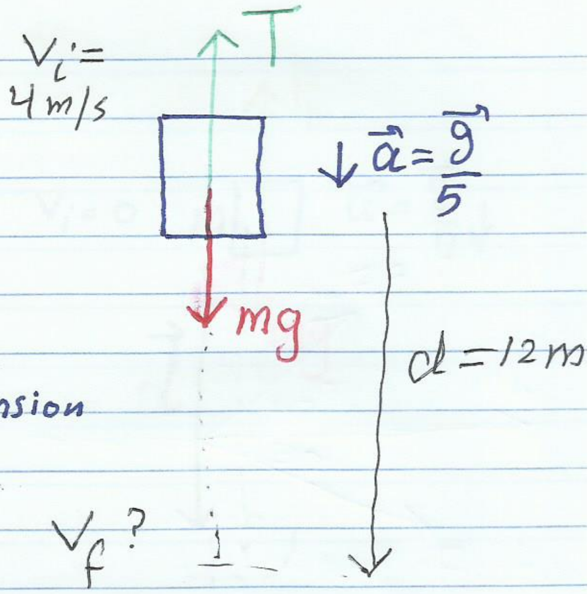
2 Force are acting on (m)

$m\vec{g}$ + $\vec{F}_{Tension}$

and is moving downward at constant $\vec{a} = \frac{g}{5}$

$d = 12\,m$

Find ① W_g ? ② $W_{Tension\ force}$ ③ V_f ?



1) $W_g = (mg)(\cos 0)\,d$

$= (500)(9.8)(1)(12)$

$= 5.88 \times 10^4\ J$

2) $W_{Tension}$? You have to find Tension

From Newton's 2^{nd} Law

$$\sum F_y = m\vec{a}$$

$$T - mg = m\left(-\frac{g}{5}\right)$$

$$T = mg - \frac{mg}{5} = \frac{4}{5}mg = \left(\frac{4}{5}\right)(4900)$$

$T = 3920\,N$

$$W_T = T d \cos 180$$

$= (3920)(12)(-1)$

$= -4.7 \times 10^4\ J$

3)

$$W_{net} = W_g + W_T = K_f - K_i$$

$$5.88 \times 10^4 + -4.7 \times 10^4 = K_f - \frac{1}{2}(500)(4)^2$$

$$1.18 \times 10^4 = K_f - 0.4 \times 10^4$$

$K_f = 1.58 \times 10^4\ J$ $= \frac{1}{2} m V_f^2$

$$V_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(1.58 \times 10^4)}{500}}$$

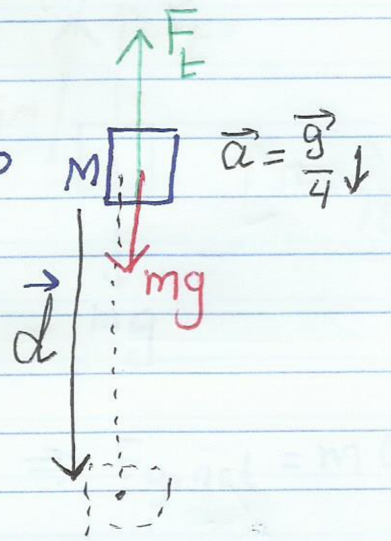
$V_f = 7.9\ m/s$

(7-21)

$\boxed{M}$ $v_i = 0$, descends at constant $\vec{a} = \frac{\vec{g}}{4}$

displacemend $= \vec{d}$, downward

Find W_T ? W_g ? K_f, v_f ?



