Formula Sheet for Math332 Final Exam - First Semester 2024/2025.

1. $\mathcal{L}\{e^{kt}\} = \frac{1}{s-k}, \quad s > k.$
2. $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, \quad s > 0.$
3. $\mathcal{L}\{\sin kt\} = \frac{k}{s^2+k^2}, \quad s > 0.$
4. $\mathcal{L}\{\cos kt\} = \frac{s}{s^2+k^2}, \quad s > 0.$
5. $\mathcal{L}\{\sinh kt\} = \frac{k}{s^2-k^2}, \quad s > |k|.$
6. $\mathcal{L}\{\cosh kt\} = \frac{s}{s^2-k^2}, \quad s > |k|.$
7. $\mathcal{L}\{y'(t)\} = sY(s) - y(0)$ **AND** $\mathcal{L}\{\frac{\partial u(x,t)}{\partial t}\}\} = sU(x,s) - u(x,0).$
8. $\mathcal{L}\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$ **AND** $\mathcal{L}\{\frac{\partial^2 u(x,t)}{\partial t^2}\} = s^2U(x,s) - su(x,0) - u_t(x,0).$
9. $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x).$
10. $\frac{d}{dx}[x^{-n} J_n(x)] = -x^{-n} J_{n+1}(x).$
11. The **Fourier-Bessel series** of f defined on the interval $(0, b)$ is

$$f(x) = \sum_{i=1}^{\infty} c_i J_n(\alpha_i x),, \quad \text{where} \quad c_i = \frac{2}{b^2 J_{n+1}^2(\alpha_i b)} \int_0^b x J_n(\alpha_i x) f(x) dx$$

and α_i are defined by $J_n(\alpha b) = 0$.

12. **Fourier transform:** $\mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{-i\alpha x} dx = F(\alpha).$
13. **Inverse Fourier transform:** $\mathcal{F}^{-1}\{F(\alpha)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\alpha) e^{i\alpha x} d\alpha = f(x).$
14. $\mathcal{F}\{f''(x)\} = -\alpha^2 F(\alpha)$ **AND** $\mathcal{F}\{\frac{\partial^2 u(x,t)}{\partial x^2}\} = -\alpha^2 U(\alpha, t).$
15. The **Fourier-Legendre series** of f defined on the interval $(-1, 1)$ is

$$f(x) = \sum_{n=0}^{\infty} c_n P_n(x), \quad \text{where}$$

$$c_n = \frac{2n+1}{2} \int_{-1}^{1} f(x) P_n(x) dx, \qquad P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} \left[(x^2-1)^n\right], \quad n = 0, 1, 2, ...$$

$P_0(x) = 1$
$P_1(x) = x$
$P_2(x) = \frac{1}{2}(3x^2 - 1)$
$P_3(x) = \frac{1}{2}(5x^3 - 3x)$
$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3).$