- Any Quantity must be written as $X \pm \Delta X$

  $\Delta X \equiv$ uncertainty

- Physical quantity $\Rightarrow$ measured & calculated quantities.

- Sources of errors :

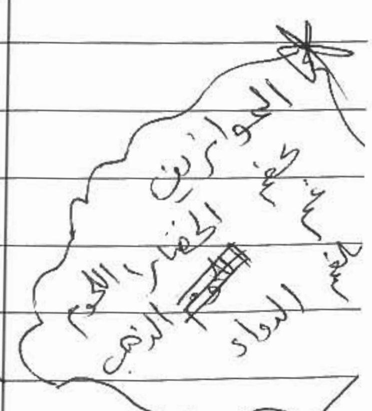
1. Instruments (precision, calibration)
2. Enviroment (Temp, ..
3. The way the experiment is done , 5. Experimenter.
4. The way the physical quantity is measured

- True value of a physical quantity can never obtained

  Errors $\neq$ Zero but we can make it as small as possible.

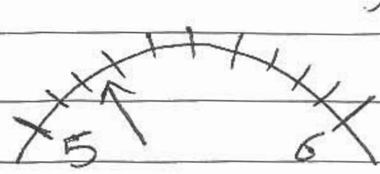
Errors

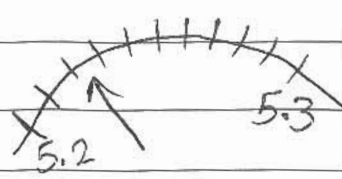
Random Errors	Systematic errors
• always exist	• bigger or smaller than the true value
• Can be reduced	• Can be eliminated
• Can not be eliminated or avoided.	
• below & above the true value	



* Random Errors

  → How finally the scale is divided?





5.22 ± 0.01 Volt

5.2 ± 0.1 Volt

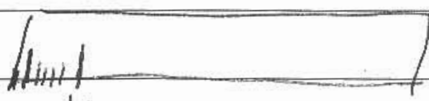
من المستحيل الوصول إلى القياس الحقيقي ولكننا نحاول الاقتراب منه حسب أداة القياس

[2] Systematic Errors
- Experimenter
- Instruments that not calbirated.

• Calbiration ⇒ adjust the instrument to read zero when it is not used.

But if instrument calbiration can't be done, the systematic error should be added or subtracted from the measurements.

⇒ Voltmeter read (−0.2 V) when it is not connected to any thing

⇒ Ruler  1cm $\boxed{0.4\ cm}$

⇒ Looking perpendicularly to the scale.

☆ Precision & Accuracy

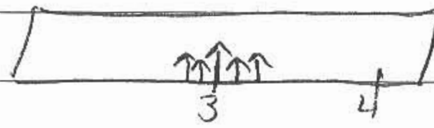
دقة → Random

صحة → Systematic

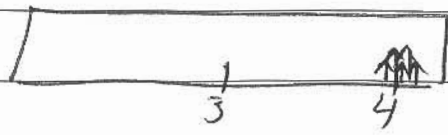
Small Random → high Precision
negligible (small) Systematic → high accuracy

⇒ True value is the value accepted by the community of physicist, because it is value obtained by experienced, skillful & trust worthly experiments.

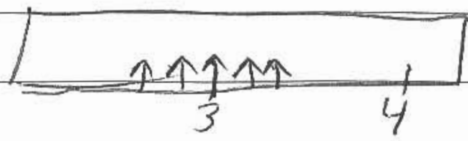
example — True value $\boxed{3}$

trail ①


accurate

trail ②


precise but not accurate

trail ③


Law precision

example — تاسقط طبشورة

Ruler	Vernier caliper	micrometer
1mm	$\frac{1}{20}$ mm = 0.05 mm	$\frac{1}{100}$ mm = 0.01 mm
Law precision		high precision.

* measured physical quantities uncertainties
  - → one value by estimation (quantity measured once)
  - → many values by std of the mean.

- The best estimate of the true value = mean, average value

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

- Sample standard deviation — الانحراف المعياري

$$\sigma_s = \left[\frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2\right]^{1/2}$$

σ_s = uncertainty in only one measurement, a sample measurement
Any one measurements, does not differ from another by more than σ_s
probably

• The uncertainty in the mean, standard deviation of the mean $\sigma_m = \sigma_s / \sqrt{N}$

The mean value ~~probability~~ does not differ from the true value by more than σ_m

(How is the avg value close to the true ?)

※ Acomparison between measured and accepted values

(Result) $X \mp \Delta X$

True value lies between $\bar{X} - \Delta X$ & $\bar{X} + \Delta X$

$$D = | R_{true} - R_{measured} | \leqslant 2 \Delta X$$

Reset all — shift mode 3 =

SD — mode 2

M+ ادخال القياسات متبوعة ب

shift 2 1 = $\bar{X}$

shift 2 3 = σ_s

$\sigma_s / \sqrt{N} = \sigma_m$

example → The length of a book (30.1, 30.3, 29.8, 30.0, 30.1) cm

$\bar{L} = 30.06$ cm

$\sigma_s = 0.1816$ cm

$\sigma_m = 0.0812$ cm

$L = (30.06 \pm 0.08)$ cm

example

A يا $V = 338 \pm 2$ m/s   ب $V = (325 \pm 5)$ m/s

$V = 331$ m/s

* Significant figures الأرقام المعنوية

- All physical quantity must be written in a suitable significant figures.

