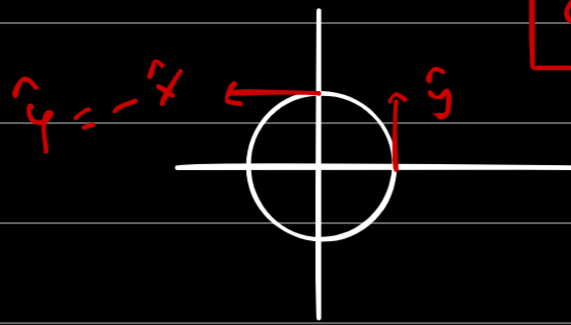




$$\hat{r} = -\sin\phi \hat{x} + \cos\phi \hat{y}$$

Example: Using spherical coordinate system
 evaluate the volume and surface
 area of a sphere of radius R

$$\begin{aligned} 0 &\leq r \leq R \\ 0 &\leq \theta \leq \pi \\ 0 &\leq \phi \leq 2\pi \end{aligned}$$

$$\begin{aligned} V &= \int_V d\tau = \int_0^R \int_0^{2\pi} \int_0^\pi r^2 \sin\theta d\theta d\phi dr \\ &= 2\pi \int_0^\pi \sin\theta d\theta \int_0^R r^2 dr \\ &= 2\pi \left(-\cos\theta \Big|_0^\pi \right) \left(\frac{r^3}{3} \Big|_0^R \right) \\ &= 2\pi (2) \left(\frac{R^3}{3} \right) = \frac{4}{3} \pi R^3 \end{aligned}$$

* Evaluate $\int_{-\infty}^{+\infty} \delta(x^2 + 1) dx = 0$

$\uparrow$
 no roots

$$\int_{-\infty}^{+\infty} \delta(x^2-1) dx = \int_{-\infty}^{+\infty} \frac{\delta(x-1)}{|f'(1)|} dx + \int_{-\infty}^{+\infty} \frac{\delta(x+1)}{|f'(-1)|} dx$$

$$f'(x) = 2x$$

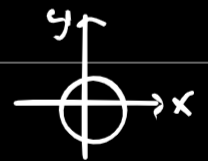
$$x_1 = 1, x_2 = -1$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$\int_0^{\infty} \delta(x^2-1) dx = \frac{1}{2} \checkmark$$

Example: Write the charge distribution of a uniformly charged ring of radius and total charge Q in terms of δ -function.
(ring is centered about the origin and lie in the xy -plane)

Cartesian,



$$\rho(x, y, z) = k_1 \delta(z) \delta(\sqrt{x^2+y^2}-R)$$

↑

$$\int \rho(x, y, z) dx dy dz = Q$$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} k_1 \delta(z) \delta(\sqrt{x^2+y^2}-R) dx dy dz$$

$$k_1 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \delta(\sqrt{x^2+y^2}-R) dx dy = Q$$

$$k_1 \int_0^R \int_0^{2\pi} \delta(s-R) s ds d\varphi = Q$$

$$2\pi k_1 R = Q \Rightarrow k_1 = \frac{Q}{2\pi R}$$

$$\varphi(x, y, z) = \frac{Q}{2\pi R} \delta(\sqrt{x^2+y^2}-R) \delta(z)$$