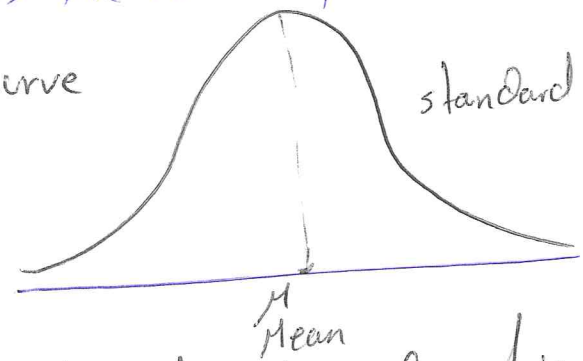
6.2 Normal Probability Distribution "Continuous"

* The most important prob. distribution for describing a continuous random variable is the normal prob. distribution.

Bell-shaped curve for the normal distribution



* The normal prob. density function

$$f(x) = \frac{1}{6\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$ where

μ = Mean
6 = standard deviation

$\pi = 3.14159$
$e = 2.71828$

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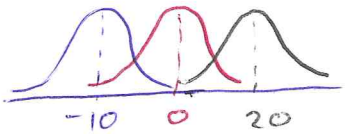
* Properities of the normal distribution:

[1] The normal curve has two parameters
- μ that determines the location of the normal distribution
- 6 that determines the shape of the normal distribution

[2] The highest point on the normal curve is at $x = \underline{\text{Mean}}$ which is also the <u>median</u> and the <u>mode</u> of the distribution

[3] Can be +, −, 0

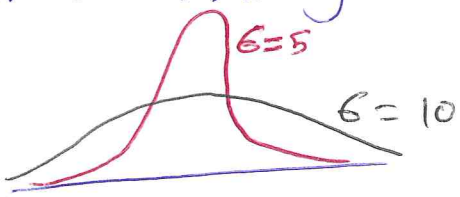


[4] The normal distribution is symmetric (skewness = 0): That is the shape of the normal curve to the left of the mean is a mirror image of the shape of the normal curve to the right of the mean. The tails of the normal curve extend to infinity in both directions and never touch the horizontal axis.

[5] The standard deviation determines how flat and wide the normal curve is.

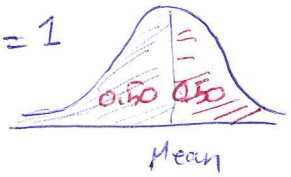
Large σ $\Rightarrow$ wider, flatter curves, showing more variability in the data

Small σ $\Rightarrow$ thinner curves, showing less variability in the data.



[6] Probabilities for the normal random variable are given by areas under the normal curve.

- The total area under the normal curve = 1
- Since the distribution is symmetric $\Rightarrow$



[7] The percentages of values (based on the empirical Rule) in some common used intervals:

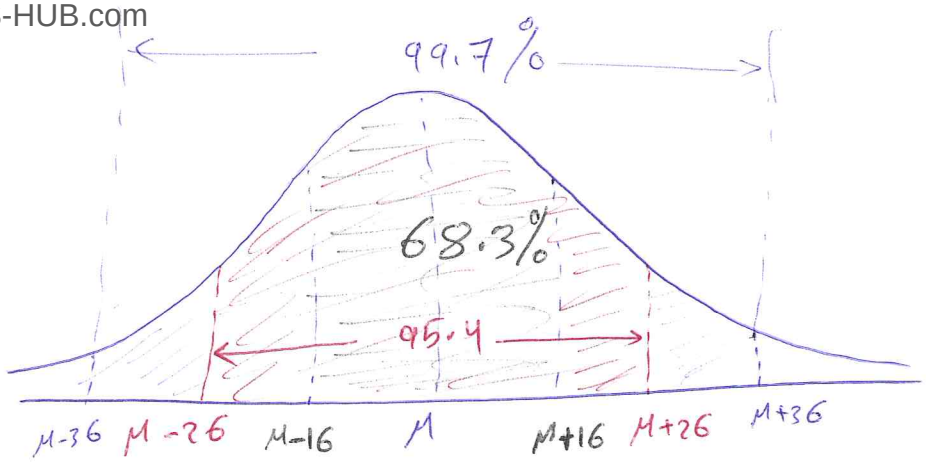
[a] 68.3 % of the values of a normal random variable are within ± 1 standard deviation of its mean.

