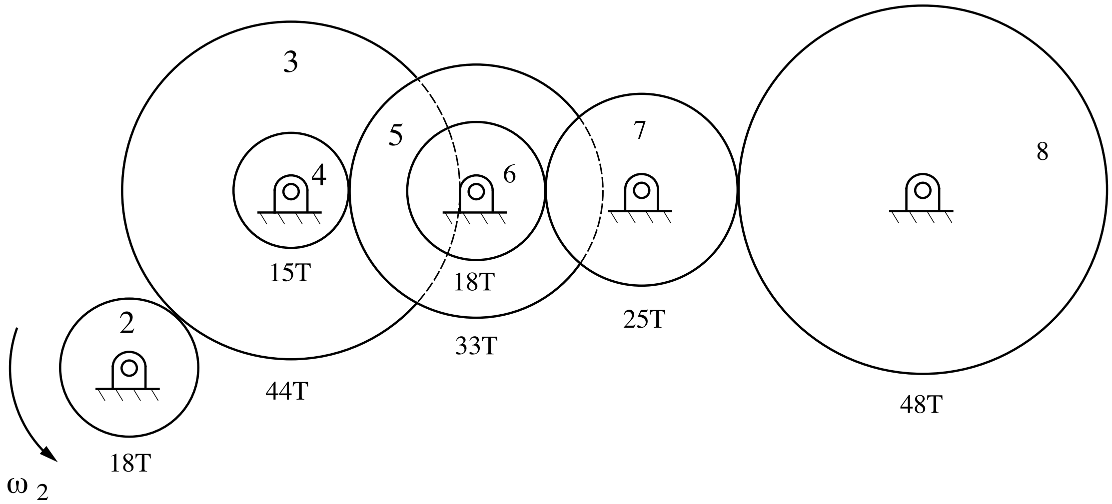
Problem 12.1

Find the angular velocity of gear 8 if the angular velocity of gear 2 is 800 rpm in the direction shown.



Solution:

The velocity ratio for the gear train is given by

$$\frac{\omega_8}{\omega_2} = \frac{N_2}{N_3}\frac{N_4}{N_5}\frac{N_6}{N_7}\frac{N_7}{N_8} = \frac{18}{44}\frac{15}{33}\frac{18}{25}\frac{25}{48} = 0.0697$$

Therefore,

$$\omega_8 = \omega_2\frac{N_2}{N_3}\frac{N_4}{N_5}\frac{N_6}{N_7}\frac{N_7}{N_8} = 800(0.0697) = 55.78 \text{ rpm CCW}$$

Problem 12.2

Find the velocity of gear 8 in Problem 12.1 if the angular velocity of the driver (gear 2) is 300 rpm in the clockwise direction.

Solution:

The velocity ratio for the gear train is given by

$$\frac{\omega_8}{\omega_2} = \frac{N_2}{N_3}\frac{N_4}{N_5}\frac{N_6}{N_7}\frac{N_7}{N_8} = \frac{18}{44}\frac{15}{33}\frac{18}{25}\frac{25}{48} = 0.0697$$

Therefore,

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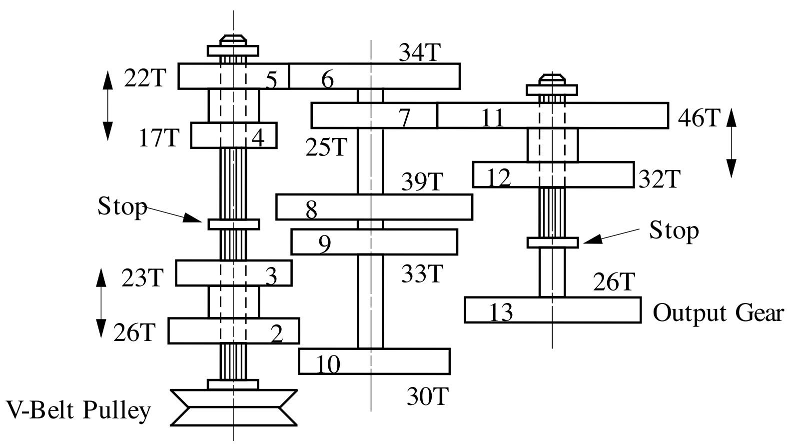
$$\omega_8 = \omega_2 \frac{N_2}{N_3}\frac{N_4}{N_5}\frac{N_6}{N_7}\frac{N_7}{N_8} = 300(0.0697) = 20.92 \text{ rpm CW}$$

Problem 12.3

The gear train given is for a machine tool. Power is input to the gear box through the pulley indicated, and the output power to the machine table is through gear 13. Gears 2 and 3, 4 and 5, and 11 and 12 are compound gears that can move axially on splined shafts to mesh with various different gears so that various combinations of overall gear ratios (ω_{13}/ω_2) can be produced. Determine the number of ratios possible and the overall gear ratio for each possibility.



Solution:

The possible meshes between the first and second shafts (counting from the left) are:

a) Gear 2 with gear 10

b) Gear 3 with gear 9

c) Gear 4 with gear 8

d) Gear 5 with gear 6

The possible meshes between the second and third shafts (counting from the left) are:

e) Gear 7 with gear 11

f) Gear 8 with gear 12

The number of combinations is given by

$$n = 4(2) = 8$$

The different meshes and gear ratios are given in the following table.

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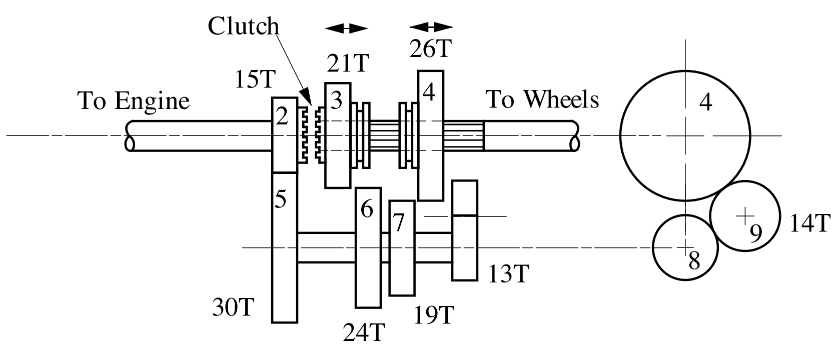
Table 1: Different cases for transmission

Case	Meshes	Gear Ratio	Gear Ratio Value
1	a, e	$\frac{N_2}{N_{10}}\frac{N_7}{N_{11}}$	$\frac{26}{30}\frac{25}{46} = 0.471$
2	a, f	$\frac{N_2}{N_{10}}\frac{N_8}{N_{12}}$	$\frac{26}{30}\frac{39}{32} = 1.056$
3	b, e	$\frac{N_3}{N_9}\frac{N_7}{N_{11}}$	$\frac{23}{33}\frac{25}{46} = 0.379$
4	b, f	$\frac{N_3}{N_9}\frac{N_8}{N_{12}}$	$\frac{23}{33}\frac{39}{32} = 0.849$
5	c, e	$\frac{N_4}{N_8}\frac{N_7}{N_{11}}$	$\frac{17}{39}\frac{25}{46} = 0.237$
6	c, f	$\frac{N_4}{N_8}\frac{N_8}{N_{12}}$	$\frac{17}{39}\frac{39}{32} = 0.531$
7	d, e	$\frac{N_5}{N_6}\frac{N_7}{N_{11}}$	$\frac{22}{34}\frac{25}{46} = 0.351$
8	d, f	$\frac{N_5}{N_6}\frac{N_8}{N_{12}}$	$\frac{22}{34}\frac{39}{32} = 0.789$

Problem 12.4

A simple three-speed transmission is shown. The power flow is as follows: (a) first gear: gear 4 is shifted to mesh with gear 7; power flows through gears 2, 5, 7, 4. (b) Second gear: gear 3 is shifted to mesh with gear 6; power flows through gears 2,5,6,3. (c) Third gear: gear 3 is shifted so that the clutch teeth on gear 3 mesh with those on gear 2; a direct drive results. (d) Reverse gear: gear 4 is shifted to mesh with gear 9; power flows through gears 2, 5, 8, 9, 4. An automobile with this transmission has a differential ratio of 3:1 and a tire outside diameter of 24 in. Determine the engine speed for the car under the following conditions: (i) first gear and the automobile is traveling at 15 mph; (ii) third gear and the automobile is traveling at 55 mph; (iii) reverse gear and the automobile is traveling at 3.5 mph.



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Solution:

Case 1: First gear and automobile traveling at 15 mph. In this gear, the speed ratio is

$$\frac{\omega_2}{\omega_4} = \frac{N_5}{N_2}\frac{N_4}{N_7}\frac{3}{1} = (-1)^2\frac{30}{15}\frac{26}{19}\frac{3}{1} = 8.196$$

If the wheel radius is 1 ft, the wheel angular velocity is

$$\omega_w = \frac{v}{r} = \frac{15\,\text{mph}}{1\,\text{ft}}\frac{5280\,\text{ft}}{1\text{mile}}\frac{1\,\text{hr}}{60\,\text{min}}\frac{1\text{rev}}{2\pi\text{rad}} = 210\text{ rpm}$$

Therefore, the engine speed must be

$$\omega_e = \omega_w\frac{N_5}{N_2}\frac{N_4}{N_7}\frac{3}{1} = 8.196(210) = 1721\,\text{rpm}$$

Case 2: Third gear and automobile traveling at 55 mph. In this gear, the speed ratio is

$$\frac{\omega_2}{\omega_4} = 1$$

If the wheel radius is 1 ft, the wheel angular velocity is

$$\omega_w = \frac{v}{r} = \frac{55\,\text{mph}}{1\text{ ft}}\frac{5280\,\text{ft}}{1\text{mile}}\frac{1\,\text{hr}}{60\text{ min}}\frac{1\text{rev}}{2\pi\,\text{rad}} = 770.3\,\text{rpm}$$

Therefore, the engine speed must be

$$\omega_e = \frac{3}{1}\omega_w = (3)770.3 = 2311\,\text{rpm}$$

Case 3: Reverse gear and automobile traveling at 3.5 mph. In this gear, the speed ratio is

$$\omega_e = \omega_w\frac{N_5}{N_2}\frac{N_9}{N_8}\frac{N_4}{N_9}\frac{3}{1} = (-1)^3\frac{30}{15}\frac{14}{13}\frac{25}{14}\frac{3}{1} = -\omega_w 11.54\text{ rpm}$$

If the wheel radius is 1 ft, the wheel angular velocity is

$$\omega_w = \frac{v}{r} = \frac{3.5\text{mph}}{1\text{ ft}}\frac{5280\,\text{ft}}{1\text{mile}}\frac{1\text{ hr}}{60\text{ min}}\frac{1\text{rev}}{2\pi\,\text{rad}} = 49.02\,\text{rpm}$$

Therefore, the engine speed must be

$$\omega_e = \omega_w\frac{N_5}{N_2}\frac{N_9}{N_8}\frac{N_4}{N_9}\frac{3}{1} = 49.02(11.54) = 565.7\text{ rpm}$$

Problem 12.5

Part of the gear train for a machine tool is shown. Compound gears 2 and 3 slide on a splined shaft so that gear 3 can mesh with gear 4 or gear 2 can mesh with gear 6. Also, compound gears 14 and

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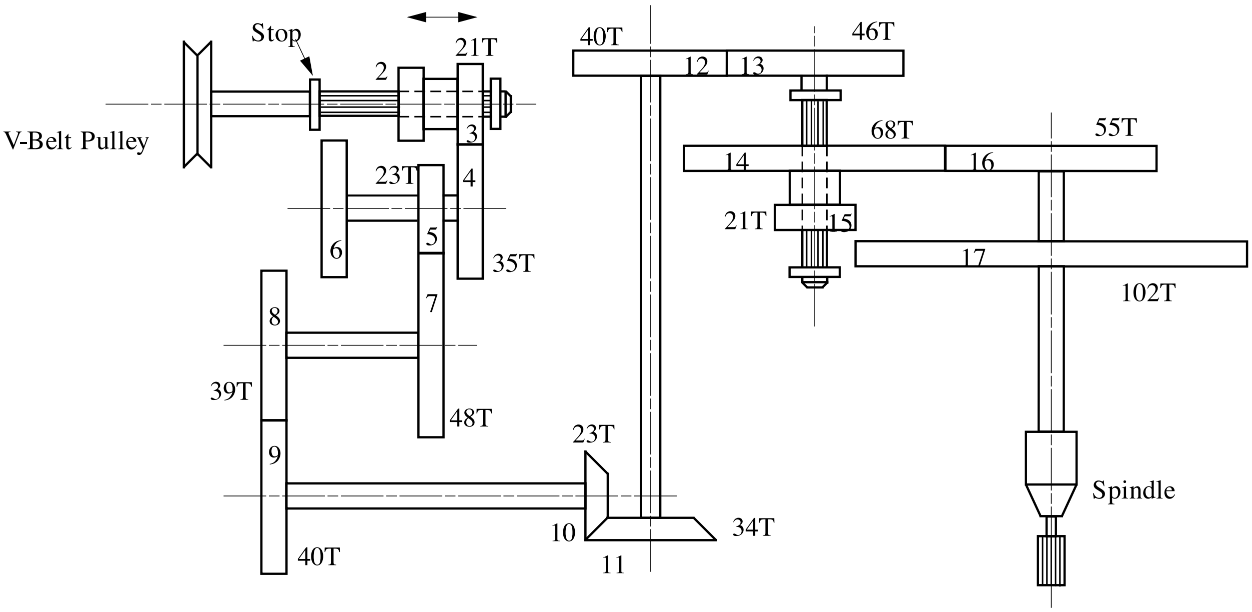
15 slide on a splined shaft so that gear 14 can mesh with gear 16 or gear 15 can mesh with gear 17. (a) If gear 3 meshes with gear 4, what are the two possible spindle speeds for a motor speed of 1800 rpm? (b) Now assume that gear 14 meshes with gear 16, and gear 2 meshes with gear 6. Gears 2, 3, 4, and 6 are standard and have the same diametral pitch. What are the tooth numbers on gears 2 and 6 if the spindle speed is 130±3 rpm?



Solution:

The angular velocity ratio for the transmission is:

$$\frac{\omega_o}{\omega_i} = y\left(\frac{N_5}{N_7}\frac{N_8}{N_9}\frac{N_{10}}{N_{11}}\frac{N_{12}}{N_{13}}\right)x$$

where

$$\omega_o = \omega_{16} = \omega_{17},$$

$$\omega_i = \omega_2 = \omega_3$$

and

$$x = \frac{N_{14}}{N_{16}}$$

or

$$x = \frac{N_{15}}{N_{17}}$$

and

$$y = \frac{N_3}{N_4}$$

or

$$y = \frac{N_2}{N_6}$$

Case a) $y = \frac{N_3}{N_4}$

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For $x = \frac{N_{14}}{N_{16}}$, the output velocity is

$$\omega_o = \omega_i\left(\frac{N_3}{N_4}\frac{N_5}{N_7}\frac{N_8}{N_9}\frac{N_{10}}{N_{11}}\frac{N_{12}}{N_{13}}\right)x = 1800\left(\frac{21}{35}\frac{23}{48}\frac{39}{40}\frac{23}{34}\frac{40}{46}\right)x$$
$$= 296.80x = 296.80\frac{N_{14}}{N_{16}} = 296.80\frac{68}{55} = 366.95 \text{ rpm}$$

For $x = \frac{N_{15}}{N_{17}}$, the output velocity is

$$\omega_o = \omega_i\left(\frac{N_3}{N_4}\frac{N_5}{N_7}\frac{N_8}{N_9}\frac{N_{10}}{N_{11}}\frac{N_{12}}{N_{13}}\right)x = 296.80\frac{N_{15}}{N_{17}} = 296.80\frac{21}{102} = 61.11 \text{ rpm}$$

Case a) $y = \frac{N_2}{N_6}$ and $x = \frac{N_{14}}{N_{16}}$ but N_2 and N_6 are to be determined.