$W_g = mgd\cos 0$

$W_g = +Mgd$ J

W_T ? You have to find $F_{tention}$

$\sum F_y = ma_y$ (Newton's 2^{nd} Law)

$F_t - mg = m(-\frac{g}{4})$

$F_t = Mg + -\frac{Mg}{4}$

$F_t = \frac{3}{4}Mg$

$W_{Tension} = F_t d\cos 180 = (\frac{3}{4}Mg)(d)\cos 180$

$W_T = -\frac{3}{4}Mgd$

$W_{net} = K_f - K_i$, $K_i = 0$

$Mgd - \frac{3}{4}Mgd = K_f$

$K_f = \frac{1}{4}Mgd = \frac{1}{2}mv_f^2$

$v_f = \sqrt{gd/2}$

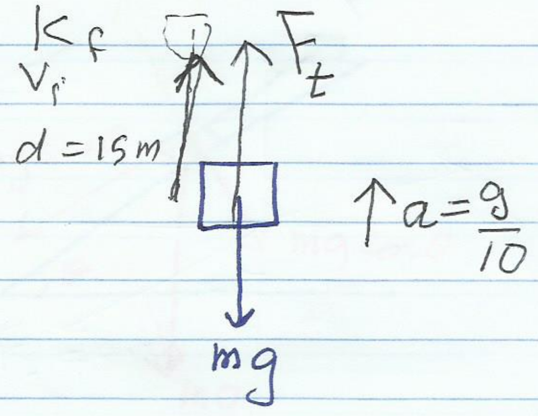
(7 – 15)

$m = 72\,kg$

$d = 15\,m$ in $+y$

$a = \frac{g}{10}$ upward

$a_y = +\frac{g}{10}\ m/s^2$



Find W_T? W_{mg}? K_f? v_f?

$\sum \vec{F}_{net} = m\vec{a}$ Newton's 2^{nd} Law $\Rightarrow F_{y,net} = m a_y$

$F_t - mg = m(+\frac{g}{10})$

$F_t = mg + \frac{1}{10} mg = \frac{11}{10} mg$

$W_{Tensinon} = (F_t)(d)\cos 0$

$= (\frac{11}{10} mg)(d)(1)$

$= \frac{11\,mgd}{10} = 11.6\ J \times 10^3 = 11.6\,kJ$

$= 1.16 \times 10^4\,J$

$W_{mg} = mgd(\cos 180)$

$= -mgd$

$= -10.6\ J \times 10^3 = -10.6\,kJ = -1.06 \times 10^4\,J$

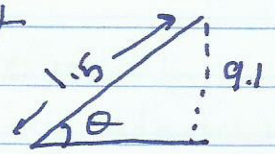
$W_{net} = K_f - K_i \quad , K_i = 0$

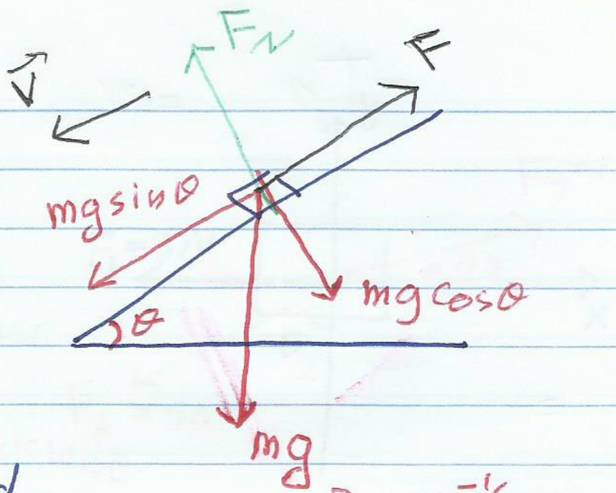
$10^3 \times (11.6 + {}^{-}10.6) = K_f \qquad K_f \approx 1 \times 10^3\,J$

$K_f = 1\,kJ = \frac{1}{2} m v_f^2$

$v_f =$ ~~[illegible]~~ $5.3\ m/s$

(7 − 76) $m = 45\,kg$ slides down frictionless incline of 1.5 m long + 0.91 m high at constant speed





a) Find F from the Worker?