- The s.f are all the figures upto and including the figure which is uncertain

Example 3.4 → 2 s.f 3.45 → 3 s.f 34 → 2 s.f
0.01 → 1 s.f 0010 → 1 s.f 00102 → 3 s.f
1.00 → 3 s.f 1.20 → 3 s.f 100 → 1 s.f

1) All non-zero digits are significant
2) Zero's between non-zero digits are s.f 303 → 3 s.f

Zero's
- with decimal pt
  - Zeros to the left of the 1st non zero digit ⇓ not s.f
  - Zeros on the right (s.f)
- without decimal pt ⇓ not s.f 80, 100 ⇒ 1 s.f
  - 8.0×10 (2 s.f)

1000 ± 20 ⇒ 3 s.f , 4.70 ± 0.03

* Uncertainty should be rounded to 1 s.f unless the leading digit in the uncertainty is one you must left the uncertainty with 2 s.f

- Each measured value must be written in a suitable significant figures

$\Rightarrow$ s.f reflect the precision
$3.4 \mp \underline{\underline{0.2}}$

$\Rightarrow$ $L = (77.2 \mp 0.1)$ cm high precision
$L = (76.7 \mp 0.5)$ cm high accuracy
$L_{accepte} = 76.9$ cm

* Rounding

1. If 1st non-s.f > 5 $\longrightarrow$ last s.f $+1$ (round up)
2. If 1st non-s.f < 5 $\longrightarrow$ last s.f remains the same
3. If the 1st non-s.f $= 5$ and no numbers after it
   Last s.f odd $\longrightarrow$ round up
   Last s.f even $\longrightarrow$ keep the same (round down)
4. If the 1st non-s.f $= 5$ and there's numbers after it you must round up all the time

Example

$4.37 \xrightarrow{2\ s.f} 4.4$
$4.32 \xrightarrow{2\ s.f} 4.3$
$4.35 \xrightarrow{2\ s.f} 4.4$
$4.25 \xrightarrow{2\ s.f} 4.2$
$4.352 \xrightarrow{2\ s.f} 4.4$
$4.3565 \xrightarrow{3\ s.f} 4.36$
$4.2251 \xrightarrow{3\ s.f} 4.23$

$299 \xrightarrow{2\ s.f} 3.0 \times 10^2$

* Addition and Subtraction
(the fewest decimal places)
Limits the number of decimal place in the result

$1.21 + 2 = 3$

* Multiplication and division / square root function
(the fewest s.f limits the number of s.f of the Result.

* Trigonometric functions
# of s.f of the result must equal the number of s.f in Θ $R = \sin\Theta$, $R = \cos\Theta$

* Natural Logarithm $\ln x$ } # of s.f in the result
exponential e^x } equal to # of s.f of (

Example $\cos(35°) = 0.81915 = 0.82$

$$\sqrt{\frac{2.3 \times 4.57}{1.2}} = \sqrt{8.75916} = 2.96 = 3.0$$

$\ln(0.2) = -1.609 \sim -2$

* Uncertainties in Calculated values
1. Addition and subtraction $(+/-)$
$R = X \pm y$, $\Delta R = \Delta X + \Delta y$

(Errors always add)

Constants like π ⇒ we don't account it as s.f

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مهم

(2) Multiplication and Division ($\times$, $\div$)

$$A = xy = R$$

$$\frac{\Delta A}{A} = \frac{x\,\Delta y}{A} + \frac{y\,\Delta x}{A}$$

$$\frac{\Delta A}{A} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

If we take $R = X/Y = xy^{-1}$

$$\frac{\Delta R}{R} = \frac{x(-1\,y^{-2}\,\Delta y)}{R} = \frac{y^{-1}\,\Delta x}{R}$$

$$\frac{\Delta R}{R} = \frac{\Delta y}{y} + \frac{\Delta x}{x}$$

In general $R = XYZ$, $R = XY/Z$

$$\frac{\Delta R}{R} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y} + \frac{\Delta Z}{Z}$$

(3) Constant multiplier

$+, -$ $\times / \div$

$R = ax + by$

$\Delta R = a\Delta x + b\Delta y$

(constant remains)

$R = axy$

$\Delta R = ax\Delta y + ay\Delta x$

$$\frac{\Delta R}{R} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$$

(constant disappeared)

example

$A = (57 \pm 2)$ cm , $B = (23 \pm 2)$ cm

① $R = A - 2B$

$R = 11$ cm , $\Delta R = \Delta A + 2\Delta B = 6$ cm

$R = (11 \pm 6)$ cm

② $R = \dfrac{6x}{y}$, $R = 14.869$ $(x = A,\ y = B)$

$$\frac{\Delta R}{R} = \left[\frac{\Delta x}{x} + \frac{\Delta y}{y}\right] \Rightarrow \Delta R = 1.8$$