In this case, $\omega_o \cong 130$ and $\omega_i = 1800$. Then,

$$\omega_o = \frac{N_2}{N_6}\omega_i\left(\frac{N_5}{N_7}\frac{N_8}{N_9}\frac{N_{10}}{N_{11}}\frac{N_{12}}{N_{13}}\frac{N_{14}}{N_{16}}\right)$$

or

$$\frac{N_2}{N_6} = \frac{\omega_o}{\omega_i\left(\frac{N_5}{N_7}\frac{N_8}{N_9}\frac{N_{10}}{N_{11}}\frac{N_{12}}{N_{13}}\frac{N_{14}}{N_{16}}\right)} = \frac{130}{1800\left(\frac{23}{48}\frac{39}{40}\frac{23}{34}\frac{40}{46}\frac{68}{55}\right)} = 0.21256$$

The center distance must be the same for both gears 3 and 4 and 2 and 6 if the diametral pitches are the same. Therefore,

$$d_2 + d_6 = d_3 + d_4$$

Therefore,

$$\frac{N_2}{P_d} + \frac{N_6}{P_d} = \frac{N_3}{P_d} + \frac{N_4}{P_d}$$

or

$$N_2 + N_6 = N_3 + N_4 = 21 + 35 = 56 \qquad (1)$$

Also,

$$\frac{N_2}{N_6} = 0.21256 \Rightarrow N_2 = (0.21256)N_6 \qquad (2)$$

Combining the two equations,

$$N_2 + N_6 = N_6(1 + 0.21256) = (1.21256)N_6 = 56$$

Therefore,

$$N_6 = \frac{56}{1.21256} = 46.18$$

Because the tooth numbers must be integers,

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$N_6 = 46$

and

$N_2 = 10$

The actual output speed would be

$$\omega_o = \omega_i\left(\frac{N_2}{N_6}\frac{N_5}{N_7}\frac{N_8}{N_9}\frac{N_{10}}{N_{11}}\frac{N_{12}}{N_{13}}\frac{N_{14}}{N_{16}}\right) = 1800\left(\frac{10}{46}\frac{23}{48}\frac{39}{40}\frac{23}{34}\frac{40}{46}\frac{68}{55}\right) = 132.95 \text{ rpm}$$

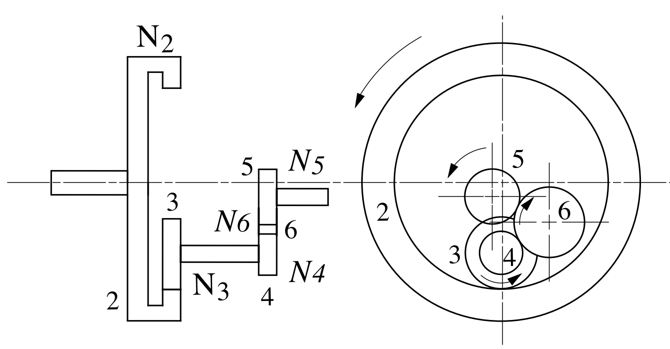
This is within the allowable limits; however, the small number of teeth on gear 2 is likely to result in undercutting unless helical gears are used.

Problem 12.6

An internal gear having 160 teeth and rotating counterclockwise at 30 rpm is connected through a gear train to an external gear, which rotates at 120 rpm in the counterclockwise direction. Using the minimum number of gears, select gears from the following list that will satisfy the design requirements. Tooth numbers for the available gears are 20, 22, 25, 30, 32, 34, 35, 40, 50, 55, 60, and 64. There is only one gear with each tooth number, and each gear has the same diametral pitch.

Solution:

The simplest gear train which will satisfy the design requirements is shown below. The idler gear (gear 6) may be any of the gears available. For example, we can use $N_6 = 30$.



The angular velocity ratio for the gear train is:

$$\frac{\omega_5}{\omega_2} = \left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right)$$

For the values given,

$$\left(\frac{1}{N_3}\right)\left(\frac{N_4}{N_5}\right) = \frac{1}{N_2}\frac{\omega_5}{\omega_2} = \frac{1}{160}\frac{120}{30} = 0.025$$

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We must therefore look for different combinations of tooth numbers from the set given to satisfy this requirement. This can be done directly or a simple program can be written to consideration all possible combinations. Such a MATLAB program is given below.

```
% Problem 12.6

N=[20, 22, 25, 30, 32, 34, 35, 40, 50, 55, 60, 64]

for i=1:1:12
   N4=N(i);
   for j=1:1:12
      N3=N(j);
      for k=1:1:12
         N5=N(k);
         if (N4~= N3) & (N4~=N5) & (N3~= N5)
            product = N4/(N3*N5);
            if product == 0.025
               [product, N4, N3, N5]
            end
         end
      end
   end
end
```

The results from the program are five sets of tooth numbers:

$N_4 = 32$, $N_3 = 64$, and $N_5 = 20$
$N_4 = 40$, $N_3 = 25$, and $N_5 = 64$
$N_4 = 40$, $N_3 = 32$, and $N_5 = 50$
$N_4 = 40$, $N_3 = 50$, and $N_5 = 32$
$N_4 = 40$, $N_3 = 64$, and $N_5 = 25$

Any of the sets will satisfy the requirements of the problem.

Problem 12.7

Resolve Problem 12.6 if the external gear is concentric with the internal gear (the rotation axis is the same for both gears) and the external gear rotates clockwise.

Solution:

For a concentric gear box, the shaft of gear 5 must be concentric with that of gear 2. If the axis of rotation is to be the same for both gears, we can easily make the gear box a concentric gear box by using the same tooth numbers as were determined in Problem 12.6 but locating the output shaft to be concentric with the input shaft.

We can use this simple solution because $N_2 - N_3 > N_4 + N_5$. Because all of the gears have the same diametral pitch, $d_2 - d_3 > d_4 + d_5$. This arrangement is shown in below.

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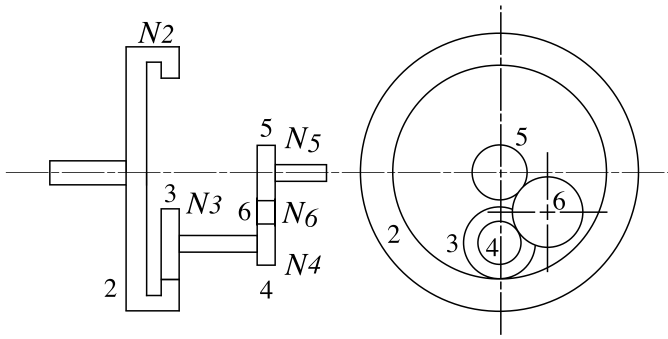

The idler gear (gear 6) can be any of the gears available. For example, we can use $N_6 = 22$.

Problem 12.8

Resolve Problem 12.6 if the external gear is concentric with the internal gear and the external gear rotates counterclockwise.

Solution:

For this problem, we can use two idler gears in series or no idler gears. Using no idler gears is obviously simpler if it will work. The geometry would be as shown in Fig. P12.8a. For this geometry to work, $N_2 - N_3 = N_4 + N_5$. From the solution to problem 12.6, the solution must be chosen among

$N_4 = 32$, $N_3 = 64$, and $N_5 = 20$
$N_4 = 40$, $N_3 = 25$, and $N_5 = 64$
$N_4 = 40$, $N_3 = 32$, and $N_5 = 50$
$N_4 = 40$, $N_3 = 50$, and $N_5 = 32$
$N_4 = 40$, $N_3 = 64$, and $N_5 = 25$

and $N_2 = 160$.

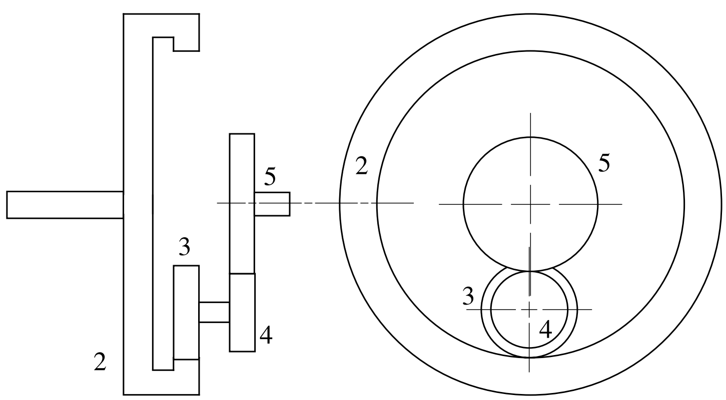

Fig. P12.8a

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Therefore, $N_2 - N_3 > N_4 + N_5$
One solution is to use two idler gears as shown in Fig. P12.8b. Tooth numbers which will work are, $N_4 = 40$, $N_3 = 50$, $N_5 = 32$, $N_6 = 34$, and $N_7 = 32$. If there were more latitude on the choice of tooth numbers and/or diametral pitch, it might be possible to develop a design along the lines of Fig. 12.8a which would be considerably less expensive to manufacture.

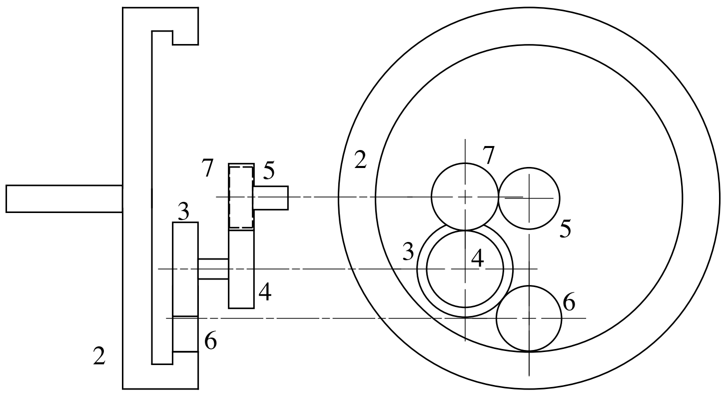


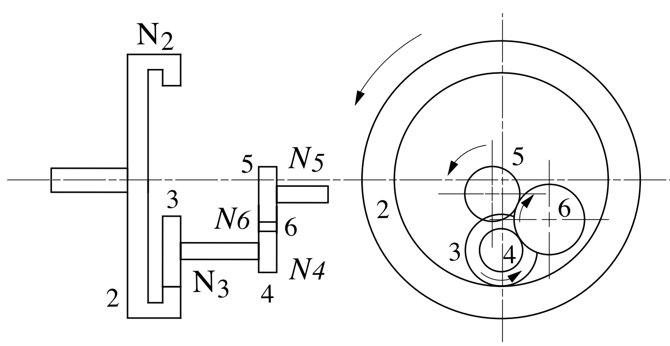
Fig. P12.8b

Problem 12.9

Resolve Problem 12.6 if the external gear rotates at 50 rpm.

Solution:

The simplest gear train which will satisfy the design requirements is shown below. The idler gear (gear 6) may be any of the gears available. For example, we can use $N_6 = 30$.



The angular velocity ratio for the gear train is:

$$\frac{\omega_5}{\omega_2} = \left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right)$$

For the values given,

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$$\left(\frac{1}{N_3}\right)\left(\frac{N_4}{N_5}\right) = \frac{1}{N_2}\frac{\omega_5}{\omega_2} = \frac{1}{160}\frac{120}{50} = 0.015$$

We must therefore look for different combinations of tooth numbers from the set given to satisfy this requirement. This can be done directly or a simple program can be written to consideration all possible combinations. Such a MATLAB program is given below.

```
% Problem 12.6

N=[20, 22, 25, 30, 32, 34, 35, 40, 50, 55, 60, 64]

for i=1:1:12
   N4=N(i);
   for j=1:1:12
      N3=N(j);
      for k=1:1:12
         N5=N(k);
         if (N4~= N3) & (N4~=N5) & (N3~= N5)
            product = N4/(N3*N5);
            if product == 0.025
               [product, N4, N3, N5]
            end
         end
      end
   end
end
```

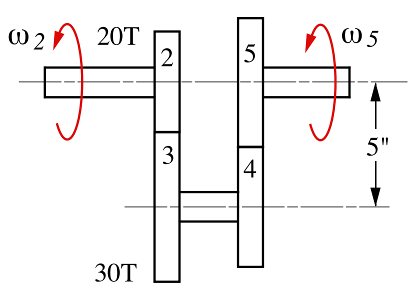
The results from the program are two sets of tooth numbers:

$N_4 = 30$, $N_3 = 40$, and $N_5 = 50$
$N_4 = 30$, $N_3 = 50$, and $N_5 = 40$

Any of the sets will satisfy the requirements of the problem.

Problem 12.10

A gear reducer is to be designed as shown in the figure. Determine the diametral pitch and number of teeth on gears 4 and 5 if the speed of gear 2 (ω_2) is to be 10 times the speed of gear 5(ω_5
pitches of the two gears should be as nearly equal as possible, and no gear should have fewer than 15 teeth.