$\theta = \sin^{-1}\left(\frac{0.91}{1.5}\right) = 37^\circ$

$\sum \vec{F} = m\vec{a}$ Newton's 2^{nd} Law

$\sum \vec{F} = 0 \qquad \vec{a} = 0,\ \vec{v} = \text{constant}$

$\sum F_x = 0 \Longrightarrow F - mg\sin\theta = 0$

$F = mg\sin\theta = 265\,N$

b) $W_F = Fd\cos\phi$

$= (265)(1.5)\cos 180$

$= (-)398\,J$

c) $W_g = (mg\sin\theta)(d)\cos 0$

$= +398\,J$

d) $W_{F_N} = F_N\, d \cos 90$

$= 0$

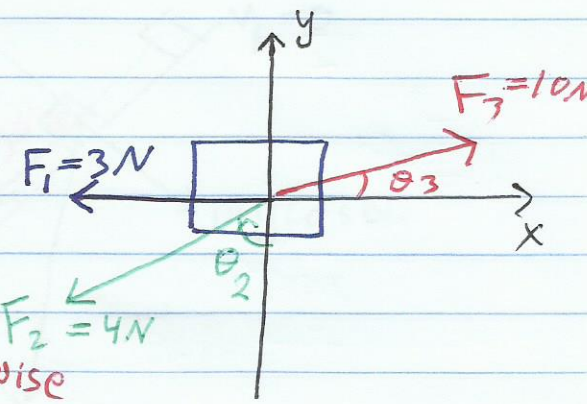
e) $\vec{F}_{Net} = 0\ ,\ \vec{v} = \text{constant}$

(7 – 14) 3 Horizontal ~~surface~~ Forces

$\vec{F_1} = 3N$; $-x$

$\vec{F_2} = 4N$, $\theta_2 = 50°$ with $-y$ clockwise

$\vec{F_3} = 10N$; $\theta_3 = 35°$ with $+x$ counterclockwise



$d = 4m$

Find W_{net} ?

$$\sum \vec{F}_{net} = \vec{F_1} + \vec{F_2} + \vec{F_3}$$

, mg in $(-\hat{k})$

F_N in $+\hat{k}$

$$F_{net,x} = F_1 \cos 180 + F_2 \cos 220 + F_3 \cos 35$$
$$= -3 + {}^{-}3.06 + 8.19$$
$$= 2.13\ N$$

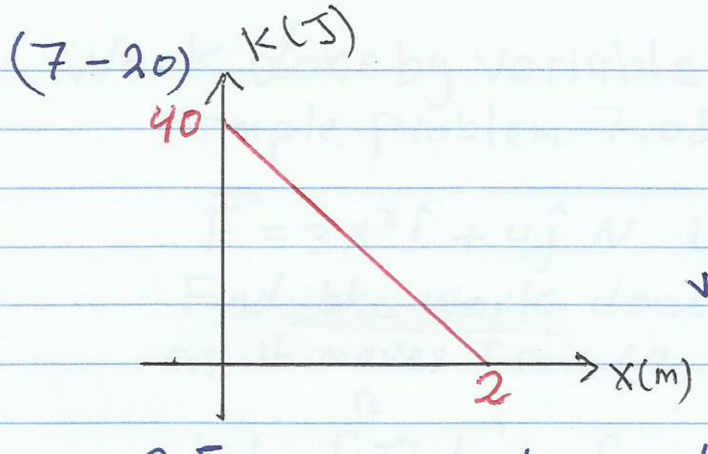
$$F_{net,y} = F_1 \sin 180 + F_2 \sin 220 + F_3 \sin 35$$
$$= 0 + {}^{-}2.57 + 5.74$$
$$= 3.17\ N$$