[b] 95.4 % : : : : : : : :

: : ± 2 : : : : :

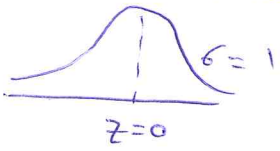
[c] 99.7 % : : : : : : :

: : ± 3 : : : : : :



Standard Normal prob. distribution: ($\mu = 0$ and $\sigma = 1$)

- A random variable (z) that has a normal distribution with mean zero and standard deviation of one is called a standard Normal prob. distribution.



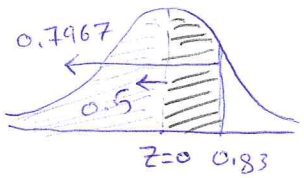
- standard normal density function:

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

Table page 581+582 gives area of $P(z \le a)$

- We will use the area under the standard normal distribution to find prob. that a normal random variable is within any specific interval.

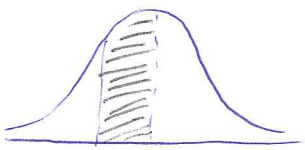
Example: (Q12 page 240) Given that z is a standard normal random variable, compute the following probabilities:



[a] $P(0 \le z \le 0.83) = P(z \le 0.83) - P(z \le 0)$

$= 0.7967 - 0.5$

$= 0.2967 = P(0 < z < 0.83)$

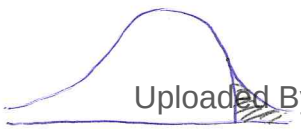


[b] $P(-1.57 \le z \le 0) = P(-1.57 < z < 0)$

$= P(z \le 0) - P(z \le -1.57)$

$= 0.5 - 0.0582$

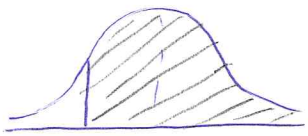
$= 0.4418$



[c] $P(z > 0.44) = 1 - P(z \le 0.44)$

$= 1 - 0.6700$

$= 0.3300$

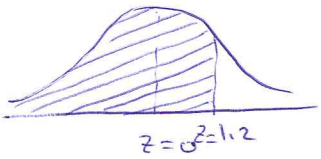


[d] $P(z \ge -0.23) = 1 - P(z < -0.23)$

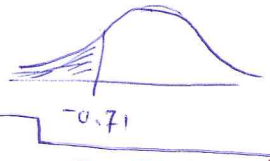
$= 1 - 0.4090$

$= 0.5910$

[e] $P(z < 1.20) = 0.8849$



[f] $P(z \leq -0.71) = 0.2389$



Computing Probabilities for any Normal Prob. distribution.

* We compute the probabilities using the standard normal distribution with the standardized values $\underline{z}$ - scores.
* If we have a random variable that follows a normal distribution with mean μ and standard deviation σ, then we can convert this normal random variable x to a standard normal random variable z using the formula:

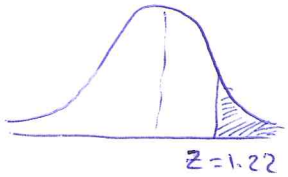
$$z = \frac{x - \mu}{\sigma}$$

Example (Q18 page 241) The average stock price for companies is \$30, and the standard deviation is \$8.20. Assume the stock prices are normally distributed. Let x be the stock price

[a] what is the prob. a company will have a stock price of at least \$40? $\mu = 30$, $\sigma = 8.20$

$$z = \frac{x - \mu}{\sigma} = \frac{40 - 30}{8.20} = \frac{10}{8.20} = 1.22$$

$$P(\text{stock price} \geq 40) = P(z \geq 1.22) = 1 - P(z < 1.22)$$
$$= 1 - 0.8888$$
$$= 0.1112$$



[b] what is the prob. a company will have a stock price no higher than \$20?

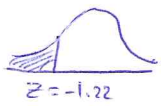
$\Rightarrow P(\text{stock price} \leq 20) = P(z \leq -1.22)$

$= 0.1112$

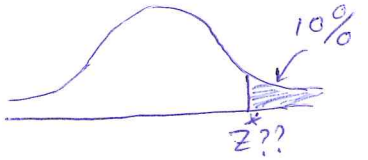
$z = \frac{x - \mu}{6}$

$= \frac{20 - 30}{8.2}$

$= -1.22$



[c] How high does a stock price have to be to put a company in the top 10%?