Solution:

The angular velocity ratio for the gear train is:

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$$\frac{\omega_5}{\omega_2} = \left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right) = \frac{1}{10} = 0.1$$

Then,

$$\left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right) = 0.1 = \left(\frac{20}{30}\right)\left(\frac{N_4}{N_5}\right) \Rightarrow \left(\frac{N_4}{N_5}\right) = 0.15 \qquad (1)$$

and

$$N_5 = N_4 / 0.15$$

Because a concentric gear train is involved,

$$d_2 + d_3 = d_4 + d_5 \qquad (2)$$

Also, for the gears to mesh properly,

$$P_{d2} = P_{d3} \Rightarrow \frac{N_2}{d_2} = \frac{N_3}{d_3}$$

and

$$P_{d4} = P_{d5} \Rightarrow \frac{N_4}{d_4} = \frac{N_5}{d_5}$$

Therefore, Eq. (2) can be written as

$$\frac{N_2 + N_3}{P_{d2}} = \frac{N_4 + N_5}{P_{d4}}$$

Substituting the known values,

$$\frac{20 + 30}{P_{d2}} = \frac{N_4 / 0.15 + N_4}{P_{d4}}$$

or

$$\frac{50}{P_{d2}} = \frac{N_4(1.15 / 0.15)}{P_{d4}}$$

and

$$N_4 = \frac{7.5}{1.15}\frac{P_{d4}}{P_{d2}} \qquad (3)$$

If we can make the diametral pitches equal,

$$N_4 = \frac{7.5}{1.15} = 6.52$$

which is not an integer and which is less than 15. Therefore, the diametral pitches must be different . If we are to look at several different diametral pitches, it is convenient to write a program to look at different choices. A MATLAB program for this is given in the following.

```
% Problem 12.10

P=[1, 1.25, 1.5, 1.75,2,2.5,3,3.5,4,6,8,10,12,14,16,18,20,...
   24,28,32,36,40,44,48,52,56];

N2=20;
N3=30;
```


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```
rat=0.1;
fact=rat*N3/N2
for i=1:1:26
   Pd2=P(i);
   for j=1:1:26
      Pd4=P(j);
      for k=15:1:200
         N4=k;
         N5=N4/fact;
         N4T=(N2+N3)/(1+1/fact)*(Pd4/Pd2);
         if abs(N4T-N4)<0.1 & fix(N5)==N5
            [N4,N5,Pd2, Pd4]
         end
      end
   end
end
```

The following eight sets of values are returned by the program.

N_4	N_5	P_{d_2}	P_{d_4}
87	580	1.5	20
174	1160	1.5	40
15	100	1.75	4
87	580	3	40
15	100	3.5	8
24	160	12	44
15	100	14	32
18	120	16	44

The solutions which have diametral pitches which are most similar are:

N_4	N_5	P_{d_2}	P_{d_4}
15	100	1.75	4
15	100	3.5	8
24	160	12	44
15	100	14	32
18	120	16	44

The final selection can be made based on considerations other than kinematics. In particular, power requirements need to be considered.

Problem 12.11

Resolve Problem 12.10 if ω_2 is to be 8 times the speed of gear 5(ω_5).

Solution:

The angular velocity ratio for the gear train is:

$$\frac{\omega_5}{\omega_2} = \left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right) = \frac{1}{8}$$

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Then,

$$\left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right) = \frac{1}{8} = \left(\frac{20}{30}\right)\left(\frac{N_4}{N_5}\right) \Rightarrow \left(\frac{N_4}{N_5}\right) = 0.1875 \qquad (1)$$

and

$$N_5 = N_4 / 0.1875$$

Because a concentric gear train is involved,

$$d_2 + d_3 = d_4 + d_5 \qquad (2)$$

Also, for the gears to mesh properly,

$$P_{d_2} = P_{d_3} \Rightarrow \frac{N_2}{d_2} = \frac{N_3}{d_3}$$

and

$$P_{d_4} = P_{d_5} \Rightarrow \frac{N_4}{d_4} = \frac{N_5}{d_5}$$

Therefore, Eq. (2) can be written as

$$\frac{N_2 + N_3}{P_{d_2}} = \frac{N_4 + N_5}{P_{d_4}}$$

Substituting the known values,

$$\frac{20 + 30}{P_{d_2}} = \frac{N_4 / 0.1875 + N_4}{P_{d_4}}$$

or

$$\frac{50}{P_{d_2}} = \frac{N_4(1.1875 / 0.1875)}{P_{d_4}}$$

and

$$N_4 = \frac{9.375}{1.1875}\frac{P_{d_4}}{P_{d_2}} \qquad (3)$$

If we can make the diametral pitches equal,

$$N_4 = \frac{9.375}{1.1875} = 7.89$$

which is not an integer and which is less than 15. Therefore, the diametral pitches must be different . If we are to look at several different diametral pitches, it is convenient to write a program to look at different choices. A MATLAB program for this is given in the following.

```
% Problem 12.11

P=[1, 1.25, 1.5, 1.75,2,2.5,3,3.5,4,6,8,10,12,14,16,18,20,...
   24,28,32,36,40,44,48,52,56];

N2=20;
N3=30;
rat=1/8;
```


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```
fact=rat*N3/N2
for i=1:1:26
   Pd2=P(i);
   for j=1:1:26
      Pd4=P(j);
      for k=15:1:200
         N4=k;
         N5=N4/fact;
         N4T=(N2+N3)/(1+1/fact)*(Pd4/Pd2);
         if abs(N4T-N4)<0.1 & fix(N5)==N5
            [N4,N5,Pd2, Pd4]
         end
      end
   end
end
```

The following 13 sets of values are returned by the program.

N_4	N_5	P_{d2}	P_{d4}
21	112	1.5	4
18	96	1.75	4
27	144	1.75	6
36	192	1.75	8
21	112	3	8
18	96	3.5	8
27	144	3.5	12
36	192	3.5	16
21	112	6	16
21	112	12	32
18	96	14	32
27	144	14	48
21	112	18	48

The solutions which have diametral pitches which are most similar are:

N_4	N_5	P_{d2}	P_{d4}
21	112	1.5	4
18	96	3.5	8
21	112	6	16
21	112	12	32
18	96	14	32
21	112	18	48

The final selection can be made based on considerations other than kinematics. In particular, power requirements need to be considered.

Problem 12.12

Resolve Problem 12.10 if ω_2 is to be 6.5 times the speed of gear 5(ω_5).

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Solution:

The angular velocity ratio for the gear train is:

$$\frac{\omega_5}{\omega_2} = \left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right) = \frac{1}{6.5}$$

Then,

$$\left(\frac{N_2}{N_3}\right)\left(\frac{N_4}{N_5}\right) = \frac{1}{6.5} = \left(\frac{20}{30}\right)\left(\frac{N_4}{N_5}\right) \Rightarrow \left(\frac{N_4}{N_5}\right) = \frac{3}{13} \qquad (1)$$

and

$$N_5 = N_4(13/3)$$

Because a concentric gear train is involved,

$$d_2 + d_3 = d_4 + d_5 \qquad (2)$$

Also, for the gears to mesh properly,

$$P_{d_2} = P_{d_3} \Rightarrow \frac{N_2}{d_2} = \frac{N_3}{d_3}$$

and

$$P_{d_4} = P_{d_5} \Rightarrow \frac{N_4}{d_4} = \frac{N_5}{d_5}$$

Therefore, Eq. (2) can be written as

$$\frac{N_2 + N_3}{P_{d_2}} = \frac{N_4 + N_5}{P_{d_4}}$$

Substituting the known values,

$$\frac{20 + 30}{P_{d_2}} = \frac{N_4(13/3) + N_4}{P_{d_4}}$$

or

$$\frac{50}{P_{d_2}} = \frac{N_4(16/3)}{P_{d_4}}$$

and

$$N_4 = \frac{150}{16}\frac{P_{d_4}}{P_{d_2}} \qquad (3)$$

If we can make the diametral pitches equal,

$$N_4 = \frac{150}{16} = 9.37$$

which is not an integer and which is less than 15. Therefore, the diametral pitches must be different . If we are to look at several different diametral pitches, it is convenient to write a program to look at different choices. There are likely to be a number of solutions to this problem, and for this reason, we wil apply the additional constraint that $P_{d_4} < 2P_{d_2}$. A MATLAB program for this problem is giv en in the following.

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```
% Problem 12.12

P=[1, 1.25, 1.5, 1.75,2,2.5,3,3.5,4,6,8,10,12,14,16,18,20,...
   24,28,32,36,40,44,48,52,56];

N2=20;
N3=30;
rat=1/6.5;
fact=rat*N3/N2;
for i=1:1:26
   Pd2=P(i);
   for j=1:1:26
      Pd4=P(j);
      for k=15:1:200
         N4=k;
         N5=N4/fact;
         N4T=(N2+N3)/(1+1/fact)*(Pd4/Pd2);
         if abs(N4T-N4)<0.1 & fix(N5)==N5 & Pd4<2.0*Pd2
            [N4,N5,Pd2, Pd4]
         end
      end
   end
end
```

The following 4 sets of values are returned by the program.

N_4	N_5	P_{d2}	P_{d4}
15	65	1.25	2
15	65	2.5	4
15	65	10	16
15	65	20	32

Other solutions can be obtained if we allow the diametral pitches to vary more relative to each other. The final selection can be made based on considerations other than kinematics. In particular, power requirements need to be considered.

Problem 12.13

The gear train shown is a candidate for the spindle drive of a gear hobbing machine. The gear blank and the worm gear (gear 10) are mounted on the same shaft and rotate together. If the gear blank is to be driven clockwise, determine the hand of the hob. Also determine the velocity ratio (ω_8 / ω_6) to cut 72 teeth on the gear blank.

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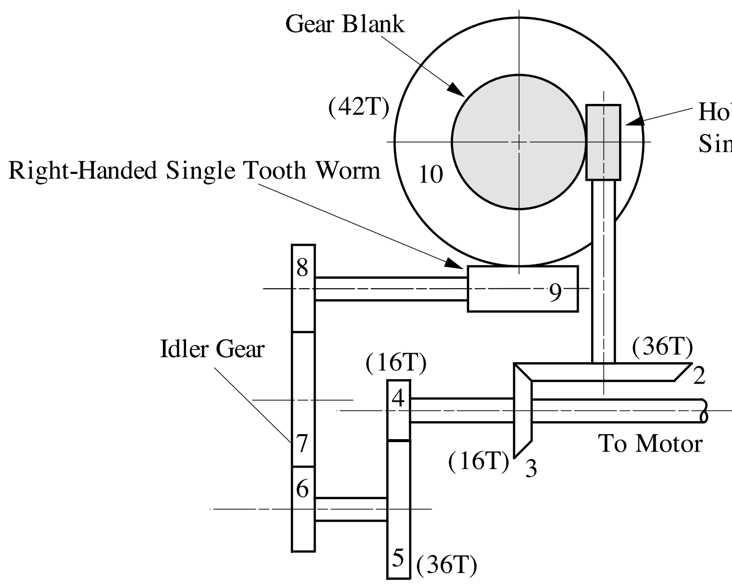

Solution:

To determine the hand for the hob, start from gear 10 and determine the direction of rotation of gear 2. This is as shown in the Fig. P12.13. From the figure, the hob must be left handed.

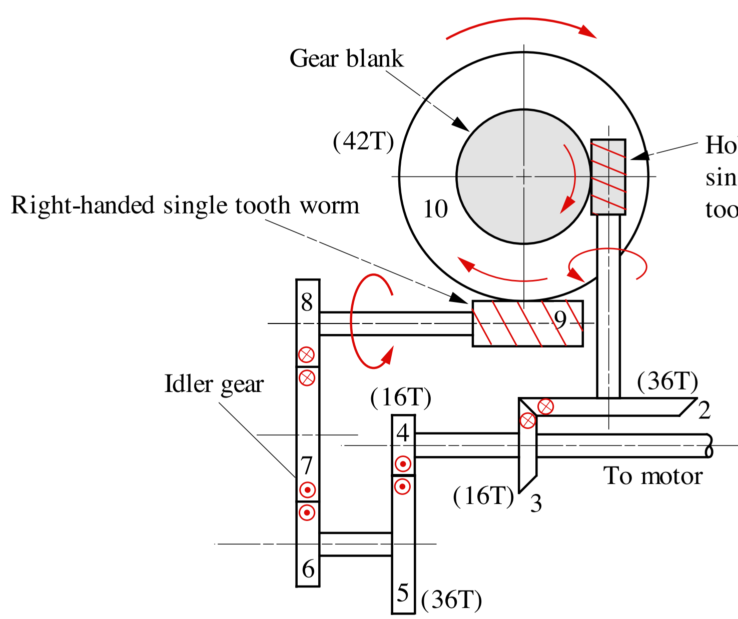

Fig. P12.13

For the velocity analysis, treat gear 10 (and the worm blank) as the input and assume that its angular velocity is 1 rpm. Then the hob and gear 2 must rotate at 72 rpm. Then,

$$\frac{\omega_2}{\omega_{10}} = \left(\frac{N_{10}}{N_9}\right)\left(\frac{N_8}{N_6}\right)\left(\frac{N_5}{N_4}\right)\left(\frac{N_3}{N_2}\right) = \frac{72}{1}$$

Substituting the known values for the tooth numbers,

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$$\left(\frac{42}{1}\right)\left(\frac{N_8}{N_6}\right)\left(\frac{36}{16}\right)\left(\frac{16}{36}\right) = 72$$

or

$$\left(\frac{N_8}{N_6}\right) = \frac{72}{42} = 1.7143$$

Because,

$$\left(\frac{\omega_6}{\omega_8}\right) = \left(\frac{N_8}{N_6}\right) = 1.7143$$

Then,

$$\left(\frac{\omega_8}{\omega_6}\right) = 0.5833$$

Problem 12.14

Assume that the input shaft of a transmission rotates clockwise at 1800 rpm. The output shaft is driven at 160 rpm in the counterclockwise direction. None of the gears in the transmission is to be an idler, and the gear ratio at any given mesh is not to exceed 3:1. Gears are available that have all tooth numbers between 13 and 85; however, only one gear is available with each tooth number. Select the appropriate gears for the transmission, and sketch the configuration designed. Label the gears and tooth numbers.

Solution:

The transmission ration is (1800/160) or 11.25. If no mesh ratio can exceed three, we need to have at least three gear meshes. Because the angular velocity ratio changes sign at each gear mesh, we can solve this problem with three gear meshes. A schematic of the solution is shown in Fig. P12.14 .

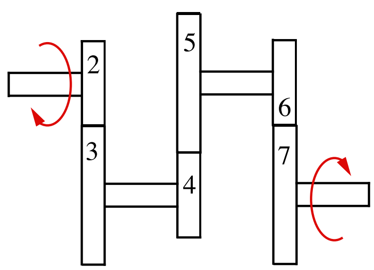

Fig. P12.14

From the transmission ratio,

$$\frac{\omega_2}{\omega_7} = \left(\frac{N_3}{N_2}\right)\left(\frac{N_5}{N_4}\right)\left(\frac{N_6}{N_7}\right) = \frac{1800}{160} = \frac{45}{4}$$

We must select different values for the gear tooth numbers to determine the combinations which will work. Again, it is convenient to do this with a computer program which can check all possible combinations of gears. A MATLAB program for doing this is given in the following.

```
% Problem 12.14
```


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```
rat=1800/160;
for N2=13:1:85
   N3max=3*N2;
   if N3max>85; N3max=85; end
   for N3=N2+1:1:N3max
      for N4=13:1:85
         N5max=3*N4;
         if N5max>85; N5max=85; end
         for N5=N4+1:1:N5max
            for N6=13:1:85
               N7max=3*N6;
               if N7max>85; N7max=85; end
               for N7=N6+1:1:N7max
                  fact=(N3/N2)*(N5/N4)*(N7/N6);
                  diff=abs(rat-fact);
                  if diff<0.00000004
                     [N2, N3, N4, N5, N6, N7]
                  end
               end
            end
         end
      end
   end
end
```

A large number of combinations of tooth numbers satisfy the problem. Some of the combinations are given in the following table.

N_2	N_3	N_4	N_5	N_6	N_7
13	18	14	39	24	70
13	18	14	42	24	65
13	18	15	45	24	65
13	18	16	44	22	65
13	18	16	45	18	52
13	18	16	45	27	78
13	18	16	46	23	65
13	18	16	48	24	65
13	18	17	51	24	65
13	18	18	52	16	45
13	18	19	57	24	65
13	18	20	55	22	65
13	18	20	60	24	65
13	18	21	63	24	65
13	18	22	65	16	44

Problem 12.15

Resolve Problem 12.14 if the output shaft rotates at 210 rpm in the counterclockwise direction.