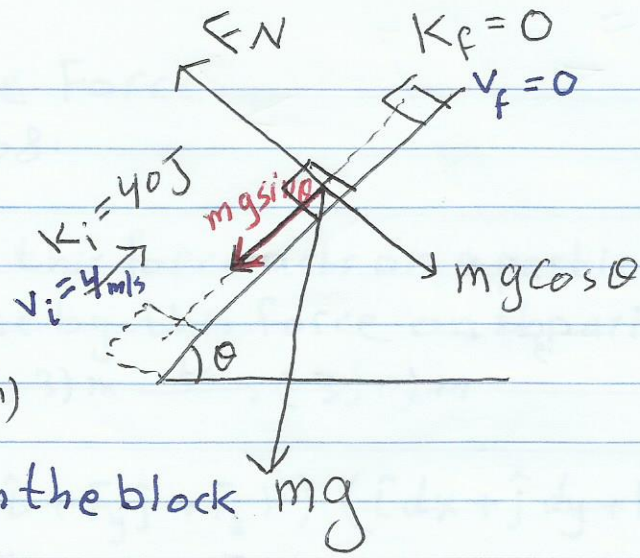
$$F_{Net} = \sqrt{(2.13)^2 + (3.17)^2}$$
$$= 3.82N$$

F_{Net} + d are in the same direction

$$W_{net} = F_{net}\, d \cos 0$$
$$= 15.3\ J$$

(7-20)





2 Force are acting on the block
mg & F_N

$\sum \vec{F}_{net} = m\vec{a}$

$F_{net,x} = -mg\sin\theta$

$F_{net,y} = 0 \Rightarrow F_N - mg\cos\theta = 0$

$F_N = mg\cos\theta$

Find F_N?

$W_{net} = K_f - K_i$, $K_i = 40$, $K_f = 0$ from the curve

$(mg\sin\theta)(x)\sin 180 = 0 - 4$, $x = 2m$, from the graph

$(mg\sin\theta)(2)(-1) = 0 - 4$

$-2mg\sin\theta = -4$, $\sin\theta = \frac{2}{mg}$

to find m $K_i = \frac{1}{2}mv_i^2$, $K_i = 40J$, $v_i = 4 m/s$

$m = \frac{2K_i}{v_i^2} = \frac{2(40)}{(4)^2} = 5kg$

$\sin\theta = \frac{2}{mg} = \frac{2}{5(9.8)} = 0.0408$

$\theta = 2.3°$

$F_N = mg\cos\theta = (5)(9.8)(\cos 2.3)$

$\approx 49N$

Work done by variable Force:

Sample Problem 7.08:

$\vec{F} = 3x^2\hat{i} + 4\hat{j}$ N this force acts on a particle

Find the work done by this force on the particle as it moves from (2, 3) m $\xrightarrow{\text{to}}$ (3, 0) m

$$W = \int_{r_i}^{r_f} \vec{F}\cdot\vec{dr} = \int (F_x\hat{i} + F_y\hat{j} + F_z\hat{k})\cdot(\hat{i}dx + \hat{j}dy + \hat{k}dz)$$

$$= \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

$$W = \int_2^3 3x^2 dx + \int_3^0 4 dy$$

$$= \left[3\frac{x^3}{3}\right]_2^3 + \left[4y\right]_3^0$$

$$= \left[x^3\right]_2^3 + \left[4y\right]_3^0 = \left[(3)^3 - (2)^3\right] + 4\left[0-3\right]$$

$$W = 27 - 8 - 12 = 7 \text{ J}$$

Positive Work means the kinetic energy of the Particle increases.

(7-61) $\vec{F} = 2x\hat{i} + 3\hat{j}$ N

How much work is done by the force as it move the Particle From $\vec{r_i} = 2\hat{i} + 3\hat{j}$ m $\longrightarrow$ $\vec{r_f} = -4\hat{i} - 3\hat{j}$ m

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy = \int_2^{-4} 2x\,dx + \int_3^{-3} 3\,dy$$

$$= \left[x^2\right]_2^{-4} + \left[3y\right]_3^{-3} = \left[(-4)^2 - (2)^2\right] + 3\left[-3-3\right]$$

$$= 16 - 4 - 18$$

$$= -6 \text{ J}$$

negative Work means kinetic energy is decreasing

Work done by spring Force

$F_{spring} = -kx$ Hooke's Law

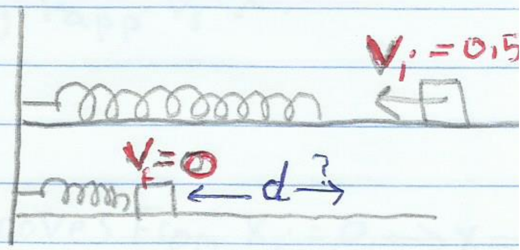
, $k = N/m$ (spring constant) (Force constant)