$R = (14.9 \pm 1.8)$ ~~cm~~

$\boxed{4}$ Raising to apower

$R = x^n y^m$

$$\Delta R = |x^n m y^{m-1} \Delta y| + |y^m n x^{n-1} \Delta x|$$

$$\frac{\Delta R}{R} = \left|m \frac{\Delta y}{y}\right| + \left|n \frac{\Delta x}{x}\right|$$

example $R = Z^2 Y^3 / X^4 = Z^2 Y^3 X^{-4}$

$$\frac{\Delta R}{R} = 2\frac{\Delta Z}{Z} + 3\frac{\Delta y}{y} + 4\frac{\Delta X}{X}$$

$\boxed{5}$ other functions

Trigonometric
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★*

Natural length
$R = \ln X$
$\Delta R = \dfrac{\Delta X}{X}$

Exponential
$R = e^x$
$\Delta R = e^x \Delta x$

$R = e^{2x} \Longrightarrow \Delta R = e^{2x}\, 2\, \Delta X$

(**) If the angle is measured in degrees, You should convert it to radius by multiply it by $\pi/180$

<u>example</u> $\theta = 80^\circ \pm 1^\circ$, $\sin\theta = ??$

$\sin 80 = 0.984810$

$$\Delta R = \cos\theta \; \Delta\theta \; \frac{\pi}{180}$$

$$\cos 80 \; \frac{\pi}{180} * 1 = 0.00303 \sim 0.003$$

$$\sin\theta = (0.985 \pm 0.003)$$

ii) General Rule

$R(x, y, z)$

$$\Delta R = \left|\frac{\partial R}{\partial x}\right| \Delta x + \left|\frac{\partial R}{\partial y}\right| \Delta y + \left|\frac{\partial R}{\partial z}\right| \Delta z$$

<u>example</u> $R = x^2 y^3 \sin(x+z)$

let $\sin(x+z) = A$

$$R = x^2 y^3 A$$

$$\frac{\Delta R}{R} = 2\frac{\Delta x}{x} + \frac{3\Delta y}{y} + \frac{\Delta A}{A}$$

$$A = \sin(x+z)$$

let $B = x+z$, $\Delta B = \Delta x + \Delta z$

$$\Delta A = \cos(x+z)\,[\Delta x + \Delta z]$$

<u>General Rule</u>

$$\Delta R = \left|2xy^3 \sin(x+z) + x^2 y^3 \cos(x+z)\right| \Delta x + \left|3x^2 y^2 \sin(x+z)\right| \Delta y + \left|x^2 y^3 \cos(x+z)\right| \Delta z.$$

example $d = \{20.1, 20.2, 19.8, 20.2\}$ Cm

Volume of the Sphere

$(\bar{d} \pm \Delta d) = (20.075 \pm 0.0946)$

$\sigma_5 = 0.18929$

$(\bar{d} \pm \Delta d) = (20.08 \pm 0.09)$ cm

$V = \frac{4}{3}\pi\left(\frac{20.08}{2}\right)^3 = 4237.1079$ cm^3

~ 4237 cm^3

$\frac{\Delta V}{V} = 3\frac{\Delta d}{\bar{d}} = 3\left(\frac{0.09}{20.08}\right)$

$\Delta V = 56.97 \sim 60$ cm^3

$(V \pm \Delta V) = (4240 \pm 60)$ cm^3

$r = (10.05, 10.1, 9.9, 10.1)$

$(r \pm \Delta r) = (10.0375 \pm 0.0473)$

(10.04 ± 0.05)

$V = \frac{4}{3}\pi r^3 = 4237.108$ cm^3

$\Delta V = V\, 3\frac{\Delta r}{r} \sim 59.8995 \sim 60$

(4240 ± 60) cm^3

SIGNIFICANT FIGURES

1) All non-zero digits are significant,
   12.3 has 3 S.F, 549 has 3 S.F.

2) Zeros between non-zero digits are significant,
   1.03 has 3 S.F, 4023 has 4 S.F.

3) Zeros at the end of a number are significant (for numbers with decimal point),
   2.00 has 3 S.F, 0.050 has 2 S.F.

4) Zeros to the left of the first non zero digit are not significant,
   0.84 has 2 S.F.

5) Zeros at the end of a number without a decimal point are ambiguous,
   80 may have 1 or 2 S.F.
   This number must reported in scientific notation, 8×10^1 has 1 S.F but 8.0×10^1 has 2 S.F.

Adding or subtracting quantities:

36.3 + 56.32 = 92.62 = 92.3 (the number with less decimals)

12.4 + 16.57 +17.4312 = 46.4012 = 46.4

18.641 - 12.33= 6.311 = 6.31 (the number with less decimals)

Multiplying or dividing measured quantities:

6.8 * 5.32 = 36.176 = 36 (the number with less S.F)

5.733 * 5.14* 7.2111 = 212.493955 = 212

17.2 ÷ 13.33 = 1.29032258 = 1.29 (the number with less S.F)

Note : when we use exact numbers in calculation like π (3.14) or any other constants we don't account it as significant figures.