Solution:

The transmission ration is (1800/210) or 8.57. We could satisfy the mesh ratio with two reductions ;however, the direction of the output would be opposite that required. Therefore, we need to have

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three gear meshes. A schematic of the solution is shown in Fig. P12.15.

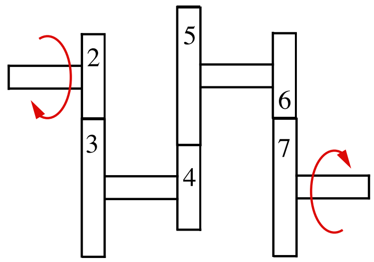

Fig. P12.15

From the transmission ratio,

$$\frac{\omega_2}{\omega_7} = \left(\frac{N_3}{N_2}\right)\left(\frac{N_5}{N_4}\right)\left(\frac{N_6}{N_7}\right) = \frac{1800}{210} = \frac{60}{7}$$

We must select different values for the gear tooth numbers to determine the combinations which will work. Again, it is convenient to do this with a computer program which can check all possible combinations of gears. A MATLAB program for doing this is given in the following.

```
% Problem 12.15

rat=1800/210;
for N2=13:1:85
   N3max=3*N2;
   if N3max>85; N3max=85; end
   for N3=N2+1:1:N3max
      for N4=13:1:85
         N5max=3*N4;
         if N5max>85; N5max=85; end
         for N5=N4+1:1:N5max
            for N6=13:1:85
               N7max=3*N6;
               if N7max>85; N7max=85; end
               for N7=N6+1:1:N7max
                  fact=(N3/N2)*(N5/N4)*(N7/N6);
                  diff=abs(rat-fact);
                  if diff<0.00000004
                     [N2, N3, N4, N5, N6, N7]
                  end
               end
            end
         end
      end
   end
end
```

A large number of combinations of tooth numbers satisfy the problem. Some of the combinations are given in the following table.

N_2	N_3	N_4	N_5	N_6	N_7
13	14	21	60	28	78
13	14	28	78	21	60

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13	15	13	39	21	52
13	15	14	36	18	52
13	15	14	36	27	78
13	15	14	38	19	52
13	15	14	39	18	48
13	15	14	39	21	56
13	15	14	39	24	64
13	15	14	39	27	72
13	15	14	39	30	80
13	15	14	40	20	52
13	15	14	40	25	65
13	15	14	40	30	78
13	15	14	42	21	52

Problem 12.16

Resolve Problem 12.14 if the output shaft rotates at 200 rpm in the clockwise direction.

Solution:

The transmission ration is (1800/200) or 9. We can satisfy both the mesh ratio and the input/output directions with two reductions. A schematic of the solution is shown in Fig. P12.16.

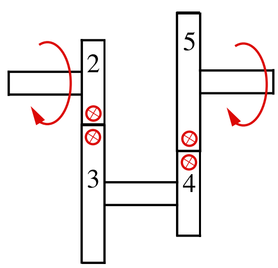

Fig. P12.16

From the transmission ratio,

$$\frac{\omega_2}{\omega_5} = \left(\frac{N_3}{N_2}\right)\left(\frac{N_5}{N_4}\right) = \frac{1800}{200} = 9$$

We can satisfy this requirement by making $N_3 = 3N_2$ and $N_5 = 3N_4$. There are a large number of possibilities from the set of gears available. Six examples are given in the following table.

N_2	N_3	N_4	N_5
13	39	14	42
13	39	15	45
13	39	16	48
13	39	17	51
13	39	18	54
13	39	19	57

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Problem 12.17

A simple gear reduction is to be used to generate a gear ratio equal to π. Make up a table of possible gear ratios where the maximum number of teeth on either gear is 100. This can be conveniently done using a simple computer program. Identify the gear set that most closely approximates the desired ratio. What is the error?

Solution:

We can write the gear ratio to be determined as

$$\left(\frac{N_2}{N_3}\right) = \pi$$

This problem can be solved easily using MATLAB. For this, we can increment N_3 from 1 to $100/\pi$ or approximately 32. We will increment N_2 from $2.9N_3$ to $3.3N_3$. The program is given in the following. The results are printed only when the error is less than 0.35 percent.

```
% Problem 12.18
format compact;

for N3=1:1:32
   N2min=fix(2.9*N3);
   N2max=fix(3.3*N3);
   for N2=N2min:1:N2max
      rat=N2/N3;
      error=100*(pi-rat)/pi;
      if abs(error) < 0.35;
         [N2, N3, error]
      end
   end
end
```

The results are given in the following.

N_2	N_3	N_2 / N_3	Percent Error
22	7	3.1429	-0.0402
44	14	3.1429	-0.0402
47	15	3.1333	0.2629
63	20	3.1500	-0.2676
66	21	3.1429	-0.0402
69	22	3.1364	0.1664
85	27	3.1481	-0.2087
88	28	3.1429	-0.0402
91	29	3.1379	0.1166
94	30	3.1333	0.2629

There are four gear sets which have the lowest error. The error value is 0.0402 percent.

Problem 12.18

A simple gear reduction is to be used to generate the gear ratio 0.467927. Make up a table of

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possible gear ratios where the maximum number of teeth on either gear is 100. Identify the gear set that most closely approximates the desired ratio. What is the error?

<u>Solution:</u>

We can write the gear ratio to be determined as

$$\left(\frac{N_2}{N_3}\right) = 0.467927$$

This problem can be solved easily using MATLAB. For this, we can increment N_3 from 2 to 46.7 or approximately 47. We will increment N_2 from $0.4N_3$ to $0.5N_3$. The program is given in the following. The results are printed only when the error is less than 0.35 percent.

```
% Problem 12.19
format compact;

for N3=2:1:47
   N2min=fix(0.4*N3);
   N2max=fix(0.5*N3);
   for N2=N2min:1:N2max
      rat=N2/N3;
      error=100*(0.467927-rat)/0.467927;
      if abs(error) < 0.35;
         [N2, N3,rat, error]
      end
   end
end
```

The results are given in the following. There are five gear sets identified by the program.

N_2	N_3	N_2 / N_3	Percent Error
7	15	0.4667	0.2693
14	30	0.4667	0.2693
15	32	0.4688	-0.1759
21	45	0.4667	0.2693
22	47	0.4681	-0.0338

The gear set with the lowest error is the last one in the table.

Problem 12.19

A simple gear reduction is to be used to generate a gear ratio equal to $\sqrt{2}$. Make up a table of possible gear ratios where the maximum number of teeth on either gear is 100. Identify the gear set that most closely approximates the desired ratio. What is the error?

<u>Solution:</u>

We can write the gear ratio to be determined as

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$$\left(\frac{N_2}{N_3}\right) = \sqrt{2}$$

This problem can be solved easily using MATLAB. For this, we can increment N_3 from 1 to $100 / \sqrt{2}$ or approximately 71. We will increment N_2 from $1.4 N_3$ to $1.5 N_3$
n in the following. The results are printed only when the error is less than 0.1 percent.

```
% Problem 12.20
format compact;

for N3=1:1:71
   N2min=fix(1.4*N3);
   N2max=fix(1.5*N3);
   for N2=N2min:1:N2max
      rat=N2/N3;
      error=100*(sqrt(2)-rat)/sqrt(2);
      if abs(error) < 0.1;
         [N2, N3,rat,error]
      end
   end
end
```

The results are given in the following. There are seven gear sets identified by the program.

N_2	N_3	N_2 / N_3	Percent Error
41	29	1.4138	0.0297
58	41	1.4146	-0.0297
65	46	1.4130	0.0827
75	53	1.4151	-0.0623
82	58	1.4138	0.0297
92	65	1.4154	-0.0828
99	70	1.4143	-0.0051

The gear set with the lowest error is the last one in the table.

Problem 12.20

An alternative gear train is shown below as a candidate for the spindle drive of a gear hobbing machine. The gear blank and the worm gear (gear 9) are mounted on the same shaft and rotate together. If the gear blank is to be driven clockwise, determine the hand of the hob. Next determine the velocity ratio (ω_3 / ω_5) to cut 75 teeth on the gear blank. Finally, select gears 3 and 5 which will satisfy the ratio. Gears are available which have all of the tooth numbers from 15 to 40.

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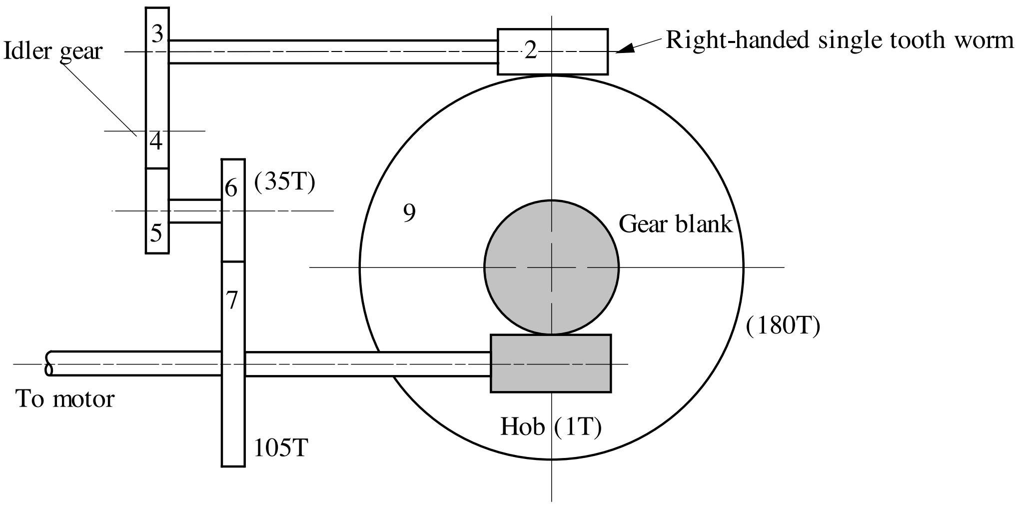

Solution:

To determine the hand for the hob, start from gear 2 and determine the direction of rotation of gear 7. This is as shown in the Fig. P10.18. From the figure, the hob must be right handed.

For the velocity analysis, treat gear 9 (and the worm blank) as the input and assume that its angular velocity is 1 rpm. Then the hob and gear 7 must rotate at 75 rpm. Then,

$$\frac{\omega_9}{\omega_7} = \left(\frac{N_7}{N_6}\right)\left(\frac{N_5}{N_3}\right)\left(\frac{N_2}{N_9}\right) = \frac{1}{75}$$

Substituting the known values for the tooth numbers,

$$\left(\frac{105}{35}\right)\left(\frac{N_5}{N_3}\right)\left(\frac{1}{180}\right) = \frac{1}{75}$$

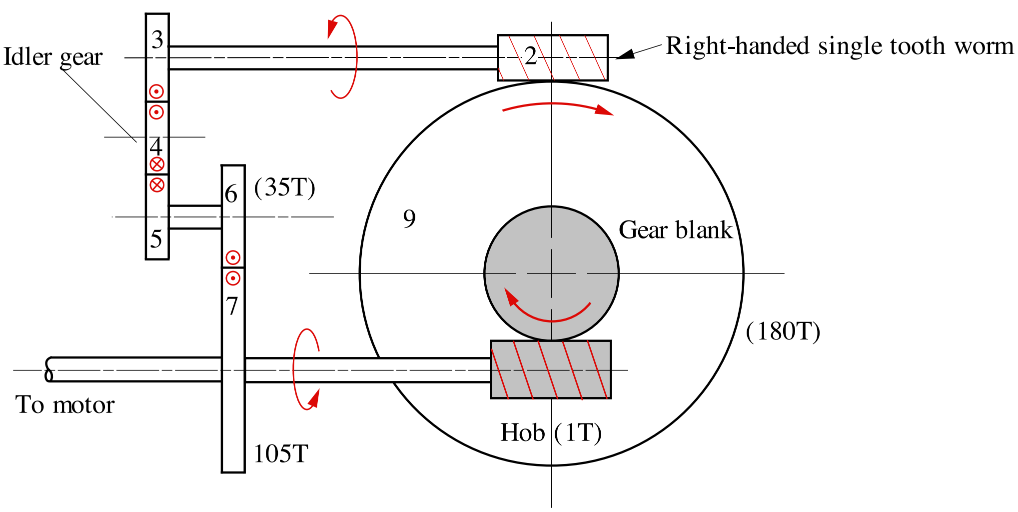

Fig. P10.17

or

$$\left(\frac{N_5}{N_3}\right) = 0.8$$

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Now,

$$\left(\frac{\omega_3}{\omega_5}\right) = \left(\frac{N_5}{N_3}\right) = 0.8$$

To determine the gear numbers, we need only search the available gears tooth numbers to find a nu mber which is 0.8 times another number. Any number ending in 0 or 5 will work for N_3. The allo wable values are:

N_3	N_5
40	32
35	28
30	24
25	20
20	16
15	12
10	8
5	4

If spur gears are used, the tooth numbers of 12 and lower are likely to result in undercutting or interference.

Problem 12.21

A simple gear reduction is to be used to generate the gear ratio equal to 2.105399. Make up a table of possible gear ratios where the maximum number of teeth on all gears is 100. Identify the gear set which most closely approximates the desired ratio. Note that this can be done most easily with a computer program. What is the error?