$P(z \geq z^*) = 0.10$

$1 - P(z < z^*) = 0.10 \Rightarrow P(z < z^*) = 0.90$ from the table

$\Rightarrow z^* = 1.28$

$z^* = \frac{x^* - \mu}{6}$, x^* is the stock price

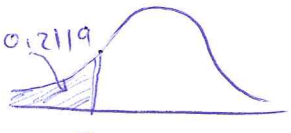
$1.28 = \frac{x^* - 30}{8.2} \iff x^* - 30 = 10.5$

$\iff x^* = 40.5$ A stock price of \$40.5 or higher will put the company in the top 10%.

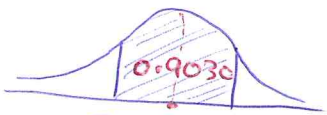
Example (Q15 page 241) Given that z is a standard normal random variable. Find z for each situation

[a] The area to the left of z is 0.2119

$z = -0.80$

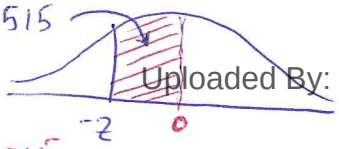


[b] The area between -z and z is 0.9030



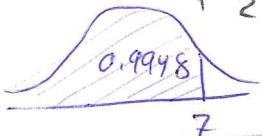
$\Rightarrow$ Thus, $z = 1.66$

$\frac{0.9030}{2} = 0.4515$

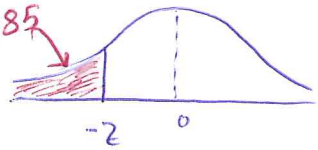


[d] The area to the left of z is 0.9948

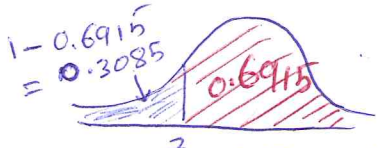
$z = 2.56$



$0.5 - 0.4515 = 0.0485$

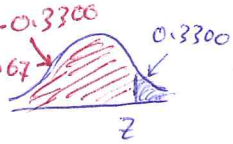


[e] The area to the right of z is 0.6915



$\Rightarrow z = -0.5$

(F) The area to the right of z is 0.3300



$\Rightarrow z = 0.44$

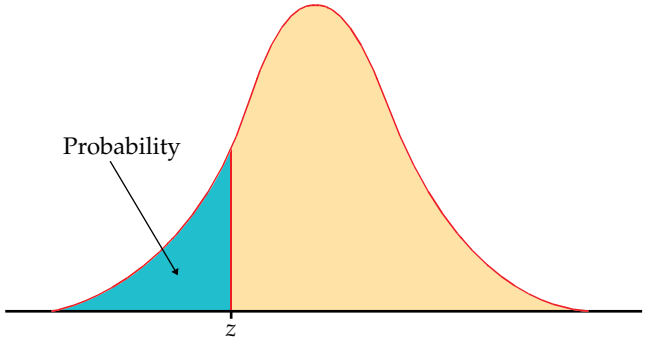


Table entry for z is the area under the standard normal curve to the left of z.

TABLE A

Standard normal probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Table entry for z is the area under the standard normal curve to the left of z.

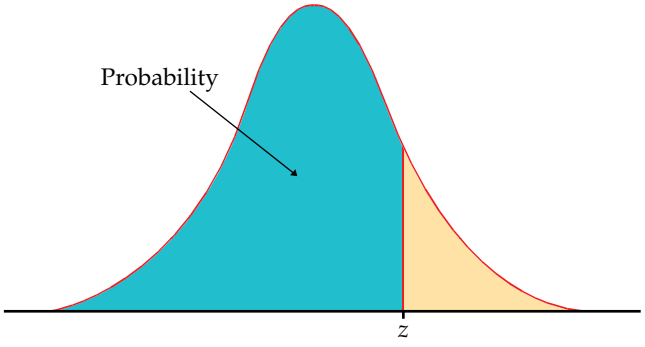


TABLE A

Standard normal probabilities (continued)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998