Work done by spring Force $W_s = \int_{x_i}^{x_f} -kx\,dx$

$W_s = -\frac{1}{2}k(x_f^2 - x_i^2)$ Joul

Sample Problem (7.06)

$m = 0.4\,kg$ moves at $V = 0.5\,m/s$ toward a spring, $k = 750\,N/m$ whem m momentarly stopped by the spring, by what distance d is the spring compressed



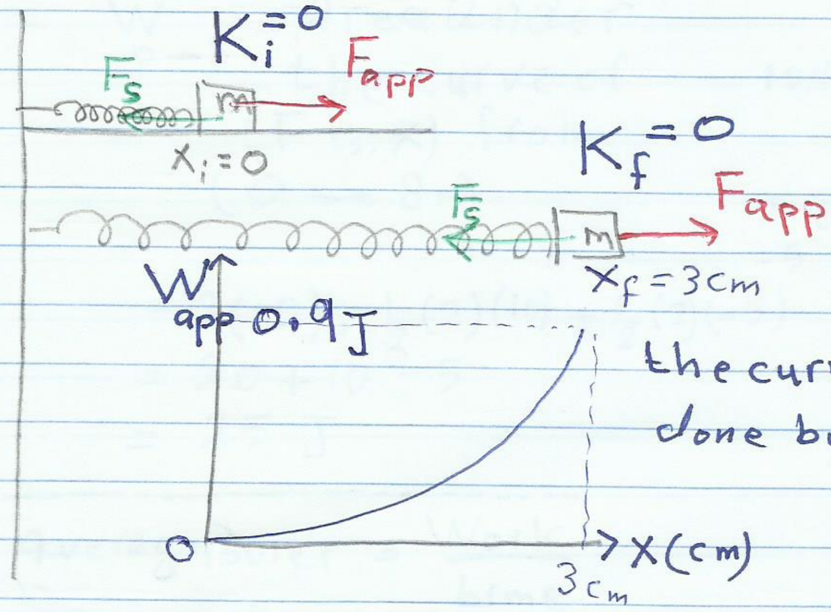
$W_s = \int_{x_i}^{x_f} -kx\,dx = \Delta K = K_f - K_i$

$x_i = 0 \longrightarrow x_f = d$, $K_f = 0$, $K_i = \frac{1}{2}mv_i^2$

$-\frac{1}{2}kd^2 = -\frac{1}{2}mv_i^2$

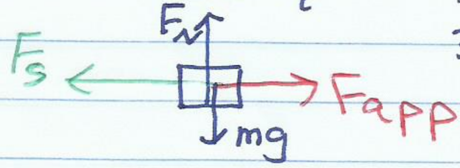
$d = v_i\sqrt{\frac{m}{k}} = 0.5\sqrt{\frac{0.4}{750}} = 1.2\times10^{-2}\,m = 1.2\,cm$

Problem (7 – 29)



the curve is the work done by F_{app} Vs x.

4 Forces are acting on (m) as it moves from $x_i = 0 \rightarrow x_f = 3cm$



$\sum \vec{F_y} = 0$, $F_N = mg$

No work done by F_N, mg, $\theta = 90°$

$W_a + W_s = \Delta K$, $K_i = 0$, $K_f = 0 \Rightarrow$

$W_a = -W_s$ Always when $K_i = K_f$

from the graph

$W_a = 0.9 J$, When $x = 3cm \Rightarrow$

$W_s = \frac{1}{2} k x^2 \Rightarrow k = \frac{2W_s}{x^2} = \frac{2(0.9)}{(3\times10^{-2})^2}$