Solution:

We can write the gear ratio to be determined as

$$\left(\frac{N_2}{N_3}\right) = \sqrt{2}$$

This problem can be solved easily using MATLAB. For this, we can increment N_3 from 1 to $100/2.105399$ or approximately 48. We will increment N_2 from $2N_3$ to $2.2N_3$ given in the following. The results are printed only when the error is less than 0.1 percent.

```
% Problem 10.21
format compact;

for N3=1:1:48
   N2min=fix(2*N3);
   N2max=fix(12.2*N3);
   for N2=N2min:1:N2max
      rat=N2/N3;
      error=100*(2.105399-rat)/2.105399;
      if abs(error) < 0.1;
         [N2, N3,rat, error]
      end
   end
```


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```
end
```

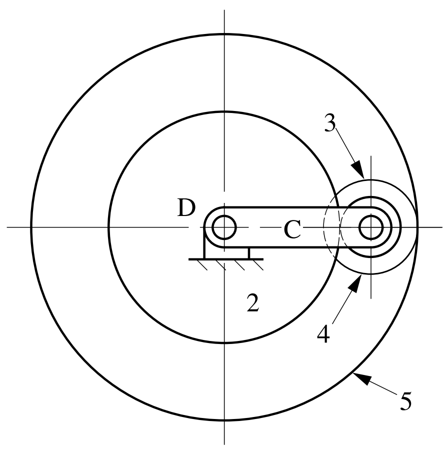
The results are given in the following. There are five gear sets identified by the program.

N_2	N_3	N_2 / N_3	Percent Error
40	19	2.1053	0.0065
59	28	2.1071	-0.0828
61	29	2.1034	0.0927
80	38	2.1053	0.0065
9	47	2.1064	-0.0467

Two gear sets have the lowest error which is 0.0065 percent.

Problem 12.22

In the gear train shown, gears 3 and 4 are integral. Gear 3 meshes with gear 2, and gear 4 meshes with gear 5. If gear 2 is fixed and $\omega_5 = 100$ rpm counterclockwise, determine the angular velocity of the carrier.



$N_2 = 60T$
$N_3 = 16T$
$N_4 = 24T$
$N_5 = 100T$

Solution:

There are four gears (2, 3, 4, and 5) which can rotate about fixed axes in the system. We will include gear 2 in this list but ultimately will use the fact that its velocity is zero. We will solve the problem by writing relative velocity equations for all of the gears which have shafts that can rotate in fixed bearings. After rearranging, the resulting equations are:

$${}^{C}\omega_2 = {}^{1}\omega_2 - {}^{1}\omega_C \qquad (1)$$

$${}^{C}\omega_3 = {}^{1}\omega_3 - {}^{1}\omega_C \qquad (2)$$

$${}^{C}\omega_4 = {}^{1}\omega_4 - {}^{1}\omega_C \qquad (3)$$

and

$${}^{C}\omega_5 = {}^{1}\omega_5 - {}^{1}\omega_C \qquad (4)$$

The angular velocity ratio of gears 2 and 5 relative to the carrier is

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$$\frac{{}^{C}\omega_2}{{}^{C}\omega_5} = \frac{N_5}{N_4}\left(-\frac{N_3}{N_2}\right) = -\frac{N_5}{N_4}\frac{N_3}{N_2} \tag{5}$$

Now, divide Eq. (1) by Eq. (2) and equate the result with Eq. (5). This gives

$$\frac{{}^{1}\omega_2 - {}^{1}\omega_C}{{}^{1}\omega_5 - {}^{1}\omega_C} = -\frac{N_5}{N_4}\frac{N_3}{N_2} \tag{6}$$

Equation (6) is the equation necessary for analyzing the planetary gear train. From the problem statement, we know that ${}^{1}\omega_2 = 0$ and ${}^{1}\omega_5 = 100$. With these known values, only ${}^{1}\omega_C$ in Eq. (6). Substituting the known values into Eq. (6) gives

$$\frac{0 - {}^{1}\omega_C}{100 - {}^{1}\omega_C} = -\frac{100}{24}\frac{16}{60} = -\frac{10}{9}$$

Solving gives

$${}^{1}\omega_C = 52.63 \text{ rpm counter-clockwise}$$

Problem 12.23

Resolve Problem 12.22 if gear 5 is fixed and $\omega_2 = 100$ rpm counterclockwise.

<u>Solution:</u>

There are four gears (2, 3, 4, and 5) which can rotate about fixed axes in the system. We will include gear 2 in this list but ultimately will use the fact that its velocity is zero. We will solve the problem by writing relative velocity equations for all of the gears which have shafts that can rotate in fixed bearings. After rearranging, the resulting equations are:

$${}^{C}\omega_2 = {}^{1}\omega_2 - {}^{1}\omega_C \tag{1}$$

$${}^{C}\omega_3 = {}^{1}\omega_3 - {}^{1}\omega_C \tag{2}$$

$${}^{C}\omega_4 = {}^{1}\omega_4 - {}^{1}\omega_C \tag{3}$$

and

$${}^{C}\omega_5 = {}^{1}\omega_5 - {}^{1}\omega_C \tag{4}$$

The angular velocity ratio of gears 2 and 5 relative to the carrier is

$$\frac{{}^{C}\omega_2}{{}^{C}\omega_5} = \frac{N_5}{N_4}\left(-\frac{N_3}{N_2}\right) = -\frac{N_5}{N_4}\frac{N_3}{N_2} \tag{5}$$

Now, divide Eq. (1) by Eq. (2) and equate the result with Eq. (5). This gives

$$\frac{{}^{1}\omega_2 - {}^{1}\omega_C}{{}^{1}\omega_5 - {}^{1}\omega_C} = -\frac{N_5}{N_4}\frac{N_3}{N_2} \tag{6}$$

Equation (6) is the equation necessary for analyzing the planetary gear train. From the problem

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statement, we know that ${}^1\omega_2 = 100$ and ${}^1\omega_5 = 0$. With these known values, only ${}^1\omega_C$ in Eq. (6). Substituting the known values into Eq. (6) gives

$$\frac{100 - {}^1\omega_C}{0 - {}^1\omega_C} = -\frac{100}{24}\frac{16}{60} = -\frac{10}{9}$$

Solving gives

$${}^1\omega_C = 47.37 \text{ rpm counter-clockwise}$$

Problem 12.24

Resolve Problem 12.22 when $N_2 = 70T$, $N_3 = 35T$, $N_4 = 15T$ and $N_5 = 120$.

Solution:

There are four gears (2, 3, 4, and 5) which can rotate about fixed axes in the system. We will include gear 2 in this list but ultimately will use the fact that its velocity is zero. We will solve the problem by writing relative velocity equations for all of the gears which have shafts that can rotate in fixed bearings. After rearranging, the resulting equations are:

$${}^C\omega_2 = {}^1\omega_2 - {}^1\omega_C \quad (1)$$

$${}^C\omega_3 = {}^1\omega_3 - {}^1\omega_C \quad (2)$$

$${}^C\omega_4 = {}^1\omega_4 - {}^1\omega_C \quad (3)$$

and

$${}^C\omega_5 = {}^1\omega_5 - {}^1\omega_C \quad (4)$$

The angular velocity ratio of gears 2 and 5 relative to the carrier is

$$\frac{{}^C\omega_2}{{}^C\omega_5} = \frac{N_5}{N_4}\left(-\frac{N_3}{N_2}\right) = -\frac{N_5}{N_4}\frac{N_3}{N_2} \quad (5)$$

Now, divide Eq. (1) by Eq. (2) and equate the result with Eq. (5). This gives

$$\frac{{}^1\omega_2 - {}^1\omega_C}{{}^1\omega_5 - {}^1\omega_C} = -\frac{N_5}{N_4}\frac{N_3}{N_2} \quad (6)$$

Equation (6) is the equation necessary for analyzing the planetary gear train. From the problem statement, we know that ${}^1\omega_2 = 0$ and ${}^1\omega_5 = 100$. With these known values, only ${}^1\omega_C$ in Eq. (6). Substituting the known values into Eq. (6) gives

$$\frac{100 - {}^1\omega_C}{0 - {}^1\omega_C} = -\frac{120}{15}\frac{35}{70} = -4$$

Solving gives

$${}^1\omega_C = 20 \text{ rpm counter-clockwise}$$

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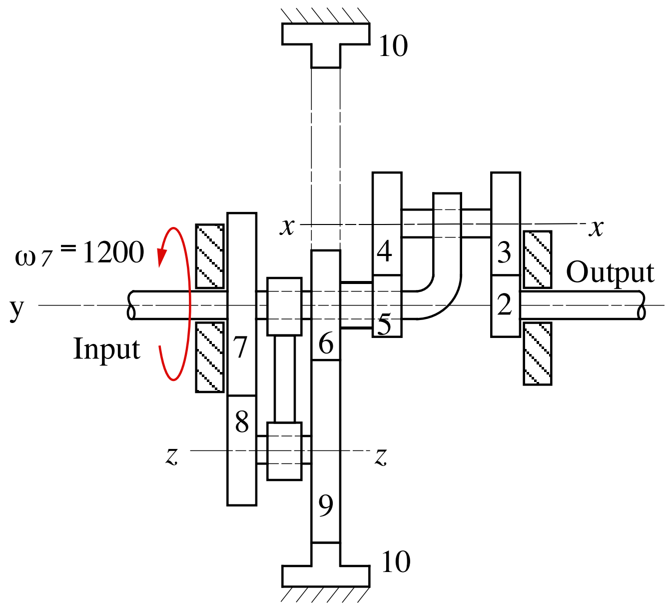
Problem 12.25

In the figure given, axis y-y is fixed while axes x-x and z-z move with the arm. Gear 7 is fixed to the carrier. Gears 3 and 4, 5 and 6, and 8 and 9 are fixed together, respectively. Gears 3 and 4 move with planetary motion. If the tooth numbers are $N_2 = 16$T, $N_3 = 20$T, $N_4 = 22$T, $N_5 = 14$T, $N_6 = 15$T, $N_7 = 36$T, $N_8 = 20$T, $N_9 = 41$T, and $N_{10} = 97$T, determine the speed and direction of the output shaft.



<u>Solution:</u>

There are five gears (2, 5, 6, 7, and 10) which can rotate about fixed axes in the system. Again, we will include the fixed ring gear in the equations and will set the velocity to zero once the equations are developed. As in the previous examples, we will solve the problem by writing relative velocity equations for all of the gears which have shafts that can rotate in fixed bearings. However, we must separate the two stages of the planetary drives when we write the equations. The first stage includes gears 6, 7, 8, 9, and 10 and the carrier. The second stage includes gears 2, 3, 4, and 5, and the second carrier which is fixed to gear 7.

The first stage can be analyzed independently of the second stage to determine the velocity of gear 6 . The second stage can then be analyzed to determine the velocity of gear 2.

The first stage relative velocity equations are:

$$^{C}\omega_7 = {}^{1}\omega_7 - {}^{1}\omega_C \quad (1)$$

$$^{C}\omega_6 = {}^{1}\omega_6 - {}^{1}\omega_C \quad (2)$$

$$^{C}\omega_{10} = {}^{1}\omega_{10} - {}^{1}\omega_C \quad (3)$$

The angular velocity ratio of gears 6 and 7 relative to the first carrier is

$$\frac{^{C}\omega_6}{^{C}\omega_7} = \frac{{}^{1}\omega_6 - {}^{1}\omega_C}{{}^{1}\omega_7 - {}^{1}\omega_C} = \frac{N_7}{N_8}\frac{N_9}{N_6} \quad (4)$$

and the angular velocity ratio of gears 10 and 7 relative to the first carrier is

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$$\frac{{}^{C}\omega_{10}}{{}^{C}\omega_7} = \frac{{}^{1}\omega_{10} - {}^{1}\omega_C}{{}^{1}\omega_7 - {}^{1}\omega_C} = -\frac{N_7}{N_8}\frac{N_9}{N_{10}} \tag{5}$$

We are given that ${}^{1}\omega_{10} = 0$ and ${}^{1}\omega_7 = 1200$. We can therefore solve Eq. (5) for ${}^{1}\omega_C$. Then,

$$\frac{0 - {}^{1}\omega_C}{1200 - {}^{1}\omega_C} = -\frac{36}{20}\frac{41}{97} = -0.76082$$

or

$${}^{1}\omega_C = 518.5$$

We can now solve Eq. (4). Substituting the known values,

$$\frac{{}^{1}\omega_6 - 518.5}{1200 - 518.5} = \frac{36}{20}\frac{41}{15} = 4.92$$

Then,

$${}^{1}\omega_6 = 3871.5 = {}^{1}\omega_5$$

We can now analyze the second stage in exactly the same manner as was done for the first stage except that now the gears are 2, 3, 4, and 5, and the carrier is 7. The second stage relative velocity equations are:

$${}^{7}\omega_2 = {}^{1}\omega_2 - {}^{1}\omega_7$$

$${}^{7}\omega_5 = {}^{1}\omega_5 - {}^{1}\omega_7$$

The angular velocity ratio of gears 2 and 5 relative to the carrier (member 7) is

$$\frac{{}^{7}\omega_2}{{}^{7}\omega_5} = \frac{{}^{1}\omega_2 - {}^{1}\omega_7}{{}^{1}\omega_5 - {}^{1}\omega_7} = \frac{N_5}{N_4}\frac{N_3}{N_2}$$

Substituting the known values,

$$\frac{{}^{1}\omega_2 - 1200}{3871.5 - 1200} = \frac{14}{22}\frac{20}{16} = 0.79545$$

Solving for ${}^{1}\omega_2$ gives

$${}^{1}\omega_2 = 3325.0 \text{ rpm CCW (same direction as that of } {}^{1}\omega_7\text{)}$$

Problem 12.26

Resolve Problem 12.25 when N_2 = 16T, N_3 = 20T, N_4 = 16T, N_5 = 20T, N_6 = 15T, N_7 = 40T, N_8 = 15T, N_9 = 40T, and N_{10} = 95T.

Solution:

There are five gears (2, 5, 6, 7, and 10) which can rotate about fixed axes in the system. Again, we will include the fixed ring gear in the equations and will set the velocity to zero once the equations are developed. As in the previous examples, we will solve the problem by writing relative velocity

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equations for all of the gears which have shafts that can rotate in fixed bearings. However, we must separate the two stages of the planetary drives when we write the equations. The first stage includes gears 6, 7, 8, 9, and 10 and the carrier. The second stage includes gears 2, 3, 4, and 5, and the second carrier which is fixed to gear 7.

The first stage can be analyzed independently of the second stage to determine the velocity of gear 6 . The second stage can then be analyzed to determine the velocity of gear 2.

The first stage relative velocity equations are:

$$ {}^{C}\omega_7 = {}^{1}\omega_7 - {}^{1}\omega_C \tag{1} $$

$$ {}^{C}\omega_6 = {}^{1}\omega_6 - {}^{1}\omega_C \tag{2} $$

$$ {}^{C}\omega_{10} = {}^{1}\omega_{10} - {}^{1}\omega_C \tag{3} $$

The angular velocity ratio of gears 6 and 7 relative to the first carrier is

$$ \frac{{}^{C}\omega_6}{{}^{C}\omega_7} = \frac{{}^{1}\omega_6 - {}^{1}\omega_C}{{}^{1}\omega_7 - {}^{1}\omega_C} = \frac{N_7}{N_8}\frac{N_9}{N_6} \tag{4} $$

and the angular velocity ratio fo gears 10 and 7 relative to the first carrier is

$$ \frac{{}^{C}\omega_{10}}{{}^{C}\omega_7} = \frac{{}^{1}\omega_{10} - {}^{1}\omega_C}{{}^{1}\omega_7 - {}^{1}\omega_C} = -\frac{N_7}{N_8}\frac{N_9}{N_{10}} \tag{5} $$

We are given that ${}^{1}\omega_{10} = 0$ and ${}^{1}\omega_7 = 1200$. We can therefore solve Eq. (5) for ${}^{1}\omega_C$. Then,

$$ \frac{0 - {}^{1}\omega_C}{1200 - {}^{1}\omega_C} = -\frac{40}{15}\frac{40}{95} = -1.1228 $$

or

$$ {}^{1}\omega_C = 634.71 $$

We can now solve Eq. (4). Substituting the known values,

$$ \frac{{}^{1}\omega_6 - 634.71}{1200 - 634.71} = \frac{40}{15}\frac{40}{15} = 7.1111 $$

Then,

$$ {}^{1}\omega_6 = 4654.5 = {}^{1}\omega_5 $$

We can now analyze the second stage in exactly the same manner as was done for the first stage except that now the gears are 2, 3, 4, and 5, and the carrier is 7. The second stage relative velocity equations are:

$$ {}^{7}\omega_2 = {}^{1}\omega_2 - {}^{1}\omega_7 $$

$$ {}^{7}\omega_5 = {}^{1}\omega_5 - {}^{1}\omega_7 $$

The angular velocity ratio of gears 2 and 5 relative to the carrier (member 7) is

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$$\frac{{}^7\omega_2}{{}^7\omega_5} = \frac{{}^1\omega_2 - {}^1\omega_7}{{}^1\omega_5 - {}^1\omega_7} = \frac{N_5}{N_4}\frac{N_3}{N_2}$$

Substituting the known values,

$$\frac{{}^1\omega_2 - 1200}{4654.5 - 1200} = \frac{20}{16}\frac{20}{16} = 1.5625$$

Solving for ${}^1\omega_2$ gives

$${}^1\omega_2 = 6597.7 \text{ rpm CCW (same direction as that of } {}^1\omega_7\text{)}$$

Problem 12.27

Resolve Problem 12.25 when N_2 = 14T, N_3 = 30T, N_4 = 14T, N_5 = 30T, N_6 = 15T, N_7 = 60T, N_8 = 15T, N_9 = 60T, and N_{10} = 135T.

Solution:

There are five gears (2, 5, 6, 7, and 10) which can rotate about fixed axes in the system. Again, we will include the fixed ring gear in the equations and will set the velocity to zero once the equations are developed. As in the previous examples, we will solve the problem by writing relative velocity equations for all of the gears which have shafts that can rotate in fixed bearings. However, we must separate the two stages of the planetary drives when we write the equations. The first stage includes gears 6, 7, 8, 9, and 10 and the carrier. The second stage includes gears 2, 3, 4, and 5, and the
second carrier which is fixed to gear 7.

The first stage can be analyzed independently of the second stage to determine the velocity of gear 6. The second stage can then be analyzed to determine the velocity of gear 2.

The first stage relative velocity equations are:

$${}^C\omega_7 = {}^1\omega_7 - {}^1\omega_C \tag{1}$$

$${}^C\omega_6 = {}^1\omega_6 - {}^1\omega_C \tag{2}$$

$${}^C\omega_{10} = {}^1\omega_{10} - {}^1\omega_C \tag{3}$$

The angular velocity ratio of gears 6 and 7 relative to the first carrier is

$$\frac{{}^C\omega_6}{{}^C\omega_7} = \frac{{}^1\omega_6 - {}^1\omega_C}{{}^1\omega_7 - {}^1\omega_C} = \frac{N_7}{N_8}\frac{N_9}{N_6} \tag{4}$$

and the angular velocity ratio fo gears 10 and 7 relative to the first carrier is

$$\frac{{}^C\omega_{10}}{{}^C\omega_7} = \frac{{}^1\omega_{10} - {}^1\omega_C}{{}^1\omega_7 - {}^1\omega_C} = -\frac{N_7}{N_8}\frac{N_9}{N_{10}} \tag{5}$$

We are given that ${}^1\omega_{10} = 0$ and ${}^1\omega_7 = 1200$. We can therefore solve Eq. (5) for ${}^1\omega_C$. Then,

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$$\frac{0 - {}^1\omega_C}{1200 - {}^1\omega_C} = -\frac{60}{15}\frac{60}{135} = -1.7778$$

or

$${}^1\omega_C = 768$$

We can now solve Eq. (4). Substituting the known values,

$$\frac{{}^1\omega_6 - 768}{1200 - 768} = \frac{60}{15}\frac{60}{15} = 16$$

Then,

$${}^1\omega_6 = 7680 = {}^1\omega_5$$

We can now analyze the second stage in exactly the same manner as was done for the first stage except that now the gears are 2, 3, 4, and 5, and the carrier is 7. The second stage relative velocity equations are:

$${}^7\omega_2 = {}^1\omega_2 - {}^1\omega_7$$

$${}^7\omega_5 = {}^1\omega_5 - {}^1\omega_7$$

The angular velocity ratio of gears 2 and 5 relative to the carrier (member 7) is

$$\frac{{}^7\omega_2}{{}^7\omega_5} = \frac{{}^1\omega_2 - {}^1\omega_7}{{}^1\omega_5 - {}^1\omega_7} = \frac{N_5}{N_4}\frac{N_3}{N_2}$$

Substituting the known values,

$$\frac{{}^1\omega_2 - 1200}{7680 - 1200} = \frac{30}{14}\frac{30}{14} = 4.5918$$

Solving for ${}^1\omega_2$ gives

$${}^1\omega_2 = 30{,}955 \text{ rpm CCW (same direction as that of } {}^1\omega_7\text{)}$$

Problem 12.28

In the gear train shown, gears 2 and 4, 6 and 7, and 3 and 9 are fixed together. If the angular velocity of the carrier is given, determine the angular velocity of gear 9.

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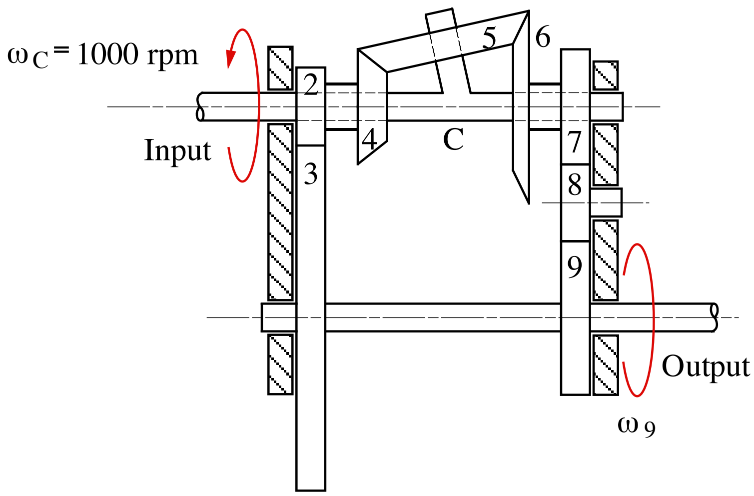

$N_2 = 20T$
$N_3 = 90T$
$N_4 = 30T$
$N_5 = 90T$
$N_6 = 50T$
$N_7 = 30T$
$N_8 = 20T$
$N_9 = 40T$

Solution:

There are six gears (2, 3, 4, 6, 7, and 9) which can rotate about fixed axes in the system. As in Examples 12.3 and 12.4, we will solve the problem by writing relative velocity equations for all of the gears which have shafts that can rotate in fixed bearings. The resulting equations are:

$$^1\omega_4 = {}^1\omega_2 \tag{1}$$

$$^1\omega_6 = {}^1\omega_7 \tag{2}$$

$$^1\omega_3 = {}^1\omega_9 \tag{3}$$

$$^1\omega_4 = {}^1\omega_C + {}^C\omega_4 \tag{4}$$

and

$$^1\omega_6 = {}^1\omega_C + {}^C\omega_6 \tag{5}$$

The angular velocity ratios of the gears which have fixed centerlines can be related through the gear numbers as follows

$$\frac{^1\omega_2}{^1\omega_3} = -\frac{N_3}{N_2} \tag{6}$$

and

$$\frac{^1\omega_7}{^1\omega_9} = \frac{N_9}{N_8}\frac{N_8}{N_7} = \frac{N_9}{N_7} \tag{7}$$

Also relative to the carrier,

$$\frac{^C\omega_4}{^C\omega_6} = -\frac{N_6}{N_5}\frac{N_5}{N_4} = -\frac{N_6}{N_4} \tag{8}$$

In Eq. (8), we determine the sign by inspection; namely, gear 4 rotates in the opposite direction to that of gear 6 relative to the coupler.

Combining Eqs. (4), (6), and (7) gives

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$$\frac{{}^1\omega_4 - {}^1\omega_C}{{}^1\omega_6 - {}^1\omega_C} = -\frac{N_6}{N_4} \tag{9}$$

From Eqs. (1-3), (6), (7),

$${}^1\omega_4 = {}^1\omega_2 = -{}^1\omega_3 \frac{N_3}{N_2}$$

and

$${}^1\omega_6 = {}^1\omega_7 = {}^1\omega_9 \frac{N_9}{N_7} = {}^1\omega_3 \frac{N_9}{N_7}$$

Equation (9) can then be written as

$$\frac{{}^1\omega_4 - {}^1\omega_C}{{}^1\omega_6 - {}^1\omega_C} = -\frac{N_6}{N_4} = \frac{-{}^1\omega_3 \frac{N_3}{N_2} - {}^1\omega_C}{{}^1\omega_3 \frac{N_9}{N_7} - {}^1\omega_C}$$

or

$$-{}^1\omega_3 \frac{N_3}{N_2} - {}^1\omega_C = -\frac{N_6}{N_4}\left({}^1\omega_3 \frac{N_9}{N_7} - {}^1\omega_C\right) = -{}^1\omega_3 \frac{N_9}{N_7}\frac{N_6}{N_4} + {}^1\omega_C \frac{N_6}{N_4}$$

or

$${}^1\omega_3\left(\frac{N_9}{N_7}\frac{N_6}{N_4} - \frac{N_3}{N_2}\right) = {}^1\omega_C\left(1 + \frac{N_6}{N_4}\right)$$

Finally,

$${}^1\omega_3 = {}^1\omega_C \frac{\left(1 + \frac{N_6}{N_4}\right)}{\left(\frac{N_9}{N_7}\frac{N_6}{N_4} - \frac{N_3}{N_2}\right)}$$

For the values given in the problem,

$${}^1\omega_3 = {}^1\omega_9 = {}^1\omega_C \frac{\left(1 + \frac{N_6}{N_4}\right)}{\left(\frac{N_9}{N_7}\frac{N_6}{N_4} - \frac{N_3}{N_2}\right)} = 1000 \frac{\left(1 + \frac{50}{30}\right)}{\left(\frac{40}{30}\frac{50}{30} - \frac{90}{20}\right)} = -1170 \text{ rpm}$$

The value is negative so ${}^1\omega_9$ is rotating clockwise when viewed from the left.

Problem 12.29

Resolve Problem 12.28 if N_2 = 10T, N_3 = 100T, N_7 = 20T, N_8 = 10T and N_9 = 70T.

Solution:

There are six gears (2, 3, 4, 6, 7, and 9) which can rotate about fixed axes in the system. As in Examples 12.3 and 12.4, we will solve the problem by writing relative velocity equations for all of the gears which have shafts that can rotate in fixed bearings. The resulting equations are:

$${}^1\omega_4 = {}^1\omega_2 \tag{1}$$

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$$ {}^1\omega_6 = {}^1\omega_7 \tag{2} $$

$$ {}^1\omega_3 = {}^1\omega_9 \tag{3} $$

$$ {}^1\omega_4 = {}^1\omega_C + {}^C\omega_4 \tag{4} $$

and

$$ {}^1\omega_6 = {}^1\omega_C + {}^C\omega_6 \tag{5} $$

The angular velocity ratios of the gears which have fixed centerlines can be related through the gear numbers as follows

$$ \frac{{}^1\omega_2}{{}^1\omega_3} = -\frac{N_3}{N_2} \tag{6} $$

and

$$ \frac{{}^1\omega_7}{{}^1\omega_9} = \frac{N_9}{N_8}\frac{N_8}{N_7} = \frac{N_9}{N_7} \tag{7} $$

Also relative to the carrier,

$$ \frac{{}^C\omega_4}{{}^C\omega_6} = -\frac{N_6}{N_5}\frac{N_5}{N_4} = -\frac{N_6}{N_4} \tag{8} $$

In Eq. (8), we determine the sign by inspection; namely, gear 4 rotates in the opposite direction to that of gear 6 relative to the coupler.