$K = 2000 \; N/m$

a) find W_s? from $x_i = 5cm \rightarrow x_f = 4cm$

$W_s = -\frac{1}{2}k(x_f^2 - x_i^2) = (-\frac{1}{2})(2\times10^3)[(0.04)^2 - (0.05)^2]$

$= +0.9 J$

b) W_s? from $x_i = 5cm \rightarrow x_f = -2cm$

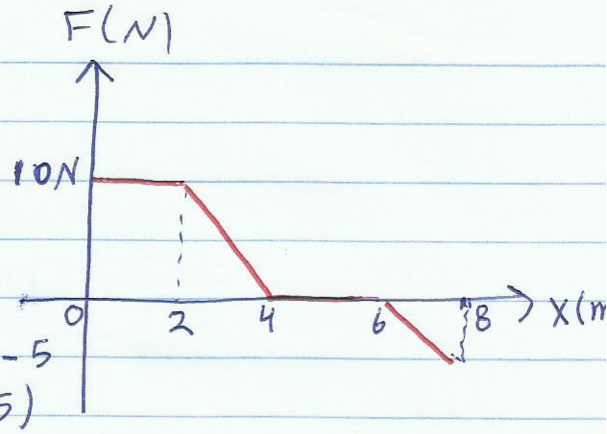
$W_s = -\frac{1}{2}k(x_f^2 - x_i^2) = -\frac{1}{2}(2000((-0.02)^2 - (0.05)^2))$

$= 2.1 J$

c) W_s? from $x_i = 5cm \rightarrow x_f = -5cm$ $W_s = 0$

(7-36)

$W_{0\to 8}$ = Area under the curve of (F vs. x) from (0 → 8m)



$= 2(10) + \frac{1}{2}(2)(10) + \frac{1}{2}(2)(-5)$

$= 20 + 10 - 5$

$= 25\ J$

Averag Power $= \dfrac{\text{Work}}{\text{time}}$

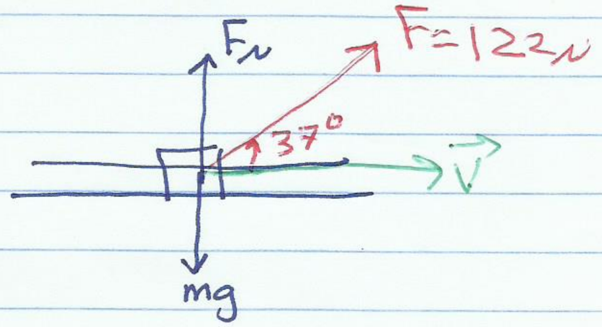
$P_{avg} = \dfrac{W}{\Delta t}$ J/s = Watt

$P_{instantaneous} = \dfrac{dW}{dt}$ Watt

$P_{inst} = \vec{F}\cdot\vec{V}$ Watt

(7-45) $m = 100kg$

$V = 5\ m/s$



$\vec{P} = \vec{F}\cdot\vec{V}$

$= 122(5)(\cos 37)$

$= 487$ Watt

(7-47) $\vec{F} = 2N\hat{i} + 4N\hat{j} + 6N\hat{k}$ $\quad m = 4kg$

at $t = 0$, $\vec{d}_i = 0.5m\hat{i} + 0.75m\hat{j} + 0.2m\hat{k}$

at $t = 12s$, $\vec{d}_f = 7.5m\hat{i} + 12m\hat{j} + 7.2m\hat{k}$

a) W ?

$W = \vec{F}\cdot\Delta\vec{r}$, $\quad \Delta\vec{r} = \vec{d}_f - \vec{d}_i = 7\hat{i} + 11.25\hat{j} + 7\hat{k}\ m$

$= (2\hat{i} + 4\hat{j} + 6\hat{k})\cdot(7\hat{i} + 11.25\hat{j} + 7\hat{k})$

$= 14 + 45 + 42 = 101\ J$

b) $P_{avg} = \dfrac{W}{\Delta t} = \dfrac{101}{12} = 8.42$ watt