Combining Eqs. (4), (5), and (8) gives

$$ \frac{{}^1\omega_4 - {}^1\omega_C}{{}^1\omega_6 - {}^1\omega_C} = -\frac{N_6}{N_4} \tag{9} $$

From Eqs. (1-3), (6), (7),

$$ {}^1\omega_4 = {}^1\omega_2 = -{}^1\omega_3\frac{N_3}{N_2} $$

and

$$ {}^1\omega_6 = {}^1\omega_7 = {}^1\omega_9\frac{N_9}{N_7} = {}^1\omega_3\frac{N_9}{N_7} $$

Equation (9) can then be written as

$$ \frac{{}^1\omega_4 - {}^1\omega_C}{{}^1\omega_6 - {}^1\omega_C} = -\frac{N_6}{N_4} = \frac{-{}^1\omega_3\frac{N_3}{N_2} - {}^1\omega_C}{{}^1\omega_3\frac{N_9}{N_7} - {}^1\omega_C} $$

or

$$ -{}^1\omega_3\frac{N_3}{N_2} - {}^1\omega_C = -\frac{N_6}{N_4}\left({}^1\omega_3\frac{N_9}{N_7} - {}^1\omega_C\right) = -{}^1\omega_3\frac{N_9}{N_7}\frac{N_6}{N_4} + {}^1\omega_C\frac{N_6}{N_4} $$

or

$$ {}^1\omega_3\left(\frac{N_9}{N_7}\frac{N_6}{N_4} - \frac{N_3}{N_2}\right) = {}^1\omega_C\left(1 + \frac{N_6}{N_4}\right) $$

Finally,

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$$^1\omega_3 = {^1\omega_C}\frac{\left(1+\frac{N_6}{N_4}\right)}{\left(\frac{N_9}{N_7}\frac{N_6}{N_4}-\frac{N_3}{N_2}\right)}$$

For the values given in the problem,

$$^1\omega_3 = {^1\omega_9} = {^1\omega_C}\frac{\left(1+\frac{N_6}{N_4}\right)}{\left(\frac{N_9}{N_7}\frac{N_6}{N_4}-\frac{N_3}{N_2}\right)} = 1000\frac{\left(1+\frac{50}{30}\right)}{\left(\frac{70}{20}\frac{50}{30}-\frac{100}{10}\right)} = -640 \text{ rpm}$$

The value is negative so $^1\omega_9$ is rotating clockwise when viewed from the left.

Problem 12.30

Resolve Problem 12.28 but assume that the shaft connecting gears 3 and 9 is the input shaft and the shaft of the carrier is the output shaft. Assume ω_9 = 500 rpm counterclockwise and compute ω_C .

Solution:

There are six gears (2, 3, 4, 6, 7, and 9) which can rotate about fixed axes in the system. As in Examples 12.3 and 12.4, we will solve the problem by writing relative velocity equations for all of the gears which have shafts that can rotate in fixed bearings. The resulting equations are:

$$^1\omega_4 = {^1\omega_2} \quad (1)$$

$$^1\omega_6 = {^1\omega_7} \quad (2)$$

$$^1\omega_3 = {^1\omega_9} \quad (3)$$

$$^1\omega_4 = {^1\omega_C} + {^C\omega_4} \quad (4)$$

and

$$^1\omega_6 = {^1\omega_C} + {^C\omega_6} \quad (5)$$

The angular velocity ratios of the gears which have fixed centerlines can be related through the gear numbers as follows

$$\frac{^1\omega_2}{^1\omega_3} = -\frac{N_3}{N_2} \Rightarrow {^1\omega_2} = -\frac{N_3}{N_2}{^1\omega_3} = -\frac{N_3}{N_2}{^1\omega_9} = {^1\omega_4} \quad (6)$$

and

$$\frac{^1\omega_7}{^1\omega_9} = \frac{N_9}{N_8}\frac{N_8}{N_7} = \frac{N_9}{N_7} \Rightarrow {^1\omega_7} = \frac{N_9}{N_7}{^1\omega_9} = {^1\omega_6} \quad (7)$$

Also relative to the carrier,

$$\frac{^C\omega_4}{^C\omega_6} = -\frac{N_6}{N_5}\frac{N_5}{N_4} = -\frac{N_6}{N_4} \quad (8)$$

In Eq. (8), we determine the sign by inspection; namely, gear 4 rotates in the opposite direction to that of gear 6 relative to the coupler.

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Combining Eqs. (4), (5), and (8) gives

$$\frac{{}^1\omega_4 - {}^1\omega_C}{{}^1\omega_6 - {}^1\omega_C} = -\frac{N_6}{N_4} \tag{9}$$

From Eqs. (6) and (7),

$${}^1\omega_4 = -\frac{N_3}{N_2}{}^1\omega_9$$

and

$${}^1\omega_6 = {}^1\omega_7 = {}^1\omega_9\frac{N_9}{N_7}$$

Equation (9) can then be written as

$$\frac{{}^1\omega_4 - {}^1\omega_C}{{}^1\omega_6 - {}^1\omega_C} = -\frac{N_6}{N_4} = \frac{-\frac{N_3}{N_2}{}^1\omega_9 - {}^1\omega_C}{\frac{N_9}{N_7}{}^1\omega_9 - {}^1\omega_C}$$

or

$$\frac{N_3}{N_2}{}^1\omega_9 + {}^1\omega_C = \frac{N_6}{N_4}\left(\frac{N_9}{N_7}{}^1\omega_9 - {}^1\omega_C\right)$$

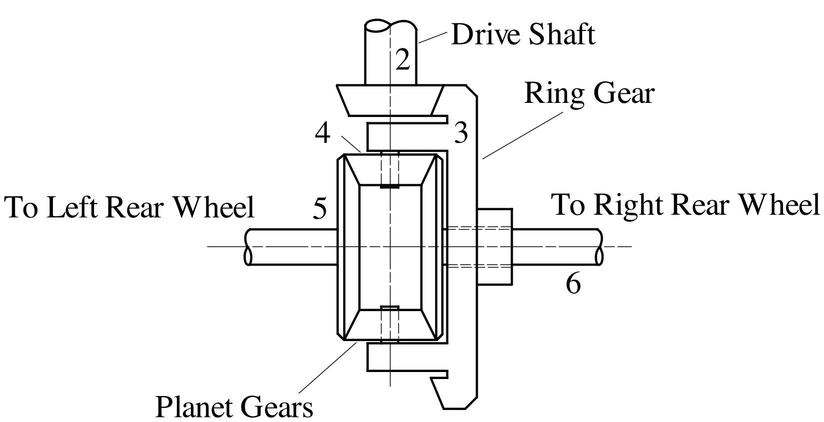
or

$${}^1\omega_C = \frac{\left(\frac{N_6}{N_4}\frac{N_9}{N_7} - \frac{N_3}{N_2}\right){}^1\omega_9}{1 + \frac{N_6}{N_4}} = \frac{\left(\frac{50}{30}\frac{40}{30} - \frac{90}{20}\right)500}{1 + \frac{50}{30}} = -427.1 \text{ rpm}$$

The value is negative so ${}^1\omega_C$ is rotating clockwise when viewed from the left.

Problem 12.31

The differential for a rear wheel-driven vehicle is shown schematically. If the drive shaft turns at 90 0 rpm, what is the speed of the vehicle if neither wheel slips and the outside diameter of the wheels is 24 in?



$N_2 = 28T$
$N_3 = 92T$
$N_4 = 22T$
$N_5 = 30T$
$N_6 = 30T$

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Solution:

From the problem statement, we know that

$$ {}^1\omega_5 = {}^1\omega_6 $$

and

$$ {}^1\omega_2 = 900 \text{ rpm.} $$

Gears 2 and 3 rotate relative to the frame. Therefore,

$$ {}^1\omega_3 = {}^1\omega_2 \frac{N_2}{N_3} \tag{1} $$

where we are dealing only with speeds since a direction of rotation was not given for gear 2. Therefore, we do not have a sign for ${}^1\omega_3$. Gear 3 is fixed to the carrier of the planetary drive. Considering gears 5 and 6,

$$ {}^1\omega_5 = {}^1\omega_3 + {}^3\omega_5 $$

and

$$ {}^1\omega_6 = {}^1\omega_3 + {}^3\omega_6 $$

If ${}^1\omega_5 = {}^1\omega_6$, then

$$ {}^1\omega_3 + {}^3\omega_5 = {}^1\omega_3 + {}^3\omega_6 $$

and

$$ {}^3\omega_5 = {}^3\omega_6 \tag{1} $$

Relative to the carrier,

$$ \frac{{}^3\omega_5}{{}^3\omega_6} = -\frac{N_6}{N_4}\frac{N_4}{N_5} = -\frac{N_6}{N_5} $$

or

$$ {}^3\omega_5 = -\frac{N_6}{N_5}{}^3\omega_6 = -\frac{30}{30}{}^3\omega_6 = -{}^3\omega_6 \tag{2} $$

Equations (1) and (2) cannot be satisfied simultaneously unless ${}^3\omega_5 = {}^3\omega_6 = 0$. Therefore,

$$ {}^1\omega_5 = {}^1\omega_6 = {}^1\omega_3 = {}^1\omega_2 \frac{N_2}{N_3} = 900\frac{28}{92} = 273.9 \text{ rpm.} $$

The car velocity (for a 1-foot radius wheel) is given by

$$ v = {}^1\omega_5 r_{wheel} = 273.9\frac{2\pi}{60}(1) = 28.68 \text{ ft/sec} = 19.56 \text{ mph} $$

Problem 12.32

Assume that the vehicle in Problem 12.31 is stopped so that the right wheel sits on a small icy patch and can spin freely while the left wheel does not spin. Determine the angular velocity of the right wheel if the angular speed of the drive shaft is 500 rpm.

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Solution:

From the problem statement, we know that

$$^1\omega_5 = 0$$

and

$$^1\omega_2 = 500 \text{ rpm.}$$

Gears 2 and 3 rotate relative to the frame. Therefore,

$$^1\omega_3 = {}^1\omega_2 \frac{N_2}{N_3} \tag{1}$$

where we are dealing only with speeds since a direction of rotation was not given for gear 2. Therefore, we do not have a sign for $^1\omega_3$. Gear 3 is fixed to the carrier of the planetary drive. Relative to the carrier (gear 3),

$$^3\omega_5 = {}^1\omega_5 - {}^1\omega_3$$

and

$$^3\omega_6 = {}^1\omega_6 - {}^1\omega_3$$

Then,

$$\frac{^3\omega_5}{^3\omega_6} = \frac{^1\omega_5 - {}^1\omega_3}{^1\omega_6 - {}^1\omega_3} = -\frac{N_6}{N_4}\frac{N_4}{N_5} = -\frac{N_6}{N_5} \tag{2}$$

Here the minus sign means that the rotation direction for gear 5 is opposite to that for gear 6 relative to the carrier.

Combining Eqs. (1) and (2),

$$\frac{^1\omega_5 - {}^1\omega_3}{^1\omega_6 - {}^1\omega_3} = \frac{^1\omega_5 - {}^1\omega_2 \frac{N_2}{N_3}}{^1\omega_6 - {}^1\omega_2 \frac{N_2}{N_3}} = -\frac{N_6}{N_5}$$

Then,

$$^1\omega_5 - {}^1\omega_2 \frac{N_2}{N_3} = -\frac{N_6}{N_5}\left({}^1\omega_6 - {}^1\omega_2 \frac{N_2}{N_3}\right)$$

or

$$-\frac{N_6}{N_5}{}^1\omega_6 = {}^1\omega_5 - {}^1\omega_2\left(\frac{N_2}{N_3} + \frac{N_2}{N_3}\frac{N_6}{N_5}\right)$$

Now, substituting for the known values and solving for $^1\omega_6$.

$$^1\omega_6 = \frac{^1\omega_5 - {}^1\omega_2\left(\frac{N_2}{N_3} + \frac{N_2}{N_3}\frac{N_6}{N_5}\right)}{-\frac{N_6}{N_5}} = \frac{0 - 500\left(\frac{28}{92} + \frac{28}{92}\frac{30}{30}\right)}{-\frac{30}{30}} = 500(2)\frac{28}{92} = 304.3 \text{ rpm}$$

Problem 12.33

Assume that the vehicle in Problem 12.31 is traveling at 35 mph and turns around a curve with a

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radius of 50 ft from the centerline of the vehicle. The center-to-center distance between the treads of the right and left wheels is 60 in. Compute the rotational speed of each rear wheel, the rotational speed of the ring gear, and the rotational speed of the drive shaft.

<u>Solution:</u>

A schematic of the rear wheels is shown in Fig. P12.34a.

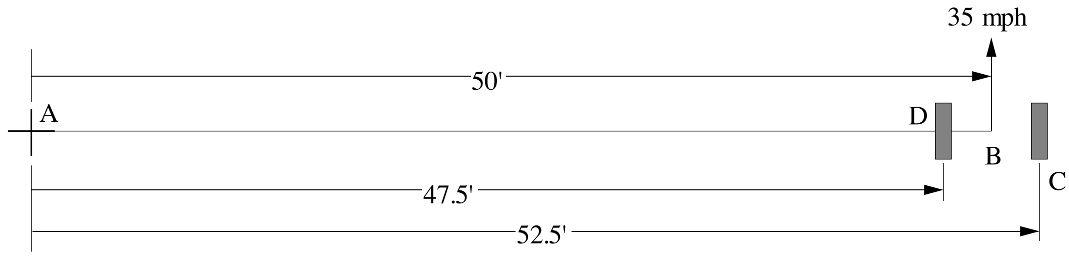

Fig. P12.24a

The magnitude of the angular velocity of the vector $\mathbf{r}_{B/A}$ is

$$|\omega| = |V| / |r_{B/C}| = (35\,m/h)(5280\,f/m)(1h/3600\,s)/(50\,f) = 1.0267 \text{ rad/s.}$$

The velocity of the right wheel axle is

$$|V_R| = |\omega||r_{C/A}| = (1.0267)(52.5) = 53.90 \text{ f/s}$$

and for the left wheel axle is

$$|V_R| = |\omega||r_{D/A}| = (1.0267)(47.5) = 48.7683 \text{ f/s.}$$

The angular velocity of hte right wheel (gear 6) for a 1-foot radius wheel is

$$|{}^1\omega_6| = |V_R| / |r_W| = 53.90/(1) = 53.90 \text{ rad/s}$$

and for the left wheel (gear 5),

$$|{}^1\omega_5| = |V_L| / |r_W| = 48.7683/(1) = 48.7683 \text{ rad/s}$$

Both ${}^1\omega_5$ and ${}^1\omega_6$
frame. Therefore,

$$ {}^1\omega_2 = {}^1\omega_3 \frac{N_3}{N_2} \qquad (1)$$

Gear 3 is fixed to the carrier of the planetary drive. Relative to the carrier (gear 3),

$$ {}^1\omega_5 = {}^1\omega_3 + {}^3\omega_5 \Rightarrow {}^1\omega_5 - {}^3\omega_5 = {}^1\omega_3 \qquad (2)$$

and

$$ {}^1\omega_6 = {}^1\omega_3 + {}^3\omega_6 \Rightarrow {}^1\omega_6 - {}^3\omega_6 = {}^1\omega_3 \qquad (3)$$

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Also, relative to the carrier,

$$\frac{^3\omega_5}{^3\omega_6} = -\frac{N_6}{N_4}\frac{N_4}{N_5} = -\frac{N_6}{N_5} \Rightarrow {}^3\omega_5 = -\frac{N_6}{N_5}{}^3\omega_6 \tag{4}$$

From Eqs. (2) and (3),

$${}^1\omega_5 - {}^3\omega_5 = {}^1\omega_6 - {}^3\omega_6,$$

and using Eq. (4),

$${}^1\omega_5 + \frac{N_6}{N_5}{}^3\omega_6 = {}^1\omega_6 - {}^3\omega_6$$

or

$$({}^1\omega_5 - {}^1\omega_6) = -\left(1 + \frac{N_6}{N_5}\right){}^3\omega_6$$

or

$${}^3\omega_6 = -({}^1\omega_5 - {}^1\omega_6) / \left(1 + \frac{N_6}{N_5}\right) = -(48.7683 - 53.90) / (1 + 1/1) = 2.5659 \text{ rad/s CCW}$$

From Eq. (3),

$${}^1\omega_3 = ({}^1\omega_6 - {}^3\omega_6) = -(53.90 - 2.5659) = 51.3342 \text{ rad/s CCW}$$

From Eq. (1),

$${}^1\omega_2 = \frac{N_3}{N_2}{}^1\omega_3 = 51.3342\frac{92}{28} = 124.28 \text{ rad/s.}$$

If gear 3 rotates CCW (when viewed from the left side), then by inspection, gear 2 rotates CW when viewed from the rear of the vehicle. See Fig. P12.34b.

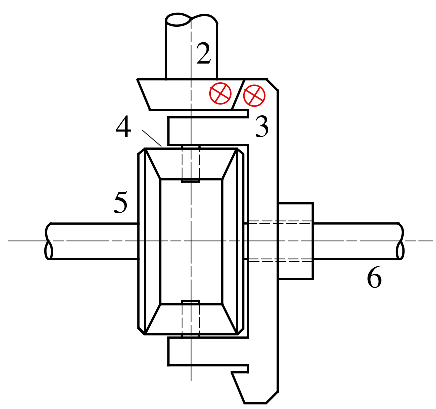

Fig. P12.34b

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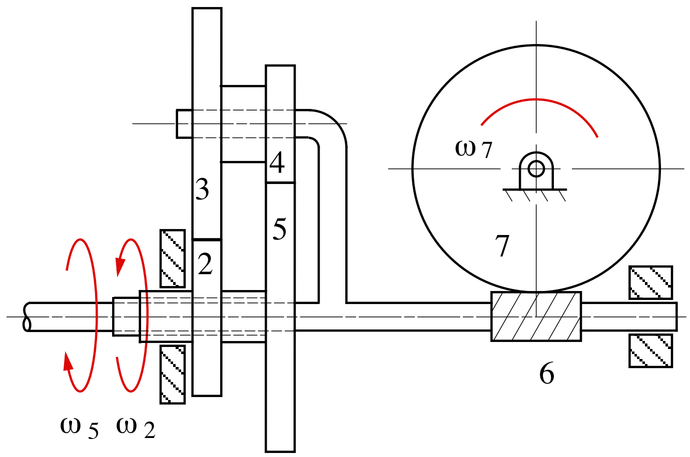
Problem 12.34

In the mechanism shown below, derive an expression for the angular velocity of gear 7 (ω_7) in terms of ω_2 and ω_5 and the tooth numbers N_2, N_3, N_4, N_5, N_6, and N_7. Take counterclockwise viewing from the left as positive for the rotation of gears 2, 3, 4, 5, and 6. Viewed from the front of the page, take counterclockwise as the positive direction for gear 7.



Solution:

Gears 2, 5, and 6 all rotate about fixed axes, and gear 6 is fixed to the carrier of the planetary drive. Therefore,

$$^6\omega_2 = {}^1\omega_2 - {}^1\omega_6 \tag{1}$$

and

$$^6\omega_5 = {}^1\omega_5 - {}^1\omega_6 \tag{2}$$

Dividing Eq. (1) by Eq. (2),

$$\frac{^6\omega_5}{^6\omega_2} = \frac{^1\omega_5 - {}^1\omega_6}{^1\omega_2 - {}^1\omega_6} \tag{3}$$

Relative to the carrier,

$$\frac{^6\omega_5}{^6\omega_2} = \frac{N_2}{N_3}\frac{N_4}{N_5} \tag{4}$$

Combining Eqs. (3) and (4),

$$^1\omega_5 - {}^1\omega_6 = \left({}^1\omega_2 - {}^1\omega_6\right)\frac{N_2}{N_3}\frac{N_4}{N_5}$$

Then,

$$\left(1 - \frac{N_2}{N_3}\frac{N_4}{N_5}\right){}^1\omega_6 = {}^1\omega_5 - \frac{N_2}{N_3}\frac{N_4}{N_5}{}^1\omega_2$$

and

$$^1\omega_6 = \left[{}^1\omega_5 - \frac{N_2}{N_3}\frac{N_4}{N_5}{}^1\omega_2\right] \Big/ \left[1 - \frac{N_2}{N_3}\frac{N_4}{N_5}\right]$$

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If ${}^1\omega_6$ is positive CCW when viewed from the left, then ${}^1\omega_7$ will be CW or negative. Then,

$$\frac{{}^1\omega_7}{{}^1\omega_6} = -\frac{N_6}{N_7}$$

or

$${}^1\omega_7 = -\frac{N_6}{N_7}{}^1\omega_6 = -\frac{N_6}{N_7}\left[{}^1\omega_5 - \frac{N_2}{N_3}\frac{N_4}{N_5}{}^1\omega_2\right] \Big/ \left[1 - \frac{N_2}{N_3}\frac{N_4}{N_5}\right]$$

Problem 12.35

In Problem 12.34, assume that $\omega_2 = 100$ rpm, $\omega_5 = -60$ rpm, $N_2 = 40T$, $N_3 = 60T$, $N_4 = 30T$, $N_5 = 70T$, $N_6 = 8T$, and $N_7 = 50T$. Determine the angular velocity of both gears 6 and 7.

Solution:

Gears 2, 5, and 6 all rotate about fixed axes, and gear 6 is fixed to the carrier of the planetary drive. Therefore,

$${}^6\omega_2 = {}^1\omega_2 - {}^1\omega_6 \qquad (1)$$

and

$${}^6\omega_5 = {}^1\omega_5 - {}^1\omega_6 \qquad (2)$$

Dividing Eq. (1) by Eq. (2),

$$\frac{{}^6\omega_5}{{}^6\omega_2} = \frac{{}^1\omega_5 - {}^1\omega_6}{{}^1\omega_2 - {}^1\omega_6} \qquad (3)$$

Relative to the carrier,

$$\frac{{}^6\omega_5}{{}^6\omega_2} = \frac{N_2}{N_3}\frac{N_4}{N_5} \qquad (4)$$

Combining Eqs. (3) and (4),

$${}^1\omega_5 - {}^1\omega_6 = \left({}^1\omega_2 - {}^1\omega_6\right)\frac{N_2}{N_3}\frac{N_4}{N_5}$$

Then,

$$\left(1 - \frac{N_2}{N_3}\frac{N_4}{N_5}\right){}^1\omega_6 = {}^1\omega_5 - \frac{N_2}{N_3}\frac{N_4}{N_5}{}^1\omega_2$$

and

$${}^1\omega_6 = \left[{}^1\omega_5 - \frac{N_2}{N_3}\frac{N_4}{N_5}{}^1\omega_2\right] \Big/ \left[1 - \frac{N_2}{N_3}\frac{N_4}{N_5}\right]$$

Then,

$${}^1\omega_6 = \left[{}^1\omega_5 - \frac{N_2}{N_3}\frac{N_4}{N_5}{}^1\omega_2\right] \Big/ \left[1 - \frac{N_2}{N_3}\frac{N_4}{N_5}\right] = \left[-60 - \frac{40}{60}\frac{30}{70}100\right] \Big/ \left[1 - \frac{40}{60}\frac{30}{70}\right] = -124 \text{ rad/s CW}$$

And,

$${}^1\omega_7 = -\frac{N_6}{N_7}{}^1\omega_6 = -\frac{8}{50}(-124) = 19.84 \text{ rad/s CCW}$$

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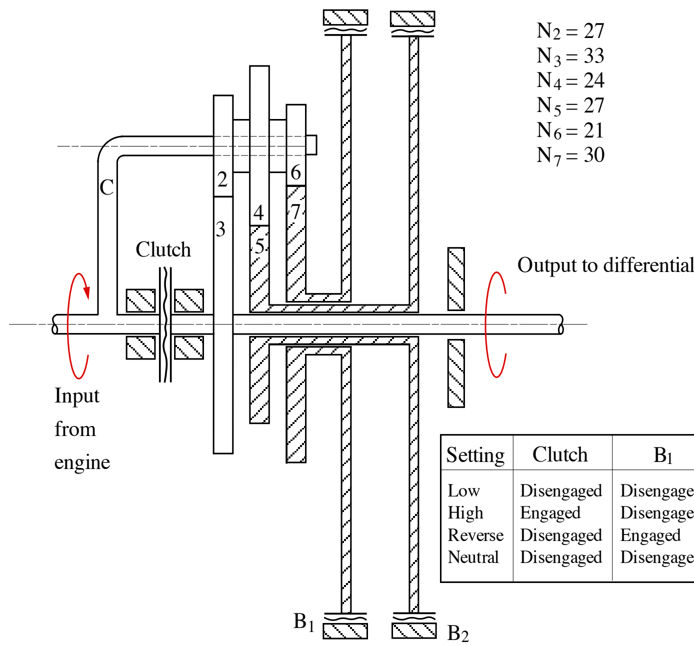
Problem 12.36[1]

The figure shows a schematic diagram of a semiautomatic transmission from the Model-T automobile. This was the forerunner of today's automatic transmission. A plate clutch, two banded clutches, and a system of pedals and levers (used to engage and disengage these plate and band clutches) operated in the proper sequence is shown in the table below. Determine the output/input speed ratio for each condition.



Setting	Clutch	B_1	B_2
Low	Disengaged	Disengaged	Engaged
High	Engaged	Disengaged	Disengaged
Reverse	Disengaged	Engaged	Disengaged
Neutral	Disengaged	Disengaged	Disengaged

Solution:

Gears 3, 5, and 7 rotate about a fixed axis. The carrier (member C) is the input member and the out put is gear 3. Looking at gears 3, 5, and 7,

$${}^C\omega_3 = {}^1\omega_3 - {}^1\omega_C \quad (1)$$

$${}^C\omega_5 = {}^1\omega_5 - {}^1\omega_C \quad (2)$$

$${}^C\omega_7 = {}^1\omega_7 - {}^1\omega_C \quad (3)$$

Also,

$$\frac{{}^C\omega_3}{{}^C\omega_5} = \frac{N_5}{N_4}\frac{N_2}{N_3}$$

and

$$\frac{{}^C\omega_3}{{}^C\omega_7} = \frac{N_7}{N_6}\frac{N_2}{N_3}$$

Combining Eqs. (1), (2), and (4),

[1] Problem courtesy of Mike Stanisik, Notre Dame University

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$$\frac{{}_C\omega_3}{{}_C\omega_5} = \frac{{}^1\omega_3 - {}^1\omega_C}{{}^1\omega_5 - {}^1\omega_C} = \frac{N_5}{N_4}\frac{N_2}{N_3} \Rightarrow {}^1\omega_3 - {}^1\omega_C = ({}^1\omega_5 - {}^1\omega_C)\frac{N_5}{N_4}\frac{N_2}{N_3} \tag{6}$$

And combining Eqs. (1), (3), and (5),

$$\frac{{}_C\omega_3}{{}_C\omega_7} = \frac{{}^1\omega_3 - {}^1\omega_C}{{}^1\omega_7 - {}^1\omega_C} = \frac{N_7}{N_6}\frac{N_2}{N_3} \Rightarrow {}^1\omega_3 - {}^1\omega_C = ({}^1\omega_7 - {}^1\omega_C)\frac{N_7}{N_6}\frac{N_2}{N_3} \tag{7}$$

Low Setting: $\boxed{{}^1\omega_5 = 0}$

From Eq. (6),

$${}^1\omega_3 - {}^1\omega_C = (0 - {}^1\omega_C)\frac{N_5}{N_4}\frac{N_2}{N_3}$$

or

$${}^1\omega_3 = {}^1\omega_C\left[1 - \frac{N_5}{N_4}\frac{N_2}{N_3}\right]$$

and

$$\frac{{}^1\omega_3}{{}^1\omega_C} = 1 - \frac{N_5}{N_4}\frac{N_2}{N_3} = 1 - \frac{27}{24}\frac{27}{33} = 0.0795$$

High Setting: ${}^1\omega_3 = {}^1\omega_C$

For this case,

$$\frac{{}^1\omega_3}{{}^1\omega_C} = 1$$

Reverse Setting: ${}^1\omega_7 = 0$

From Eq. (7),

$${}^1\omega_3 - {}^1\omega_C = (0 - {}^1\omega_C)\frac{N_7}{N_6}\frac{N_2}{N_3}$$

or

$${}^1\omega_3 = {}^1\omega_C\left(1 - \frac{N_7}{N_6}\frac{N_2}{N_3}\right)$$

or

$$\frac{{}^1\omega_3}{{}^1\omega_C} = 1 - \frac{N_7}{N_6}\frac{N_2}{N_3} = 1 - \frac{30}{21}\frac{27}{33} = -0.1688$$

Neutral Setting:

In the neutral setting, all of the clutches are disengaged. The transmission then has two degrees of freedom. Therefore, ${}^1\omega_3$ is independent of shaft C. If the rear wheels are stationary, ${}^1\omega_3 = 0$ independently of the engine speed.

Problem 12.37

In problem 12.36, if the engine rotates at 400 rpm, determine the angular velocity of gear 5 when the transmission is in low gear.

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Solution:

Gears 3, 5, and 7 rotate about a fixed axis. The carrier (member C) is the input member and the output is gear 3. Looking at gears 3, 5, and 7,

$$ {}^{C}\omega_3 = {}^{1}\omega_3 - {}^{1}\omega_C \tag{1}$$

$$ {}^{C}\omega_5 = {}^{1}\omega_5 - {}^{1}\omega_C \tag{2}$$

$$ {}^{C}\omega_7 = {}^{1}\omega_7 - {}^{1}\omega_C \tag{3}$$

Also,

$$\frac{{}^{C}\omega_3}{{}^{C}\omega_5} = \frac{N_5}{N_4}\frac{N_2}{N_3}$$

and

$$\frac{{}^{C}\omega_3}{{}^{C}\omega_7} = \frac{N_7}{N_6}\frac{N_2}{N_3}$$

Combining Eqs. (1), (2), and (4),

$$\frac{{}^{C}\omega_3}{{}^{C}\omega_5} = \frac{{}^{1}\omega_3 - {}^{1}\omega_C}{{}^{1}\omega_5 - {}^{1}\omega_C} = \frac{N_5}{N_4}\frac{N_2}{N_3} \Rightarrow {}^{1}\omega_3 - {}^{1}\omega_C = \left({}^{1}\omega_5 - {}^{1}\omega_C\right)\frac{N_5}{N_4}\frac{N_2}{N_3} \tag{6}$$

For the low setting, ${}^{1}\omega_5 = 0$. Therefore,

$${}^{1}\omega_3 - {}^{1}\omega_C = \left(0 - {}^{1}\omega_C\right)\frac{N_5}{N_4}\frac{N_2}{N_3}$$

or

$${}^{1}\omega_3 = {}^{1}\omega_C\left[1 - \frac{N_5}{N_4}\frac{N_2}{N_3}\right] = 400\left[1 - \frac{27}{24}\frac{27}{33}\right] = 31.82 \text{ rpm}$$

Problem 12.38

In the mechanism shown, let the input be gear 2 and assume that all of the gear tooth numbers (N_2, N_3, N_4, N_5, N_6, N_7, and N_8) are known. Derive an expression for the angular velocity of gear 8.

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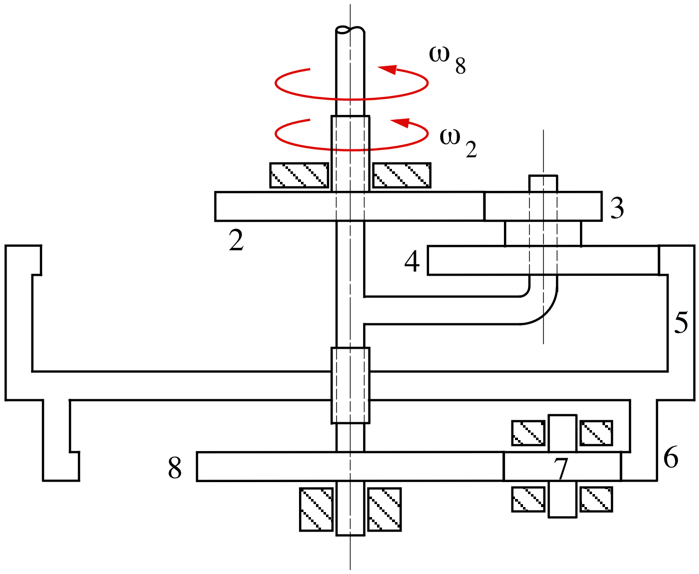

Solution:

Treat all angular velocities as positive if they are counterclockwise when viewed from the top. Gear s 2, 5, and 8 all rotate about fixed axes. Gears 5 and 6 are fixed together as are gears 3 and 4. Gear 8 is fixed to the carrier.

For gears 2 and 5,

$${}^8\omega_2 = {}^1\omega_2 - {}^1\omega_8 \qquad (1)$$

and

$${}^8\omega_5 = {}^1\omega_5 - {}^1\omega_8 \qquad (2)$$

Also,

$${}^1\omega_5 = {}^1\omega_8 \frac{N_8}{N_6} \qquad (3)$$

and

$$\frac{{}^8\omega_2}{{}^8\omega_5} = -\frac{N_3}{N_2}\frac{N_5}{N_4} \qquad (4)$$

Combining Eqs. (1), (2), and (4)

$$\frac{{}^8\omega_2}{{}^8\omega_5} = \frac{{}^1\omega_2 - {}^1\omega_8}{{}^1\omega_5 - {}^1\omega_8} = -\frac{N_3}{N_2}\frac{N_5}{N_4}$$

or

$${}^1\omega_2 - {}^1\omega_8 = -\frac{N_3}{N_2}\frac{N_5}{N_4}\left({}^1\omega_5 - {}^1\omega_8\right)$$

Using Eq. (3) to eliminate ${}^1\omega_5$,

$${}^1\omega_2 = {}^1\omega_8 - \frac{N_3}{N_2}\frac{N_5}{N_4}\left[{}^1\omega_8\frac{N_8}{N_6} - {}^1\omega_8\right] = {}^1\omega_8\left\{1 - \frac{N_3}{N_2}\frac{N_5}{N_4}\left[\frac{N_8}{N_6} - 1\right]\right\}$$

Then,

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$$^{1}\omega_{8} = {}^{1}\omega_{2} \Big/ \left\{1 - \frac{N_3}{N_2}\frac{N_5}{N_4}\left[\frac{N_8}{N_6} - 1\right]\right\}$$

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